Computer Algebra
Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/326-7.1.2

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 12:43am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**191**]. This is test number [326].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (191)	0.00 (0)
Rubi	97.91 (187)	2.09 (4)
Maple	73.82 (141)	26.18 (50)
Maxima	40.84 (78)	59.16 (113)
Sympy	38.74 (74)	61.26 (117)
Reduce	38.22 (73)	61.78 (118)
Fricas	35.08 (67)	64.92 (124)
Mupad	30.89 (59)	69.11 (132)
Giac	30.89 (59)	69.11 (132)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

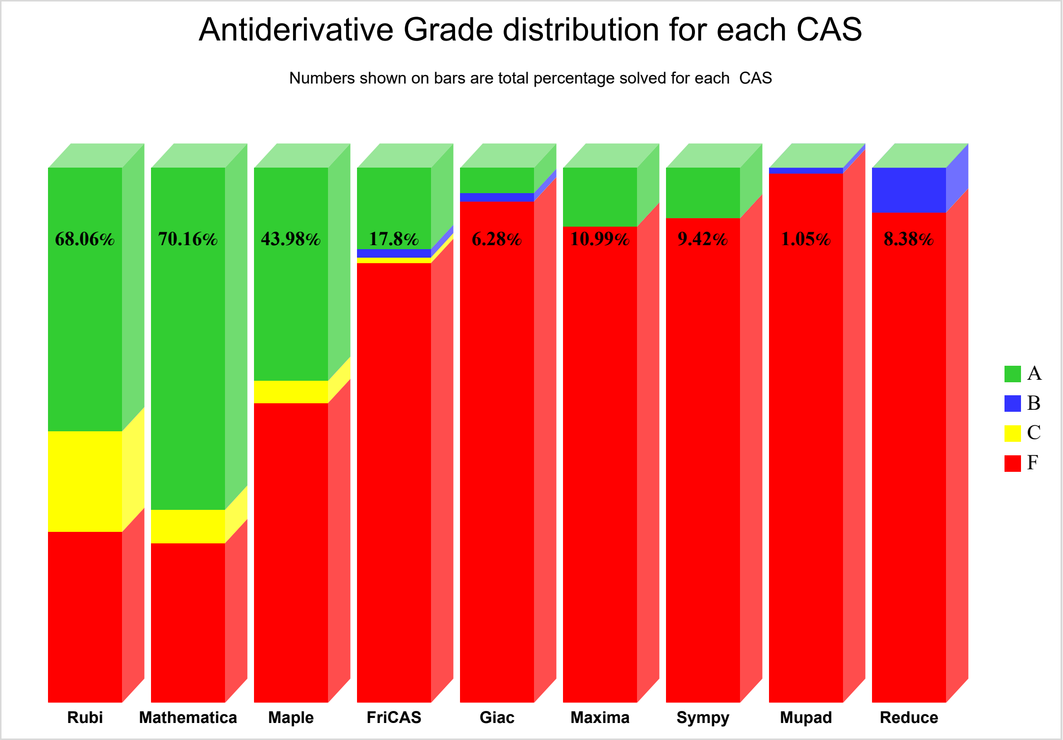
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

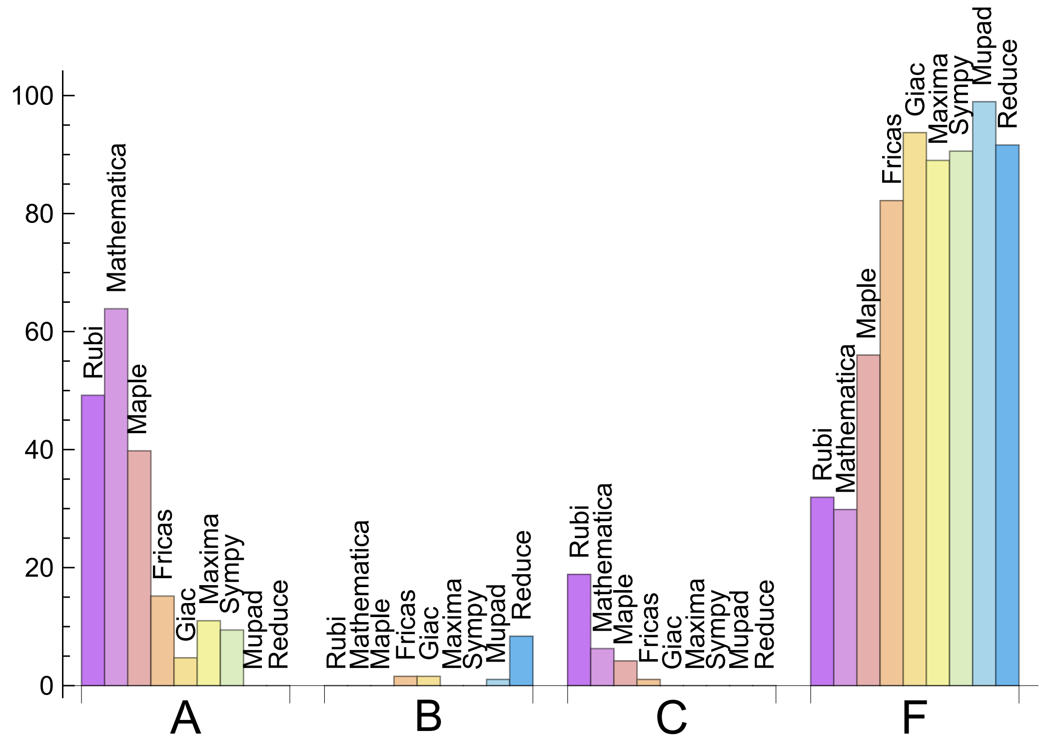
System	% A grade	% B grade	% C grade	% F grade
Mathematica	63.874	0.000	6.283	29.843
Rubi	49.215	0.000	18.848	31.937
Maple	39.791	0.000	4.188	56.021
Fricas	15.183	1.571	1.047	82.199
Maxima	10.995	0.000	0.000	89.005
Sympy	9.424	0.000	0.000	90.576
Giac	4.712	1.571	0.000	93.717
Mupad	0.000	1.047	0.000	98.953
Reduce	0.000	8.377	0.000	91.623

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	4	100.00	0.00	0.00
Maple	50	100.00	0.00	0.00
Fricas	124	39.52	0.00	60.48
Maxima	113	100.00	0.00	0.00
Sympy	117	97.44	2.56	0.00
Reduce	118	100.00	0.00	0.00
Giac	132	62.12	3.79	34.09
Mupad	132	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Maxima	0.35
Giac	0.35
Rubi	0.62
Maple	0.62
Mathematica	2.14
Mupad	2.64
Sympy	5.68
Reduce	11.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	15.14	1.05	14.00	1.00
Giac	28.53	1.19	16.00	1.00
Sympy	32.93	0.96	15.00	0.96
Reduce	36.25	1.87	22.00	1.20
Maple	60.63	0.99	37.00	0.89
Fricas	66.31	9.72	50.00	1.20
Mathematica	87.84	14.20	56.00	1.00
Rubi	99.57	1.08	76.00	1.00
Maxima	141.31	10.59	23.50	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

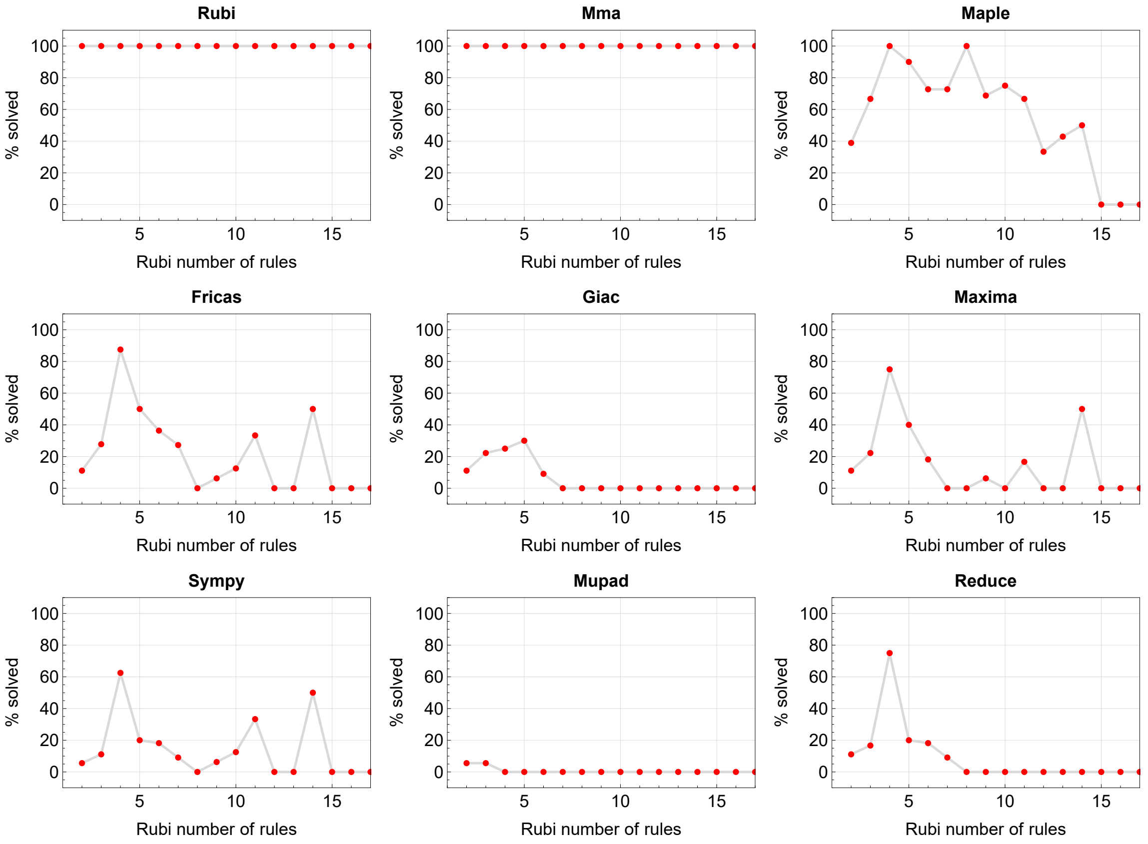


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

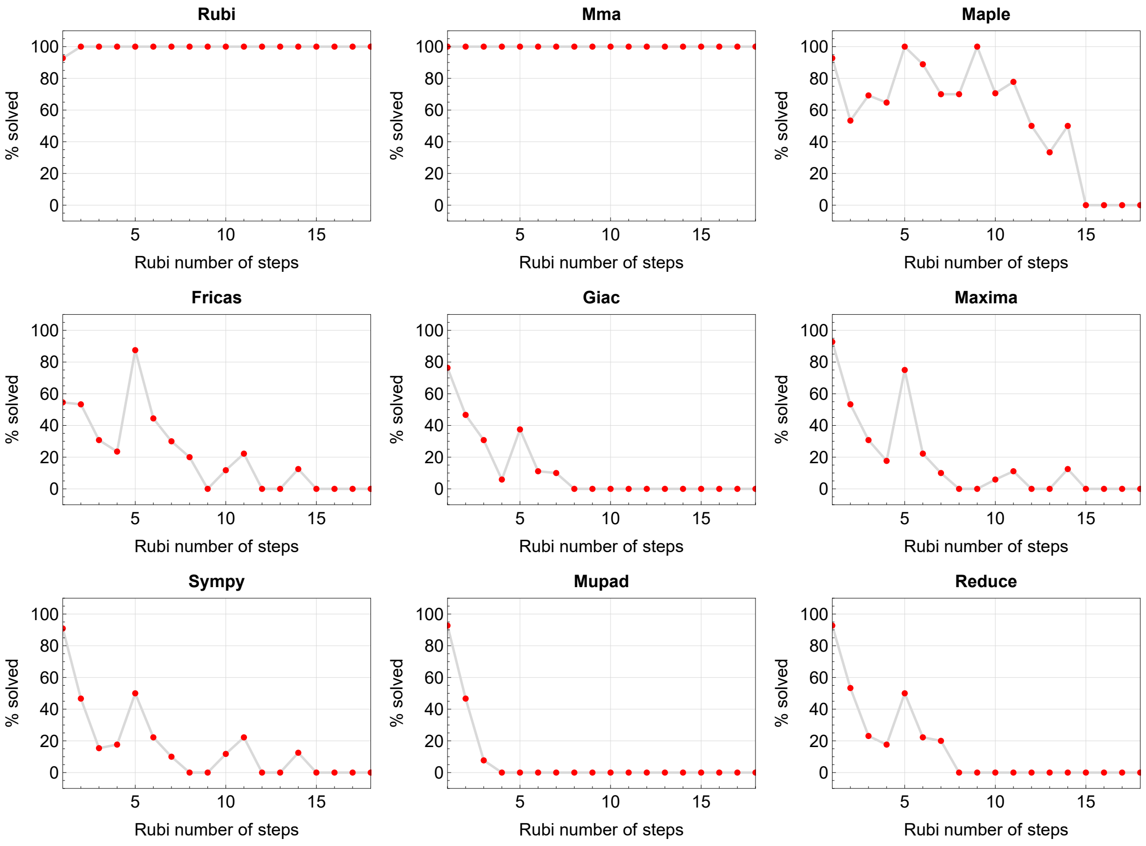


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

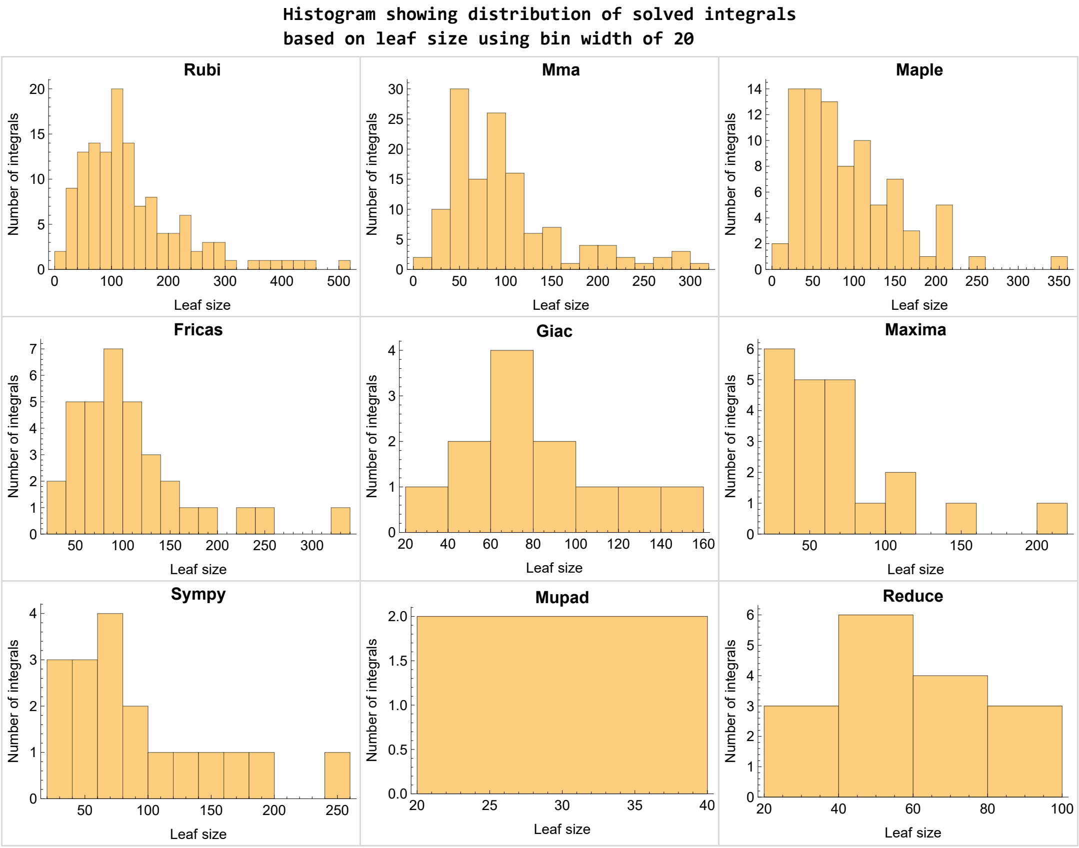


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

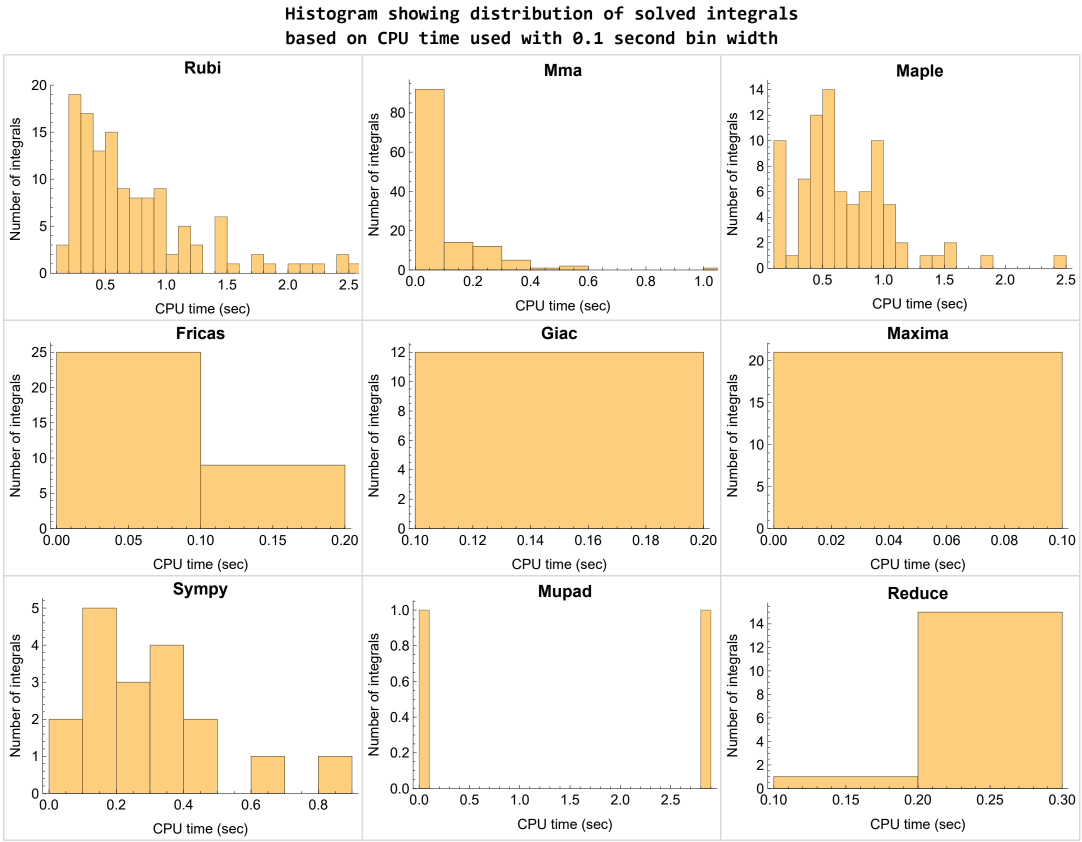


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

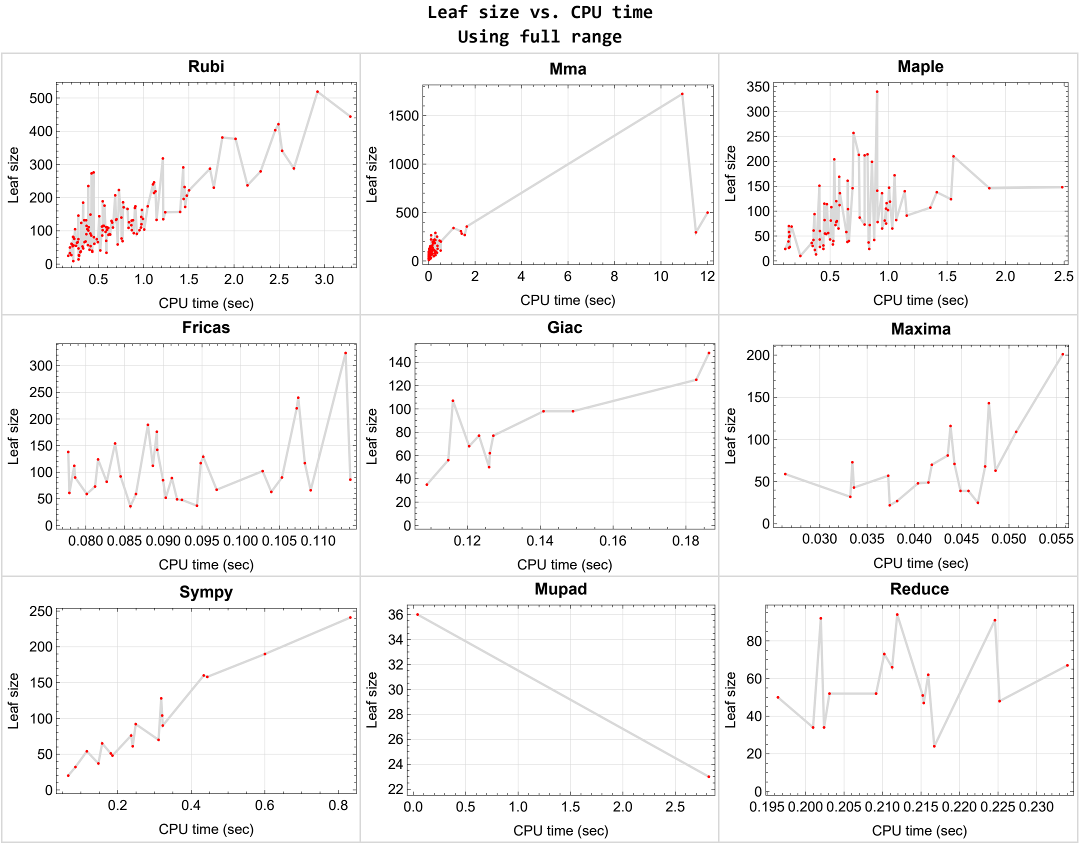


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{5, 6, 52, 53, 61, 62, 68, 69, 75, 76, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 111, 117, 123, 129, 130, 136, 141, 146, 150, 151, 155, 156, 160, 161, 165, 166, 170, 171, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{1, 3\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

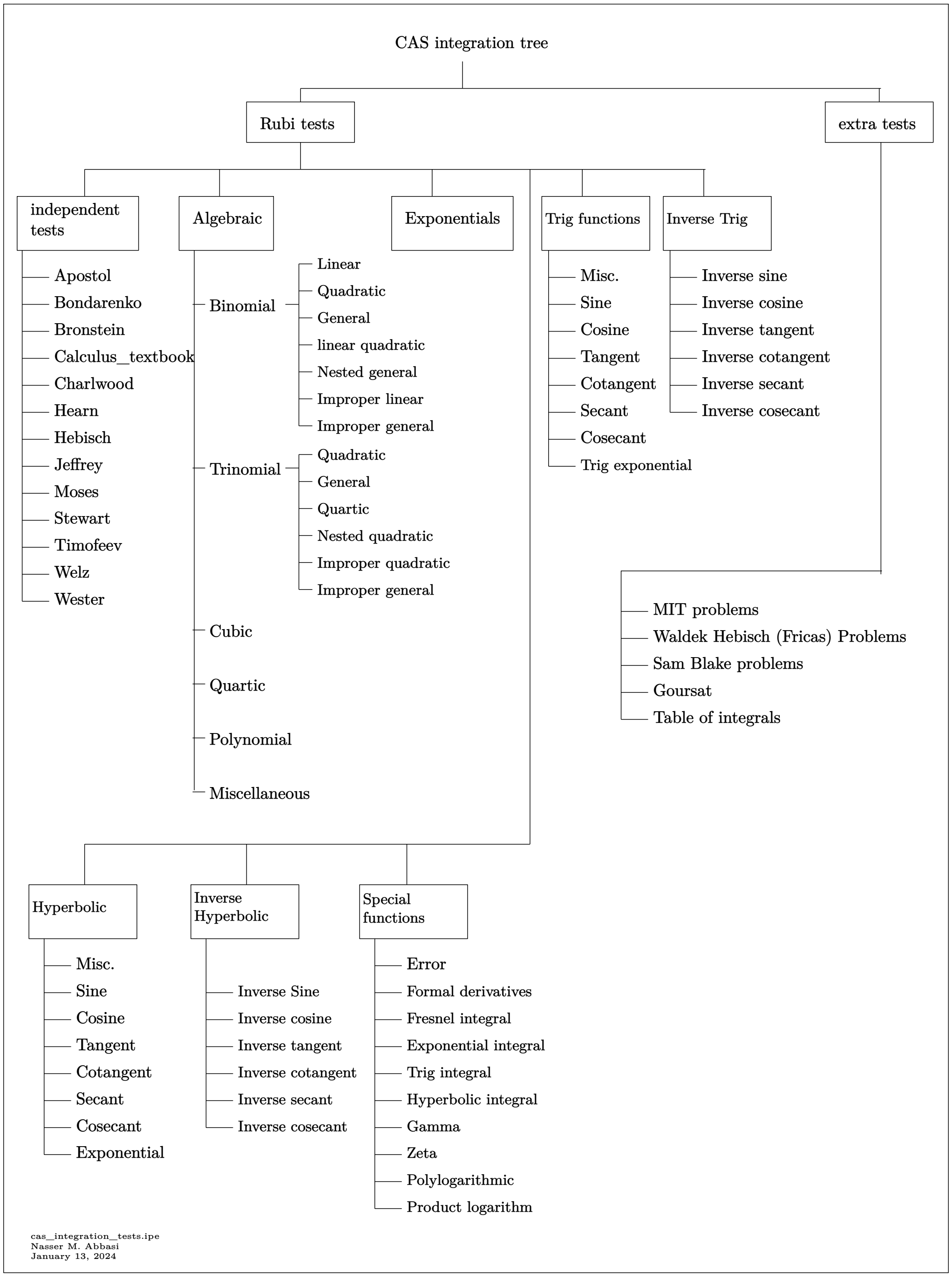
Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

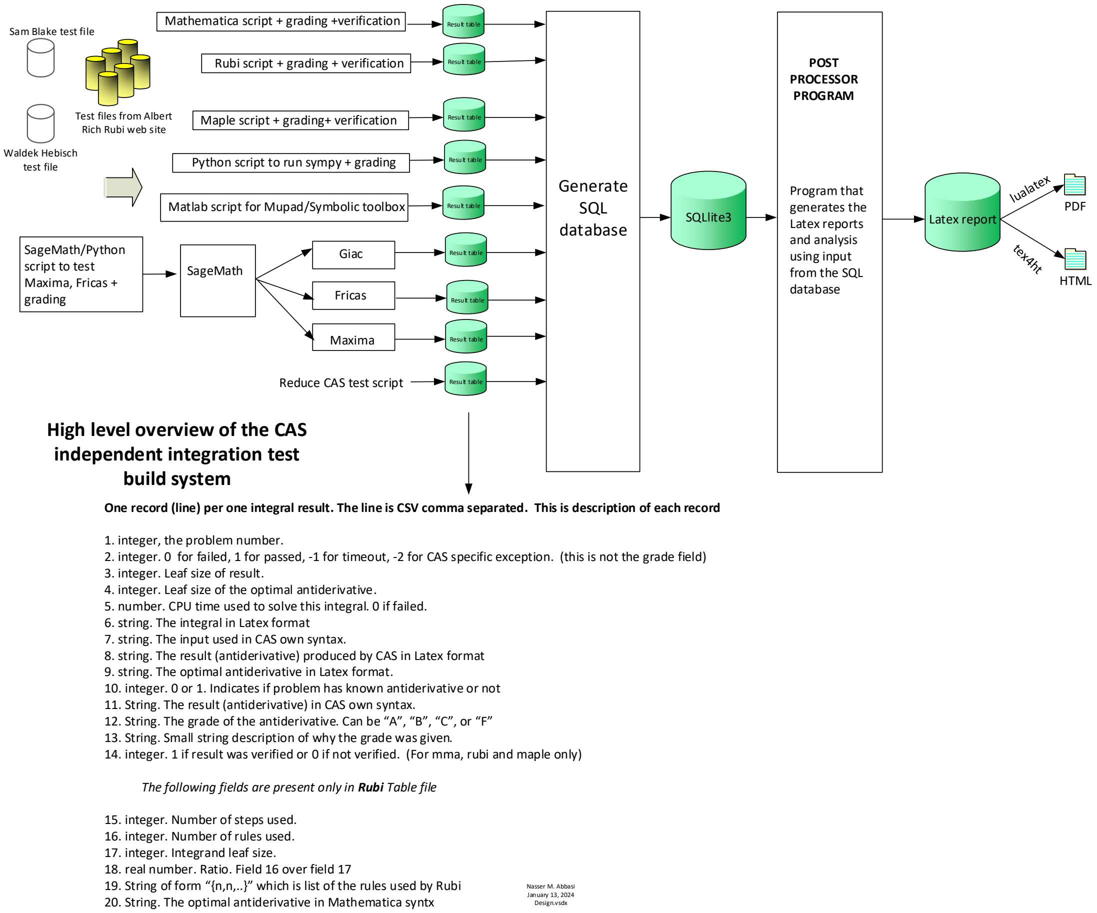


cas_integration_tests.ipe
Nasser M. Abbasi
January 13, 2024

Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 103, 107, 109, 112, 114, 116, 119, 121, 124, 125, 126, 128, 131, 132, 133, 134, 138, 140, 142, 144, 148, 152, 154, 158, 162, 164, 167, 168, 172, 174, 181, 182, 184 }

B grade { }

C grade { 12, 22, 23, 25, 31, 32, 33, 34, 35, 41, 42, 43, 44, 106, 108, 110, 113, 115, 118, 120, 122, 127, 135, 137, 139, 143, 145, 147, 149, 153, 157, 159, 163, 169, 173, 183 }

F normal fail { 1, 2, 3, 4 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 102, 103, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 167, 168, 169, 172, 173, 174, 181, 182, 183, 184 }

B grade { }

C grade { 1, 2, 3, 4, 17, 43, 77, 78, 79, 80, 81, 82 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 109, 110, 115, 116, 121, 122, 127, 128, 134, 135, 139, 140, 144, 145 }

B grade { }

C grade { 77, 78, 79, 80, 81, 82, 183, 184 }

F normal fail { 1, 2, 3, 4, 83, 84, 85, 86, 87, 102, 103, 106, 107, 108, 112, 113, 114, 118, 119, 120, 124, 125, 126, 131, 132, 133, 137, 138, 142, 143, 147, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 167, 168, 169, 172, 173, 174, 181, 182 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 4, 7, 8, 9, 10, 11, 14, 16, 18, 19, 20, 21, 24, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 77, 78, 79, 80, 81, 82 }

B grade { 13, 15, 17 }

C grade { 1, 2 }

F normal fail { 3, 12, 22, 23, 25, 31, 32, 33, 34, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 102, 103, 181, 182, 183, 184 }

F(-1) timedout fail { }

F(-2) exception fail { 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

Maxima

A grade { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 26, 28, 30, 36, 38, 40 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 12, 22, 23, 25, 27, 29, 31, 32, 33, 34, 35, 37, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 103, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 167, 168, 169, 172, 173, 174, 181, 182, 183, 184 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 10, 11, 14, 15, 16, 17, 21, 30, 40 }

B grade { 13, 24, 26 }

C grade { }

F normal fail { 3, 4, 12, 22, 23, 25, 29, 31, 32, 34, 41, 42, 44, 45, 47, 49, 50, 51, 54, 56, 58, 59, 60, 63, 65, 66, 67, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 102, 103, 106, 108, 109, 110, 112, 114, 115, 116, 124, 126, 127, 128, 131, 133, 134, 135, 138, 139, 140, 143, 144, 145, 147, 148, 149, 153, 154, 162, 163, 164, 167, 168, 169, 172, 173, 174, 182, 183, 184 }

F(-1) timedout fail { 177, 178, 187, 188, 191 }

F(-2) exception fail { 1, 2, 7, 8, 9, 18, 19, 20, 27, 28, 33, 35, 36, 37, 38, 39, 43, 46, 48, 55, 57, 64, 71, 84, 89, 95, 99, 107, 113, 118, 119, 120, 121, 122, 125, 132, 137, 142, 152, 157, 158, 159, 170, 175, 181 }

Mupad

A grade { }

B grade { 10, 11 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 103, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 167, 168, 169, 172, 173, 174, 181, 182, 183, 184 }

F(-2) exception fail { }

Sympy

A grade { 7, 8, 9, 10, 11, 18, 19, 20, 21, 27, 28, 29, 30, 36, 37, 38, 39, 40 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 103, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 167, 168, 169, 172, 173, 174, 181, 182, 183, 184 }

F(-1) timedout fail { 77, 118, 187 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 20, 21, 29, 30, 39, 40 }

C grade { }

F normal fail { 1, 2, 3, 4, 12, 18, 19, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 103, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 167, 168, 169, 172, 173, 174, 181, 182, 183, 184 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	C	F	F(-2)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	499	0	0	324	0	0	22	0
N.S.	1	0.00	499.00	0.00	0.00	324.00	0.00	0.00	22.00	0.00
time (sec)	N/A	0.000	12.007	0.000	0.000	0.114	0.000	0.000	200.033	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	C	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	307	0	0	240	0	0	22	0
N.S.	1	0.00	307.00	0.00	0.00	240.00	0.00	0.00	22.00	0.00
time (sec)	N/A	0.000	1.390	0.000	0.000	0.107	0.000	0.000	200.034	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	1724	0	0	0	0	0	20	0
N.S.	1	0.00	1724.00	0.00	0.00	0.00	0.00	0.00	20.00	0.00
time (sec)	N/A	0.000	10.917	0.000	0.000	0.000	0.000	0.000	200.039	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	A	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	295	0	0	220	0	0	19	0
N.S.	1	0.00	1.76	0.00	0.00	1.31	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	11.505	0.000	0.000	0.107	0.000	0.000	200.046	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	99	33	19	22	22	22
N.S.	1	1.00	1.10	1.00	4.95	1.65	0.95	1.10	1.10	1.10
time (sec)	N/A	0.355	0.242	0.115	0.531	0.091	3.224	0.153	200.027	3.453

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	105	33	20	22	22	22
N.S.	1	1.00	1.10	1.00	5.25	1.65	1.00	1.10	1.10	1.10
time (sec)	N/A	0.227	0.251	0.110	0.455	0.084	1.933	0.149	200.034	3.323

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	50	69	68	61	70	0	66	0
N.S.	1	1.06	0.69	0.96	0.94	0.85	0.97	0.00	0.92	0.00
time (sec)	N/A	0.233	0.029	0.171	0.047	0.078	0.311	0.000	0.211	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	49	58	59	59	61	0	67	0
N.S.	1	1.21	0.73	0.87	0.88	0.88	0.91	0.00	1.00	0.00
time (sec)	N/A	0.223	0.014	0.149	0.026	0.086	0.241	0.000	0.234	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	56	41	50	48	52	48	0	48	0
N.S.	1	1.08	0.79	0.96	0.92	1.00	0.92	0.00	0.92	0.00
time (sec)	N/A	0.226	0.020	0.156	0.040	0.090	0.186	0.000	0.225	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	50	40	39	39	48	37	68	47	36
N.S.	1	1.14	0.91	0.89	0.89	1.09	0.84	1.55	1.07	0.82
time (sec)	N/A	0.185	0.009	0.144	0.046	0.092	0.148	0.120	0.215	0.041

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	25	37	20	35	24	23
N.S.	1	1.00	1.00	0.96	1.00	1.48	0.80	1.40	0.96	0.92
time (sec)	N/A	0.166	0.003	0.117	0.047	0.094	0.066	0.109	0.217	2.820

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	57	43	94	0	0	0	0	10	0
N.S.	1	1.33	1.00	2.19	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.384	0.004	0.367	0.000	0.000	0.000	0.000	0.193	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	22	90	0	56	50	0
N.S.	1	1.00	1.00	0.96	0.81	3.33	0.00	2.07	1.85	0.00
time (sec)	N/A	0.201	0.003	0.148	0.037	0.105	0.000	0.115	0.196	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	28	27	36	0	50	34	0
N.S.	1	1.00	0.85	0.85	0.82	1.09	0.00	1.52	1.03	0.00
time (sec)	N/A	0.184	0.006	0.156	0.038	0.086	0.000	0.126	0.202	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	48	43	117	0	77	73	0
N.S.	1	1.00	1.00	0.89	0.80	2.17	0.00	1.43	1.35	0.00
time (sec)	N/A	0.216	0.009	0.150	0.034	0.095	0.000	0.123	0.210	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	40	50	49	49	0	77	52	0
N.S.	1	1.09	0.71	0.89	0.88	0.88	0.00	1.38	0.93	0.00
time (sec)	N/A	0.202	0.010	0.155	0.041	0.092	0.000	0.127	0.209	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	49	70	63	129	0	107	92	0
N.S.	1	1.06	0.64	0.91	0.82	1.68	0.00	1.39	1.19	0.00
time (sec)	N/A	0.219	0.011	0.148	0.049	0.095	0.000	0.116	0.202	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	113	72	87	109	92	90	0	12	0
N.S.	1	1.18	0.75	0.91	1.14	0.96	0.94	0.00	0.12	0.00
time (sec)	N/A	0.539	0.033	0.753	0.051	0.084	0.323	0.000	0.209	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	92	59	72	70	82	76	0	12	0
N.S.	1	1.15	0.74	0.90	0.88	1.02	0.95	0.00	0.15	0.00
time (sec)	N/A	0.411	0.045	0.839	0.042	0.083	0.237	0.000	0.202	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	66	53	43	81	73	51	0	51	0
N.S.	1	1.12	0.90	0.73	1.37	1.24	0.86	0.00	0.86	0.00
time (sec)	N/A	0.494	0.023	0.500	0.044	0.081	0.181	0.000	0.215	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	40	34	36	32	59	32	62	34	0
N.S.	1	1.18	1.00	1.06	0.94	1.74	0.94	1.82	1.00	0.00
time (sec)	N/A	0.431	0.011	0.343	0.033	0.080	0.085	0.126	0.201	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	77	60	151	0	0	0	0	12	0
N.S.	1	1.28	1.00	2.52	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.756	0.006	0.409	0.000	0.000	0.000	0.000	0.218	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	59	75	108	0	0	0	0	12	0
N.S.	1	1.18	1.50	2.16	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.713	0.156	0.511	0.000	0.000	0.000	0.000	0.222	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	72	39	67	0	98	12	0
N.S.	1	1.00	1.00	1.67	0.91	1.56	0.00	2.28	0.28	0.00
time (sec)	N/A	0.402	0.025	0.546	0.045	0.097	0.000	0.149	0.201	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	125	136	0	0	0	0	12	0
N.S.	1	1.04	1.26	1.37	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.596	0.386	0.584	0.000	0.000	0.000	0.000	0.204	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	64	112	71	85	0	148	12	0
N.S.	1	1.01	0.75	1.32	0.84	1.00	0.00	1.74	0.14	0.00
time (sec)	N/A	0.421	0.046	0.581	0.044	0.090	0.000	0.186	0.211	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	240	110	141	0	142	160	0	12	0
N.S.	1	1.47	0.67	0.87	0.00	0.87	0.98	0.00	0.07	0.00
time (sec)	N/A	1.103	0.055	0.903	0.000	0.089	0.434	0.000	0.203	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	171	93	116	116	124	128	0	12	0
N.S.	1	1.30	0.70	0.88	0.88	0.94	0.97	0.00	0.09	0.00
time (sec)	N/A	0.779	0.043	0.983	0.044	0.082	0.318	0.000	0.210	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	115	80	58	0	112	92	0	91	0
N.S.	1	1.19	0.82	0.60	0.00	1.15	0.95	0.00	0.94	0.00
time (sec)	N/A	0.561	0.033	0.639	0.000	0.079	0.249	0.000	0.225	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	66	58	55	57	90	54	98	52	0
N.S.	1	1.14	1.00	0.95	0.98	1.55	0.93	1.69	0.90	0.00
time (sec)	N/A	0.347	0.013	0.456	0.037	0.079	0.116	0.141	0.203	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	102	83	204	0	0	0	0	12	0
N.S.	1	1.23	1.00	2.46	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.547	0.006	0.536	0.000	0.000	0.000	0.000	0.215	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	117	161	0	0	0	0	12	0
N.S.	1	1.11	1.39	1.92	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.891	0.090	0.650	0.000	0.000	0.000	0.000	0.231	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	102	80	146	0	0	0	0	12	0
N.S.	1	1.10	0.86	1.57	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.954	0.265	0.692	0.000	0.000	0.000	0.000	0.254	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	268	212	0	0	0	0	12	0
N.S.	1	1.04	1.77	1.40	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.404	1.554	0.795	0.000	0.000	0.000	0.000	0.212	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	174	107	213	0	0	0	0	12	0
N.S.	1	1.09	0.67	1.34	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.045	0.516	0.746	0.000	0.000	0.000	0.000	0.230	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	403	148	210	201	189	241	0	12	0
N.S.	1	1.65	0.61	0.86	0.82	0.77	0.99	0.00	0.05	0.00
time (sec)	N/A	2.455	0.066	1.555	0.056	0.088	0.832	0.000	0.233	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	288	133	172	0	176	190	0	12	0
N.S.	1	1.48	0.69	0.89	0.00	0.91	0.98	0.00	0.06	0.00
time (sec)	N/A	2.662	0.054	1.051	0.000	0.089	0.600	0.000	0.226	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	222	112	140	143	154	158	0	12	0
N.S.	1	1.37	0.69	0.86	0.88	0.95	0.98	0.00	0.07	0.00
time (sec)	N/A	1.502	0.051	1.138	0.048	0.084	0.444	0.000	0.220	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	131	94	73	0	138	104	0	94	0
N.S.	1	1.19	0.85	0.66	0.00	1.25	0.95	0.00	0.85	0.00
time (sec)	N/A	0.867	0.034	0.795	0.000	0.078	0.321	0.000	0.212	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	67	65	73	112	65	125	62	0
N.S.	1	1.21	1.00	0.97	1.09	1.67	0.97	1.87	0.93	0.00
time (sec)	N/A	0.450	0.018	0.572	0.033	0.089	0.158	0.183	0.216	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	125	97	257	0	0	0	0	12	0
N.S.	1	1.29	1.00	2.65	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.652	0.007	0.700	0.000	0.000	0.000	0.000	0.199	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	135	161	214	0	0	0	0	12	0
N.S.	1	1.12	1.34	1.78	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.701	0.160	0.820	0.000	0.000	0.000	0.000	0.201	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	113	199	0	0	0	0	12	0
N.S.	1	1.11	1.05	1.84	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.976	0.232	0.858	0.000	0.000	0.000	0.000	0.205	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	237	355	340	0	0	0	0	12	0
N.S.	1	1.06	1.59	1.52	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.149	1.641	0.902	0.000	0.000	0.000	0.000	0.208	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	40	40	0	0	0	0	12	0
N.S.	1	0.85	0.73	0.73	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.332	0.013	0.528	0.000	0.000	0.000	0.000	0.209	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	38	33	33	0	0	0	0	12	0
N.S.	1	0.88	0.77	0.77	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.305	0.102	0.524	0.000	0.000	0.000	0.000	0.224	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	31	31	0	0	0	0	12	0
N.S.	1	0.88	0.76	0.76	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.300	0.009	0.443	0.000	0.000	0.000	0.000	0.208	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	24	24	0	0	0	0	12	0
N.S.	1	0.93	0.83	0.83	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.284	0.079	0.443	0.000	0.000	0.000	0.000	0.212	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	22	22	0	0	0	0	12	0
N.S.	1	0.93	0.81	0.81	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.287	0.007	0.370	0.000	0.000	0.000	0.000	0.205	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	0	10	0
N.S.	1	1.00	1.00	0.93	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.280	0.025	0.380	0.000	0.000	0.000	0.000	0.207	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	8	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.222	0.010	0.246	0.000	0.000	0.000	0.000	0.223	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.184	0.123	0.151	0.090	0.074	0.279	0.122	0.193	2.621

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.183	0.697	0.205	0.084	0.070	0.299	0.122	0.209	2.667

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	75	85	104	0	0	0	0	12	0
N.S.	1	0.91	1.04	1.27	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.279	0.362	0.653	0.000	0.000	0.000	0.000	0.212	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	66	78	78	0	0	0	0	12	0
N.S.	1	0.94	1.11	1.11	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.270	0.030	0.553	0.000	0.000	0.000	0.000	0.198	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	64	60	80	0	0	0	0	12	0
N.S.	1	0.94	0.88	1.18	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.263	0.298	0.558	0.000	0.000	0.000	0.000	0.221	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	56	54	0	0	0	0	12	0
N.S.	1	0.98	1.00	0.96	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.252	0.021	0.466	0.000	0.000	0.000	0.000	0.231	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	49	56	0	0	0	0	12	0
N.S.	1	0.98	0.91	1.04	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.324	0.223	0.484	0.000	0.000	0.000	0.000	0.213	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	28	0	0	0	0	10	0
N.S.	1	1.00	0.78	0.76	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.433	0.009	0.413	0.000	0.000	0.000	0.000	0.228	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	30	0	0	0	0	8	0
N.S.	1	1.00	0.91	0.88	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.589	0.063	0.351	0.000	0.000	0.000	0.000	0.237	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	200	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	20.00	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.284	0.626	0.150	0.178	0.079	0.341	0.125	0.230	2.603

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	237	12	12	12	12	12
N.S.	1	1.00	1.20	1.00	23.70	1.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.230	4.283	0.208	0.254	0.071	0.391	0.126	0.213	2.595

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	133	102	120	0	0	0	0	12	0
N.S.	1	1.37	1.05	1.24	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.143	0.103	0.553	0.000	0.000	0.000	0.000	0.208	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	109	69	82	0	0	0	0	12	0
N.S.	1	1.33	0.84	1.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.833	0.159	0.453	0.000	0.000	0.000	0.000	0.227	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	102	64	81	0	0	0	0	12	0
N.S.	1	1.26	0.79	1.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.869	0.119	0.506	0.000	0.000	0.000	0.000	0.223	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	69	62	43	0	0	0	0	10	0
N.S.	1	1.10	0.98	0.68	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.767	0.045	0.411	0.000	0.000	0.000	0.000	0.225	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	47	42	0	0	0	0	8	0
N.S.	1	1.08	0.94	0.84	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.413	0.016	0.358	0.000	0.000	0.000	0.000	0.198	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	697	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	69.70	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.173	0.384	0.155	0.516	0.078	0.423	0.130	0.205	2.859

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	822	12	12	12	12	12
N.S.	1	1.00	1.20	1.00	82.20	1.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.175	3.784	0.207	0.562	0.070	0.502	0.123	0.216	2.648

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	207	156	169	0	0	0	0	12	0
N.S.	1	1.34	1.01	1.09	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.686	0.380	0.578	0.000	0.000	0.000	0.000	0.205	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	174	105	114	0	0	0	0	12	0
N.S.	1	1.23	0.74	0.81	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.909	0.264	0.473	0.000	0.000	0.000	0.000	0.206	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	172	99	115	0	0	0	0	12	0
N.S.	1	1.25	0.72	0.83	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.459	0.303	0.451	0.000	0.000	0.000	0.000	0.202	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	104	84	60	0	0	0	0	10	0
N.S.	1	1.09	0.88	0.63	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.007	0.099	0.413	0.000	0.000	0.000	0.000	0.201	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	69	61	0	0	0	0	8	0
N.S.	1	1.16	0.91	0.80	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.611	0.073	0.359	0.000	0.000	0.000	0.000	0.200	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1611	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	161.10	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.176	1.663	0.160	1.379	0.073	0.523	0.119	0.219	2.630

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1885	12	12	12	12	12
N.S.	1	1.00	1.20	1.00	188.50	1.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.181	6.287	0.205	1.426	0.082	0.709	0.125	0.195	2.571

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	185	99	148	0	117	0	0	31	0
N.S.	1	1.15	0.61	0.92	0.00	0.73	0.00	0.00	0.19	0.00
time (sec)	N/A	0.331	0.024	2.484	0.000	0.108	0.000	0.000	0.234	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	276	66	146	0	102	0	0	27	0
N.S.	1	1.13	0.27	0.60	0.00	0.42	0.00	0.00	0.11	0.00
time (sec)	N/A	0.447	0.019	1.861	0.000	0.103	0.000	0.000	0.214	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	146	66	124	0	86	0	0	23	0
N.S.	1	1.12	0.51	0.95	0.00	0.66	0.00	0.00	0.18	0.00
time (sec)	N/A	0.279	0.013	1.533	0.000	0.114	0.000	0.000	0.204	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	235	46	107	0	66	0	0	24	0
N.S.	1	1.13	0.22	0.51	0.00	0.32	0.00	0.00	0.12	0.00
time (sec)	N/A	0.388	0.012	1.357	0.000	0.109	0.000	0.000	0.206	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	105	41	91	0	63	0	0	34	0
N.S.	1	1.07	0.42	0.93	0.00	0.64	0.00	0.00	0.35	0.00
time (sec)	N/A	0.242	0.012	1.155	0.000	0.104	0.000	0.000	0.219	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	273	43	138	0	89	0	0	40	0
N.S.	1	1.10	0.17	0.56	0.00	0.36	0.00	0.00	0.16	0.00
time (sec)	N/A	0.424	0.013	1.412	0.000	0.091	0.000	0.000	0.223	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	115	92	0	0	0	0	0	47	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.369	0.039	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	115	92	0	0	0	0	0	42	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.551	0.036	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	111	92	0	0	0	0	0	45	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.593	0.033	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	109	89	0	0	0	0	0	60	0
N.S.	1	1.02	0.83	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.609	0.033	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	0	0	0	0	0	67	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.586	0.036	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	279	53	17	18	67	18
N.S.	1	1.00	1.11	0.89	15.50	2.94	0.94	1.00	3.72	1.00
time (sec)	N/A	0.465	41.555	0.632	1.780	0.096	85.001	0.298	0.293	2.641

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	285	44	17	0	61	18
N.S.	1	1.00	1.11	0.89	15.83	2.44	0.94	0.00	3.39	1.00
time (sec)	N/A	0.396	91.700	0.411	1.709	0.092	6.895	0.000	0.277	2.760

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	261	50	17	18	66	18
N.S.	1	1.00	1.11	0.89	14.50	2.78	0.94	1.00	3.67	1.00
time (sec)	N/A	0.363	87.606	0.485	1.735	0.094	3.719	0.407	0.271	2.977

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	261	50	17	18	86	18
N.S.	1	1.00	1.11	0.89	14.50	2.78	0.94	1.00	4.78	1.00
time (sec)	N/A	0.376	46.158	0.441	1.874	0.086	5.575	0.585	0.271	2.895

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	20	15	18	20	18
N.S.	1	1.00	1.11	0.89	1.00	1.11	0.83	1.00	1.11	1.00
time (sec)	N/A	0.207	1.269	0.473	0.168	0.076	4.209	0.125	0.258	2.906

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	18	15	18	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.194	1.161	0.584	0.179	0.074	0.502	0.132	0.246	2.895

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	23	17	18	22	18
N.S.	1	1.00	1.11	0.89	1.00	1.28	0.94	1.00	1.22	1.00
time (sec)	N/A	0.195	0.710	0.543	0.211	0.081	1.424	0.235	0.249	2.906

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	31	17	0	27	18
N.S.	1	1.00	1.11	0.89	1.00	1.72	0.94	0.00	1.50	1.00
time (sec)	N/A	0.198	0.637	0.642	0.235	0.074	3.478	0.000	0.240	2.767

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	427	34	17	18	34	18
N.S.	1	1.00	1.11	0.89	23.72	1.89	0.94	1.00	1.89	1.00
time (sec)	N/A	0.192	4.738	0.478	0.863	0.088	11.122	0.136	0.236	2.679

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	426	32	17	18	32	18
N.S.	1	1.00	1.11	0.89	23.67	1.78	0.94	1.00	1.78	1.00
time (sec)	N/A	0.193	4.432	0.579	0.835	0.087	2.188	0.134	0.248	2.769

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	446	39	19	18	38	18
N.S.	1	1.00	1.11	0.89	24.78	2.17	1.06	1.00	2.11	1.00
time (sec)	N/A	0.202	16.308	0.587	0.768	0.100	4.444	0.288	0.252	2.755

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	472	51	19	0	44	18
N.S.	1	1.00	1.11	0.89	26.22	2.83	1.06	0.00	2.44	1.00
time (sec)	N/A	0.202	8.709	0.660	0.895	0.087	11.705	0.000	0.281	2.885

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	134	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	13.40	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.337	0.369	1.514	0.357	0.107	5.359	0.273	0.221	2.793

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	134	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	13.40	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.340	0.357	1.449	0.347	0.094	2.801	0.193	0.233	2.696

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	132	123	0	0	0	0	0	12	0
N.S.	1	0.96	0.90	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.343	0.030	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	0	0	0	0	0	10	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.207	0.017	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.181	0.159	1.227	0.108	0.082	0.384	0.121	0.233	2.704

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	268	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	26.80	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.178	0.165	1.267	0.445	0.082	0.714	0.124	0.226	2.627

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	186	152	0	0	0	0	0	11	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.773	0.084	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	132	99	0	0	0	0	0	11	0
N.S.	1	0.95	0.71	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.882	0.053	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	99	0	0	0	0	0	11	0
N.S.	1	1.05	0.82	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.839	0.053	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	52	75	0	0	0	0	9	0
N.S.	1	0.97	0.56	0.81	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.623	0.025	0.979	0.000	0.000	0.000	0.000	0.216	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	45	42	0	0	0	0	7	0
N.S.	1	1.15	0.85	0.79	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.455	0.040	0.877	0.000	0.000	0.000	0.000	0.221	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	10	12	11	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.83	1.00	0.92	1.00
time (sec)	N/A	0.176	0.139	0.385	0.211	0.000	0.335	0.254	0.220	2.712

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	421	152	0	0	0	0	0	15	0
N.S.	1	1.28	0.46	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.491	0.076	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	279	99	0	0	0	0	0	15	0
N.S.	1	1.40	0.50	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.294	0.055	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	230	99	0	0	0	0	0	15	0
N.S.	1	1.28	0.55	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.776	0.053	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	136	52	102	0	0	0	0	13	0
N.S.	1	1.11	0.43	0.84	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.993	0.016	0.995	0.000	0.000	0.000	0.000	0.243	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	47	65	0	0	0	0	12	0
N.S.	1	1.06	0.58	0.80	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.542	0.023	0.945	0.000	0.000	0.000	0.000	0.224	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	10	12	15	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.83	1.00	1.25	1.00
time (sec)	N/A	0.172	0.114	0.365	0.223	0.000	1.397	0.419	0.222	2.674

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	379	519	152	0	0	0	0	0	17	0
N.S.	1	1.37	0.40	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.923	0.082	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	341	99	0	0	0	0	0	17	0
N.S.	1	1.38	0.40	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	2.531	0.051	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	287	99	0	0	0	0	0	17	0
N.S.	1	1.37	0.47	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.734	0.049	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	163	52	136	0	0	0	0	15	0
N.S.	1	1.07	0.34	0.89	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.981	0.024	0.944	0.000	0.000	0.000	0.000	0.221	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	110	45	78	0	0	0	0	14	0
N.S.	1	1.17	0.48	0.83	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.661	0.038	0.906	0.000	0.000	0.000	0.000	0.228	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	10	12	17	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.83	1.00	1.42	1.00
time (sec)	N/A	0.173	0.115	0.381	0.241	0.000	18.013	0.444	0.240	2.682

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	149	151	0	0	0	0	0	218	0
N.S.	1	0.91	0.93	0.00	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.412	0.074	0.000	0.000	0.000	0.000	0.000	0.306	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	101	99	0	0	0	0	0	145	0
N.S.	1	0.93	0.91	0.00	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.389	0.057	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	99	0	0	0	0	0	94	0
N.S.	1	0.92	0.94	0.00	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.579	0.052	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	52	37	0	0	0	0	73	0
N.S.	1	1.11	0.83	0.59	0.00	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	0.586	0.017	0.829	0.000	0.000	0.000	0.000	0.244	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	41	47	24	0	0	0	0	57	0
N.S.	1	0.95	1.09	0.56	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.521	0.022	0.835	0.000	0.000	0.000	0.000	0.244	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.287	0.137	0.549	0.217	0.000	0.374	0.275	0.217	2.798

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	14	12	17	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.17	1.00	1.42	1.00
time (sec)	N/A	0.221	1.092	0.469	0.204	0.000	0.502	0.268	0.236	2.677

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	176	265	0	0	0	0	0	273	0
N.S.	1	0.94	1.41	0.00	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.567	0.108	0.000	0.000	0.000	0.000	0.000	0.319	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	132	126	0	0	0	0	0	184	0
N.S.	1	0.96	0.91	0.00	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.364	0.032	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	124	140	0	0	0	0	0	123	0
N.S.	1	0.95	1.08	0.00	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.310	0.080	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	73	82	0	0	0	0	90	0
N.S.	1	1.07	0.87	0.98	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.367	0.019	1.065	0.000	0.000	0.000	0.000	0.254	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	75	69	65	0	0	0	0	74	0
N.S.	1	1.17	1.08	1.02	0.00	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	0.481	0.051	1.032	0.000	0.000	0.000	0.000	0.251	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.176	0.140	0.408	0.216	0.000	0.912	0.129	0.246	2.514

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	246	184	0	0	0	0	0	203	0
N.S.	1	1.47	1.10	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	1.115	0.266	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	214	222	0	0	0	0	0	135	0
N.S.	1	1.33	1.38	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	1.114	0.106	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	135	98	119	0	0	0	0	96	0
N.S.	1	1.14	0.83	1.01	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	1.217	0.117	1.012	0.000	0.000	0.000	0.000	0.278	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	91	105	81	0	0	0	0	80	0
N.S.	1	1.08	1.25	0.96	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.924	0.071	0.967	0.000	0.000	0.000	0.000	0.250	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.284	0.136	0.387	0.225	0.000	6.562	0.134	0.209	2.559

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	318	210	0	0	0	0	0	203	0
N.S.	1	1.39	0.92	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	1.213	0.472	0.000	0.000	0.000	0.000	0.000	0.283	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	291	221	0	0	0	0	0	135	0
N.S.	1	1.31	1.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.438	0.254	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	166	118	147	0	0	0	0	96	0
N.S.	1	1.13	0.80	1.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.822	0.203	1.004	0.000	0.000	0.000	0.000	0.245	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	133	111	105	0	0	0	0	80	0
N.S.	1	1.19	0.99	0.94	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.696	0.129	0.978	0.000	0.000	0.000	0.000	0.229	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.192	0.138	0.408	0.223	0.000	63.339	0.130	0.210	2.495

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	219	215	0	0	0	0	0	15	0
N.S.	1	1.03	1.01	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.131	0.292	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	142	111	0	0	0	0	0	13	0
N.S.	1	0.98	0.77	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.973	0.056	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	101	0	0	0	0	0	11	0
N.S.	1	1.08	0.99	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.971	0.147	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	15	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	0.94	1.00
time (sec)	N/A	0.222	0.399	0.438	0.279	0.000	0.420	0.825	0.225	2.722

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	15	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	0.94	1.00
time (sec)	N/A	0.207	3.443	0.695	0.235	0.000	0.366	0.866	0.274	2.539

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	377	215	0	0	0	0	0	39	0
N.S.	1	1.34	0.76	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.016	0.219	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	196	114	0	0	0	0	0	35	0
N.S.	1	1.09	0.64	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.442	0.064	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	140	251	0	0	0	0	0	32	0
N.S.	1	1.04	1.86	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.754	0.363	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	39	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	2.44	1.00
time (sec)	N/A	0.337	0.414	0.430	0.267	0.000	3.792	1.327	0.243	2.671

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.338	2.829	0.674	0.258	0.000	2.617	1.319	200.030	2.700

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	444	198	0	0	0	0	0	68	0
N.S.	1	1.36	0.61	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	3.286	0.218	0.000	0.000	0.000	0.000	0.000	0.308	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	232	115	0	0	0	0	0	62	0
N.S.	1	1.04	0.52	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.453	0.058	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	169	282	0	0	0	0	0	58	0
N.S.	1	1.09	1.82	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.904	1.424	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	68	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	4.25	1.00
time (sec)	N/A	0.209	0.478	0.476	0.249	0.000	21.254	2.102	0.234	2.446

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.213	2.877	0.713	0.263	0.000	22.859	2.104	200.023	2.427

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	189	196	0	0	0	0	0	25	0
N.S.	1	0.97	1.01	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.547	0.175	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	117	108	0	0	0	0	0	23	0
N.S.	1	1.09	1.01	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.864	0.053	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	101	0	0	0	0	0	22	0
N.S.	1	1.01	1.15	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.631	0.068	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	25	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.56	1.00
time (sec)	N/A	0.292	0.799	0.606	0.238	0.000	0.428	0.846	0.176	2.444

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	29	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	1.81	1.00
time (sec)	N/A	0.253	3.165	0.628	0.244	0.000	0.523	0.834	0.206	2.477

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	223	290	0	0	0	0	0	39	0
N.S.	1	0.99	1.28	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.727	0.292	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	144	134	0	0	0	0	0	37	0
N.S.	1	1.07	0.99	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.510	0.089	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	130	137	0	0	0	0	0	810	0
N.S.	1	1.12	1.18	0.00	0.00	0.00	0.00	0.00	6.98	0.00
time (sec)	N/A	0.645	0.173	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	0	40	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	0.00	2.50	1.00
time (sec)	N/A	0.247	0.729	0.530	0.264	0.000	1.671	0.000	0.221	2.444

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	46	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	2.88	1.00
time (sec)	N/A	0.238	2.994	0.643	0.261	0.000	2.376	0.178	0.201	2.442

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	381	340	0	0	0	0	0	53	0
N.S.	1	1.41	1.25	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.870	1.071	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	206	200	0	0	0	0	0	51	0
N.S.	1	1.13	1.09	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.478	0.527	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	156	181	0	0	0	0	0	0	0
N.S.	1	1.09	1.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.239	0.192	0.000	0.000	0.000	0.000	0.000	0.297	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	0	55	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	0.00	3.44	1.00
time (sec)	N/A	0.342	0.736	0.508	0.283	0.000	7.475	0.000	0.191	2.502

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	63	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	3.94	1.00
time (sec)	N/A	0.332	3.196	0.638	0.265	0.000	13.884	0.179	0.239	2.482

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	0	15	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	0.00	1.25	1.00
time (sec)	N/A	0.183	0.280	0.589	0.203	0.000	58.937	0.000	0.210	2.514

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	0	11	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	0.00	0.92	1.00
time (sec)	N/A	0.184	0.327	0.592	0.231	0.000	1.238	0.000	0.188	2.553

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	137	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	11.42	1.00
time (sec)	N/A	0.185	0.338	0.592	0.208	0.000	0.526	0.282	0.239	2.581

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	176	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	14.67	1.00
time (sec)	N/A	0.174	0.313	0.595	0.225	0.000	4.104	0.144	0.236	2.554

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	111	99	0	0	0	0	0	12	0
N.S.	1	0.93	0.83	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.393	0.050	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	105	97	0	0	0	0	0	12	0
N.S.	1	0.93	0.86	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.377	0.051	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	68	57	38	0	0	0	0	10	0
N.S.	1	1.15	0.97	0.64	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.358	0.011	0.648	0.000	0.000	0.000	0.000	0.183	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	45	40	0	0	0	0	8	0
N.S.	1	0.96	0.92	0.82	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.280	0.024	0.661	0.000	0.000	0.000	0.000	0.214	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.183	0.206	0.227	0.206	0.091	0.392	0.265	0.175	2.933

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.187	0.772	0.263	0.209	0.095	0.572	0.254	0.201	2.660

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	16	0	0	16	14
N.S.	1	1.00	1.14	0.86	1.00	1.14	0.00	0.00	1.14	1.00
time (sec)	N/A	0.188	1.358	0.606	0.479	0.127	0.000	0.000	0.210	2.613

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	14	14	0	14	14
N.S.	1	1.00	1.14	0.86	1.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.182	2.153	0.725	0.495	0.125	3.738	0.000	0.235	2.619

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	20	14	14	18	14
N.S.	1	1.00	1.14	0.86	1.00	1.43	1.00	1.00	1.29	1.00
time (sec)	N/A	0.185	0.816	0.642	0.490	0.089	1.627	0.676	0.206	2.748

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	20	14	14	24	14
N.S.	1	1.00	1.14	0.86	1.00	1.43	1.00	1.00	1.71	1.00
time (sec)	N/A	0.197	0.824	0.784	0.514	0.105	12.895	0.720	0.197	2.668

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	0	16	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	0.00	1.33	1.17
time (sec)	N/A	0.189	0.381	0.366	0.215	0.101	3.652	0.000	0.185	2.676

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [112] had the largest ratio of [1.41667000000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	F	0	0	N/A	0.000	N/A
2	F	0	0	N/A	0.000	N/A
3	F	0	0	N/A	0.000	N/A
4	F	0	0	N/A	0.000	N/A
5	N/A	1	0	1.00	20	0.000
6	N/A	1	0	1.00	20	0.000
7	A	5	4	1.06	8	0.500
8	A	4	4	1.21	8	0.500
9	A	5	4	1.08	8	0.500
10	A	3	3	1.14	6	0.500
11	A	2	2	1.00	4	0.500
12	C	9	8	1.33	8	1.000
13	A	5	4	1.00	8	0.500
14	A	2	2	1.00	8	0.250
15	A	6	5	1.00	8	0.625
16	A	3	3	1.09	8	0.375
17	A	7	6	1.06	8	0.750
18	A	6	6	1.18	10	0.600
19	A	5	5	1.15	10	0.500
20	A	4	4	1.12	8	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	3	1.18	6	0.500
22	C	10	9	1.28	10	0.900
23	C	8	7	1.18	10	0.700
24	A	3	3	1.00	10	0.300
25	C	10	9	1.04	10	0.900
26	A	5	5	1.01	10	0.500
27	A	11	11	1.47	10	1.100
28	A	10	9	1.30	10	0.900
29	A	6	6	1.19	8	0.750
30	A	4	4	1.14	6	0.667
31	C	11	10	1.23	10	1.000
32	C	9	8	1.11	10	0.800
33	C	11	10	1.10	10	1.000
34	C	14	13	1.04	10	1.300
35	C	14	13	1.09	10	1.300
36	A	14	14	1.65	10	1.400
37	A	10	10	1.48	10	1.000
38	A	11	11	1.37	10	1.100
39	A	7	7	1.19	8	0.875
40	A	5	5	1.21	6	0.833
41	C	12	11	1.29	10	1.100
42	C	10	9	1.12	10	0.900
43	C	12	11	1.11	10	1.100
44	C	14	13	1.06	10	1.300
45	A	4	3	0.85	10	0.300
46	A	4	3	0.88	10	0.300
47	A	4	3	0.88	10	0.300
48	A	4	3	0.93	10	0.300
49	A	4	3	0.93	10	0.300
50	A	7	6	1.00	8	0.750
51	A	4	3	1.00	6	0.500
52	N/A	1	0	1.00	10	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	N/A	1	0	1.00	10	0.000
54	A	3	2	0.91	10	0.200
55	A	3	2	0.94	10	0.200
56	A	3	2	0.94	10	0.200
57	A	3	2	0.98	10	0.200
58	A	3	2	0.98	10	0.200
59	A	4	3	1.00	8	0.375
60	A	6	5	1.00	6	0.833
61	N/A	1	0	1.00	10	0.000
62	N/A	1	0	1.00	10	0.000
63	A	6	5	1.37	10	0.500
64	A	10	9	1.33	10	0.900
65	A	9	8	1.26	10	0.800
66	A	10	9	1.10	8	1.125
67	A	6	5	1.08	6	0.833
68	N/A	1	0	1.00	10	0.000
69	N/A	1	0	1.00	10	0.000
70	A	5	4	1.34	10	0.400
71	A	7	6	1.23	10	0.600
72	A	10	9	1.25	10	0.900
73	A	7	6	1.09	8	0.750
74	A	8	7	1.16	6	1.167
75	N/A	1	0	1.00	10	0.000
76	N/A	1	0	1.00	10	0.000
77	A	6	5	1.15	16	0.312
78	A	8	7	1.13	16	0.438
79	A	5	4	1.12	16	0.250
80	A	7	6	1.13	16	0.375
81	A	4	3	1.07	16	0.188
82	A	8	7	1.10	16	0.438
83	A	2	2	1.04	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
84	A	2	2	1.04	18	0.111
85	A	2	2	1.02	18	0.111
86	A	2	2	1.02	18	0.111
87	A	2	2	1.00	18	0.111
88	N/A	2	0	1.00	18	0.000
89	N/A	2	0	1.00	18	0.000
90	N/A	2	0	1.00	18	0.000
91	N/A	2	0	1.00	18	0.000
92	N/A	1	0	1.00	18	0.000
93	N/A	1	0	1.00	18	0.000
94	N/A	1	0	1.00	18	0.000
95	N/A	1	0	1.00	18	0.000
96	N/A	1	0	1.00	18	0.000
97	N/A	1	0	1.00	18	0.000
98	N/A	1	0	1.00	18	0.000
99	N/A	1	0	1.00	18	0.000
100	N/A	2	0	1.00	10	0.000
101	N/A	2	0	1.00	10	0.000
102	A	2	2	0.96	10	0.200
103	A	2	2	1.00	8	0.250
104	N/A	1	0	1.00	10	0.000
105	N/A	1	0	1.00	10	0.000
106	C	7	6	1.02	12	0.500
107	A	6	5	0.95	12	0.417
108	C	7	6	1.05	12	0.500
109	A	7	6	0.97	10	0.600
110	C	9	8	1.15	8	1.000
111	N/A	1	0	1.00	12	0.000
112	A	18	17	1.28	12	1.417
113	C	17	16	1.40	12	1.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
114	A	14	13	1.28	12	1.083
115	C	13	12	1.11	10	1.200
116	A	10	9	1.06	8	1.125
117	N/A	1	0	1.00	12	0.000
118	C	17	16	1.37	12	1.333
119	A	12	11	1.38	12	0.917
120	C	15	14	1.37	12	1.167
121	A	10	9	1.07	10	0.900
122	C	11	10	1.17	8	1.250
123	N/A	1	0	1.00	12	0.000
124	A	4	3	0.91	12	0.250
125	A	4	3	0.93	12	0.250
126	A	4	3	0.92	12	0.250
127	C	10	9	1.11	10	0.900
128	A	8	7	0.95	8	0.875
129	N/A	1	0	1.00	12	0.000
130	N/A	1	0	1.00	12	0.000
131	A	3	2	0.94	12	0.167
132	A	3	2	0.96	12	0.167
133	A	3	2	0.95	12	0.167
134	A	8	7	1.07	10	0.700
135	C	9	8	1.17	8	1.000
136	N/A	1	0	1.00	12	0.000
137	C	13	12	1.47	12	1.000
138	A	13	12	1.33	12	1.000
139	C	13	12	1.14	10	1.200
140	A	10	9	1.08	8	1.125
141	N/A	1	0	1.00	12	0.000
142	A	11	10	1.39	12	0.833
143	C	13	12	1.31	12	1.000
144	A	11	10	1.13	10	1.000
145	C	11	10	1.19	8	1.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	N/A	1	0	1.00	12	0.000
147	C	8	7	1.03	16	0.438
148	A	7	6	0.98	14	0.429
149	C	10	9	1.08	12	0.750
150	N/A	1	0	1.00	16	0.000
151	N/A	1	0	1.00	16	0.000
152	A	14	13	1.34	16	0.812
153	C	14	13	1.09	14	0.929
154	A	10	9	1.04	12	0.750
155	N/A	1	0	1.00	16	0.000
156	N/A	1	0	1.00	16	0.000
157	C	16	15	1.36	16	0.938
158	A	10	9	1.04	14	0.643
159	C	12	11	1.09	12	0.917
160	N/A	1	0	1.00	16	0.000
161	N/A	1	0	1.00	16	0.000
162	A	4	3	0.97	16	0.188
163	C	11	10	1.09	14	0.714
164	A	8	7	1.01	12	0.583
165	N/A	1	0	1.00	16	0.000
166	N/A	1	0	1.00	16	0.000
167	A	3	2	0.99	16	0.125
168	A	8	7	1.07	14	0.500
169	C	10	9	1.12	12	0.750
170	N/A	1	0	1.00	16	0.000
171	N/A	1	0	1.00	16	0.000
172	A	13	12	1.41	16	0.750
173	C	14	13	1.13	14	0.929
174	A	10	9	1.09	12	0.750
175	N/A	1	0	1.00	16	0.000
176	N/A	1	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
177	N/A	1	0	1.00	12	0.000
178	N/A	1	0	1.00	12	0.000
179	N/A	1	0	1.00	12	0.000
180	N/A	1	0	1.00	12	0.000
181	A	4	3	0.93	10	0.300
182	A	4	3	0.93	10	0.300
183	C	8	7	1.15	8	0.875
184	A	6	5	0.96	6	0.833
185	N/A	1	0	1.00	10	0.000
186	N/A	1	0	1.00	10	0.000
187	N/A	1	0	1.00	14	0.000
188	N/A	1	0	1.00	14	0.000
189	N/A	1	0	1.00	14	0.000
190	N/A	1	0	1.00	14	0.000
191	N/A	1	0	1.00	12	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(c+d\,\mathrm{arccosh}(ex))(a+b\,\mathrm{arcsinh}(ex))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1

$$\int x^3(c+d\,\mathrm{arccosh}(ex))(a+b\,\mathrm{arcsinh}(ex))\,dx=0$$

output

```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 1 in optimal.

Time = 12.01 (sec) , antiderivative size = 499, normalized size of antiderivative =

499.00

$$\int x^3(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx$$

$$= \frac{1}{32}\Bigg(8acx^4+\frac{bdx^2\sqrt{-1+ex}\sqrt{1+ex}\sqrt{1+e^2x^2}}{e^2}-\frac{bcx\sqrt{1+e^2x^2}(-3+2e^2x^2)}{e^3}$$

$$-\frac{adx\sqrt{-1+ex}\sqrt{1+ex}(3+2e^2x^2)}{e^3}$$

$$+\frac{dx\left(8ae^3x^3+b(3-2e^2x^2)\sqrt{1+e^2x^2}\right)\text{arccosh}(ex)}{e^3}-\frac{3bc\text{arcsinh}(ex)}{e^4}$$

$$+\frac{b\left(8ce^4x^4-dex\sqrt{-1+ex}\sqrt{1+ex}(3+2e^2x^2)+d(-3+8e^4x^4)\text{arccosh}(ex)\right)\text{arcsinh}(ex)}{e^4}$$

$$+\frac{(2+2i)bd\sqrt{\frac{i(1+ex)}{-1+ex}}\sqrt{-2+2ex}\Big(\sqrt{2}(-i+ex)\sqrt{\frac{(1+i)(i+ex)}{-1+ex}}\,\text{EllipticF}\Big(\arcsin\Big(\sqrt{\frac{(1+i)-(1-i)ex}{2-2ex}}\Big),2\Big)-}{e^4\sqrt{\frac{(1+i)-(1-i)ex}{1-ex}}\sqrt{1+ex}\sqrt{}}$$

$$-\frac{3ad\log\left(ex+\sqrt{-1+ex}\sqrt{1+ex}\right)}{e^4}\Bigg)$$

input
```
Integrate[x^3*(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
(8*a*c*x^4 + (b*d*x^2*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*Sqrt[1 + e^2*x^2])/e^2
- (b*c*x*Sqrt[1 + e^2*x^2]*(-3 + 2*e^2*x^2))/e^3 - (a*d*x*Sqrt[-1 + e*x]*S
qrt[1 + e*x]*(3 + 2*e^2*x^2))/e^3 + (d*x*(8*a*e^3*x^3 + b*(3 - 2*e^2*x^2)*
Sqrt[1 + e^2*x^2])*ArcCosh[e*x])/e^3 - (3*b*c*ArcSinh[e*x])/e^4 + (b*(8*c*
e^4*x^4 - d*e*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(3 + 2*e^2*x^2) + d*(-3 + 8*e
^4*x^4)*ArcCosh[e*x])*ArcSinh[e*x])/e^4 + ((2 + 2*I)*b*d*Sqrt[(I*(1 + e*x)
)/(-1 + e*x)]*Sqrt[-2 + 2*e*x]*(Sqrt[2]*(-I + e*x)*Sqrt[((1 + I)*(I + e*x)
)/(-1 + e*x)]*EllipticF[ArcSin[Sqrt[((1 + I) - (1 - I)*e*x)/(2 - 2*e*x)]],
 2] - 2*(-1 + e*x)*Sqrt[((1 - I)*(-I + e*x))/(-1 + e*x)]*Sqrt[(1 + e^2*x^2
)/(-1 + e*x)^2]*EllipticPi[1 + I, ArcSin[Sqrt[((1 + I) - (1 - I)*e*x)/(2 -
 2*e*x)]], 2]))/(e^4*Sqrt[((1 + I) - (1 - I)*e*x)/(1 - e*x)]*Sqrt[1 + e*x]
*Sqrt[1 + e^2*x^2]) - (3*a*d*Log[e*x + Sqrt[-1 + e*x]*Sqrt[1 + e*x]])/e^4)
/32
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)\,dx$$

$$\Big\downarrow \text{6272}$$

$$\int x^3(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)dx$$

input

```
Int[x^3*(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output

```
$Aborted
```

Maple [F]

$$\int x^3(c+d\ \text{arccosh}\,(ex))\,(a+b\ \text{arcsinh}\,(ex))\,dx$$

input

```
int(x^3*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

output

```
int(x^3*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 324.00

$$\int x^3(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx$$

$$= \frac{8\,ace^4x^4 + 2\,bd\log\left(-e^2x^2+\sqrt{e^2x^2+1}\sqrt{e^2x^2-1}\right) + 3\,bc\log\left(-ex+\sqrt{e^2x^2+1}\right) + 3\,ad\log\left(-ex+\sqrt{e}\right.}{}$$

input
```
integrate(x^3*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="fricas")
```

output
```
1/32*(8*a*c*e^4*x^4 + 2*b*d*log(-e^2*x^2 + sqrt(e^2*x^2 + 1)*sqrt(e^2*x^2
- 1)) + 3*b*c*log(-e*x + sqrt(e^2*x^2 + 1)) + 3*a*d*log(-e*x + sqrt(e^2*x^
2 - 1)) + (8*b*c*e^4*x^4 + (8*b*d*e^4*x^4 - 3*b*d)*log(e*x + sqrt(e^2*x^2
- 1)) - (2*b*d*e^3*x^3 + 3*b*d*e*x)*sqrt(e^2*x^2 - 1))*log(e*x + sqrt(e^2*
x^2 + 1)) + (8*a*d*e^4*x^4 - (2*b*d*e^3*x^3 - 3*b*d*e*x)*sqrt(e^2*x^2 + 1)
)*log(e*x + sqrt(e^2*x^2 - 1)) - (2*b*c*e^3*x^3 - sqrt(e^2*x^2 - 1)*b*d*e^
2*x^2 - 3*b*c*e*x)*sqrt(e^2*x^2 + 1) - (2*a*d*e^3*x^3 + 3*a*d*e*x)*sqrt(e^
2*x^2 - 1))/e^4
```

Sympy [F]

$$\int x^3(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx = \int x^3(a+b\operatorname{asinh}(ex))\,(c+d\operatorname{acosh}(ex))\,\,dx$$

input
```
integrate(x**3*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output
```
Integral(x**3*(a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Maxima [F]

$$\begin{aligned}&\int x^3(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx\\&=\int (d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)x^3\,dx\end{aligned}$$

input
```
integrate(x^3*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="maxima")
```

output
```
1/4*a*c*x^4 + 1/32*(8*x^4*arcsinh(e*x) - (2*sqrt(e^2*x^2 + 1)*x^3/e^2 - 3*
sqrt(e^2*x^2 + 1)*x/e^4 + 3*arcsinh(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^4))*e)*b
*c + 1/32*(8*x^4*arccosh(e*x) - (2*sqrt(e^2*x^2 - 1)*x^3/e^2 + 3*sqrt(e^2*
x^2 - 1)*x/e^4 + 3*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)
*e^4))*e)*a*d + b*d*integrate(x^3*log(e*x + sqrt(e*x + 1)*sqrt(e*x - 1))*l
og(e*x + sqrt(e^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int x^3(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx\\&=\int x^3\,(a+b\,\text{asinh}(e\,x))\,(c+d\,\text{acosh}(e\,x))\,dx\end{aligned}$$

input
```
int(x^3*(a + b*asinh(e*x))*(c + d*acosh(e*x)),x)
```

output
```
int(x^3*(a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Reduce [F]

$$\int x^3(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx=\int x^3(c+dacosh(ex))\,(a+basinh(ex))\,dx$$

input

```
int(x^3*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output

```
int(x^3*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

3.2 $\int x^2(c+d\,\mathrm{arccosh}(ex))(a+b\,\mathrm{arcsinh}(ex))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1

$$\int x^2(c+d\,\mathrm{arccosh}(ex))(a+b\,\mathrm{arcsinh}(ex))\,dx=0$$

output

```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 1 in optimal.

Time = 1.39 (sec) , antiderivative size = 307, normalized size of antiderivative = 307.00

$$\int x^2(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx$$
$$=\frac{1}{27}\left(9acx^3+\frac{2bdx\sqrt{-1+ex}\sqrt{1+ex}\sqrt{1+e^2x^2}}{e^2}-\frac{3bc(-2+e^2x^2)\sqrt{1+e^2x^2}}{e^3}\right.$$
$$-\frac{3ad\sqrt{-1+ex}\sqrt{1+ex}(2+e^2x^2)}{e^3}$$
$$+\frac{3d\left(3ae^3x^3+b(2-e^2x^2)\sqrt{1+e^2x^2}\right)\operatorname{arccosh}(ex)}{e^3}$$
$$+\frac{3b\left(3ce^3x^3-d\sqrt{-1+ex}\sqrt{1+ex}(2+e^2x^2)+3de^3x^3\operatorname{arccosh}(ex)\right)\operatorname{arcsinh}(ex)}{e^3}$$
$$\left.+\frac{10bd\sqrt{1+ex}\sqrt{-2+2ex}\sqrt{\frac{1+e^2x^2}{(-1+ex)^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\left(1+\frac{2}{-1+ex}\right)^2\right)}{e^3\sqrt{1+e^2x^2}}\right)$$

input
```
Integrate[x^2*(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
(9*a*c*x^3 + (2*b*d*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*Sqrt[1 + e^2*x^2])/e^2
- (3*b*c*(-2 + e^2*x^2)*Sqrt[1 + e^2*x^2])/e^3 - (3*a*d*Sqrt[-1 + e*x]*Sqr
t[1 + e*x]*(2 + e^2*x^2))/e^3 + (3*d*(3*a*e^3*x^3 + b*(2 - e^2*x^2)*Sqrt[1
 + e^2*x^2])*ArcCosh[e*x])/e^3 + (3*b*(3*c*e^3*x^3 - d*Sqrt[-1 + e*x]*Sqrt
[1 + e*x]*(2 + e^2*x^2) + 3*d*e^3*x^3*ArcCosh[e*x])*ArcSinh[e*x])/e^3 + (1
0*b*d*Sqrt[1 + e*x]*Sqrt[-2 + 2*e*x]*Sqrt[(1 + e^2*x^2)/(-1 + e*x)^2]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -(1 + 2/(-1 + e*x))^2])/(e^3*Sqrt[1 + e^2*x^2
]))/27
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\operatorname{arcsinh}(ex))(d\operatorname{arccosh}(ex)+c)\,dx$$
$$\Big\downarrow \text{6272}$$

$$\int x^2(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)dx$$

input
```
Int[x^2*(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^2(c+d\,\text{arccosh}\,(ex))\,(a+b\,\text{arcsinh}\,(ex))\,dx$$

input
```
int(x^2*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

output
```
int(x^2*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 240.00

$$\int x^2(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx$$
$$= \frac{9\,ace^3x^3 + 10\,bdF\left(\arcsin\left(\frac{1}{ex}\right)\,|\,-1\right) + 3\left(3\,bde^3x^3\log\left(ex+\sqrt{e^2x^2-1}\right) + 3\,bce^3x^3 - (bde^2x^2+2\,bd)\sqrt{e}\right.}{}$$

input
```
integrate(x^2*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="fricas")
```

output
```
1/27*(9*a*c*e^3*x^3 + 10*b*d*elliptic_f(arcsin(1/(e*x)), -1) + 3*(3*b*d*e^
3*x^3*log(e*x + sqrt(e^2*x^2 - 1)) + 3*b*c*e^3*x^3 - (b*d*e^2*x^2 + 2*b*d)
*sqrt(e^2*x^2 - 1))*log(e*x + sqrt(e^2*x^2 + 1)) + 3*(3*a*d*e^3*x^3 - (b*d
*e^2*x^2 - 2*b*d)*sqrt(e^2*x^2 + 1))*log(e*x + sqrt(e^2*x^2 - 1)) - (3*b*c
*e^2*x^2 - 2*sqrt(e^2*x^2 - 1)*b*d*e*x - 6*b*c)*sqrt(e^2*x^2 + 1) - 3*(a*d
*e^2*x^2 + 2*a*d)*sqrt(e^2*x^2 - 1))/e^3
```

Sympy [F]

$$\int x^2(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\int x^2(a+b\,\text{asinh}\,(ex))\,(c+d\,\text{acosh}\,(ex))\ dx$$

input
```
integrate(x**2*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output
```
Integral(x**2*(a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Maxima [F]

$$\begin{aligned}&\int x^2(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx\\&=\int(d\,\text{arcosh}\,(ex)+c)(b\,\text{arsinh}\,(ex)+a)x^2\,dx\end{aligned}$$

input
```
integrate(x^2*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="maxima")
```

output
```
1/3*a*c*x^3 + 1/9*(3*x^3*arcsinh(e*x) - e*(sqrt(e^2*x^2 + 1)*x^2/e^2 - 2*s
qrt(e^2*x^2 + 1)/e^4))*b*c + 1/9*(3*x^3*arccosh(e*x) - e*(sqrt(e^2*x^2 - 1
)*x^2/e^2 + 2*sqrt(e^2*x^2 - 1)/e^4))*a*d + b*d*integrate(x^2*log(e*x + sq
rt(e*x + 1)*sqrt(e*x - 1))*log(e*x + sqrt(e^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx$$
$$=\int x^2\,(a+b\operatorname{asinh}(e\,x))\,(c+d\operatorname{acosh}(e\,x))\,dx$$

input
```
int(x^2*(a + b*asinh(e*x))*(c + d*acosh(e*x)),x)
```

output
```
int(x^2*(a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Reduce [F]

$$\int x^2(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx=\int x^2(c+d\,acosh(ex))\,(a+b\,asinh(ex))\,dx$$

input
```
int(x^2*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output
```
int(x^2*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

3.3 $\int x(c+d\,\text{arccosh}(ex))(a+b\,\text{arcsinh}(ex))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1

$$\int x(c+d\,\text{arccosh}(ex))(a+b\,\text{arcsinh}(ex))\,dx=0$$

output

```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 1 in optimal.

Time = 10.92 (sec) , antiderivative size = 1724, normalized size of antiderivative = 1724.00

$$\int x(c+d\,\text{arccosh}(ex))(a+b\,\text{arcsinh}(ex))\,dx=\text{Too large to display}$$

input

```
Integrate[x*(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
(8*a*c*x^2 + (4*a*d*x)/(e*Sqrt[-1 + e*x]*Sqrt[1 + e*x]) - (4*a*d*e*x^3)/(S
qrt[-1 + e*x]*Sqrt[1 + e*x]) - (4*b*c*x)/(e*Sqrt[1 + e^2*x^2]) - (4*b*c*e*
x^3)/Sqrt[1 + e^2*x^2] - (2*b*d)/(e^2*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*Sqrt[1
 + e^2*x^2]) + (2*b*d*e^2*x^4)/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*Sqrt[1 + e^2*x
^2]) + 8*a*d*x^2*ArcCosh[e*x] + (4*b*c*ArcSinh[e*x])/e^2 + 8*b*c*x^2*ArcSi
nh[e*x] + (4*b*d*x*ArcSinh[e*x])/(e*Sqrt[-1 + e*x]*Sqrt[1 + e*x]) - (4*b*d
*e*x^3*ArcSinh[e*x])/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]) - (4*b*d*ArcCosh[e*x]*
ArcSinh[e*x])/e^2 + 8*b*d*x^2*ArcCosh[e*x]*ArcSinh[e*x] - (8*a*d*ArcTanh[S
qrt[(-1 + e*x)/(1 + e*x)]])/e^2 - (2*b*d*Sqrt[-1 + e^4*x^4]*ArcTanh[(e^2*x
^2)/Sqrt[-1 + e^4*x^4]])/(e^2*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*Sqrt[1 + e^2*x^
2]) - ((4*I)*b*d*Pi*ArcTanh[(Sqrt[2]*e*x)/Sqrt[3 + Cosh[2*ArcCosh[e*x]]]])
/e^2 + (4*b*d*ArcTanh[(Sqrt[2]*Sqrt[(-1 + e*x)/(1 + e*x)]*(1 + e*x))/Sqrt[
3 + Cosh[2*ArcCosh[e*x]]]])/e^2 + (Sqrt[2]*b*d*Sqrt[(-1 + e*x)/(1 + e*x)]*
Sqrt[3 + Cosh[2*ArcCosh[e*x]]])/e^2 + (Sqrt[2]*b*d*x*Sqrt[(-1 + e*x)/(1 +
e*x)]*Sqrt[3 + Cosh[2*ArcCosh[e*x]]])/e - (2*Sqrt[2]*b*d*x*ArcCosh[e*x]*Sq
rt[3 + Cosh[2*ArcCosh[e*x]]])/e - (I*b*d*Pi*Log[(E^ArcCosh[e*x] - Sqrt[(-1
 + e*x)/(1 + e*x)]*(1 + e*x) - Sqrt[1 + e^2*x^2])/E^ArcCosh[e*x]])/e^2 - (
2*b*d*ArcCosh[e*x]*Log[(E^ArcCosh[e*x] - Sqrt[(-1 + e*x)/(1 + e*x)]*(1 + e
*x) - Sqrt[1 + e^2*x^2])/E^ArcCosh[e*x]])/e^2 + (2*b*d*ArcTanh[(Sqrt[(-1 +
 e*x)/(1 + e*x)]*(1 + e*x))/Sqrt[1 + e^2*x^2]]*Log[(E^ArcCosh[e*x] - Sq...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\operatorname{arcsinh}(ex))(d\operatorname{arccosh}(ex)+c)\,dx$$

$$\Big\downarrow \text{6272}$$

$$\int x(a+b\operatorname{arcsinh}(ex))(d\operatorname{arccosh}(ex)+c)dx$$

input
```
Int[x*(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int x(c+d\ \mathrm{arccosh}\,(ex))\,(a+b\ \mathrm{arcsinh}\,(ex))\,dx$$

input
```
int(x*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

output
```
int(x*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

Fricas [F]

$$\int x(c+d\mathrm{arccosh}(ex))(a+b\mathrm{arcsinh}(ex))\,dx$$
$$=\int (d\,\mathrm{arcosh}\,(ex)+c)(b\,\mathrm{arsinh}\,(ex)+a)x\,dx$$

input
```
integrate(x*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="fricas")
```

output
```
integral(a*d*x*arccosh(e*x) + a*c*x + (b*d*x*arccosh(e*x) + b*c*x)*arcsinh
(e*x), x)
```

Sympy [F]

$$\int x(c+d\mathrm{arccosh}(ex))(a+b\mathrm{arcsinh}(ex))\,dx=\int x(a+b\,\mathrm{asinh}\,(ex))\,(c+d\,\mathrm{acosh}\,(ex))\,\,dx$$

input
```
integrate(x*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output
```
Integral(x*(a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Maxima [F]

$$\int x(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx$$
$$=\int (d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)x\,dx$$

input
```
integrate(x*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="maxima")
```

output
```
1/2*a*c*x^2 + 1/4*(2*x^2*arcsinh(e*x) - e*(sqrt(e^2*x^2 + 1)*x/e^2 - arcsi
nh(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^2)))*b*c + 1/4*(2*x^2*arccosh(e*x) - e*(s
qrt(e^2*x^2 - 1)*x/e^2 + log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqr
t(e^2)*e^2)))*a*d + b*d*integrate(x*log(e*x + sqrt(e*x + 1)*sqrt(e*x - 1))
*log(e*x + sqrt(e^2*x^2 + 1)), x)
```

Giac [F]

$$\int x(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx$$
$$=\int (d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)x\,dx$$

input
```
integrate(x*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="giac")
```

output
```
integrate((d*arccosh(e*x) + c)*(b*arcsinh(e*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx=\int x\,(a+b\,\mathrm{asinh}(e\,x))\,\left(c+d\,\mathrm{acosh}(e\,x)\right)\,dx$$

input

```
int(x*(a + b*asinh(e*x))*(c + d*acosh(e*x)),x)
```

output

```
int(x*(a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Reduce [F]

$$\int x(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx=\int x(c+dacosh(ex))\,(a+basinh(ex))\,dx$$

input

```
int(x*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output

```
int(x*(c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

3.4 $\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 168

$$\begin{aligned}\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx = &-\frac{b\sqrt{1+e^2x^2}(c+d\text{arccosh}(ex))}{e}\\ &-\frac{d\sqrt{-1+ex}\sqrt{1+ex}(a+b\text{arcsinh}(ex))}{e}\\ &+x(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\\ &+\frac{2bd\sqrt{1-e^2x^2}E(\arcsin(ex)|-1)}{e\sqrt{-1+ex}\sqrt{1+ex}}\\ &-\frac{2bd\sqrt{1-e^2x^2}\,\text{EllipticF}(\arcsin(ex),-1)}{e\sqrt{-1+ex}\sqrt{1+ex}}\end{aligned}$$

output

```
-b*(e^2*x^2+1)^(1/2)*(c+d*arccosh(e*x))/e-d*(e*x-1)^(1/2)*(e*x+1)^(1/2)*(a
+b*arcsinh(e*x))/e+x*(c+d*arccosh(e*x))*(a+b*arcsinh(e*x))+2*b*d*(-e^2*x^2
+1)^(1/2)*EllipticE(e*x,I)/e/(e*x-1)^(1/2)/(e*x+1)^(1/2)-2*b*d*(-e^2*x^2+1
)^(1/2)*EllipticF(e*x,I)/e/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.50 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.76

$$\int (c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))\,dx=acx-\frac{ad\sqrt{-1+ex}\sqrt{1+ex}}{e}$$
$$-\frac{bc\sqrt{1+e^2x^2}}{e}+adx\operatorname{arccosh}(ex)-\frac{bd\sqrt{1+e^2x^2}\operatorname{arccosh}(ex)}{e}$$
$$+bcx\operatorname{arcsinh}(ex)+\frac{bd\left(-\sqrt{-1+ex}\sqrt{1+ex}+ex\operatorname{arccosh}(ex)\right)\operatorname{arcsinh}(ex)}{e}$$
$$-\frac{\sqrt{2}bd(-1+ex)^{5/2}\Bigg(-\sqrt{2}\left(1+\frac{4}{(-1+ex)^3}+\frac{6}{(-1+ex)^2}+\frac{4}{-1+ex}\right)+\frac{2\sqrt{i\left(1+\frac{2}{-1+ex}\right)}\sqrt{1+\frac{2}{(-1+ex)^2}+\frac{2}{-1+ex}}E\Big(\arcsin\big(}{-1+ex}}{e\sqrt{1+ex}\sqrt{2+2(-1+ex)+(-1+ex)^2}}$$

input
```
Integrate[(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
a*c*x - (a*d*Sqrt[-1 + e*x]*Sqrt[1 + e*x])/e - (b*c*Sqrt[1 + e^2*x^2])/e +
 a*d*x*ArcCosh[e*x] - (b*d*Sqrt[1 + e^2*x^2]*ArcCosh[e*x])/e + b*c*x*ArcSi
nh[e*x] + (b*d*(-(Sqrt[-1 + e*x]*Sqrt[1 + e*x]) + e*x*ArcCosh[e*x])*ArcSin
h[e*x])/e - (Sqrt[2]*b*d*(-1 + e*x)^(5/2)*(-(Sqrt[2]*(1 + 4/(-1 + e*x)^3 +
 6/(-1 + e*x)^2 + 4/(-1 + e*x))) + (2*Sqrt[I*(1 + 2/(-1 + e*x))]*Sqrt[1 +
2/(-1 + e*x)^2 + 2/(-1 + e*x)]*EllipticE[ArcSin[Sqrt[(1 - I) - (2*I)/(-1 +
 e*x)]/Sqrt[2]], 2])/(-1 + e*x)))/(e*Sqrt[1 + e*x]*Sqrt[2 + 2*(-1 + e*x) +
 (-1 + e*x)^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\operatorname{arcsinh}(ex))(d\operatorname{arccosh}(ex)+c)\,dx$$
$$\Big\downarrow \text{6272}$$

$$\int (a + b\text{arcsinh}(ex))(d\text{arccosh}(ex) + c)dx$$

input
```
Int[(c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int (c + d\ \text{arccosh}\,(ex))\,(a + b\ \text{arcsinh}\,(ex))\,dx$$

input
```
int((c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

output
```
int((c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.31

$$\int (c + d\text{arccosh}(ex))(a + b\text{arcsinh}(ex))\,dx = \frac{ace^3x^2 - \sqrt{e^2x^2-1}ade^2x + 2\,bdxE(\text{arcsin}\left(\frac{1}{ex}\right)\,|-1) - 2\,bdxF(\text{arcsin}\left(\frac{1}{ex}\right)\,|-1) + \left(bde^3x^2\log\left(ex + \surd\right.\right.}{}$$

input
```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="fricas")
```

output
```
(a*c*e^3*x^2 - sqrt(e^2*x^2 - 1)*a*d*e^2*x + 2*b*d*x*elliptic_e(arcsin(1/(
e*x)), -1) - 2*b*d*x*elliptic_f(arcsin(1/(e*x)), -1) + (b*d*e^3*x^2*log(e*
x + sqrt(e^2*x^2 - 1)) + b*c*e^3*x^2 - sqrt(e^2*x^2 - 1)*b*d*e^2*x)*log(e*
x + sqrt(e^2*x^2 + 1)) + (a*d*e^3*x^2 - sqrt(e^2*x^2 + 1)*b*d*e^2*x)*log(e
*x + sqrt(e^2*x^2 - 1)) - (b*c*e^2*x - 2*sqrt(e^2*x^2 - 1)*b*d*e)*sqrt(e^2
*x^2 + 1))/(e^3*x)
```

Sympy [F]

$$\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\int (a+b\operatorname{asinh}{(ex)})\,(c+d\operatorname{acosh}{(ex)})\,\,dx$$

input
```
integrate((c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output
```
Integral((a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Maxima [F]

$$\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\int (d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)\,dx$$

input
```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="maxima")
```

output
```
a*c*x + b*d*integrate(log(e*x + sqrt(e*x + 1)*sqrt(e*x - 1))*log(e*x + sqr
t(e^2*x^2 + 1)), x) + (e*x*arcsinh(e*x) - sqrt(e^2*x^2 + 1))*b*c/e + (e*x*
arccosh(e*x) - sqrt(e^2*x^2 - 1))*a*d/e
```

Giac [F]

$$\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\int (d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)\,dx$$

input
```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x)),x, algorithm="giac")
```

output
```
integrate((d*arccosh(e*x) + c)*(b*arcsinh(e*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\int (a+b\,\mathrm{asinh}(e\,x))\,\left(c+d\,\mathrm{acosh}(e\,x)\right)\,dx$$

input
```
int((a + b*asinh(e*x))*(c + d*acosh(e*x)),x)
```

output
```
int((a + b*asinh(e*x))*(c + d*acosh(e*x)), x)
```

Reduce [F]

$$\int (c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))\,dx=\int (c+dacosh(ex))\,(a+basinh(ex))\,dx$$

input
```
int((c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

output
```
int((c+d*acosh(e*x))*(a+b*asinh(e*x)),x)
```

3.5 $\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x}\,dx = \text{Int}\left(\frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x},x\right)$$

output

```
Defer(Int)((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x}\,dx = \int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x}\,dx$$

input

```
Integrate[((c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]))/x,x]
```

output
```
Integrate[((c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]))/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)}{x}dx$$

$$\Big\downarrow \text{ 6272}$$

$$\int \frac{(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)}{x}dx$$

input
```
Int[((c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]))/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+d\operatorname{arccosh}(ex))\,(a+b\operatorname{arcsinh}(ex))}{x}dx$$

input
```
int((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x,x)
```

output
```
int((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x}\,dx$$
$$=\int \frac{(d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)}{x}\,dx$$

input

```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x,x, algorithm="fricas")
```

output

```
integral((a*d*arccosh(e*x) + a*c + (b*d*arccosh(e*x) + b*c)*arcsinh(e*x))/
x, x)
```

Sympy [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x}\,dx=\int \frac{(a+b\operatorname{asinh}(ex))\,(c+d\operatorname{acosh}(ex))}{x}\,dx$$

input

```
integrate((c+d*acosh(e*x))*(a+b*asinh(e*x))/x,x)
```

output

```
Integral((a + b*asinh(e*x))*(c + d*acosh(e*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \frac{(c + d\text{arccosh}(ex))(a + b\text{arcsinh}(ex))}{x} \, dx = \int \frac{(d\,\text{arcosh}\,(ex) + c)(b\,\text{arsinh}\,(ex) + a)}{x} \, dx$$

input
```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x,x, algorithm="maxima")
```

output
```
a*c*log(x) + integrate(b*d*log(e*x + sqrt(e*x + 1)*sqrt(e*x - 1))*log(e*x
+ sqrt(e^2*x^2 + 1))/x + a*d*log(e*x + sqrt(e*x + 1)*sqrt(e*x - 1))/x + b*
c*log(e*x + sqrt(e^2*x^2 + 1))/x, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c + d\text{arccosh}(ex))(a + b\text{arcsinh}(ex))}{x} \, dx = \int \frac{(d\,\text{arcosh}\,(ex) + c)(b\,\text{arsinh}\,(ex) + a)}{x} \, dx$$

input
```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x,x, algorithm="giac")
```

output
```
integrate((d*arccosh(e*x) + c)*(b*arcsinh(e*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))}{x}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(e\,x)\right)\,\left(c+d\,\mathrm{acosh}(e\,x)\right)}{x}\,dx$$

input
```
int(((a + b*asinh(e*x))*(c + d*acosh(e*x)))/x,x)
```

output
```
int(((a + b*asinh(e*x))*(c + d*acosh(e*x)))/x, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))}{x}\,dx=\int \frac{\left(c+d\,acosh(ex)\right)\left(a+b\,asinh(ex)\right)}{x}dx$$

input
```
int((c+d*acosh(e*x))*(a+b*asinh(e*x))/x,x)
```

output
```
int((c+d*acosh(e*x))*(a+b*asinh(e*x))/x,x)
```

3.6 $\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx = \text{Int}\left(\frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2},x\right)$$

output
```
Defer(Int)((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx = \int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx$$

input
```
Integrate[((c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]))/x^2,x]
```

output
```
Integrate[((c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]))/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)}{x^2}dx$$
$$\Big\downarrow \text{6272}$$
$$\int \frac{(a+b\text{arcsinh}(ex))(d\text{arccosh}(ex)+c)}{x^2}dx$$

input
```
Int[((c + d*ArcCosh[e*x])*(a + b*ArcSinh[e*x]))/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+d\,\text{arccosh}\,(ex))\,(a+b\,\text{arcsinh}\,(ex))}{x^2}dx$$

input
```
int((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x^2,x)
```

output
```
int((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx = \int \frac{(d\,\text{arcosh}\,(ex)+c)(b\,\text{arsinh}\,(ex)+a)}{x^2}\,dx$$

input
```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*d*arccosh(e*x) + a*c + (b*d*arccosh(e*x) + b*c)*arcsinh(e*x))/
x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx = \int \frac{(a+b\,\text{asinh}\,(ex))\,(c+d\,\text{acosh}\,(ex))}{x^2}\,dx$$

input
```
integrate((c+d*acosh(e*x))*(a+b*asinh(e*x))/x**2,x)
```

output
```
Integral((a + b*asinh(e*x))*(c + d*acosh(e*x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx = \int \frac{(d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)}{x^2}\,dx$$

input

```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x^2,x, algorithm="maxima")
```

output

```
-(e*arcsinh(1/(sqrt(e^2)*abs(x))) + arcsinh(e*x)/x)*b*c - (e*arcsin(1/(sqr
t(e^2)*abs(x))) + arccosh(e*x)/x)*a*d + b*d*integrate(log(e*x + sqrt(e*x +
 1)*sqrt(e*x - 1))*log(e*x + sqrt(e^2*x^2 + 1))/x^2, x) - a*c/x
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\text{arccosh}(ex))(a+b\text{arcsinh}(ex))}{x^2}\,dx = \int \frac{(d\operatorname{arcosh}(ex)+c)(b\operatorname{arsinh}(ex)+a)}{x^2}\,dx$$

input

```
integrate((c+d*arccosh(e*x))*(a+b*arcsinh(e*x))/x^2,x, algorithm="giac")
```

output

```
integrate((d*arccosh(e*x) + c)*(b*arcsinh(e*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(e\,x)\right)\,\left(c+d\,\mathrm{acosh}(e\,x)\right)}{x^2}\,dx$$

input

```
int(((a + b*asinh(e*x))*(c + d*acosh(e*x)))/x^2,x)
```

output

```
int(((a + b*asinh(e*x))*(c + d*acosh(e*x)))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d\operatorname{arccosh}(ex))(a+b\operatorname{arcsinh}(ex))}{x^2}\,dx=\int \frac{\left(c+dacosh(ex)\right)\left(a+basinh(ex)\right)}{x^2}dx$$

input

```
int((c+d*acosh(e*x))*(a+b*asinh(e*x))/x^2,x)
```

output

```
int((c+d*acosh(e*x))*(a+b*asinh(e*x))/x^2,x)
```

3.7 $\int x^4 \textbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 72

$$\int x^4 \operatorname{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}}{5a^5} + \frac{2(1+a^2x^2)^{3/2}}{15a^5} - \frac{(1+a^2x^2)^{5/2}}{25a^5} + \frac{1}{5}x^5 \operatorname{arcsinh}(ax)$$

output

```
-1/5*(a^2*x^2+1)^(1/2)/a^5+2/15*(a^2*x^2+1)^(3/2)/a^5-1/25*(a^2*x^2+1)^(5/
2)/a^5+1/5*x^5*arcsinh(a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int x^4 \operatorname{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}(8-4a^2x^2+3a^4x^4)}{75a^5} + \frac{1}{5}x^5 \operatorname{arcsinh}(ax)$$

input

```
Integrate[x^4*ArcSinh[a*x],x]
```

output

```
-1/75*(Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4))/a^5 + (x^5*ArcSinh[a
*x])/5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6191, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{arcsinh}(ax)\,dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)-\frac{1}{5}a\int \frac{x^5}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)-\frac{1}{10}a\int \frac{x^4}{\sqrt{a^2x^2+1}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)-\frac{1}{10}a\int \left(\frac{\left(a^2x^2+1\right)^{3/2}}{a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}+\frac{1}{a^4\sqrt{a^2x^2+1}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)-\frac{1}{10}a\left(\frac{2\left(a^2x^2+1\right)^{5/2}}{5a^6}-\frac{4\left(a^2x^2+1\right)^{3/2}}{3a^6}+\frac{2\sqrt{a^2x^2+1}}{a^6}\right)$$

input
```
Int[x^4*ArcSinh[a*x],x]
```

output
```
-1/10*(a*((2*Sqrt[1 + a^2*x^2])/a^6 - (4*(1 + a^2*x^2)^(3/2))/(3*a^6) + (2
*(1 + a^2*x^2)^(5/2))/(5*a^6))) + (x^5*ArcSinh[a*x])/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{x^{5} a^{5} \operatorname{arcsinh}\left(x a\right)}{5}-\frac{x^{4} a^{4} \sqrt{a^{2} x^{2}+1}}{25}+\frac{4 x^{2} a^{2} \sqrt{a^{2} x^{2}+1}}{75}-\frac{8 \sqrt{a^{2} x^{2}+1}}{75}}{a^{5}}$	69
default	$\frac{\frac{x^{5} a^{5} \operatorname{arcsinh}\left(x a\right)}{5}-\frac{x^{4} a^{4} \sqrt{a^{2} x^{2}+1}}{25}+\frac{4 x^{2} a^{2} \sqrt{a^{2} x^{2}+1}}{75}-\frac{8 \sqrt{a^{2} x^{2}+1}}{75}}{a^{5}}$	69
parts	$\frac{x^{5} \operatorname{arcsinh}\left(x a\right)}{5}-\frac{a \left(\frac{x^{4} \sqrt{a^{2} x^{2}+1}}{5 a^{2}}-\frac{4 \left(\frac{x^{2} \sqrt{a^{2} x^{2}+1}}{3 a^{2}}-\frac{2 \sqrt{a^{2} x^{2}+1}}{3 a^{4}}\right)}{5 a^{2}}\right)}{5}$	75
orering	$\frac{\left(27 a^{6} x^{6}-4 a^{4} x^{4}+16 a^{2} x^{2}+32\right) \operatorname{arcsinh}\left(x a\right)}{75 a^{6} x}-\frac{\left(3 a^{4} x^{4}-4 a^{2} x^{2}+8\right) \left(a^{2} x^{2}+1\right) \left(4 x^{3} \operatorname{arcsinh}\left(x a\right)+\frac{x^{4} a}{\sqrt{a^{2} x^{2}+1}}\right)}{75 a^{6} x^{4}}$	101

input
```
int(x^4*arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*x^5*a^5*arcsinh(x*a)-1/25*x^4*a^4*(a^2*x^2+1)^(1/2)+4/75*x^2*a^
2*(a^2*x^2+1)^(1/2)-8/75*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int x^4\text{arcsinh}(ax)\,dx=\frac{15\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)-(3\,a^4x^4-4\,a^2x^2+8)\sqrt{a^2x^2+1}}{75\,a^5}$$

input
```
integrate(x^4*arcsinh(a*x),x, algorithm="fricas")
```

output
```
1/75*(15*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1)) - (3*a^4*x^4 - 4*a^2*x^2 + 8
)*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int x^4\text{arcsinh}(ax)\,dx=\begin{cases}\frac{x^5\,\text{asinh}\,(ax)}{5}-\frac{x^4\sqrt{a^2x^2+1}}{25a}+\frac{4x^2\sqrt{a^2x^2+1}}{75a^3}-\frac{8\sqrt{a^2x^2+1}}{75a^5} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*asinh(a*x),x)
```

output
```
Piecewise((x**5*asinh(a*x)/5 - x**4*sqrt(a**2*x**2 + 1)/(25*a) + 4*x**2*sq
rt(a**2*x**2 + 1)/(75*a**3) - 8*sqrt(a**2*x**2 + 1)/(75*a**5), Ne(a, 0)),
(0, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x^4 \text{arcsinh}(ax)\,dx = \frac{1}{5}\,x^5\,\text{arsinh}\,(ax) - \frac{1}{75}\left(\frac{3\,\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\,\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a$$

input

```
integrate(x^4*arcsinh(a*x),x, algorithm="maxima")
```

output

```
1/5*x^5*arcsinh(a*x) - 1/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
+ 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^4*arcsinh(a*x),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)\,dx = \int x^4\,\text{asinh}(a\,x)\,dx$$

input

```
int(x^4*asinh(a*x),x)
```

output
```
int(x^4*asinh(a*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int x^4 \text{arcsinh}(ax)\,dx = \frac{15 asinh(ax)\,a^5x^5 - 3\sqrt{a^2x^2+1}\,a^4x^4 + 4\sqrt{a^2x^2+1}\,a^2x^2 - 8\sqrt{a^2x^2+1}}{75a^5}$$

input
```
int(x^4*asinh(a*x),x)
```

output
```
(15*asinh(a*x)*a**5*x**5 - 3*sqrt(a**2*x**2 + 1)*a**4*x**4 + 4*sqrt(a**2*x
**2 + 1)*a**2*x**2 - 8*sqrt(a**2*x**2 + 1))/(75*a**5)
```

3.8 $\int x^3 \mathbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int x^3\text{arcsinh}(ax)\,dx=\frac{3x\sqrt{1+a^2x^2}}{32a^3}-\frac{x^3\sqrt{1+a^2x^2}}{16a}-\frac{3\text{arcsinh}(ax)}{32a^4}+\frac{1}{4}x^4\text{arcsinh}(ax)$$

output

```
3/32*x*(a^2*x^2+1)^(1/2)/a^3-1/16*x^3*(a^2*x^2+1)^(1/2)/a-3/32*arcsinh(a*x
)/a^4+1/4*x^4*arcsinh(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x^3\text{arcsinh}(ax)\,dx=\frac{ax(3-2a^2x^2)\sqrt{1+a^2x^2}+(-3+8a^4x^4)\,\text{arcsinh}(ax)}{32a^4}$$

input

```
Integrate[x^3*ArcSinh[a*x],x]
```

output

```
(a*x*(3 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + (-3 + 8*a^4*x^4)*ArcSinh[a*x])/(3
2*a^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6191, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}(ax) \, dx$$

$$\Big\downarrow \; 6191$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\int \frac{x^4}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\int \frac{x^2}{\sqrt{a^2x^2+1}}dx}{4a^2}\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{4a^2}\right)$$

$$\Big\downarrow \; 222$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{4a^2}\right)$$

input

```
Int[x^3*ArcSinh[a*x],x]
```

output

```
(x^4*ArcSinh[a*x])/4 - (a*((x^3*Sqrt[1 + a^2*x^2])/(4*a^2) - (3*((x*Sqrt[1
 + a^2*x^2])/(2*a^2) - ArcSinh[a*x]/(2*a^3)))/(4*a^2)))/4
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{x^4 a^4 \operatorname{arcsinh}(xa)}{4}-\frac{x^3 a^3 \sqrt{a^2x^2+1}}{16}+\frac{3xa\sqrt{a^2x^2+1}}{32}-\frac{3 \operatorname{arcsinh}(xa)}{32}}{a^4}$	58
default	$\frac{\frac{x^4 a^4 \operatorname{arcsinh}(xa)}{4}-\frac{x^3 a^3 \sqrt{a^2x^2+1}}{16}+\frac{3xa\sqrt{a^2x^2+1}}{32}-\frac{3 \operatorname{arcsinh}(xa)}{32}}{a^4}$	58
orering	$\frac{(14a^4x^4-3a^2x^2-12) \operatorname{arcsinh}(xa)}{32a^4}-\frac{(2a^2x^2-3)(a^2x^2+1)\left(3x^2 \operatorname{arcsinh}(xa)+\frac{x^3 a}{\sqrt{a^2x^2+1}}\right)}{32x^2a^4}$	82
parts	$\frac{x^4 \operatorname{arcsinh}(xa)}{4}-\frac{a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{4a^2}\right)}{4}$	90

input
```
int(x^3*arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*x^4*a^4*arcsinh(x*a)-1/16*x^3*a^3*(a^2*x^2+1)^(1/2)+3/32*x*a*(a
^2*x^2+1)^(1/2)-3/32*arcsinh(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{arcsinh}(ax)\,dx=\frac{(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)-(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}}{32\,a^4}$$

input
```
integrate(x^3*arcsinh(a*x),x, algorithm="fricas")
```

output
```
1/32*((8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1)) - (2*a^3*x^3 - 3*a*x)*s
qrt(a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int x^3 \operatorname{arcsinh}(ax)\,dx=\begin{cases}\frac{x^4\operatorname{asinh}(ax)}{4}-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}-\frac{3\operatorname{asinh}(ax)}{32a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*asinh(a*x),x)
```

output
```
Piecewise((x**4*asinh(a*x)/4 - x**3*sqrt(a**2*x**2 + 1)/(16*a) + 3*x*sqrt(
a**2*x**2 + 1)/(32*a**3) - 3*asinh(a*x)/(32*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^3 \text{arcsinh}(ax)\,dx = \frac{1}{4}\,x^4 \operatorname{arsinh}(ax) - \frac{1}{32}\left(\frac{2\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\sqrt{a^2x^2+1}x}{a^4} + \frac{3\operatorname{arsinh}(ax)}{a^5}\right)a$$

input

```
integrate(x^3*arcsinh(a*x),x, algorithm="maxima")
```

output

```
1/4*x^4*arcsinh(a*x) - 1/32*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2
+ 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3*arcsinh(a*x),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(ax)\,dx = \int x^3\,\text{asinh}(a\,x)\;dx$$

input

```
int(x^3*asinh(a*x),x)
```

output
```
int(x^3*asinh(a*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x^3\text{arcsinh}(ax)\,dx$$
$$= \frac{8asinh(ax)\,a^4x^4 - 2\sqrt{a^2x^2+1}\,a^3x^3 + 3\sqrt{a^2x^2+1}\,ax - 3\log\left(\sqrt{a^2x^2+1}+ax\right)}{32a^4}$$

input
```
int(x^3*asinh(a*x),x)
```

output
```
(8*asinh(a*x)*a**4*x**4 - 2*sqrt(a**2*x**2 + 1)*a**3*x**3 + 3*sqrt(a**2*x*
*2 + 1)*a*x - 3*log(sqrt(a**2*x**2 + 1) + a*x))/(32*a**4)
```

3.9 $\int x^2 \textbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 52

$$\int x^2\operatorname{arcsinh}(ax)\,dx=\frac{\sqrt{1+a^2x^2}}{3a^3}-\frac{(1+a^2x^2)^{3/2}}{9a^3}+\frac{1}{3}x^3\operatorname{arcsinh}(ax)$$

output

```
1/3*(a^2*x^2+1)^(1/2)/a^3-1/9*(a^2*x^2+1)^(3/2)/a^3+1/3*x^3*arcsinh(a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int x^2\operatorname{arcsinh}(ax)\,dx=\frac{1}{9}\left(\frac{(2-a^2x^2)\sqrt{1+a^2x^2}}{a^3}+3x^3\operatorname{arcsinh}(ax)\right)$$

input

```
Integrate[x^2*ArcSinh[a*x],x]
```

output

```
(((2 - a^2*x^2)*Sqrt[1 + a^2*x^2])/a^3 + 3*x^3*ArcSinh[a*x])/9
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6191, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}(ax)\,dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{3}a\int \frac{x^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\int \frac{x^2}{\sqrt{a^2x^2+1}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\int \left(\frac{\sqrt{a^2x^2+1}}{a^2}-\frac{1}{a^2\sqrt{a^2x^2+1}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2\left(a^2x^2+1\right)^{3/2}}{3a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)$$

input

```
Int[x^2*ArcSinh[a*x],x]
```

output

```
-1/6*(a*((-2*Sqrt[1 + a^2*x^2])/a^4 + (2*(1 + a^2*x^2)^(3/2))/(3*a^4))) +
(x^3*ArcSinh[a*x])/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{x^{3} a^{3} \operatorname{arcsinh}(x a)}{3}-\frac{x^{2} a^{2} \sqrt{a^{2} x^{2}+1}}{9}+\frac{2 \sqrt{a^{2} x^{2}+1}}{9}}{a^{3}}$	50
default	$\frac{\frac{x^{3} a^{3} \operatorname{arcsinh}(x a)}{3}-\frac{x^{2} a^{2} \sqrt{a^{2} x^{2}+1}}{9}+\frac{2 \sqrt{a^{2} x^{2}+1}}{9}}{a^{3}}$	50
parts	$\frac{x^{3} \operatorname{arcsinh}(x a)}{3}-\frac{a \left(\frac{x^{2} \sqrt{a^{2} x^{2}+1}}{3 a^{2}}-\frac{2 \sqrt{a^{2} x^{2}+1}}{3 a^{4}}\right)}{3}$	50
orering	$\frac{\left(5 a^{4} x^{4}-2 a^{2} x^{2}-4\right) \operatorname{arcsinh}(x a)}{9 a^{4} x}-\frac{\left(a^{2} x^{2}-2\right) \left(a^{2} x^{2}+1\right) \left(2 x \operatorname{arcsinh}(x a)+\frac{x^{2} a}{\sqrt{a^{2} x^{2}+1}}\right)}{9 a^{4} x^{2}}$	82

input
```
int(x^2*arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output

```
1/a^3*(1/3*x^3*a^3*arcsinh(x*a)-1/9*x^2*a^2*(a^2*x^2+1)^(1/2)+2/9*(a^2*x^2
+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{arcsinh}(ax)\,dx = \frac{3\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}(a^2x^2-2)}{9\,a^3}$$

input

```
integrate(x^2*arcsinh(a*x),x, algorithm="fricas")
```

output

```
1/9*(3*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*(a^2*x^2 -
 2))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^2 \operatorname{arcsinh}(ax)\,dx = \begin{cases} \frac{x^3\operatorname{asinh}(ax)}{3}-\frac{x^2\sqrt{a^2x^2+1}}{9a}+\frac{2\sqrt{a^2x^2+1}}{9a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input

```
integrate(x**2*asinh(a*x),x)
```

output

```
Piecewise((x**3*asinh(a*x)/3 - x**2*sqrt(a**2*x**2 + 1)/(9*a) + 2*sqrt(a**
2*x**2 + 1)/(9*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^2\text{arcsinh}(ax)\,dx = \frac{1}{3}\,x^3\operatorname{arsinh}\left(ax\right) - \frac{1}{9}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)$$

input

```
integrate(x^2*arcsinh(a*x),x, algorithm="maxima")
```

output

```
1/3*x^3*arcsinh(a*x) - 1/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 +
 1)/a^4)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*arcsinh(a*x),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)\,dx = \int x^2\,\text{asinh}(a\,x)\,dx$$

input

```
int(x^2*asinh(a*x),x)
```

output

```
int(x^2*asinh(a*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^2 \mathrm{arcsinh}(ax)\, dx = \frac{3asinh(ax)\, a^3x^3 - \sqrt{a^2x^2+1}\, a^2x^2 + 2\sqrt{a^2x^2+1}}{9a^3}$$

input
```
int(x^2*asinh(a*x),x)
```

output
```
(3*asinh(a*x)*a**3*x**3 - sqrt(a**2*x**2 + 1)*a**2*x**2 + 2*sqrt(a**2*x**2
 + 1))/(9*a**3)
```

3.10 $\int x\textbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 44

$$\int x\text{arcsinh}(ax)\,dx=-\frac{x\sqrt{1+a^2x^2}}{4a}+\frac{\text{arcsinh}(ax)}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)$$

output

```
-1/4*x*(a^2*x^2+1)^(1/2)/a+1/4*arcsinh(a*x)/a^2+1/2*x^2*arcsinh(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x\text{arcsinh}(ax)\,dx=\frac{-ax\sqrt{1+a^2x^2}+(1+2a^2x^2)\,\text{arcsinh}(ax)}{4a^2}$$

input

```
Integrate[x*ArcSinh[a*x],x]
```

output

```
(-(a*x*Sqrt[1 + a^2*x^2]) + (1 + 2*a^2*x^2)*ArcSinh[a*x])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6191, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)\,dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\int \frac{x^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)$$

input

```
Int[x*ArcSinh[a*x],x]
```

output

```
(x^2*ArcSinh[a*x])/2 - (a*((x*Sqrt[1 + a^2*x^2])/(2*a^2) - ArcSinh[a*x]/(2
*a^3)))/2
```

Defintions of rubi rules used

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{x^2 a^2 \operatorname{arcsinh}(xa)}{2}-\frac{xa\sqrt{a^2x^2+1}}{4}+\frac{\operatorname{arcsinh}(xa)}{4}}{a^2}$	39
default	$\frac{\frac{x^2 a^2 \operatorname{arcsinh}(xa)}{2}-\frac{xa\sqrt{a^2x^2+1}}{4}+\frac{\operatorname{arcsinh}(xa)}{4}}{a^2}$	39
orering	$\frac{\left(3a^2x^2+2\right)\operatorname{arcsinh}(xa)}{4a^2}-\frac{\left(a^2x^2+1\right)\left(\operatorname{arcsinh}(xa)+\frac{xa}{\sqrt{a^2x^2+1}}\right)}{4a^2}$	54
parts	$\frac{x^2\operatorname{arcsinh}(xa)}{2}-\frac{a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{2}$	65

input

```
int(x*arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output

```
1/a^2*(1/2*x^2*a^2*arcsinh(x*a)-1/4*x*a*(a^2*x^2+1)^(1/2)+1/4*arcsinh(x*a)
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int x\text{arcsinh}(ax)\,dx = -\frac{\sqrt{a^2x^2+1}ax - (2\,a^2x^2+1)\log\left(ax+\sqrt{a^2x^2+1}\right)}{4\,a^2}$$

input

```
integrate(x*arcsinh(a*x),x, algorithm="fricas")
```

output

```
-1/4*(sqrt(a^2*x^2 + 1)*a*x - (2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x\text{arcsinh}(ax)\,dx = \begin{cases} \frac{x^2\operatorname{asinh}(ax)}{2} - \frac{x\sqrt{a^2x^2+1}}{4a} + \frac{\operatorname{asinh}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input

```
integrate(x*asinh(a*x),x)
```

output

```
Piecewise((x**2*asinh(a*x)/2 - x*sqrt(a**2*x**2 + 1)/(4*a) + asinh(a*x)/(4
*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x\text{arcsinh}(ax)\,dx = \frac{1}{2}\,x^2\operatorname{arsinh}(ax) - \frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\operatorname{arsinh}(ax)}{a^3}\right)$$

input

```
integrate(x*arcsinh(a*x),x, algorithm="maxima")
```

output

```
1/2*x^2*arcsinh(a*x) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int x\text{arcsinh}(ax)\,dx = \frac{1}{2}\,x^2\log\left(ax+\sqrt{a^2x^2+1}\right) - \frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2}+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{a^2|a|}\right)$$

input
```
integrate(x*arcsinh(a*x),x, algorithm="giac")
```

output
```
1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1)) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 + lo
g(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^2*abs(a)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x\text{arcsinh}(ax)\,dx = x\,\text{asinh}(a\,x)\,\left(\frac{x}{2}+\frac{1}{4\,a^2\,x}\right)-\frac{x\,\sqrt{a^2\,x^2+1}}{4\,a}$$

input
```
int(x*asinh(a*x),x)
```

output
```
x*asinh(a*x)*(x/2 + 1/(4*a^2*x)) - (x*(a^2*x^2 + 1)^(1/2))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int x\text{arcsinh}(ax)\,dx = \frac{2asinh(ax)\,a^2x^2-\sqrt{a^2x^2+1}\,ax+\log\left(\sqrt{a^2x^2+1}+ax\right)}{4a^2}$$

input
```
int(x*asinh(a*x),x)
```

output

```
(2*asinh(a*x)*a**2*x**2 - sqrt(a**2*x**2 + 1)*a*x + log(sqrt(a**2*x**2 + 1
) + a*x))/(4*a**2)
```

3.11 $\int \mathbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \operatorname{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}}{a} + x\operatorname{arcsinh}(ax)$$

output

```
-(a^2*x^2+1)^(1/2)/a+x*arcsinh(a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}}{a} + x\operatorname{arcsinh}(ax)$$

input

```
Integrate[ArcSinh[a*x],x]
```

output

```
-(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)\,dx$$

$$\Big\downarrow \text{ 6187}$$

$$x\operatorname{arcsinh}(ax)-a\int \frac{x}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 241}$$

$$x\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}$$

input
```
Int[ArcSinh[a*x],x]
```

output
```
-(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{\sqrt{a^2x^2+1}}{a}+x\,\operatorname{arcsinh}(xa)$	24
orering	$-\frac{\sqrt{a^2x^2+1}}{a}+x\,\operatorname{arcsinh}(xa)$	24
derivativedivides	$\frac{xa\,\operatorname{arcsinh}(xa)-\sqrt{a^2x^2+1}}{a}$	26
default	$\frac{xa\,\operatorname{arcsinh}(xa)-\sqrt{a^2x^2+1}}{a}$	26

input

```
int(arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output

```
-(a^2*x^2+1)^(1/2)/a+x*arcsinh(x*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \operatorname{arcsinh}(ax)\,dx = \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}}{a}$$

input

```
integrate(arcsinh(a*x),x, algorithm="fricas")
```

output

```
(a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \operatorname{arcsinh}(ax)\,dx = \begin{cases} x\operatorname{asinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input

```
integrate(asinh(a*x),x)
```

output
```
Piecewise((x*asinh(a*x) - sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)\,dx = \frac{ax\operatorname{arsinh}(ax) - \sqrt{a^2x^2+1}}{a}$$

input
```
integrate(arcsinh(a*x),x, algorithm="maxima")
```

output
```
(a*x*arcsinh(a*x) - sqrt(a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \operatorname{arcsinh}(ax)\,dx = x\log\left(ax + \sqrt{a^2x^2+1}\right) - \frac{\sqrt{a^2x^2+1}}{a}$$

input
```
integrate(arcsinh(a*x),x, algorithm="giac")
```

output
```
x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \operatorname{arcsinh}(ax)\,dx = x\operatorname{asinh}(a\,x) - \frac{\sqrt{a^2\,x^2+1}}{a}$$

input
```
int(asinh(a*x),x)
```

output
```
x*asinh(a*x) - (a^2*x^2 + 1)^(1/2)/a
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \operatorname{arcsinh}(ax)\,dx = \frac{asinh(ax)\,ax - \sqrt{a^2x^2+1}}{a}$$

input

```
int(asinh(a*x),x)
```

output

```
(asinh(a*x)*a*x - sqrt(a**2*x**2 + 1))/a
```

3.12 $\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1 - e^{2\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right)$$

output
```
-1/2*arcsinh(a*x)^2+arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+1/2*polyl
og(2,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1 - e^{2\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right)$$

input
```
Integrate[ArcSinh[a*x]/x,x]
```

output
```
-1/2*ArcSinh[a*x]^2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2
, E^(2*ArcSinh[a*x])]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)}{x} \, dx$$

$$\Big\downarrow \text{ 6190}$$

$$\int \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{ax} d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\operatorname{arcsinh}(ax)\tan\left(\frac{\pi}{2}+i\operatorname{arcsinh}(ax)\right)d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \operatorname{arcsinh}(ax)\tan\left(i\operatorname{arcsinh}(ax)+\frac{\pi}{2}\right)d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{ 4199}$$

$$-i\left(2i\int -\frac{e^{2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax)}{1-e^{2\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)-\frac{1}{2}i\operatorname{arcsinh}(ax)^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(-2i\int \frac{e^{2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax)}{1-e^{2\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)-\frac{1}{2}i\operatorname{arcsinh}(ax)^2\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)$$

$$\Big\downarrow \textcolor{blue}{2715}$$

$$-i\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\log\left(1-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{2}i\text{arcsi}$$

$$\Big\downarrow \textcolor{blue}{2838}$$

$$-i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)$$

input
```
Int[ArcSinh[a*x]/x,x]
```

output
```
(-I)*((-1/2*I)*ArcSinh[a*x]^2 - (2*I)*(-1/2*(ArcSinh[a*x]*Log[1 - E^(2*Arc
Sinh[a*x])]) - PolyLog[2, E^(2*ArcSinh[a*x])]/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(xa)^2}{2}+\operatorname{arcsinh}(xa)\ln\left(1+xa+\sqrt{a^2x^2+1}\right)+\operatorname{polylog}\left(2,-xa-\sqrt{a^2x^2+1}\right)$
default	$-\frac{\operatorname{arcsinh}(xa)^2}{2}+\operatorname{arcsinh}(xa)\ln\left(1+xa+\sqrt{a^2x^2+1}\right)+\operatorname{polylog}\left(2,-xa-\sqrt{a^2x^2+1}\right)$

input

```
int(arcsinh(x*a)/x,x,method=_RETURNVERBOSE)
```

output

```
-1/2*arcsinh(x*a)^2+arcsinh(x*a)*ln(1+x*a+(a^2*x^2+1)^(1/2))+polylog(2,-x*
a-(a^2*x^2+1)^(1/2))+arcsinh(x*a)*ln(1-x*a-(a^2*x^2+1)^(1/2))+polylog(2,x*
a+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x} \, dx = \int \frac{\operatorname{arsinh}(ax)}{x} \, dx$$

input
```
integrate(arcsinh(a*x)/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x} \, dx = \int \frac{\operatorname{asinh}(ax)}{x} \, dx$$

input
```
integrate(asinh(a*x)/x,x)
```

output
```
Integral(asinh(a*x)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x} \, dx = \int \frac{\operatorname{arsinh}(ax)}{x} \, dx$$

input
```
integrate(arcsinh(a*x)/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)}{x}\,dx$$

input
```
integrate(arcsinh(a*x)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x}\,dx$$

input
```
int(asinh(a*x)/x,x)
```

output
```
int(asinh(a*x)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{asinh(ax)}{x}dx$$

input
```
int(asinh(a*x)/x,x)
```

output
```
int(asinh(a*x)/x,x)
```

3.13 $\int \frac{\text{arcsinh}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\text{arcsinh}(ax)}{x^2}\,dx = -\frac{\text{arcsinh}(ax)}{x} - a\,\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-arcsinh(a*x)/x-a*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)}{x^2}\,dx = -\frac{\text{arcsinh}(ax)}{x} - a\,\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[ArcSinh[a*x]/x^2,x]
```

output
```
-(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{6191, 243, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^2}dx$$

$$\Big\downarrow \text{6191}$$

$$a\int \frac{1}{x\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{\text{arcsinh}(ax)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}-\frac{\text{arcsinh}(ax)}{x}$$

$$\Big\downarrow \text{221}$$

$$-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}$$

input
```
Int[ArcSinh[a*x]/x^2,x]
```

output
```
-(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{\operatorname{arcsinh}(xa)}{x}-a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)$	26
derivativedivides	$a\left(-\frac{\operatorname{arcsinh}(xa)}{xa}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)$	30
default	$a\left(-\frac{\operatorname{arcsinh}(xa)}{xa}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)$	30

input
```
int(arcsinh(x*a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(x*a)/x-a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.33

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -\frac{ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right) - (x-1)\log\left(ax+\sqrt{a^2x^2+1}\right) - x}{x}$$

input

```
integrate(arcsinh(a*x)/x^2,x, algorithm="fricas")
```

output

```
-(a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a*x*log(-a*x + sqrt(a^2*x^2 + 1)
 - 1) - (x - 1)*log(a*x + sqrt(a^2*x^2 + 1)) - x*log(-a*x + sqrt(a^2*x^2 +
 1)))/x
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^2}\,dx$$

input

```
integrate(asinh(a*x)/x**2,x)
```

output

```
Integral(asinh(a*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -a\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\operatorname{arsinh}(ax)}{x}$$

input

```
integrate(arcsinh(a*x)/x^2,x, algorithm="maxima")
```

output
```
-a*arcsinh(1/(a*abs(x))) - arcsinh(a*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = & -\frac{1}{2}a\Big(\log\left(\sqrt{a^2x^2+1}+1\right)-\log\left(\sqrt{a^2x^2+1}-1\right)\Big)\\ & -\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}\end{aligned}$$

input
```
integrate(arcsinh(a*x)/x^2,x, algorithm="giac")
```

output
```
-1/2*a*(log(sqrt(a^2*x^2 + 1) + 1) - log(sqrt(a^2*x^2 + 1) - 1)) - log(a*x
 + sqrt(a^2*x^2 + 1))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^2}\,dx$$

input
```
int(asinh(a*x)/x^2,x)
```

output
```
int(asinh(a*x)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\text{arcsinh}(ax)}{x^2}\,dx = \frac{-asinh(ax) + \log\left(\sqrt{a^2x^2+1}+ax-1\right)ax - \log\left(\sqrt{a^2x^2+1}+ax+1\right)ax}{x}$$

input

```
int(asinh(a*x)/x^2,x)
```

output

```
( - asinh(a*x) + log(sqrt(a**2*x**2 + 1) + a*x - 1)*a*x - log(sqrt(a**2*x*
*2 + 1) + a*x + 1)*a*x)/x
```

3.14 $\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{a\sqrt{1+a^2x^2}}{2x} - \frac{\operatorname{arcsinh}(ax)}{2x^2}$$

output
```
-1/2*a*(a^2*x^2+1)^(1/2)/x-1/2*arcsinh(a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{ax\sqrt{1+a^2x^2}+\operatorname{arcsinh}(ax)}{2x^2}$$

input
```
Integrate[ArcSinh[a*x]/x^3,x]
```

output
```
-1/2*(a*x*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6191, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^3} dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)}{2x^2}$$

$$\Big\downarrow \text{242}$$

$$-\frac{a\sqrt{a^2x^2+1}}{2x}-\frac{\text{arcsinh}(ax)}{2x^2}$$

input

```
Int[ArcSinh[a*x]/x^3,x]
```

output

```
-1/2*(a*Sqrt[1 + a^2*x^2])/x - ArcSinh[a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{a\sqrt{a^{2}x^{2}+1}}{2x}-\frac{\operatorname{arcsinh}(xa)}{2x^{2}}$	28
derivativedivides	$a^{2}\left(-\frac{\operatorname{arcsinh}(xa)}{2x^{2}a^{2}}-\frac{\sqrt{a^{2}x^{2}+1}}{2xa}\right)$	37
default	$a^{2}\left(-\frac{\operatorname{arcsinh}(xa)}{2x^{2}a^{2}}-\frac{\sqrt{a^{2}x^{2}+1}}{2xa}\right)$	37
orering	$\frac{\left(-\frac{3}{2}a^{2}x^{3}-2x\right)\operatorname{arcsinh}(xa)}{x^{3}}-\frac{\left(a^{2}x^{2}+1\right)x^{2}\left(\frac{a}{\sqrt{a^{2}x^{2}+1}\,x^{3}}-\frac{3\operatorname{arcsinh}(xa)}{x^{4}}\right)}{2}$	62

input
```
int(arcsinh(x*a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(a^2*x^2+1)^(1/2)/x-1/2*arcsinh(x*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)}{x^{3}}\,dx=-\frac{\sqrt{a^{2}x^{2}+1}ax+\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}{2\,x^{2}}$$

input
```
integrate(arcsinh(a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(a^2*x^2 + 1)*a*x + log(a*x + sqrt(a^2*x^2 + 1)))/x^2
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^3}\,dx$$

input
```
integrate(asinh(a*x)/x**3,x)
```

output
```
Integral(asinh(a*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{\sqrt{a^2x^2+1}a}{2\,x} - \frac{\operatorname{arsinh}(ax)}{2\,x^2}$$

input
```
integrate(arcsinh(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*sqrt(a^2*x^2 + 1)*a/x - 1/2*arcsinh(a*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \frac{a|a|}{\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1} - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(a*x)/x^3,x, algorithm="giac")
```

output
```
a*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1) - 1/2*log(a*x + sqrt(a^2*x
^2 + 1))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \int \frac{\mathrm{asinh}(a\,x)}{x^3}\,dx$$

input

```
int(asinh(a*x)/x^3,x)
```

output

```
int(asinh(a*x)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \frac{-asinh(ax) - \sqrt{a^2x^2+1}\,ax - a^2x^2}{2x^2}$$

input

```
int(asinh(a*x)/x^3,x)
```

output

```
( - (asinh(a*x) + sqrt(a**2*x**2 + 1)*a*x + a**2*x**2))/(2*x**2)
```

3.15 $\int \frac{\operatorname{arcsinh}(ax)}{x^4} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4} \, dx = -\frac{a\sqrt{1+a^2x^2}}{6x^2} - \frac{\operatorname{arcsinh}(ax)}{3x^3} + \frac{1}{6}a^3 \operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output

```
-1/6*a*(a^2*x^2+1)^(1/2)/x^2-1/3*arcsinh(a*x)/x^3+1/6*a^3*arctanh((a^2*x^2
+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4} \, dx = -\frac{a\sqrt{1+a^2x^2}}{6x^2} - \frac{\operatorname{arcsinh}(ax)}{3x^3} + \frac{1}{6}a^3 \operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

input

```
Integrate[ArcSinh[a*x]/x^4,x]
```

output

```
-1/6*(a*Sqrt[1 + a^2*x^2])/x^2 - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[
1 + a^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6191, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^4}dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{3}a\int \frac{1}{x^3\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}a\int \frac{1}{x^4\sqrt{a^2x^2+1}}dx^2-\frac{\text{arcsinh}(ax)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}a\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{\sqrt{a^2x^2+1}}{x^2}\right)-\frac{\text{arcsinh}(ax)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}a\left(-\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}-\frac{\sqrt{a^2x^2+1}}{x^2}\right)-\frac{\text{arcsinh}(ax)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{x^2}\right)-\frac{\text{arcsinh}(ax)}{3x^3}$$

input
```
Int[ArcSinh[a*x]/x^4,x]
```

output
```
-1/3*ArcSinh[a*x]/x^3 + (a*(-(Sqrt[1 + a^2*x^2]/x^2) + a^2*ArcTanh[Sqrt[1
+ a^2*x^2]]))/6
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\operatorname{arcsinh}(xa)}{3x^{3}}+\frac{a\left(-\frac{\sqrt{a^{2}x^{2}+1}}{2x^{2}}+\frac{a^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{2}\right)}{3}$	48
derivativedivides	$a^{3}\left(-\frac{\operatorname{arcsinh}(xa)}{3x^{3}a^{3}}-\frac{\sqrt{a^{2}x^{2}+1}}{6x^{2}a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{6}\right)$	51
default	$a^{3}\left(-\frac{\operatorname{arcsinh}(xa)}{3x^{3}a^{3}}-\frac{\sqrt{a^{2}x^{2}+1}}{6x^{2}a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{6}\right)$	51

input

```
int(arcsinh(x*a)/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*arcsinh(x*a)/x^3+1/3*a*(-1/2/x^2*(a^2*x^2+1)^(1/2)+1/2*a^2*arctanh(1/
(a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(44) = 88$.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{arcsinh}(ax)}{x^{4}}\,dx$$

$$=\frac{a^{3}x^{3}\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)-a^{3}x^{3}\log\left(-ax+\sqrt{a^{2}x^{2}+1}-1\right)+2\,x^{3}\log\left(-ax+\sqrt{a^{2}x^{2}+1}\right)-}{6\,x^{3}}$$

input

```
integrate(arcsinh(a*x)/x^4,x, algorithm="fricas")
```

output

```
1/6*(a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^3*x^3*log(-a*x + sqrt(a
^2*x^2 + 1) - 1) + 2*x^3*log(-a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)
*a*x + 2*(x^3 - 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x^3
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx=\int \frac{\operatorname{asinh}{\left(ax \right)}}{x^{4}}\,dx$$

input

```
integrate(asinh(a*x)/x**4,x)
```

output

```
Integral(asinh(a*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx=\frac{1}{6}\left(a^{2} \operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{\sqrt{a^{2} x^{2}+1}}{x^{2}}\right)a-\frac{\operatorname{arsinh}\left(ax\right)}{3\,x^{3}}$$

input

```
integrate(arcsinh(a*x)/x^4,x, algorithm="maxima")
```

output

```
1/6*(a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a - 1/3*arcsinh(a*
x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\begin{aligned}&\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx\\&=-\frac{1}{12}\,a^{3}\left(\frac{2\,\sqrt{a^{2} x^{2}+1}}{a^{2} x^{2}}-\log\left(\sqrt{a^{2} x^{2}+1}+1\right)+\log\left(\sqrt{a^{2} x^{2}+1}-1\right)\right)\\&\quad-\frac{\log\left(ax+\sqrt{a^{2} x^{2}+1}\right)}{3\,x^{3}}\end{aligned}$$

input

```
integrate(arcsinh(a*x)/x^4,x, algorithm="giac")
```

output
```
-1/12*a^3*(2*sqrt(a^2*x^2 + 1)/(a^2*x^2) - log(sqrt(a^2*x^2 + 1) + 1) + lo
g(sqrt(a^2*x^2 + 1) - 1)) - 1/3*log(a*x + sqrt(a^2*x^2 + 1))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^4}\,dx$$

input
```
int(asinh(a*x)/x^4,x)
```

output
```
int(asinh(a*x)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx$$
$$= \frac{-2asinh(ax) - \sqrt{a^2x^2+1}\,ax - \log\left(\sqrt{a^2x^2+1}+ax-1\right)a^3x^3 + \log\left(\sqrt{a^2x^2+1}+ax+1\right)a^3x^3}{6x^3}$$

input
```
int(asinh(a*x)/x^4,x)
```

output
```
( - 2*asinh(a*x) - sqrt(a**2*x**2 + 1)*a*x - log(sqrt(a**2*x**2 + 1) + a*x
 - 1)*a**3*x**3 + log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**3*x**3)/(6*x**3)
```

3.16 $\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = -\frac{a\sqrt{1+a^2x^2}}{12x^3} + \frac{a^3\sqrt{1+a^2x^2}}{6x} - \frac{\operatorname{arcsinh}(ax)}{4x^4}$$

output
```
-1/12*a*(a^2*x^2+1)^(1/2)/x^3+1/6*a^3*(a^2*x^2+1)^(1/2)/x-1/4*arcsinh(a*x)
/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{ax\sqrt{1+a^2x^2}(-1+2a^2x^2) - 3\operatorname{arcsinh}(ax)}{12x^4}$$

input
```
Integrate[ArcSinh[a*x]/x^5,x]
```

output
```
(a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2) - 3*ArcSinh[a*x])/(12*x^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6191, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)}{x^5} \, dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{4}a\int \frac{1}{x^4\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)}{4x^4}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{4}a\left(-\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}}{3x^3}\right)-\frac{\text{arcsinh}(ax)}{4x^4}$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{4}a\left(\frac{2a^2\sqrt{a^2x^2+1}}{3x}-\frac{\sqrt{a^2x^2+1}}{3x^3}\right)-\frac{\text{arcsinh}(ax)}{4x^4}$$

input

```
Int[ArcSinh[a*x]/x^5,x]
```

output

```
(a*(-1/3*Sqrt[1 + a^2*x^2]/x^3 + (2*a^2*Sqrt[1 + a^2*x^2])/(3*x)))/4 - Arc
Sinh[a*x]/(4*x^4)
```

Defintions of rubi rules used

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\operatorname{arcsinh}\left(x a \right)}{4 x^{4}}+\frac{a \left(-\frac{\sqrt{a^{2} x^{2}+1}}{3 x^{3}}+\frac{2 a^{2} \sqrt{a^{2} x^{2}+1}}{3 x}\right)}{4}$	50
derivativedivides	$a^{4} \left(-\frac{\operatorname{arcsinh}\left(x a \right)}{4 x^{4} a^{4}}-\frac{\sqrt{a^{2} x^{2}+1}}{12 x^{3} a^{3}}+\frac{\sqrt{a^{2} x^{2}+1}}{6 x a}\right)$	56
default	$a^{4} \left(-\frac{\operatorname{arcsinh}\left(x a \right)}{4 x^{4} a^{4}}-\frac{\sqrt{a^{2} x^{2}+1}}{12 x^{3} a^{3}}+\frac{\sqrt{a^{2} x^{2}+1}}{6 x a}\right)$	56
orering	$\frac{\left(\frac{5}{6} a^{4} x^{5}+\frac{5}{12} a^{2} x^{3}-\frac{2}{3} x \right) \operatorname{arcsinh}\left(x a \right)}{x^{5}}+\frac{\left(2 a^{2} x^{2}-1\right) \left(a^{2} x^{2}+1\right) x^{2} \left(\frac{a}{\sqrt{a^{2} x^{2}+1}\, x^{5}}-\frac{5 \operatorname{arcsinh}\left(x a \right)}{x^{6}}\right)}{12}$	80

input
```
int(arcsinh(x*a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arcsinh(x*a)/x^4+1/4*a*(-1/3/x^3*(a^2*x^2+1)^(1/2)+2/3*a^2/x*(a^2*x^2
+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{(2\,a^3x^3 - ax)\sqrt{a^2x^2+1} - 3\,\log\left(ax + \sqrt{a^2x^2+1}\right)}{12\,x^4}$$

input

```
integrate(arcsinh(a*x)/x^5,x, algorithm="fricas")
```

output

```
1/12*((2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) - 3*log(a*x + sqrt(a^2*x^2 + 1))
)/x^4
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^5}\,dx$$

input

```
integrate(asinh(a*x)/x**5,x)
```

output

```
Integral(asinh(a*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{1}{12}\left(\frac{2\,\sqrt{a^2x^2+1}a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)a - \frac{\operatorname{arsinh}(ax)}{4\,x^4}$$

input

```
integrate(arcsinh(a*x)/x^5,x, algorithm="maxima")
```

output

```
1/12*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*a - 1/4*arcsinh(a
*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)a^3|a|}{3\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3} - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)}{4\,x^4}$$

input

```
integrate(arcsinh(a*x)/x^5,x, algorithm="giac")
```

output

```
1/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^3*abs(a)/((x*abs(a) - sqrt(
a^2*x^2 + 1))^2 - 1)^3 - 1/4*log(a*x + sqrt(a^2*x^2 + 1))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^5}\,dx$$

input

```
int(asinh(a*x)/x^5,x)
```

output

```
int(asinh(a*x)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{-3asinh(ax) + 2\sqrt{a^2x^2+1}\,a^3x^3 - \sqrt{a^2x^2+1}\,ax - 2a^4x^4}{12x^4}$$

input

```
int(asinh(a*x)/x^5,x)
```

output

```
( - 3*asinh(a*x) + 2*sqrt(a**2*x**2 + 1)*a**3*x**3 - sqrt(a**2*x**2 + 1)*a
*x - 2*a**4*x**4)/(12*x**4)
```

3.17 $\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 77

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{a\sqrt{1+a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1+a^2x^2}}{40x^2} - \frac{\operatorname{arcsinh}(ax)}{5x^5} - \frac{3}{40}a^5\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output

```
-1/20*a*(a^2*x^2+1)^(1/2)/x^4+3/40*a^3*(a^2*x^2+1)^(1/2)/x^2-1/5*arcsinh(a
*x)/x^5-3/40*a^5*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{\operatorname{arcsinh}(ax)}{5x^5} - \frac{1}{5}a^5\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1+a^2x^2\right)$$

input

```
Integrate[ArcSinh[a*x]/x^6,x]
```

output
```
-1/5*ArcSinh[a*x]/x^5 - (a^5*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2, 3, 3
/2, 1 + a^2*x^2])/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6191, 243, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{5}a\int \frac{1}{x^5\sqrt{a^2x^2+1}}dx-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}a\int \frac{1}{x^6\sqrt{a^2x^2+1}}dx^2-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\int \frac{1}{x^4\sqrt{a^2x^2+1}}dx^2-\frac{\sqrt{a^2x^2+1}}{2x^4}\right)-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{\sqrt{a^2x^2+1}}{x^2}\right)-\frac{\sqrt{a^2x^2+1}}{2x^4}\right)-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\left(-\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}-\frac{\sqrt{a^2x^2+1}}{x^2}\right)-\frac{\sqrt{a^2x^2+1}}{2x^4}\right)-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\left(a^2\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{x^2}\right)-\frac{\sqrt{a^2x^2+1}}{2x^4}\right)-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

input

```
Int[ArcSinh[a*x]/x^6,x]
```

output

```
-1/5*ArcSinh[a*x]/x^5 + (a*(-1/2*Sqrt[1 + a^2*x^2]/x^4 - (3*a^2*(-(Sqrt[1
+ a^2*x^2]/x^2) + a^2*ArcTanh[Sqrt[1 + a^2*x^2]]))/4))/10
```

Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$a^{5} \left(-\frac{\operatorname{arcsinh}\left(x a \right)}{5 x^{5} a^{5}}-\frac{\sqrt{a^{2} x^{2}+1}}{20 x^{4} a^{4}}+\frac{3 \sqrt{a^{2} x^{2}+1}}{40 x^{2} a^{2}}-\frac{3 \,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{40}\right)$	70
default	$a^{5} \left(-\frac{\operatorname{arcsinh}\left(x a \right)}{5 x^{5} a^{5}}-\frac{\sqrt{a^{2} x^{2}+1}}{20 x^{4} a^{4}}+\frac{3 \sqrt{a^{2} x^{2}+1}}{40 x^{2} a^{2}}-\frac{3 \,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{40}\right)$	70
parts	$-\frac{\operatorname{arcsinh}\left(x a \right)}{5 x^{5}}+\frac{a \left(-\frac{\sqrt{a^{2} x^{2}+1}}{4 x^{4}}-\frac{3 a^{2} \left(-\frac{\sqrt{a^{2} x^{2}+1}}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{2}\right)}{4}\right)}{5}$	70

input
```
int(arcsinh(x*a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/5/x^5/a^5*arcsinh(x*a)-1/20/x^4/a^4*(a^2*x^2+1)^(1/2)+3/40/x^2/a^2
*(a^2*x^2+1)^(1/2)-3/40*arctanh(1/(a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{3\,a^{5}x^{5}\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)-3\,a^{5}x^{5}\log\left(-ax+\sqrt{a^{2}x^{2}+1}-1\right)-8\,x^{5}\log(-ax+\sqrt{a^{2}x^{2}+1}}{40\,x^{5}}$$

input
```
integrate(arcsinh(a*x)/x^6,x, algorithm="fricas")
```

output
```
-1/40*(3*a^5*x^5*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(-a*x +
sqrt(a^2*x^2 + 1) - 1) - 8*x^5*log(-a*x + sqrt(a^2*x^2 + 1)) - 8*(x^5 - 1)
*log(a*x + sqrt(a^2*x^2 + 1)) - (3*a^3*x^3 - 2*a*x)*sqrt(a^2*x^2 + 1))/x^5
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = \int \frac{\operatorname{asinh}{(ax)}}{x^6}\,dx$$

input
```
integrate(asinh(a*x)/x**6,x)
```

output
```
Integral(asinh(a*x)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{1}{40}\left(3\,a^4 \operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{3\,\sqrt{a^2x^2+1}a^2}{x^2} + \frac{2\,\sqrt{a^2x^2+1}}{x^4}\right)a - \frac{\operatorname{arsinh}{(ax)}}{5\,x^5}$$

input
```
integrate(arcsinh(a*x)/x^6,x, algorithm="maxima")
```

output
```
-1/40*(3*a^4*arcsinh(1/(a*abs(x))) - 3*sqrt(a^2*x^2 + 1)*a^2/x^2 + 2*sqrt(
a^2*x^2 + 1)/x^4)*a - 1/5*arcsinh(a*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx$$
$$= -\frac{3\,a^6 \log\left(\sqrt{a^2x^2+1}+1\right) - 3\,a^6 \log\left(\sqrt{a^2x^2+1}-1\right) - \frac{2\left(3\,(a^2x^2+1)^{\frac{3}{2}}a^6 - 5\,\sqrt{a^2x^2+1}a^6\right)}{a^4x^4}}{80\,a} - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{5\,x^5}$$

input
```
integrate(arcsinh(a*x)/x^6,x, algorithm="giac")
```

output
```
-1/80*(3*a^6*log(sqrt(a^2*x^2 + 1) + 1) - 3*a^6*log(sqrt(a^2*x^2 + 1) - 1)
 - 2*(3*(a^2*x^2 + 1)^(3/2)*a^6 - 5*sqrt(a^2*x^2 + 1)*a^6)/(a^4*x^4))/a -
1/5*log(a*x + sqrt(a^2*x^2 + 1))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^6}\,dx$$

input
```
int(asinh(a*x)/x^6,x)
```

output
```
int(asinh(a*x)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx$$
$$= \frac{-8asinh(ax) + 3\sqrt{a^2x^2+1}\,a^3x^3 - 2\sqrt{a^2x^2+1}\,ax + 3\log\left(\sqrt{a^2x^2+1} + ax - 1\right)a^5x^5 - 3\log\left(\sqrt{a^2x^2+\right.}{40x^5}$$

input
```
int(asinh(a*x)/x^6,x)
```

output
```
( - 8*asinh(a*x) + 3*sqrt(a**2*x**2 + 1)*a**3*x**3 - 2*sqrt(a**2*x**2 + 1)
*a*x + 3*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**5*x**5 - 3*log(sqrt(a**2*x*
*2 + 1) + a*x + 1)*a**5*x**5)/(40*x**5)
```

3.18 $\int x^3 \operatorname{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 96

$$\int x^3 \operatorname{arcsinh}(ax)^2\,dx = -\frac{3x^2}{32a^2}+\frac{x^4}{32}+\frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{16a^3} - \frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{8a}-\frac{3\operatorname{arcsinh}(ax)^2}{32a^4}+\frac{1}{4}x^4\operatorname{arcsinh}(ax)^2$$

output

```
-3/32*x^2/a^2+1/32*x^4+3/16*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a^3-1/8*x^3*(
a^2*x^2+1)^(1/2)*arcsinh(a*x)/a-3/32*arcsinh(a*x)^2/a^4+1/4*x^4*arcsinh(a*
x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

$$\int x^3 \operatorname{arcsinh}(ax)^2\,dx = \frac{a^2x^2(-3+a^2x^2)-2ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\operatorname{arcsinh}(ax)+(-3+8a^4x^4)\operatorname{arcsinh}(ax)^2}{32a^4}$$

input

```
Integrate[x^3*ArcSinh[a*x]^2,x]
```

output
```
(a^2*x^2*(-3 + a^2*x^2) - 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh
[a*x] + (-3 + 8*a^4*x^4)*ArcSinh[a*x]^2)/(32*a^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6191, 6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arcsinh}(ax)^2\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int x^3dx}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{3\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}\right)$$

input

```
Int[x^3*ArcSinh[a*x]^2,x]
```

output

```
(x^4*ArcSinh[a*x]^2)/4 - (a*(-1/16*x^4/a + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x])/(4*a^2) - (3*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2
) - ArcSinh[a*x]^2/(4*a^3)))/(4*a^2)))/2
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{x^{4}a^{4}\operatorname{arcsinh}(xa)^{2}}{4}-\frac{x^{3}a^{3}\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}}{8}+\frac{3\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}\,xa}{16}-\frac{3\operatorname{arcsinh}(xa)^{2}}{32}+\frac{a^{4}x^{4}}{32}-\frac{3a^{2}x^{2}}{32}-\frac{3}{32}}{a^{4}}$
default	$\frac{\frac{x^{4}a^{4}\operatorname{arcsinh}(xa)^{2}}{4}-\frac{x^{3}a^{3}\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}}{8}+\frac{3\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}\,xa}{16}-\frac{3\operatorname{arcsinh}(xa)^{2}}{32}+\frac{a^{4}x^{4}}{32}-\frac{3a^{2}x^{2}}{32}-\frac{3}{32}}{a^{4}}$
orering	$\frac{(37a^{4}x^{4}-21a^{2}x^{2}-60)\operatorname{arcsinh}(xa)^{2}}{64a^{4}}-\frac{(9a^{4}x^{4}-11a^{2}x^{2}-24)\left(3x^{2}\operatorname{arcsinh}(xa)^{2}+\frac{2x^{3}\operatorname{arcsinh}(xa)a}{\sqrt{a^{2}x^{2}+1}}\right)}{64x^{2}a^{4}}+\frac{(a^{2}x^{2}-3)\,(a}{}$

input
```
int(x^3*arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*x^4*a^4*arcsinh(x*a)^2-1/8*x^3*a^3*arcsinh(x*a)*(a^2*x^2+1)^(1/
2)+3/16*arcsinh(x*a)*(a^2*x^2+1)^(1/2)*x*a-3/32*arcsinh(x*a)^2+1/32*a^4*x^
4-3/32*a^2*x^2-3/32)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^{3}\operatorname{arcsinh}(ax)^{2}\,dx$$
$$=\frac{a^{4}x^{4}-3\,a^{2}x^{2}+(8\,a^{4}x^{4}-3)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}-2\,(2\,a^{3}x^{3}-3\,ax)\sqrt{a^{2}x^{2}+1}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}{32\,a^{4}}$$

input
```
integrate(x^3*arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
1/32*(a^4*x^4 - 3*a^2*x^2 + (8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^2
 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a
^4
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int x^3 \text{arcsinh}(ax)^2\,dx = \begin{cases} \frac{x^4 \operatorname{asinh}^2(ax)}{4} + \frac{x^4}{32} - \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a} - \frac{3x^2}{32a^2} + \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{16a^3} - \frac{3\operatorname{asinh}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*asinh(a*x)**2,x)
```

output
```
Piecewise((x**4*asinh(a*x)**2/4 + x**4/32 - x**3*sqrt(a**2*x**2 + 1)*asinh
(a*x)/(8*a) - 3*x**2/(32*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(16*a*
*3) - 3*asinh(a*x)**2/(32*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\begin{aligned}\int x^3 \text{arcsinh}(ax)^2\,dx &= \frac{1}{4}x^4 \operatorname{arsinh}(ax)^2 + \frac{1}{32}\left(\frac{x^4}{a^2} - \frac{3x^2}{a^4} + \frac{3\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^6}\right)a^2 \\ &\quad - \frac{1}{16}\left(\frac{2\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\sqrt{a^2x^2+1}x}{a^4} + \frac{3\operatorname{arsinh}(ax)}{a^5}\right)a \operatorname{arsinh}(ax)\end{aligned}$$

input
```
integrate(x^3*arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arcsinh(a*x)^2 + 1/32*(x^4/a^2 - 3*x^2/a^4 + 3*log(a*x + sqrt(a^2*
x^2 + 1))^2/a^6)*a^2 - 1/16*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2
+ 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*a*arcsinh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3\text{arcsinh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{arcsinh}(ax)^2\,dx = \int x^3\,\text{asinh}(a\,x)^2\,dx$$

input
```
int(x^3*asinh(a*x)^2,x)
```

output
```
int(x^3*asinh(a*x)^2, x)
```

Reduce [F]

$$\int x^3 \operatorname{arcsinh}(ax)^2\,dx = \int asinh(ax)^2\,x^3 dx$$

input
```
int(x^3*asinh(a*x)^2,x)
```

output
```
int(asinh(a*x)**2*x**3,x)
```

3.19 $\int x^2 \text{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^2\text{arcsinh}(ax)^2\,dx = -\frac{4x}{9a^2}+\frac{2x^3}{27}+\frac{4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{9a}+\frac{1}{3}x^3\text{arcsinh}(ax)^2$$

output

```
-4/9*x/a^2+2/27*x^3+4/9*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a^3-2/9*x^2*(a^2*x^
2+1)^(1/2)*arcsinh(a*x)/a+1/3*x^3*arcsinh(a*x)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int x^2\text{arcsinh}(ax)^2\,dx = \frac{1}{27}\left(2x\left(-\frac{6}{a^2}+x^2\right)-\frac{6(-2+a^2x^2)\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{a^3} + 9x^3\text{arcsinh}(ax)^2\right)$$

input

```
Integrate[x^2*ArcSinh[a*x]^2,x]
```

output
```
(2*x*(-6/a^2 + x^2) - (6*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^
3 + 9*x^3*ArcSinh[a*x]^2)/27
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6191, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}(ax)^2 \, dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}\right)$$

input

```
Int[x^2*ArcSinh[a*x]^2,x]
```

output

```
(x^3*ArcSinh[a*x]^2)/3 - (2*a*(-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh
[a*x])/(3*a^2) - (2*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2))/(3*a^
2)))/3
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{x^{3}a^{3}\operatorname{arcsinh}(xa)^{2}}{3}+\frac{4\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}}{9}-\frac{2x^{2}a^{2}\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}}{9}-\frac{4xa}{9}+\frac{2x^{3}a^{3}}{27}}{a^{3}}$
default	$\frac{\frac{x^{3}a^{3}\operatorname{arcsinh}(xa)^{2}}{3}+\frac{4\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}}{9}-\frac{2x^{2}a^{2}\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}}{9}-\frac{4xa}{9}+\frac{2x^{3}a^{3}}{27}}{a^{3}}$
orering	$\frac{(19a^{4}x^{4}-24a^{2}x^{2}-48)\operatorname{arcsinh}(xa)^{2}}{27a^{4}x}-\frac{(6a^{4}x^{4}-17a^{2}x^{2}-30)\left(2x\operatorname{arcsinh}(xa)^{2}+\frac{2x^{2}\operatorname{arcsinh}(xa)a}{\sqrt{a^{2}x^{2}+1}}\right)}{27x^{2}a^{4}}+\frac{(a^{2}x^{2}-6)(a^{2}}{}$

input
```
int(x^2*arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*x^3*a^3*arcsinh(x*a)^2+4/9*arcsinh(x*a)*(a^2*x^2+1)^(1/2)-2/9*x
^2*a^2*arcsinh(x*a)*(a^2*x^2+1)^(1/2)-4/9*x*a+2/27*x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int x^{2}\operatorname{arcsinh}(ax)^{2}\,dx$$
$$=\frac{9\,a^{3}x^{3}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}+2\,a^{3}x^{3}-6\,\sqrt{a^{2}x^{2}+1}(a^{2}x^{2}-2)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)-12\,ax}{27\,a^{3}}$$

input
```
integrate(x^2*arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a^3*x^3 - 6*sqrt(a^2*x^
2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1)) - 12*a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int x^2 \operatorname{arcsinh}(ax)^2\,dx = \begin{cases} \frac{x^3 \operatorname{asinh}^2(ax)}{3} + \frac{2x^3}{27} - \frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a} - \frac{4x}{9a^2} + \frac{4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input

```
integrate(x**2*asinh(a*x)**2,x)
```

output

```
Piecewise((x**3*asinh(a*x)**2/3 + 2*x**3/27 - 2*x**2*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(9*a) - 4*x/(9*a**2) + 4*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**3)
, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int x^2 \operatorname{arcsinh}(ax)^2\,dx = \frac{1}{3}x^3 \operatorname{arsinh}(ax)^2 - \frac{2}{9}a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax) + \frac{2(a^2x^3 - 6x)}{27a^2}$$

input

```
integrate(x^2*arcsinh(a*x)^2,x, algorithm="maxima")
```

output

```
1/3*x^3*arcsinh(a*x)^2 - 2/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
 + 1)/a^4)*arcsinh(a*x) + 2/27*(a^2*x^3 - 6*x)/a^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \text{arcsinh}(ax)^2 \, dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*arcsinh(a*x)^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(ax)^2 \, dx = \int x^2 \operatorname{asinh}(a\,x)^2 \, dx$$

input

```
int(x^2*asinh(a*x)^2,x)
```

output

```
int(x^2*asinh(a*x)^2, x)
```

Reduce [F]

$$\int x^2 \text{arcsinh}(ax)^2 \, dx = \int asinh(ax)^2 \, x^2 dx$$

input

```
int(x^2*asinh(a*x)^2,x)
```

output

```
int(asinh(a*x)**2*x**2,x)
```

3.20 $\int x\text{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x\text{arcsinh}(ax)^2\,dx=\frac{x^2}{4}-\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a}+\frac{\text{arcsinh}(ax)^2}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^2$$

output

```
1/4*x^2-1/2*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a+1/4*arcsinh(a*x)^2/a^2+1/2*
x^2*arcsinh(a*x)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x\text{arcsinh}(ax)^2\,dx=\frac{a^2x^2-2ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)+(1+2a^2x^2)\,\text{arcsinh}(ax)^2}{4a^2}$$

input

```
Integrate[x*ArcSinh[a*x]^2,x]
```

output

```
(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (1 + 2*a^2*x^2)*ArcSinh[
a*x]^2)/(4*a^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6191, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)^2\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)$$

input
```
Int[x*ArcSinh[a*x]^2,x]
```

output
```
(x^2*ArcSinh[a*x]^2)/2 - a*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(2*a^2) - ArcSinh[a*x]^2/(4*a^3))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{\cosh(2 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)^2}{4}-\frac{\sinh(2 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)}{4}+\frac{\cosh(2 \operatorname{arcsinh}(xa))}{8}}{a^2}$
default	$\frac{\frac{\cosh(2 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)^2}{4}-\frac{\sinh(2 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)}{4}+\frac{\cosh(2 \operatorname{arcsinh}(xa))}{8}}{a^2}$
orering	$\frac{(7a^2x^2+6) \operatorname{arcsinh}(xa)^2}{8a^2}-\frac{(3a^2x^2+4)\left(\operatorname{arcsinh}(xa)^2+\frac{2x \operatorname{arcsinh}(xa)a}{\sqrt{a^2x^2+1}}\right)}{8a^2}+\frac{x(a^2x^2+1)\Bigg(\frac{4 \operatorname{arcsinh}(xa)a}{\sqrt{a^2x^2+1}}+\frac{2x a^2}{a^2x^2+1}-}{8a^2}$

input
```
int(x*arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*cosh(2*arcsinh(x*a))*arcsinh(x*a)^2-1/4*sinh(2*arcsinh(x*a))*ar
csinh(x*a)+1/8*cosh(2*arcsinh(x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x\text{arcsinh}(ax)^2\,dx$$
$$=\frac{a^2x^2-2\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)+\left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^2}$$

input
```
integrate(x*arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + (2*a
^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x\text{arcsinh}(ax)^2\,dx=\begin{cases}\frac{x^2\operatorname{asinh}^2(ax)}{2}+\frac{x^2}{4}-\frac{x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a}+\frac{\operatorname{asinh}^2(ax)}{4a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x*asinh(a*x)**2,x)
```

output
```
Piecewise((x**2*asinh(a*x)**2/2 + x**2/4 - x*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(2*a) + asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int x\text{arcsinh}(ax)^2\,dx = \frac{1}{2}\,x^2\,\text{arsinh}\left(ax\right)^2 + \frac{1}{4}\,a^2\left(\frac{x^2}{a^2} - \frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{a^4}\right) - \frac{1}{2}\,a\left(\frac{\sqrt{a^2x^2 + 1}x}{a^2} - \frac{\text{arsinh}\left(ax\right)}{a^3}\right)\text{arsinh}\left(ax\right)$$

input
```
integrate(x*arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arcsinh(a*x)^2 + 1/4*a^2*(x^2/a^2 - log(a*x + sqrt(a^2*x^2 + 1))^2
/a^4) - 1/2*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)*arcsinh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^2\,dx=\int x\,\text{asinh}(a\,x)^2\,dx$$

input
```
int(x*asinh(a*x)^2,x)
```

output
```
int(x*asinh(a*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x\text{arcsinh}(ax)^2\,dx=\frac{2asinh(ax)^2\,a^2x^2+asinh(ax)^2-2\sqrt{a^2x^2+1}\,asinh(ax)\,ax+a^2x^2}{4a^2}$$

input
```
int(x*asinh(a*x)^2,x)
```

output
```
(2*asinh(a*x)**2*a**2*x**2 + asinh(a*x)**2 - 2*sqrt(a**2*x**2 + 1)*asinh(a
*x)*a*x + a**2*x**2)/(4*a**2)
```

3.21 $\int \operatorname{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \operatorname{arcsinh}(ax)^2\,dx = 2x - \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + x\operatorname{arcsinh}(ax)^2$$

output

```
2*x-2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a+x*arcsinh(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)^2\,dx = 2x - \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + x\operatorname{arcsinh}(ax)^2$$

input

```
Integrate[ArcSinh[a*x]^2,x]
```

output

```
2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^2 \, dx$$

$$\Big\downarrow \text{6187}$$

$$x\text{arcsinh}(ax)^2 - 2a\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x\text{arcsinh}(ax)^2 - 2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2} - \frac{\int 1dx}{a}\right)$$

$$\Big\downarrow \text{24}$$

$$x\text{arcsinh}(ax)^2 - 2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2} - \frac{x}{a}\right)$$

input
```
Int[ArcSinh[a*x]^2,x]
```

output
```
x*ArcSinh[a*x]^2 - 2*a*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(xa)^2 xa-2\operatorname{arcsinh}(xa)\sqrt{a^2x^2+1}+2xa}{a}$	36
default	$\frac{\operatorname{arcsinh}(xa)^2 xa-2\operatorname{arcsinh}(xa)\sqrt{a^2x^2+1}+2xa}{a}$	36
orering	$x\operatorname{arcsinh}\left(xa\right)^2-\frac{2\operatorname{arcsinh}(xa)}{a\sqrt{a^2x^2+1}}+\frac{x\left(a^2x^2+1\right)\left(\frac{2a^2}{a^2x^2+1}-\frac{2\operatorname{arcsinh}(xa)a^3x}{\left(a^2x^2+1\right)^{\frac{3}{2}}}\right)}{a^2}$	82

input
```
int(arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arcsinh(x*a)^2*x*a-2*arcsinh(x*a)*(a^2*x^2+1)^(1/2)+2*x*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \operatorname{arcsinh}(ax)^2\,dx = \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,ax-2\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a}$$

input
```
integrate(arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
(a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*x - 2*sqrt(a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \operatorname{arcsinh}(ax)^2\,dx = \begin{cases} x\operatorname{asinh}^2(ax) + 2x - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)**2,x)
```

output
```
Piecewise((x*asinh(a*x)**2 + 2*x - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(
a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \operatorname{arcsinh}(ax)^2\,dx = x\operatorname{arsinh}(ax)^2 + 2\,x - \frac{2\,\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a}$$

input
```
integrate(arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
x*arcsinh(a*x)^2 + 2*x - 2*sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

$$\int \operatorname{arcsinh}(ax)^2\,dx = x\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 2\,a\left(\frac{x}{a} - \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)$$

input
```
integrate(arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x +
sqrt(a^2*x^2 + 1))/a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^2\,dx = \int \operatorname{asinh}(a\,x)^2\,dx$$

input
```
int(asinh(a*x)^2,x)
```

output
```
int(asinh(a*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)^2\,dx = \frac{asinh(ax)^2\,ax - 2\sqrt{a^2x^2+1}\,asinh(ax) + 2ax}{a}$$

input
```
int(asinh(a*x)^2,x)
```

output
```
(asinh(a*x)**2*a*x - 2*sqrt(a**2*x**2 + 1)*asinh(a*x) + 2*a*x)/a
```

3.22 $\int \frac{\text{arcsinh}(ax)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\text{arcsinh}(ax)^2}{x}\,dx = -\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right) + \text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)$$

output

```
-1/3*arcsinh(a*x)^3+arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+arcsinh
(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-1/2*polylog(3,(a*x+(a^2*x^2+1)^
(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax)^2}{x}\,dx = -\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right) + \text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)$$

input

```
Integrate[ArcSinh[a*x]^2/x,x]
```

output
```
-1/3*ArcSinh[a*x]^3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh
[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6190, 3042, 26, 4199, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x} \, dx$$

$$\Big\downarrow \text{6190}$$

$$\int \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{ax} d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\operatorname{arcsinh}(ax)^2 \tan\left(\frac{\pi}{2}+i\operatorname{arcsinh}(ax)\right) d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{26}$$

$$-i\int \operatorname{arcsinh}(ax)^2 \tan\left(i\operatorname{arcsinh}(ax)+\frac{\pi}{2}\right) d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{4199}$$

$$-i\left(2i\int -\frac{e^{2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax)^2}{1-e^{2\operatorname{arcsinh}(ax)}} d\operatorname{arcsinh}(ax)-\frac{1}{3}i\operatorname{arcsinh}(ax)^3\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(-2i\int \frac{e^{2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax)^2}{1-e^{2\operatorname{arcsinh}(ax)}} d\operatorname{arcsinh}(ax)-\frac{1}{3}i\operatorname{arcsinh}(ax)^3\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\Bigg(-2i\Bigg(\int \text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right)\Bigg)-\frac{1}{3}i\text{arcsin}$$

$$\Big\downarrow \; 3011$$

$$-i\Bigg(-2i\Bigg(\frac{1}{2}\int \text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)^2$$

$$\Big\downarrow \; 2720$$

$$-i\Bigg(-2i\Bigg(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}$$

$$\Big\downarrow \; 7143$$

$$-i\Bigg(-2i\Bigg(-\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)+\frac{1}{4}\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)^2\log\Big(1-e^{2\text{arcs}}$$

input

```
Int[ArcSinh[a*x]^2/x,x]
```

output

```
(-I)*((-1/3*I)*ArcSinh[a*x]^3 - (2*I)*(-1/2*(ArcSinh[a*x]^2*Log[1 - E^(2*A
rcSinh[a*x])]) - (ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])])/2 + PolyLog
[3, E^(2*ArcSinh[a*x])]/4))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.52

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(xa)^{3}}{3}+\operatorname{arcsinh}\left(xa\right)^{2}\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)+2\operatorname{arcsinh}\left(xa\right)\operatorname{polylog}\left(2,-x\right.$
default	$-\frac{\operatorname{arcsinh}(xa)^{3}}{3}+\operatorname{arcsinh}\left(xa\right)^{2}\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)+2\operatorname{arcsinh}\left(xa\right)\operatorname{polylog}\left(2,-x\right.$

input

```
int(arcsinh(x*a)^2/x,x,method=_RETURNVERBOSE)
```

output

```
-1/3*arcsinh(x*a)^3+arcsinh(x*a)^2*ln(1+x*a+(a^2*x^2+1)^(1/2))+2*arcsinh(x
*a)*polylog(2,-x*a-(a^2*x^2+1)^(1/2))-2*polylog(3,-x*a-(a^2*x^2+1)^(1/2))+
arcsinh(x*a)^2*ln(1-x*a-(a^2*x^2+1)^(1/2))+2*arcsinh(x*a)*polylog(2,x*a+(a
^2*x^2+1)^(1/2))-2*polylog(3,x*a+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^2}{x}\,dx$$

input

```
integrate(arcsinh(a*x)^2/x,x, algorithm="fricas")
```

output

```
integral(arcsinh(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx=\int \frac{\operatorname{asinh}^{2}\left(ax\right)}{x}\,dx$$

input

```
integrate(asinh(a*x)**2/x,x)
```

output

```
Integral(asinh(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x}\,dx$$

input

```
integrate(arcsinh(a*x)^2/x,x, algorithm="maxima")
```

output

```
integrate(arcsinh(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x}\,dx$$

input

```
integrate(arcsinh(a*x)^2/x,x, algorithm="giac")
```

output

```
integrate(arcsinh(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x}\,dx$$

input

```
int(asinh(a*x)^2/x,x)
```

output

```
int(asinh(a*x)^2/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{asinh(ax)^2}{x}dx$$

input
```
int(asinh(a*x)^2/x,x)
```

output
```
int(asinh(a*x)**2/x,x)
```

3.23 $\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = -\frac{\operatorname{arcsinh}(ax)^2}{x} - 4a\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) - 2a\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + 2a\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)$$

output
```
-arcsinh(a*x)^2/x-4*a*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*a*poly
log(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = a\left(-\operatorname{arcsinh}(ax)\left(\frac{\operatorname{arcsinh}(ax)}{ax} - 2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) + 2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right)\right) + 2\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) - 2\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right)\right)$$

input
```
Integrate[ArcSinh[a*x]^2/x^2,x]
```

output
```
a*(-(ArcSinh[a*x]*(ArcSinh[a*x]/(a*x) - 2*Log[1 - E^(-ArcSinh[a*x])] + 2*L
og[1 + E^(-ArcSinh[a*x])])) + 2*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*PolyLog
[2, E^(-ArcSinh[a*x])])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6191, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{6191}$$

$$2a\int \frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^2}{x}$$

$$\Big\downarrow \text{6231}$$

$$2a\int \frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\text{arcsinh}(ax)^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{arcsinh}(ax)^2}{x}+2a\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{26}$$

$$-\frac{\text{arcsinh}(ax)^2}{x}+2ia\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \text{4670}$$

$$-\frac{\text{arcsinh}(ax)^2}{x}+$$
$$2ia\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arcsinh}(ax)\text{arctanh}\Big(e^{\text{ar}}\right.$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\text{arcsinh}(ax)^2}{x}+$$

$$2ia\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}+2i\text{ar}\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\text{arcsinh}(ax)^2}{x}+$$

$$2ia\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\right)$$

input
```
Int[ArcSinh[a*x]^2/x^2,x]
```

output
```
-(ArcSinh[a*x]^2/x) + (2*I)*a*((2*I)*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]]
+ I*PolyLog[2, -E^ArcSinh[a*x]] - I*PolyLog[2, E^ArcSinh[a*x]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.16

method	result
derivativedivides	$a\Big(-\frac{\operatorname{arcsinh}(xa)^2}{xa}-2\operatorname{arcsinh}(xa)\ln\left(1+xa+\sqrt{a^2x^2+1}\right)-2\operatorname{polylog}\left(2,-xa-\sqrt{a^2x}\right.$
default	$a\Big(-\frac{\operatorname{arcsinh}(xa)^2}{xa}-2\operatorname{arcsinh}(xa)\ln\left(1+xa+\sqrt{a^2x^2+1}\right)-2\operatorname{polylog}\left(2,-xa-\sqrt{a^2x}\right.$

input
```
int(arcsinh(x*a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-arcsinh(x*a)^2/x/a-2*arcsinh(x*a)*ln(1+x*a+(a^2*x^2+1)^(1/2))-2*polylo
g(2,-x*a-(a^2*x^2+1)^(1/2))+2*arcsinh(x*a)*ln(1-x*a-(a^2*x^2+1)^(1/2))+2*p
olylog(2,x*a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx=\int \frac{\operatorname{asinh}^2\left(ax\right)}{x^2}\,dx$$

input
```
integrate(asinh(a*x)**2/x**2,x)
```

output
```
Integral(asinh(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
-log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2 + sqrt(a^2*x^2 +
1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^2}\,dx$$

input

```
integrate(arcsinh(a*x)^2/x^2,x, algorithm="giac")
```

output

```
integrate(arcsinh(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^2}\,dx$$

input

```
int(asinh(a*x)^2/x^2,x)
```

output

```
int(asinh(a*x)^2/x^2, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{asinh(ax)^2}{x^2}dx$$

input

```
int(asinh(a*x)^2/x^2,x)
```

output

```
int(asinh(a*x)**2/x**2,x)
```

3.24 $\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} - \frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a^2\log(x)$$

output

```
-a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/x-1/2*arcsinh(a*x)^2/x^2+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} - \frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a^2\log(x)$$

input

```
Integrate[ArcSinh[a*x]^2/x^3,x]
```

output

```
-((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log
[x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6191, 6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 6191}$$

$$a\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{a^2x^2+1}}dx-\frac{\operatorname{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 6215}$$

$$a\left(a\int \frac{1}{x}dx-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{x}\right)-\frac{\operatorname{arcsinh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 14}$$

$$a\left(a\log(x)-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{x}\right)-\frac{\operatorname{arcsinh}(ax)^2}{2x^2}$$

input

```
Int[ArcSinh[a*x]^2/x^3,x]
```

output

```
-1/2*ArcSinh[a*x]^2/x^2 + a*(-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log
[x])
```

Defintions of rubi rules used

rule 14

```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

method	result
derivativedivides	$a^2 \left(-2 \operatorname{arcsinh}(xa) - \frac{\operatorname{arcsinh}(xa)\left(-2a^2x^2+2xa\sqrt{a^2x^2+1}+\operatorname{arcsinh}(xa)\right)}{2x^2a^2} + \ln\left(\left(xa+\sqrt{a^2x^2+1}\right.\right.\right.$
default	$a^2 \left(-2 \operatorname{arcsinh}(xa) - \frac{\operatorname{arcsinh}(xa)\left(-2a^2x^2+2xa\sqrt{a^2x^2+1}+\operatorname{arcsinh}(xa)\right)}{2x^2a^2} + \ln\left(\left(xa+\sqrt{a^2x^2+1}\right.\right.\right.$

input
```
int(arcsinh(x*a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-2*arcsinh(x*a)-1/2*arcsinh(x*a)*(-2*a^2*x^2+2*x*a*(a^2*x^2+1)^(1/2)+
arcsinh(x*a))/x^2/a^2+ln((x*a+(a^2*x^2+1)^(1/2))^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \frac{2\,a^2x^2\log\left(x\right) - 2\,\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right) - \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

input
```
integrate(arcsinh(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*x^2*log(x) - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1
)) - log(a*x + sqrt(a^2*x^2 + 1))^2)/x^2
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x^3}\,dx$$

input
```
integrate(asinh(a*x)**2/x**3,x)
```

output
```
Integral(asinh(a*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = a^2\log\left(x\right) - \frac{\sqrt{a^2x^2+1}a\operatorname{arsinh}\left(ax\right)}{x} - \frac{\operatorname{arsinh}\left(ax\right)^2}{2\,x^2}$$

input
```
integrate(arcsinh(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
a^2*log(x) - sqrt(a^2*x^2 + 1)*a*arcsinh(a*x)/x - 1/2*arcsinh(a*x)^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = -\left(a\log\left(-x|a|+\sqrt{a^2x^2+1}\right) - a\log(|x|) - \frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}\right)a - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

input

```
integrate(arcsinh(a*x)^2/x^3,x, algorithm="giac")
```

output

```
-(a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - a*log(abs(x)) - 2*abs(a)*log(a*x
+ sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1))*a - 1/2*log(a
*x + sqrt(a^2*x^2 + 1))^2/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^3}\,dx$$

input

```
int(asinh(a*x)^2/x^3,x)
```

output

```
int(asinh(a*x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \int \frac{asinh(ax)^2}{x^3}dx$$

input

```
int(asinh(a*x)^2/x^3,x)
```

output

```
int(asinh(a*x)**2/x**3,x)
```

3.25 $\int \frac{\mathbf{arcsinh}(ax)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 99

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = {} & -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x^2}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3} \\ & +\frac{2}{3}a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\ & +\frac{1}{3}a^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)-\frac{1}{3}a^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-1/3*a^2/x-1/3*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/x^2-1/3*arcsinh(a*x)^2/x^3
+2/3*a^3*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/3*a^3*polylog(2,-a*
x-(a^2*x^2+1)^(1/2))-1/3*a^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = -\frac{a^2x^2+ax\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)+\operatorname{arcsinh}(ax)^2+a^3x^3\operatorname{arcsinh}(ax)\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right)-a^3x^3\mathrm{arcsi}}{3x^3}$$

input
```
Integrate[ArcSinh[a*x]^2/x^4,x]
```

output
```
-1/3*(a^2*x^2 + a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2 + a^3*
x^3*ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] - a^3*x^3*ArcSinh[a*x]*Log[1 +
 E^(-ArcSinh[a*x])] + a^3*x^3*PolyLog[2, -E^(-ArcSinh[a*x])] - a^3*x^3*Pol
yLog[2, E^(-ArcSinh[a*x])])/x^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6191, 6224, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2}{3}a\int \frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{a^2x^2+1}}dx-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6224}$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx+\frac{1}{2}a\int \frac{1}{x^2}dx-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{2x^2}\right)-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{2x^2}-\frac{a}{2x}\right)-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6231}$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\operatorname{arcsinh}(ax)}{ax}d\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{2x^2}-\frac{a}{2x}\right)-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{arcsinh}(ax)^2}{3x^3}+$$
$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int i\text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}\right)$$

$$\Big\downarrow 26$$

$$-\frac{\text{arcsinh}(ax)^2}{3x^3}+$$
$$\frac{2}{3}a\left(-\frac{1}{2}ia^2\int \text{arcsinh}(ax)\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2x^2}-\frac{a}{2x}\right)$$

$$\Big\downarrow 4670$$

$$-\frac{\text{arcsinh}(ax)^2}{3x^3}+$$
$$\frac{2}{3}a\left(-\frac{1}{2}ia^2\left(i\int \log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-i\int \log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+2i\text{arcsinh}(ax)\text{arct}\right.\right.$$

$$\Big\downarrow 2715$$

$$-\frac{\text{arcsinh}(ax)^2}{3x^3}+$$
$$\frac{2}{3}a\left(-\frac{1}{2}ia^2\left(i\int e^{-\text{arcsinh}(ax)}\log\left(1-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}\log\left(1+e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax}\right.\right.$$

$$\Big\downarrow 2838$$

$$-\frac{\text{arcsinh}(ax)^2}{3x^3}+$$
$$\frac{2}{3}a\left(-\frac{1}{2}ia^2\left(2i\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)-i\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)\right)-\right.$$

input
```
Int[ArcSinh[a*x]^2/x^4,x]
```

output
```
-1/3*ArcSinh[a*x]^2/x^3 + (2*a*(-1/2*a/x - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(2*x^2) - (I/2)*a^2*((2*I)*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + I*Poly
Log[2, -E^ArcSinh[a*x]] - I*PolyLog[2, E^ArcSinh[a*x]])))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

method	result
derivativedivides	$a^{3} \left(-\frac{\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}\,xa+\operatorname{arcsinh}(xa)^{2}+a^{2}x^{2}}{3x^{3}a^{3}}+\frac{\operatorname{arcsinh}(xa)\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)}{3}+\frac{\operatorname{polylog}\left(2,-xa-\right.}{3}\right.$
default	$a^{3} \left(-\frac{\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}\,xa+\operatorname{arcsinh}(xa)^{2}+a^{2}x^{2}}{3x^{3}a^{3}}+\frac{\operatorname{arcsinh}(xa)\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)}{3}+\frac{\operatorname{polylog}\left(2,-xa-\right.}{3}\right.$

input

```
int(arcsinh(x*a)^2/x^4,x,method=_RETURNVERBOSE)
```

output

```
a^3*(-1/3*(arcsinh(x*a)*(a^2*x^2+1)^(1/2)*x*a+arcsinh(x*a)^2+a^2*x^2)/x^3/
a^3+1/3*arcsinh(x*a)*ln(1+x*a+(a^2*x^2+1)^(1/2))+1/3*polylog(2,-x*a-(a^2*x
^2+1)^(1/2))-1/3*arcsinh(x*a)*ln(1-x*a-(a^2*x^2+1)^(1/2))-1/3*polylog(2,x*
a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^4}\,dx$$

input

```
integrate(arcsinh(a*x)^2/x^4,x, algorithm="fricas")
```

output

```
integral(arcsinh(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x^4}\,dx$$

input

```
integrate(asinh(a*x)**2/x**4,x)
```

output

```
Integral(asinh(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^4}\,dx$$

input

```
integrate(arcsinh(a*x)^2/x^4,x, algorithm="maxima")
```

output

```
-1/3*log(a*x + sqrt(a^2*x^2 + 1))^2/x^3 + integrate(2/3*(a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^6 + a*x^4 + (a^2
*x^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^4}\,dx$$

input
```
int(asinh(a*x)^2/x^4,x)
```

output
```
int(asinh(a*x)^2/x^4, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{asinh(ax)^2}{x^4}dx$$

input
```
int(asinh(a*x)^2/x^4,x)
```

output
```
int(asinh(a*x)**2/x**4,x)
```

3.26 $\int \frac{\text{arcsinh}(ax)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{\text{arcsinh}(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} - \frac{a\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{6x^3} + \frac{a^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{3x} - \frac{\text{arcsinh}(ax)^2}{4x^4} - \frac{1}{3}a^4\log(x)$$

output
```
-1/12*a^2/x^2-1/6*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/x^3+1/3*a^3*(a^2*x^2+1)
^(1/2)*arcsinh(a*x)/x-1/4*arcsinh(a*x)^2/x^4-1/3*a^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{\text{arcsinh}(ax)^2}{x^5}\,dx = -\frac{a^2x^2 - 2ax\sqrt{1+a^2x^2}\,(-1+2a^2x^2)\,\text{arcsinh}(ax) + 3\text{arcsinh}(ax)^2 + 4a^4x^4\log(x)}{12x^4}$$

input
```
Integrate[ArcSinh[a*x]^2/x^5,x]
```

output
```
-1/12*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2)*ArcSinh[a*x] + 3
*ArcSinh[a*x]^2 + 4*a^4*x^4*Log[x])/x^4
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{6191, 6224, 15, 6215, 14\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^2}{x^5}dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}a\int \frac{\text{arcsinh}(ax)}{x^4\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6224}$$

$$\frac{1}{2}a\left(-\frac{2}{3}a^2\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{a^2x^2+1}}dx+\frac{1}{3}a\int \frac{1}{x^3}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3x^3}\right)-\frac{\text{arcsinh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}a\left(-\frac{2}{3}a^2\int \frac{\text{arcsinh}(ax)}{x^2\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3x^3}-\frac{a}{6x^2}\right)-\frac{\text{arcsinh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6215}$$

$$\frac{1}{2}a\left(-\frac{2}{3}a^2\left(a\int \frac{1}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3x^3}-\frac{a}{6x^2}\right)-\frac{\text{arcsinh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}a\left(-\frac{2}{3}a^2\left(a\log(x)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3x^3}-\frac{a}{6x^2}\right)-\frac{\text{arcsinh}(ax)^2}{4x^4}$$

input
```
Int[ArcSinh[a*x]^2/x^5,x]
```

output
```
-1/4*ArcSinh[a*x]^2/x^4 + (a*(-1/6*a/x^2 - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(3*x^3) - (2*a^2*(-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]))/3))
/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

method	result
derivativedivides	$a^{4} \left(\frac{2 \operatorname{arcsinh}(xa)}{3}-\frac{-4 x^{3} a^{3} \operatorname{arcsinh}(xa) \sqrt{a^{2} x^{2}+1}+4 x^{4} a^{4} \operatorname{arcsinh}(xa)+2 \operatorname{arcsinh}(xa) \sqrt{a^{2} x^{2}+1}\, xa+3 \operatorname{arcsinh}(xa}{12 x^{4} a^{4}}\right.$
default	$a^{4} \left(\frac{2 \operatorname{arcsinh}(xa)}{3}-\frac{-4 x^{3} a^{3} \operatorname{arcsinh}(xa) \sqrt{a^{2} x^{2}+1}+4 x^{4} a^{4} \operatorname{arcsinh}(xa)+2 \operatorname{arcsinh}(xa) \sqrt{a^{2} x^{2}+1}\, xa+3 \operatorname{arcsinh}(xa}{12 x^{4} a^{4}}\right.$

input
```
int(arcsinh(x*a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(2/3*arcsinh(x*a)-1/12*(-4*x^3*a^3*arcsinh(x*a)*(a^2*x^2+1)^(1/2)+4*x^
4*a^4*arcsinh(x*a)+2*arcsinh(x*a)*(a^2*x^2+1)^(1/2)*x*a+3*arcsinh(x*a)^2+a
^2*x^2)/x^4/a^4-1/3*ln((x*a+(a^2*x^2+1)^(1/2))^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{2}}{x^{5}}\, dx = -\frac{4\, a^{4} x^{4} \log\left(x\right) + a^{2} x^{2} - 2\, {\left(2\, a^{3} x^{3} - a x\right)} \sqrt{a^{2} x^{2} + 1} \log\left(a x + \sqrt{a^{2} x^{2} + 1}\right) + 3\, \log\left(a x + \sqrt{a^{2} x^{2} + 1}\right)^{2}}{12\, x^{4}}$$

input
```
integrate(arcsinh(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(4*a^4*x^4*log(x) + a^2*x^2 - 2*(2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1)*
log(a*x + sqrt(a^2*x^2 + 1)) + 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/x^4
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{x^5}\,dx$$

input
```
integrate(asinh(a*x)**2/x**5,x)
```

output
```
Integral(asinh(a*x)**2/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{1}{12}\left(4a^2\log(x) + \frac{1}{x^2}\right)a^2 + \frac{1}{6}\left(\frac{2\sqrt{a^2x^2+1}a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)a\operatorname{arsinh}(ax) - \frac{\operatorname{arsinh}(ax)^2}{4x^4}$$

input
```
integrate(arcsinh(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/12*(4*a^2*log(x) + 1/x^2)*a^2 + 1/6*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a
^2*x^2 + 1)/x^3)*a*arcsinh(a*x) - 1/4*arcsinh(a*x)^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{1}{12}\left(2a^3\log\left(x^2\right) - 4a^3\log\left(-x|a| + \sqrt{a^2x^2+1}\right) - \frac{8\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)a^2|a|\log\left(ax + \sqrt{\ }\right.}{\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3}\right. - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)^2}{4x^4}$$

input
```
integrate(arcsinh(a*x)^2/x^5,x, algorithm="giac")
```

output
```
-1/12*(2*a^3*log(x^2) - 4*a^3*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - 8*(3*(x
*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1)
)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^3 - (2*a^3*x^2 - a)/x^2)*a - 1/4*
log(a*x + sqrt(a^2*x^2 + 1))^2/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^5}\,dx$$

input
```
int(asinh(a*x)^2/x^5,x)
```

output
```
int(asinh(a*x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = \int \frac{\mathit{asinh}(ax)^2}{x^5}dx$$

input
```
int(asinh(a*x)^2/x^5,x)
```

output
```
int(asinh(a*x)**2/x**5,x)
```

3.27 $\int x^3\text{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\begin{aligned}\int x^3\text{arcsinh}(ax)^3\,dx = {} & \frac{45x\sqrt{1+a^2x^2}}{256a^3}-\frac{3x^3\sqrt{1+a^2x^2}}{128a}-\frac{45\text{arcsinh}(ax)}{256a^4}\\ & -\frac{9x^2\text{arcsinh}(ax)}{32a^2}+\frac{3}{32}x^4\text{arcsinh}(ax)\\ & +\frac{9x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{32a^3}-\frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a}\\ & -\frac{3\text{arcsinh}(ax)^3}{32a^4}+\frac{1}{4}x^4\text{arcsinh}(ax)^3\end{aligned}$$

output
```
45/256*x*(a^2*x^2+1)^(1/2)/a^3-3/128*x^3*(a^2*x^2+1)^(1/2)/a-45/256*arcsin
h(a*x)/a^4-9/32*x^2*arcsinh(a*x)/a^2+3/32*x^4*arcsinh(a*x)+9/32*x*(a^2*x^2
+1)^(1/2)*arcsinh(a*x)^2/a^3-3/16*x^3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/a-3
/32*arcsinh(a*x)^3/a^4+1/4*x^4*arcsinh(a*x)^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \frac{3ax(15-2a^2x^2)\sqrt{1+a^2x^2}+3(-15-24a^2x^2+8a^4x^4)\operatorname{arcsinh}(ax)-24ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\,\mathrm{arc}}{256a^4}$$

input
```
Integrate[x^3*ArcSinh[a*x]^3,x]
```

output
```
(3*a*x*(15 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^
4)*ArcSinh[a*x] - 24*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2
 + 8*(-3 + 8*a^4*x^4)*ArcSinh[a*x]^3)/(256*a^4)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6191, 6227, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$\Big\downarrow \ 6191$$
$$\frac{1}{4}x^4\operatorname{arcsinh}(ax)^3-\frac{3}{4}a\int \frac{x^4\operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \ 6227$$
$$\frac{1}{4}x^4\operatorname{arcsinh}(ax)^3-$$
$$\frac{3}{4}a\left(-\frac{3\int \frac{x^2\operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int x^3\operatorname{arcsinh}(ax)dx}{2a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{4a^2}\right)$$
$$\Big\downarrow \ 6191$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\int\frac{x^4}{\sqrt{a^2x^2+1}}dx}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\int\frac{x^2}{\sqrt{a^2x^2+1}}dx}{4a^2}\right)}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int\frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{4a^2}\right)}{2a}+\frac{x^3\sqrt{a^2x^2+1}\text{ar}}{4a^2}\right.$$

$$\Big\downarrow \; 222$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\text{arcs}\right.}{4a^2}\right.}{2a}\right.$$

$$\Big\downarrow \; 6227$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\left(-\frac{\int\frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int x\text{arcsinh}(ax)dx}{a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x^4\text{arcs}}{}\right.$$

$\downarrow$ 6191

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\left(-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\int\frac{x^2}{\sqrt{a^2x^2+1}}dx}{a}-\frac{\int\frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(a}{4a^2}\right.$$

$\downarrow$ 262

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\left(-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int\frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{a}-\frac{\int\frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)}{4a^2}+x^3\sqrt{a^2x^2\right.$$

$\downarrow$ 222

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(-\frac{3\left(-\frac{\int\frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}\right)}{4a^2}+x^3\sqrt{a^2x^2+\right.$$

$\downarrow$ 6198

$$\frac{1}{4}x^4\text{arcsinh}(ax)^3-\frac{3}{4}a\left(\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a^2}-\frac{3\left(-\frac{\text{arcsinh}(ax)^3}{6a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\text{arcsi}\right.}{a}\right.}{4a^2}\right.$$

input

```
Int[x^3*ArcSinh[a*x]^3,x]
```

output

```
(x^4*ArcSinh[a*x]^3)/4 - (3*a*((x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*a
^2) - ((x^4*ArcSinh[a*x])/4 - (a*((x^3*Sqrt[1 + a^2*x^2])/(4*a^2) - (3*((x
*Sqrt[1 + a^2*x^2])/(2*a^2) - ArcSinh[a*x]/(2*a^3)))/(4*a^2)))/4)/(2*a) -
(3*((x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3)
- ((x^2*ArcSinh[a*x])/2 - (a*((x*Sqrt[1 + a^2*x^2])/(2*a^2) - ArcSinh[a*x]
/(2*a^3)))/2)/a))/(4*a^2)))/4
```

Defintions of rubi rules used

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{x^4a^4\operatorname{arcsinh}(xa)^3}{4}-\frac{3x^3a^3\operatorname{arcsinh}(xa)^2\sqrt{a^2x^2+1}}{16}+\frac{9\operatorname{arcsinh}(xa)^2\sqrt{a^2x^2+1}\,xa}{32}-\frac{3\operatorname{arcsinh}(xa)^3}{32}+\frac{3x^4a^4\operatorname{arcsinh}(xa)}{32}-}{a^4}$
default	$\frac{\frac{x^4a^4\operatorname{arcsinh}(xa)^3}{4}-\frac{3x^3a^3\operatorname{arcsinh}(xa)^2\sqrt{a^2x^2+1}}{16}+\frac{9\operatorname{arcsinh}(xa)^2\sqrt{a^2x^2+1}\,xa}{32}-\frac{3\operatorname{arcsinh}(xa)^3}{32}+\frac{3x^4a^4\operatorname{arcsinh}(xa)}{32}-}{a^4}$
orering	$\frac{\left(350a^6x^6-399a^4x^4-1800a^2x^2-1080\right)\operatorname{arcsinh}(xa)^3}{512a^6x^2}-\frac{\left(110a^6x^6-263a^4x^4-1020a^2x^2-630\right)\left(3x^2\operatorname{arcsinh}(xa)^3+\right.}{512a^6x^4}$

input
```
int(x^3*arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*x^4*a^4*arcsinh(x*a)^3-3/16*x^3*a^3*arcsinh(x*a)^2*(a^2*x^2+1)^
(1/2)+9/32*arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)*x*a-3/32*arcsinh(x*a)^3+3/32*x
^4*a^4*arcsinh(x*a)-3/128*x^3*a^3*(a^2*x^2+1)^(1/2)+45/256*x*a*(a^2*x^2+1)
^(1/2)+27/256*arcsinh(x*a)-9/32*(a^2*x^2+1)*arcsinh(x*a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \frac{8\,(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-24\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+3\,(8\,a^4x^4}{256\,a^4}$$

input
```
integrate(x^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
1/256*(8*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 24*(2*a^3*x^3 -
3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 3*(8*a^4*x^4 - 2
4*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^3*x^3 - 15*a*x)*sqrt
(a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx$$
$$= \begin{cases} \frac{x^4\operatorname{asinh}^3(ax)}{4}+\frac{3x^4\operatorname{asinh}(ax)}{32}-\frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{16a}-\frac{3x^3\sqrt{a^2x^2+1}}{128a}-\frac{9x^2\operatorname{asinh}(ax)}{32a^2}+\frac{9x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{32a^3}+ \\ 0 \end{cases}$$

input
```
integrate(x**3*asinh(a*x)**3,x)
```

output
```
Piecewise((x**4*asinh(a*x)**3/4 + 3*x**4*asinh(a*x)/32 - 3*x**3*sqrt(a**2*
x**2 + 1)*asinh(a*x)**2/(16*a) - 3*x**3*sqrt(a**2*x**2 + 1)/(128*a) - 9*x*
*2*asinh(a*x)/(32*a**2) + 9*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(32*a**3)
+ 45*x*sqrt(a**2*x**2 + 1)/(256*a**3) - 3*asinh(a*x)**3/(32*a**4) - 45*asi
nh(a*x)/(256*a**4), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x^3\text{arcsinh}(ax)^3\,dx = \int x^3\,\operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate(x^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/4*(a^3*x^6 + sqrt(a^2
*x^2 + 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x +
 (a^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3\text{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{arcsinh}(ax)^3\,dx = \int x^3\,\operatorname{asinh}(a\,x)^3\,dx$$

input
```
int(x^3*asinh(a*x)^3,x)
```

output
```
int(x^3*asinh(a*x)^3, x)
```

Reduce [F]

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx = \int asinh(ax)^3\,x^3dx$$

input

```
int(x^3*asinh(a*x)^3,x)
```

output

```
int(asinh(a*x)**3*x**3,x)
```

3.28 $\int x^2 \operatorname{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int x^2 \operatorname{arcsinh}(ax)^3\,dx = \frac{14\sqrt{1+a^2x^2}}{9a^3} - \frac{2(1+a^2x^2)^{3/2}}{27a^3} - \frac{4x\operatorname{arcsinh}(ax)}{3a^2} + \frac{2}{9}x^3\operatorname{arcsinh}(ax) + \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{3a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^3$$

output

```
14/9*(a^2*x^2+1)^(1/2)/a^3-2/27*(a^2*x^2+1)^(3/2)/a^3-4/3*x*arcsinh(a*x)/a
^2+2/9*x^3*arcsinh(a*x)+2/3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/a^3-1/3*x^2*(
a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/a+1/3*x^3*arcsinh(a*x)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

$$\int x^2 \operatorname{arcsinh}(ax)^3\,dx = \frac{-2(-20+a^2x^2)\sqrt{1+a^2x^2} + 6ax(-6+a^2x^2)\operatorname{arcsinh}(ax) - 9(-2+a^2x^2)\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2 + 9}{27a^3}$$

input

```
Integrate[x^2*ArcSinh[a*x]^3,x]
```

output

```
(-2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] + 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]
- 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 9*a^3*x^3*ArcSinh[a*
x]^3)/(27*a^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6191, 6227, 6191, 243, 53, 2009, 6213, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}(ax)^3\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\int x^2\text{arcsinh}(ax)dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}\right)$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{3}a\int \frac{x^3}{\sqrt{a^2x^2+1}}dx\right)}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\int\frac{x^2}{\sqrt{a^2x^2+1}}dx^2\right)}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}\right)$$

$$\Big\downarrow\ 53$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\int\left(\frac{\sqrt{a^2x^2+1}}{a^2}-\frac{1}{a^2\sqrt{a^2x^2+1}}\right)dx^2\right)}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}\right.$$

$$\Big\downarrow\ 2009$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2\left(a^2x^2+1\right)^{3/2}}{3a^4}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\right)}{3a}\right)$$

$$\Big\downarrow\ 6213$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\int\text{arcsinh}(ax)dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(ax)-\frac{1}{6}a\left(\frac{2(a^2x^2}{3\ldots}\right.\right.}{3a}\right.$$

$$\Big\downarrow\ 6187$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3-a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\text{arcsinh}(ax)-a\int\frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2}-\frac{2\left(\frac{1}{3}x^3\text{arcsinh}(a\right.}{}\right.$$

$$\Big\downarrow\ 241$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^3 - a\left(\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3a^2} - \frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{a^2} - \frac{2\left(x\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)}{3a^2} - 2\left(\frac{1}{3}x^3\text{arcsinh}(ax) - \frac{1}{6}\right.\right.$$

input

```
Int[x^2*ArcSinh[a*x]^3,x]
```

output

```
(x^3*ArcSinh[a*x]^3)/3 - a*((x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2)
 - (2*(-1/6*(a*((-2*Sqrt[1 + a^2*x^2])/a^4 + (2*(1 + a^2*x^2)^(3/2))/(3*a^
4))) + (x^3*ArcSinh[a*x])/3))/(3*a) - (2*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
2)/a^2 - (2*(-(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]))/a))/(3*a^2))
```

Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{x^3 a^3 \operatorname{arcsinh}(xa)^3}{3}+\frac{2 \operatorname{arcsinh}(xa)^2 \sqrt{a^2x^2+1}}{3}-\frac{x^2 a^2 \operatorname{arcsinh}(xa)^2 \sqrt{a^2x^2+1}}{3}-\frac{4xa \operatorname{arcsinh}(xa)}{3}+\frac{40\sqrt{a^2x^2+1}}{27}+\frac{2x^3a^3 \operatorname{arcsinh}(xa}{9}}{a^3}$
default	$\frac{\frac{x^3 a^3 \operatorname{arcsinh}(xa)^3}{3}+\frac{2 \operatorname{arcsinh}(xa)^2 \sqrt{a^2x^2+1}}{3}-\frac{x^2 a^2 \operatorname{arcsinh}(xa)^2 \sqrt{a^2x^2+1}}{3}-\frac{4xa \operatorname{arcsinh}(xa)}{3}+\frac{40\sqrt{a^2x^2+1}}{27}+\frac{2x^3a^3 \operatorname{arcsinh}(xa}{9}}{a^3}$
orering	$\frac{5(13a^6x^6-40a^4x^4-152a^2x^2-96) \operatorname{arcsinh}(xa)^3}{81a^6x^3}-\frac{(25a^6x^6-166a^4x^4-572a^2x^2-360)\Bigg(2x \operatorname{arcsinh}(xa)^3+\frac{3x^2 \operatorname{arcsinh}}{\sqrt{a^2x^2}}}{81a^6x^4}$

input
```
int(x^2*arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*x^3*a^3*arcsinh(x*a)^3+2/3*arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)-1/3
*x^2*a^2*arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)-4/3*x*a*arcsinh(x*a)+40/27*(a^2*
x^2+1)^(1/2)+2/9*x^3*a^3*arcsinh(x*a)-2/27*x^2*a^2*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int x^2 \text{arcsinh}(ax)^3\,dx$$
$$= \frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 9\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 6\,(a^3x^3-6\,ax)\log\,(ax}{27\,a^3}$$

input
```
integrate(x^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*
x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*x^3 - 6*a*x)*log(a*x + sq
rt(a^2*x^2 + 1)) - 2*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int x^2 \text{arcsinh}(ax)^3\,dx$$
$$= \begin{cases} \frac{x^3\operatorname{asinh}^3(ax)}{3} + \frac{2x^3\operatorname{asinh}(ax)}{9} - \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a} - \frac{2x^2\sqrt{a^2x^2+1}}{27a} - \frac{4x\operatorname{asinh}(ax)}{3a^2} + \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^3} + \frac{40\sqrt{a^2x^2}}{27a^3} \\ 0 \end{cases}$$

input
```
integrate(x**2*asinh(a*x)**3,x)
```

output
```
Piecewise((x**3*asinh(a*x)**3/3 + 2*x**3*asinh(a*x)/9 - x**2*sqrt(a**2*x**
2 + 1)*asinh(a*x)**2/(3*a) - 2*x**2*sqrt(a**2*x**2 + 1)/(27*a) - 4*x*asinh
(a*x)/(3*a**2) + 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**3) + 40*sqrt(a*
*2*x**2 + 1)/(27*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

$$\int x^2\text{arcsinh}(ax)^3\,dx = \frac{1}{3}\,x^3\,\text{arsinh}\,(ax)^3 - \frac{1}{3}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)^2 - \frac{2}{27}\,a\left(\frac{\sqrt{a^2x^2+1}x^2 - \frac{20\,\sqrt{a^2x^2+1}}{a^2}}{a^2} - \frac{3\,(a^2x^3-6\,x)\,\text{arsinh}\,(ax)}{a^3}\right)$$

input
```
integrate(x^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
1/3*x^3*arcsinh(a*x)^3 - 1/3*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
 + 1)/a^4)*arcsinh(a*x)^2 - 2/27*a*((sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x
^2 + 1)/a^2)/a^2 - 3*(a^2*x^3 - 6*x)*arcsinh(a*x)/a^3)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\operatorname{arcsinh}(ax)^3\,dx=\int x^2\,\mathrm{asinh}(a\,x)^3\,dx$$

input
```
int(x^2*asinh(a*x)^3,x)
```

output
```
int(x^2*asinh(a*x)^3, x)
```

Reduce [F]

$$\int x^2\operatorname{arcsinh}(ax)^3\,dx=\int asinh(ax)^3\,x^2dx$$

input
```
int(x^2*asinh(a*x)^3,x)
```

output
```
int(asinh(a*x)**3*x**2,x)
```

3.29 $\int x\text{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 8, antiderivative size = 97

$$\int x\text{arcsinh}(ax)^3\,dx = -\frac{3x\sqrt{1+a^2x^2}}{8a}+\frac{3\text{arcsinh}(ax)}{8a^2}+\frac{3}{4}x^2\text{arcsinh}(ax)-\frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a}+\frac{\text{arcsinh}(ax)^3}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^3$$

output

```
-3/8*x*(a^2*x^2+1)^(1/2)/a+3/8*arcsinh(a*x)/a^2+3/4*x^2*arcsinh(a*x)-3/4*x
*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/a+1/4*arcsinh(a*x)^3/a^2+1/2*x^2*arcsinh
(a*x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int x\text{arcsinh}(ax)^3\,dx = \frac{-3ax\sqrt{1+a^2x^2}+(3+6a^2x^2)\,\text{arcsinh}(ax)-6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2+(2+4a^2x^2)\,\text{arcsinh}(ax)^3}{8a^2}$$

input

```
Integrate[x*ArcSinh[a*x]^3,x]
```

output
```
(-3*a*x*Sqrt[1 + a^2*x^2] + (3 + 6*a^2*x^2)*ArcSinh[a*x] - 6*a*x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^2 + (2 + 4*a^2*x^2)*ArcSinh[a*x]^3)/(8*a^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6191, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)^3\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^3-\frac{3}{2}a\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^3-\frac{3}{2}a\left(-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int x\text{arcsinh}(ax)dx}{a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^3-\frac{3}{2}a\left(-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\int \frac{x^2}{\sqrt{a^2x^2+1}}dx}{a}-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^3-\frac{3}{2}a\left(-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{a}-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^3-$$
$$\frac{3}{2}a\left(-\frac{\int \frac{\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^3-$$
$$\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^3}{6a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2a^2}-\frac{\frac{1}{2}x^2\text{arcsinh}(ax)-\frac{1}{2}a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\text{arcsinh}(ax)}{2a^3}\right)}{a}\right)$$

input

```
Int[x*ArcSinh[a*x]^3,x]
```

output

```
(x^2*ArcSinh[a*x]^3)/2 - (3*a*((x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2
) - ArcSinh[a*x]^3/(6*a^3) - ((x^2*ArcSinh[a*x])/2 - (a*((x*Sqrt[1 + a^2*x
^2])/(2*a^2) - ArcSinh[a*x]/(2*a^3)))/2)/a))/2
```

Defintions of rubi rules used

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{\frac{\cosh\left(2 \operatorname{arcsinh}\left(x a\right)\right) \operatorname{arcsinh}\left(x a\right)^{3}}{4}-\frac{3 \sinh\left(2 \operatorname{arcsinh}\left(x a\right)\right) \operatorname{arcsinh}\left(x a\right)^{2}}{8}+\frac{3 \cosh\left(2 \operatorname{arcsinh}\left(x a\right)\right) \operatorname{arcsinh}\left(x a\right)}{8}-\frac{3 \sinh\left(2 \operatorname{arcsinh}\left(x a\right)\right)}{16}}{a^{2}}$
default	$\frac{\frac{\cosh\left(2 \operatorname{arcsinh}\left(x a\right)\right) \operatorname{arcsinh}\left(x a\right)^{3}}{4}-\frac{3 \sinh\left(2 \operatorname{arcsinh}\left(x a\right)\right) \operatorname{arcsinh}\left(x a\right)^{2}}{8}+\frac{3 \cosh\left(2 \operatorname{arcsinh}\left(x a\right)\right) \operatorname{arcsinh}\left(x a\right)}{8}-\frac{3 \sinh\left(2 \operatorname{arcsinh}\left(x a\right)\right)}{16}}{a^{2}}$
orering	$\frac{\left(15 a^{4} x^{4}+20 a^{2} x^{2}+8\right) \operatorname{arcsinh}\left(x a\right)^{3}}{16 a^{4} x^{2}}-\frac{\left(7 a^{4} x^{4}+16 a^{2} x^{2}+8\right) \left(\operatorname{arcsinh}\left(x a\right)^{3}+\frac{3 x \operatorname{arcsinh}\left(x a\right)^{2} a}{\sqrt{a^{2} x^{2}+1}}\right)}{16 x^{2} a^{4}}+ \left(a^{2} x^{2}+1\right) \left(a^{2} x^{2}+2\right.$

input
```
int(x*arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*cosh(2*arcsinh(x*a))*arcsinh(x*a)^3-3/8*sinh(2*arcsinh(x*a))*ar
csinh(x*a)^2+3/8*cosh(2*arcsinh(x*a))*arcsinh(x*a)-3/16*sinh(2*arcsinh(x*a
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int x\operatorname{arcsinh}(ax)^3\,dx =$$

$$-\frac{6\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+3\sqrt{a^2x^2+1}ax-3\,(}{8\,a^2}$$

input
```
integrate(x*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
-1/8*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*(2*a^2*x^
2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2
*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int x\operatorname{arcsinh}(ax)^3\,dx$$

$$=\begin{cases}\frac{x^2\operatorname{asinh}^3(ax)}{2}+\frac{3x^2\operatorname{asinh}(ax)}{4}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a}-\frac{3x\sqrt{a^2x^2+1}}{8a}+\frac{\operatorname{asinh}^3(ax)}{4a^2}+\frac{3\operatorname{asinh}(ax)}{8a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x*asinh(a*x)**3,x)
```

output
```
Piecewise((x**2*asinh(a*x)**3/2 + 3*x**2*asinh(a*x)/4 - 3*x*sqrt(a**2*x**2
 + 1)*asinh(a*x)**2/(4*a) - 3*x*sqrt(a**2*x**2 + 1)/(8*a) + asinh(a*x)**3/
(4*a**2) + 3*asinh(a*x)/(8*a**2), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^3\,dx = \int x\,\text{arsinh}\,(ax)^3\,dx$$

input
```
integrate(x*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/2*(a^3*x^4 + sqrt(a^2
*x^2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x +
 (a^2*x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int x\text{arcsinh}(ax)^3\,dx = \int x\,\text{arsinh}\,(ax)^3\,dx$$

input
```
integrate(x*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x*arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^3\,dx = \int x\,\text{asinh}(a\,x)^3\,dx$$

input
```
int(x*asinh(a*x)^3,x)
```

output
```
int(x*asinh(a*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int x\text{arcsinh}(ax)^3\,dx$$

$$= \frac{4asinh(ax)^3\,a^2x^2+2asinh(ax)^3-6\sqrt{a^2x^2+1}\,asinh(ax)^2\,ax+6asinh(ax)\,a^2x^2-3\sqrt{a^2x^2+1}\,ax+3\,\mathrm{lo}}{8a^2}$$

input

```
int(x*asinh(a*x)^3,x)
```

output

```
(4*asinh(a*x)**3*a**2*x**2 + 2*asinh(a*x)**3 - 6*sqrt(a**2*x**2 + 1)*asinh
(a*x)**2*a*x + 6*asinh(a*x)*a**2*x**2 - 3*sqrt(a**2*x**2 + 1)*a*x + 3*log(
sqrt(a**2*x**2 + 1) + a*x))/(8*a**2)
```

3.30 $\int \operatorname{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 6, antiderivative size = 58

$$\int \operatorname{arcsinh}(ax)^3\,dx = -\frac{6\sqrt{1+a^2x^2}}{a} + 6x\operatorname{arcsinh}(ax) - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} + x\operatorname{arcsinh}(ax)^3$$

output

```
-6*(a^2*x^2+1)^(1/2)/a+6*x*arcsinh(a*x)-3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2
/a+x*arcsinh(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)^3\,dx = -\frac{6\sqrt{1+a^2x^2}}{a} + 6x\operatorname{arcsinh}(ax) - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} + x\operatorname{arcsinh}(ax)^3$$

input

```
Integrate[ArcSinh[a*x]^3,x]
```

output

```
(-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh
[a*x]^2)/a + x*ArcSinh[a*x]^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6187, 6213, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^3\,dx$$

$$\Big\downarrow \text{6187}$$

$$x\operatorname{arcsinh}(ax)^3-3a\int \frac{x\operatorname{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\int \operatorname{arcsinh}(ax)dx}{a}\right)$$

$$\Big\downarrow \text{6187}$$

$$x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-a\int \frac{x}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)$$

$$\Big\downarrow \text{241}$$

$$x\operatorname{arcsinh}(ax)^3-3a\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a^2}-\frac{2\left(x\operatorname{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)$$

input

```
Int[ArcSinh[a*x]^3,x]
```

output

```
x*ArcSinh[a*x]^3 - 3*a*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/a^2 - (2*(-(Sqr
t[1 + a^2*x^2]/a) + x*ArcSinh[a*x]))/a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\operatorname{arcsinh}(xa)^3 xa-3\operatorname{arcsinh}(xa)^2\sqrt{a^2x^2+1}+6xa\ \operatorname{arcsinh}(xa)-6\sqrt{a^2x^2+1}}{a}$
default	$\frac{\operatorname{arcsinh}(xa)^3 xa-3\operatorname{arcsinh}(xa)^2\sqrt{a^2x^2+1}+6xa\ \operatorname{arcsinh}(xa)-6\sqrt{a^2x^2+1}}{a}$
orering	$x\operatorname{arcsinh}(xa)^3-\frac{3(a^2x^2+2)\operatorname{arcsinh}(xa)^2}{a\sqrt{a^2x^2+1}}-\frac{2(a^2x^2+1)x\left(\frac{6\ \operatorname{arcsinh}(xa)a^2}{a^2x^2+1}-\frac{3\operatorname{arcsinh}(xa)^2a^3x}{(a^2x^2+1)^{\frac{3}{2}}}\right)}{a^2}-\frac{(a^2x^2+}{}$

input
```
int(arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arcsinh(x*a)^3*x*a-3*arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)+6*x*a*arcsinh(x
*a)-6*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time $= 0.08$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 1.55$

$$\int \operatorname{arcsinh}(ax)^3\,dx$$
$$= \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 6\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) - 3\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\,\sqrt{a^2x^2}}{a}$$

input
```
integrate(arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
(a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 + 6*a*x*log(a*x + sqrt(a^2*x^2 + 1)) -
 3*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*x^2 + 1))
/a
```

Sympy [A] (verification not implemented)

Time $= 0.12$ (sec) , antiderivative size $= 54$, normalized size of antiderivative $= 0.93$

$$\int \operatorname{arcsinh}(ax)^3\,dx$$
$$= \begin{cases} x\operatorname{asinh}^3(ax) + 6x\operatorname{asinh}(ax) - \frac{3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{a} - \frac{6\sqrt{a^2x^2+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)**3,x)
```

output
```
Piecewise((x*asinh(a*x)**3 + 6*x*asinh(a*x) - 3*sqrt(a**2*x**2 + 1)*asinh(
a*x)**2/a - 6*sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \operatorname{arcsinh}(ax)^3\,dx = x\operatorname{arsinh}(ax)^3 - \frac{3\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^2}{a} + \frac{6\left(ax\operatorname{arsinh}(ax) - \sqrt{a^2x^2+1}\right)}{a}$$

input

```
integrate(arcsinh(a*x)^3,x, algorithm="maxima")
```

output

```
x*arcsinh(a*x)^3 - 3*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a + 6*(a*x*arcsinh(a
*x) - sqrt(a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\begin{aligned}&\int \operatorname{arcsinh}(ax)^3\,dx \\ &= x\log\left(ax+\sqrt{a^2x^2+1}\right)^3 \\ &\quad -3a\left(\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2} - \frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right) - \frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\end{aligned}$$

input

```
integrate(arcsinh(a*x)^3,x, algorithm="giac")
```

output

```
x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a
^2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)
/a)/a)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^3\,dx = \int \operatorname{asinh}(a\,x)^3\,dx$$

input
```
int(asinh(a*x)^3,x)
```

output
```
int(asinh(a*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \operatorname{arcsinh}(ax)^3\,dx = \frac{asinh(ax)^3\,ax - 3\sqrt{a^2x^2+1}\,asinh(ax)^2 + 6asinh(ax)\,ax - 6\sqrt{a^2x^2+1}}{a}$$

input
```
int(asinh(a*x)^3,x)
```

output
```
(asinh(a*x)**3*a*x - 3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2 + 6*asinh(a*x)*a*
x - 6*sqrt(a**2*x**2 + 1))/a
```

3.31 $\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = & -\frac{1}{4}\operatorname{arcsinh}(ax)^4+\operatorname{arcsinh}(ax)^3\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\ & +\frac{3}{2}\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\ & -\frac{3}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\\ & +\frac{3}{4}\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-1/4*arcsinh(a*x)^4+arcsinh(a*x)^3*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*arc
sinh(a*x)^2*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*arcsinh(a*x)*polylog(
3,(a*x+(a^2*x^2+1)^(1/2))^2)+3/4*polylog(4,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = -\frac{1}{4}\operatorname{arcsinh}(ax)^4 + \operatorname{arcsinh}(ax)^3\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + \frac{3}{2}\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) - \frac{3}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right) + \frac{3}{4}\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)$$

input

```
Integrate[ArcSinh[a*x]^3/x,x]
```

output

```
-1/4*ArcSinh[a*x]^4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcS
inh[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3,
E^(2*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6190, 3042, 26, 4199, 25, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx$$

$$\Big\downarrow \text{6190}$$

$$\int \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{ax}\,d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\text{arcsinh}(ax)^3\tan\left(\frac{\pi}{2}+i\text{arcsinh}(ax)\right)d\text{arcsinh}(ax)$$

$$\Big\downarrow \; 26$$

$$-i\int \text{arcsinh}(ax)^3\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)$$

$$\Big\downarrow \; 4199$$

$$-i\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^3}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{4}i\text{arcsinh}(ax)^4\right)$$

$$\Big\downarrow \; 25$$

$$-i\left(-2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^3}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{4}i\text{arcsinh}(ax)^4\right)$$

$$\Big\downarrow \; 2620$$

$$-i\Bigg(-2i\Bigg(\frac{3}{2}\int \text{arcsinh}(ax)^2\log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)\Bigg)-\frac{1}{4}i\text{arc}$$

$$\Big\downarrow \; 3011$$

$$-i\Bigg(-2i\Bigg(\frac{3}{2}\Bigg(\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)\Bigg)$$

$$\Big\downarrow \; 7163$$

$$-i\Bigg(-2i\Bigg(\frac{3}{2}\Bigg(-\frac{1}{2}\int \text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsin}$$

$$\Big\downarrow \; 2720$$

$$-i\Bigg(-2i\Bigg(\frac{3}{2}\Bigg(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\Big(2,e^{2\text{arcsinh}(ax}$$

$$\Big\downarrow \; 7143$$

$$-i\Bigg(-2i\Bigg(\frac{3}{2}\Bigg(-\frac{1}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{4}\,\text{PolyLog}\Big($$

input

```
Int[ArcSinh[a*x]^3/x,x]
```

output
```
(-I)*((-1/4*I)*ArcSinh[a*x]^4 - (2*I)*(-1/2*(ArcSinh[a*x]^3*Log[1 - E^(2*A
rcSinh[a*x])]) + (3*(-1/2*(ArcSinh[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])
+ (ArcSinh[a*x]*PolyLog[3, E^(2*ArcSinh[a*x])])/2 - PolyLog[4, E^(2*ArcSin
h[a*x])]/4))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.46

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(xa)^4}{4}+\operatorname{arcsinh}(xa)^3\ln(1+xa+\sqrt{a^2x^2+1})+3\operatorname{arcsinh}(xa)^2\operatorname{polylog}(2,-x$
default	$-\frac{\operatorname{arcsinh}(xa)^4}{4}+\operatorname{arcsinh}(xa)^3\ln(1+xa+\sqrt{a^2x^2+1})+3\operatorname{arcsinh}(xa)^2\operatorname{polylog}(2,-x$

input
```
int(arcsinh(x*a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arcsinh(x*a)^4+arcsinh(x*a)^3*ln(1+x*a+(a^2*x^2+1)^(1/2))+3*arcsinh(x
*a)^2*polylog(2,-x*a-(a^2*x^2+1)^(1/2))-6*arcsinh(x*a)*polylog(3,-x*a-(a^2
*x^2+1)^(1/2))+6*polylog(4,-x*a-(a^2*x^2+1)^(1/2))+arcsinh(x*a)^3*ln(1-x*a
-(a^2*x^2+1)^(1/2))+3*arcsinh(x*a)^2*polylog(2,x*a+(a^2*x^2+1)^(1/2))-6*ar
csinh(x*a)*polylog(3,x*a+(a^2*x^2+1)^(1/2))+6*polylog(4,x*a+(a^2*x^2+1)^(1
/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{x}\,dx$$

input
```
integrate(asinh(a*x)**3/x,x)
```

output
```
Integral(asinh(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x}\,dx$$

input
```
int(asinh(a*x)^3/x,x)
```

output
```
int(asinh(a*x)^3/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{asinh(ax)^3}{x}dx$$

input

```
int(asinh(a*x)^3/x,x)
```

output

```
int(asinh(a*x)**3/x,x)
```

3.32 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = & -\frac{\operatorname{arcsinh}(ax)^3}{x} - 6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ & - 6a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ & + 6a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ & + 6a\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) - 6a\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-arcsinh(a*x)^3/x-6*a*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-6*a*ar
csinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*
x+(a^2*x^2+1)^(1/2))+6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*a*polylog(3,a
*x+(a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = a\bigg(-\frac{\operatorname{arcsinh}(ax)^3}{ax} + 3\operatorname{arcsinh}(ax)^2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) - 3\operatorname{arcsinh}(ax)^2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) + 6\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) - 6\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right) + 6\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) - 6\operatorname{PolyLog}\left(3,e^{-\operatorname{arcsinh}(ax)}\right)\bigg)$$

input
```
Integrate[ArcSinh[a*x]^3/x^2,x]
```

output
```
a*(-(ArcSinh[a*x]^3/(a*x)) + 3*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] -
 3*ArcSinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 6*ArcSinh[a*x]*PolyLog[2, -
E^(-ArcSinh[a*x])] - 6*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])] + 6*Poly
Log[3, -E^(-ArcSinh[a*x])] - 6*PolyLog[3, E^(-ArcSinh[a*x])])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6191, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx$$

$$\Big\downarrow \text{6191}$$

$$3a\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx - \frac{\operatorname{arcsinh}(ax)^3}{x}$$

$$\Big\downarrow \text{6231}$$

$$3a\int \frac{\operatorname{arcsinh}(ax)^2}{ax}d\operatorname{arcsinh}(ax)-\frac{\operatorname{arcsinh}(ax)^3}{x}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+3a\int i\operatorname{arcsinh}(ax)^2\csc(i\operatorname{arcsinh}(ax))d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \; 26$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+3ia\int \operatorname{arcsinh}(ax)^2\csc(i\operatorname{arcsinh}(ax))d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \; 4670$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+$$

$$3ia\Bigl(2i\int \operatorname{arcsinh}(ax)\log\Bigl(1-e^{\operatorname{arcsinh}(ax)}\Bigr)d\operatorname{arcsinh}(ax)-2i\int \operatorname{arcsinh}(ax)\log\Bigl(1+e^{\operatorname{arcsinh}(ax)}\Bigr)d\operatorname{arcsinh}(ax)+$$

$$\Big\downarrow \; 3011$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+$$

$$3ia\Bigl(-2i\Bigl(\int \operatorname{PolyLog}\Bigl(2,-e^{\operatorname{arcsinh}(ax)}\Bigr)d\operatorname{arcsinh}(ax)-\operatorname{arcsinh}(ax)\operatorname{PolyLog}\Bigl(2,-e^{\operatorname{arcsinh}(ax)}\Bigr)\Bigr)+2i\Bigl(\int \operatorname{PolyLo}$$

$$\Big\downarrow \; 2720$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+$$

$$3ia\Bigl(-2i\Bigl(\int e^{-\operatorname{arcsinh}(ax)}\operatorname{PolyLog}\Bigl(2,-e^{\operatorname{arcsinh}(ax)}\Bigr)de^{\operatorname{arcsinh}(ax)}-\operatorname{arcsinh}(ax)\operatorname{PolyLog}\Bigl(2,-e^{\operatorname{arcsinh}(ax)}\Bigr)\Bigr)+2i\Bigl($$

$$\Big\downarrow \; 7143$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+$$

$$3ia\Bigl(2i\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\Bigl(e^{\operatorname{arcsinh}(ax)}\Bigr)-2i\Bigl(\operatorname{PolyLog}\Bigl(3,-e^{\operatorname{arcsinh}(ax)}\Bigr)-\operatorname{arcsinh}(ax)\operatorname{PolyLog}\Bigl(2,-e^{\operatorname{arcsinh}(ax)}$$

input
```
Int[ArcSinh[a*x]^3/x^2,x]
```

output
```
-(ArcSinh[a*x]^3/x) + (3*I)*a*((2*I)*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]
] - (2*I)*(-(ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]]) + PolyLog[3, -E^Arc
Sinh[a*x]]) + (2*I)*(-(ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]]) + PolyLog[
3, E^ArcSinh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.92

method	result
derivativedivides	$a\left(-\frac{\operatorname{arcsinh}(xa)^3}{xa} - 3\operatorname{arcsinh}(xa)^2\ln\left(1+xa+\sqrt{a^2x^2+1}\right) - 6\operatorname{arcsinh}(xa)\operatorname{polylog}(2$
default	$a\left(-\frac{\operatorname{arcsinh}(xa)^3}{xa} - 3\operatorname{arcsinh}(xa)^2\ln\left(1+xa+\sqrt{a^2x^2+1}\right) - 6\operatorname{arcsinh}(xa)\operatorname{polylog}(2$

input
```
int(arcsinh(x*a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-arcsinh(x*a)^3/x/a-3*arcsinh(x*a)^2*ln(1+x*a+(a^2*x^2+1)^(1/2))-6*arcs
inh(x*a)*polylog(2,-x*a-(a^2*x^2+1)^(1/2))+6*polylog(3,-x*a-(a^2*x^2+1)^(1
/2))+3*arcsinh(x*a)^2*ln(1-x*a-(a^2*x^2+1)^(1/2))+6*arcsinh(x*a)*polylog(2
,x*a+(a^2*x^2+1)^(1/2))-6*polylog(3,x*a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{x^2}\,dx$$

input
```
integrate(asinh(a*x)**3/x**2,x)
```

output
```
Integral(asinh(a*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
-log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2 + sqrt(a^2*x^2 +
1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^4 + a*x^2 + (a^2*x^3 +
 x)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx=\int \frac{{\mathrm{asinh}\left(a\,x\right)}^3}{x^2}\,dx$$

input
```
int(asinh(a*x)^3/x^2,x)
```

output
```
int(asinh(a*x)^3/x^2, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx=\int \frac{\mathit{asinh}(ax)^{3}}{x^{2}}dx$$

input
```
int(asinh(a*x)^3/x^2,x)
```

output
```
int(asinh(a*x)**3/x**2,x)
```

3.33 $\int \frac{\textbf{arcsinh}(ax)^3}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = -\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2 - \frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x} - \frac{\operatorname{arcsinh}(ax)^3}{2x^2} + 3a^2\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + \frac{3}{2}a^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)$$

output

```
-3/2*a^2*arcsinh(a*x)^2-3/2*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/x-1/2*arcsi
nh(a*x)^3/x^2+3*a^2*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*a^2*p
olylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = -\frac{\operatorname{arcsinh}(ax)^3 - 3ax\left(\operatorname{arcsinh}(ax)\left(\left(ax-\sqrt{1+a^2x^2}\right)\operatorname{arcsinh}(ax) + 2ax\log\left(1-e^{-2\operatorname{arcsinh}(ax)}\right)\right) - ax\,\mathrm{P}}{2x^2}$$

input

```
Integrate[ArcSinh[a*x]^3/x^3,x]
```

output

```
-1/2*(ArcSinh[a*x]^3 - 3*a*x*(ArcSinh[a*x]*((a*x - Sqrt[1 + a^2*x^2])*ArcS
inh[a*x] + 2*a*x*Log[1 - E^(-2*ArcSinh[a*x])]) - a*x*PolyLog[2, E^(-2*ArcS
inh[a*x])]))/x^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6191, 6215, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{3}{2}a\int \frac{\text{arcsinh}(ax)^2}{x^2\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 6215}$$

$$\frac{3}{2}a\left(2a\int \frac{\text{arcsinh}(ax)}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}\right)-\frac{\text{arcsinh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 6190}$$

$$\frac{3}{2}a\left(2a\int \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}\right)-\frac{\text{arcsinh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}+2a\int -i\text{arcsinh}(ax)\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)\right)$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\int \text{arcsinh}(ax)\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)\right)$$

$$\Big\downarrow 4199$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)\right)$$

$$\Big\downarrow 25$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)\right)$$

$$\Big\downarrow 2620$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2a}\right.\right.\right.\right.$$

$$\Big\downarrow 2715$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\log\left(1-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh}(ax\right.\right.\right.$$

$$\Big\downarrow 2838$$

$$-\frac{\text{arcsinh}(ax)^3}{2x^2}+$$
$$\frac{3}{2}a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)\right.\right.\right.$$

input

```
Int[ArcSinh[a*x]^3/x^3,x]
```

output

```
-1/2*ArcSinh[a*x]^3/x^2 + (3*a*(-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x) -
(2*I)*a*((-1/2*I)*ArcSinh[a*x]^2 - (2*I)*(-1/2*(ArcSinh[a*x]*Log[1 - E^(2*
ArcSinh[a*x])]) - PolyLog[2, E^(2*ArcSinh[a*x])]/4))))/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.57

method	result
derivativedivides	$a^{2}\left(-\frac{\operatorname{arcsinh}\left(xa\right)^{2}\left(3xa\sqrt{a^{2}x^{2}+1}-3a^{2}x^{2}+\operatorname{arcsinh}\left(xa\right)\right)}{2x^{2}a^{2}}-3\operatorname{arcsinh}\left(xa\right)^{2}+3\operatorname{arcsinh}\left(xa\right)\ln\left(1+\right.\right.$
default	$a^{2}\left(-\frac{\operatorname{arcsinh}\left(xa\right)^{2}\left(3xa\sqrt{a^{2}x^{2}+1}-3a^{2}x^{2}+\operatorname{arcsinh}\left(xa\right)\right)}{2x^{2}a^{2}}-3\operatorname{arcsinh}\left(xa\right)^{2}+3\operatorname{arcsinh}\left(xa\right)\ln\left(1+\right.\right.$

input
```
int(arcsinh(x*a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arcsinh(x*a)^2*(3*x*a*(a^2*x^2+1)^(1/2)-3*a^2*x^2+arcsinh(x*a))/
x^2/a^2-3*arcsinh(x*a)^2+3*arcsinh(x*a)*ln(1+x*a+(a^2*x^2+1)^(1/2))+3*poly
log(2,-x*a-(a^2*x^2+1)^(1/2))+3*arcsinh(x*a)*ln(1-x*a-(a^2*x^2+1)^(1/2))+3
*polylog(2,x*a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{x^3}\,dx$$

input
```
integrate(asinh(a*x)**3/x**3,x)
```

output
```
Integral(asinh(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(a*x + sqrt(a^2*x^2 + 1))^3/x^2 + integrate(3/2*(a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^5 + a*x^3 + (a
^2*x^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{{\mathrm{asinh}\left(a\,x\right)}^3}{x^3}\,dx$$

input
```
int(asinh(a*x)^3/x^3,x)
```

output
```
int(asinh(a*x)^3/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{asinh(ax)^3}{x^3}dx$$

input
```
int(asinh(a*x)^3/x^3,x)
```

output
```
int(asinh(a*x)**3/x**3,x)
```

3.34 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 151

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = {} & -\frac{a^2\operatorname{arcsinh}(ax)}{x}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2}-\frac{\operatorname{arcsinh}(ax)^3}{3x^3}\\ & +a^3\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)-a^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)\\ & +a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ & -a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ & -a^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)+a^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-a^2*arcsinh(a*x)/x-1/2*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/x^2-1/3*arcsinh
(a*x)^3/x^3+a^3*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-a^3*arctanh(
(a^2*x^2+1)^(1/2))+a^3*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^3*
arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-a^3*polylog(3,-a*x-(a^2*x^2+
1)^(1/2))+a^3*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \frac{1}{48}a^3\Bigg(& -24\operatorname{arcsinh}(ax)\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
& + 4\operatorname{arcsinh}(ax)^3\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
& - 6\operatorname{arcsinh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
& - ax\operatorname{arcsinh}(ax)^3\operatorname{csch}^4\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
& - 24\operatorname{arcsinh}(ax)^2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) \\
& + 24\operatorname{arcsinh}(ax)^2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) \\
& + 48\log\left(\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\right) \\
& - 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) \\
& + 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right) \\
& - 48\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) + 48\operatorname{PolyLog}\left(3,e^{-\operatorname{arcsinh}(ax)}\right) \\
& - 6\operatorname{arcsinh}(ax)^2\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
& - \frac{16\operatorname{arcsinh}(ax)^3\sinh^4\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)}{a^3x^3} \\
& + 24\operatorname{arcsinh}(ax)\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
& - 4\operatorname{arcsinh}(ax)^3\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\Bigg)
\end{aligned}
$$

input
```
Integrate[ArcSinh[a*x]^3/x^4,x]
```

output

```
(a^3*(-24*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] + 4*ArcSinh[a*x]^3*Coth[ArcSin
h[a*x]/2] - 6*ArcSinh[a*x]^2*Csch[ArcSinh[a*x]/2]^2 - a*x*ArcSinh[a*x]^3*C
sch[ArcSinh[a*x]/2]^4 - 24*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 24*
ArcSinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 48*Log[Tanh[ArcSinh[a*x]/2]] -
 48*ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] + 48*ArcSinh[a*x]*PolyLog[
2, E^(-ArcSinh[a*x])] - 48*PolyLog[3, -E^(-ArcSinh[a*x])] + 48*PolyLog[3,
E^(-ArcSinh[a*x])] - 6*ArcSinh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 - (16*ArcSinh
[a*x]^3*Sinh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]*Tanh[ArcSinh[a
*x]/2] - 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/48
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {6191, 6224, 6191, 243, 73, 221, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{6191}$$

$$a\int \frac{\text{arcsinh}(ax)^2}{x^3\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{6224}$$

$$a\left(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx+a\int \frac{\text{arcsinh}(ax)}{x^2}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}\right)-\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{6191}$$

$$a\left(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx+a\left(a\int \frac{1}{x\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}\right)-\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$a\Bigg(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx+a\Bigg(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{\text{arcsinh}(ax)}{x}\Bigg)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}\Bigg)-\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$\Big\downarrow$ 73

$$a\Bigg(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx+a\Bigg(\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}-\frac{\text{arcsinh}(ax)}{x}\Bigg)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}\Bigg)-\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$\Big\downarrow$ 221

$$a\Bigg(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{x\sqrt{a^2x^2+1}}dx+a\Bigg(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\Bigg)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}\Bigg)-\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$\Big\downarrow$ 6231

$$a\Bigg(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^2}{ax}d\text{arcsinh}(ax)+a\Bigg(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\Bigg)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x^2}$$
$$\frac{\text{arcsinh}(ax)^3}{3x^3}$$

$\Big\downarrow$ 3042

$$-\frac{\text{arcsinh}(ax)^3}{3x^3}+$$
$$a\Bigg(-\frac{1}{2}a^2\int i\text{arcsinh}(ax)^2\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)+a\Bigg(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\Bigg)-$$

$\Big\downarrow$ 26

$$-\frac{\text{arcsinh}(ax)^3}{3x^3}+$$
$$a\Bigg(-\frac{1}{2}ia^2\int \text{arcsinh}(ax)^2\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)+a\Bigg(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\text{arcsinh}(ax)}{x}\Bigg)-$$

$\Big\downarrow$ 4670

$$-\frac{\operatorname{arcsinh}(ax)^3}{3x^3}+$$
$$a\left(-\frac{1}{2}ia^2\left(2i\int \operatorname{arcsinh}(ax)\log\left(1-e^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)-2i\int \operatorname{arcsinh}(ax)\log\left(1+e^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3011}$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{3x^3}+$$
$$a\left(-\frac{1}{2}ia^2\left(-2i\left(\int \operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)-\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\right)+2i\left(\int\right.\right.\right.$$

$$\Big\downarrow \textcolor{blue}{2720}$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{3x^3}+$$
$$a\left(-\frac{1}{2}ia^2\left(-2i\left(\int e^{-\operatorname{arcsinh}(ax)}\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)de^{\operatorname{arcsinh}(ax)}-\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\right)\right.\right.$$

$$\Big\downarrow \textcolor{blue}{7143}$$

$$-\frac{\operatorname{arcsinh}(ax)^3}{3x^3}+$$
$$a\left(-\frac{1}{2}ia^2\left(2i\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)-2i\left(\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)-\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arc}}\right.\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^3/x^4,x]
```

output
```
-1/3*ArcSinh[a*x]^3/x^3 + a*(-1/2*(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x^2 +
 a*(-(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]) - (I/2)*a^2*((2*I)*A
rcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - (2*I)*(-(ArcSinh[a*x]*PolyLog[2, -
E^ArcSinh[a*x]]) + PolyLog[3, -E^ArcSinh[a*x]]) + (2*I)*(-(ArcSinh[a*x]*Po
lyLog[2, E^ArcSinh[a*x]]) + PolyLog[3, E^ArcSinh[a*x]])))
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.40

method	result
derivativedivides	$a^{3} \left(-\frac{\operatorname{arcsinh}(xa) \left(3 \operatorname{arcsinh}(xa) \sqrt{a^{2} x^{2}+1}\, xa+2 \operatorname{arcsinh}(xa)^{2}+6 a^{2} x^{2}\right)}{6 x^{3} a^{3}}+\frac{\operatorname{arcsinh}(xa)^{2} \ln\left(1+xa+\sqrt{a^{2} x^{2}+1}\right)}{2}+\right.$
default	$a^{3} \left(-\frac{\operatorname{arcsinh}(xa) \left(3 \operatorname{arcsinh}(xa) \sqrt{a^{2} x^{2}+1}\, xa+2 \operatorname{arcsinh}(xa)^{2}+6 a^{2} x^{2}\right)}{6 x^{3} a^{3}}+\frac{\operatorname{arcsinh}(xa)^{2} \ln\left(1+xa+\sqrt{a^{2} x^{2}+1}\right)}{2}+\right.$

input

```
int(arcsinh(x*a)^3/x^4,x,method=_RETURNVERBOSE)
```

output

```
a^3*(-1/6/x^3/a^3*arcsinh(x*a)*(3*arcsinh(x*a)*(a^2*x^2+1)^(1/2)*x*a+2*arc
sinh(x*a)^2+6*a^2*x^2)+1/2*arcsinh(x*a)^2*ln(1+x*a+(a^2*x^2+1)^(1/2))+arcs
inh(x*a)*polylog(2,-x*a-(a^2*x^2+1)^(1/2))-polylog(3,-x*a-(a^2*x^2+1)^(1/2
))-1/2*arcsinh(x*a)^2*ln(1-x*a-(a^2*x^2+1)^(1/2))-arcsinh(x*a)*polylog(2,x
*a+(a^2*x^2+1)^(1/2))+polylog(3,x*a+(a^2*x^2+1)^(1/2))-2*arctanh(x*a+(a^2*
x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\, dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{3}}{x^{4}}\, dx$$

input

```
integrate(arcsinh(a*x)^3/x^4,x, algorithm="fricas")
```

output

```
integral(arcsinh(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{x^4}\,dx$$

input
```
integrate(asinh(a*x)**3/x**4,x)
```

output
```
Integral(asinh(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
-1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/x^3 + integrate((a^3*x^2 + sqrt(a^2*x^
2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^6 + a*x^4 + (a^2*x
^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x^4}\,dx$$

input
```
int(asinh(a*x)^3/x^4,x)
```

output
```
int(asinh(a*x)^3/x^4, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{asinh(ax)^3}{x^4}dx$$

input
```
int(asinh(a*x)^3/x^4,x)
```

output
```
int(asinh(a*x)**3/x**4,x)
```

3.35 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 159

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = &-\frac{a^3\sqrt{1+a^2x^2}}{4x}-\frac{a^2\operatorname{arcsinh}(ax)}{4x^2}+\frac{1}{2}a^4\operatorname{arcsinh}(ax)^2\\ &-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4x^3}+\frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}\\ &-\frac{\operatorname{arcsinh}(ax)^3}{4x^4}-a^4\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\ &-\frac{1}{2}a^4\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-1/4*a^3*(a^2*x^2+1)^(1/2)/x-1/4*a^2*arcsinh(a*x)/x^2+1/2*a^4*arcsinh(a*x)
^2-1/4*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2/x^3+1/2*a^3*(a^2*x^2+1)^(1/2)*ar
csinh(a*x)^2/x-1/4*arcsinh(a*x)^3/x^4-a^4*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+
1)^(1/2))^2)-1/2*a^4*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \frac{1}{4}\left(-\frac{\operatorname{arcsinh}(ax)^3}{x^4} + a^4\left(-\frac{\sqrt{1+a^2x^2}\left(1+\left(-2+\frac{1}{a^2x^2}\right)\operatorname{arcsinh}(ax)^2\right)}{ax} - \operatorname{arcsinh}(ax)\left(\frac{1}{a^2x^2}+2\operatorname{arcsinh}(ax)+4\log\left(1-e^{-2\operatorname{arcsinh}(ax)}\right)\right) + 2\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(ax)}\right)\right)\right)$$

input

```
Integrate[ArcSinh[a*x]^3/x^5,x]
```

output

```
(-(ArcSinh[a*x]^3/x^4) + a^4*(-((Sqrt[1 + a^2*x^2]*(1 + (-2 + 1/(a^2*x^2))
*ArcSinh[a*x]^2))/(a*x)) - ArcSinh[a*x]*(1/(a^2*x^2) + 2*ArcSinh[a*x] + 4*
Log[1 - E^(-2*ArcSinh[a*x])]) + 2*PolyLog[2, E^(-2*ArcSinh[a*x])]))/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {6191, 6224, 6191, 242, 6215, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{3}{4}a\int \frac{\operatorname{arcsinh}(ax)^2}{x^4\sqrt{a^2x^2+1}}dx - \frac{\operatorname{arcsinh}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 6224}$$

$$\frac{3}{4}a\left(-\frac{2}{3}a^2\int\frac{\text{arcsinh}(ax)^2}{x^2\sqrt{a^2x^2+1}}dx+\frac{2}{3}a\int\frac{\text{arcsinh}(ax)}{x^3}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3x^3}\right)-\frac{\text{arcsinh}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{3}{4}a\left(-\frac{2}{3}a^2\int\frac{\text{arcsinh}(ax)^2}{x^2\sqrt{a^2x^2+1}}dx+\frac{2}{3}a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)}{2x^2}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3x^3}\right)-\frac{\text{arcsinh}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{3}{4}a\left(-\frac{2}{3}a^2\int\frac{\text{arcsinh}(ax)^2}{x^2\sqrt{a^2x^2+1}}dx+\frac{2}{3}a\left(-\frac{a\sqrt{a^2x^2+1}}{2x}-\frac{\text{arcsinh}(ax)}{2x^2}\right)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{3x^3}\right)-\frac{\text{arcsinh}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 6215}$$

$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(2a\int\frac{\text{arcsinh}(ax)}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}\right)+\frac{2}{3}a\left(-\frac{a\sqrt{a^2x^2+1}}{2x}-\frac{\text{arcsinh}(ax)}{2x^2}\right)-\frac{\sqrt{a^2x^2+}}{}\right.$$
$$-\frac{\text{arcsinh}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 6190}$$

$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(2a\int\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}\right)+\frac{2}{3}a\left(-\frac{a\sqrt{a^2x^2+1}}{2x}-\frac{\text{arcsi}}{2}\right.\right.$$
$$-\frac{\text{arcsinh}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}+2a\int-i\text{arcsinh}(ax)\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)\right)+\frac{2}{3}a\left(-\frac{a}{}\right.\right.$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\int \text{arcsinh}(ax)\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)\right)+\frac{2}{3}a\left(-\frac{a\sqrt{}}{}\right.\right.$$

$$\Big\downarrow \ 4199$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)\right)\right.$$

$$\Big\downarrow \ 25$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\text{arcsinh}(ax)^2\right)\right)\right.$$

$$\Big\downarrow \ 2620$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)\log\left(\right.\right.\right.\right.\right.$$

$$\Big\downarrow \ 2715$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\log\left(1-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}-\frac{1}{2}\text{arcs}\right.\right.\right.\right.$$

$$\Big\downarrow \ 2838$$

$$-\frac{\text{arcsinh}(ax)^3}{4x^4}+$$
$$\frac{3}{4}a\left(-\frac{2}{3}a^2\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{x}-2ia\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsi}}\right.\right.\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^3/x^5,x]
```

output
```
-1/4*ArcSinh[a*x]^3/x^4 + (3*a*(-1/3*(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x^
3 + (2*a*(-1/2*(a*Sqrt[1 + a^2*x^2])/x - ArcSinh[a*x]/(2*x^2)))/3 - (2*a^2
*(-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x) - (2*I)*a*((-1/2*I)*ArcSinh[a*x]
^2 - (2*I)*(-1/2*(ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])]) - PolyLog[2, E
^(2*ArcSinh[a*x])]/4))))/3))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6224

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34

method	result
derivativedivides	$a^{4} \left(-\frac{-2 x^{3} a^{3} \operatorname{arcsinh}(xa)^{2} \sqrt{a^{2} x^{2}+1}+2 x^{4} a^{4} \operatorname{arcsinh}(xa)^{2}+\operatorname{arcsinh}(xa)^{2} \sqrt{a^{2} x^{2}+1}\, xa+x^{3} a^{3} \sqrt{a^{2} x^{2}+1}-a^{4} x^{4}+\mathrm{arcsi}}{4 x^{4} a^{4}}\right.$
default	$a^{4} \left(-\frac{-2 x^{3} a^{3} \operatorname{arcsinh}(xa)^{2} \sqrt{a^{2} x^{2}+1}+2 x^{4} a^{4} \operatorname{arcsinh}(xa)^{2}+\operatorname{arcsinh}(xa)^{2} \sqrt{a^{2} x^{2}+1}\, xa+x^{3} a^{3} \sqrt{a^{2} x^{2}+1}-a^{4} x^{4}+\mathrm{arcsi}}{4 x^{4} a^{4}}\right.$

input
```
int(arcsinh(x*a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/4*(-2*x^3*a^3*arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)+2*x^4*a^4*arcsinh(x
*a)^2+arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)*x*a+x^3*a^3*(a^2*x^2+1)^(1/2)-a^4*x
^4+arcsinh(x*a)^3+arcsinh(x*a)*x^2*a^2)/x^4/a^4+arcsinh(x*a)^2-arcsinh(x*a
)*ln(1+x*a+(a^2*x^2+1)^(1/2))-polylog(2,-x*a-(a^2*x^2+1)^(1/2))-arcsinh(x*
a)*ln(1-x*a-(a^2*x^2+1)^(1/2))-polylog(2,x*a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^5}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^5,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{x^5}\,dx$$

input
```
integrate(asinh(a*x)**3/x**5,x)
```

output
```
Integral(asinh(a*x)**3/x**5, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x^5}\,dx$$

input
```
integrate(arcsinh(a*x)^3/x^5,x, algorithm="maxima")
```

output
```
-1/4*log(a*x + sqrt(a^2*x^2 + 1))^3/x^4 + integrate(3/4*(a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^7 + a*x^5 + (a
^2*x^6 + x^4)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx=\int \frac{\operatorname{asinh}(a\,x)^3}{x^5}\,dx$$

input
```
int(asinh(a*x)^3/x^5,x)
```

output
```
int(asinh(a*x)^3/x^5, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx=\int \frac{asinh(ax)^3}{x^5}dx$$

input
```
int(asinh(a*x)^3/x^5,x)
```

output
```
int(asinh(a*x)**3/x**5,x)
```

3.36 $\int x^4 \textbf{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 244

$$\begin{aligned}\int x^4 \text{arcsinh}(ax)^4 \, dx = {} & \frac{16576x}{5625a^4} - \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{5625a^5} \\ & + \frac{1088x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{625a} \\ & + \frac{32x\text{arcsinh}(ax)^2}{25a^4} - \frac{16x^3\text{arcsinh}(ax)^2}{75a^2} + \frac{12}{125}x^5\text{arcsinh}(ax)^2 \\ & - \frac{32\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{75a^5} + \frac{16x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{75a^3} \\ & - \frac{4x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^4\end{aligned}$$

output

```
16576/5625*x/a^4-1088/16875*x^3/a^2+24/3125*x^5-16576/5625*(a^2*x^2+1)^(1/
2)*arcsinh(a*x)/a^5+1088/5625*x^2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a^3-24/62
5*x^4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a+32/25*x*arcsinh(a*x)^2/a^4-16/75*x^
3*arcsinh(a*x)^2/a^2+12/125*x^5*arcsinh(a*x)^2-32/75*(a^2*x^2+1)^(1/2)*arc
sinh(a*x)^3/a^5+16/75*x^2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/a^3-4/25*x^4*(a
^2*x^2+1)^(1/2)*arcsinh(a*x)^3/a+1/5*x^5*arcsinh(a*x)^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.61

$$\int x^4\text{arcsinh}(ax)^4\,dx$$
$$=\frac{8ax(31080-680a^2x^2+81a^4x^4)-120\sqrt{1+a^2x^2}(2072-136a^2x^2+27a^4x^4)\,\text{arcsinh}(ax)+900ax(120}{8}$$

input
```
Integrate[x^4*ArcSinh[a*x]^4,x]
```

output
```
(8*a*x*(31080 - 680*a^2*x^2 + 81*a^4*x^4) - 120*Sqrt[1 + a^2*x^2]*(2072 -
136*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] + 900*a*x*(120 - 20*a^2*x^2 + 9*a^4
*x^4)*ArcSinh[a*x]^2 - 4500*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4)*
ArcSinh[a*x]^3 + 16875*a^5*x^5*ArcSinh[a*x]^4)/(84375*a^5)
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.65, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {6191, 6227, 6191, 6227, 15, 6191, 6213, 6187, 6213, 24, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\text{arcsinh}(ax)^4\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\int \frac{x^5\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-$$
$$\frac{4}{5}a\left(-\frac{4\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{5a^2}-\frac{3\int x^4\text{arcsinh}(ax)^2dx}{5a}+\frac{x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{5a^2}\right)$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{3\left(\frac{1}{5}x^5\text{arcsinh}(ax)^2-\frac{2}{5}a\int\frac{x^5\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{5a}-\frac{4\int\frac{x^3\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{5a^2}+\frac{x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{5a^2}\right)$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{4\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2\text{arcsinh}(ax)^2dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{1}{5}x^5\text{arcsinh}(ax)^2-\frac{2}{5}a\left(-\frac{4\int}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{4\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2\text{arcsinh}(ax)^2dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{1}{5}x^5\text{arcsinh}(ax)^2-\frac{2}{5}a\left(-\frac{4\int}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{4\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{1}{5}x^5\text{arcsin}\right.}{}\right.$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\int \text{arcsinh}(ax)^2dx}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(a}{3a^2}\right.}{5a^2}\right.$$

$$\Big\downarrow \; 6187$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\right.}{5a^2}\right.$$

$$\Big\downarrow \; 6213$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-\frac{4}{5}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int \frac{x^3\text{arcsi}}{\sqrt{a^2x}}\right.}{5a^2}\right.$$

$$\Big\downarrow \; 24$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)^4-$$

$$\frac{4}{5}a\left(-\frac{4\left(-\frac{\frac{1}{3}x^3\operatorname{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\operatorname{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\operatorname{arcsinh}(ax)^2\right.}{}\right.}{3a^2}\right.}{5a^2}\right.$$

$$\Big\downarrow 6227$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)^4-$$

$$\frac{4}{5}a\left(-\frac{4\left(-\frac{\frac{1}{3}x^3\operatorname{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int\frac{x\operatorname{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{3a^2}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\sqrt{}\right.}{}\right.}{5a^2}\right.$$

$$\Big\downarrow 15$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)^4-$$

$$\frac{4}{5}a\left(-\frac{4\left(-\frac{\frac{1}{3}x^3\operatorname{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int\frac{x\operatorname{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{5a^2}\right.$$

$$\Big\downarrow \text{6213}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-$$

$$\frac{4}{5}a\left(-\frac{4\left(-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}\right.}{5a^2}\right.$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^4-$$

$$\frac{4}{5}a\left(\frac{x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{5a^2}-\frac{4\left(\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{a}}{}\right.\right.}{a}\right.}{3a^2}\right.}{}\right.$$

input

```
Int[x^4*ArcSinh[a*x]^4,x]
```

output

```
(x^5*ArcSinh[a*x]^4)/5 - (4*a*((x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(5*a
^2) - (3*((x^5*ArcSinh[a*x]^2)/5 - (2*a*(-1/25*x^5/a + (x^4*Sqrt[1 + a^2*x
^2]*ArcSinh[a*x])/(5*a^2) - (4*(-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x])/(3*a^2) - (2*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2))/(3*a
^2)))/(5*a^2)))/5))/(5*a) - (4*((x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*
a^2) - ((x^3*ArcSinh[a*x]^2)/3 - (2*a*(-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]
*ArcSinh[a*x])/(3*a^2) - (2*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
))/(3*a^2)))/3)/a - (2*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2 - (3*(x*Arc
Sinh[a*x]^2 - 2*a*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2)))/a))/(3
*a^2)))/(5*a^2)))/5
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
 + b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{x^{5}a^{5}\,\mathrm{arcsinh}(xa)^{4}}{5}-\frac{32\,\mathrm{arcsinh}(xa)^{3}\sqrt{a^{2}x^{2}+1}}{75}-\frac{4x^{4}a^{4}\,\mathrm{arcsinh}(xa)^{3}\sqrt{a^{2}x^{2}+1}}{25}+\frac{16\,\mathrm{arcsinh}(xa)^{3}\sqrt{a^{2}x^{2}+1}\,x^{2}a^{2}}{75}+\frac{32\,\mathrm{arcsinh}(xa)^{2}}{25}$
default	$\frac{x^{5}a^{5}\,\mathrm{arcsinh}(xa)^{4}}{5}-\frac{32\,\mathrm{arcsinh}(xa)^{3}\sqrt{a^{2}x^{2}+1}}{75}-\frac{4x^{4}a^{4}\,\mathrm{arcsinh}(xa)^{3}\sqrt{a^{2}x^{2}+1}}{25}+\frac{16\,\mathrm{arcsinh}(xa)^{3}\sqrt{a^{2}x^{2}+1}\,x^{2}a^{2}}{75}+\frac{32\,\mathrm{arcsinh}(xa)^{2}}{25}$
orering	$\frac{\left(170181a^{8}x^{8}-190880a^{6}x^{6}+9375680a^{4}x^{4}+37873920a^{2}x^{2}+29836800\right)\mathrm{arcsinh}(xa)^{4}}{253125a^{8}x^{3}}-\frac{2(26730a^{8}x^{8}-61339a^{6}x^{6}+3}{}$

input
```
int(x^4*arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*x^5*a^5*arcsinh(x*a)^4-32/75*arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)-4
/25*x^4*a^4*arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)+16/75*arcsinh(x*a)^3*(a^2*x^2
+1)^(1/2)*x^2*a^2+32/25*arcsinh(x*a)^2*x*a-16576/5625*arcsinh(x*a)*(a^2*x^
2+1)^(1/2)+16576/5625*x*a+12/125*arcsinh(x*a)^2*x^5*a^5-24/625*arcsinh(x*a
)*(a^2*x^2+1)^(1/2)*x^4*a^4+1088/5625*x^2*a^2*arcsinh(x*a)*(a^2*x^2+1)^(1/
2)+24/3125*x^5*a^5-1088/16875*x^3*a^3-16/75*x^3*a^3*arcsinh(x*a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int x^4 \text{arcsinh}(ax)^4\,dx$$
$$= \frac{16875\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^4+648\,a^5x^5-5440\,a^3x^3-4500\left(3\,a^4x^4-4\,a^2x^2+8\right)\sqrt{a^2x^2+1}\log\left(a\right.}{}$$

input
```
integrate(x^4*arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
1/84375*(16875*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^4 + 648*a^5*x^5 - 5440
*a^3*x^3 - 4500*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sq
rt(a^2*x^2 + 1))^3 + 900*(9*a^5*x^5 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt
(a^2*x^2 + 1))^2 - 120*(27*a^4*x^4 - 136*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1)
*log(a*x + sqrt(a^2*x^2 + 1)) + 248640*a*x)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99

$$\int x^4 \text{arcsinh}(ax)^4\,dx$$
$$= \begin{cases} \frac{x^5\operatorname{asinh}^4(ax)}{5}+\frac{12x^5\operatorname{asinh}^2(ax)}{125}+\frac{24x^5}{3125}-\frac{4x^4\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{25a}-\frac{24x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{625a}-\frac{16x^3\operatorname{asinh}^2(ax)}{75a^2}-\frac{1088x^3}{16875a^2} \\ 0 \end{cases}$$

input
```
integrate(x**4*asinh(a*x)**4,x)
```

output
```
Piecewise((x**5*asinh(a*x)**4/5 + 12*x**5*asinh(a*x)**2/125 + 24*x**5/3125
 - 4*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(25*a) - 24*x**4*sqrt(a**2*x**
2 + 1)*asinh(a*x)/(625*a) - 16*x**3*asinh(a*x)**2/(75*a**2) - 1088*x**3/(1
6875*a**2) + 16*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**3) + 1088*x*
*2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**3) + 32*x*asinh(a*x)**2/(25*a**
4) + 16576*x/(5625*a**4) - 32*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**5)
- 16576*sqrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.82

$$\int x^4 \text{arcsinh}(ax)^4 \, dx = \frac{1}{5} x^5 \operatorname{arsinh}(ax)^4$$
$$- \frac{4}{75} \left(\frac{3\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{a^2x^2+1}}{a^6} \right) a \operatorname{arsinh}(ax)^3$$
$$- \frac{4}{84375} \left(2a \left(\frac{15 \left(27\sqrt{a^2x^2+1}a^2x^4 - 136\sqrt{a^2x^2+1}x^2 + \frac{2072\sqrt{a^2x^2+1}}{a^2} \right) \operatorname{arsinh}(ax)}{a^5} - \frac{81\,a^4x^5 - 680}{a} \right.\right.$$

input
```
integrate(x^4*arcsinh(a*x)^4,x, algorithm="maxima")
```

output
```
1/5*x^5*arcsinh(a*x)^4 - 4/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^3 - 4/84375*(2*a*
(15*(27*sqrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(
a^2*x^2 + 1)/a^2)*arcsinh(a*x)/a^5 - (81*a^4*x^5 - 680*a^2*x^3 + 31080*x)/
a^6) - 225*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)^2/a^5)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arcsinh(a*x)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4\text{arcsinh}(ax)^4\,dx=\int x^4\,\mathrm{asinh}(a\,x)^4\,dx$$

input
```
int(x^4*asinh(a*x)^4,x)
```

output
```
int(x^4*asinh(a*x)^4, x)
```

Reduce [F]

$$\int x^4\text{arcsinh}(ax)^4\,dx=\int asinh(ax)^4\,x^4dx$$

input
```
int(x^4*asinh(a*x)^4,x)
```

output
```
int(asinh(a*x)**4*x**4,x)
```

3.37 $\int x^3 \text{arcsinh}(ax)^4\,dx$

Optimal result

Integrand size = 10, antiderivative size = 194

$$\begin{aligned}\int x^3\text{arcsinh}(ax)^4\,dx = &-\frac{45x^2}{128a^2}+\frac{3x^4}{128}+\frac{45x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{64a^3}\\ &-\frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{32a}-\frac{45\text{arcsinh}(ax)^2}{128a^4}\\ &-\frac{9x^2\text{arcsinh}(ax)^2}{16a^2}+\frac{3}{16}x^4\text{arcsinh}(ax)^2\\ &+\frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{8a^3}-\frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{4a}\\ &-\frac{3\text{arcsinh}(ax)^4}{32a^4}+\frac{1}{4}x^4\text{arcsinh}(ax)^4\end{aligned}$$

output

```
-45/128*x^2/a^2+3/128*x^4+45/64*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a^3-3/32*
x^3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a-45/128*arcsinh(a*x)^2/a^4-9/16*x^2*ar
csinh(a*x)^2/a^2+3/16*x^4*arcsinh(a*x)^2+3/8*x*(a^2*x^2+1)^(1/2)*arcsinh(a
*x)^3/a^3-1/4*x^3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/a-3/32*arcsinh(a*x)^4/a
^4+1/4*x^4*arcsinh(a*x)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.69

$$\int x^3\text{arcsinh}(ax)^4\,dx$$
$$=\frac{3a^2x^2(-15+a^2x^2)-6ax\sqrt{1+a^2x^2}(-15+2a^2x^2)\,\text{arcsinh}(ax)+3(-15-24a^2x^2+8a^4x^4)\,\text{arcsinh}(ax}{128a^4}$$

input
```
Integrate[x^3*ArcSinh[a*x]^4,x]
```

output
```
(3*a^2*x^2*(-15 + a^2*x^2) - 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*Arc
Sinh[a*x] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4)*ArcSinh[a*x]^2 - 16*a*x*Sqrt[
1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcSinh[
a*x]^4)/(128*a^4)
```

Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6191, 6227, 6191, 6227, 15, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arcsinh}(ax)^4\,dx$$
$$\Big\downarrow \text{6191}$$
$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-a\int\frac{x^4\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{6227}$$
$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-$$
$$a\left(-\frac{3\int\frac{x^2\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{3\int x^3\text{arcsinh}(ax)^2dx}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}\right)$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{4a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a^2}\right)$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-a\left(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\text{arcsinh}(ax)^2dx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{ar}}{\sqrt{}}}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-a\left(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\text{arcsinh}(ax)^2dx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{ar}}{\sqrt{}}}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-a\left(-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(a\right.}{}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-$$

$$a\left(-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}-3\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2\right.}{}\right.\right.$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-$$

$$a\left(-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right.}{4a}\right.$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^4-$$

$$a\left(-\frac{3\left(\frac{1}{4}x^4\text{arcsinh}(ax)^2-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{4a^2}-\frac{x^4}{16a}\right)\right)}{4a}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x^4\operatorname{arcsinh}(ax)^4 -$$
$$a\left(\frac{x^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{4a^2} - \frac{3\left(-\frac{\operatorname{arcsinh}(ax)^4}{8a^3} + \frac{x\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{2a^2} - \frac{3\left(\frac{1}{2}x^2\operatorname{arcsinh}(ax)^2 - a\left(-\frac{\operatorname{arcsinh}(ax)^2}{4a^3} + \right.\right.}{2a}\right.}{4a^2}\right.$$

input
```
Int[x^3*ArcSinh[a*x]^4,x]
```

output
```
(x^4*ArcSinh[a*x]^4)/4 - a*((x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(4*a^2)
 - (3*((x^4*ArcSinh[a*x]^2)/4 - (a*(-1/16*x^4/a + (x^3*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x])/(4*a^2) - (3*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])
/(2*a^2) - ArcSinh[a*x]^2/(4*a^3)))/(4*a^2)))/2))/(4*a) - (3*((x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3) - (3*((x^2*ArcSi
nh[a*x]^2)/2 - a*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2)
- ArcSinh[a*x]^2/(4*a^3))))/(2*a)))/(4*a^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\dfrac{\frac{x^4a^4\operatorname{arcsinh}(xa)^4}{4}-\frac{x^3a^3\operatorname{arcsinh}(xa)^3\sqrt{a^2x^2+1}}{4}+\frac{3\operatorname{arcsinh}(xa)^3\sqrt{a^2x^2+1}\,xa}{8}-\frac{3\operatorname{arcsinh}(xa)^4}{32}+\frac{3x^4a^4\operatorname{arcsinh}(xa)^2}{16}-3x^3a^3\,\mathrm{ar}}{a^4}$
default	$\dfrac{\frac{x^4a^4\operatorname{arcsinh}(xa)^4}{4}-\frac{x^3a^3\operatorname{arcsinh}(xa)^3\sqrt{a^2x^2+1}}{4}+\frac{3\operatorname{arcsinh}(xa)^3\sqrt{a^2x^2+1}\,xa}{8}-\frac{3\operatorname{arcsinh}(xa)^4}{32}+\frac{3x^4a^4\operatorname{arcsinh}(xa)^2}{16}-3x^3a^3\,\mathrm{ar}}{a^4}$
orering	$\dfrac{(781a^6x^6-1605a^4x^4-9360a^2x^2-7560)\operatorname{arcsinh}(xa)^4}{1024a^6x^2}-\dfrac{(285a^6x^6-1213a^4x^4-6120a^2x^2-4860)\left(3x^2\operatorname{arcsinh}(xa)^4\right.}{1024a^6x^4}$

input

```
int(x^3*arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output

```
1/a^4*(1/4*x^4*a^4*arcsinh(x*a)^4-1/4*x^3*a^3*arcsinh(x*a)^3*(a^2*x^2+1)^(
1/2)+3/8*arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)*x*a-3/32*arcsinh(x*a)^4+3/16*x^4
*a^4*arcsinh(x*a)^2-3/32*x^3*a^3*arcsinh(x*a)*(a^2*x^2+1)^(1/2)+45/64*arcs
inh(x*a)*(a^2*x^2+1)^(1/2)*x*a+27/128*arcsinh(x*a)^2+3/128*a^4*x^4-45/128*
a^2*x^2-45/128-9/16*arcsinh(x*a)^2*(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx$$

$$= \frac{3\,a^4x^4 + 4\,(8\,a^4x^4 - 3)\log\left(ax + \sqrt{a^2x^2+1}\right)^4 - 16\,(2\,a^3x^3 - 3\,ax)\sqrt{a^2x^2+1}\log\left(ax + \sqrt{a^2x^2+1}\right)^3 -}{}$$

input
```
integrate(x^3*arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 16*(
2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 45*a
^2*x^2 + 3*(8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 -
6*(2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx$$

$$= \begin{cases} \frac{x^4\operatorname{asinh}^4(ax)}{4} + \frac{3x^4\operatorname{asinh}^2(ax)}{16} + \frac{3x^4}{128} - \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a} - \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a} - \frac{9x^2\operatorname{asinh}^2(ax)}{16a^2} - \frac{45x^2}{128a^2} + \frac{3x\sqrt{}}{} \\ 0 \end{cases}$$

input
```
integrate(x**3*asinh(a*x)**4,x)
```

output
```
Piecewise((x**4*asinh(a*x)**4/4 + 3*x**4*asinh(a*x)**2/16 + 3*x**4/128 - x
**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(4*a) - 3*x**3*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(32*a) - 9*x**2*asinh(a*x)**2/(16*a**2) - 45*x**2/(128*a**2) + 3
**x*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(8*a**3) + 45*x*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(64*a**3) - 3*asinh(a*x)**4/(32*a**4) - 45*asinh(a*x)**2/(128*a*
*4), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x^3 \operatorname{arcsinh}(ax)^4 \, dx = \int x^3 \operatorname{arsinh}\left(ax\right)^4 \, dx$$

input
```
integrate(x^3*arcsinh(a*x)^4,x, algorithm="maxima")
```

output
```
1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate((a^3*x^6 + sqrt(a^2*x^2
 + 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^
2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \operatorname{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arcsinh}(ax)^4 \, dx = \int x^3 \operatorname{asinh}(a\,x)^4 \,dx$$

input
```
int(x^3*asinh(a*x)^4,x)
```

output
```
int(x^3*asinh(a*x)^4, x)
```

Reduce [F]

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx = \int asinh(ax)^4\,x^3dx$$

input

```
int(x^3*asinh(a*x)^4,x)
```

output

```
int(asinh(a*x)**4*x**3,x)
```

3.38 $\int x^2\textbf{arcsinh}(ax)^4\,dx$

Optimal result

Integrand size = 10, antiderivative size = 162

$$\begin{aligned}\int x^2\text{arcsinh}(ax)^4\,dx = &-\frac{160x}{27a^2}+\frac{8x^3}{81}+\frac{160\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a^3}\\ &-\frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a}-\frac{8x\text{arcsinh}(ax)^2}{3a^2}\\ &+\frac{4}{9}x^3\text{arcsinh}(ax)^2+\frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a^3}\\ &-\frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a}+\frac{1}{3}x^3\text{arcsinh}(ax)^4\end{aligned}$$

output
```
-160/27*x/a^2+8/81*x^3+160/27*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a^3-8/27*x^2*
(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a-8/3*x*arcsinh(a*x)^2/a^2+4/9*x^3*arcsinh(
a*x)^2+8/9*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/a^3-4/9*x^2*(a^2*x^2+1)^(1/2)*
arcsinh(a*x)^3/a+1/3*x^3*arcsinh(a*x)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.69

$$\int x^2\text{arcsinh}(ax)^4\,dx$$
$$=\frac{8ax(-60+a^2x^2)-24(-20+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)+36ax(-6+a^2x^2)\,\text{arcsinh}(ax)^2-36(-2+}{81a^3}$$

input
```
Integrate[x^2*ArcSinh[a*x]^4,x]
```

output
```
(8*a*x*(-60 + a^2*x^2) - 24*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
 + 36*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 - 36*(-2 + a^2*x^2)*Sqrt[1 + a^2*x
^2]*ArcSinh[a*x]^3 + 27*a^3*x^3*ArcSinh[a*x]^4)/(81*a^3)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6191, 6227, 6191, 6213, 6187, 6213, 24, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}(ax)^4\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-$$
$$\frac{4}{3}a\left(-\frac{2\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2\text{arcsinh}(ax)^2dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}\right)$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}\right)$$

$$\Big\downarrow \text{6213}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\Bigg(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\int\text{arcsinh}(ax)^2dx}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{a}}{3a^2}$$

$$\Big\downarrow \text{6187}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\Bigg(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\int\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}}{a}$$

$$\Big\downarrow \text{6213}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\Bigg(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3}{}}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\Bigg(-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\int\frac{x^3\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\Bigg(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\big(x\text{arc}}{}}{}$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\left(-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2dx}{3a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax}{3a^2}\right.$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\left(-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\right.$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\left(-\frac{\frac{1}{3}x^3\text{arcsinh}(ax)^2-\frac{2}{3}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)}{3a^2}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3a^2}-\frac{x^3}{9a}\right)}{a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcs}}{3a^2}\right.$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^4-\frac{4}{3}a\left(\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3a^2}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)}{3a^2}-\frac{\frac{1}{3}x}{}\right.$$

input

```
Int[x^2*ArcSinh[a*x]^4,x]
```

output

```
(x^3*ArcSinh[a*x]^4)/3 - (4*a*((x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a
^2) - ((x^3*ArcSinh[a*x]^2)/3 - (2*a*(-1/9*x^3/a + (x^2*Sqrt[1 + a^2*x^2]*
ArcSinh[a*x])/(3*a^2) - (2*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2)
)/(3*a^2)))/3)/a - (2*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2 - (3*(x*ArcS
inh[a*x]^2 - 2*a*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2)))/a))/(3*
a^2)))/3
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{x^{3} a^{3} \operatorname{arcsinh}\left(x a\right)^{4}}{3}+\frac{8 \operatorname{arcsinh}\left(x a\right)^{3} \sqrt{a^{2} x^{2}+1}}{9}-\frac{4 \operatorname{arcsinh}\left(x a\right)^{3} \sqrt{a^{2} x^{2}+1}\, x^{2} a^{2}}{9}-\frac{8 \operatorname{arcsinh}\left(x a\right)^{2} x a}{3}+\frac{160 \operatorname{arcsinh}\left(x a\right) \sqrt{a^{2} x^{2}+1}}{27}-}{a^{3}}$
default	$\frac{\frac{x^{3} a^{3} \operatorname{arcsinh}\left(x a\right)^{4}}{3}+\frac{8 \operatorname{arcsinh}\left(x a\right)^{3} \sqrt{a^{2} x^{2}+1}}{9}-\frac{4 \operatorname{arcsinh}\left(x a\right)^{3} \sqrt{a^{2} x^{2}+1}\, x^{2} a^{2}}{9}-\frac{8 \operatorname{arcsinh}\left(x a\right)^{2} x a}{3}+\frac{160 \operatorname{arcsinh}\left(x a\right) \sqrt{a^{2} x^{2}+1}}{27}-}{a^{3}}$
orering	$\frac{\left(211 a^{6} x^{6}-1440 a^{4} x^{4}-9360 a^{2} x^{2}-8640\right) \operatorname{arcsinh}\left(x a\right)^{4}}{243 a^{6} x^{3}}-\frac{2\left(45 a^{6} x^{6}-649 a^{4} x^{4}-3810 a^{2} x^{2}-3420\right)\left(2 x \operatorname{arcsinh}\left(x a\right)^{4}+\right.}{243 a^{6} x^{4}}$

input

```
int(x^2*arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output

```
1/a^3*(1/3*x^3*a^3*arcsinh(x*a)^4+8/9*arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)-4/9
*arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)*x^2*a^2-8/3*arcsinh(x*a)^2*x*a+160/27*ar
csinh(x*a)*(a^2*x^2+1)^(1/2)-160/27*x*a+4/9*x^3*a^3*arcsinh(x*a)^2-8/27*x^
2*a^2*arcsinh(x*a)*(a^2*x^2+1)^(1/2)+8/81*x^3*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int x^2 \operatorname{arcsinh}(ax)^4\,dx$$

$$= \frac{27\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^4+8\,a^3x^3-36\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+36\,(a^3x^3-}{81\,a^3}$$

input
```
integrate(x^2*arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
1/81*(27*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^4 + 8*a^3*x^3 - 36*sqrt(a^2*
x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 36*(a^3*x^3 - 6*a*
x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 24*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20)*lo
g(a*x + sqrt(a^2*x^2 + 1)) - 480*a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

$$\int x^2\text{arcsinh}(ax)^4\,dx$$

$$= \begin{cases} \frac{x^{3}\operatorname{asinh}^{4}(ax)}{3} + \frac{4x^{3}\operatorname{asinh}^{2}(ax)}{9} + \frac{8x^{3}}{81} - \frac{4x^{2}\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{3}(ax)}{9a} - \frac{8x^{2}\sqrt{a^{2}x^{2}+1}\operatorname{asinh}(ax)}{27a} - \frac{8x\operatorname{asinh}^{2}(ax)}{3a^{2}} - \frac{160x}{27a^{2}} + 8\sqrt{a^{2}} \\ 0 \end{cases}$$

input
```
integrate(x**2*asinh(a*x)**4,x)
```

output
```
Piecewise((x**3*asinh(a*x)**4/3 + 4*x**3*asinh(a*x)**2/9 + 8*x**3/81 - 4*x
**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - 8*x**2*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(27*a) - 8*x*asinh(a*x)**2/(3*a**2) - 160*x/(27*a**2) + 8*sqrt(a
**2*x**2 + 1)*asinh(a*x)**3/(9*a**3) + 160*sqrt(a**2*x**2 + 1)*asinh(a*x)/
(27*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int x^2\text{arcsinh}(ax)^4\,dx$$

$$= \frac{1}{3}\,x^{3}\,\operatorname{arsinh}\left(ax\right)^{4} - \frac{4}{9}\,a\left(\frac{\sqrt{a^{2}x^{2}+1}x^{2}}{a^{2}} - \frac{2\sqrt{a^{2}x^{2}+1}}{a^{4}}\right)\operatorname{arsinh}\left(ax\right)^{3}$$

$$- \frac{4}{81}\left(2\,a\left(\frac{3\left(\sqrt{a^{2}x^{2}+1}x^{2} - \frac{20\sqrt{a^{2}x^{2}+1}}{a^{2}}\right)\operatorname{arsinh}\left(ax\right)}{a^{3}} - \frac{a^{2}x^{3}-60\,x}{a^{4}}\right) - \frac{9\left(a^{2}x^{3}-6\,x\right)\operatorname{arsinh}\left(ax\right)^{2}}{a^{3}}\right)$$

input
```
integrate(x^2*arcsinh(a*x)^4,x, algorithm="maxima")
```

output
```
1/3*x^3*arcsinh(a*x)^4 - 4/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
 + 1)/a^4)*arcsinh(a*x)^3 - 4/81*(2*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(
a^2*x^2 + 1)/a^2)*arcsinh(a*x)/a^3 - (a^2*x^3 - 60*x)/a^4) - 9*(a^2*x^3 -
6*x)*arcsinh(a*x)^2/a^3)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \operatorname{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arcsinh(a*x)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}(ax)^4 \, dx = \int x^2 \operatorname{asinh}(a\,x)^4 \, dx$$

input
```
int(x^2*asinh(a*x)^4,x)
```

output
```
int(x^2*asinh(a*x)^4, x)
```

Reduce [F]

$$\int x^2 \text{arcsinh}(ax)^4\,dx = \int asinh(ax)^4\,x^2dx$$

input

```
int(x^2*asinh(a*x)^4,x)
```

output

```
int(asinh(a*x)**4*x**2,x)
```

3.39 $\int x\text{arcsinh}(ax)^4\,dx$

Optimal result

Integrand size = 8, antiderivative size = 110

$$\int x\text{arcsinh}(ax)^4\,dx = \frac{3x^2}{4} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a} + \frac{3\text{arcsinh}(ax)^2}{4a^2} + \frac{3}{2}x^2\text{arcsinh}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a} + \frac{\text{arcsinh}(ax)^4}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^4$$

output
```
3/4*x^2-3/2*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a+3/4*arcsinh(a*x)^2/a^2+3/2*
x^2*arcsinh(a*x)^2-x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/a+1/4*arcsinh(a*x)^4
/a^2+1/2*x^2*arcsinh(a*x)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int x\text{arcsinh}(ax)^4\,dx = \frac{3a^2x^2 - 6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax) + (3+6a^2x^2)\,\text{arcsinh}(ax)^2 - 4ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3 + (1+2a^2}{4a^2}$$

input
```
Integrate[x*ArcSinh[a*x]^4,x]
```

output
```
(3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*ArcSin
h[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + (1 + 2*a^2*x^2)*ArcSin
h[a*x]^4)/(4*a^2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6191, 6227, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)^4\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\int\frac{x^2\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\left(-\frac{\int\frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\text{arcsinh}(ax)^2dx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)$$

$$\Big\downarrow \text{6191}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\int\frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}\right)$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2}}{}\right.$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\left(-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2a}-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+}}{}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^4-2a\left(-\frac{\text{arcsinh}(ax)^4}{8a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2a^2}-\frac{3\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right.\right.}{2a}\right.$$

input

```
Int[x*ArcSinh[a*x]^4,x]
```

output

```
(x^2*ArcSinh[a*x]^4)/2 - 2*a*((x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*a^2)
 - ArcSinh[a*x]^4/(8*a^3) - (3*((x^2*ArcSinh[a*x]^2)/2 - a*(-1/4*x^2/a + (
x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3))))/(2*a
))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{\frac{\cosh(2\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)^4}{4}-\frac{\sinh(2\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)^3}{2}+\frac{3\cosh(2\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)^2}{4}-\frac{3\sinh(2\operatorname{arcsinh}(xa))\,\mathrm{ar}}{4}}{a^2}$
default	$\frac{\frac{\cosh(2\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)^4}{4}-\frac{\sinh(2\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)^3}{2}+\frac{3\cosh(2\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)^2}{4}-\frac{3\sinh(2\operatorname{arcsinh}(xa))\,\mathrm{ar}}{4}}{a^2}$
orering	$\frac{(31a^4x^4+60a^2x^2+40)\operatorname{arcsinh}(xa)^4}{32a^4x^2}-\frac{(15a^4x^4+52a^2x^2+40)\left(\operatorname{arcsinh}(xa)^4+\frac{4x\operatorname{arcsinh}(xa)^3a}{\sqrt{a^2x^2+1}}\right)}{32x^2a^4}+\frac{(5a^4x^4+22a^2x}{}$

input
```
int(x*arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*cosh(2*arcsinh(x*a))*arcsinh(x*a)^4-1/2*sinh(2*arcsinh(x*a))*ar
csinh(x*a)^3+3/4*cosh(2*arcsinh(x*a))*arcsinh(x*a)^2-3/4*sinh(2*arcsinh(x*
a))*arcsinh(x*a)+3/8*cosh(2*arcsinh(x*a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int x\text{arcsinh}(ax)^4\,dx = -\frac{4\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3-(2\,a^2x^2+1)\log\left(ax+\sqrt{a^2x^2+1}\right)^4-3\,a^2x^2+6\sqrt{a^2x^2+1}a}{4\,a^2}$$

input
```
integrate(x*arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - (2*a^2*x^2
+ 1)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 3*a^2*x^2 + 6*sqrt(a^2*x^2 + 1)*a*x*
log(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 +
1))^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int x\text{arcsinh}(ax)^4\,dx = \begin{cases} \frac{x^2\operatorname{asinh}^4(ax)}{2}+\frac{3x^2\operatorname{asinh}^2(ax)}{2}+\frac{3x^2}{4}-\frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a}+\frac{\operatorname{asinh}^4(ax)}{4a^2}+\frac{3\operatorname{asinh}^2(ax)}{4a^2} & \text{for} \\ 0 & \text{oth} \end{cases}$$

input
```
integrate(x*asinh(a*x)**4,x)
```

output

```
Piecewise((x**2*asinh(a*x)**4/2 + 3*x**2*asinh(a*x)**2/2 + 3*x**2/4 - x*sq
rt(a**2*x**2 + 1)*asinh(a*x)**3/a - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*
a) + asinh(a*x)**4/(4*a**2) + 3*asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, Tru
e))
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^4\,dx=\int x\,\text{arsinh}\left(ax\right)^4\,dx$$

input

```
integrate(x*arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2*(a^3*x^4 + sqrt(a^2*x
^2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax)^4\,dx=\text{Exception raised: TypeError}$$

input

```
integrate(x*arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^4\,dx=\int x\,\text{asinh}(a\,x)^4\,dx$$

input
```
int(x*asinh(a*x)^4,x)
```

output
```
int(x*asinh(a*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int x\text{arcsinh}(ax)^4\,dx$$

$$=\frac{2asinh(ax)^4\,a^2x^2+asinh(ax)^4-4\sqrt{a^2x^2+1}\,asinh(ax)^3\,ax+6asinh(ax)^2\,a^2x^2+3asinh(ax)^2-6\sqrt{a^2}}{4a^2}$$

input
```
int(x*asinh(a*x)^4,x)
```

output
```
(2*asinh(a*x)**4*a**2*x**2 + asinh(a*x)**4 - 4*sqrt(a**2*x**2 + 1)*asinh(a
*x)**3*a*x + 6*asinh(a*x)**2*a**2*x**2 + 3*asinh(a*x)**2 - 6*sqrt(a**2*x**
2 + 1)*asinh(a*x)*a*x + 3*a**2*x**2)/(4*a**2)
```

3.40 $\int \operatorname{arcsinh}(ax)^4\,dx$

Optimal result

Integrand size = 6, antiderivative size = 67

$$\int \operatorname{arcsinh}(ax)^4\,dx = 24x - \frac{24\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + 12x\operatorname{arcsinh}(ax)^2 - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{a} + x\operatorname{arcsinh}(ax)^4$$

output

```
24*x-24*(a^2*x^2+1)^(1/2)*arcsinh(a*x)/a+12*x*arcsinh(a*x)^2-4*(a^2*x^2+1)
^(1/2)*arcsinh(a*x)^3/a+x*arcsinh(a*x)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)^4\,dx = 24x - \frac{24\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + 12x\operatorname{arcsinh}(ax)^2 - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{a} + x\operatorname{arcsinh}(ax)^4$$

input

```
Integrate[ArcSinh[a*x]^4,x]
```

output
```
24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6187, 6213, 6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^4\,dx$$

$$\Big\downarrow \text{ 6187}$$

$$x\text{arcsinh}(ax)^4-4a\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6213}$$

$$x\text{arcsinh}(ax)^4-4a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\int \text{arcsinh}(ax)^2dx}{a}\right)$$

$$\Big\downarrow \text{ 6187}$$

$$x\text{arcsinh}(ax)^4-4a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\int \frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{a}\right)$$

$$\Big\downarrow \text{ 6213}$$

$$x\text{arcsinh}(ax)^4-4a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{\int 1dx}{a}\right)\right)}{a}\right)$$

$$\Big\downarrow \text{ 24}$$

$$x\text{arcsinh}(ax)^4-4a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a^2}-\frac{3\left(x\text{arcsinh}(ax)^2-2a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{a^2}-\frac{x}{a}\right)\right)}{a}\right)$$

input
```
Int[ArcSinh[a*x]^4,x]
```

output
```
x*ArcSinh[a*x]^4 - 4*a*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2 - (3*(x*Arc
Sinh[a*x]^2 - 2*a*(-(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2)))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\operatorname{arcsinh}(xa)^4 xa-4\operatorname{arcsinh}(xa)^3\sqrt{a^2x^2+1}+12\operatorname{arcsinh}(xa)^2 xa-24\operatorname{arcsinh}(xa)\sqrt{a^2x^2+1}+24xa}{a}$
default	$\frac{\operatorname{arcsinh}(xa)^4 xa-4\operatorname{arcsinh}(xa)^3\sqrt{a^2x^2+1}+12\operatorname{arcsinh}(xa)^2 xa-24\operatorname{arcsinh}(xa)\sqrt{a^2x^2+1}+24xa}{a}$
orering	$x\operatorname{arcsinh}(xa)^4-\frac{8\operatorname{arcsinh}(xa)^3}{a\sqrt{a^2x^2+1}}+\frac{x(5a^2x^2+2)\left(\frac{12\operatorname{arcsinh}(xa)^2a^2}{a^2x^2+1}-\frac{4\operatorname{arcsinh}(xa)^3a^3x}{(a^2x^2+1)^{\frac{3}{2}}}\right)}{a^2}+\frac{(5a^4x^4+4a^2x^2-}{}$

input
```
int(arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arcsinh(x*a)^4*x*a-4*arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)+12*arcsinh(x*a)
^2*x*a-24*arcsinh(x*a)*(a^2*x^2+1)^(1/2)+24*x*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^4+12\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-4\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3+24\,ax}{a}$$

input
```
integrate(arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
(a*x*log(a*x + sqrt(a^2*x^2 + 1))^4 + 12*a*x*log(a*x + sqrt(a^2*x^2 + 1))^
2 - 4*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 24*a*x - 24*sqrt(
a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \operatorname{arcsinh}(ax)^4\,dx$$
$$= \begin{cases} x\operatorname{asinh}^4(ax)+12x\operatorname{asinh}^2(ax)+24x-\frac{4\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a}-\frac{24\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a*x)**4,x)
```

output
```
Piecewise((x*asinh(a*x)**4 + 12*x*asinh(a*x)**2 + 24*x - 4*sqrt(a**2*x**2
+ 1)*asinh(a*x)**3/a - 24*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a, 0)), (0,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \operatorname{arcsinh}(ax)^4\,dx = x \operatorname{arsinh}(ax)^4 - \frac{4\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^3}{a} + 12\left(\frac{x\operatorname{arsinh}(ax)^2}{a} + \frac{2\left(x - \frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a}\right)}{a}\right)a$$

input

```
integrate(arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
x*arcsinh(a*x)^4 - 4*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a + 12*(x*arcsinh(a*
x)^2/a + 2*(x - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a)/a)*a
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \operatorname{arcsinh}(ax)^4\,dx = x\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - 4\left(\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2} - \frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 2a\left(\frac{x}{a} - \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right.}{a^2}\right.\right.}{a}\right.$$

input

```
integrate(arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
x*log(a*x + sqrt(a^2*x^2 + 1))^4 - 4*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2
*x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a
^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2))/a)*a
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^4\,dx = \int \operatorname{asinh}(a\,x)^4\,dx$$

input
```
int(asinh(a*x)^4,x)
```

output
```
int(asinh(a*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{asinh(ax)^4\,ax - 4\sqrt{a^2x^2+1}\,asinh(ax)^3 + 12asinh(ax)^2\,ax - 24\sqrt{a^2x^2+1}\,asinh(ax) + 24ax}{a}$$

input
```
int(asinh(a*x)^4,x)
```

output
```
(asinh(a*x)**4*a*x - 4*sqrt(a**2*x**2 + 1)*asinh(a*x)**3 + 12*asinh(a*x)**
2*a*x - 24*sqrt(a**2*x**2 + 1)*asinh(a*x) + 24*a*x)/a
```

3.41 $\int \frac{\operatorname{arcsinh}(ax)^4}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x} dx = & -\frac{1}{5}\operatorname{arcsinh}(ax)^5 + \operatorname{arcsinh}(ax)^4 \log\left(1 - e^{2\operatorname{arcsinh}(ax)}\right) \\ & + 2\operatorname{arcsinh}(ax)^3 \operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right) \\ & - 3\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(3, e^{2\operatorname{arcsinh}(ax)}\right) \\ & + 3\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(4, e^{2\operatorname{arcsinh}(ax)}\right) \\ & - \frac{3}{2} \operatorname{PolyLog}\left(5, e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-1/5*arcsinh(a*x)^5+arcsinh(a*x)^4*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+2*arcsi
nh(a*x)^3*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3*arcsinh(a*x)^2*polylog(3,
(a*x+(a^2*x^2+1)^(1/2))^2)+3*arcsinh(a*x)*polylog(4,(a*x+(a^2*x^2+1)^(1/2)
)^2)-3/2*polylog(5,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx =& -\frac{1}{5}\operatorname{arcsinh}(ax)^5+\operatorname{arcsinh}(ax)^4\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\&+2\operatorname{arcsinh}(ax)^3\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\&-3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\\&+3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)\\&-\frac{3}{2}\operatorname{PolyLog}\left(5,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

input

```
Integrate[ArcSinh[a*x]^4/x,x]
```

output

```
-1/5*ArcSinh[a*x]^5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSi
nh[a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(
2*ArcSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*Poly
Log[5, E^(2*ArcSinh[a*x])])/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6190, 3042, 26, 4199, 25, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}dx$$

$$\Big\downarrow \text{6190}$$

$$\int \frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^4}{ax}d\operatorname{arcsinh}(ax)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\text{arcsinh}(ax)^4\tan\left(\frac{\pi}{2}+i\text{arcsinh}(ax)\right)d\text{arcsinh}(ax)$$

$$\Big\downarrow \ 26$$

$$-i\int \text{arcsinh}(ax)^4\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)$$

$$\Big\downarrow \ 4199$$

$$-i\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^4}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{5}i\text{arcsinh}(ax)^5\right)$$

$$\Big\downarrow \ 25$$

$$-i\left(-2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^4}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{5}i\text{arcsinh}(ax)^5\right)$$

$$\Big\downarrow \ 2620$$

$$-i\bigg(-2i\bigg(2\int \text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^4\log\left(1-e^{2\text{arcsinh}(ax)}\right)\bigg)-\frac{1}{5}i\text{arcs}$$

$$\Big\downarrow \ 3011$$

$$-i\bigg(-2i\bigg(2\bigg(\frac{3}{2}\int \text{arcsinh}(ax)^2\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^3\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)$$

$$\Big\downarrow \ 7163$$

$$-i\bigg(-2i\bigg(2\bigg(\frac{3}{2}\bigg(\frac{1}{2}\text{arcsinh}(ax)^2\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)-\int \text{arcsinh}(ax)\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax$$

$$\Big\downarrow \ 7163$$

$$-i\bigg(-2i\bigg(2\bigg(\frac{3}{2}\bigg(\frac{1}{2}\int \text{PolyLog}\left(4,e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+\frac{1}{2}\text{arcsinh}(ax)^2\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsi}$$

$$\Big\downarrow \ 2720$$

$$-i\bigg(-2i\bigg(2\bigg(\frac{3}{2}\bigg(\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(4,e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)^2\text{PolyLog}\bigg(3,e^{2\text{arcsinh}(a}$$

$$\Big\downarrow \ 7143$$

$$-i\bigg(-2i\bigg(2\bigg(\frac{3}{2}\bigg(\frac{1}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(3,e^{2\text{arcsinh}(ax)}\right)-\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(4,e^{2\text{arcsinh}(ax)}\right)+\frac{1}{4}\,\text{PolyLog}$$

input

```
Int[ArcSinh[a*x]^4/x,x]
```

output

```
(-I)*((-1/5*I)*ArcSinh[a*x]^5 - (2*I)*(-1/2*(ArcSinh[a*x]^4*Log[1 - E^(2*A
rcSinh[a*x])]) + 2*(-1/2*(ArcSinh[a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])]) +
 (3*((ArcSinh[a*x]^2*PolyLog[3, E^(2*ArcSinh[a*x])])/2 - (ArcSinh[a*x]*Pol
yLog[4, E^(2*ArcSinh[a*x])])/2 + PolyLog[5, E^(2*ArcSinh[a*x])]/4))/2)))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.65

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(xa)^{5}}{5}+\operatorname{arcsinh}\left(xa\right)^{4}\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)+4\operatorname{arcsinh}\left(xa\right)^{3}\operatorname{polylog}\left(2,-\right.$
default	$-\frac{\operatorname{arcsinh}(xa)^{5}}{5}+\operatorname{arcsinh}\left(xa\right)^{4}\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)+4\operatorname{arcsinh}\left(xa\right)^{3}\operatorname{polylog}\left(2,-\right.$

input
```
int(arcsinh(x*a)^4/x,x,method=_RETURNVERBOSE)
```

output
```
-1/5*arcsinh(x*a)^5+arcsinh(x*a)^4*ln(1+x*a+(a^2*x^2+1)^(1/2))+4*arcsinh(x
*a)^3*polylog(2,-x*a-(a^2*x^2+1)^(1/2))-12*arcsinh(x*a)^2*polylog(3,-x*a-(
a^2*x^2+1)^(1/2))+24*arcsinh(x*a)*polylog(4,-x*a-(a^2*x^2+1)^(1/2))-24*pol
ylog(5,-x*a-(a^2*x^2+1)^(1/2))+arcsinh(x*a)^4*ln(1-x*a-(a^2*x^2+1)^(1/2))+
4*arcsinh(x*a)^3*polylog(2,x*a+(a^2*x^2+1)^(1/2))-12*arcsinh(x*a)^2*polylo
g(3,x*a+(a^2*x^2+1)^(1/2))+24*arcsinh(x*a)*polylog(4,x*a+(a^2*x^2+1)^(1/2)
)-24*polylog(5,x*a+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int\frac{\operatorname{arcsinh}(ax)^{4}}{x}\,dx=\int\frac{\operatorname{arsinh}\left(ax\right)^{4}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^4/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = \int \frac{\operatorname{asinh}^4(ax)}{x}\,dx$$

input
```
integrate(asinh(a*x)**4/x,x)
```

output
```
Integral(asinh(a*x)**4/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)^4}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^4/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)^4}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^4}{x}\,dx$$

input
```
int(asinh(a*x)^4/x,x)
```

output
```
int(asinh(a*x)^4/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = \int \frac{asinh(ax)^4}{x}dx$$

input
```
int(asinh(a*x)^4/x,x)
```

output
```
int(asinh(a*x)**4/x,x)
```

3.42 $\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 120

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = &-\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &- 12a\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+ 12a\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &+ 24a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)\\ &- 24a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\\ &- 24a\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right) + 24a\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-arcsinh(a*x)^4/x-8*a*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-12*a*a
rcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+12*a*arcsinh(a*x)^2*polylo
g(2,a*x+(a^2*x^2+1)^(1/2))+24*a*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1
/2))-24*a*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))-24*a*polylog(4,-a*
x-(a^2*x^2+1)^(1/2))+24*a*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \frac{1}{2}a\left(\pi^4 - 2\operatorname{arcsinh}(ax)^4 - \frac{2\operatorname{arcsinh}(ax)^4}{ax} - 8\operatorname{arcsinh}(ax)^3\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) + 8\operatorname{arcsinh}(ax)^3\log\left(1-e^{\operatorname{arcsinh}(ax)}\right) + 24\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) + 24\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) + 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) - 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,-e^{-\operatorname{arcsinh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\right)$$

input
```
Integrate[ArcSinh[a*x]^4/x^2,x]
```

output
```
(a*(Pi^4 - 2*ArcSinh[a*x]^4 - (2*ArcSinh[a*x]^4)/(a*x) - 8*ArcSinh[a*x]^3*
Log[1 + E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24
*ArcSinh[a*x]^2*PolyLog[2, -E^(-ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog
[2, E^ArcSinh[a*x]] + 48*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*
ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])
] + 48*PolyLog[4, E^ArcSinh[a*x]]))/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6191, 6231, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx$$

$$\Big\downarrow \; 6191$$

$$4a\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^4}{x}$$

$$\Big\downarrow \; 6231$$

$$4a\int \frac{\text{arcsinh}(ax)^3}{ax}d\text{arcsinh}(ax)-\frac{\text{arcsinh}(ax)^4}{x}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\text{arcsinh}(ax)^4}{x}+4a\int i\text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \; 26$$

$$-\frac{\text{arcsinh}(ax)^4}{x}+4ia\int \text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)$$

$$\Big\downarrow \; 4670$$

$$-\frac{\text{arcsinh}(ax)^4}{x}+$$
$$4ia\Bigg(3i\int \text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int \text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax$$

$$\Big\downarrow \; 3011$$

$$-\frac{\text{arcsinh}(ax)^4}{x}+$$
$$4ia\Bigg(-3i\Bigg(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)\Bigg)+$$

$$\Big\downarrow \; 7163$$

$$-\frac{\text{arcsinh}(ax)^4}{x}+$$
$$4ia\Bigg(-3i\Bigg(2\Bigg(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\Bigg)-\text{arcsinh}(ax$$

$$\Big\downarrow \; 2720$$

$$-\frac{\text{arcsinh}(ax)^4}{x}+$$
$$4ia\Bigg(-3i\Bigg(2\Bigg(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}(ax)}\Bigg)-$$

$$\Big\downarrow \; 7143$$

$$-\frac{\operatorname{arcsinh}(ax)^4}{x}+$$

$$4ia\Big(2i\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\Big(e^{\operatorname{arcsinh}(ax)}\Big)-3i\Big(2\Big(\operatorname{arcsinh}(ax)\operatorname{PolyLog}\Big(3,-e^{\operatorname{arcsinh}(ax)}\Big)-\operatorname{PolyLog}\Big(4,-e^{\operatorname{arcsinh}(}$$

input
```
Int[ArcSinh[a*x]^4/x^2,x]
```

output
```
-(ArcSinh[a*x]^4/x) + (4*I)*a*((2*I)*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]
] - (3*I)*(-(ArcSinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]
*PolyLog[3, -E^ArcSinh[a*x]] - PolyLog[4, -E^ArcSinh[a*x]])) + (3*I)*(-(Ar
cSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, E^Ar
cSinh[a*x]] - PolyLog[4, E^ArcSinh[a*x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.78

method	result
derivativedivides	$a\Big(-\frac{\operatorname{arcsinh}(xa)^4}{xa}-4\operatorname{arcsinh}(xa)^3\ln\left(1+xa+\sqrt{a^2x^2+1}\right)-12\operatorname{arcsinh}(xa)^2\operatorname{polylog}($
default	$a\Big(-\frac{\operatorname{arcsinh}(xa)^4}{xa}-4\operatorname{arcsinh}(xa)^3\ln\left(1+xa+\sqrt{a^2x^2+1}\right)-12\operatorname{arcsinh}(xa)^2\operatorname{polylog}($

input
```
int(arcsinh(x*a)^4/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-arcsinh(x*a)^4/x/a-4*arcsinh(x*a)^3*ln(1+x*a+(a^2*x^2+1)^(1/2))-12*arc
sinh(x*a)^2*polylog(2,-x*a-(a^2*x^2+1)^(1/2))+24*arcsinh(x*a)*polylog(3,-x
*a-(a^2*x^2+1)^(1/2))-24*polylog(4,-x*a-(a^2*x^2+1)^(1/2))+4*arcsinh(x*a)^
3*ln(1-x*a-(a^2*x^2+1)^(1/2))+12*arcsinh(x*a)^2*polylog(2,x*a+(a^2*x^2+1)^
(1/2))-24*arcsinh(x*a)*polylog(3,x*a+(a^2*x^2+1)^(1/2))+24*polylog(4,x*a+(
a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\, dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{4}}{x^{2}}\, dx$$

input
```
integrate(arcsinh(a*x)^4/x^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^4/x^2, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\, dx = \int \frac{\operatorname{asinh}^{4}\left(ax\right)}{x^{2}}\, dx$$

input
```
integrate(asinh(a*x)**4/x**2,x)
```

output
```
Integral(asinh(a*x)**4/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^2}\,dx$$

input

```
integrate(arcsinh(a*x)^4/x^2,x, algorithm="maxima")
```

output

```
-log(a*x + sqrt(a^2*x^2 + 1))^4/x + integrate(4*(a^3*x^2 + sqrt(a^2*x^2 +
1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^4 + a*x^2 + (a^2*x^3 +
 x)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^2}\,dx$$

input

```
integrate(arcsinh(a*x)^4/x^2,x, algorithm="giac")
```

output

```
integrate(arcsinh(a*x)^4/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)^4}{x^2}\,dx$$

input

```
int(asinh(a*x)^4/x^2,x)
```

output

```
int(asinh(a*x)^4/x^2, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{asinh(ax)^4}{x^2}dx$$

input

```
int(asinh(a*x)^4/x^2,x)
```

output

```
int(asinh(a*x)**4/x**2,x)
```

3.43 $\int \frac{\textbf{arcsinh}(ax)^4}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 108

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = {} & -2a^2\operatorname{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} \\ & - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} + 6a^2\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) \\ & + 6a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) \\ & - 3a^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-2*a^2*arcsinh(a*x)^3-2*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/x-1/2*arcsinh(a
*x)^4/x^2+6*a^2*arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+6*a^2*arcsi
nh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3*a^2*polylog(3,(a*x+(a^2*x^2
+1)^(1/2))^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = -\frac{\operatorname{arcsinh}(ax)^4}{2x^2}+\frac{1}{4}a^2\Bigg(& i\pi^3-8\operatorname{arcsinh}(ax)^3\\ &-\frac{8\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{ax}+24\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\ &+24\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\ &-12\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\Bigg)\end{aligned}$$

input

```
Integrate[ArcSinh[a*x]^4/x^3,x]
```

output

```
-1/2*ArcSinh[a*x]^4/x^2 + (a^2*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^3)/(a*x) + 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x]
)] + 24*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*A
rcSinh[a*x])]))/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6191, 6215, 6190, 3042, 26, 4199, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}dx$$

$$\Big\downarrow \text{ 6191}$$

$$2a\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{a^2x^2+1}}dx-\frac{\operatorname{arcsinh}(ax)^4}{2x^2}$$

$$\Big\downarrow \text{6215}$$

$$2a\left(3a\int \frac{\text{arcsinh}(ax)^2}{x}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}\right)-\frac{\text{arcsinh}(ax)^4}{2x^2}$$

$$\Big\downarrow \text{6190}$$

$$2a\left(3a\int \frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{ax}d\text{arcsinh}(ax)-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}\right)-\frac{\text{arcsinh}(ax)^4}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}+3a\int -i\text{arcsinh}(ax)^2\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{\text{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\int \text{arcsinh}(ax)^2\tan\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)d\text{arcsinh}(ax)\right)$$

$$\Big\downarrow \text{4199}$$

$$-\frac{\text{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\left(2i\int -\frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{\text{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\left(-2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1-e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)\right)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\text{arcsinh}(ax)^4}{2x^2}+$$
$$2a\Bigg(-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{x}-3ia\Bigg(-2i\Bigg(\int \text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\frac{1}{2}\text{arcsinh}(ax)^2\,\text{lo}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\operatorname{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{x}-3ia\left(-2i\left(\frac{1}{2}\int \operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)d\operatorname{arcsinh}(ax)-\frac{1}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\right.\right.\right.$$

$$\Big\downarrow \ 2720$$

$$-\frac{\operatorname{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{x}-3ia\left(-2i\left(\frac{1}{4}\int e^{-2\operatorname{arcsinh}(ax)}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)de^{2\operatorname{arcsinh}(ax)}-\frac{1}{2}\text{arcsinh(}\right.\right.\right.$$

$$\Big\downarrow \ 7143$$

$$-\frac{\operatorname{arcsinh}(ax)^4}{2x^2}+$$
$$2a\left(-\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{x}-3ia\left(-2i\left(-\frac{1}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)+\frac{1}{4}\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax}\right.\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^4/x^3,x]
```

output
```
-1/2*ArcSinh[a*x]^4/x^2 + 2*a*(-((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x) - (
3*I)*a*((-1/3*I)*ArcSinh[a*x]^3 - (2*I)*(-1/2*(ArcSinh[a*x]^2*Log[1 - E^(2
*ArcSinh[a*x])]) - (ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])])/2 + PolyL
og[3, E^(2*ArcSinh[a*x])]/4)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

method	result
derivativedivides	$a^2\bigg(-\frac{\operatorname{arcsinh}(xa)^3\left(-4a^2x^2+4xa\sqrt{a^2x^2+1}+\operatorname{arcsinh}(xa)\right)}{2x^2a^2}-4\operatorname{arcsinh}(xa)^3+6\operatorname{arcsinh}(xa)^2\ln\big(1$
default	$a^2\bigg(-\frac{\operatorname{arcsinh}(xa)^3\left(-4a^2x^2+4xa\sqrt{a^2x^2+1}+\operatorname{arcsinh}(xa)\right)}{2x^2a^2}-4\operatorname{arcsinh}(xa)^3+6\operatorname{arcsinh}(xa)^2\ln\big(1$

input
```
int(arcsinh(x*a)^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arcsinh(x*a)^3*(-4*a^2*x^2+4*x*a*(a^2*x^2+1)^(1/2)+arcsinh(x*a))
/x^2/a^2-4*arcsinh(x*a)^3+6*arcsinh(x*a)^2*ln(1+x*a+(a^2*x^2+1)^(1/2))+12*
arcsinh(x*a)*polylog(2,-x*a-(a^2*x^2+1)^(1/2))-12*polylog(3,-x*a-(a^2*x^2+
1)^(1/2))+6*arcsinh(x*a)^2*ln(1-x*a-(a^2*x^2+1)^(1/2))+12*arcsinh(x*a)*pol
ylog(2,x*a+(a^2*x^2+1)^(1/2))-12*polylog(3,x*a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^4/x^3, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{asinh}^4\left(ax\right)}{x^3}\,dx$$

input
```
integrate(asinh(a*x)**4/x**3,x)
```

output
```
Integral(asinh(a*x)**4/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^3}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(a*x + sqrt(a^2*x^2 + 1))^4/x^2 + integrate(2*(a^3*x^2 + sqrt(a^2*
x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^5 + a*x^3 + (a^2
*x^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^4/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x)^4}{x^3}\,dx$$

input
```
int(asinh(a*x)^4/x^3,x)
```

output
```
int(asinh(a*x)^4/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{asinh(ax)^4}{x^3}dx$$

input
```
int(asinh(a*x)^4/x^3,x)
```

output
```
int(asinh(a*x)**4/x**3,x)
```

3.44 $\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 223

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = &-\frac{2a^2\operatorname{arcsinh}(ax)^2}{x}-\frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3x^2}\\ &-\frac{\operatorname{arcsinh}(ax)^4}{3x^3}-8a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &+\frac{4}{3}a^3\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &-4a^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+4a^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &-2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &-4a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+4a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\\ &+4a^3\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right)-4a^3\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

output

```
-2*a^2*arcsinh(a*x)^2/x-2/3*a*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/x^2-1/3*arc
sinh(a*x)^4/x^3-8*a^3*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+4/3*a^3*
arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-4*a^3*polylog(2,-a*x-(a^2*x^
2+1)^(1/2))+2*a^3*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+4*a^3*p
olylog(2,a*x+(a^2*x^2+1)^(1/2))-2*a^3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^
2+1)^(1/2))-4*a^3*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+4*a^3*arc
sinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+4*a^3*polylog(4,-a*x-(a^2*x^2+1
)^(1/2))-4*a^3*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.59

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \frac{1}{24}a^3\Bigg(&-2\pi^4+4\operatorname{arcsinh}(ax)^4 \\
&-24\operatorname{arcsinh}(ax)^2\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
&+2\operatorname{arcsinh}(ax)^4\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
&-4\operatorname{arcsinh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
&-\frac{1}{2}ax\operatorname{arcsinh}(ax)^4\operatorname{csch}^4\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
&+96\operatorname{arcsinh}(ax)\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) \\
&-96\operatorname{arcsinh}(ax)\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) \\
&+16\operatorname{arcsinh}(ax)^3\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) \\
&-16\operatorname{arcsinh}(ax)^3\log\left(1-e^{\operatorname{arcsinh}(ax)}\right) \\
&-48\left(-2+\operatorname{arcsinh}(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) \\
&-96\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right) \\
&-48\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\
&-96\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) \\
&+96\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) \\
&-96\operatorname{PolyLog}\left(4,-e^{-\operatorname{arcsinh}(ax)}\right)-96\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right) \\
&-4\operatorname{arcsinh}(ax)^3\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
&-\frac{8\operatorname{arcsinh}(ax)^4\sinh^4\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)}{a^3x^3} \\
&+24\operatorname{arcsinh}(ax)^2\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right) \\
&-2\operatorname{arcsinh}(ax)^4\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\Bigg)
\end{aligned}
$$

input

```
Integrate[ArcSinh[a*x]^4/x^4,x]
```

output

```
(a^3*(-2*Pi^4 + 4*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*Coth[ArcSinh[a*x]/2]
+ 2*ArcSinh[a*x]^4*Coth[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Csch[ArcSinh[a*
x]/2]^2 - (a*x*ArcSinh[a*x]^4*Csch[ArcSinh[a*x]/2]^4)/2 + 96*ArcSinh[a*x]*
Log[1 - E^(-ArcSinh[a*x])] - 96*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] +
16*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 16*ArcSinh[a*x]^3*Log[1 - E
^ArcSinh[a*x]] - 48*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-ArcSinh[a*x])] -
 96*PolyLog[2, E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh
[a*x]] - 96*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] + 96*ArcSinh[a*x]*
PolyLog[3, E^ArcSinh[a*x]] - 96*PolyLog[4, -E^(-ArcSinh[a*x])] - 96*PolyLo
g[4, E^ArcSinh[a*x]] - 4*ArcSinh[a*x]^3*Sech[ArcSinh[a*x]/2]^2 - (8*ArcSin
h[a*x]^4*Sinh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]^2*Tanh[ArcSin
h[a*x]/2] - 2*ArcSinh[a*x]^4*Tanh[ArcSinh[a*x]/2]))/24
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6191, 6224, 6191, 6231, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^4}{x^4}dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{4}{3}a\int \frac{\text{arcsinh}(ax)^3}{x^3\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^4}{3x^3}$$

$$\Big\downarrow \text{6224}$$

$$\frac{4}{3}a\left(-\frac{1}{2}a^2\int \frac{\text{arcsinh}(ax)^3}{x\sqrt{a^2x^2+1}}dx+\frac{3}{2}a\int \frac{\text{arcsinh}(ax)^2}{x^2}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}\right)-\frac{\text{arcsinh}(ax)^4}{3x^3}$$

$$\Big\downarrow \text{6191}$$

$$\frac{4}{3}a\left(\frac{3}{2}a\left(2a\int\frac{\text{arcsinh}(ax)}{x\sqrt{a^2x^2+1}}dx-\frac{\text{arcsinh}(ax)^2}{x}\right)-\frac{1}{2}a^2\int\frac{\text{arcsinh}(ax)^3}{x\sqrt{a^2x^2+1}}dx-\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{2x^2}\right)-\frac{\text{arcsinh}(ax)^4}{3x^3}$$

$$\Big\downarrow \text{6231}$$

$$\frac{4}{3}a\left(-\frac{1}{2}a^2\int\frac{\text{arcsinh}(ax)^3}{ax}d\text{arcsinh}(ax)+\frac{3}{2}a\left(2a\int\frac{\text{arcsinh}(ax)}{ax}d\text{arcsinh}(ax)-\frac{\text{arcsinh}(ax)^2}{x}\right)-\frac{\sqrt{a^2x^2+1}\text{ar}}{2x}\right.$$
$$\frac{\text{arcsinh}(ax)^4}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}a^2\int i\text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)+\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2a\int i\text{arcsinh}(ax)\csc(i\text{arcsi}\right.\right.$$

$$\Big\downarrow \text{26}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\int\text{arcsinh}(ax)^3\csc(i\text{arcsinh}(ax))d\text{arcsinh}(ax)+\frac{3}{2}a\left(-\frac{\text{arcsinh}(ax)^2}{x}+2ia\int\text{arcsinh}(ax)\csc(i\text{arcsi}\right.\right.$$

$$\Big\downarrow \text{4670}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(3i\int\text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int\text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arc}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(3i\int\text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int\text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arc}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(3i\int\text{arcsinh}(ax)^2\log\left(1-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-3i\int\text{arcsinh}(ax)^2\log\left(1+e^{\text{arcsinh}(ax)}\right)d\text{arc}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(-3i\left(2\int \text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\text{arc}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\int e^{-\text{arcsinh}(ax)}\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)de^{\text{arcsinh}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\text{arcsinh}(ax)^4}{3x^3}+$$
$$\frac{4}{3}a\left(-\frac{1}{2}ia^2\left(2i\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)-3i\left(2\left(\text{arcsinh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right)-\text{PolyLog}\left(4,-\right.\right.\right.\right.\right.$$

input
```
Int[ArcSinh[a*x]^4/x^4,x]
```

output
```
-1/3*ArcSinh[a*x]^4/x^3 + (4*a*(-1/2*(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x^
2 + (3*a*(-(ArcSinh[a*x]^2/x) + (2*I)*a*((2*I)*ArcSinh[a*x]*ArcTanh[E^ArcS
inh[a*x]] + I*PolyLog[2, -E^ArcSinh[a*x]] - I*PolyLog[2, E^ArcSinh[a*x]]))
)/2 - (I/2)*a^2*((2*I)*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - (3*I)*(-(A
rcSinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, -E
^ArcSinh[a*x]] - PolyLog[4, -E^ArcSinh[a*x]])) + (3*I)*(-(ArcSinh[a*x]^2*P
olyLog[2, E^ArcSinh[a*x]]) + 2*(ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] -
PolyLog[4, E^ArcSinh[a*x]])))))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.52

method	result
derivativedivides	$a^{3}\left(-\frac{\operatorname{arcsinh}(xa)^{2}\left(2\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}\,xa+\operatorname{arcsinh}(xa)^{2}+6a^{2}x^{2}\right)}{3x^{3}a^{3}}+\frac{2\operatorname{arcsinh}(xa)^{3}\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)}{3}+\right.$
default	$a^{3}\left(-\frac{\operatorname{arcsinh}(xa)^{2}\left(2\operatorname{arcsinh}(xa)\sqrt{a^{2}x^{2}+1}\,xa+\operatorname{arcsinh}(xa)^{2}+6a^{2}x^{2}\right)}{3x^{3}a^{3}}+\frac{2\operatorname{arcsinh}(xa)^{3}\ln\left(1+xa+\sqrt{a^{2}x^{2}+1}\right)}{3}+\right.$

input
```
int(arcsinh(x*a)^4/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/x^3/a^3*arcsinh(x*a)^2*(2*arcsinh(x*a)*(a^2*x^2+1)^(1/2)*x*a+arc
sinh(x*a)^2+6*a^2*x^2)+2/3*arcsinh(x*a)^3*ln(1+x*a+(a^2*x^2+1)^(1/2))+2*ar
csinh(x*a)^2*polylog(2,-x*a-(a^2*x^2+1)^(1/2))-4*arcsinh(x*a)*polylog(3,-x
*a-(a^2*x^2+1)^(1/2))+4*polylog(4,-x*a-(a^2*x^2+1)^(1/2))-2/3*arcsinh(x*a)
^3*ln(1-x*a-(a^2*x^2+1)^(1/2))-2*arcsinh(x*a)^2*polylog(2,x*a+(a^2*x^2+1)^
(1/2))+4*arcsinh(x*a)*polylog(3,x*a+(a^2*x^2+1)^(1/2))-4*polylog(4,x*a+(a^
2*x^2+1)^(1/2))-4*arcsinh(x*a)*ln(1+x*a+(a^2*x^2+1)^(1/2))-4*polylog(2,-x*
a-(a^2*x^2+1)^(1/2))+4*arcsinh(x*a)*ln(1-x*a-(a^2*x^2+1)^(1/2))+4*polylog(
2,x*a+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{4}}{x^{4}}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^{4}}{x^{4}}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x^4,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^4/x^4, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{asinh}^4(ax)}{x^4}\,dx$$

input
```
integrate(asinh(a*x)**4/x**4,x)
```

output
```
Integral(asinh(a*x)**4/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{arsinh}(ax)^4}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x^4,x, algorithm="maxima")
```

output
```
-1/3*log(a*x + sqrt(a^2*x^2 + 1))^4/x^3 + integrate(4/3*(a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^6 + a*x^4 + (a
^2*x^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{arsinh}(ax)^4}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x)^4/x^4,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^4/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx=\int \frac{{\mathrm{asinh}\left(a\,x\right)}^4}{x^4}\,dx$$

input

```
int(asinh(a*x)^4/x^4,x)
```

output

```
int(asinh(a*x)^4/x^4, x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx=\int \frac{asinh(ax)^{4}}{x^{4}}dx$$

input

```
int(asinh(a*x)^4/x^4,x)
```

output

```
int(asinh(a*x)**4/x**4,x)
```

3.45 $\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = -\frac{5\operatorname{Chi}(\operatorname{arcsinh}(ax))}{64a^7} + \frac{9\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{64a^7} - \frac{5\operatorname{Chi}(5\operatorname{arcsinh}(ax))}{64a^7} + \frac{\operatorname{Chi}(7\operatorname{arcsinh}(ax))}{64a^7}$$

output

```
-5/64*Chi(arcsinh(a*x))/a^7+9/64*Chi(3*arcsinh(a*x))/a^7-5/64*Chi(5*arcsin
h(a*x))/a^7+1/64*Chi(7*arcsinh(a*x))/a^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \frac{-5\operatorname{Chi}(\operatorname{arcsinh}(ax)) + 9\operatorname{Chi}(3\operatorname{arcsinh}(ax)) - 5\operatorname{Chi}(5\operatorname{arcsinh}(ax)) + \operatorname{Chi}(7\operatorname{arcsinh}(ax))}{64a^7}$$

input

```
Integrate[x^6/ArcSinh[a*x],x]
```

output
```
(-5*CoshIntegral[ArcSinh[a*x]] + 9*CoshIntegral[3*ArcSinh[a*x]] - 5*CoshIn
tegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = $\{6195, 5971, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{\int \frac{a^6x^6\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^7}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{9\cosh(3\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}-\frac{5\cosh(5\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{\cosh(7\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}-\frac{5\sqrt{a^2x^2+1}}{64\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^7}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{5}{64}\text{Chi}(\text{arcsinh}(ax))+\frac{9}{64}\text{Chi}(3\text{arcsinh}(ax))-\frac{5}{64}\text{Chi}(5\text{arcsinh}(ax))+\frac{1}{64}\text{Chi}(7\text{arcsinh}(ax))}{a^7}$$

input
```
Int[x^6/ArcSinh[a*x],x]
```

output
```
((-5*CoshIntegral[ArcSinh[a*x]])/64 + (9*CoshIntegral[3*ArcSinh[a*x]])/64
- (5*CoshIntegral[5*ArcSinh[a*x]])/64 + CoshIntegral[7*ArcSinh[a*x]]/64)/a
^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{5\,\mathrm{Chi}(\operatorname{arcsinh}(xa))}{64}+\frac{9\,\mathrm{Chi}(3\operatorname{arcsinh}(xa))}{64}-\frac{5\,\mathrm{Chi}(5\operatorname{arcsinh}(xa))}{64}+\frac{\mathrm{Chi}(7\operatorname{arcsinh}(xa))}{64}}{a^7}$	40
default	$\frac{-\frac{5\,\mathrm{Chi}(\operatorname{arcsinh}(xa))}{64}+\frac{9\,\mathrm{Chi}(3\operatorname{arcsinh}(xa))}{64}-\frac{5\,\mathrm{Chi}(5\operatorname{arcsinh}(xa))}{64}+\frac{\mathrm{Chi}(7\operatorname{arcsinh}(xa))}{64}}{a^7}$	40

input
```
int(x^6/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(-5/64*Chi(arcsinh(x*a))+9/64*Chi(3*arcsinh(x*a))-5/64*Chi(5*arcsinh
(x*a))+1/64*Chi(7*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input

```
integrate(x^6/arcsinh(a*x),x, algorithm="fricas")
```

output

```
integral(x^6/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{asinh}\left(ax\right)}\,dx$$

input

```
integrate(x**6/asinh(a*x),x)
```

output

```
Integral(x**6/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input

```
integrate(x^6/arcsinh(a*x),x, algorithm="maxima")
```

output

```
integrate(x^6/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input

```
integrate(x^6/arcsinh(a*x),x, algorithm="giac")
```

output

```
integrate(x^6/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input

```
int(x^6/asinh(a*x),x)
```

output

```
int(x^6/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{asinh\left(ax\right)}dx$$

input

```
int(x^6/asinh(a*x),x)
```

output

```
int(x**6/asinh(a*x),x)
```

3.46 $\int \frac{x^5}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{x^5}{\text{arcsinh}(ax)}\,dx = \frac{5\text{Shi}(2\text{arcsinh}(ax))}{32a^6} - \frac{\text{Shi}(4\text{arcsinh}(ax))}{8a^6} + \frac{\text{Shi}(6\text{arcsinh}(ax))}{32a^6}$$

output
```
5/32*Shi(2*arcsinh(a*x))/a^6-1/8*Shi(4*arcsinh(a*x))/a^6+1/32*Shi(6*arcsin
h(a*x))/a^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\text{arcsinh}(ax)}\,dx = \frac{5\text{Shi}(2\text{arcsinh}(ax)) - 4\text{Shi}(4\text{arcsinh}(ax)) + \text{Shi}(6\text{arcsinh}(ax))}{32a^6}$$

input
```
Integrate[x^5/ArcSinh[a*x],x]
```

output
```
(5*SinhIntegral[2*ArcSinh[a*x]] - 4*SinhIntegral[4*ArcSinh[a*x]] + SinhInt
egral[6*ArcSinh[a*x]])/(32*a^6)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\text{arcsinh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{\int \frac{a^5x^5\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^6}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{5\sinh(2\text{arcsinh}(ax))}{32\text{arcsinh}(ax)}-\frac{\sinh(4\text{arcsinh}(ax))}{8\text{arcsinh}(ax)}+\frac{\sinh(6\text{arcsinh}(ax))}{32\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{5}{32}\text{Shi}(2\text{arcsinh}(ax))-\frac{1}{8}\text{Shi}(4\text{arcsinh}(ax))+\frac{1}{32}\text{Shi}(6\text{arcsinh}(ax))}{a^6}$$

input

```
Int[x^5/ArcSinh[a*x],x]
```

output

```
((5*SinhIntegral[2*ArcSinh[a*x]])/32 - SinhIntegral[4*ArcSinh[a*x]]/8 + Si
nhIntegral[6*ArcSinh[a*x]]/32)/a^6
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{5\,\mathrm{Shi}(2\,\mathrm{arcsinh}(xa))}{32}-\frac{\mathrm{Shi}(4\,\mathrm{arcsinh}(xa))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arcsinh}(xa))}{32}}{a^6}$	33
default	$\frac{\frac{5\,\mathrm{Shi}(2\,\mathrm{arcsinh}(xa))}{32}-\frac{\mathrm{Shi}(4\,\mathrm{arcsinh}(xa))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arcsinh}(xa))}{32}}{a^6}$	33

input
```
int(x^5/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(5/32*Shi(2*arcsinh(x*a))-1/8*Shi(4*arcsinh(x*a))+1/32*Shi(6*arcsinh
(x*a)))
```

Fricas [F]

$$\int \frac{x^5}{\mathrm{arcsinh}(ax)}\,dx=\int \frac{x^5}{\mathrm{arsinh}\,(ax)}\,dx$$

input
```
integrate(x^5/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x^5/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^5}{\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate(x**5/asinh(a*x),x)
```

output
```
Integral(x**5/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^5}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^5/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/arcsinh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/arcsinh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^5}{\operatorname{asinh}(a\,x)}\,dx$$

input
```
int(x^5/asinh(a*x),x)
```

output
```
int(x^5/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^5}{asinh\,(ax)}dx$$

input
```
int(x^5/asinh(a*x),x)
```

output
```
int(x**5/asinh(a*x),x)
```

3.47 $\int \frac{x^4}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{x^4}{\text{arcsinh}(ax)}\,dx = \frac{\text{Chi}(\text{arcsinh}(ax))}{8a^5} - \frac{3\text{Chi}(3\text{arcsinh}(ax))}{16a^5} + \frac{\text{Chi}(5\text{arcsinh}(ax))}{16a^5}$$

output

```
1/8*Chi(arcsinh(a*x))/a^5-3/16*Chi(3*arcsinh(a*x))/a^5+1/16*Chi(5*arcsinh(
a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\text{arcsinh}(ax)}\,dx = \frac{2\text{Chi}(\text{arcsinh}(ax)) - 3\text{Chi}(3\text{arcsinh}(ax)) + \text{Chi}(5\text{arcsinh}(ax))}{16a^5}$$

input

```
Integrate[x^4/ArcSinh[a*x],x]
```

output

```
(2*CoshIntegral[ArcSinh[a*x]] - 3*CoshIntegral[3*ArcSinh[a*x]] + CoshInteg
ral[5*ArcSinh[a*x]])/(16*a^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\text{arcsinh}(ax)} \, dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int \frac{a^4x^4\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^5}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(-\frac{3\cosh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\cosh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\sqrt{a^2x^2+1}}{8\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\text{Chi}(\text{arcsinh}(ax))-\frac{3}{16}\text{Chi}(3\text{arcsinh}(ax))+\frac{1}{16}\text{Chi}(5\text{arcsinh}(ax))}{a^5}$$

input

```
Int[x^4/ArcSinh[a*x],x]
```

output

```
(CoshIntegral[ArcSinh[a*x]]/8 - (3*CoshIntegral[3*ArcSinh[a*x]])/16 + Cosh
Integral[5*ArcSinh[a*x]]/16)/a^5
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{\mathrm{Chi}(\operatorname{arcsinh}(xa))}{8}-\frac{3\ \mathrm{Chi}(3\ \operatorname{arcsinh}(xa))}{16}+\frac{\mathrm{Chi}(5\ \operatorname{arcsinh}(xa))}{16}}{a^{5}}$	31
default	$\frac{\frac{\mathrm{Chi}(\operatorname{arcsinh}(xa))}{8}-\frac{3\ \mathrm{Chi}(3\ \operatorname{arcsinh}(xa))}{16}+\frac{\mathrm{Chi}(5\ \operatorname{arcsinh}(xa))}{16}}{a^{5}}$	31

input
```
int(x^4/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/8*Chi(arcsinh(x*a))-3/16*Chi(3*arcsinh(x*a))+1/16*Chi(5*arcsinh(x
*a)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^4/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x^4/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{asinh}(ax)}\,dx$$

input
```
integrate(x**4/asinh(a*x),x)
```

output
```
Integral(x**4/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x^4/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x^4/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input

```
int(x^4/asinh(a*x),x)
```

output

```
int(x^4/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{asinh\left(ax\right)}dx$$

input

```
int(x^4/asinh(a*x),x)
```

output

```
int(x**4/asinh(a*x),x)
```

3.48 $\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx=-\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{4a^4}+\frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{8a^4}$$

output
```
-1/4*Shi(2*arcsinh(a*x))/a^4+1/8*Shi(4*arcsinh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx=\frac{-2\operatorname{Shi}(2\operatorname{arcsinh}(ax))+\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{8a^4}$$

input
```
Integrate[x^3/ArcSinh[a*x],x]
```

output
```
(-2*SinhIntegral[2*ArcSinh[a*x]] + SinhIntegral[4*ArcSinh[a*x]])/(8*a^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int \frac{a^3x^3\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{\sinh(4\text{arcsinh}(ax))}{8\text{arcsinh}(ax)}-\frac{\sinh(2\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\text{Shi}(4\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(2\text{arcsinh}(ax))}{a^4}$$

input
```
Int[x^3/ArcSinh[a*x],x]
```

output
```
(-1/4*SinhIntegral[2*ArcSinh[a*x]] + SinhIntegral[4*ArcSinh[a*x]]/8)/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Shi}(2 \operatorname{arcsinh}(xa))}{4}+\frac{\operatorname{Shi}(4 \operatorname{arcsinh}(xa))}{8}}{a^4}$	24
default	$\frac{-\frac{\operatorname{Shi}(2 \operatorname{arcsinh}(xa))}{4}+\frac{\operatorname{Shi}(4 \operatorname{arcsinh}(xa))}{8}}{a^4}$	24

input
```
int(x^3/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*Shi(2*arcsinh(x*a))+1/8*Shi(4*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^3/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x^3/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{asinh}(ax)}\,dx$$

input
```
integrate(x**3/asinh(a*x),x)
```

output
```
Integral(x**3/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x^3/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/arcsinh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input

```
int(x^3/asinh(a*x),x)
```

output

```
int(x^3/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\mathit{asinh}\left(ax\right)}dx$$

input

```
int(x^3/asinh(a*x),x)
```

output

```
int(x**3/asinh(a*x),x)
```

3.49 $\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = -\frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{4a^3} + \frac{\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{4a^3}$$

output
```
-1/4*Chi(arcsinh(a*x))/a^3+1/4*Chi(3*arcsinh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \frac{-\operatorname{Chi}(\operatorname{arcsinh}(ax)) + \operatorname{Chi}(3\operatorname{arcsinh}(ax))}{4a^3}$$

input
```
Integrate[x^2/ArcSinh[a*x],x]
```

output
```
(-CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{\int \frac{a^2x^2\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{\cosh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}-\frac{\sqrt{a^2x^2+1}}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{4}\text{Chi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Chi}(\text{arcsinh}(ax))}{a^3}$$

input

```
Int[x^2/ArcSinh[a*x],x]
```

output

```
(-1/4*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]/4)/a^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{4}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(xa))}{4}}{a^3}$	22
default	$\frac{-\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{4}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(xa))}{4}}{a^3}$	22

input
```
int(x^2/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/4*Chi(arcsinh(x*a))+1/4*Chi(3*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{asinh}(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x),x)
```

output
```
Integral(x**2/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x^2/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x^2/asinh(a*x),x)
```

output
```
int(x^2/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{asinh\left(ax\right)}dx$$

input
```
int(x^2/asinh(a*x),x)
```

output
```
int(x**2/asinh(a*x),x)
```

3.50 $\int \frac{x}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{x}{\text{arcsinh}(ax)}\,dx = \frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^2}$$

output

```
1/2*Shi(2*arcsinh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{\text{arcsinh}(ax)}\,dx = \frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^2}$$

input

```
Integrate[x/ArcSinh[a*x],x]
```

output

```
SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6195, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int \frac{ax\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \frac{\sinh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sinh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sin(2i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^2}$$

input
```
Int[x/ArcSinh[a*x],x]
```

output
```
SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(xa))}{2a^2}$	13
default	$\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(xa))}{2a^2}$	13

input
```
int(x/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*Shi(2*arcsinh(x*a))/a^2
```

Fricas [F]

$$\int \frac{x}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{x}{\mathrm{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{x}{\mathrm{asinh}\left(ax\right)}\,dx$$

input
```
integrate(x/asinh(a*x),x)
```

output
```
Integral(x/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(x/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(x/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x/asinh(a*x),x)
```

output
```
int(x/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{asinh\,(ax)}dx$$

input
```
int(x/asinh(a*x),x)
```

output
```
int(x/asinh(a*x),x)
```

3.51 $\int \frac{1}{\text{arcsinh}(ax)} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{\text{arcsinh}(ax)} \, dx = \frac{\text{Chi}(\text{arcsinh}(ax))}{a}$$

output

```
Chi(arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\text{arcsinh}(ax)} \, dx = \frac{\text{Chi}(\text{arcsinh}(ax))}{a}$$

input

```
Integrate[ArcSinh[a*x]^(-1),x]
```

output

```
CoshIntegral[ArcSinh[a*x]]/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6189, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\text{Chi}(\text{arcsinh}(ax))}{a}$$

input
```
Int[ArcSinh[a*x]^(-1),x]
```

output
```
CoshIntegral[ArcSinh[a*x]]/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{a}$	10
default	$\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{a}$	10

input
```
int(1/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
Chi(arcsinh(x*a))/a
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(1/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}{\left(ax \right)}}\,dx$$

input
```
integrate(1/asinh(a*x),x)
```

output
```
Integral(1/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(1/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(1/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(1/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)}\,dx$$

input
```
int(1/asinh(a*x),x)
```

output
```
int(1/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{asinh\,(ax)}dx$$

input
```
int(1/asinh(a*x),x)
```

output
```
int(1/asinh(a*x),x)
```

3.52 $\int \frac{1}{x\mathbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx=\text{Int}\left(\frac{1}{x\text{arcsinh}(ax)},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx=\int \frac{1}{x\text{arcsinh}(ax)}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x\text{arcsinh}(ax)}dx$$

input
```
Int[1/(x*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ \text{arcsinh}\left(xa\right)}dx$$

input
```
int(1/x/arcsinh(x*a),x)
```

output
```
int(1/x/arcsinh(x*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{asinh}\,(ax)}\,dx$$

input
```
integrate(1/x/asinh(a*x),x)
```

output
```
Integral(1/(x*asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/x/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)}\,dx$$

input
```
int(1/(x*asinh(a*x)),x)
```

output
```
int(1/(x*asinh(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{asinh\left(ax\right)x}dx$$

input
```
int(1/x/asinh(a*x),x)
```

output
```
int(1/(asinh(a*x)*x),x)
```

3.53 $\int \frac{1}{x^2\textbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\text{arcsinh}(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/arcsinh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)}\,dx$$

input
```
Integrate[1/(x^2*ArcSinh[a*x]),x]
```

output
```
Integrate[1/(x^2*ArcSinh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{1}{x^2\text{arcsinh}(ax)}dx$$

input
```
Int[1/(x^2*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\,\text{arcsinh}\,(xa)}dx$$

input
```
int(1/x^2/arcsinh(x*a),x)
```

output
```
int(1/x^2/arcsinh(x*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x^2*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate(1/x**2/asinh(a*x),x)
```

output
```
Integral(1/(x**2*asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x),x, algorithm="maxima")
```

output

```
integrate(1/(x^2*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\,\text{arsinh}\,(ax)}\,dx$$

input

```
integrate(1/x^2/arcsinh(a*x),x, algorithm="giac")
```

output

```
integrate(1/(x^2*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\,\text{asinh}\,(a\,x)}\,dx$$

input

```
int(1/(x^2*asinh(a*x)),x)
```

output

```
int(1/(x^2*asinh(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{asinh\,(ax)\,x^2}dx$$

input

```
int(1/x^2/asinh(a*x),x)
```

output

```
int(1/(asinh(a*x)*x**2),x)
```

3.54 $\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx = -\frac{x^6\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)} - \frac{5\text{Shi}(\text{arcsinh}(ax))}{64a^7} + \frac{27\text{Shi}(3\text{arcsinh}(ax))}{64a^7} - \frac{25\text{Shi}(5\text{arcsinh}(ax))}{64a^7} + \frac{7\text{Shi}(7\text{arcsinh}(ax))}{64a^7}$$

output
```
-x^6*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)-5/64*Shi(arcsinh(a*x))/a^7+27/64*Shi
(3*arcsinh(a*x))/a^7-25/64*Shi(5*arcsinh(a*x))/a^7+7/64*Shi(7*arcsinh(a*x)
)/a^7
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx = -\frac{64a^6x^6\sqrt{1+a^2x^2} + 5\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax)) - 27\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax)) + 25\text{arcsinh}(a}{64a^7\text{arcsinh}(ax)}$$

input
```
Integrate[x^6/ArcSinh[a*x]^2,x]
```

output

```
-1/64*(64*a^6*x^6*Sqrt[1 + a^2*x^2] + 5*ArcSinh[a*x]*SinhIntegral[ArcSinh[
a*x]] - 27*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 25*ArcSinh[a*x]*Sin
hIntegral[5*ArcSinh[a*x]] - 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]])/(
a^7*ArcSinh[a*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{\int \left(-\frac{5ax}{64\text{arcsinh}(ax)}+\frac{27\sinh(3\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}-\frac{25\sinh(5\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{7\sinh(7\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^7}-\frac{x^6\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{5}{64}\text{Shi}(\text{arcsinh}(ax))+\frac{27}{64}\text{Shi}(3\text{arcsinh}(ax))-\frac{25}{64}\text{Shi}(5\text{arcsinh}(ax))+\frac{7}{64}\text{Shi}(7\text{arcsinh}(ax))}{a^7}-\frac{x^6\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input

```
Int[x^6/ArcSinh[a*x]^2,x]
```

output

```
-((x^6*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + ((-5*SinhIntegral[ArcSinh[a*
x]])/64 + (27*SinhIntegral[3*ArcSinh[a*x]])/64 - (25*SinhIntegral[5*ArcSin
h[a*x]])/64 + (7*SinhIntegral[7*ArcSinh[a*x]])/64)/a^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{5\sqrt{a^2x^2+1}}{64\,\mathrm{arcsinh}(xa)}-\frac{5\,\mathrm{Shi}(\mathrm{arcsinh}(xa))}{64}-\frac{9\cosh(3\,\mathrm{arcsinh}(xa))}{64\,\mathrm{arcsinh}(xa)}+\frac{27\,\mathrm{Shi}(3\,\mathrm{arcsinh}(xa))}{64}+\frac{5\cosh(5\,\mathrm{arcsinh}(xa))}{64\,\mathrm{arcsinh}(xa)}-\frac{25\,\mathrm{Shi}(5\,\mathrm{arcsinh}(xa}{64}}{a^7}$
default	$\frac{\frac{5\sqrt{a^2x^2+1}}{64\,\mathrm{arcsinh}(xa)}-\frac{5\,\mathrm{Shi}(\mathrm{arcsinh}(xa))}{64}-\frac{9\cosh(3\,\mathrm{arcsinh}(xa))}{64\,\mathrm{arcsinh}(xa)}+\frac{27\,\mathrm{Shi}(3\,\mathrm{arcsinh}(xa))}{64}+\frac{5\cosh(5\,\mathrm{arcsinh}(xa))}{64\,\mathrm{arcsinh}(xa)}-\frac{25\,\mathrm{Shi}(5\,\mathrm{arcsinh}(xa}{64}}{a^7}$

input
```
int(x^6/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(5/64/arcsinh(x*a)*(a^2*x^2+1)^(1/2)-5/64*Shi(arcsinh(x*a))-9/64/arc
sinh(x*a)*cosh(3*arcsinh(x*a))+27/64*Shi(3*arcsinh(x*a))+5/64/arcsinh(x*a)
*cosh(5*arcsinh(x*a))-25/64*Shi(5*arcsinh(x*a))-1/64/arcsinh(x*a)*cosh(7*a
rcsinh(x*a))+7/64*Shi(7*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^6}{\mathrm{arcsinh}(ax)^2}\,dx=\int \frac{x^6}{\mathrm{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^6/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^6/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{asinh}^2(ax)}\,dx$$

input
```
integrate(x**6/asinh(a*x)**2,x)
```

output
```
Integral(x**6/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(x^6/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^3*x^9 + a*x^7 + (a^2*x^8 + x^6)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((7*a^5*x^1
0 + 14*a^3*x^8 + 7*a*x^6 + (7*a^3*x^8 + 5*a*x^6)*(a^2*x^2 + 1) + (14*a^4*x
^9 + 19*a^2*x^7 + 6*x^5)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*
x^2 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqr
t(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^6/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^6/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input
```
int(x^6/asinh(a*x)^2,x)
```

output
```
int(x^6/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{asinh\left(ax\right)^2}dx$$

input
```
int(x^6/asinh(a*x)^2,x)
```

output
```
int(x**6/asinh(a*x)**2,x)
```

3.55 $\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{x^5\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} + \frac{5\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{16a^6} - \frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{2a^6} + \frac{3\operatorname{Chi}(6\operatorname{arcsinh}(ax))}{16a^6}$$

output
```
-x^5*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)+5/16*Chi(2*arcsinh(a*x))/a^6-1/2*Chi
(4*arcsinh(a*x))/a^6+3/16*Chi(6*arcsinh(a*x))/a^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{-10\operatorname{arcsinh}(ax)\operatorname{Chi}(2\operatorname{arcsinh}(ax)) + 16\operatorname{arcsinh}(ax)\operatorname{Chi}(4\operatorname{arcsinh}(ax)) - 6\operatorname{arcsinh}(ax)\operatorname{Chi}(6\operatorname{arcsinh}(a}{32a^6\operatorname{arcsinh}(ax)}$$

input
```
Integrate[x^5/ArcSinh[a*x]^2,x]
```

output
```
-1/32*(-10*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] + 16*ArcSinh[a*x]*Cos
hIntegral[4*ArcSinh[a*x]] - 6*ArcSinh[a*x]*CoshIntegral[6*ArcSinh[a*x]] +
5*Sinh[2*ArcSinh[a*x]] - 4*Sinh[4*ArcSinh[a*x]] + Sinh[6*ArcSinh[a*x]])/(a
^6*ArcSinh[a*x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\text{arcsinh}(ax)^2} \, dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{\int \left(\frac{5\cosh(2\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}-\frac{\cosh(4\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}+\frac{3\cosh(6\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{5}{16}\text{Chi}(2\text{arcsinh}(ax))-\frac{1}{2}\text{Chi}(4\text{arcsinh}(ax))+\frac{3}{16}\text{Chi}(6\text{arcsinh}(ax))}{a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input
```
Int[x^5/ArcSinh[a*x]^2,x]
```

output
```
-((x^5*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + ((5*CoshIntegral[2*ArcSinh[a
*x]])/16 - CoshIntegral[4*ArcSinh[a*x]]/2 + (3*CoshIntegral[6*ArcSinh[a*x]
])/16)/a^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))    Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{5\sinh(2\,\mathrm{arcsinh}(xa))}{32\,\mathrm{arcsinh}(xa)}+\frac{5\,\mathrm{Chi}(2\,\mathrm{arcsinh}(xa))}{16}+\frac{\sinh(4\,\mathrm{arcsinh}(xa))}{8\,\mathrm{arcsinh}(xa)}-\frac{\mathrm{Chi}(4\,\mathrm{arcsinh}(xa))}{2}-\frac{\sinh(6\,\mathrm{arcsinh}(xa))}{32\,\mathrm{arcsinh}(xa)}+\frac{3\,\mathrm{Chi}(6\,\mathrm{arcsinh}}{16}}{a^6}$
default	$\frac{-\frac{5\sinh(2\,\mathrm{arcsinh}(xa))}{32\,\mathrm{arcsinh}(xa)}+\frac{5\,\mathrm{Chi}(2\,\mathrm{arcsinh}(xa))}{16}+\frac{\sinh(4\,\mathrm{arcsinh}(xa))}{8\,\mathrm{arcsinh}(xa)}-\frac{\mathrm{Chi}(4\,\mathrm{arcsinh}(xa))}{2}-\frac{\sinh(6\,\mathrm{arcsinh}(xa))}{32\,\mathrm{arcsinh}(xa)}+\frac{3\,\mathrm{Chi}(6\,\mathrm{arcsinh}}{16}}{a^6}$

input
```
int(x^5/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(-5/32/arcsinh(x*a)*sinh(2*arcsinh(x*a))+5/16*Chi(2*arcsinh(x*a))+1/
8/arcsinh(x*a)*sinh(4*arcsinh(x*a))-1/2*Chi(4*arcsinh(x*a))-1/32/arcsinh(x
*a)*sinh(6*arcsinh(x*a))+3/16*Chi(6*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^5}{\mathrm{arcsinh}(ax)^2}\,dx=\int \frac{x^5}{\mathrm{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^5/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^5/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^5}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

input
```
integrate(x**5/asinh(a*x)**2,x)
```

output
```
Integral(x**5/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^5}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^5/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^3*x^8 + a*x^6 + (a^2*x^7 + x^5)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((6*a^5*x^9
 + 12*a^3*x^7 + 6*a*x^5 + 2*(3*a^3*x^7 + 2*a*x^5)*(a^2*x^2 + 1) + (12*a^4*
x^8 + 16*a^2*x^6 + 5*x^4)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3
*x^2 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sq
rt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^5}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input
```
int(x^5/asinh(a*x)^2,x)
```

output
```
int(x^5/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^5}{\mathit{asinh}\left(ax\right)^2}dx$$

input
```
int(x^5/asinh(a*x)^2,x)
```

output
```
int(x**5/asinh(a*x)**2,x)
```

3.56 $\int \frac{x^4}{\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{x^4}{\text{arcsinh}(ax)^2}\,dx = -\frac{x^4\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)} + \frac{\text{Shi}(\text{arcsinh}(ax))}{8a^5} - \frac{9\text{Shi}(3\text{arcsinh}(ax))}{16a^5} + \frac{5\text{Shi}(5\text{arcsinh}(ax))}{16a^5}$$

output

```
-x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)+1/8*Shi(arcsinh(a*x))/a^5-9/16*Shi(3
*arcsinh(a*x))/a^5+5/16*Shi(5*arcsinh(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\text{arcsinh}(ax)^2}\,dx = \frac{-\frac{16a^4x^4\sqrt{1+a^2x^2}}{\text{arcsinh}(ax)} + 2\text{Shi}(\text{arcsinh}(ax)) - 9\text{Shi}(3\text{arcsinh}(ax)) + 5\text{Shi}(5\text{arcsinh}(ax))}{16a^5}$$

input

```
Integrate[x^4/ArcSinh[a*x]^2,x]
```

output
```
((-16*a^4*x^4*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + 2*SinhIntegral[ArcSinh[a*x
]] - 9*SinhIntegral[3*ArcSinh[a*x]] + 5*SinhIntegral[5*ArcSinh[a*x]])/(16*
a^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{\int \left(\frac{ax}{8\text{arcsinh}(ax)}-\frac{9\sinh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{5\sinh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\text{Shi}(\text{arcsinh}(ax))-\frac{9}{16}\text{Shi}(3\text{arcsinh}(ax))+\frac{5}{16}\text{Shi}(5\text{arcsinh}(ax))}{a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input
```
Int[x^4/ArcSinh[a*x]^2,x]
```

output
```
-((x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (SinhIntegral[ArcSinh[a*x]]/
8 - (9*SinhIntegral[3*ArcSinh[a*x]])/16 + (5*SinhIntegral[5*ArcSinh[a*x]])
/16)/a^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{8 \operatorname{arcsinh}\left(x a\right)}+\frac{\operatorname{Shi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{8}+\frac{3 \cosh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{16 \operatorname{arcsinh}\left(x a\right)}-\frac{9 \operatorname{Shi}\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{16}-\frac{\cosh\left(5 \operatorname{arcsinh}\left(x a\right)\right)}{16 \operatorname{arcsinh}\left(x a\right)}+\frac{5 \operatorname{Shi}\left(5 \operatorname{arcsinh}\left(x a\right)\right)}{16}}{a^{5}}$
default	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{8 \operatorname{arcsinh}\left(x a\right)}+\frac{\operatorname{Shi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{8}+\frac{3 \cosh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{16 \operatorname{arcsinh}\left(x a\right)}-\frac{9 \operatorname{Shi}\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{16}-\frac{\cosh\left(5 \operatorname{arcsinh}\left(x a\right)\right)}{16 \operatorname{arcsinh}\left(x a\right)}+\frac{5 \operatorname{Shi}\left(5 \operatorname{arcsinh}\left(x a\right)\right)}{16}}{a^{5}}$

input
```
int(x^4/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-1/8/arcsinh(x*a)*(a^2*x^2+1)^(1/2)+1/8*Shi(arcsinh(x*a))+3/16/arcs
inh(x*a)*cosh(3*arcsinh(x*a))-9/16*Shi(3*arcsinh(x*a))-1/16/arcsinh(x*a)*c
osh(5*arcsinh(x*a))+5/16*Shi(5*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{x^{4}}{\operatorname{arsinh}\left(a x\right)^{2}}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^4/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^4}{\operatorname{asinh}^2(ax)}\,dx$$

input
```
integrate(x**4/asinh(a*x)**2,x)
```

output
```
Integral(x**4/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^3*x^7 + a*x^5 + (a^2*x^6 + x^4)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^5*x^8
 + 10*a^3*x^6 + 5*a*x^4 + (5*a^3*x^6 + 3*a*x^4)*(a^2*x^2 + 1) + (10*a^4*x^
7 + 13*a^2*x^5 + 4*x^3)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x
^2 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt
(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input
```
int(x^4/asinh(a*x)^2,x)
```

output
```
int(x^4/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^4}{asinh\left(ax\right)^2}dx$$

input
```
int(x^4/asinh(a*x)^2,x)
```

output
```
int(x**4/asinh(a*x)**2,x)
```

3.57 $\int \frac{x^3}{\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{x^3}{\text{arcsinh}(ax)^2}\,dx=-\frac{x^3\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}-\frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^4}+\frac{\text{Chi}(4\text{arcsinh}(ax))}{2a^4}$$

output

```
-x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)-1/2*Chi(2*arcsinh(a*x))/a^4+1/2*Chi(
4*arcsinh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\text{arcsinh}(ax)^2}\,dx = -\frac{4\text{arcsinh}(ax)\text{Chi}(2\text{arcsinh}(ax))-4\text{arcsinh}(ax)\text{Chi}(4\text{arcsinh}(ax))-2\sinh(2\text{arcsinh}(ax))+\sinh(4\text{arc}}{8a^4\text{arcsinh}(ax)}$$

input

```
Integrate[x^3/ArcSinh[a*x]^2,x]
```

output
```
-1/8*(4*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] - 4*ArcSinh[a*x]*CoshInt
egral[4*ArcSinh[a*x]] - 2*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])/(a^
4*ArcSinh[a*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arcsinh}(ax)^2}dx$$
$$\Big\downarrow \text{6193}$$
$$\frac{\int \left(\frac{\cosh(4\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}-\frac{\cosh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{2}\text{Chi}(4\text{arcsinh}(ax))-\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input
```
Int[x^3/ArcSinh[a*x]^2,x]
```

output
```
-((x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (-1/2*CoshIntegral[2*ArcSinh
[a*x]] + CoshIntegral[4*ArcSinh[a*x]]/2)/a^4
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{\sinh(2\,\mathrm{arcsinh}(xa))}{4\,\mathrm{arcsinh}(xa)}-\frac{\mathrm{Chi}(2\,\mathrm{arcsinh}(xa))}{2}-\frac{\sinh(4\,\mathrm{arcsinh}(xa))}{8\,\mathrm{arcsinh}(xa)}+\frac{\mathrm{Chi}(4\,\mathrm{arcsinh}(xa))}{2}}{a^4}$	54
default	$\frac{\frac{\sinh(2\,\mathrm{arcsinh}(xa))}{4\,\mathrm{arcsinh}(xa)}-\frac{\mathrm{Chi}(2\,\mathrm{arcsinh}(xa))}{2}-\frac{\sinh(4\,\mathrm{arcsinh}(xa))}{8\,\mathrm{arcsinh}(xa)}+\frac{\mathrm{Chi}(4\,\mathrm{arcsinh}(xa))}{2}}{a^4}$	54

input

```
int(x^3/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/a^4*(1/4/arcsinh(x*a)*sinh(2*arcsinh(x*a))-1/2*Chi(2*arcsinh(x*a))-1/8/a
rcsinh(x*a)*sinh(4*arcsinh(x*a))+1/2*Chi(4*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^3}{\mathrm{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\mathrm{arsinh}\,(ax)^2}\,dx$$

input

```
integrate(x^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

output

```
integral(x^3/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\operatorname{asinh}^2(ax)}\,dx$$

input

```
integrate(x**3/asinh(a*x)**2,x)
```

output

```
Integral(x**3/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^2}\,dx$$

input

```
integrate(x^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

output

```
-(a^3*x^6 + a*x^4 + (a^2*x^5 + x^3)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((4*a^5*x^7
 + 8*a^3*x^5 + 4*a*x^3 + 2*(2*a^3*x^5 + a*x^3)*(a^2*x^2 + 1) + (8*a^4*x^6
+ 10*a^2*x^4 + 3*x^2)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a
^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

input
```
int(x^3/asinh(a*x)^2,x)
```

output
```
int(x^3/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\mathit{asinh}\left(ax\right)^2}dx$$

input
```
int(x^3/asinh(a*x)^2,x)
```

output
```
int(x**3/asinh(a*x)**2,x)
```

3.58 $\int \frac{x^2}{\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} - \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{4a^3} + \frac{3\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{4a^3}$$

output

```
-x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)-1/4*Shi(arcsinh(a*x))/a^3+3/4*Shi(3*
arcsinh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{\frac{4a^2x^2\sqrt{1+a^2x^2}}{\operatorname{arcsinh}(ax)} + \operatorname{Shi}(\operatorname{arcsinh}(ax)) - 3\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{4a^3}$$

input

```
Integrate[x^2/ArcSinh[a*x]^2,x]
```

output

```
-1/4*((4*a^2*x^2*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + SinhIntegral[ArcSinh[a*
x]] - 3*SinhIntegral[3*ArcSinh[a*x]])/a^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arcsinh}(ax)^2}\,dx$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{\int \left(\frac{3\sinh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}-\frac{ax}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input

```
Int[x^2/ArcSinh[a*x]^2,x]
```

output

```
-((x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (-1/4*SinhIntegral[ArcSinh[a
*x]] + (3*SinhIntegral[3*ArcSinh[a*x]])/4)/a^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\frac{\sqrt{a^{2} x^{2}+1}}{4 \operatorname{arcsinh}\left(x a\right)}-\frac{\operatorname{Shi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{4}-\frac{\cosh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{4 \operatorname{arcsinh}\left(x a\right)}+\frac{3 \operatorname{Shi}\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{4}}{a^{3}}$	56
default	$\frac{\frac{\sqrt{a^{2} x^{2}+1}}{4 \operatorname{arcsinh}\left(x a\right)}-\frac{\operatorname{Shi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{4}-\frac{\cosh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{4 \operatorname{arcsinh}\left(x a\right)}+\frac{3 \operatorname{Shi}\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{4}}{a^{3}}$	56

input
```
int(x^2/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/4/arcsinh(x*a)*(a^2*x^2+1)^(1/2)-1/4*Shi(arcsinh(x*a))-1/4/arcsin
h(x*a)*cosh(3*arcsinh(x*a))+3/4*Shi(3*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{x^{2}}{\operatorname{arsinh}\left(a x\right)^{2}} \, dx$$

input
```
integrate(x^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{x^{2}}{\operatorname{asinh}^{2}{\left(a x \right)}}\, dx$$

input
```
integrate(x**2/asinh(a*x)**2,x)
```

output
```
Integral(x**2/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input

```
integrate(x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

output

```
-(a^3*x^5 + a*x^3 + (a^2*x^4 + x^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((3*a^5*x^6
 + 6*a^3*x^4 + 3*a*x^2 + (3*a^3*x^4 + a*x^2)*(a^2*x^2 + 1) + (6*a^4*x^5 +
7*a^2*x^3 + 2*x)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*
a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^
2 + 1))), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input

```
integrate(x^2/arcsinh(a*x)^2,x, algorithm="giac")
```

output

```
integrate(x^2/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input

```
int(x^2/asinh(a*x)^2,x)
```

output

```
int(x^2/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\mathit{asinh}\left(ax\right)^2}dx$$

input

```
int(x^2/asinh(a*x)^2,x)
```

output

```
int(x**2/asinh(a*x)**2,x)
```

3.59 $\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{x\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{a^2}$$

output
```
-x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)+Chi(2*arcsinh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax)) - \frac{\sinh(2\operatorname{arcsinh}(ax))}{2\operatorname{arcsinh}(ax)}}{a^2}$$

input
```
Integrate[x/ArcSinh[a*x]^2,x]
```

output
```
(CoshIntegral[2*ArcSinh[a*x]] - Sinh[2*ArcSinh[a*x]]/(2*ArcSinh[a*x]))/a^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6193, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{\int \frac{\cosh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}+\frac{\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{ 3782}$$

$$\frac{\text{Chi}(2\text{arcsinh}(ax))}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input

```
Int[x/ArcSinh[a*x]^2,x]
```

output

```
-((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + CoshIntegral[2*ArcSinh[a*x]]/a
^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\mathrm{arcsinh}(xa))}{2\,\mathrm{arcsinh}(xa)}+\mathrm{Chi}(2\,\mathrm{arcsinh}(xa))}{a^2}$	28
default	$\frac{-\frac{\sinh(2\,\mathrm{arcsinh}(xa))}{2\,\mathrm{arcsinh}(xa)}+\mathrm{Chi}(2\,\mathrm{arcsinh}(xa))}{a^2}$	28

input
```
int(x/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/2/arcsinh(x*a)*sinh(2*arcsinh(x*a))+Chi(2*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input

```
integrate(x/arcsinh(a*x)^2,x, algorithm="fricas")
```

output

```
integral(x/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

input

```
integrate(x/asinh(a*x)**2,x)
```

output

```
Integral(x/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input

```
integrate(x/arcsinh(a*x)^2,x, algorithm="maxima")
```

output

```
-(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*
x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((2*a^5*x^5 +
 2*(a^2*x^2 + 1)*a^3*x^3 + 4*a^3*x^3 + 2*a*x + (4*a^4*x^4 + 4*a^2*x^2 + 1)
*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*x^2 + 2*(a^4
*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input

```
integrate(x/arcsinh(a*x)^2,x, algorithm="giac")
```

output

```
integrate(x/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input

```
int(x/asinh(a*x)^2,x)
```

output

```
int(x/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{asinh\left(ax\right)^2}dx$$

input

```
int(x/asinh(a*x)^2,x)
```

output

```
int(x/asinh(a*x)**2,x)
```

3.60 $\int \frac{1}{\operatorname{arcsinh}(ax)^2} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2} \, dx = -\frac{\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} + \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{a}$$

output
```
-(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)+Shi(arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2} \, dx = \frac{-\frac{\sqrt{1+a^2x^2}}{\operatorname{arcsinh}(ax)} + \operatorname{Shi}(\operatorname{arcsinh}(ax))}{a}$$

input
```
Integrate[ArcSinh[a*x]^(-2),x]
```

output
```
(-(Sqrt[1 + a^2*x^2]/ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6188, 6234, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6188}$$

$$a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int \frac{ax}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}+\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}-\frac{i\int \frac{\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(\text{arcsinh}(ax))}{a}-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

input

```
Int[ArcSinh[a*x]^(-2),x]
```

output

```
-(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/a
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6188

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{a^2x^2+1}}{\operatorname{arcsinh}(xa)}+\operatorname{Shi}(\operatorname{arcsinh}(xa))}{a}$	30
default	$\frac{-\frac{\sqrt{a^2x^2+1}}{\operatorname{arcsinh}(xa)}+\operatorname{Shi}(\operatorname{arcsinh}(xa))}{a}$	30

input

```
int(1/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/arcsinh(x*a)*(a^2*x^2+1)^(1/2)+Shi(arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

input
```
integrate(1/asinh(a*x)**2,x)
```

output
```
Integral(asinh(a*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x
 + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((a^4*x^4 + 2*a^2*x^2 + (a^
2*x^2 + 1)*(a^2*x^2 - 1) + (2*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^4*
x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 +
 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{2}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{2}}\,dx$$

input
```
int(1/asinh(a*x)^2,x)
```

output
```
int(1/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{asinh\left(ax\right)^{2}}dx$$

input

```
int(1/asinh(a*x)^2,x)
```

output

```
int(1/asinh(a*x)**2,x)
```

3.61 $\int \frac{1}{x\mathbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^2},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^2}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]^2),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{1}{x\text{arcsinh}(ax)^2}dx$$

input
```
Int[1/(x*ArcSinh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}\left(x a\right)^{2}}dx$$

input
```
int(1/x/arcsinh(x*a)^2,x)
```

output
```
int(1/x/arcsinh(x*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2} \, dx = \int \frac{1}{x \,\text{arsinh}\left(ax\right)^2} \, dx$$

input
```
integrate(1/x/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^2} \, dx = \int \frac{1}{x \,\text{asinh}^2\left(ax\right)} \, dx$$

input
```
integrate(1/x/asinh(a*x)**2,x)
```

output
```
Integral(1/(x*asinh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 20.00

$$\int \frac{1}{x\text{arcsinh}(ax)^2} \, dx = \int \frac{1}{x \,\text{arsinh}\left(ax\right)^2} \, dx$$

input
```
integrate(1/x/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^3 + sqrt(a^2*x^2 + 1)*a^2*x
^2 + a*x)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((2*(a^2*x^2 + 1)*a*x +
 (2*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^5*x^6 + (a^2*x^2 + 1)*a^3*x^4 + 2*
a^3*x^4 + a*x^2 + 2*(a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(
a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^2}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)^2}\,dx$$

input
```
int(1/(x*asinh(a*x)^2),x)
```

output
```
int(1/(x*asinh(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{asinh\left(ax\right)^2 x}dx$$

input

```
int(1/x/asinh(a*x)^2,x)
```

output

```
int(1/(asinh(a*x)**2*x),x)
```

3.62 $\int \frac{1}{x^2\mathbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx = \text{Int}\left(\frac{1}{x^2\text{arcsinh}(ax)^2},x\right)$$

output
```
Defer(Int)(1/x^2/arcsinh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*ArcSinh[a*x]^2),x]
```

output
```
Integrate[1/(x^2*ArcSinh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2} \, dx$$
$$\Big\downarrow \ 6196$$
$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2}dx$$

input
```
Int[1/(x^2*ArcSinh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}\left(xa\right)^2}dx$$

input
```
int(1/x^2/arcsinh(x*a)^2,x)
```

output
```
int(1/x^2/arcsinh(x*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2 \operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2 \operatorname{asinh}^2(ax)}\,dx$$

input
```
integrate(1/x**2/asinh(a*x)**2,x)
```

output
```
Integral(1/(x**2*asinh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 237, normalized size of antiderivative = 23.70

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2 \operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

output

```
-(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^4 + sqrt(a^2*x^2 + 1)*a^2*x
^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^5*x^5 + 2*a^3*x^3
 + (a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + a*x + (2*a^4*x^4 + 5*a^2*x^2 + 2)*sqr
t(a^2*x^2 + 1))/((a^5*x^7 + (a^2*x^2 + 1)*a^3*x^5 + 2*a^3*x^5 + a*x^3 + 2*
(a^4*x^6 + a^2*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\,\text{arsinh}\left(ax\right)^2}\,dx$$

input

```
integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="giac")
```

output

```
integrate(1/(x^2*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\,\text{asinh}\left(a\,x\right)^2}\,dx$$

input

```
int(1/(x^2*asinh(a*x)^2),x)
```

output

```
int(1/(x^2*asinh(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{asinh\,(ax)^2\,x^2}dx$$

input

```
int(1/x^2/asinh(a*x)^2,x)
```

output

```
int(1/(asinh(a*x)**2*x**2),x)
```

3.63 $\int \frac{x^4}{\text{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\begin{aligned}\int \frac{x^4}{\text{arcsinh}(ax)^3}\,dx = &-\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2}-\frac{2x^3}{a^2\text{arcsinh}(ax)}\\ &-\frac{5x^5}{2\text{arcsinh}(ax)}+\frac{\text{Chi}(\text{arcsinh}(ax))}{16a^5}\\ &-\frac{27\text{Chi}(3\text{arcsinh}(ax))}{32a^5}+\frac{25\text{Chi}(5\text{arcsinh}(ax))}{32a^5}\end{aligned}$$

output

```
-1/2*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2-2*x^3/a^2/arcsinh(a*x)-5/2*x^5
/arcsinh(a*x)+1/16*Chi(arcsinh(a*x))/a^5-27/32*Chi(3*arcsinh(a*x))/a^5+25/
32*Chi(5*arcsinh(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\text{arcsinh}(ax)^3}\,dx = -\frac{16a^4x^4\sqrt{1+a^2x^2}+64a^3x^3\text{arcsinh}(ax)+80a^5x^5\text{arcsinh}(ax)-2\text{arcsinh}(ax)^2\text{Chi}(\text{arcsinh}(ax))+27a}{32a^5\text{arcsinh}(ax)^2}$$

input
```
Integrate[x^4/ArcSinh[a*x]^3,x]
```

output
```
-1/32*(16*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5
*ArcSinh[a*x] - 2*ArcSinh[a*x]^2*CoshIntegral[ArcSinh[a*x]] + 27*ArcSinh[a
*x]^2*CoshIntegral[3*ArcSinh[a*x]] - 25*ArcSinh[a*x]^2*CoshIntegral[5*ArcS
inh[a*x]])/(a^5*ArcSinh[a*x]^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6194, 6233, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\text{arcsinh}(ax)^3}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{5}{2}a\int \frac{x^5}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx+\frac{2\int \frac{x^3}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx}{a}-\frac{x^4\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6233}$$

$$\frac{5}{2}a\left(\frac{5\int \frac{x^4}{\text{arcsinh}(ax)}dx}{a}-\frac{x^5}{a\text{arcsinh}(ax)}\right)+\frac{2\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)}dx}{a}-\frac{x^3}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6195}$$

$$\frac{2\left(\frac{3\int \frac{a^2x^2\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^4}-\frac{x^3}{a\text{arcsinh}(ax)}\right)}{a}+\frac{5}{2}a\left(\frac{5\int \frac{a^4x^4\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^6}-\frac{x^5}{a\text{arcsinh}(ax)}\right)-\frac{x^4\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{5}{2}a\left(\frac{5\int\left(-\frac{3\cosh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\cosh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\sqrt{a^2x^2+1}}{8\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^6}-\frac{x^5}{a\text{arcsinh}(ax)}\right)+\frac{2\left(\frac{3\int\left(\frac{\cosh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}-\frac{\sqrt{a^2x^2+1}}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^4}-\frac{x^3}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{2}a\left(\frac{5\left(\frac{1}{8}\text{Chi}(\text{arcsinh}(ax))-\frac{3}{16}\text{Chi}(3\text{arcsinh}(ax))+\frac{1}{16}\text{Chi}(5\text{arcsinh}(ax))\right)}{a^6}-\frac{x^5}{a\text{arcsinh}(ax)}\right)+\frac{2\left(\frac{3\left(\frac{1}{4}\text{Chi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Chi}(\text{arcsinh}(ax))\right)}{a^4}-\frac{x^3}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

input

```
Int[x^4/ArcSinh[a*x]^3,x]
```

output

```
-1/2*(x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) + (2*(-(x^3/(a*ArcSinh[a*x
])) + (3*(-1/4*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]/4
))/a^4))/a + (5*a*(-(x^5/(a*ArcSinh[a*x])) + (5*(CoshIntegral[ArcSinh[a*x]
]/8 - (3*CoshIntegral[3*ArcSinh[a*x]])/16 + CoshIntegral[5*ArcSinh[a*x]]/1
6))/a^6))/2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{16 \operatorname{arcsinh}\left(x a\right)^{2}}-\frac{x a}{16 \operatorname{arcsinh}\left(x a\right)}+\frac{\operatorname{Chi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{16}+\frac{3 \cosh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{32 \operatorname{arcsinh}\left(x a\right)^{2}}+\frac{9 \sinh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{32 \operatorname{arcsinh}\left(x a\right)}-\frac{27 \operatorname{Chi}\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{32}-}{a^{5}}$
default	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{16 \operatorname{arcsinh}\left(x a\right)^{2}}-\frac{x a}{16 \operatorname{arcsinh}\left(x a\right)}+\frac{\operatorname{Chi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{16}+\frac{3 \cosh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{32 \operatorname{arcsinh}\left(x a\right)^{2}}+\frac{9 \sinh\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{32 \operatorname{arcsinh}\left(x a\right)}-\frac{27 \operatorname{Chi}\left(3 \operatorname{arcsinh}\left(x a\right)\right)}{32}-}{a^{5}}$

input
```
int(x^4/arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-1/16/arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)-1/16/arcsinh(x*a)*x*a+1/16*C
hi(arcsinh(x*a))+3/32/arcsinh(x*a)^2*cosh(3*arcsinh(x*a))+9/32/arcsinh(x*a
)*sinh(3*arcsinh(x*a))-27/32*Chi(3*arcsinh(x*a))-1/32/arcsinh(x*a)^2*cosh(
5*arcsinh(x*a))-5/32/arcsinh(x*a)*sinh(5*arcsinh(x*a))+25/32*Chi(5*arcsinh
(x*a)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^4/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{asinh}^3\left(ax\right)}\,dx$$

input
```
integrate(x**4/asinh(a*x)**3,x)
```

output
```
Integral(x**4/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^3,x, algorithm="maxima")
```

output

```
-1/2*(a^8*x^11 + 3*a^6*x^9 + 3*a^4*x^7 + a^2*x^5 + (a^5*x^8 + a^3*x^6)*(a^
2*x^2 + 1)^(3/2) + (3*a^6*x^9 + 5*a^4*x^7 + 2*a^2*x^5)*(a^2*x^2 + 1) + (5*
a^8*x^11 + 15*a^6*x^9 + 15*a^4*x^7 + 5*a^2*x^5 + (5*a^5*x^8 + 8*a^3*x^6 +
3*a*x^4)*(a^2*x^2 + 1)^(3/2) + (15*a^6*x^9 + 31*a^4*x^7 + 20*a^2*x^5 + 4*x
^3)*(a^2*x^2 + 1) + (15*a^7*x^10 + 38*a^5*x^8 + 32*a^3*x^6 + 9*a*x^4)*sqrt
(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^10 + 7*a^5*x^8 + 5*
a^3*x^6 + a*x^4)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^
(3/2)*a^5*x^3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*
(a^7*x^5 + 2*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 +
1))^2) + integrate(1/2*(25*a^10*x^12 + 100*a^8*x^10 + 150*a^6*x^8 + 100*a^
4*x^6 + 25*a^2*x^4 + (25*a^6*x^8 + 24*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^2
 + (100*a^7*x^9 + 172*a^5*x^7 + 87*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^(3/2)
 + 3*(50*a^8*x^10 + 124*a^6*x^8 + 105*a^4*x^6 + 35*a^2*x^4 + 4*x^2)*(a^2*x
^2 + 1) + (100*a^9*x^11 + 324*a^7*x^9 + 381*a^5*x^7 + 193*a^3*x^5 + 36*a*x
^3)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x^2 + 1)^2*a^6*x^4 +
6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^8
*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9*x^7 + 3*a^7*x^5 +
 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input

```
integrate(x^4/arcsinh(a*x)^3,x, algorithm="giac")
```

output

```
integrate(x^4/arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

input
```
int(x^4/asinh(a*x)^3,x)
```

output
```
int(x^4/asinh(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{asinh\left(ax\right)^3}dx$$

input
```
int(x^4/asinh(a*x)^3,x)
```

output
```
int(x**4/asinh(a*x)**3,x)
```

3.64 $\int \frac{x^3}{\text{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^3}{\text{arcsinh}(ax)^3}\,dx = -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\text{arcsinh}(ax)} - \frac{2x^4}{\text{arcsinh}(ax)} - \frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^4} + \frac{\text{Shi}(4\text{arcsinh}(ax))}{a^4}$$

output
```
-1/2*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2-3/2*x^2/a^2/arcsinh(a*x)-2*x^4
/arcsinh(a*x)-1/2*Shi(2*arcsinh(a*x))/a^4+Shi(4*arcsinh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\text{arcsinh}(ax)^3}\,dx = -\frac{\frac{a^2x^2\left(ax\sqrt{1+a^2x^2}+(3+4a^2x^2)\text{arcsinh}(ax)\right)}{\text{arcsinh}(ax)^2} + \text{Shi}(2\text{arcsinh}(ax)) - 2\text{Shi}(4\text{arcsinh}(ax))}{2a^4}$$

input
```
Integrate[x^3/ArcSinh[a*x]^3,x]
```

output

```
-1/2*((a^2*x^2*(a*x*Sqrt[1 + a^2*x^2] + (3 + 4*a^2*x^2)*ArcSinh[a*x]))/Arc
Sinh[a*x]^2 + SinhIntegral[2*ArcSinh[a*x]] - 2*SinhIntegral[4*ArcSinh[a*x]
])/a^4
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6194, 6233, 6195, 5971, 27, 2009, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arcsinh}(ax)^3}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{3\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx}{2a}+2a\int \frac{x^4}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6233}$$

$$\frac{3\left(\frac{2\int \frac{x}{\text{arcsinh}(ax)}dx}{a}-\frac{x^2}{a\text{arcsinh}(ax)}\right)}{2a}+2a\left(\frac{4\int \frac{x^3}{\text{arcsinh}(ax)}dx}{a}-\frac{x^4}{a\text{arcsinh}(ax)}\right)-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6195}$$

$$\frac{3\left(\frac{2\int \frac{ax\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}-\frac{x^2}{a\text{arcsinh}(ax)}\right)}{2a}+2a\left(\frac{4\int \frac{a^3x^3\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{5971}$$

$$2a\left(\frac{4\int\left(\frac{\sinh(4\text{arcsinh}(ax))}{8\text{arcsinh}(ax)}-\frac{\sinh(2\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)+\frac{3\left(\frac{2\int\frac{\sinh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}-\frac{x^2}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow 27$$

$$2a\left(\frac{4\int\left(\frac{\sinh(4\text{arcsinh}(ax))}{8\text{arcsinh}(ax)}-\frac{\sinh(2\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)+\frac{3\left(\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}-\frac{x^2}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow 2009$$

$$\frac{3\left(\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}-\frac{x^2}{a\text{arcsinh}(ax)}\right)}{2a}+2a\left(\frac{4\left(\frac{1}{8}\text{Shi}(4\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(2\text{arcsinh}(ax))\right)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(-\frac{x^2}{a\text{arcsinh}(ax)}+\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}\right)}{2a}+2a\left(\frac{4\left(\frac{1}{8}\text{Shi}(4\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(2\text{arcsinh}(ax))\right)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow 26$$

$$\frac{3\left(-\frac{x^2}{a\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}\right)}{2a}+2a\left(\frac{4\left(\frac{1}{8}\text{Shi}(4\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(2\text{arcsinh}(ax))\right)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow 3779$$

$$2a\left(\frac{4\left(\frac{1}{8}\text{Shi}(4\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(2\text{arcsinh}(ax))\right)}{a^5}-\frac{x^4}{a\text{arcsinh}(ax)}\right)+\frac{3\left(\frac{\text{Shi}(2\text{arcsinh}(ax))}{a^3}-\frac{x^2}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

input
```
Int[x^3/ArcSinh[a*x]^3,x]
```

output
```
-1/2*(x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) + (3*(-(x^2/(a*ArcSinh[a*x
])) + SinhIntegral[2*ArcSinh[a*x]]/a^3))/(2*a) + 2*a*(-(x^4/(a*ArcSinh[a*x
])) + (4*(-1/4*SinhIntegral[2*ArcSinh[a*x]] + SinhIntegral[4*ArcSinh[a*x]]
/8))/a^5)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\sinh(2\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)^{2}}+\frac{\cosh(2\operatorname{arcsinh}(xa))}{4\operatorname{arcsinh}(xa)}-\frac{\operatorname{Shi}(2\operatorname{arcsinh}(xa))}{2}-\frac{\sinh(4\operatorname{arcsinh}(xa))}{16\operatorname{arcsinh}(xa)^{2}}-\frac{\cosh(4\operatorname{arcsinh}(xa))}{4\operatorname{arcsinh}(xa)}+\operatorname{Shi}(4\operatorname{arcsinh}(xa))}{a^{4}}$
default	$\frac{\frac{\sinh(2\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)^{2}}+\frac{\cosh(2\operatorname{arcsinh}(xa))}{4\operatorname{arcsinh}(xa)}-\frac{\operatorname{Shi}(2\operatorname{arcsinh}(xa))}{2}-\frac{\sinh(4\operatorname{arcsinh}(xa))}{16\operatorname{arcsinh}(xa)^{2}}-\frac{\cosh(4\operatorname{arcsinh}(xa))}{4\operatorname{arcsinh}(xa)}+\operatorname{Shi}(4\operatorname{arcsinh}(xa))}{a^{4}}$

input
```
int(x^3/arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8/arcsinh(x*a)^2*sinh(2*arcsinh(x*a))+1/4/arcsinh(x*a)*cosh(2*arc
sinh(x*a))-1/2*Shi(2*arcsinh(x*a))-1/16/arcsinh(x*a)^2*sinh(4*arcsinh(x*a)
)-1/4/arcsinh(x*a)*cosh(4*arcsinh(x*a))+Shi(4*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\operatorname{asinh}^3\left(ax\right)}\,dx$$

input
```
integrate(x**3/asinh(a*x)**3,x)
```

output
```
Integral(x**3/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^3,x, algorithm="maxima")
```

output

```
-1/2*(a^8*x^10 + 3*a^6*x^8 + 3*a^4*x^6 + a^2*x^4 + (a^5*x^7 + a^3*x^5)*(a^
2*x^2 + 1)^(3/2) + (3*a^6*x^8 + 5*a^4*x^6 + 2*a^2*x^4)*(a^2*x^2 + 1) + (4*
a^8*x^10 + 12*a^6*x^8 + 12*a^4*x^6 + 4*a^2*x^4 + 2*(2*a^5*x^7 + 3*a^3*x^5
+ a*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(4*a^6*x^8 + 8*a^4*x^6 + 5*a^2*x^4 + x^2)
*(a^2*x^2 + 1) + (12*a^7*x^9 + 30*a^5*x^7 + 25*a^3*x^5 + 7*a*x^3)*sqrt(a^2
*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^9 + 7*a^5*x^7 + 5*a^3*x
^5 + a*x^3)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)
*a^5*x^3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*
x^5 + 2*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2
) + integrate(1/2*(16*a^10*x^11 + 64*a^8*x^9 + 96*a^6*x^7 + 64*a^4*x^5 + 1
6*a^2*x^3 + 4*(4*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^2 + (64*a^7*x^8 + 100*
a^5*x^6 + 42*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^(3/2) + 6*(16*a^8*x^9 + 38*a
^6*x^7 + 30*a^4*x^5 + 9*a^2*x^3 + x)*(a^2*x^2 + 1) + (64*a^9*x^10 + 204*a^
7*x^8 + 234*a^5*x^6 + 115*a^3*x^4 + 21*a*x^2)*sqrt(a^2*x^2 + 1))/((a^10*x^
8 + 4*a^8*x^6 + (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x
^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2
*x^2 + 1) + a^2 + 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3/arcsinh(a*x)^3,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

input

```
int(x^3/asinh(a*x)^3,x)
```

output

```
int(x^3/asinh(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{asinh\left(ax\right)^3}dx$$

input

```
int(x^3/asinh(a*x)^3,x)
```

output

```
int(x**3/asinh(a*x)**3,x)
```

3.65 $\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2}-\frac{x}{a^2\text{arcsinh}(ax)}-\frac{3x^3}{2\text{arcsinh}(ax)} - \frac{\text{Chi}(\text{arcsinh}(ax))}{8a^3}+\frac{9\text{Chi}(3\text{arcsinh}(ax))}{8a^3}$$

output
```
-1/2*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2-x/a^2/arcsinh(a*x)-3/2*x^3/arc
sinh(a*x)-1/8*Chi(arcsinh(a*x))/a^3+9/8*Chi(3*arcsinh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx = -\frac{\frac{4ax\left(ax\sqrt{1+a^2x^2}+(2+3a^2x^2)\text{arcsinh}(ax)\right)}{\text{arcsinh}(ax)^2}+\text{Chi}(\text{arcsinh}(ax))-9\text{Chi}(3\text{arcsinh}(ax))}{8a^3}$$

input
```
Integrate[x^2/ArcSinh[a*x]^3,x]
```

output
```
-1/8*((4*a*x*(a*x*Sqrt[1 + a^2*x^2] + (2 + 3*a^2*x^2)*ArcSinh[a*x]))/ArcSi
nh[a*x]^2 + CoshIntegral[ArcSinh[a*x]] - 9*CoshIntegral[3*ArcSinh[a*x]])/a
^3
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6194, 6233, 6189, 3042, 3782, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx}{a}+\frac{3}{2}a\int \frac{x^3}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6233}$$

$$\frac{3}{2}a\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)}dx}{a}-\frac{x^3}{a\text{arcsinh}(ax)}\right)+\frac{\frac{\int \frac{1}{\text{arcsinh}(ax)}dx}{a}-\frac{x}{a\text{arcsinh}(ax)}}{a}-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\frac{\int \frac{\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}-\frac{x}{a\text{arcsinh}(ax)}}{a}+\frac{3}{2}a\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)}dx}{a}-\frac{x^3}{a\text{arcsinh}(ax)}\right)-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{x}{a\text{arcsinh}(ax)}+\frac{\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}}{a}+\frac{3}{2}a\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)}dx}{a}-\frac{x^3}{a\text{arcsinh}(ax)}\right)-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \; 3782$$

$$\frac{3}{2}a\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)}dx}{a}-\frac{x^3}{a\text{arcsinh}(ax)}\right)+\frac{\frac{\text{Chi}(\text{arcsinh}(ax))}{a^2}-\frac{x}{a\text{arcsinh}(ax)}}{a}-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \; 6195$$

$$\frac{3}{2}a\left(\frac{3\int \frac{a^2x^2\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^4}-\frac{x^3}{a\text{arcsinh}(ax)}\right)+\frac{\frac{\text{Chi}(\text{arcsinh}(ax))}{a^2}-\frac{x}{a\text{arcsinh}(ax)}}{a}-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \; 5971$$

$$\frac{3}{2}a\left(\frac{3\int \left(\frac{\cosh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}-\frac{\sqrt{a^2x^2+1}}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^4}-\frac{x^3}{a\text{arcsinh}(ax)}\right)+\frac{\frac{\text{Chi}(\text{arcsinh}(ax))}{a^2}-\frac{x}{a\text{arcsinh}(ax)}}{a}-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{2}a\left(\frac{3\left(\frac{1}{4}\text{Chi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Chi}(\text{arcsinh}(ax))\right)}{a^4}-\frac{x^3}{a\text{arcsinh}(ax)}\right)+\frac{\frac{\text{Chi}(\text{arcsinh}(ax))}{a^2}-\frac{x}{a\text{arcsinh}(ax)}}{a}-\frac{x^2\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

input
```
Int[x^2/ArcSinh[a*x]^3,x]
```

output
```
-1/2*(x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) + (-(x/(a*ArcSinh[a*x])) +
 CoshIntegral[ArcSinh[a*x]]/a^2)/a + (3*a*(-(x^3/(a*ArcSinh[a*x])) + (3*(-
1/4*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]/4))/a^4))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\sqrt{a^2x^2+1}}{8\operatorname{arcsinh}(xa)^2}+\frac{xa}{8\operatorname{arcsinh}(xa)}-\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{8}-\frac{\cosh(3\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)^2}-\frac{3\sinh(3\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)}+\frac{9\operatorname{Chi}(3\operatorname{arcsinh}(xa))}{8}}{a^3}$	81
default	$\frac{\frac{\sqrt{a^2x^2+1}}{8\operatorname{arcsinh}(xa)^2}+\frac{xa}{8\operatorname{arcsinh}(xa)}-\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{8}-\frac{\cosh(3\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)^2}-\frac{3\sinh(3\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)}+\frac{9\operatorname{Chi}(3\operatorname{arcsinh}(xa))}{8}}{a^3}$	81

input
```
int(x^2/arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/8/arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)+1/8/arcsinh(x*a)*x*a-1/8*Chi(a
rcsinh(x*a))-1/8/arcsinh(x*a)^2*cosh(3*arcsinh(x*a))-3/8/arcsinh(x*a)*sinh
(3*arcsinh(x*a))+9/8*Chi(3*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx=\int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}^3(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x)**3,x)
```

output
```
Integral(x**2/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^3}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^8*x^9 + 3*a^6*x^7 + 3*a^4*x^5 + a^2*x^3 + (a^5*x^6 + a^3*x^4)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^7 + 5*a^4*x^5 + 2*a^2*x^3)*(a^2*x^2 + 1) + (3*a
^8*x^9 + 9*a^6*x^7 + 9*a^4*x^5 + 3*a^2*x^3 + (3*a^5*x^6 + 4*a^3*x^4 + a*x^
2)*(a^2*x^2 + 1)^(3/2) + (9*a^6*x^7 + 17*a^4*x^5 + 10*a^2*x^3 + 2*x)*(a^2*
x^2 + 1) + (9*a^7*x^8 + 22*a^5*x^6 + 18*a^3*x^4 + 5*a*x^2)*sqrt(a^2*x^2 +
1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^8 + 7*a^5*x^6 + 5*a^3*x^4 + a*
x^2)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^
3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2
*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + int
egrate(1/2*(9*a^10*x^10 + 36*a^8*x^8 + 54*a^6*x^6 + 36*a^4*x^4 + 9*a^2*x^2
 + (9*a^6*x^6 + 4*a^4*x^4 - a^2*x^2)*(a^2*x^2 + 1)^2 + (36*a^7*x^7 + 48*a^
5*x^5 + 13*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^(3/2) + (54*a^8*x^8 + 120*a^6*x^
6 + 83*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1) + (36*a^9*x^9 + 112*a^7*x^7
 + 123*a^5*x^5 + 57*a^3*x^3 + 10*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^
8*x^6 + (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5
*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1
) + a^2 + 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*l
og(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

input
```
int(x^2/asinh(a*x)^3,x)
```

output
```
int(x^2/asinh(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{asinh\left(ax\right)^3}dx$$

input
```
int(x^2/asinh(a*x)^3,x)
```

output
```
int(x**2/asinh(a*x)**3,x)
```

3.66 $\int \frac{x}{\operatorname{arcsinh}(ax)^3} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3} \, dx = -\frac{x\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} - \frac{1}{2a^2\operatorname{arcsinh}(ax)} - \frac{x^2}{\operatorname{arcsinh}(ax)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{a^2}$$

output
```
-1/2*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2-1/2/a^2/arcsinh(a*x)-x^2/arcsinh
(a*x)+Shi(2*arcsinh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3} \, dx = -\frac{x\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} + \frac{-1-2a^2x^2}{2a^2\operatorname{arcsinh}(ax)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{a^2}$$

input
```
Integrate[x/ArcSinh[a*x]^3,x]
```

output
```
-1/2*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) + (-1 - 2*a^2*x^2)/(2*a^2*Ar
cSinh[a*x]) + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {6194, 6198, 6233, 6195, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{\int \frac{1}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}dx}{2a}+a\int \frac{x^2}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}dx-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6198}$$

$$a\int \frac{x^2}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}dx-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}$$

$$\Big\downarrow \text{6233}$$

$$a\left(\frac{2\int \frac{x}{\operatorname{arcsinh}(ax)}dx}{a}-\frac{x^2}{a\operatorname{arcsinh}(ax)}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}$$

$$\Big\downarrow \text{6195}$$

$$a\left(\frac{2\int \frac{ax\sqrt{a^2x^2+1}}{\operatorname{arcsinh}(ax)}d\operatorname{arcsinh}(ax)}{a^3}-\frac{x^2}{a\operatorname{arcsinh}(ax)}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$a\left(\frac{2\int \frac{\sinh(2\operatorname{arcsinh}(ax))}{2\operatorname{arcsinh}(ax)}d\operatorname{arcsinh}(ax)}{a^3}-\frac{x^2}{a\operatorname{arcsinh}(ax)}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}$$

$$\Big\downarrow \text{27}$$

$$a\left(\frac{\int \frac{\sinh(2\operatorname{arcsinh}(ax))}{\operatorname{arcsinh}(ax)}d\operatorname{arcsinh}(ax)}{a^3}-\frac{x^2}{a\operatorname{arcsinh}(ax)}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{x^2}{a\text{arcsinh}(ax)}+\frac{\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}-\frac{1}{2a^2\text{arcsinh}(ax)}$$

$$\Big\downarrow \color{blue}{26}$$

$$a\left(-\frac{x^2}{a\text{arcsinh}(ax)}-\frac{i\int \frac{\sin(2i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^3}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}-\frac{1}{2a^2\text{arcsinh}(ax)}$$

$$\Big\downarrow \color{blue}{3779}$$

$$a\left(\frac{\text{Shi}(2\text{arcsinh}(ax))}{a^3}-\frac{x^2}{a\text{arcsinh}(ax)}\right)-\frac{x\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}-\frac{1}{2a^2\text{arcsinh}(ax)}$$

input
```
Int[x/ArcSinh[a*x]^3,x]
```

output
```
-1/2*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) - 1/(2*a^2*ArcSinh[a*x]) + a
*(-(x^2/(a*ArcSinh[a*x])) + SinhIntegral[2*ArcSinh[a*x]]/a^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\operatorname{arcsinh}(xa))}{4\operatorname{arcsinh}(xa)^2}-\frac{\cosh(2\operatorname{arcsinh}(xa))}{2\operatorname{arcsinh}(xa)}+\operatorname{Shi}(2\operatorname{arcsinh}(xa))}{a^2}$	43
default	$\frac{-\frac{\sinh(2\operatorname{arcsinh}(xa))}{4\operatorname{arcsinh}(xa)^2}-\frac{\cosh(2\operatorname{arcsinh}(xa))}{2\operatorname{arcsinh}(xa)}+\operatorname{Shi}(2\operatorname{arcsinh}(xa))}{a^2}$	43

input

```
int(x/arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/a^2*(-1/4/arcsinh(x*a)^2*sinh(2*arcsinh(x*a))-1/2/arcsinh(x*a)*cosh(2*ar
csinh(x*a))+Shi(2*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input

```
integrate(x/arcsinh(a*x)^3,x, algorithm="fricas")
```

output

```
integral(x/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{asinh}^3\left(ax\right)}\,dx$$

input

```
integrate(x/asinh(a*x)**3,x)
```

output

```
Integral(x/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x/arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) + (2*a
^8*x^8 + 6*a^6*x^6 + 6*a^4*x^4 + 2*a^2*x^2 + 2*(a^5*x^5 + a^3*x^3)*(a^2*x^
2 + 1)^(3/2) + (6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (6
*a^7*x^7 + 14*a^5*x^5 + 11*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + s
qrt(a^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2
 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a^4*x^2 + 3
*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3 + a^3*x)
*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*a^9
*x^9 + 16*a^7*x^7 + 4*(a^2*x^2 + 1)^2*a^5*x^5 + 24*a^5*x^5 + 16*a^3*x^3 +
(16*a^6*x^6 + 16*a^4*x^4 - 3)*(a^2*x^2 + 1)^(3/2) + 24*(a^7*x^7 + 2*a^5*x^
5 + a^3*x^3)*(a^2*x^2 + 1) + 4*a*x + (16*a^8*x^8 + 48*a^6*x^6 + 48*a^4*x^4
 + 19*a^2*x^2 + 3)*sqrt(a^2*x^2 + 1))/((a^9*x^8 + 4*a^7*x^6 + (a^2*x^2 + 1
)^2*a^5*x^4 + 6*a^5*x^4 + 4*a^3*x^2 + 4*(a^6*x^5 + a^4*x^3)*(a^2*x^2 + 1)^
(3/2) + 6*(a^7*x^6 + 2*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 4*(a^8*x^7 + 3*a
^6*x^5 + 3*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2
+ 1))), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(x/arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

input

```
int(x/asinh(a*x)^3,x)
```

output

```
int(x/asinh(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\mathit{asinh}\left(ax\right)^3}dx$$

input

```
int(x/asinh(a*x)^3,x)
```

output

```
int(x/asinh(a*x)**3,x)
```

3.67 $\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 50

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = -\frac{\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} - \frac{x}{2\operatorname{arcsinh}(ax)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{2a}$$

output
```
-1/2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2-1/2*x/arcsinh(a*x)+1/2*Chi(arcsinh
(a*x))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = -\frac{\sqrt{1+a^2x^2} + ax\operatorname{arcsinh}(ax) - \operatorname{arcsinh}(ax)^2\operatorname{Chi}(\operatorname{arcsinh}(ax))}{2a\operatorname{arcsinh}(ax)^2}$$

input
```
Integrate[ArcSinh[a*x]^(-3),x]
```

output
```
-1/2*(Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] - ArcSinh[a*x]^2*CoshIntegral[A
rcSinh[a*x]])/(a*ArcSinh[a*x]^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6188, 6233, 6189, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^3}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{1}{2}a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}dx-\frac{\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6233}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{\text{arcsinh}(ax)}dx}{a}-\frac{x}{a\text{arcsinh}(ax)}\right)-\frac{\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6189}$$

$$\frac{1}{2}a\left(\frac{\int \frac{\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}-\frac{x}{a\text{arcsinh}(ax)}\right)-\frac{\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}+\frac{1}{2}a\left(-\frac{x}{a\text{arcsinh}(ax)}+\frac{\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}\right)$$

$$\Big\downarrow \text{3782}$$

$$\frac{1}{2}a\left(\frac{\text{Chi}(\text{arcsinh}(ax))}{a^2}-\frac{x}{a\text{arcsinh}(ax)}\right)-\frac{\sqrt{a^2x^2+1}}{2a\text{arcsinh}(ax)^2}$$

input
```
Int[ArcSinh[a*x]^(-3),x]
```

output
```
-1/2*Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x]^2) + (a*(-(x/(a*ArcSinh[a*x])) + Co
shIntegral[ArcSinh[a*x]]/a^2))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{a^2x^2+1}}{2\operatorname{arcsinh}(xa)^2}-\frac{xa}{2\operatorname{arcsinh}(xa)}+\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{2}}{a}$	42
default	$\frac{-\frac{\sqrt{a^2x^2+1}}{2\operatorname{arcsinh}(xa)^2}-\frac{xa}{2\operatorname{arcsinh}(xa)}+\frac{\operatorname{Chi}(\operatorname{arcsinh}(xa))}{2}}{a}$	42

input
```
int(1/arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2/arcsinh(x*a)^2*(a^2*x^2+1)^(1/2)-1/2/arcsinh(x*a)*x*a+1/2*Chi(ar
csinh(x*a)))
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^(-3), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{\operatorname{asinh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/asinh(a*x)**3,x)
```

output
```
Integral(asinh(a*x)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx=\int \frac{1}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input

```
integrate(1/arcsinh(a*x)^3,x, algorithm="maxima")
```

output

```
-1/2*(a^7*x^7 + 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^
(3/2) + (3*a^5*x^5 + 5*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + a*x + (a^7*x^7 + 3
*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - 1)*(a^2*x^2 + 1)^(3/2) + 3*(a^5*x^5 + a^
3*x^3)*(a^2*x^2 + 1) + a*x + (3*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + 1)*sqrt(
a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^6*x^6 + 7*a^4*x^4 + 5*a^
2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^7*x^6 + 3*a^5*x^4 + (a^2*x^2 + 1)^(3/2)*
a^4*x^3 + 3*a^3*x^2 + 3*(a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 3*(a^6*x^5 + 2
*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^2) +
 integrate(1/2*(a^8*x^8 + 4*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + (a^4*x^4 + 3
)*(a^2*x^2 + 1)^2 + (4*a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^(3/2) +
3*(2*a^6*x^6 + 4*a^4*x^4 + a^2*x^2 - 1)*(a^2*x^2 + 1) + (4*a^7*x^7 + 12*a^
5*x^5 + 9*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^8*x^8 + 4*a^6*x^6 + (a
^2*x^2 + 1)^2*a^4*x^4 + 6*a^4*x^4 + 4*a^2*x^2 + 4*(a^5*x^5 + a^3*x^3)*(a^2
*x^2 + 1)^(3/2) + 6*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 4*(a^7
*x^7 + 3*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(
a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx=\int \frac{1}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input

```
integrate(1/arcsinh(a*x)^3,x, algorithm="giac")
```

output

```
integrate(arcsinh(a*x)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

input

```
int(1/asinh(a*x)^3,x)
```

output

```
int(1/asinh(a*x)^3, x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{asinh\left(ax\right)^3}dx$$

input

```
int(1/asinh(a*x)^3,x)
```

output

```
int(1/asinh(a*x)**3,x)
```

3.68 $\int \frac{1}{x\mathbf{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^3},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^3}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]^3),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)^3} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x\text{arcsinh}(ax)^3}dx$$

input
```
Int[1/(x*ArcSinh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}\left(xa\right)^{3}}dx$$

input
```
int(1/x/arcsinh(x*a)^3,x)
```

output
```
int(1/x/arcsinh(x*a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\,\text{asinh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/asinh(a*x)**3,x)
```

output
```
Integral(1/(x*asinh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 697, normalized size of antiderivative = 69.70

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^3,x, algorithm="maxima")
```

output

```
-1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (2*(
a^3*x^3 + a*x)*(a^2*x^2 + 1)^(3/2) + (4*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2
+ 1) + (2*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2
*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/
((a^8*x^8 + 3*a^6*x^6 + (a^2*x^2 + 1)^(3/2)*a^5*x^5 + 3*a^4*x^4 + a^2*x^2
+ 3*(a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1) + 3*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*
sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*(a^4
*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1)^2 + (12*a^5*x^5 + 22*a^3*x^3 + 7*a*x)*(a^2
*x^2 + 1)^(3/2) + 2*(6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1)
 + (4*a^7*x^7 + 6*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^1
1 + 4*a^8*x^9 + (a^2*x^2 + 1)^2*a^6*x^7 + 6*a^6*x^7 + 4*a^4*x^5 + a^2*x^3
+ 4*(a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^9 + 2*a^6*x^7 + a^4
*x^5)*(a^2*x^2 + 1) + 4*(a^9*x^10 + 3*a^7*x^8 + 3*a^5*x^6 + a^3*x^4)*sqrt(
a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3} \, dx = \int \frac{1}{x \,\text{arsinh}\left(ax\right)^3} \, dx$$

input

```
integrate(1/x/arcsinh(a*x)^3,x, algorithm="giac")
```

output

```
integrate(1/(x*arcsinh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3} \, dx = \int \frac{1}{x \,\text{asinh}\left(a\,x\right)^3} \, dx$$

input

```
int(1/(x*asinh(a*x)^3),x)
```

output

```
int(1/(x*asinh(a*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{asinh\left(ax\right)^3 x}dx$$

input

```
int(1/x/asinh(a*x)^3,x)
```

output

```
int(1/(asinh(a*x)**3*x),x)
```

3.69 $\int \frac{1}{x^2\textbf{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2\text{arcsinh}(ax)^3}\,dx = \text{Int}\left(\frac{1}{x^2\text{arcsinh}(ax)^3},x\right)$$

output
```
Defer(Int)(1/x^2/arcsinh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*ArcSinh[a*x]^3),x]
```

output
```
Integrate[1/(x^2*ArcSinh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3} \, dx$$

$$\Big\downarrow \, 6196$$

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3} dx$$

input
```
Int[1/(x^2*ArcSinh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}\left(xa\right)^3} dx$$

input
```
int(1/x^2/arcsinh(x*a)^3,x)
```

output
```
int(1/x^2/arcsinh(x*a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(x^2*arcsinh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2\operatorname{asinh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x**2/asinh(a*x)**3,x)
```

output
```
Integral(1/(x**2*asinh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 822, normalized size of antiderivative = 82.20

$$\int \frac{1}{x^2\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2\operatorname{arsinh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

output

```
-1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (a^8
*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^
2*x^2 + 1)^(3/2) + (3*a^6*x^6 + 11*a^4*x^4 + 10*a^2*x^2 + 2)*(a^2*x^2 + 1)
 + (3*a^7*x^7 + 10*a^5*x^5 + 10*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*
x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^
2*x^2 + 1))/((a^8*x^9 + 3*a^6*x^7 + (a^2*x^2 + 1)^(3/2)*a^5*x^6 + 3*a^4*x^
5 + a^2*x^3 + 3*(a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1) + 3*(a^7*x^8 + 2*a^5*x^6
 + a^3*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate
(1/2*(a^10*x^10 + 4*a^8*x^8 + 6*a^6*x^6 + 4*a^4*x^4 + a^2*x^2 + (a^6*x^6 +
 12*a^4*x^4 + 15*a^2*x^2)*(a^2*x^2 + 1)^2 + (4*a^7*x^7 + 40*a^5*x^5 + 57*a
^3*x^3 + 18*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^8*x^8 + 16*a^6*x^6 + 25*a^4*
x^4 + 13*a^2*x^2 + 2)*(a^2*x^2 + 1) + (4*a^9*x^9 + 24*a^7*x^7 + 39*a^5*x^5
 + 25*a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^12 + 4*a^8*x^10 + (a^2*
x^2 + 1)^2*a^6*x^8 + 6*a^6*x^8 + 4*a^4*x^6 + a^2*x^4 + 4*(a^7*x^9 + a^5*x^
7)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^10 + 2*a^6*x^8 + a^4*x^6)*(a^2*x^2 + 1)
+ 4*(a^9*x^11 + 3*a^7*x^9 + 3*a^5*x^7 + a^3*x^5)*sqrt(a^2*x^2 + 1))*log(a*
x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{arsinh}(ax)^3} \, dx$$

input

```
integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

output

```
integrate(1/(x^2*arcsinh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{asinh}(a\,x)^3} \, dx$$

input
```
int(1/(x^2*asinh(a*x)^3),x)
```

output
```
int(1/(x^2*asinh(a*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3} \, dx = \int \frac{1}{asinh(ax)^3 x^2} dx$$

input
```
int(1/x^2/asinh(a*x)^3,x)
```

output
```
int(1/(asinh(a*x)**3*x**2),x)
```

3.70 $\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 155

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = &-\frac{x^4\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3}-\frac{2x^3}{3a^2\operatorname{arcsinh}(ax)^2}-\frac{5x^5}{6\operatorname{arcsinh}(ax)^2}\\&-\frac{2x^2\sqrt{1+a^2x^2}}{a^3\operatorname{arcsinh}(ax)}-\frac{25x^4\sqrt{1+a^2x^2}}{6a\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{48a^5}\\&-\frac{27\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{32a^5}+\frac{125\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{96a^5}\end{aligned}$$

output
```
-1/3*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-2/3*x^3/a^2/arcsinh(a*x)^2-5/6
*x^5/arcsinh(a*x)^2-2*x^2*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-25/6*x^4*(a^2
*x^2+1)^(1/2)/a/arcsinh(a*x)+1/48*Shi(arcsinh(a*x))/a^5-27/32*Shi(3*arcsin
h(a*x))/a^5+125/96*Shi(5*arcsinh(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx =$$

$$-\frac{32a^4x^4\sqrt{1+a^2x^2}+64a^3x^3\operatorname{arcsinh}(ax)+80a^5x^5\operatorname{arcsinh}(ax)+192a^2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2+400}{}$$

input
```
Integrate[x^4/ArcSinh[a*x]^4,x]
```

output
```
-1/96*(32*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5
*ArcSinh[a*x] + 192*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 400*a^4*x^4
*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3*SinhIntegral[ArcSinh[
a*x]] + 81*ArcSinh[a*x]^3*SinhIntegral[3*ArcSinh[a*x]] - 125*ArcSinh[a*x]^
3*SinhIntegral[5*ArcSinh[a*x]])/(a^5*ArcSinh[a*x]^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6194, 6233, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{5}{3}a\int \frac{x^5}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}dx+\frac{4\int \frac{x^3}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}dx}{3a}-\frac{x^4\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6233}$$

$$\frac{5}{3}a\left(\frac{5\int \frac{x^4}{\text{arcsinh}(ax)^2}dx}{2a}-\frac{x^5}{2a\text{arcsinh}(ax)^2}\right)+\frac{4\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)^2}dx}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)}{3a}-\frac{x^4\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \textit{6193}$$

$$\frac{5}{3}a\left(\frac{5\left(\frac{\int \left(\frac{ax}{8\text{arcsinh}(ax)}-\frac{9\sinh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{5\sinh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^5}{2a\text{arcsinh}(ax)^2}\right)+\frac{4\left(\frac{3\left(\frac{\int \left(\frac{3\sinh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}-\frac{ax}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)}{3a}-\frac{x^4\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \textit{2009}$$

$$-\frac{x^4\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+\frac{5}{3}a\left(\frac{5\left(\frac{\frac{1}{8}\text{Shi}(\text{arcsinh}(ax))-\frac{9}{16}\text{Shi}(3\text{arcsinh}(ax))+\frac{5}{16}\text{Shi}(5\text{arcsinh}(ax))}{a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^5}{2a\text{arcsinh}(ax)^2}\right)+\frac{4\left(\frac{3\left(\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)}{3a}$$

input

```
Int[x^4/ArcSinh[a*x]^4,x]
```

output

```
-1/3*(x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) + (4*(-1/2*x^3/(a*ArcSinh[
a*x]^2) + (3*(-((x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (-1/4*SinhInte
gral[ArcSinh[a*x]] + (3*SinhIntegral[3*ArcSinh[a*x]])/4)/a^3))/(2*a)))/(3*
a) + (5*a*(-1/2*x^5/(a*ArcSinh[a*x]^2) + (5*(-((x^4*Sqrt[1 + a^2*x^2])/(a*
ArcSinh[a*x])) + (SinhIntegral[ArcSinh[a*x]]/8 - (9*SinhIntegral[3*ArcSinh
[a*x]])/16 + (5*SinhIntegral[5*ArcSinh[a*x]])/16)/a^5))/(2*a)))/3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(xa)^3}-\frac{xa}{48\operatorname{arcsinh}(xa)^2}-\frac{\sqrt{a^2x^2+1}}{48\operatorname{arcsinh}(xa)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(xa))}{48}+\frac{\cosh(3\operatorname{arcsinh}(xa))}{16\operatorname{arcsinh}(xa)^3}+\frac{3\sinh(3\operatorname{arcsinh}(xa))}{32\operatorname{arcsinh}(xa)^2}+\frac{9\cosh(3\ \mathrm{a}}{32\ \mathrm{arc}}$
default	$-\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(xa)^3}-\frac{xa}{48\operatorname{arcsinh}(xa)^2}-\frac{\sqrt{a^2x^2+1}}{48\operatorname{arcsinh}(xa)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(xa))}{48}+\frac{\cosh(3\operatorname{arcsinh}(xa))}{16\operatorname{arcsinh}(xa)^3}+\frac{3\sinh(3\operatorname{arcsinh}(xa))}{32\operatorname{arcsinh}(xa)^2}+\frac{9\cosh(3\ \mathrm{a}}{32\ \mathrm{arc}}$

input
```
int(x^4/arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-1/24/arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)-1/48/arcsinh(x*a)^2*x*a-1/48
/arcsinh(x*a)*(a^2*x^2+1)^(1/2)+1/48*Shi(arcsinh(x*a))+1/16/arcsinh(x*a)^3
*cosh(3*arcsinh(x*a))+3/32/arcsinh(x*a)^2*sinh(3*arcsinh(x*a))+9/32/arcsin
h(x*a)*cosh(3*arcsinh(x*a))-27/32*Shi(3*arcsinh(x*a))-1/48/arcsinh(x*a)^3*
cosh(5*arcsinh(x*a))-5/96/arcsinh(x*a)^2*sinh(5*arcsinh(x*a))-25/96/arcsin
h(x*a)*cosh(5*arcsinh(x*a))+125/96*Shi(5*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx=\int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^4/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{asinh}^4{(ax)}}\,dx$$

input
```
integrate(x**4/asinh(a*x)**4,x)
```

output
```
Integral(x**4/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
-1/6*(2*a^13*x^15 + 10*a^11*x^13 + 20*a^9*x^11 + 20*a^7*x^9 + 10*a^5*x^7 +
 2*a^3*x^5 + 2*(a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^11 +
9*a^7*x^9 + 4*a^5*x^7)*(a^2*x^2 + 1)^2 + (25*a^13*x^15 + 125*a^11*x^13 + 2
50*a^9*x^11 + 250*a^7*x^9 + 125*a^5*x^7 + 25*a^3*x^5 + (25*a^8*x^10 + 49*a
^6*x^8 + 27*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^(5/2) + (125*a^9*x^11 + 321
*a^7*x^9 + 286*a^5*x^7 + 102*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^2 + (250*a^
10*x^12 + 794*a^8*x^10 + 946*a^6*x^8 + 519*a^4*x^6 + 129*a^2*x^4 + 12*x^2)
*(a^2*x^2 + 1)^(3/2) + 2*(125*a^11*x^13 + 473*a^9*x^11 + 696*a^7*x^9 + 497
*a^5*x^7 + 173*a^3*x^5 + 24*a*x^3)*(a^2*x^2 + 1) + (125*a^12*x^14 + 549*a^
10*x^12 + 955*a^8*x^10 + 824*a^6*x^8 + 354*a^4*x^6 + 61*a^2*x^4)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^12 + 13*a^8*x^10 +
11*a^6*x^8 + 3*a^4*x^6)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^13 + 17*a^9*x^11
 + 21*a^7*x^9 + 11*a^5*x^7 + 2*a^3*x^5)*(a^2*x^2 + 1) + (5*a^13*x^15 + 25*
a^11*x^13 + 50*a^9*x^11 + 50*a^7*x^9 + 25*a^5*x^7 + 5*a^3*x^5 + (5*a^8*x^1
0 + 8*a^6*x^8 + 3*a^4*x^6)*(a^2*x^2 + 1)^(5/2) + (25*a^9*x^11 + 57*a^7*x^9
 + 42*a^5*x^7 + 10*a^3*x^5)*(a^2*x^2 + 1)^2 + (50*a^10*x^12 + 148*a^8*x^10
 + 158*a^6*x^8 + 71*a^4*x^6 + 11*a^2*x^4)*(a^2*x^2 + 1)^(3/2) + 2*(25*a^11
*x^13 + 91*a^9*x^11 + 126*a^7*x^9 + 81*a^5*x^7 + 23*a^3*x^5 + 2*a*x^3)*(a^
2*x^2 + 1) + (25*a^12*x^14 + 108*a^10*x^12 + 183*a^8*x^10 + 151*a^6*x^8 +
60*a^4*x^6 + 9*a^2*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))...
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input

```
integrate(x^4/arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
integrate(x^4/arcsinh(a*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

input
```
int(x^4/asinh(a*x)^4,x)
```

output
```
int(x^4/asinh(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{asinh\left(ax\right)^4}dx$$

input
```
int(x^4/asinh(a*x)^4,x)
```

output
```
int(x**4/asinh(a*x)**4,x)
```

3.71 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 141

$$\begin{aligned}\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = &-\frac{x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3}-\frac{x^2}{2a^2\operatorname{arcsinh}(ax)^2}\\ &-\frac{2x^4}{3\operatorname{arcsinh}(ax)^2}-\frac{x\sqrt{1+a^2x^2}}{a^3\operatorname{arcsinh}(ax)}-\frac{8x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)}\\ &-\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3a^4}+\frac{4\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{3a^4}\end{aligned}$$

output
```
-1/3*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-1/2*x^2/a^2/arcsinh(a*x)^2-2/3
*x^4/arcsinh(a*x)^2-x*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-8/3*x^3*(a^2*x^2+
1)^(1/2)/a/arcsinh(a*x)-1/3*Chi(2*arcsinh(a*x))/a^4+4/3*Chi(4*arcsinh(a*x)
)/a^4
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{\frac{ax\left(2a^2x^2\sqrt{1+a^2x^2}+ax(3+4a^2x^2)\operatorname{arcsinh}(ax)+2\sqrt{1+a^2x^2}(3+8a^2x^2)\operatorname{arcsinh}(ax)^2\right)}{\operatorname{arcsinh}(ax)^3}+2\operatorname{Chi}(2\operatorname{arcsinh}(ax))-8\operatorname{Chi}(4\mathrm{ar}}{6a^4}$$

input

```
Integrate[x^3/ArcSinh[a*x]^4,x]
```

output

```
-1/6*((a*x*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(3 + 4*a^2*x^2)*ArcSinh[a*x]
 + 2*Sqrt[1 + a^2*x^2]*(3 + 8*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + 2
*CoshIntegral[2*ArcSinh[a*x]] - 8*CoshIntegral[4*ArcSinh[a*x]])/a^4
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6194, 6233, 6193, 2009, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}dx}{a}+\frac{4}{3}a\int \frac{x^4}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}dx-\frac{x^3\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6233}$$

$$\frac{\frac{\int \frac{x}{\text{arcsinh}(ax)^2}dx}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}}{a}+\frac{4}{3}a\left(\frac{2\int \frac{x^3}{\text{arcsinh}(ax)^2}dx}{a}-\frac{x^4}{2a\text{arcsinh}(ax)^2}\right)-\frac{x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6193}$$

$$\frac{\frac{\frac{\int \frac{\cosh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}}{a}+\frac{4}{3}a\left(\frac{2\left(\frac{\int \left(\frac{\cosh(4\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}-\frac{\cosh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4}{2a\text{arcsinh}(ax)^2}\right)-\frac{x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{\int \frac{\cosh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}}{a}-\frac{x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+\frac{4}{3}a\left(\frac{2\left(\frac{\frac{1}{2}\text{Chi}(4\text{arcsinh}(ax))-\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4}{2a\text{arcsinh}(ax)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{x^2}{2a\text{arcsinh}(ax)^2}+\frac{-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}+\frac{\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}}{a}}{a}-\frac{x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+\frac{4}{3}a\left(\frac{2\left(\frac{\frac{1}{2}\text{Chi}(4\text{arcsinh}(ax))-\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4}{2a\text{arcsinh}(ax)^2}\right)$$

$$\Big\downarrow \text{3782}$$

$$\frac{\frac{\frac{\text{Chi}(2\text{arcsinh}(ax))}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}}{a}-\frac{x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+\frac{4}{3}a\left(\frac{2\left(\frac{\frac{1}{2}\text{Chi}(4\text{arcsinh}(ax))-\frac{1}{2}\text{Chi}(2\text{arcsinh}(ax))}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{a}-\frac{x^4}{2a\text{arcsinh}(ax)^2}\right)$$

input

```
Int[x^3/ArcSinh[a*x]^4,x]
```

output

```
-1/3*(x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) + (-1/2*x^2/(a*ArcSinh[a*x
]^2) + (-((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + CoshIntegral[2*ArcSinh
[a*x]]/a^2)/a)/a + (4*a*(-1/2*x^4/(a*ArcSinh[a*x]^2) + (2*(-((x^3*Sqrt[1 +
 a^2*x^2])/(a*ArcSinh[a*x])) + (-1/2*CoshIntegral[2*ArcSinh[a*x]] + CoshIn
tegral[4*ArcSinh[a*x]]/2)/a^4))/a))/3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\sinh(2 \operatorname{arcsinh}(xa))}{12 \operatorname{arcsinh}(xa)^{3}}+\frac{\cosh(2 \operatorname{arcsinh}(xa))}{12 \operatorname{arcsinh}(xa)^{2}}+\frac{\sinh(2 \operatorname{arcsinh}(xa))}{6 \operatorname{arcsinh}(xa)}-\frac{\operatorname{Chi}(2 \operatorname{arcsinh}(xa))}{3}-\frac{\sinh(4 \operatorname{arcsinh}(xa))}{24 \operatorname{arcsinh}(xa)^{3}}-\frac{\cosh(4 \operatorname{arcsinh}(xa))}{12 \operatorname{arcsinh}(xa)^{2}}-}{a^{4}}$
default	$\frac{\frac{\sinh(2 \operatorname{arcsinh}(xa))}{12 \operatorname{arcsinh}(xa)^{3}}+\frac{\cosh(2 \operatorname{arcsinh}(xa))}{12 \operatorname{arcsinh}(xa)^{2}}+\frac{\sinh(2 \operatorname{arcsinh}(xa))}{6 \operatorname{arcsinh}(xa)}-\frac{\operatorname{Chi}(2 \operatorname{arcsinh}(xa))}{3}-\frac{\sinh(4 \operatorname{arcsinh}(xa))}{24 \operatorname{arcsinh}(xa)^{3}}-\frac{\cosh(4 \operatorname{arcsinh}(xa))}{12 \operatorname{arcsinh}(xa)^{2}}-}{a^{4}}$

input
```
int(x^3/arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/12*sinh(2*arcsinh(x*a))/arcsinh(x*a)^3+1/12/arcsinh(x*a)^2*cosh(2
*arcsinh(x*a))+1/6/arcsinh(x*a)*sinh(2*arcsinh(x*a))-1/3*Chi(2*arcsinh(x*a
))-1/24/arcsinh(x*a)^3*sinh(4*arcsinh(x*a))-1/12/arcsinh(x*a)^2*cosh(4*arc
sinh(x*a))-1/3/arcsinh(x*a)*sinh(4*arcsinh(x*a))+4/3*Chi(4*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^3/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{asinh}^4\left(ax\right)}\,dx$$

input
```
integrate(x**3/asinh(a*x)**4,x)
```

output
```
Integral(x**3/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
-1/6*(2*a^13*x^14 + 10*a^11*x^12 + 20*a^9*x^10 + 20*a^7*x^8 + 10*a^5*x^6 +
 2*a^3*x^4 + 2*(a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^10 + 9
*a^7*x^8 + 4*a^5*x^6)*(a^2*x^2 + 1)^2 + (16*a^13*x^14 + 80*a^11*x^12 + 160
*a^9*x^10 + 160*a^7*x^8 + 80*a^5*x^6 + 16*a^3*x^4 + 4*(4*a^8*x^9 + 7*a^6*x
^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^(5/2) + (80*a^9*x^10 + 192*a^7*x^8 + 154*a^5
*x^6 + 45*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^2 + (160*a^10*x^11 + 488*a^8*x^
9 + 550*a^6*x^7 + 279*a^4*x^5 + 63*a^2*x^3 + 6*x)*(a^2*x^2 + 1)^(3/2) + (1
60*a^11*x^12 + 592*a^9*x^10 + 846*a^7*x^8 + 583*a^5*x^6 + 196*a^3*x^4 + 27
*a*x^2)*(a^2*x^2 + 1) + (80*a^12*x^13 + 348*a^10*x^11 + 598*a^8*x^9 + 509*
a^6*x^7 + 216*a^4*x^5 + 37*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*
x^2 + 1))^2 + 4*(5*a^10*x^11 + 13*a^8*x^9 + 11*a^6*x^7 + 3*a^4*x^5)*(a^2*x
^2 + 1)^(3/2) + 4*(5*a^11*x^12 + 17*a^9*x^10 + 21*a^7*x^8 + 11*a^5*x^6 + 2
*a^3*x^4)*(a^2*x^2 + 1) + (4*a^13*x^14 + 20*a^11*x^12 + 40*a^9*x^10 + 40*a
^7*x^8 + 20*a^5*x^6 + 4*a^3*x^4 + 2*(2*a^8*x^9 + 3*a^6*x^7 + a^4*x^5)*(a^2
*x^2 + 1)^(5/2) + (20*a^9*x^10 + 44*a^7*x^8 + 31*a^5*x^6 + 7*a^3*x^4)*(a^2
*x^2 + 1)^2 + (40*a^10*x^11 + 116*a^8*x^9 + 121*a^6*x^7 + 53*a^4*x^5 + 8*a
^2*x^3)*(a^2*x^2 + 1)^(3/2) + (40*a^11*x^12 + 144*a^9*x^10 + 197*a^7*x^8 +
 125*a^5*x^6 + 35*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1) + (20*a^12*x^13 + 86*a^
10*x^11 + 145*a^8*x^9 + 119*a^6*x^7 + 47*a^4*x^5 + 7*a^2*x^3)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^13 + 21*a^10*x^11 + 3...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3/arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

input

```
int(x^3/asinh(a*x)^4,x)
```

output

```
int(x^3/asinh(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\mathit{asinh}\left(ax\right)^4}dx$$

input

```
int(x^3/asinh(a*x)^4,x)
```

output

```
int(x**3/asinh(a*x)**4,x)
```

3.72 $\int \frac{x^2}{\text{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 138

$$\begin{aligned}\int \frac{x^2}{\text{arcsinh}(ax)^4}\,dx = &-\frac{x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3}-\frac{x}{3a^2\text{arcsinh}(ax)^2}\\&-\frac{x^3}{2\text{arcsinh}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{3a^3\text{arcsinh}(ax)}-\frac{3x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)}\\&-\frac{\text{Shi}(\text{arcsinh}(ax))}{24a^3}+\frac{9\text{Shi}(3\text{arcsinh}(ax))}{8a^3}\end{aligned}$$

output

```
-1/3*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-1/3*x/a^2/arcsinh(a*x)^2-1/2*x
^3/arcsinh(a*x)^2-1/3*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-3/2*x^2*(a^2*x^2+
1)^(1/2)/a/arcsinh(a*x)-1/24*Shi(arcsinh(a*x))/a^3+9/8*Shi(3*arcsinh(a*x))
/a^3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{\frac{4\left(2a^2x^2\sqrt{1+a^2x^2}+ax(2+3a^2x^2)\operatorname{arcsinh}(ax)+\sqrt{1+a^2x^2}(2+9a^2x^2)\operatorname{arcsinh}(ax)^2\right)}{\operatorname{arcsinh}(ax)^3}+\operatorname{Shi}(\operatorname{arcsinh}(ax))-27\operatorname{Shi}(3\operatorname{arcsin}}{24a^3}$$

input
```
Integrate[x^2/ArcSinh[a*x]^4,x]
```

output
```
-1/24*((4*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(2 + 3*a^2*x^2)*ArcSinh[a*x]
+ Sqrt[1 + a^2*x^2]*(2 + 9*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + Sinh
Integral[ArcSinh[a*x]] - 27*SinhIntegral[3*ArcSinh[a*x]])/a^3
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6194, 6233, 6188, 6193, 2009, 6234, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx$$

$$\Big\downarrow \text{ 6194}$$

$$\frac{2\int \frac{x}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}dx}{3a}+a\int \frac{x^3}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}dx-\frac{x^2\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{ 6233}$$

$$a\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)^2}dx}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)+\frac{2\left(\frac{\int \frac{1}{\text{arcsinh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)}{3a}-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{ 6188}$$

$$\frac{2\left(\frac{a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)}{3a}+a\left(\frac{3\int \frac{x^2}{\text{arcsinh}(ax)^2}dx}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{2\left(\frac{a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)}{3a}+a\left(\frac{3\left(\frac{\int \left(\frac{3\sinh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}-\frac{ax}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(\frac{a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)}{3a}-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+a\left(\frac{3\left(\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{2\left(\frac{\frac{\int \frac{ax}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)}{3a}-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+a\left(\frac{3\left(\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\left(-\frac{x}{2a\text{arcsinh}(ax)^2}+\frac{-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}+\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}}{2a}\right)}{3a}-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+a\left(\frac{3\left(\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2\left(-\frac{x}{2a\text{arcsinh}(ax)^2}+\frac{-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}-\frac{i\int \frac{\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}}{2a}\right)}{3a}-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+a\left(\frac{3\left(\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{2\left(\frac{\frac{\text{Shi}(\text{arcsinh}(ax))}{a}-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)}{3a}-\frac{x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+a\left(\frac{3\left(\frac{\frac{3}{4}\text{Shi}(3\text{arcsinh}(ax))-\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))}{a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}\right)}{2a}-\frac{x^3}{2a\text{arcsinh}(ax)^2}\right)$$

input
```
Int[x^2/ArcSinh[a*x]^4,x]
```

output
```
-1/3*(x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) + (2*(-1/2*x/(a*ArcSinh[a*
x]^2) + (-(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]
]/a)/(2*a)))/(3*a) + a*(-1/2*x^3/(a*ArcSinh[a*x]^2) + (3*(-((x^2*Sqrt[1 +
a^2*x^2])/(a*ArcSinh[a*x])) + (-1/4*SinhIntegral[ArcSinh[a*x]] + (3*SinhIn
tegral[3*ArcSinh[a*x]])/4)/a^3))/(2*a))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\sqrt{a^2x^2+1}}{12\operatorname{arcsinh}(xa)^3}+\frac{xa}{24\operatorname{arcsinh}(xa)^2}+\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(xa)}-\frac{\operatorname{Shi}(\operatorname{arcsinh}(xa))}{24}-\frac{\cosh(3\operatorname{arcsinh}(xa))}{12\operatorname{arcsinh}(xa)^3}-\frac{\sinh(3\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)^2}-\frac{3\cosh(3\ \mathrm{arcsi}}{8\ \mathrm{arcsinh}}}{a^3}$
default	$\frac{\frac{\sqrt{a^2x^2+1}}{12\operatorname{arcsinh}(xa)^3}+\frac{xa}{24\operatorname{arcsinh}(xa)^2}+\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(xa)}-\frac{\operatorname{Shi}(\operatorname{arcsinh}(xa))}{24}-\frac{\cosh(3\operatorname{arcsinh}(xa))}{12\operatorname{arcsinh}(xa)^3}-\frac{\sinh(3\operatorname{arcsinh}(xa))}{8\operatorname{arcsinh}(xa)^2}-\frac{3\cosh(3\ \mathrm{arcsi}}{8\ \mathrm{arcsinh}}}{a^3}$

input
```
int(x^2/arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/12/arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)+1/24/arcsinh(x*a)^2*x*a+1/24/
arcsinh(x*a)*(a^2*x^2+1)^(1/2)-1/24*Shi(arcsinh(x*a))-1/12/arcsinh(x*a)^3*
cosh(3*arcsinh(x*a))-1/8/arcsinh(x*a)^2*sinh(3*arcsinh(x*a))-3/8/arcsinh(x
*a)*cosh(3*arcsinh(x*a))+9/8*Shi(3*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{asinh}^4\left(ax\right)}\,dx$$

input
```
integrate(x**2/asinh(a*x)**4,x)
```

output
```
Integral(x**2/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

output
```
-1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 +
2*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a
^7*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (9*a^13*x^13 + 45*a^11*x^11 + 90*a^9
*x^9 + 90*a^7*x^7 + 45*a^5*x^5 + 9*a^3*x^3 + (9*a^8*x^8 + 13*a^6*x^6 + 3*a
^4*x^4 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (45*a^9*x^9 + 97*a^7*x^7 + 64*a^5*
x^5 + 10*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (90*a^10*x^10 + 258*a^8*x^8 +
264*a^6*x^6 + 113*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1)^(3/2) + 2*(45*a^
11*x^11 + 161*a^9*x^9 + 219*a^7*x^7 + 141*a^5*x^5 + 44*a^3*x^3 + 6*a*x)*(a
^2*x^2 + 1) + (45*a^12*x^12 + 193*a^10*x^10 + 325*a^8*x^8 + 270*a^6*x^6 +
112*a^4*x^4 + 19*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 4*(5*a^10*x^10 + 13*a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3
/2) + 4*(5*a^11*x^11 + 17*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(
a^2*x^2 + 1) + (3*a^13*x^13 + 15*a^11*x^11 + 30*a^9*x^9 + 30*a^7*x^7 + 15*
a^5*x^5 + 3*a^3*x^3 + (3*a^8*x^8 + 4*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2
) + (15*a^9*x^9 + 31*a^7*x^7 + 20*a^5*x^5 + 4*a^3*x^3)*(a^2*x^2 + 1)^2 + (
30*a^10*x^10 + 84*a^8*x^8 + 84*a^6*x^6 + 35*a^4*x^4 + 5*a^2*x^2)*(a^2*x^2
+ 1)^(3/2) + 2*(15*a^11*x^11 + 53*a^9*x^9 + 71*a^7*x^7 + 44*a^5*x^5 + 12*a
^3*x^3 + a*x)*(a^2*x^2 + 1) + (15*a^12*x^12 + 64*a^10*x^10 + 107*a^8*x^8 +
 87*a^6*x^6 + 34*a^4*x^4 + 5*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^
2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10 + 34*a^8*x^8 + 26*a^6*x^6 +...
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(a*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

input
```
int(x^2/asinh(a*x)^4,x)
```

output
```
int(x^2/asinh(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{asinh\left(ax\right)^4}dx$$

input
```
int(x^2/asinh(a*x)^4,x)
```

output
```
int(x**2/asinh(a*x)**4,x)
```

3.73 $\int \frac{x}{\text{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 95

$$\int \frac{x}{\text{arcsinh}(ax)^4}\,dx = -\frac{x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{1}{6a^2\text{arcsinh}(ax)^2} - \frac{x^2}{3\text{arcsinh}(ax)^2} - \frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)} + \frac{2\text{Chi}(2\text{arcsinh}(ax))}{3a^2}$$

output
```
-1/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-1/6/a^2/arcsinh(a*x)^2-1/3*x^2/a
rcsinh(a*x)^2-2/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)+2/3*Chi(2*arcsinh(a*x
))/a^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x}{\text{arcsinh}(ax)^4}\,dx = -\frac{2ax\sqrt{1+a^2x^2} + (1+2a^2x^2)\,\text{arcsinh}(ax) + 4ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 - 4\text{arcsinh}(ax)^3\text{Chi}(2\text{arcsinh}}{6a^2\text{arcsinh}(ax)^3}$$

input
```
Integrate[x/ArcSinh[a*x]^4,x]
```

output
```
-1/6*(2*a*x*Sqrt[1 + a^2*x^2] + (1 + 2*a^2*x^2)*ArcSinh[a*x] + 4*a*x*Sqrt[
1 + a^2*x^2]*ArcSinh[a*x]^2 - 4*ArcSinh[a*x]^3*CoshIntegral[2*ArcSinh[a*x]
])/(a^2*ArcSinh[a*x]^3)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6194, 6198, 6233, 6193, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(ax)^4}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{\int \frac{1}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}dx}{3a}+\frac{2}{3}a\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}dx-\frac{x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6198}$$

$$\frac{2}{3}a\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}dx-\frac{x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}-\frac{1}{6a^2\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6233}$$

$$\frac{2}{3}a\left(\frac{\int \frac{x}{\text{arcsinh}(ax)^2}dx}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}\right)-\frac{x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}-\frac{1}{6a^2\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{6193}$$

$$\frac{2}{3}a\left(\frac{\frac{\int \frac{\cosh(2\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}\right)-\frac{x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}-\frac{1}{6a^2\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\left(-\frac{x^2}{2a\text{arcsinh}(ax)^2}+\frac{-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}+\frac{\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a^2}}{a}\right)-\frac{x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}-\frac{1}{6a^2\text{arcsinh}(ax)^2}$$

$$\Big\downarrow \text{3782}$$

$$\frac{2}{3}a\left(\frac{\frac{\text{Chi}(2\text{arcsinh}(ax))}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{a}-\frac{x^2}{2a\text{arcsinh}(ax)^2}\right)-\frac{x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}-\frac{1}{6a^2\text{arcsinh}(ax)^2}$$

input

```
Int[x/ArcSinh[a*x]^4,x]
```

output

```
-1/3*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) - 1/(6*a^2*ArcSinh[a*x]^2) +
 (2*a*(-1/2*x^2/(a*ArcSinh[a*x]^2) + (-((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a
*x])) + CoshIntegral[2*ArcSinh[a*x]]/a^2)/a))/3
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\operatorname{arcsinh}(xa))}{6\operatorname{arcsinh}(xa)^{3}}-\frac{\cosh(2\,\operatorname{arcsinh}(xa))}{6\operatorname{arcsinh}(xa)^{2}}-\frac{\sinh(2\,\operatorname{arcsinh}(xa))}{3\,\operatorname{arcsinh}(xa)}+\frac{2\,\operatorname{Chi}(2\,\operatorname{arcsinh}(xa))}{3}}{a^{2}}$	60
default	$\frac{-\frac{\sinh(2\,\operatorname{arcsinh}(xa))}{6\operatorname{arcsinh}(xa)^{3}}-\frac{\cosh(2\,\operatorname{arcsinh}(xa))}{6\operatorname{arcsinh}(xa)^{2}}-\frac{\sinh(2\,\operatorname{arcsinh}(xa))}{3\,\operatorname{arcsinh}(xa)}+\frac{2\,\operatorname{Chi}(2\,\operatorname{arcsinh}(xa))}{3}}{a^{2}}$	60

input
```
int(x/arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/6*sinh(2*arcsinh(x*a))/arcsinh(x*a)^3-1/6/arcsinh(x*a)^2*cosh(2*
arcsinh(x*a))-1/3/arcsinh(x*a)*sinh(2*arcsinh(x*a))+2/3*Chi(2*arcsinh(x*a)
))
```

Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{asinh}^4\left(ax\right)}\,dx$$

input
```
integrate(x/asinh(a*x)**4,x)
```

output
```
Integral(x/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(x/arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
-1/6*(2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10*a^4*x^4 +
2*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^8*x^8 + 9*a
^6*x^6 + 4*a^4*x^4)*(a^2*x^2 + 1)^2 + (4*a^12*x^12 + 20*a^10*x^10 + 40*a^8
*x^8 + 40*a^6*x^6 + 20*a^4*x^4 + 4*a^2*x^2 + 4*(a^7*x^7 + a^5*x^5)*(a^2*x^
2 + 1)^(5/2) + (20*a^8*x^8 + 36*a^6*x^6 + 16*a^4*x^4 - 3*a^2*x^2 - 3)*(a^2
*x^2 + 1)^2 + (40*a^9*x^9 + 104*a^7*x^7 + 88*a^5*x^5 + 21*a^3*x^3 - 3*a*x)
*(a^2*x^2 + 1)^(3/2) + (40*a^10*x^10 + 136*a^8*x^8 + 168*a^6*x^6 + 91*a^4*
x^4 + 22*a^2*x^2 + 3)*(a^2*x^2 + 1) + (20*a^11*x^11 + 84*a^9*x^9 + 136*a^7
*x^7 + 107*a^5*x^5 + 42*a^3*x^3 + 7*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt
(a^2*x^2 + 1))^2 + 4*(5*a^9*x^9 + 13*a^7*x^7 + 11*a^5*x^5 + 3*a^3*x^3)*(a^
2*x^2 + 1)^(3/2) + 4*(5*a^10*x^10 + 17*a^8*x^8 + 21*a^6*x^6 + 11*a^4*x^4 +
 2*a^2*x^2)*(a^2*x^2 + 1) + (2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*
a^6*x^6 + 10*a^4*x^4 + 2*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/
2) + (10*a^8*x^8 + 18*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^2 + (20
*a^9*x^9 + 52*a^7*x^7 + 47*a^5*x^5 + 17*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1)^(3/
2) + (20*a^10*x^10 + 68*a^8*x^8 + 87*a^6*x^6 + 51*a^4*x^4 + 13*a^2*x^2 + 1
)*(a^2*x^2 + 1) + (10*a^11*x^11 + 42*a^9*x^9 + 69*a^7*x^7 + 55*a^5*x^5 + 2
1*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*
a^11*x^11 + 21*a^9*x^9 + 34*a^7*x^7 + 26*a^5*x^5 + 9*a^3*x^3 + a*x)*sqrt(a
^2*x^2 + 1))/((a^12*x^10 + 5*a^10*x^8 + (a^2*x^2 + 1)^(5/2)*a^7*x^5 + 1...
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input

```
integrate(x/arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
integrate(x/arcsinh(a*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

input
```
int(x/asinh(a*x)^4,x)
```

output
```
int(x/asinh(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{asinh\left(ax\right)^4}dx$$

input
```
int(x/asinh(a*x)^4,x)
```

output
```
int(x/asinh(a*x)**4,x)
```

3.74 $\int \frac{1}{\mathrm{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 76

$$\int \frac{1}{\mathrm{arcsinh}(ax)^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3a\,\mathrm{arcsinh}(ax)^3} - \frac{x}{6\,\mathrm{arcsinh}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{6a\,\mathrm{arcsinh}(ax)} + \frac{\mathrm{Shi}(\mathrm{arcsinh}(ax))}{6a}$$

output

```
-1/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-1/6*x/arcsinh(a*x)^2-1/6*(a^2*x^2+
1)^(1/2)/a/arcsinh(a*x)+1/6*Shi(arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{1}{\mathrm{arcsinh}(ax)^4}\,dx = -\frac{2\sqrt{1+a^2x^2} + ax\,\mathrm{arcsinh}(ax) + \sqrt{1+a^2x^2}\,\mathrm{arcsinh}(ax)^2 - \mathrm{arcsinh}(ax)^3\,\mathrm{Shi}(\mathrm{arcsinh}(ax))}{6a\,\mathrm{arcsinh}(ax)^3}$$

input

```
Integrate[ArcSinh[a*x]^(-4),x]
```

output
```
-1/6*(2*Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a
*x]^2 - ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x]])/(a*ArcSinh[a*x]^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {6188, 6233, 6188, 6234, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^4}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{1}{3}a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}dx-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6233}$$

$$\frac{1}{3}a\left(\frac{\int \frac{1}{\text{arcsinh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6188}$$

$$\frac{1}{3}a\left(\frac{a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)}dx-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{3}a\left(\frac{\frac{\int \frac{ax}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+\frac{1}{3}a\left(-\frac{x}{2a\text{arcsinh}(ax)^2}+\frac{-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}+\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}}{2a}\right)$$

$$\Big\downarrow 26$$

$$-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}+\frac{1}{3}a\left(-\frac{x}{2a\text{arcsinh}(ax)^2}+\frac{-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}-\frac{i\int \frac{\sin(i\text{arcsinh}(ax))}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}}{2a}\right)$$

$$\Big\downarrow 3779$$

$$\frac{1}{3}a\left(\frac{\frac{\text{Shi}(\text{arcsinh}(ax))}{a}-\frac{\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}}{2a}-\frac{x}{2a\text{arcsinh}(ax)^2}\right)-\frac{\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^3}$$

input
```
Int[ArcSinh[a*x]^(-4),x]
```

output
```
-1/3*Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x]^3) + (a*(-1/2*x/(a*ArcSinh[a*x]^2)
+ (-(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/a)/(
2*a)))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{3 \operatorname{arcsinh}\left(x a\right)^{3}}-\frac{x a}{6 \operatorname{arcsinh}\left(x a\right)^{2}}-\frac{\sqrt{a^{2} x^{2}+1}}{6 \operatorname{arcsinh}\left(x a\right)}+\frac{\operatorname{Shi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{6}}{a}$	61
default	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{3 \operatorname{arcsinh}\left(x a\right)^{3}}-\frac{x a}{6 \operatorname{arcsinh}\left(x a\right)^{2}}-\frac{\sqrt{a^{2} x^{2}+1}}{6 \operatorname{arcsinh}\left(x a\right)}+\frac{\operatorname{Shi}\left(\operatorname{arcsinh}\left(x a\right)\right)}{6}}{a}$	61

input
```
int(1/arcsinh(x*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/3/arcsinh(x*a)^3*(a^2*x^2+1)^(1/2)-1/6/arcsinh(x*a)^2*x*a-1/6/arcs
inh(x*a)*(a^2*x^2+1)^(1/2)+1/6*Shi(arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{4}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^(-4), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{asinh}^{4}\left(ax\right)}\,dx$$

input
```
integrate(1/asinh(a*x)**4,x)
```

output
```
Integral(asinh(a*x)**(-4), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{4}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^4,x, algorithm="maxima")
```

output
```
-1/6*(2*a^11*x^11 + 10*a^9*x^9 + 20*a^7*x^7 + 20*a^5*x^5 + 10*a^3*x^3 + 2*
(a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^7*x^7 + 9*a^5*x^5 + 4*a^3
*x^3)*(a^2*x^2 + 1)^2 + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*x^7 + 10*a^5*x^5 +
 5*a^3*x^3 + (a^6*x^6 + a^4*x^4 + 3*a^2*x^2 + 3)*(a^2*x^2 + 1)^(5/2) + (5*
a^7*x^7 + 9*a^5*x^5 + 10*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 +
26*a^6*x^6 + 22*a^4*x^4 + 3*a^2*x^2 - 3)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^
9 + 17*a^7*x^7 + 18*a^5*x^5 + 5*a^3*x^3 - a*x)*(a^2*x^2 + 1) + a*x + (5*a^
10*x^10 + 21*a^8*x^8 + 31*a^6*x^6 + 20*a^4*x^4 + 6*a^2*x^2 + 1)*sqrt(a^2*x
^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^8*x^8 + 13*a^6*x^6 + 11*a
^4*x^4 + 3*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^9*x^9 + 17*a^7*x^7 + 21*a
^5*x^5 + 11*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + 2*a*x + (a^11*x^11 + 5*a^9*x^
9 + 10*a^7*x^7 + 10*a^5*x^5 + 5*a^3*x^3 + (a^6*x^6 - a^2*x^2)*(a^2*x^2 + 1
)^(5/2) + (5*a^7*x^7 + 5*a^5*x^5 - 2*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (1
0*a^8*x^8 + 20*a^6*x^6 + 10*a^4*x^4 - a^2*x^2 - 1)*(a^2*x^2 + 1)^(3/2) + 2
*(5*a^9*x^9 + 15*a^7*x^7 + 16*a^5*x^5 + 7*a^3*x^3 + a*x)*(a^2*x^2 + 1) + a
*x + (5*a^10*x^10 + 20*a^8*x^8 + 31*a^6*x^6 + 23*a^4*x^4 + 8*a^2*x^2 + 1)*
sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^10*x^10 + 21*a^8*
x^8 + 34*a^6*x^6 + 26*a^4*x^4 + 9*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^11*x
^10 + 5*a^9*x^8 + (a^2*x^2 + 1)^(5/2)*a^6*x^5 + 10*a^7*x^6 + 10*a^5*x^4 +
5*a^3*x^2 + 5*(a^7*x^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 10*(a^8*x^7 + 2*a^6...
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(1/arcsinh(a*x)^4,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(-4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

input
```
int(1/asinh(a*x)^4,x)
```

output
```
int(1/asinh(a*x)^4, x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{asinh\left(ax\right)^4}dx$$

input
```
int(1/asinh(a*x)^4,x)
```

output
```
int(1/asinh(a*x)**4,x)
```

3.75 $\int \frac{1}{x\,\textbf{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\,\text{arcsinh}(ax)^4}\,dx = \text{Int}\left(\frac{1}{x\,\text{arcsinh}(ax)^4},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\,\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{arcsinh}(ax)^4}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]^4),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]^4), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)^4} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x\text{arcsinh}(ax)^4}dx$$

input
```
Int[1/(x*ArcSinh[a*x]^4),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}\left(x a\right)^{4}}dx$$

input
```
int(1/x/arcsinh(x*a)^4,x)
```

output
```
int(1/x/arcsinh(x*a)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{asinh}^4\left(ax\right)}\,dx$$

input
```
integrate(1/x/asinh(a*x)**4,x)
```

output
```
Integral(1/(x*asinh(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 1611, normalized size of antiderivative = 161.10

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^4}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
-1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 +
2*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a
^7*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (4*(a^6*x^6 + 3*a^4*x^4 + 2*a^2*x^2)
*(a^2*x^2 + 1)^(5/2) + (16*a^7*x^7 + 46*a^5*x^5 + 37*a^3*x^3 + 7*a*x)*(a^2
*x^2 + 1)^2 + (24*a^8*x^8 + 66*a^6*x^6 + 59*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2
*x^2 + 1)^(3/2) + (16*a^9*x^9 + 42*a^7*x^7 + 39*a^5*x^5 + 16*a^3*x^3 + 3*a
*x)*(a^2*x^2 + 1) + (4*a^10*x^10 + 10*a^8*x^8 + 9*a^6*x^6 + 4*a^4*x^4 + a^
2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10
+ 13*a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^1
1 + 17*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (2*(
a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (8*a^7*x^7 + 13*a^5*x^5 + 5*a^3*x
^3)*(a^2*x^2 + 1)^2 + (12*a^8*x^8 + 27*a^6*x^6 + 19*a^4*x^4 + 4*a^2*x^2)*(
a^2*x^2 + 1)^(3/2) + (8*a^9*x^9 + 23*a^7*x^7 + 23*a^5*x^5 + 9*a^3*x^3 + a*
x)*(a^2*x^2 + 1) + (2*a^10*x^10 + 7*a^8*x^8 + 9*a^6*x^6 + 5*a^4*x^4 + a^2*
x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21
*a^10*x^10 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 +
 1))/((a^13*x^13 + 5*a^11*x^11 + (a^2*x^2 + 1)^(5/2)*a^8*x^8 + 10*a^9*x^9
+ 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + 5*(a^9*x^9 + a^7*x^7)*(a^2*x^2 + 1)^2
 + 10*(a^10*x^10 + 2*a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^1
1 + 3*a^9*x^9 + 3*a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1) + 5*(a^12*x^12 + 4*a...
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^4}\,dx$$

input

```
integrate(1/x/arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
integrate(1/(x*arcsinh(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\mathrm{asinh}\left(a\,x\right)^4}\,dx$$

input
```
int(1/(x*asinh(a*x)^4),x)
```

output
```
int(1/(x*asinh(a*x)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{asinh\left(ax\right)^4 x}dx$$

input
```
int(1/x/asinh(a*x)^4,x)
```

output
```
int(1/(asinh(a*x)**4*x),x)
```

3.76 $\int \frac{1}{x^2 \mathbf{arcsinh}(ax)^4} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^4} \, dx = \text{Int}\left(\frac{1}{x^2 \text{arcsinh}(ax)^4}, x\right)$$

output

```
Defer(Int)(1/x^2/arcsinh(a*x)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 6.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^4} \, dx = \int \frac{1}{x^2 \text{arcsinh}(ax)^4} \, dx$$

input

```
Integrate[1/(x^2*ArcSinh[a*x]^4),x]
```

output

```
Integrate[1/(x^2*ArcSinh[a*x]^4), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4} \, dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}dx$$

input
```
Int[1/(x^2*ArcSinh[a*x]^4),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \operatorname{arcsinh}\left(x a \right)^{4}}d x$$

input
```
int(1/x^2/arcsinh(x*a)^4,x)
```

output
```
int(1/x^2/arcsinh(x*a)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\,\text{arsinh}\,(ax)^4}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(1/(x^2*arcsinh(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\,\text{asinh}^4\,(ax)}\,dx$$

input
```
integrate(1/x**2/asinh(a*x)**4,x)
```

output
```
Integral(1/(x**2*asinh(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 1885, normalized size of antiderivative = 188.50

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\,\text{arsinh}\,(ax)^4}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

output

```
-1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 +
2*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a
^7*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (a^13*x^13 + 5*a^11*x^11 + 10*a^9*x^
9 + 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 + 13*a^6*x^6 + 27*a^4*x^4
+ 15*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 57*a^7*x^7 + 124*a^5*x^5
+ 90*a^3*x^3 + 18*a*x)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 98*a^8*x^8 + 220*
a^6*x^6 + 189*a^4*x^4 + 63*a^2*x^2 + 6)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^
11 + 41*a^9*x^9 + 93*a^7*x^7 + 89*a^5*x^5 + 38*a^3*x^3 + 6*a*x)*(a^2*x^2 +
 1) + (5*a^12*x^12 + 33*a^10*x^10 + 73*a^8*x^8 + 74*a^6*x^6 + 36*a^4*x^4 +
 7*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*
x^10 + 13*a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^1
1*x^11 + 17*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) -
 (a^13*x^13 + 5*a^11*x^11 + 10*a^9*x^9 + 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3
+ (a^8*x^8 + 4*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 21*
a^7*x^7 + 24*a^5*x^5 + 8*a^3*x^3)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 44*a^8
*x^8 + 64*a^6*x^6 + 37*a^4*x^4 + 7*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^1
1*x^11 + 23*a^9*x^9 + 39*a^7*x^7 + 30*a^5*x^5 + 10*a^3*x^3 + a*x)*(a^2*x^2
 + 1) + (5*a^12*x^12 + 24*a^10*x^10 + 45*a^8*x^8 + 41*a^6*x^6 + 18*a^4*x^4
 + 3*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*
x^12 + 21*a^10*x^10 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sq...
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\operatorname{arsinh}\left(ax\right)^4}\,dx$$

input

```
integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

output

```
integrate(1/(x^2*arcsinh(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx=\int \frac{1}{x^2\,\mathrm{asinh}\left(a\,x\right)^4}\,dx$$

input

```
int(1/(x^2*asinh(a*x)^4),x)
```

output

```
int(1/(x^2*asinh(a*x)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx=\int \frac{1}{asinh\left(ax\right)^4 x^2}dx$$

input

```
int(1/x^2/asinh(a*x)^4,x)
```

output

```
int(1/(asinh(a*x)**4*x**2),x)
```

3.77 $\int (dx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 161

$$\begin{aligned}\int (dx)^{5/2}(a+b\text{arcsinh}(cx))\,dx &= \frac{20bd^2\sqrt{dx}\sqrt{1+c^2x^2}}{147c^3} \\ &\quad - \frac{4b(dx)^{5/2}\sqrt{1+c^2x^2}}{49c} + \frac{2(dx)^{7/2}(a+b\text{arcsinh}(cx))}{7d} \\ &\quad - \frac{10bd^{5/2}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{147c^{7/2}\sqrt{1+c^2x^2}}\end{aligned}$$

output

```
20/147*b*d^2*(d*x)^(1/2)*(c^2*x^2+1)^(1/2)/c^3-4/49*b*(d*x)^(5/2)*(c^2*x^2
+1)^(1/2)/c+2/7*(d*x)^(7/2)*(a+b*arcsinh(c*x))/d-10/147*b*d^(5/2)*(c*x+1)*
((c^2*x^2+1)/(c*x+1)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/2)*(d*x)^(1/2)
/d^(1/2)),1/2*2^(1/2))/c^(7/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

$$\int (dx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{2d^2\sqrt{dx}\left(21ac^3x^3+10b\sqrt{1+c^2x^2}-6bc^2x^2\sqrt{1+c^2x^2}+21bc^3x^3\operatorname{arcsinh}(cx)-10b\,\mathrm{Hy}\right.}{147c^3}$$

input
```
Integrate[(d*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(2*d^2*Sqrt[d*x]*(21*a*c^3*x^3 + 10*b*Sqrt[1 + c^2*x^2] - 6*b*c^2*x^2*Sqrt
[1 + c^2*x^2] + 21*b*c^3*x^3*ArcSinh[c*x] - 10*b*Hypergeometric2F1[1/4, 1/
2, 5/4, -(c^2*x^2)]))/(147*c^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6191, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2(dx)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7d}-\frac{2bc\int \frac{(dx)^{7/2}}{\sqrt{c^2x^2+1}}dx}{7d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(dx)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{5/2}}{7c^2}-\frac{5d^2\int \frac{(dx)^{3/2}}{\sqrt{c^2x^2+1}}dx}{7c^2}\right)}{7d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(dx)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{5/2}}{7c^2}-\frac{5d^2\left(\frac{2d\sqrt{c^2x^2+1}\sqrt{dx}}{3c^2}-\frac{d^2\int \frac{1}{\sqrt{dx}\sqrt{c^2x^2+1}}dx}{3c^2}\right)}{7c^2}\right)}{7d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(dx)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{5/2}}{7c^2}-\frac{5d^2\left(\frac{2d\sqrt{c^2x^2+1}\sqrt{dx}}{3c^2}-\frac{2d\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{3c^2}\right)}{7c^2}\right)}{7d}$$

$$\Big\downarrow \text{761}$$

$$\frac{2(dx)^{7/2}(a+b\operatorname{arcsinh}(cx))}{7d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{5/2}}{7c^2}-\frac{5d^2\left(\frac{2d\sqrt{c^2x^2+1}\sqrt{dx}}{3c^2}-\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{3c^{5/2}\sqrt{c^2x^2+1}}\right)}{7c^2}\right)}{7d}$$

input

```
Int[(d*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output

```
(2*(d*x)^(7/2)*(a + b*ArcSinh[c*x]))/(7*d) - (2*b*c*((2*d*(d*x)^(5/2)*Sqrt
[1 + c^2*x^2])/(7*c^2) - (5*d^2*((2*d*Sqrt[d*x]*Sqrt[1 + c^2*x^2])/(3*c^2)
 - (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(d + c*d*x)^2]*EllipticF[
2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(3*c^(5/2)*Sqrt[1 + c^2*x^2])
))/(7*c^2)))/(7*d)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\operatorname{arcsinh}(xc)}{7}-\frac{2c\left(\frac{d^{2}(dx)^{\frac{5}{2}}\sqrt{c^{2}x^{2}+1}}{7c^{2}}-\frac{5d^{4}\sqrt{dx}\,\sqrt{c^{2}x^{2}+1}}{21c^{4}}+\frac{5d^{4}\sqrt{-ixc+1}\,\sqrt{ixc+1}\ \mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{21c^{4}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{7d}\right.}{d}$
default	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\operatorname{arcsinh}(xc)}{7}-\frac{2c\left(\frac{d^{2}(dx)^{\frac{5}{2}}\sqrt{c^{2}x^{2}+1}}{7c^{2}}-\frac{5d^{4}\sqrt{dx}\,\sqrt{c^{2}x^{2}+1}}{21c^{4}}+\frac{5d^{4}\sqrt{-ixc+1}\,\sqrt{ixc+1}\ \mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{21c^{4}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{7d}\right.}{d}$
parts	$\frac{2a(dx)^{\frac{7}{2}}}{7d}+\frac{2b\left(\frac{(dx)^{\frac{7}{2}}\operatorname{arcsinh}(xc)}{7}-\frac{2c\left(\frac{d^{2}(dx)^{\frac{5}{2}}\sqrt{c^{2}x^{2}+1}}{7c^{2}}-\frac{5d^{4}\sqrt{dx}\,\sqrt{c^{2}x^{2}+1}}{21c^{4}}+\frac{5d^{4}\sqrt{-ixc+1}\,\sqrt{ixc+1}\ \mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right.}{21c^{4}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right.}{7d}\right.}{d}$

input
```
int((d*x)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/7*a*(d*x)^(7/2)+b*(1/7*(d*x)^(7/2)*arcsinh(x*c)-2/7/d*c*(1/7/c^2*d^
2*(d*x)^(5/2)*(c^2*x^2+1)^(1/2)-5/21/c^4*d^4*(d*x)^(1/2)*(c^2*x^2+1)^(1/2)
+5/21/c^4*d^4/(I/d*c)^(1/2)*(1-I*x*c)^(1/2)*(1+I*x*c)^(1/2)/(c^2*x^2+1)^(1
/2)*EllipticF((d*x)^(1/2)*(I/d*c)^(1/2),I))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int (dx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{2\left(21\sqrt{dx}bc^{5}d^{2}x^{3}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+21\sqrt{dx}ac^{5}d^{2}x^{3}-10\sqrt{c^{2}d}bd^{2}\mathrm{weierstrassPI}\right.}{147\,c^{5}}$$

input
```
integrate((d*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
2/147*(21*sqrt(d*x)*b*c^5*d^2*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 21*sqrt(d
*x)*a*c^5*d^2*x^3 - 10*sqrt(c^2*d)*b*d^2*weierstrassPInverse(-4/c^2, 0, x)
 - 2*(3*b*c^4*d^2*x^2 - 5*b*c^2*d^2)*sqrt(c^2*x^2 + 1)*sqrt(d*x))/c^5
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^{5/2}(a + b\text{arcsinh}(cx))\,dx = \int (dx)^{\frac{5}{2}}\,(b\,\text{arsinh}\,(cx) + a)\,dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
2/7*(d*x)^(7/2)*a/d + 1/294*(84*d^(5/2)*x^(7/2)*log(c*x + sqrt(c^2*x^2 + 1
)) - c^2*d^(5/2)*(8*(3*c^2*x^(7/2) - 7*x^(3/2))/c^4 - 21*(I*sqrt(2)*(log(1
/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqr
t(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) + 1))/c^(3/2) + I*sqrt(2)*(lo
g(1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*
sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c))/sqrt(c) + 1))/c^(3/2) + sqrt(2)*lo
g(c*x + sqrt(2)*sqrt(c)*sqrt(x) + 1)/c^(3/2) - sqrt(2)*log(c*x - sqrt(2)*s
qrt(c)*sqrt(x) + 1)/c^(3/2))/c^4) - 588*c*d^(5/2)*integrate(1/7*x^(7/2)/(c
^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2)), x))*b
```

Giac [F]

$$\int (dx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (dx)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)*(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\operatorname{asinh}(c\,x))\ (d\,x)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d*x)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d*x)^(5/2), x)
```

Reduce [F]

$$\int (dx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{\sqrt{d}\,d^2\left(2\sqrt{x}\,a\,x^3+7\left(\int\sqrt{x}\,asinh(cx)\,x^2dx\right)b\right)}{7}$$

input
```
int((d*x)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(2*sqrt(x)*a*x**3 + 7*int(sqrt(x)*asinh(c*x)*x**2,x)*b))/7
```

3.78 $\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 245

$$\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=-\frac{4b(dx)^{3/2}\sqrt{1+c^2x^2}}{25c}+\frac{12bd\sqrt{dx}\sqrt{1+c^2x^2}}{25c^2(1+cx)}+\frac{2(dx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{5d}-\frac{12bd^{3/2}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)|\frac{1}{2}\right)}{25c^{5/2}\sqrt{1+c^2x^2}}+\frac{6bd^{3/2}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{25c^{5/2}\sqrt{1+c^2x^2}}$$

output

```
-4/25*b*(d*x)^(3/2)*(c^2*x^2+1)^(1/2)/c+12/25*b*d*(d*x)^(1/2)*(c^2*x^2+1)^
(1/2)/c^2/(c*x+1)+2/5*(d*x)^(5/2)*(a+b*arcsinh(c*x))/d-12/25*b*d^(3/2)*(c*
x+1)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/2)*(d*x)^(1
/2)/d^(1/2))),1/2*2^(1/2))/c^(5/2)/(c^2*x^2+1)^(1/2)+6/25*b*d^(3/2)*(c*x+1
)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/2)*(d*x)^(1/
2)/d^(1/2)),1/2*2^(1/2))/c^(5/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.27

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{2(dx)^{3/2}\left(5acx-2b\sqrt{1+c^2x^2}+5bcx\text{arcsinh}(cx)+2b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\right.\right.}{25c}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(2*(d*x)^(3/2)*(5*a*c*x - 2*b*Sqrt[1 + c^2*x^2] + 5*b*c*x*ArcSinh[c*x] + 2
*b*Hypergeometric2F1[1/2, 3/4, 7/4, -(c^2*x^2)]))/(25*c)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6191, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\int \frac{(dx)^{5/2}}{\sqrt{c^2x^2+1}}dx}{5d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{3/2}}{5c^2}-\frac{3d^2\int \frac{\sqrt{dx}}{\sqrt{c^2x^2+1}}dx}{5c^2}\right)}{5d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{3/2}}{5c^2}-\frac{6d\int\frac{dx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{5c^2}\right)}{5d}$$

$$\Big\downarrow \; 834$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{3/2}}{5c^2}-\frac{6d\left(\frac{d\int\frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}-\frac{d\int\frac{d-cdx}{d\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{5c^2}\right)}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{3/2}}{5c^2}-\frac{6d\left(\frac{d\int\frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}-\frac{\int\frac{d-cdx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{5c^2}\right)}{5d}$$

$$\Big\downarrow \; 761$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{3/2}}{5c^2}-\frac{6d\left(\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{2c^{3/2}\sqrt{c^2x^2+1}}-\frac{\int\frac{d-cdx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{5c^2}\right)}{5d}$$

$$\Big\downarrow \; 1510$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))}{5d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}(dx)^{3/2}}{5c^2}-\frac{6d\left(\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{2c^{3/2}\sqrt{c^2x^2+1}}-\frac{\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{c}\sqrt{c^2x^2+1}}-d^2}{c}\right)}{5c^2}\right)}{5d}$$

input

```
Int[(d*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output

```
(2*(d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(5*d) - (2*b*c*((2*d*(d*x)^(3/2)*Sqrt
[1 + c^2*x^2])/(5*c^2) - (6*d*(-((-((d^2*Sqrt[d*x]*Sqrt[1 + c^2*x^2])/(d +
 c*d*x)) + (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(d + c*d*x)^2]*El
lipticE[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(Sqrt[c]*Sqrt[1 + c^2
*x^2]))/c) + (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(d + c*d*x)^2]*
EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(2*c^(3/2)*Sqrt[1 +
 c^2*x^2])))/(5*c^2)))/(5*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\operatorname{arcsinh}(xc)}{5}-\frac{2c\left(\frac{d^{2}(dx)^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}}{5c^{2}}-\frac{3id^{3}\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\left(\mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}}\right.\right.}{5c^{3}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{5d}\right.}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\operatorname{arcsinh}(xc)}{5}-\frac{2c\left(\frac{d^{2}(dx)^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}}{5c^{2}}-\frac{3id^{3}\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\left(\mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}}\right.\right.}{5c^{3}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{5d}\right.}{d}$
parts	$\frac{2a(dx)^{\frac{5}{2}}}{5d}+\frac{2b\left(\frac{(dx)^{\frac{5}{2}}\operatorname{arcsinh}(xc)}{5}-\frac{2c\left(\frac{d^{2}(dx)^{\frac{3}{2}}\sqrt{c^{2}x^{2}+1}}{5c^{2}}-\frac{3id^{3}\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\left(\mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\right.\right.}{5c^{3}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{5d}\right.}{d}$

input
```
int((d*x)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/5*(d*x)^(5/2)*a+b*(1/5*(d*x)^(5/2)*arcsinh(x*c)-2/5/d*c*(1/5/c^2*d^
2*(d*x)^(3/2)*(c^2*x^2+1)^(1/2)-3/5*I/c^3*d^3/(I/d*c)^(1/2)*(1-I*x*c)^(1/2
)*(1+I*x*c)^(1/2)/(c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(I/d*c)^(1/2),I
)-EllipticE((d*x)^(1/2)*(I/d*c)^(1/2),I)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.42

$$\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{2\left(5\sqrt{dx}bc^3dx^2\log\left(cx+\sqrt{c^2x^2+1}\right)+5\sqrt{dx}ac^3dx^2-2\sqrt{c^2x^2+1}\sqrt{dx}bc^2dx-6\sqrt{c}\right.}{25\,c^3}$$

input
```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
2/25*(5*sqrt(d*x)*b*c^3*d*x^2*log(c*x + sqrt(c^2*x^2 + 1)) + 5*sqrt(d*x)*a
*c^3*d*x^2 - 2*sqrt(c^2*x^2 + 1)*sqrt(d*x)*b*c^2*d*x - 6*sqrt(c^2*d)*b*d*w
eierstrassZeta(-4/c^2, 0, weierstrassPInverse(-4/c^2, 0, x)))/c^3
```

Sympy [F]

$$\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (dx)^{\frac{3}{2}}\left(a+b\operatorname{asinh}(cx)\right)\,dx$$

input
```
integrate((d*x)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*x)**(3/2)*(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int (dx)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)dx$$

input

```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output

```
2/5*(d*x)^(5/2)*a/d + 1/50*(20*d^(3/2)*x^(5/2)*log(c*x + sqrt(c^2*x^2 + 1)
) - c^2*d^(3/2)*(8*(c^2*x^(5/2) - 5*sqrt(x))/c^4 - 5*(I*sqrt(2)*(log(1/2*I
*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqrt(2)
*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) + 1))/sqrt(c) + I*sqrt(2)*(log(1/
2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqrt
(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c))/sqrt(c) + 1))/sqrt(c) - sqrt(2)*log(c*
x + sqrt(2)*sqrt(c)*sqrt(x) + 1)/sqrt(c) + sqrt(2)*log(c*x - sqrt(2)*sqrt(
c)*sqrt(x) + 1)/sqrt(c))/c^4) - 100*c*d^(3/2)*integrate(1/5*x^(5/2)/(c^3*x
^3 + c*x + (c^2*x^2 + 1)^(3/2)), x))*b
```

Giac [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int (dx)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)dx$$

input

```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output

```
integrate((d*x)^(3/2)*(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,x\right)}^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d*x)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{d}\,d\left(2\sqrt{x}\,a\,x^{2}+5\left(\int \sqrt{x}\,asinh(cx)\,xdx\right)b\right)}{5}$$

input
```
int((d*x)^(3/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*d*(2*sqrt(x)*a*x**2 + 5*int(sqrt(x)*asinh(c*x)*x,x)*b))/5
```

3.79 $\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\begin{aligned}\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))\,dx &= -\frac{4b\sqrt{dx}\sqrt{1+c^2x^2}}{9c}+\frac{2(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3d}\\ &\quad+\frac{2b\sqrt{d}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{9c^{3/2}\sqrt{1+c^2x^2}}\end{aligned}$$

output

```
-4/9*b*(d*x)^(1/2)*(c^2*x^2+1)^(1/2)/c+2/3*(d*x)^(3/2)*(a+b*arcsinh(c*x))/
d+2/9*b*d^(1/2)*(c*x+1)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(c^(1/2)*(d*x)^(1/2)/d^(1/2)),1/2*2^(1/2))/c^(3/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.51

$$\begin{aligned}&\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))\,dx\\ &=\frac{2\sqrt{dx}\left(3acx-2b\sqrt{1+c^2x^2}+3bcx\operatorname{arcsinh}(cx)+2b\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^2x^2\right)\right)}{9c}\end{aligned}$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(2*Sqrt[d*x]*(3*a*c*x - 2*b*Sqrt[1 + c^2*x^2] + 3*b*c*x*ArcSinh[c*x] + 2*b
*Hypergeometric2F1[1/4, 1/2, 5/4, -(c^2*x^2)]))/(9*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6191, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2(dx)^{3/2}(a+b\text{arcsinh}(cx))}{3d}-\frac{2bc\int \frac{(dx)^{3/2}}{\sqrt{c^2x^2+1}}dx}{3d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(dx)^{3/2}(a+b\text{arcsinh}(cx))}{3d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}\sqrt{dx}}{3c^2}-\frac{d^2\int \frac{1}{\sqrt{dx}\sqrt{c^2x^2+1}}dx}{3c^2}\right)}{3d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(dx)^{3/2}(a+b\text{arcsinh}(cx))}{3d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}\sqrt{dx}}{3c^2}-\frac{2d\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{3c^2}\right)}{3d}$$

$$\Big\downarrow \text{761}$$

$$\frac{2(dx)^{3/2}(a+b\text{arcsinh}(cx))}{3d}-\frac{2bc\left(\frac{2d\sqrt{c^2x^2+1}\sqrt{dx}}{3c^2}-\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{3c^{5/2}\sqrt{c^2x^2+1}}\right)}{3d}$$

input

```
Int[Sqrt[d*x]*(a + b*ArcSinh[c*x]),x]
```

output

```
(2*(d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(3*d) - (2*b*c*((2*d*Sqrt[d*x]*Sqrt[1
 + c^2*x^2])/(3*c^2) - (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(d +
c*d*x)^2]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(3*c^(5/2
)*Sqrt[1 + c^2*x^2])))/(3*d)
```

Defintions of rubi rules used

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\operatorname{arcsinh}(xc)}{3}-\frac{2c\left(\frac{d^{2}\sqrt{dx}\,\sqrt{c^{2}x^{2}+1}}{3c^{2}}-\frac{d^{2}\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{3c^{2}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{3d}\right)}{d}$	124
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\operatorname{arcsinh}(xc)}{3}-\frac{2c\left(\frac{d^{2}\sqrt{dx}\,\sqrt{c^{2}x^{2}+1}}{3c^{2}}-\frac{d^{2}\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{3c^{2}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{3d}\right)}{d}$	124
parts	$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b\left(\frac{(dx)^{\frac{3}{2}}\operatorname{arcsinh}(xc)}{3}-\frac{2c\left(\frac{d^{2}\sqrt{dx}\,\sqrt{c^{2}x^{2}+1}}{3c^{2}}-\frac{d^{2}\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{3c^{2}\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{3d}\right)}{d}$	126

input
```
int((d*x)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/3*(d*x)^(3/2)*a+b*(1/3*(d*x)^(3/2)*arcsinh(x*c)-2/3/d*c*(1/3/c^2*d^
2*(d*x)^(1/2)*(c^2*x^2+1)^(1/2)-1/3/c^2*d^2/(I/d*c)^(1/2)*(1-I*x*c)^(1/2)*
(1+I*x*c)^(1/2)/(c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(I/d*c)^(1/2),I)))
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{2\Big(3\sqrt{dx}bc^{3}x\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+3\sqrt{dx}ac^{3}x-2\sqrt{c^{2}x^{2}+1}\sqrt{dx}bc^{2}+2\sqrt{c^{2}d}b\text{weierstrassPInverse}\big(}{9\,c^{3}}$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(d*x)*b*c^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*sqrt(d*x)*a*c^3*
x - 2*sqrt(c^2*x^2 + 1)*sqrt(d*x)*b*c^2 + 2*sqrt(c^2*d)*b*weierstrassPInve
rse(-4/c^2, 0, x))/c^3
```

Sympy [F]

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{dx}\,(a+b\operatorname{asinh}{(cx)})\,\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*x)*(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{dx}(b\operatorname{arsinh}{(cx)}+a)\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
-1/18*(c^2*sqrt(d)*(8*x^(3/2)/c^2 + 3*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*s
qrt(x) + sqrt(2)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) +
 sqrt(2)*sqrt(c))/sqrt(c) + 1))/c^(3/2) + I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*
c*sqrt(x) - sqrt(2)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x
) - sqrt(2)*sqrt(c))/sqrt(c) + 1))/c^(3/2) + sqrt(2)*log(c*x + sqrt(2)*sqr
t(c)*sqrt(x) + 1)/c^(3/2) - sqrt(2)*log(c*x - sqrt(2)*sqrt(c)*sqrt(x) + 1)
/c^(3/2))/c^2) - 12*sqrt(d)*x^(3/2)*log(c*x + sqrt(c^2*x^2 + 1)) + 36*c*sq
rt(d)*integrate(1/3*x^(3/2)/(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2)), x))*b +
 2/3*(d*x)^(3/2)*a/d
```

Giac [F]

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{dx}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\operatorname{asinh}(c\,x))\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*asinh(c*x))*(d*x)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{d}\,\left(2\sqrt{x}\,ax+3\left(\int \sqrt{x}\,asinh(cx)\,dx\right)b\right)}{3}$$

input
```
int((d*x)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*(2*sqrt(x)*a*x + 3*int(sqrt(x)*asinh(c*x),x)*b))/3
```

3.80 $\int \frac{a+b\,\mathbf{arcsinh}(cx)}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 208

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{dx}}\,dx = &-\frac{4b\sqrt{dx}\sqrt{1+c^2x^2}}{d(1+cx)}+\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}\\&+\frac{4b(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\,E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{c}\sqrt{d}\sqrt{1+c^2x^2}}\\&-\frac{2b(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{\sqrt{c}\sqrt{d}\sqrt{1+c^2x^2}}\end{aligned}$$

output

```
-4*b*(d*x)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c*x+1)+2*(d*x)^(1/2)*(a+b*arcsinh(c*
x))/d+4*b*(c*x+1)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*EllipticE(sin(2*arctan(c^(
1/2)*(d*x)^(1/2)/d^(1/2))),1/2*2^(1/2))/c^(1/2)/d^(1/2)/(c^2*x^2+1)^(1/2)-
2*b*(c*x+1)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/2)
*(d*x)^(1/2)/d^(1/2)),1/2*2^(1/2))/c^(1/2)/d^(1/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{dx}}\,dx = \frac{2x\left(3(a+b\operatorname{arcsinh}(cx))-2bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-c^2x^2\right)\right)}{3\sqrt{dx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d*x],x]
```

output
```
(2*x*(3*(a + b*ArcSinh[c*x]) - 2*b*c*x*Hypergeometric2F1[1/2, 3/4, 7/4, -(
c^2*x^2)]))/(3*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6191, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}-\frac{2bc\int \frac{\sqrt{dx}}{\sqrt{c^2x^2+1}}dx}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}-\frac{4bc\int \frac{dx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{d^2}$$

$$\Big\downarrow \text{834}$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}-\frac{4bc\left(\frac{d\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}-\frac{d\int \frac{d-cdx}{d\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{d^2}$$

$$\Big\downarrow 27$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}-\frac{4bc\left(\frac{d\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}-\frac{\int \frac{d-cdx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{d^2}$$

$$\Big\downarrow 761$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}-\frac{4bc\left(\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{2c^{3/2}\sqrt{c^2x^2+1}}-\frac{\int \frac{d-cdx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{d^2}$$

$$\Big\downarrow 1510$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{d}-\frac{4bc\left(\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{2c^{3/2}\sqrt{c^2x^2+1}}-\frac{\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{c}\sqrt{c^2x^2+1}}-\frac{d^2\sqrt{c^2x^2+1}\sqrt{dx}}{cdx+d}}{c}\right)}{d^2}$$

input

```
Int[(a + b*ArcSinh[c*x])/Sqrt[d*x],x]
```

output

```
(2*Sqrt[d*x]*(a + b*ArcSinh[c*x]))/d - (4*b*c*(-((-((d^2*Sqrt[d*x]*Sqrt[1
+ c^2*x^2])/(d + c*d*x)) + (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(
d + c*d*x)^2]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(Sqrt
[c]*Sqrt[1 + c^2*x^2]))/c) + (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)
/(d + c*d*x)^2]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(2*
c^(3/2)*Sqrt[1 + c^2*x^2])))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

method	result	size
derivativedivides	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\operatorname{arcsinh}(xc)-\frac{2i\sqrt{-ixc+1}\,\sqrt{ixc+1}\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)\right)}{\sqrt{\frac{ic}{d}}\,\sqrt{c^2x^2+1}}\right)}{d}$	107
default	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\operatorname{arcsinh}(xc)-\frac{2i\sqrt{-ixc+1}\,\sqrt{ixc+1}\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)\right)}{\sqrt{\frac{ic}{d}}\,\sqrt{c^2x^2+1}}\right)}{d}$	107
parts	$\frac{2a\sqrt{dx}}{d}+\frac{2b\left(\sqrt{dx}\,\operatorname{arcsinh}(xc)-\frac{2i\sqrt{-ixc+1}\,\sqrt{ixc+1}\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)\right)}{\sqrt{\frac{ic}{d}}\,\sqrt{c^2x^2+1}}\right)}{d}$	110

input
```
int((a+b*arcsinh(x*c))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x)^(1/2)*a+b*((d*x)^(1/2)*arcsinh(x*c)-2*I/(I/d*c)^(1/2)*(1-I*x*c)
^(1/2)*(1+I*x*c)^(1/2)/(c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(I/d*c)^(1
/2),I)-EllipticE((d*x)^(1/2)*(I/d*c)^(1/2),I))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.32

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{dx}}\,dx$$
$$=\frac{2\Bigl(\sqrt{dx}bc\log\bigl(cx+\sqrt{c^2x^2+1}\bigr)+\sqrt{dx}ac+2\sqrt{c^2d}b\,\mathrm{weierstrassZeta}\bigl(-\frac{4}{c^2},0,\mathrm{weierstrassPInverse}\bigl(-\frac{4}{c^2},0,}{cd}$$

input
```
integrate((a+b*arcsinh(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(d*x)*b*c*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(d*x)*a*c + 2*sqrt(c^2
*d)*b*weierstrassZeta(-4/c^2, 0, weierstrassPInverse(-4/c^2, 0, x)))/(c*d)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{dx}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{dx}}\,dx = \int \frac{b\,\text{arsinh}(cx)+a}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(c^2*((I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/s
qrt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) +
 1))/sqrt(c) + I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c)
)/sqrt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c))/sqrt(c
) + 1))/sqrt(c) - sqrt(2)*log(c*x + sqrt(2)*sqrt(c)*sqrt(x) + 1)/sqrt(c) +
 sqrt(2)*log(c*x - sqrt(2)*sqrt(c)*sqrt(x) + 1)/sqrt(c))/(c^2*sqrt(d)) + 8
*sqrt(x)/(c^2*sqrt(d))) + 4*c*integrate(sqrt(x)/(c^3*sqrt(d)*x^3 + c*sqrt(
d)*x + (c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1)), x) - 4*sqrt(x)*log(
c*x + sqrt(c^2*x^2 + 1))/sqrt(d))*b + 2*sqrt(d*x)*a/d
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{dx}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{dx}}\,dx = \int \frac{a+b\operatorname{asinh}(c\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d*x)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,a+\left(\int \frac{asinh(cx)}{\sqrt{x}}dx\right)b}{\sqrt{d}}$$

input
```
int((a+b*asinh(c*x))/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*a + int(asinh(c*x)/sqrt(x),x)*b)/sqrt(d)
```

3.81 $\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx = -\frac{2(a+b\text{arcsinh}(cx))}{d\sqrt{dx}} + \frac{2b\sqrt{c}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{d^{3/2}\sqrt{1+c^2x^2}}$$

output
```
(-2*a-2*b*arcsinh(c*x))/d/(d*x)^(1/2)+2*b*c^(1/2)*(c*x+1)*((c^2*x^2+1)/(c*
x+1)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2)),1/2*2^
(1/2))/d^(3/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx = -\frac{2x\left(a+b\text{arcsinh}(cx)-2bcx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^2x^2\right)\right)}{(dx)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d*x)^(3/2),x]
```

output
```
(-2*x*(a + b*ArcSinh[c*x] - 2*b*c*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(c^2
*x^2)]))/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6191, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \; 6191$$

$$\frac{2bc\int \frac{1}{\sqrt{dx}\sqrt{c^2x^2+1}}dx}{d}-\frac{2(a+b\text{arcsinh}(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \; 266$$

$$\frac{4bc\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{d^2}-\frac{2(a+b\text{arcsinh}(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \; 761$$

$$\frac{2b\sqrt{c}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{d^{5/2}\sqrt{c^2x^2+1}}-\frac{2(a+b\text{arcsinh}(cx))}{d\sqrt{dx}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcSinh[c*x]))/(d*Sqrt[d*x]) + (2*b*Sqrt[c]*(d + c*d*x)*Sqrt[(d
^2 + c^2*d^2*x^2)/(d + c*d*x)^2]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sq
rt[d]], 1/2])/(d^(5/2)*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\operatorname{arcsinh}(xc)}{\sqrt{dx}}+\frac{2c\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{d\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{d}$	91
default	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\operatorname{arcsinh}(xc)}{\sqrt{dx}}+\frac{2c\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{d\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{d}$	91
parts	$-\frac{2a}{\sqrt{dx}\,d}+\frac{2b\left(-\frac{\operatorname{arcsinh}(xc)}{\sqrt{dx}}+\frac{2c\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)}{d\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{d}$	93

input
```
int((a+b*arcsinh(x*c))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-a/(d*x)^(1/2)+b*(-1/(d*x)^(1/2)*arcsinh(x*c)+2/d*c/(I/d*c)^(1/2)*(1-
I*x*c)^(1/2)*(1+I*x*c)^(1/2)/(c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(I/d*
c)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx = -\frac{2\left(\sqrt{dx}bc\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2d}bx\text{weierstrassPInverse}\left(-\frac{4}{c^2},0,x\right)+\sqrt{dx}ac\right)}{cd^2x}$$

input
```
integrate((a+b*arcsinh(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(d*x)*b*c*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d)*b*x*weierst
rassPInverse(-4/c^2, 0, x) + sqrt(d*x)*a*c)/(c*d^2*x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(dx)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

output

```
-1/2*(c^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sq
rt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) +
1))/(c^(3/2)*d^(3/2)) + I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2
)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c
))/sqrt(c) + 1))/(c^(3/2)*d^(3/2)) + sqrt(2)*log(c*x + sqrt(2)*sqrt(c)*sqr
t(x) + 1)/(c^(3/2)*d^(3/2)) - sqrt(2)*log(c*x - sqrt(2)*sqrt(c)*sqrt(x) +
1)/(c^(3/2)*d^(3/2))) - 4*c*integrate(1/((c^3*d^(3/2)*x^3 + c*d^(3/2)*x +
(c^2*d^(3/2)*x^2 + d^(3/2))*sqrt(c^2*x^2 + 1))*sqrt(x)), x) + 4*log(c*x +
sqrt(c^2*x^2 + 1))/(d^(3/2)*sqrt(x)))*b - 2*a/(sqrt(d*x)*d)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(dx)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

output

```
integrate((b*arcsinh(c*x) + a)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(dx)^{3/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}(c\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d*x)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{x}\left(\int \frac{asinh(cx)}{\sqrt{x}\,x}dx\right)b - 2a}{\sqrt{x}\,\sqrt{d}\,d}$$

input
```
int((a+b*asinh(c*x))/(d*x)^(3/2),x)
```

output
```
(sqrt(x)*int(asinh(c*x)/(sqrt(x)*x),x)*b - 2*a)/(sqrt(x)*sqrt(d)*d)
```

3.82 $\int \frac{a+b\mathbf{arcsinh}(cx)}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 248

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{(dx)^{5/2}}\,dx &= -\frac{4bc\sqrt{1+c^2x^2}}{3d^2\sqrt{dx}}+\frac{4bc^2\sqrt{dx}\sqrt{1+c^2x^2}}{3d^3(1+cx)} \\ &\quad -\frac{2(a+b\operatorname{arcsinh}(cx))}{3d(dx)^{3/2}}-\frac{4bc^{3/2}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\,E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)|\frac{1}{2}\right)}{3d^{5/2}\sqrt{1+c^2x^2}} \\ &\quad +\frac{2bc^{3/2}(1+cx)\sqrt{\frac{1+c^2x^2}{(1+cx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{3d^{5/2}\sqrt{1+c^2x^2}}\end{aligned}$$

output

```
-4/3*b*c*(c^2*x^2+1)^(1/2)/d^2/(d*x)^(1/2)+4/3*b*c^2*(d*x)^(1/2)*(c^2*x^2+
1)^(1/2)/d^3/(c*x+1)-2/3*(a+b*arcsinh(c*x))/d/(d*x)^(3/2)-4/3*b*c^(3/2)*(c
*x+1)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/2)*(d*x)^(
1/2)/d^(1/2))),1/2*2^(1/2))/d^(5/2)/(c^2*x^2+1)^(1/2)+2/3*b*c^(3/2)*(c*x+1
)*((c^2*x^2+1)/(c*x+1)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/2)*(d*x)^(1/
2)/d^(1/2)),1/2*2^(1/2))/d^(5/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(dx)^{5/2}}\,dx = -\frac{2x\left(a+b\operatorname{arcsinh}(cx)+2bcx\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-c^2x^2\right)\right)}{3(dx)^{5/2}}$$

input

```
Integrate[(a + b*ArcSinh[c*x])/(d*x)^(5/2),x]
```

output

```
(-2*x*(a + b*ArcSinh[c*x] + 2*b*c*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^
2*x^2)]))/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6191, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2bc\int \frac{1}{(dx)^{3/2}\sqrt{c^2x^2+1}}dx}{3d}-\frac{2(a+b\operatorname{arcsinh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2bc\left(\frac{c^2\int \frac{\sqrt{dx}}{\sqrt{c^2x^2+1}}dx}{d^2}-\frac{2\sqrt{c^2x^2+1}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\operatorname{arcsinh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bc\left(\frac{2c^2\int \frac{dx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{d^3}-\frac{2\sqrt{c^2x^2+1}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arcsinh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{d\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}-\frac{d\int \frac{d-cdx}{d\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{d^3}-\frac{2\sqrt{c^2x^2+1}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arcsinh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{d\int \frac{1}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}-\frac{\int \frac{d-cdx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{d^3}-\frac{2\sqrt{c^2x^2+1}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arcsinh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{2c^{3/2}\sqrt{c^2x^2+1}}-\frac{\int \frac{d-cdx}{\sqrt{c^2x^2+1}}d\sqrt{dx}}{c}\right)}{d^3}-\frac{2\sqrt{c^2x^2+1}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arcsinh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),\frac{1}{2}\right)}{2c^{3/2}\sqrt{c^2x^2+1}}-\frac{\frac{\sqrt{d}(cdx+d)\sqrt{\frac{c^2d^2x^2+d^2}{(cdx+d)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{c}\sqrt{c^2x^2+1}}-\frac{d^2\sqrt{c^2x^2+1}\sqrt{dx}}{cdx+d}}{c}\right)}{d^3}-\frac{2\sqrt{c}}{d}\right)}{3d}$$

$$\frac{2(a+b\text{arcsinh}(cx))}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcSinh[c*x]))/(3*d*(d*x)^(3/2)) + (2*b*c*((-2*Sqrt[1 + c^2*x^2
])/(d*Sqrt[d*x]) + (2*c^2*(-((-((d^2*Sqrt[d*x]*Sqrt[1 + c^2*x^2])/(d + c*d
*x)) + (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(d + c*d*x)^2]*Ellipt
icE[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(Sqrt[c]*Sqrt[1 + c^2*x^2
]))/c) + (Sqrt[d]*(d + c*d*x)*Sqrt[(d^2 + c^2*d^2*x^2)/(d + c*d*x)^2]*Elli
pticF[2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], 1/2])/(2*c^(3/2)*Sqrt[1 + c^2
*x^2])))/d^3))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}(xc)}{3(dx)^{\frac{3}{2}}}+\frac{2c\left(-\frac{\sqrt{c^{2}x^{2}+1}}{\sqrt{dx}}+\frac{ic\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)\right)}{d\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{3d}\right)}{d}$
default	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}(xc)}{3(dx)^{\frac{3}{2}}}+\frac{2c\left(-\frac{\sqrt{c^{2}x^{2}+1}}{\sqrt{dx}}+\frac{ic\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)\right)}{d\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{3d}\right)}{d}$
parts	$-\frac{2a}{3(dx)^{\frac{3}{2}}d}+\frac{2b\left(-\frac{\operatorname{arcsinh}(xc)}{3(dx)^{\frac{3}{2}}}+\frac{2c\left(-\frac{\sqrt{c^{2}x^{2}+1}}{\sqrt{dx}}+\frac{ic\sqrt{-ixc+1}\,\sqrt{ixc+1}\,\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{ic}{d}},i\right)\right)}{d\sqrt{\frac{ic}{d}}\,\sqrt{c^{2}x^{2}+1}}\right)}{3d}\right.}{d}$

input
```
int((a+b*arcsinh(x*c))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/3*a/(d*x)^(3/2)+b*(-1/3/(d*x)^(3/2)*arcsinh(x*c)+2/3/d*c*(-(c^2*x^
2+1)^(1/2)/(d*x)^(1/2)+I*c/d/(I/d*c)^(1/2)*(1-I*x*c)^(1/2)*(1+I*x*c)^(1/2)
/(c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(I/d*c)^(1/2),I)-EllipticE((d*x)
^(1/2)*(I/d*c)^(1/2),I)))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.36

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{5/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{c^2d}bcx^2\text{weierstrassZeta}\left(-\frac{4}{c^2},0,\text{weierstrassPInverse}\left(-\frac{4}{c^2},0,x\right)\right)+2\sqrt{c^2x^2+1}\sqrt{dx}bcx+\sqrt{dx}b\,\text{lo}}{3\,d^3x^2}$$

input
```
integrate((a+b*arcsinh(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(c^2*d)*b*c*x^2*weierstrassZeta(-4/c^2, 0, weierstrassPInverse
(-4/c^2, 0, x)) + 2*sqrt(c^2*x^2 + 1)*sqrt(d*x)*b*c*x + sqrt(d*x)*b*log(c*
x + sqrt(c^2*x^2 + 1)) + sqrt(d*x)*a)/(d^3*x^2)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{5/2}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d*x)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(dx)^{\frac{5}{2}}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

output

```
-1/6*(c^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sq
rt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) + sqrt(2)*sqrt(c))/sqrt(c) +
1))/(sqrt(c)*d^(5/2)) + I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2
)*sqrt(c))/sqrt(c) + 1) - log(-1/2*I*sqrt(2)*(2*c*sqrt(x) - sqrt(2)*sqrt(c
))/sqrt(c) + 1))/(sqrt(c)*d^(5/2)) - sqrt(2)*log(c*x + sqrt(2)*sqrt(c)*sqr
t(x) + 1)/(sqrt(c)*d^(5/2)) + sqrt(2)*log(c*x - sqrt(2)*sqrt(c)*sqrt(x) +
1)/(sqrt(c)*d^(5/2))) - 12*c*integrate(1/3/((c^3*d^(5/2)*x^3 + c*d^(5/2)*x
 + (c^2*d^(5/2)*x^2 + d^(5/2))*sqrt(c^2*x^2 + 1))*x^(3/2)), x) + 4*log(c*x
 + sqrt(c^2*x^2 + 1))/(d^(5/2)*x^(3/2)))*b - 2/3*a/((d*x)^(3/2)*d)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(dx)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(dx)^{\frac{5}{2}}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

output

```
integrate((b*arcsinh(c*x) + a)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(dx)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d\,x\right)}^{5/2}} \,d x$$

input
```
int((a + b*asinh(c*x))/(d*x)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(dx)^{5/2}}\,dx=\frac{3\sqrt{x}\,\left(\int \frac{asinh(cx)}{\sqrt{x}\,x^{2}}d x\right) b x-2 a}{3 \sqrt{x}\,\sqrt{d}\,d^{2} x}$$

input
```
int((a+b*asinh(c*x))/(d*x)^(5/2),x)
```

output
```
(3*sqrt(x)*int(asinh(c*x)/(sqrt(x)*x**2),x)*b*x - 2*a)/(3*sqrt(x)*sqrt(d)*
d**2*x)
```

3.83 $\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\begin{aligned}\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))^2}{5d}\\ &-\frac{8bc(dx)^{7/2}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},-c^2x^2\right)}{35d^2}\\ &+\frac{16b^2c^2(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-c^2x^2\right)}{315d^3}\end{aligned}$$

output
```
2/5*(d*x)^(5/2)*(a+b*arcsinh(c*x))^2/d-8/35*b*c*(d*x)^(7/2)*(a+b*arcsinh(c
*x))*hypergeom([1/2, 7/4],[11/4],-c^2*x^2)/d^2+16/315*b^2*c^2*(d*x)^(9/2)*
hypergeom([1, 9/4, 9/4],[11/4, 13/4],-c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{2}{315}x(dx)^{3/2}\bigg(9(a+b\text{arcsinh}(cx))\bigg(7(a+b\text{arcsinh}(cx))-4bcx\,\text{Hypergeometric2F1}\bigg($$

input

```
Integrate[(d*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output

```
(2*x*(d*x)^(3/2)*(9*(a + b*ArcSinh[c*x])*(7*(a + b*ArcSinh[c*x]) - 4*b*c*x
*Hypergeometric2F1[1/2, 7/4, 11/4, -(c^2*x^2)]) + 8*b^2*c^2*x^2*Hypergeome
tricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c^2*x^2)]))/315
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))^2}{5d}-\frac{4bc\int \frac{(dx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{5d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))^2}{5d}-\frac{4bc\left(\frac{2(dx)^{7/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{7d}-\frac{4bc(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-c^2x^2\right)}{63d^2}\right)}{5d}$$

input

```
Int[(d*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output

```
(2*(d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*d) - (4*b*c*((2*(d*x)^(7/2)*(a +
 b*ArcSinh[c*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c^2*x^2)])/(7*d) - (4
*b*c*(d*x)^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c^2*x^2)
])/(63*d^2)))/(5*d)
```

Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Maple [F]

$$\int (dx)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(xc))^2 \, dx$$

input
```
int((d*x)^(3/2)*(a+b*arcsinh(x*c))^2,x)
```

output
```
int((d*x)^(3/2)*(a+b*arcsinh(x*c))^2,x)
```

Fricas [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2 \, dx = \int (dx)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^2 \, dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*d*x*arcsinh(c*x)^2 + 2*a*b*d*x*arcsinh(c*x) + a^2*d*x)*sqrt(
d*x), x)
```

Sympy [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (dx)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

input

```
integrate((d*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output

```
Integral((d*x)**(3/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (dx)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2\,dx$$

input

```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output

```
2/5*b^2*d^(3/2)*x^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2/5*(d*x)^(5/2)*a
^2/d + integrate(2/5*(((5*a*b*d^(3/2) - 2*b^2*d^(3/2))*c^2*x^2 + 5*a*b*d^(
3/2))*sqrt(c^2*x^2 + 1)*x^(3/2) + ((5*a*b*d^(3/2) - 2*b^2*d^(3/2))*c^3*x^3
 + (5*a*b*d^(3/2) - 2*b^2*d^(3/2))*c*x)*x^(3/2))*log(c*x + sqrt(c^2*x^2 +
1))/(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (dx)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2\,dx$$

input

```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output

```
integrate((d*x)^(3/2)*(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,x\right)}^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d*x)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\frac{\sqrt{d}\,d\left(2\sqrt{x}\,a^2x^2+10\left(\int \sqrt{x}\,asinh(cx)\,xdx\right)ab+5\left(\int \sqrt{x}\,asinh(cx)^2\,xdx\right)b^2\right)}{5}$$

input
```
int((d*x)^(3/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(d)*d*(2*sqrt(x)*a**2*x**2 + 10*int(sqrt(x)*asinh(c*x)*x,x)*a*b + 5*i
nt(sqrt(x)*asinh(c*x)**2*x,x)*b**2))/5
```

3.84 $\int \sqrt{dx}(a+b\,\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\begin{aligned}\int \sqrt{dx}(a+b\,\text{arcsinh}(cx))^2\,dx &= \frac{2(dx)^{3/2}(a+b\,\text{arcsinh}(cx))^2}{3d} \\ &\quad - \frac{8bc(dx)^{5/2}(a+b\,\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-c^2x^2\right)}{15d^2} \\ &\quad + \frac{16b^2c^2(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-c^2x^2\right)}{105d^3}\end{aligned}$$

output

```
2/3*(d*x)^(3/2)*(a+b*arcsinh(c*x))^2/d-8/15*b*c*(d*x)^(5/2)*(a+b*arcsinh(c
*x))*hypergeom([1/2, 5/4],[9/4],-c^2*x^2)/d^2+16/105*b^2*c^2*(d*x)^(7/2)*h
ypergeom([1, 7/4, 7/4],[9/4, 11/4],-c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{2}{105}x\sqrt{dx}\left(7(a+b\operatorname{arcsinh}(cx))\left(5(a+b\operatorname{arcsinh}(cx)) - 4bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-c^2x^2\right)\right) + 8b^2c^2x^2\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-c^2x^2\right)\right)$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(2*x*Sqrt[d*x]*(7*(a + b*ArcSinh[c*x])*(5*(a + b*ArcSinh[c*x]) - 4*b*c*x*H
ypergeometric2F1[1/2, 5/4, 9/4, -(c^2*x^2)]) + 8*b^2*c^2*x^2*Hypergeometri
cPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c^2*x^2)]))/105
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3d} - \frac{4bc\int \frac{(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{2(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{3d} - \frac{4bc\left(\frac{2(dx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{5d} - \frac{4bc(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-c^2x^2\right)}{35d^2}\right)}{3d}$$

input

```
Int[Sqrt[d*x]*(a + b*ArcSinh[c*x])^2,x]
```

output

```
(2*(d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*d) - (4*b*c*((2*(d*x)^(5/2)*(a +
 b*ArcSinh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c^2*x^2)])/(5*d) - (4*
b*c*(d*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c^2*x^2)])
/(35*d^2)))/(3*d)
```

Defintions of rubi rules used

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Maple [F]

$$\int \sqrt{dx}\,(a+b\ \mathrm{arcsinh}\,(xc))^{2}\,dx$$

input
```
int((d*x)^(1/2)*(a+b*arcsinh(x*c))^2,x)
```

output
```
int((d*x)^(1/2)*(a+b*arcsinh(x*c))^2,x)
```

Fricas [F]

$$\int \sqrt{dx}(a+b\mathrm{arcsinh}(cx))^{2}\,dx=\int \sqrt{dx}\left(b\,\mathrm{arsinh}\,(cx)+a\right)^{2}\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(d*x), x)
```

Sympy [F]

$$\int \sqrt{dx}(a+b\mathrm{arcsinh}(cx))^{2}\,dx=\int \sqrt{dx}\left(a+b\,\mathrm{asinh}\,(cx)\right)^{2}\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(d*x)*(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{dx}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
2/3*b^2*sqrt(d)*x^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2/3*(d*x)^(3/2)*a
^2/d + integrate(2/3*(sqrt(c^2*x^2 + 1)*((3*a*b*c^2*sqrt(d) - 2*b^2*c^2*sq
rt(d))*x^2 + 3*a*b*sqrt(d))*sqrt(x) + ((3*a*b*c^3*sqrt(d) - 2*b^2*c^3*sqrt
(d))*x^3 + (3*a*b*c*sqrt(d) - 2*b^2*c*sqrt(d))*x)*sqrt(x))*log(c*x + sqrt(
c^2*x^2 + 1))/(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d*x)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{\sqrt{d}\,\left(2\sqrt{x}\,a^2x+6\left(\int\sqrt{x}\,asinh(cx)\,dx\right)ab+3\left(\int\sqrt{x}\,asinh(cx)^2\,dx\right)b^2\right)}{3}$$

input
```
int((d*x)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(d)*(2*sqrt(x)*a**2*x + 6*int(sqrt(x)*asinh(c*x),x)*a*b + 3*int(sqrt(
x)*asinh(c*x)**2,x)*b**2))/3
```

3.85 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^2}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{dx}}\,dx
&= \frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2}{d} \\
&\quad - \frac{8bc(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-c^2x^2\right)}{3d^2} \\
&\quad + \frac{16b^2c^2(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-c^2x^2\right)}{15d^3}
\end{aligned}
$$

output

```
2*(d*x)^(1/2)*(a+b*arcsinh(c*x))^2/d-8/3*b*c*(d*x)^(3/2)*(a+b*arcsinh(c*x)
)*hypergeom([1/2, 3/4],[7/4],-c^2*x^2)/d^2+16/15*b^2*c^2*(d*x)^(5/2)*hyper
geom([1, 5/4, 5/4],[7/4, 9/4],-c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{dx}}\,dx$$
$$= \frac{2x\left(5(a+b\operatorname{arcsinh}(cx))\left(3(a+b\operatorname{arcsinh}(cx))-4bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-c^2x^2\right)\right)+8b^2c^2x^2\,{}_3F_2}{15\sqrt{dx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d*x],x]
```

output
```
(2*x*(5*(a + b*ArcSinh[c*x])*(3*(a + b*ArcSinh[c*x]) - 4*b*c*x*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(c^2*x^2)]) + 8*b^2*c^2*x^2*HypergeometricPFQ[{1, 5
/4, 5/4}, {7/4, 9/4}, -(c^2*x^2)]))/(15*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2}{d}-\frac{4bc\int \frac{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{2\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2}{d}-\frac{4bc\left(\frac{2(dx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{3d}-\frac{4bc(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-c^2x^2\right)}{15d^2}\right)}{d}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*ArcSinh[c*x])^2)/d - (4*b*c*((2*(d*x)^(3/2)*(a + b*Arc
Sinh[c*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c^2*x^2)])/(3*d) - (4*b*c*(d
*x)^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, -(c^2*x^2)])/(15*d^
2)))/d
```

Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(xc))^2}{\sqrt{dx}}\,dx$$

input
```
int((a+b*arcsinh(x*c))^2/(d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{dx}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(1/2),x, algorithm="fricas")
```

output

```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(d*x)/(d*x),
x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{\sqrt{dx}}\,dx$$

input

```
integrate((a+b*asinh(c*x))**2/(d*x)**(1/2),x)
```

output

```
Integral((a + b*asinh(c*x))**2/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{dx}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(1/2),x, algorithm="maxima")
```

output

```
2*b^2*sqrt(x)*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(d) + 2*sqrt(d*x)*a^2/d +
 integrate(2*((a*b*c^3*sqrt(d) - 2*b^2*c^3*sqrt(d))*x^3 + (a*b*c*sqrt(d) -
 2*b^2*c*sqrt(d))*x + sqrt(c^2*x^2 + 1)*((a*b*c^2*sqrt(d) - 2*b^2*c^2*sqrt
(d))*x^2 + a*b*sqrt(d)))*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)*sqr
t(c^2*x^2 + 1)*sqrt(x) + (c^3*d*x^3 + c*d*x)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{dx}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(1/2),x, algorithm="giac")
```

output

```
integrate((b*arcsinh(c*x) + a)^2/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d\,x}}\,dx$$

input

```
int((a + b*asinh(c*x))^2/(d*x)^(1/2),x)
```

output

```
int((a + b*asinh(c*x))^2/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,a^2+2\left(\int \frac{\mathit{asinh}(cx)}{\sqrt{x}}dx\right)ab+\left(\int \frac{\mathit{asinh}(cx)^2}{\sqrt{x}}dx\right)b^2}{\sqrt{d}}$$

input

```
int((a+b*asinh(c*x))^2/(d*x)^(1/2),x)
```

output

```
(2*sqrt(x)*a**2 + 2*int(asinh(c*x)/sqrt(x),x)*a*b + int(asinh(c*x)**2/sqrt
(x),x)*b**2)/sqrt(d)
```

3.86 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx = & -\frac{2(a+b\text{arcsinh}(cx))^2}{d\sqrt{dx}} \\ & + \frac{8bc\sqrt{dx}(a+b\text{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^2x^2\right)}{d^2} \\ & - \frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-c^2x^2\right)}{3d^3}\end{aligned}$$

output
```
-2*(a+b*arcsinh(c*x))^2/d/(d*x)^(1/2)+8*b*c*(d*x)^(1/2)*(a+b*arcsinh(c*x))
*hypergeom([1/4, 1/2],[5/4],-c^2*x^2)/d^2-16/3*b^2*c^2*(d*x)^(3/2)*hyperge
om([3/4, 3/4, 1],[5/4, 7/4],-c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx =$$

$$-\frac{2x\big(3(a+b\text{arcsinh}(cx))\,\big(a+b\text{arcsinh}(cx)-4bcx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^2x^2\right)\big)+8b^2c^2x^2\,{}_3F_2\big(\frac{3}{4},}{3(dx)^{3/2}}$$

input

```
Integrate[(a + b*ArcSinh[c*x])^2/(d*x)^(3/2),x]
```

output

```
(-2*x*(3*(a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 4*b*c*x*Hypergeometric
2F1[1/4, 1/2, 5/4, -(c^2*x^2)]) + 8*b^2*c^2*x^2*HypergeometricPFQ[{3/4, 3/
4, 1}, {5/4, 7/4}, -(c^2*x^2)]))/(3*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{4bc\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{dx}\sqrt{c^2x^2+1}}dx}{d}-\frac{2(a+b\text{arcsinh}(cx))^2}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 6232}$$

$$\frac{4bc\left(\frac{2\sqrt{dx}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^2x^2\right)(a+b\text{arcsinh}(cx))}{d}-\frac{4bc(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-c^2x^2\right)}{3d^2}\right)}{d}-\frac{2(a+b\text{arcsinh}(cx))^2}{d\sqrt{dx}}$$

input

```
Int[(a + b*ArcSinh[c*x])^2/(d*x)^(3/2),x]
```

output

```
(-2*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d*x]) + (4*b*c*((2*Sqrt[d*x]*(a + b*Ar
cSinh[c*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c^2*x^2)])/d - (4*b*c*(d*x)
^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, -(c^2*x^2)])/(3*d^2)))
/d
```

Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(a+b\,\operatorname{arcsinh}\left(xc\right)\right)^{2}}{\left(dx\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^2/(d*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{\left(b\,\operatorname{arsinh}\left(cx\right)+a\right)^{2}}{\left(dx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(d*x)/(d^2*x^
2), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(d^(3/2)*sqrt(x)) - 2*a^2/(sqrt(d*x)
*d) + integrate(2*((a*b*c^3*sqrt(d) + 2*b^2*c^3*sqrt(d))*x^3 + (a*b*c*sqrt
(d) + 2*b^2*c*sqrt(d))*x + sqrt(c^2*x^2 + 1)*((a*b*c^2*sqrt(d) + 2*b^2*c^2
*sqrt(d))*x^2 + a*b*sqrt(d)))*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d^2*x^2 +
 d^2)*sqrt(c^2*x^2 + 1)*x^(3/2) + (c^3*d^2*x^3 + c*d^2*x)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2}{\left(d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d*x)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{3/2}}\,dx=\frac{2\sqrt{x}\left(\int \frac{asinh(cx)}{\sqrt{x}\,x}dx\right)ab+\sqrt{x}\left(\int \frac{asinh(cx)^2}{\sqrt{x}\,x}dx\right)b^2-2a^2}{\sqrt{x}\,\sqrt{d}\,d}$$

input
```
int((a+b*asinh(c*x))^2/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*int(asinh(c*x)/(sqrt(x)*x),x)*a*b + sqrt(x)*int(asinh(c*x)**2/(
sqrt(x)*x),x)*b**2 - 2*a**2)/(sqrt(x)*sqrt(d)*d)
```

3.87 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx &= -\frac{2(a+b\operatorname{arcsinh}(cx))^2}{3d(dx)^{3/2}}\\ &\quad-\frac{8bc(a+b\operatorname{arcsinh}(cx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-c^2x^2\right)}{3d^2\sqrt{dx}}\\ &\quad+\frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-c^2x^2\right)}{3d^3}\end{aligned}$$

output
```
-2/3*(a+b*arcsinh(c*x))^2/d/(d*x)^(3/2)-8/3*b*c*(a+b*arcsinh(c*x))*hyperge
om([-1/4, 1/2],[3/4],-c^2*x^2)/d^2/(d*x)^(1/2)+16/3*b^2*c^2*(d*x)^(1/2)*hy
pergeom([1/4, 1/4, 1],[3/4, 5/4],-c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx = \frac{x(-2(a+b\operatorname{arcsinh}(cx))\,(a+b\operatorname{arcsinh}(cx)+4bcx\,\text{Hypergeometric2F1}\,(-\frac{1}{4},\frac{1}{2}}{3(dx)^{5/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d*x)^(5/2),x]
```

output
```
(x*(-2*(a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] + 4*b*c*x*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(c^2*x^2)]) + 16*b^2*c^2*x^2*HypergeometricPFQ[{1/4, 1/
4, 1}, {3/4, 5/4}, -(c^2*x^2)]))/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{5/2}}dx$$

$$\Big\downarrow \; 6191$$

$$\frac{4bc\int \frac{a+b\operatorname{arcsinh}(cx)}{(dx)^{3/2}\sqrt{c^2x^2+1}}dx}{3d}-\frac{2(a+b\operatorname{arcsinh}(cx))^2}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 6232$$

$$\frac{4bc\left(\frac{4bc\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-c^2x^2\right)}{d^2}-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-c^2x^2\right)(a+b\operatorname{arcsinh}(cx))}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\operatorname{arcsinh}(cx))^2}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcSinh[c*x])^2)/(3*d*(d*x)^(3/2)) + (4*b*c*((-2*(a + b*ArcSinh
[c*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^2*x^2)])/(d*Sqrt[d*x]) + (4*b
*c*Sqrt[d*x]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c^2*x^2)])/d^2
))/(3*d)
```

Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(xc))^2}{(dx)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^2/(d*x)^(5/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(d*x)/(d^3*x^
3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(a+b\operatorname{asinh}{(cx)})^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d*x)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(d^(5/2)*x^(3/2)) - 2/3*a^2/((d*x)
^(3/2)*d) + integrate(2/3*((3*a*b*c^3*sqrt(d) + 2*b^2*c^3*sqrt(d))*x^3 + (
3*a*b*c*sqrt(d) + 2*b^2*c*sqrt(d))*x + sqrt(c^2*x^2 + 1)*((3*a*b*c^2*sqrt(
d) + 2*b^2*c^2*sqrt(d))*x^2 + 3*a*b*sqrt(d)))*log(c*x + sqrt(c^2*x^2 + 1))
/((c^2*d^3*x^2 + d^3)*sqrt(c^2*x^2 + 1)*x^(5/2) + (c^3*d^3*x^3 + c*d^3*x)*
x^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,x\right)}^{5/2}} \,d x$$

input
```
int((a + b*asinh(c*x))^2/(d*x)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(dx)^{5/2}}\,dx=\frac{6\sqrt{x}\,\left(\int \frac{asinh(cx)}{\sqrt{x}\,x^{2}}d x \right) a b x +3\sqrt{x}\,\left(\int \frac{asinh(cx)^{2}}{\sqrt{x}\,x^{2}}d x \right) b^{2} x -2 a^{2}}{3 \sqrt{x}\,\sqrt{d}\,d^{2} x}$$

input
```
int((a+b*asinh(c*x))^2/(d*x)^(5/2),x)
```

output
```
(6*sqrt(x)*int(asinh(c*x)/(sqrt(x)*x**2),x)*a*b*x + 3*sqrt(x)*int(asinh(c*
x)**2/(sqrt(x)*x**2),x)*b**2*x - 2*a**2)/(3*sqrt(x)*sqrt(d)*d**2*x)
```

3.88 $\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\text{Int}\left((dx)^{3/2}(a+b\text{arcsinh}(cx))^3,x\right)$$

output

```
Defer(Int)((d*x)^(3/2)*(a+b*arcsinh(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 41.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx$$

input

```
Integrate[(d*x)^(3/2)*(a + b*ArcSinh[c*x])^3,x]
```

output

```
Integrate[(d*x)^(3/2)*(a + b*ArcSinh[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))^3}{5d}-\frac{6bc\int \frac{(dx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{2(dx)^{5/2}(a+b\text{arcsinh}(cx))^3}{5d}-\frac{6bc\int \frac{(dx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{5d}$$

input
```
Int[(d*x)^(3/2)*(a + b*ArcSinh[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (dx)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(xc))^3\,dx$$

input
```
int((d*x)^(3/2)*(a+b*arcsinh(x*c))^3,x)
```

output
```
int((d*x)^(3/2)*(a+b*arcsinh(x*c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\,(b\,\text{arsinh}\,(cx)+a)^3\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*d*x*arcsinh(c*x)^3 + 3*a*b^2*d*x*arcsinh(c*x)^2 + 3*a^2*b*d*
x*arcsinh(c*x) + a^3*d*x)*sqrt(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 85.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^3\;dx$$

input
```
integrate((d*x)**(3/2)*(a+b*asinh(c*x))**3,x)
```

output
```
Integral((d*x)**(3/2)*(a + b*asinh(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 279, normalized size of antiderivative = 15.50

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\,(b\,\text{arsinh}\,(cx)+a)^3\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x))^3,x, algorithm="maxima")
```

output

```
2/5*b^3*d^(3/2)*x^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))^3 + 2/5*(d*x)^(5/2)*a
^3/d + integrate(3/5*((((5*a*b^2*d^(3/2) - 2*b^3*d^(3/2))*c^2*x^2 + 5*a*b^
2*d^(3/2))*sqrt(c^2*x^2 + 1)*x^(3/2) + ((5*a*b^2*d^(3/2) - 2*b^3*d^(3/2))*
c^3*x^3 + (5*a*b^2*d^(3/2) - 2*b^3*d^(3/2))*c*x)*x^(3/2))*log(c*x + sqrt(c
^2*x^2 + 1))^2 + 5*((a^2*b*c^2*d^(3/2)*x^2 + a^2*b*d^(3/2))*sqrt(c^2*x^2 +
 1)*x^(3/2) + (a^2*b*c^3*d^(3/2)*x^3 + a^2*b*c*d^(3/2)*x)*x^(3/2))*log(c*x
 + sqrt(c^2*x^2 + 1)))/(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^{3}\,dx$$

input

```
integrate((d*x)^(3/2)*(a+b*arcsinh(c*x))^3,x, algorithm="giac")
```

output

```
integrate((d*x)^(3/2)*(b*arcsinh(c*x) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2}(a+b\text{arcsinh}(cx))^3\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^3\,{\left(d\,x\right)}^{3/2}\,dx$$

input

```
int((a + b*asinh(c*x))^3*(d*x)^(3/2),x)
```

output

```
int((a + b*asinh(c*x))^3*(d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int (dx)^{3/2}(a+b\operatorname{arcsinh}(cx))^3\,dx = \frac{\sqrt{d}\,d\big(2\sqrt{x}\,a^3x^2+15\big(\int\sqrt{x}\,asinh(cx)\,xdx\big)\,a^2b+5\big(\int\sqrt{x}\,asinh(cx)^3\,xdx\big)\,b^3+15\big(\int}{5}$$

input

```
int((d*x)^(3/2)*(a+b*asinh(c*x))^3,x)
```

output

```
(sqrt(d)*d*(2*sqrt(x)*a**3*x**2 + 15*int(sqrt(x)*asinh(c*x)*x,x)*a**2*b +
5*int(sqrt(x)*asinh(c*x)**3*x,x)*b**3 + 15*int(sqrt(x)*asinh(c*x)**2*x,x)*
a*b**2))/5
```

3.89 $\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^3\,dx=\operatorname{Int}\left(\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^3,x\right)$$

output

```
Defer(Int)((d*x)^(1/2)*(a+b*arcsinh(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 91.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^3\,dx=\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^3\,dx$$

input

```
Integrate[Sqrt[d*x]*(a + b*ArcSinh[c*x])^3,x]
```

output

```
Integrate[Sqrt[d*x]*(a + b*ArcSinh[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^3\,dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{2(dx)^{3/2}(a+b\text{arcsinh}(cx))^3}{3d}-\frac{2bc\int \frac{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{d}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{2(dx)^{3/2}(a+b\text{arcsinh}(cx))^3}{3d}-\frac{2bc\int \frac{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{d}$$

input
```
Int[Sqrt[d*x]*(a + b*ArcSinh[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \sqrt{dx}\,(a+b\,\text{arcsinh}\,(xc))^3\,dx$$

input
```
int((d*x)^(1/2)*(a+b*arcsinh(x*c))^3,x)
```

output
```
int((d*x)^(1/2)*(a+b*arcsinh(x*c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^3\,dx=\int \sqrt{dx}(b\,\text{arsinh}\,(cx)+a)^3\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(c*x)^3 + 3*a*b^2*arcsinh(c*x)^2 + 3*a^2*b*arcsinh(c*
x) + a^3)*sqrt(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 6.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^3\,dx=\int \sqrt{dx}\left(a+b\,\text{asinh}\,(cx)\right)^3\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*asinh(c*x))**3,x)
```

output
```
Integral(sqrt(d*x)*(a + b*asinh(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 285, normalized size of antiderivative = 15.83

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^3\,dx=\int \sqrt{dx}(b\,\text{arsinh}\,(cx)+a)^3\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x))^3,x, algorithm="maxima")
```

output
```
2/3*b^3*sqrt(d)*x^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))^3 + 2/3*(d*x)^(3/2)*a
^3/d + integrate(((sqrt(c^2*x^2 + 1)*(3*a*b^2*sqrt(d) + (3*a*b^2*c^2*sqrt(
d) - 2*b^3*c^2*sqrt(d))*x^2)*sqrt(x) + ((3*a*b^2*c^3*sqrt(d) - 2*b^3*c^3*s
qrt(d))*x^3 + (3*a*b^2*c*sqrt(d) - 2*b^3*c*sqrt(d))*x)*sqrt(x))*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 3*((a^2*b*c^2*sqrt(d)*x^2 + a^2*b*sqrt(d))*sqrt(c^2
*x^2 + 1)*sqrt(x) + (a^2*b*c^3*sqrt(d)*x^3 + a^2*b*c*sqrt(d)*x)*sqrt(x))*l
og(c*x + sqrt(c^2*x^2 + 1)))/(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^3\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(1/2)*(a+b*arcsinh(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}(a+b\text{arcsinh}(cx))^3\,dx = \int (a+b\,\text{asinh}(c\,x))^3\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*asinh(c*x))^3*(d*x)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^3*(d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int \sqrt{dx}(a+b\operatorname{arcsinh}(cx))^3\,dx$$
$$=\frac{\sqrt{d}\,\left(2\sqrt{x}\,a^3x+9\left(\int\sqrt{x}\,asinh(cx)\,dx\right)a^2b+3\left(\int\sqrt{x}\,asinh(cx)^3\,dx\right)b^3+9\left(\int\sqrt{x}\,asinh(cx)^2\,dx\right)a\,b^2\right)}{3}$$

input
```
int((d*x)^(1/2)*(a+b*asinh(c*x))^3,x)
```

output
```
(sqrt(d)*(2*sqrt(x)*a**3*x + 9*int(sqrt(x)*asinh(c*x),x)*a**2*b + 3*int(sq
rt(x)*asinh(c*x)**3,x)*b**3 + 9*int(sqrt(x)*asinh(c*x)**2,x)*a*b**2))/3
```

3.90 $\int \frac{(a+b\operatorname{arcsinh}(cx))^3}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^3}{\sqrt{dx}}\,dx = \operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(cx))^3}{\sqrt{dx}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^3/(d*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 87.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(cx))^3}{\sqrt{dx}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^3/Sqrt[d*x],x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^3/Sqrt[d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{2\sqrt{dx}(a+b\text{arcsinh}(cx))^3}{d}-\frac{6bc\int \frac{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{d}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{2\sqrt{dx}(a+b\text{arcsinh}(cx))^3}{d}-\frac{6bc\int \frac{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{d}$$

input
```
Int[(a + b*ArcSinh[c*x])^3/Sqrt[d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^3}{\sqrt{dx}}dx$$

input
```
int((a+b*arcsinh(x*c))^3/(d*x)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^3/(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(c*x)^3 + 3*a*b^2*arcsinh(c*x)^2 + 3*a^2*b*arcsinh(c*
x) + a^3)*sqrt(d*x)/(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^3}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**3/(d*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**3/sqrt(d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 261, normalized size of antiderivative = 14.50

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{\sqrt{dx}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^3/(d*x)^(1/2),x, algorithm="maxima")
```

output

```
2*b^3*sqrt(x)*log(c*x + sqrt(c^2*x^2 + 1))^3/sqrt(d) + 2*sqrt(d*x)*a^3/d +
 integrate(3*(((a*b^2*c^3 - 2*b^3*c^3)*x^3 + (a*b^2*c - 2*b^3*c)*x + sqrt(
c^2*x^2 + 1)*(a*b^2 + (a*b^2*c^2 - 2*b^3*c^2)*x^2))*log(c*x + sqrt(c^2*x^2
 + 1))^2 + (a^2*b*c^3*x^3 + a^2*b*c*x + (a^2*b*c^2*x^2 + a^2*b)*sqrt(c^2*x
^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)))/((c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c
^2*x^2 + 1)*sqrt(x) + (c^3*sqrt(d)*x^3 + c*sqrt(d)*x)*sqrt(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{\sqrt{dx}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^3/(d*x)^(1/2),x, algorithm="giac")
```

output

```
integrate((b*arcsinh(c*x) + a)^3/sqrt(d*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^3}{\sqrt{d\,x}}\,dx$$

input

```
int((a + b*asinh(c*x))^3/(d*x)^(1/2),x)
```

output

```
int((a + b*asinh(c*x))^3/(d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{dx}}\,dx$$
$$= \frac{2\sqrt{x}\,a^3+3\left(\int \frac{asinh(cx)}{\sqrt{x}}dx\right)a^2b+\left(\int \frac{asinh(cx)^3}{\sqrt{x}}dx\right)b^3+3\left(\int \frac{asinh(cx)^2}{\sqrt{x}}dx\right)a\,b^2}{\sqrt{d}}$$

input

```
int((a+b*asinh(c*x))^3/(d*x)^(1/2),x)
```

output

```
(2*sqrt(x)*a**3 + 3*int(asinh(c*x)/sqrt(x),x)*a**2*b + int(asinh(c*x)**3/s
qrt(x),x)*b**3 + 3*int(asinh(c*x)**2/sqrt(x),x)*a*b**2)/sqrt(d)
```

3.91 $\int \frac{(a+b\textbf{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^3/(d*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 46.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^3/(d*x)^(3/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^3/(d*x)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{6bc\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}\sqrt{c^2x^2+1}}dx}{d}-\frac{2(a+b\text{arcsinh}(cx))^3}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{6bc\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{dx}\sqrt{c^2x^2+1}}dx}{d}-\frac{2(a+b\text{arcsinh}(cx))^3}{d\sqrt{dx}}$$

input

```
Int[(a + b*ArcSinh[c*x])^3/(d*x)^(3/2),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(xc))^3}{(dx)^{\frac{3}{2}}}dx$$

input

```
int((a+b*arcsinh(x*c))^3/(d*x)^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^3/(d*x)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(c*x)^3 + 3*a*b^2*arcsinh(c*x)^2 + 3*a^2*b*arcsinh(c*
x) + a^3)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))^3}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**3/(d*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**3/(d*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 261, normalized size of antiderivative = 14.50

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^3/(d*x)^(3/2),x, algorithm="maxima")
```

output

```
-2*b^3*log(c*x + sqrt(c^2*x^2 + 1))^3/(d^(3/2)*sqrt(x)) - 2*a^3/(sqrt(d*x)
*d) + integrate(3*(((a*b^2*c^3 + 2*b^3*c^3)*x^3 + (a*b^2*c + 2*b^3*c)*x +
sqrt(c^2*x^2 + 1)*(a*b^2 + (a*b^2*c^2 + 2*b^3*c^2)*x^2))*log(c*x + sqrt(c^
2*x^2 + 1))^2 + (a^2*b*c^3*x^3 + a^2*b*c*x + (a^2*b*c^2*x^2 + a^2*b)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)))/((c^2*d^(3/2)*x^2 + d^(3/2))*s
qrt(c^2*x^2 + 1)*x^(3/2) + (c^3*d^(3/2)*x^3 + c*d^(3/2)*x)*x^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*arcsinh(c*x))^3/(d*x)^(3/2),x, algorithm="giac")
```

output

```
integrate((b*arcsinh(c*x) + a)^3/(d*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^3}{{\left(d\,x\right)}^{3/2}}\,dx$$

input

```
int((a + b*asinh(c*x))^3/(d*x)^(3/2),x)
```

output

```
int((a + b*asinh(c*x))^3/(d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.78

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(dx)^{3/2}}\,dx = \frac{3\sqrt{x}\left(\int \frac{asinh(cx)}{\sqrt{x}\,x}dx\right)a^2b+\sqrt{x}\left(\int \frac{asinh(cx)^3}{\sqrt{x}\,x}dx\right)b^3+3\sqrt{x}\left(\int \frac{asinh(cx)^2}{\sqrt{x}\,x}dx\right)a\,b^2 -}{\sqrt{x}\,\sqrt{d}\,d}$$

input

```
int((a+b*asinh(c*x))^3/(d*x)^(3/2),x)
```

output

```
(3*sqrt(x)*int(asinh(c*x)/(sqrt(x)*x),x)*a**2*b + sqrt(x)*int(asinh(c*x)**
3/(sqrt(x)*x),x)*b**3 + 3*sqrt(x)*int(asinh(c*x)**2/(sqrt(x)*x),x)*a*b**2
- 2*a**3)/(sqrt(x)*sqrt(d)*d)
```

3.92 $\int \frac{(dx)^{3/2}}{a+b\,\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^{3/2}}{a+b\,\mathrm{arcsinh}(cx)}\,dx=\mathrm{Int}\left(\frac{(dx)^{3/2}}{a+b\,\mathrm{arcsinh}(cx)},x\right)$$

output

```
Defer(Int)((d*x)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\,\mathrm{arcsinh}(cx)}\,dx=\int \frac{(dx)^{3/2}}{a+b\,\mathrm{arcsinh}(cx)}\,dx$$

input

```
Integrate[(d*x)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

output

```
Integrate[(d*x)^(3/2)/(a + b*ArcSinh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)} \, dx$$
$$\Big\downarrow \text{ 6196}$$
$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[(d*x)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^{\frac{3}{2}}}{a+b\,\text{arcsinh}\left(xc\right)}dx$$

input
```
int((d*x)^(3/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int((d*x)^(3/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{a+b\operatorname{asinh}\left(cx\right)}\,dx$$

input
```
integrate((d*x)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d*x)**(3/2)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d\,x)^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d*x)^(3/2)/(a + b*asinh(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,x}{asinh\left(cx\right)b+a}dx\right)d$$

input

```
int((d*x)^(3/2)/(a+b*asinh(c*x)),x)
```

output

```
sqrt(d)*int((sqrt(x)*x)/(asinh(c*x)*b + a),x)*d
```

3.93 $\int \frac{\sqrt{dx}}{a+b\,\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt{dx}}{a+b\operatorname{arcsinh}(cx)}\,dx=\operatorname{Int}\left(\frac{\sqrt{dx}}{a+b\operatorname{arcsinh}(cx)},x\right)$$

output

```
Defer(Int)((d*x)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{dx}}{a+b\operatorname{arcsinh}(cx)}\,dx=\int \frac{\sqrt{dx}}{a+b\operatorname{arcsinh}(cx)}\,dx$$

input

```
Integrate[Sqrt[d*x]/(a + b*ArcSinh[c*x]),x]
```

output

```
Integrate[Sqrt[d*x]/(a + b*ArcSinh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)} \, dx$$

$$\Big\downarrow \ 6196$$

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[Sqrt[d*x]/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{dx}}{a+b \,\text{arcsinh}\left(xc\right)}dx$$

input
```
int((d*x)^(1/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int((d*x)^(1/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{dx}}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{dx}}{a+b\,\text{asinh}\left(cx\right)}\,dx$$

input
```
integrate((d*x)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*x)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{dx}}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/(b*arcsinh(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{dx}}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{d\,x}}{a+b\,\text{asinh}\left(c\,x\right)}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d*x)^(1/2)/(a + b*asinh(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\mathrm{arcsinh}(cx)}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}}{asinh\left(cx\right)b+a}dx\right)$$

input
```
int((d*x)^(1/2)/(a+b*asinh(c*x)),x)
```

output
```
sqrt(d)*int(sqrt(x)/(asinh(c*x)*b + a),x)
```

3.94 $\int \frac{1}{\sqrt{dx}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}\,dx=\operatorname{Int}\left(\frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(d*x)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))} \, dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/(Sqrt[d*x]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{dx}\,\left(a+b\,\operatorname{arcsinh}\left(xc\right)\right)}dx$$

input
```
int(1/(d*x)^(1/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(d*x)^(1/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{dx}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b*d*x*arcsinh(c*x) + a*d*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{dx}\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{dx}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{dx}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{d\,x}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d*x)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{dx}(a + b\text{arcsinh}(cx))}\,dx = \frac{\int \frac{1}{\sqrt{x}\,asinh(cx)b+\sqrt{x}\,a}dx}{\sqrt{d}}$$

input

```
int(1/(d*x)^(1/2)/(a+b*asinh(c*x)),x)
```

output

```
int(1/(sqrt(x)*asinh(c*x)*b + sqrt(x)*a),x)/sqrt(d)
```

3.95 $\int \frac{1}{(dx)^{3/2}(a+b\,\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(dx)^{3/2}(a+b\,\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{(dx)^{3/2}(a+b\,\text{arcsinh}(cx))},x\right)$$

output

```
Defer(Int)(1/(d*x)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(dx)^{3/2}(a+b\,\text{arcsinh}(cx))}\,dx=\int \frac{1}{(dx)^{3/2}(a+b\,\text{arcsinh}(cx))}\,dx$$

input

```
Integrate[1/((d*x)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output

```
Integrate[1/((d*x)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6196}$$
$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d*x)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\,\text{arcsinh}\left(xc\right)\right)}dx$$

input
```
int(1/(d*x)^(3/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(d*x)^(3/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)}\,dx$$

input

```
integrate(1/(d*x)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output

```
integral(sqrt(d*x)/(b*d^2*x^2*arcsinh(c*x) + a*d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

input

```
integrate(1/(d*x)**(3/2)/(a+b*asinh(c*x)),x)
```

output

```
Integral(1/((d*x)**(3/2)*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)}\,dx$$

input

```
integrate(1/(d*x)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,x\right)}^{3/2}} \,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d*x)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\int \frac{1}{\sqrt{x}\,asinh(cx)bx+\sqrt{x}\,ax}dx}{\sqrt{d}\,d}$$

input
```
int(1/(d*x)^(3/2)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(sqrt(x)*asinh(c*x)*b*x + sqrt(x)*a*x),x)/(sqrt(d)*d)
```

3.96 $\int \frac{(dx)^{3/2}}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output

```
Defer(Int)((d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input

```
Integrate[(d*x)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

output

```
Integrate[(d*x)^(3/2)/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \; 6196$$

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(d*x)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+b\,\text{arcsinh}\left(xc\right)\right)^{2}}dx$$

input
```
int((d*x)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int((d*x)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((d*x)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d*x)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 427, normalized size of antiderivative = 23.72

$$\int \frac{(dx)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output

```
-((c^2*d^(3/2)*x^2 + d^(3/2))*sqrt(c^2*x^2 + 1)*x^(3/2) + (c^3*d^(3/2)*x^3
 + c*d^(3/2)*x)*x^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*
c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2
*x^2 + 1))) + integrate(1/2*((5*c^3*d^(3/2)*x^3 + c*d^(3/2)*x)*(c^2*x^2 +
1)*x^(3/2) + (10*c^4*d^(3/2)*x^4 + 11*c^2*d^(3/2)*x^2 + 3*d^(3/2))*sqrt(c^
2*x^2 + 1)*x^(3/2) + 5*(c^5*d^(3/2)*x^5 + 2*c^3*d^(3/2)*x^3 + c*d^(3/2)*x)
*x^(3/2))/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c
*x + (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x +
2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input

```
integrate((d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output

```
integrate((d*x)^(3/2)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\int \frac{(d\,x)^{3/2}}{(a+b\operatorname{asinh}(c\,x))^2}\,dx$$

input

```
int((d*x)^(3/2)/(a + b*asinh(c*x))^2,x)
```

output

```
int((d*x)^(3/2)/(a + b*asinh(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(dx)^{3/2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,x}{asinh\left(cx\right)^{2}b^{2}+2asinh\left(cx\right)ab+a^{2}}dx\right)d$$

input
```
int((d*x)^(3/2)/(a+b*asinh(c*x))^2,x)
```

output
```
sqrt(d)*int((sqrt(x)*x)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*
d
```

3.97 $\int \frac{\sqrt{dx}}{(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)((d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[d*x]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[Sqrt[d*x]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx \\ \Big\downarrow \text{ 6196} \\ \displaystyle\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}dx \end{array}$$

input
```
Int[Sqrt[d*x]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{dx}}{\left(a+b\ \text{arcsinh}\left(xc\right)\right)^{2}}dx$$

input
```
int((d*x)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int((d*x)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{dx}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{dx}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input

```
integrate((d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output

```
integral(sqrt(d*x)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{dx}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{dx}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

input

```
integrate((d*x)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output

```
Integral(sqrt(d*x)/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 426, normalized size of antiderivative = 23.67

$$\int \frac{\sqrt{dx}}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{dx}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input

```
integrate((d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output

```
-((c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1)*sqrt(x) + (c^3*sqrt(d)*x^3
 + c*sqrt(d)*x)*sqrt(x))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*
c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2
*x^2 + 1))) + integrate(1/2*((3*c^3*sqrt(d)*x^3 - c*sqrt(d)*x)*(c^2*x^2 +
1)*sqrt(x) + (6*c^4*sqrt(d)*x^4 + 5*c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^
2 + 1)*sqrt(x) + 3*(c^5*sqrt(d)*x^5 + 2*c^3*sqrt(d)*x^3 + c*sqrt(d)*x)*sqr
t(x))/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x +
 (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b
^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
+ 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{dx}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input

```
integrate((d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output

```
integrate(sqrt(d*x)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{d\,x}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input

```
int((d*x)^(1/2)/(a + b*asinh(c*x))^2,x)
```

output

```
int((d*x)^(1/2)/(a + b*asinh(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{dx}}{(a+b\text{arcsinh}(cx))^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}}{asinh\left(cx\right)^{2}b^{2}+2asinh\left(cx\right)ab+a^{2}}dx\right)$$

input
```
int((d*x)^(1/2)/(a+b*asinh(c*x))^2,x)
```

output
```
sqrt(d)*int(sqrt(x)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)
```

3.98 $\int \frac{1}{\sqrt{dx}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}dx$$

input

```
Int[1/(Sqrt[d*x]*(a + b*ArcSinh[c*x])^2),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{dx}\,\left(a+b\,\text{arcsinh}\left(xc\right)\right)^{2}}dx$$

input

```
int(1/(d*x)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

output

```
int(1/(d*x)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b^2*d*x*arcsinh(c*x)^2 + 2*a*b*d*x*arcsinh(c*x) + a^2*
d*x), x)
```

Sympy [N/A]

Not integrable

Time = 4.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}\left(a+b\,\text{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 446, normalized size of antiderivative = 24.78

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*a*b*c^2*sqrt(d)*
x^(3/2) + (sqrt(c^2*x^2 + 1)*b^2*c^2*sqrt(d)*x^(3/2) + (b^2*c^3*sqrt(d)*x^
2 + b^2*c*sqrt(d))*sqrt(x))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*sqrt(d
)*x^2 + a*b*c*sqrt(d))*sqrt(x)) + integrate(1/2*(c^5*sqrt(d)*x^5 + 2*c^3*s
qrt(d)*x^3 + c*sqrt(d)*x + (c^3*sqrt(d)*x^3 - 3*c*sqrt(d)*x)*(c^2*x^2 + 1)
 + (2*c^4*sqrt(d)*x^4 - c^2*sqrt(d)*x^2 - sqrt(d))*sqrt(c^2*x^2 + 1))/((c^
2*x^2 + 1)*a*b*c^3*d*x^(7/2) + 2*(a*b*c^4*d*x^4 + a*b*c^2*d*x^2)*sqrt(c^2*
x^2 + 1)*sqrt(x) + ((c^2*x^2 + 1)*b^2*c^3*d*x^(7/2) + 2*(b^2*c^4*d*x^4 + b
^2*c^2*d*x^2)*sqrt(c^2*x^2 + 1)*sqrt(x) + (b^2*c^5*d*x^5 + 2*b^2*c^3*d*x^3
 + b^2*c*d*x)*sqrt(x))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*d*x^5 + 2*a
*b*c^3*d*x^3 + a*b*c*d*x)*sqrt(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\sqrt{dx}{\left(b\,\text{arsinh}\left(cx\right)+a\right)}^2}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,x}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d*x)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{dx}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{\sqrt{x}\,asinh(cx)^2b^2+2\sqrt{x}\,asinh(cx)ab+\sqrt{x}\,a^2}dx}{\sqrt{d}}$$

input
```
int(1/(d*x)^(1/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(sqrt(x)*asinh(c*x)**2*b**2 + 2*sqrt(x)*asinh(c*x)*a*b + sqrt(x)*a**
2),x)/sqrt(d)
```

3.99 $\int \frac{1}{(dx)^{3/2}(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output

```
Defer(Int)(1/(d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input

```
Integrate[1/((d*x)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output

```
Integrate[1/((d*x)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \text{6196}$$
$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d*x)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(dx)^{\frac{3}{2}} \left(a+b \,\text{arcsinh}\left(xc\right)\right)^{2}}dx$$

input
```
int(1/(d*x)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(d*x)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b^2*d^2*x^2*arcsinh(c*x)^2 + 2*a*b*d^2*x^2*arcsinh(c*x
) + a^2*d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 472, normalized size of antiderivative = 26.22

$$\int \frac{1}{(dx)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input

```
integrate(1/(d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output

```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*a*b*c^2*d^(3/2)*
x^(5/2) + (a*b*c^3*d^(3/2)*x^2 + a*b*c*d^(3/2))*x^(3/2) + (sqrt(c^2*x^2 +
1)*b^2*c^2*d^(3/2)*x^(5/2) + (b^2*c^3*d^(3/2)*x^2 + b^2*c*d^(3/2))*x^(3/2)
)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(1/2*(c^5*sqrt(d)*x^5 + 2*c^3*s
qrt(d)*x^3 + c*sqrt(d)*x + (c^3*sqrt(d)*x^3 + 5*c*sqrt(d)*x)*(c^2*x^2 + 1)
 + (2*c^4*sqrt(d)*x^4 + 7*c^2*sqrt(d)*x^2 + 3*sqrt(d))*sqrt(c^2*x^2 + 1))/
((c^2*x^2 + 1)*a*b*c^3*d^2*x^(9/2) + 2*(a*b*c^4*d^2*x^4 + a*b*c^2*d^2*x^2)
*sqrt(c^2*x^2 + 1)*x^(3/2) + (a*b*c^5*d^2*x^5 + 2*a*b*c^3*d^2*x^3 + a*b*c*
d^2*x)*x^(3/2) + ((c^2*x^2 + 1)*b^2*c^3*d^2*x^(9/2) + 2*(b^2*c^4*d^2*x^4 +
 b^2*c^2*d^2*x^2)*sqrt(c^2*x^2 + 1)*x^(3/2) + (b^2*c^5*d^2*x^5 + 2*b^2*c^3
*d^2*x^3 + b^2*c*d^2*x)*x^(3/2))*log(c*x + sqrt(c^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(dx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(1/(d*x)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(d\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d*x)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{(dx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{\sqrt{x}\, asinh(cx)^2 b^2 x+2\sqrt{x}\, asinh(cx) abx+\sqrt{x}\, a^2 x}dx}{\sqrt{d}\, d}$$

input
```
int(1/(d*x)^(3/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(sqrt(x)*asinh(c*x)**2*b**2*x + 2*sqrt(x)*asinh(c*x)*a*b*x + sqrt(x)
*a**2*x),x)/(sqrt(d)*d)
```

3.100 $\int x^m \mathbf{arcsinh}(ax)^4\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \text{arcsinh}(ax)^4\,dx = \text{Int}\left(x^m \text{arcsinh}(ax)^4, x\right)$$

output

```
Defer(Int)(x^m*arcsinh(a*x)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^4\,dx = \int x^m \text{arcsinh}(ax)^4\,dx$$

input

```
Integrate[x^m*ArcSinh[a*x]^4,x]
```

output

```
Integrate[x^m*ArcSinh[a*x]^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arcsinh}(ax)^4 \, dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)^4}{m+1} - \frac{4a \int \frac{x^{m+1}\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{m+1}$$

$$\Big\downarrow \text{6239}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)^4}{m+1} - \frac{4a \int \frac{x^{m+1}\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{m+1}$$

input
```
Int[x^m*ArcSinh[a*x]^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arcsinh}(xa)^4 \, dx$$

input
```
int(x^m*arcsinh(x*a)^4,x)
```

output
```
int(x^m*arcsinh(x*a)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arsinh}\left(ax\right)^4 \, dx$$

input
```
integrate(x^m*arcsinh(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)^4, x)
```

Sympy [N/A]

Not integrable

Time = 5.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{asinh}^4\left(ax\right) dx$$

input
```
integrate(x**m*asinh(a*x)**4,x)
```

output
```
Integral(x**m*asinh(a*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 13.40

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arsinh}\left(ax\right)^4 \, dx$$

input
```
integrate(x^m*arcsinh(a*x)^4,x, algorithm="maxima")
```

output
```
x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^4/(m + 1) - integrate(4*(sqrt(a^2*x^2 +
 1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3
*(m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)),
 x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arsinh}\left(ax\right)^4 \, dx$$

input
```
integrate(x^m*arcsinh(a*x)^4,x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)^4, x)
```

Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{asinh}(a\,x)^4 \, dx$$

input
```
int(x^m*asinh(a*x)^4,x)
```

output
```
int(x^m*asinh(a*x)^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^4 \, dx = \int x^m asinh(ax)^4 \, dx$$

input

```
int(x^m*asinh(a*x)^4,x)
```

output

```
int(x**m*asinh(a*x)**4,x)
```

3.101 $\int x^m \textbf{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \operatorname{Int}\left(x^m \operatorname{arcsinh}(ax)^3, x\right)$$

output

```
Defer(Int)(x^m*arcsinh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arcsinh}(ax)^3\,dx$$

input

```
Integrate[x^m*ArcSinh[a*x]^3,x]
```

output

```
Integrate[x^m*ArcSinh[a*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arcsinh}(ax)^3 \, dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)^3}{m+1}-\frac{3a\int \frac{x^{m+1}\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{m+1}$$

$$\Big\downarrow \text{6239}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)^3}{m+1}-\frac{3a\int \frac{x^{m+1}\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}dx}{m+1}$$

input
```
Int[x^m*ArcSinh[a*x]^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arcsinh}(xa)^3 \, dx$$

input
```
int(x^m*arcsinh(x*a)^3,x)
```

output
```
int(x^m*arcsinh(x*a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{asinh}^3\left(ax\right)dx$$

input
```
integrate(x**m*asinh(a*x)**3,x)
```

output
```
Integral(x**m*asinh(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 13.40

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^3/(m + 1) - integrate(3*(sqrt(a^2*x^2 +
 1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3
*(m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)),
 x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m\,\operatorname{asinh}(a\,x)^3\,dx$$

input
```
int(x^m*asinh(a*x)^3,x)
```

output
```
int(x^m*asinh(a*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m asinh(ax)^3\,dx$$

input
```
int(x^m*asinh(a*x)^3,x)
```

output
```
int(x**m*asinh(a*x)**3,x)
```

3.102 $\int x^m \operatorname{arcsinh}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 137

$$\int x^m \operatorname{arcsinh}(ax)^2 \, dx = \frac{x^{1+m}\operatorname{arcsinh}(ax)^2}{1+m} - \frac{2ax^{2+m}\operatorname{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2} + \frac{2a^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};-a^2x^2\right)}{6+11m+6m^2+m^3}$$

output

```
x^(1+m)*arcsinh(a*x)^2/(1+m)-2*a*x^(2+m)*arcsinh(a*x)*hypergeom([1/2, 1+1/
2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)+2*a^2*x^(3+m)*hypergeom([1, 3/2+1/2*m
, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],-a^2*x^2)/(m^3+6*m^2+11*m+6)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int x^m \operatorname{arcsinh}(ax)^2 \, dx = \frac{x^{1+m}\left((3+m)\operatorname{arcsinh}(ax)\left((2+m)\operatorname{arcsinh}(ax) - 2ax\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)\right) + 2a^2}{(1+m)(2+m)(3+m)}$$

input
```
Integrate[x^m*ArcSinh[a*x]^2,x]
```

output
```
(x^(1 + m)*((3 + m)*ArcSinh[a*x]*((2 + m)*ArcSinh[a*x] - 2*a*x*Hypergeomet
ric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]) + 2*a^2*x^2*HypergeometricP
FQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, -(a^2*x^2)]))/((1 + m)
*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arcsinh}(ax)^2 \, dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)^2}{m+1} - \frac{2a\int \frac{x^{m+1}\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{m+1}$$

$$\Big\downarrow \text{6232}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)^2}{m+1} - \frac{2a\left(\frac{x^{m+2}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2} - \frac{ax^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-a^2x^2\right)}{m^2+5m+6}\right)}{m+1}$$

input
```
Int[x^m*ArcSinh[a*x]^2,x]
```

output
```
(x^(1 + m)*ArcSinh[a*x]^2)/(1 + m) - (2*a*((x^(2 + m)*ArcSinh[a*x]*Hyperge
ometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + m) - (a*x^(3 + m)*
HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, -(a^2*x
^2)])/(6 + 5*m + m^2)))/(1 + m)
```

Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^{m} \operatorname{arcsinh}\left(x a\right)^{2}dx$$

input
```
int(x^m*arcsinh(x*a)^2,x)
```

output
```
int(x^m*arcsinh(x*a)^2,x)
```

Fricas [F]

$$\int x^m \operatorname{arcsinh}(ax)^2\,dx = \int x^{m} \operatorname{arsinh}\left(a x\right)^{2}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int x^m \text{arcsinh}(ax)^2\,dx = \int x^m \operatorname{asinh}^2(ax)\,dx$$

input
```
integrate(x**m*asinh(a*x)**2,x)
```

output
```
Integral(x**m*asinh(a*x)**2, x)
```

Maxima [F]

$$\int x^m \text{arcsinh}(ax)^2\,dx = \int x^m \operatorname{arsinh}(ax)^2\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^2/(m + 1) - integrate(2*(sqrt(a^2*x^2 +
 1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*(
m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x
)
```

Giac [F]

$$\int x^m \text{arcsinh}(ax)^2\,dx = \int x^m \operatorname{arsinh}(ax)^2\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \operatorname{arcsinh}(ax)^2 \, dx = \int x^m \operatorname{asinh}(a\,x)^2 \, dx$$

input
```
int(x^m*asinh(a*x)^2,x)
```

output
```
int(x^m*asinh(a*x)^2, x)
```

Reduce [F]

$$\int x^m \operatorname{arcsinh}(ax)^2 \, dx = \int x^m asinh(ax)^2 \, dx$$

input
```
int(x^m*asinh(a*x)^2,x)
```

output
```
int(x**m*asinh(a*x)**2,x)
```

3.103 $\int x^m \text{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x^m \text{arcsinh}(ax)\,dx = \frac{x^{1+m}\text{arcsinh}(ax)}{1+m} - \frac{ax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}$$

output
```
x^(1+m)*arcsinh(a*x)/(1+m)-a*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a
^2*x^2)/(m^2+3*m+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x^m \text{arcsinh}(ax)\,dx = \frac{x^{1+m}\left((2+m)\text{arcsinh}(ax) - ax\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[x^m*ArcSinh[a*x],x]
```

output
```
(x^(1 + m)*((2 + m)*ArcSinh[a*x] - a*x*Hypergeometric2F1[1/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{6191, 278\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arcsinh}(ax)\,dx$$

$$\Big\downarrow \text{6191}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)}{m+1}-\frac{a\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}\text{arcsinh}(ax)}{m+1}-\frac{ax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcSinh[a*x],x]
```

output
```
(x^(1 + m)*ArcSinh[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2
+ m)/2, (4 + m)/2, -(a^2*x^2)])/((1 + m)*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{m} \operatorname{arcsinh}\left(x a \right)d x$$

input
```
int(x^m*arcsinh(x*a),x)
```

output
```
int(x^m*arcsinh(x*a),x)
```

Fricas [F]

$$\int x^m \text{arcsinh}(ax)\, dx = \int x^{m} \operatorname{arsinh}\left(a x\right)\,{d x}$$

input
```
integrate(x^m*arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x^m*arcsinh(a*x), x)
```

Sympy [F]

$$\int x^m \text{arcsinh}(ax)\,dx = \int x^m \operatorname{asinh}(ax)\,dx$$

input
```
integrate(x**m*asinh(a*x),x)
```

output
```
Integral(x**m*asinh(a*x), x)
```

Maxima [F]

$$\int x^m \text{arcsinh}(ax)\,dx = \int x^m \operatorname{arsinh}(ax)\;dx$$

input
```
integrate(x^m*arcsinh(a*x),x, algorithm="maxima")
```

output
```
-a^2*integrate(x^2*x^m/(a^2*(m + 1)*x^2 + m + 1), x) - a*integrate(x*x^m/(
a^3*(m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1
)), x) + x*x^m*log(a*x + sqrt(a^2*x^2 + 1))/(m + 1)
```

Giac [F]

$$\int x^m \text{arcsinh}(ax)\,dx = \int x^m \operatorname{arsinh}(ax)\;dx$$

input
```
integrate(x^m*arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \text{arcsinh}(ax)\,dx = \int x^m \,\text{asinh}(a\,x)\;dx$$

input

```
int(x^m*asinh(a*x),x)
```

output

```
int(x^m*asinh(a*x), x)
```

Reduce [F]

$$\int x^m \text{arcsinh}(ax)\,dx = \int x^m asinh(ax)\,dx$$

input

```
int(x^m*asinh(a*x),x)
```

output

```
int(x**m*asinh(a*x),x)
```

3.104 $\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(\frac{x^m}{\operatorname{arcsinh}(ax)},x\right)$$

output
```
Defer(Int)(x^m/arcsinh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx$$

input
```
Integrate[x^m/ArcSinh[a*x],x]
```

output
```
Integrate[x^m/ArcSinh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx \\ \Big\downarrow \; 6196 \\ \displaystyle\int \frac{x^m}{\operatorname{arcsinh}(ax)}dx \end{array}$$

input

```
Int[x^m/ArcSinh[a*x],x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(xa)}dx$$

input

```
int(x^m/arcsinh(x*a),x)
```

output

```
int(x^m/arcsinh(x*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^m/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/arcsinh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate(x**m/asinh(a*x),x)
```

output
```
Integral(x**m/asinh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^m/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/arcsinh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^m/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/arcsinh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x^m/asinh(a*x),x)
```

output
```
int(x^m/asinh(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{asinh\left(ax\right)}dx$$

input
```
int(x^m/asinh(a*x),x)
```

output
```
int(x**m/asinh(a*x),x)
```

3.105 $\int \frac{x^m}{\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m}{\operatorname{arcsinh}(ax)^2},x\right)$$

output

```
Defer(Int)(x^m/arcsinh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx$$

input

```
Integrate[x^m/ArcSinh[a*x]^2,x]
```

output

```
Integrate[x^m/ArcSinh[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\text{arcsinh}(ax)^2}\, dx$$
$$\Big\downarrow \text{6196}$$
$$\int \frac{x^m}{\text{arcsinh}(ax)^2}dx$$

input
```
Int[x^m/ArcSinh[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}\left(xa\right)^2}dx$$

input
```
int(x^m/arcsinh(x*a)^2,x)
```

output
```
int(x^m/arcsinh(x*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\text{arcsinh}(ax)^2}\,dx=\int \frac{x^m}{\text{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/arcsinh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\text{arcsinh}(ax)^2}\,dx=\int \frac{x^m}{\text{asinh}^2\left(ax\right)}\,dx$$

input
```
integrate(x**m/asinh(a*x)**2,x)
```

output
```
Integral(x**m/asinh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 26.80

$$\int \frac{x^m}{\text{arcsinh}(ax)^2}\,dx=\int \frac{x^m}{\text{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^2*x^2 + 1)^(3/2)*x^m + (a^3*x^3 + a*x)*x^m)/((a^3*x^2 + sqrt(a^2*x^2
+ 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate(((a^3*(m + 1)*x^
3 + a*(m - 1)*x)*(a^2*x^2 + 1)*x^m + (2*a^4*(m + 1)*x^4 + a^2*(3*m + 1)*x^
2 + m)*sqrt(a^2*x^2 + 1)*x^m + (a^5*(m + 1)*x^5 + 2*a^3*(m + 1)*x^3 + a*(m
 + 1)*x)*x^m)/((a^5*x^5 + (a^2*x^2 + 1)*a^3*x^3 + 2*a^3*x^3 + a*x + 2*(a^4
*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/arcsinh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

input
```
int(x^m/asinh(a*x)^2,x)
```

output
```
int(x^m/asinh(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{asinh\left(ax\right)^2}dx$$

input
```
int(x^m/asinh(a*x)^2,x)
```

output
```
int(x**m/asinh(a*x)**2,x)
```

3.106 $\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 182

$$\begin{aligned}\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx &= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5}\\&\quad-\frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5}+\frac{\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5}\\&\quad-\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5}\\&\quad-\frac{\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5}\end{aligned}$$

output

```
1/5*x^5*arcsinh(a*x)^(1/2)+1/32*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^5-1/192
*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^5+1/1600*5^(1/2)*Pi^(1
/2)*erf(5^(1/2)*arcsinh(a*x)^(1/2))/a^5-1/32*Pi^(1/2)*erfi(arcsinh(a*x)^(1
/2))/a^5+1/192*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*x)^(1/2))/a^5-1/160
0*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*arcsinh(a*x)^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$\int x^4\sqrt{\text{arcsinh}(ax)}\,dx$$

$$= \frac{\frac{3\sqrt{5}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-5\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\frac{25\sqrt{3}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+\frac{150\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-\text{arcsinh}(\right.}{\sqrt{-\text{arcsinh}(ax)}}}{2400a^5}$$

input
```
Integrate[x^4*Sqrt[ArcSinh[a*x]],x]
```

output
```
((3*Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -5*ArcSinh[a*x]])/Sqrt[-ArcSinh[
a*x]] + (25*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]])/Sqrt[
ArcSinh[a*x]] + (150*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[-A
rcSinh[a*x]] - 150*Gamma[3/2, ArcSinh[a*x]] + 25*Sqrt[3]*Gamma[3/2, 3*ArcS
inh[a*x]] - 3*Sqrt[5]*Gamma[3/2, 5*ArcSinh[a*x]])/(2400*a^5)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6192, 6234, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{\text{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{1}{10}a\int \frac{x^5}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{a^5x^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}$$

$$\Big\downarrow \; 26$$

$$\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int \frac{\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}$$

$$\Big\downarrow \; 3793$$

$$\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int \left(\frac{5iax}{8\sqrt{\text{arcsinh}(ax)}}-\frac{5i\sinh(3\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{i\sinh(5\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{10a^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\Big(-\frac{5}{16}i\sqrt{\pi}\text{erf}\Big(\sqrt{\text{arcsinh}(ax)}\Big)+\frac{5}{32}i\sqrt{\frac{\pi}{3}}\text{erf}\Big(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Big)-\frac{1}{32}i\sqrt{\frac{\pi}{5}}\text{erf}\Big(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\Big)+\frac{5}{16}i\sqrt{\pi}\text{erfi}\Big(\sqrt{\ }}{10a^5}$$

input

```
Int[x^4*Sqrt[ArcSinh[a*x]],x]
```

output

```
(x^5*Sqrt[ArcSinh[a*x]])/5 + ((I/10)*(((-5*I)/16)*Sqrt[Pi]*Erf[Sqrt[ArcSin
h[a*x]]] + ((5*I)/32)*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]] - (I/32)*
Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]] + ((5*I)/16)*Sqrt[Pi]*Erfi[Sqrt
[ArcSinh[a*x]]] - ((5*I)/32)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]] +
 (I/32)*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]]))/a^5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^4\sqrt{\operatorname{arcsinh}(xa)}dx$$

input
```
int(x^4*arcsinh(x*a)^(1/2),x)
```

output
```
int(x^4*arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^4\sqrt{\operatorname{asinh}(ax)}\,dx$$

input

```
integrate(x**4*asinh(a*x)**(1/2),x)
```

output

```
Integral(x**4*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^4\sqrt{\operatorname{arsinh}(ax)}\,dx$$

input

```
integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output

```
integrate(x^4*sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^4\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^4\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x^4*asinh(a*x)^(1/2),x)
```

output
```
int(x^4*asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int \sqrt{asinh\left(ax\right)}\,x^4dx$$

input
```
int(x^4*asinh(a*x)^(1/2),x)
```

output
```
int(sqrt(asinh(a*x))*x**4,x)
```

3.107 $\int x^3 \sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 139

$$\begin{aligned}\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = &-\frac{3\sqrt{\operatorname{arcsinh}(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)}\\ &-\frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^4}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^4}\end{aligned}$$

output

```
-3/32*arcsinh(a*x)^(1/2)/a^4+1/4*x^4*arcsinh(a*x)^(1/2)-1/256*Pi^(1/2)*erf
(2*arcsinh(a*x)^(1/2))/a^4+1/64*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^
(1/2))/a^4-1/256*Pi^(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a^4+1/64*2^(1/2)*Pi^(
1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx$$
$$=\frac{\frac{\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-4\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+\frac{4\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}-4\sqrt{2}\Gamma\left(\frac{3}{2},2\operatorname{arcsinh}(ax)\right)+\Gamma\left(\frac{3}{2},4}{128a^4}$$

input
```
Integrate[x^3*Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + (4
*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] - 4*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] + Gamma[3/2, 4*ArcSinh[a*x]])/(1
28*a^4)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6192, 6234, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)}-\frac{1}{8}a\int \frac{x^4}{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)}-\frac{\int \frac{a^4x^4}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{8a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{\sin(i\text{arcsinh}(ax))^4}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^4}$$

$$\Big\downarrow \; 3793$$

$$\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\\ \frac{\int \left(-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}+\frac{3}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\\ \frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsin}}\right.}{8a^4}$$

input
```
Int[x^3*Sqrt[ArcSinh[a*x]],x]
```

output
```
(x^4*Sqrt[ArcSinh[a*x]])/4 - ((3*Sqrt[ArcSinh[a*x]])/4 + (Sqrt[Pi]*Erf[2*S
qrt[ArcSinh[a*x]]])/32 - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4 +
(Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[A
rcSinh[a*x]]])/4)/(8*a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^{3}\sqrt{\operatorname{arcsinh}\left(xa\right)}dx$$

input
```
int(x^3*arcsinh(x*a)^(1/2),x)
```

output
```
int(x^3*arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^{3}\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^3 \sqrt{\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate(x**3*asinh(a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x^3 \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^3 \sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*sqrt(arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^3\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x^3*asinh(a*x)^(1/2),x)
```

output
```
int(x^3*asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int \sqrt{asinh\left(ax\right)}\,x^3dx$$

input
```
int(x^3*asinh(a*x)^(1/2),x)
```

output
```
int(sqrt(asinh(a*x))*x**3,x)
```

3.108 $\int x^2 \sqrt{\text{arcsinh}(ax)}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 120

$$\begin{aligned}\int x^2 \sqrt{\text{arcsinh}(ax)}\, dx = {} & \frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^3} \\ & + \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{48a^3} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^3} \\ & - \frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{48a^3}\end{aligned}$$

output

```
1/3*x^3*arcsinh(a*x)^(1/2)-1/16*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^3+1/144
*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^3+1/16*Pi^(1/2)*erfi(a
rcsinh(a*x)^(1/2))/a^3-1/144*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*x)^(1
/2))/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

$$=\frac{\frac{\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+\frac{9\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}+9\Gamma\left(\frac{3}{2},\operatorname{arcsinh}(ax)\right)-\sqrt{3}\Gamma\left(\frac{3}{2},3\mathrm{ar}\right.}{72a^3}$$

input
```
Integrate[x^2*Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*
x]] + (9*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]]
 + 9*Gamma[3/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[3/2, 3*ArcSinh[a*x]])/(72*a^
3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6192, 6234, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)}-\frac{1}{6}a\int \frac{x^3}{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)}-\frac{\int \frac{a^3x^3}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{6a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{i\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int \frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int \left(\frac{3iax}{4\sqrt{\text{arcsinh}(ax)}}-\frac{i\sinh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{6a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-$$

$$\frac{i\left(-\frac{3}{8}i\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}i\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)+\frac{3}{8}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}i\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arc}}\right.\right.}{6a^3}$$

input

```
Int[x^2*Sqrt[ArcSinh[a*x]],x]
```

output

```
(x^3*Sqrt[ArcSinh[a*x]])/3 - ((I/6)*(((-3*I)/8)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[
a*x]]] + (I/8)*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]] + ((3*I)/8)*Sqrt
[Pi]*Erfi[Sqrt[ArcSinh[a*x]]] - (I/8)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh
[a*x]]]))/a^3
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \sqrt{\operatorname{arcsinh}(xa)}dx$$

input

```
int(x^2*arcsinh(x*a)^(1/2),x)
```

output

```
int(x^2*arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^2 \sqrt{\operatorname{asinh}(ax)}\,dx$$

input
```
integrate(x**2*asinh(a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x^2 \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^2 \sqrt{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^2\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^2\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x^2*asinh(a*x)^(1/2),x)
```

output
```
int(x^2*asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int\sqrt{asinh\left(ax\right)}\,x^2dx$$

input
```
int(x^2*asinh(a*x)^(1/2),x)
```

output
```
int(sqrt(asinh(a*x))*x**2,x)
```

3.109 $\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=\frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}$$

output
```
1/4*arcsinh(a*x)^(1/2)/a^2+1/2*x^2*arcsinh(a*x)^(1/2)-1/32*2^(1/2)*Pi^(1/2
)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^2-1/32*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*a
rcsinh(a*x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=\frac{\frac{\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+\Gamma\left(\frac{3}{2},2\operatorname{arcsinh}(ax)\right)}{8\sqrt{2}a^2}$$

input
```
Integrate[x*Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Ga
mma[3/2, 2*ArcSinh[a*x]])/(8*Sqrt[2]*a^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6192, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int \frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{a^2x^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int -\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}+\frac{\int \frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{1}{2\sqrt{\text{arcsinh}(ax)}}-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{4a^2}+\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\sqrt{\operatorname{arcsinh}(ax)}}{4a^2}$$

input
```
Int[x*Sqrt[ArcSinh[a*x]],x]
```

output
```
(x^2*Sqrt[ArcSinh[a*x]])/2 - (-Sqrt[ArcSinh[a*x]] + (Sqrt[Pi/2]*Erf[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4
)/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{2}\left(8\sqrt{2}\,\sqrt{\operatorname{arcsinh}(xa)}\,\sqrt{\pi}\,a^{2}x^{2}+4\sqrt{2}\,\sqrt{\operatorname{arcsinh}(xa)}\,\sqrt{\pi}-\pi\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(xa)}\right)-\pi\,\operatorname{erfi}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(xa)}\right)\right)}{32\sqrt{\pi}\,a^{2}}$	75

input

```
int(x*arcsinh(x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/32*2^(1/2)*(8*2^(1/2)*arcsinh(x*a)^(1/2)*Pi^(1/2)*a^2*x^2+4*2^(1/2)*arcs
inh(x*a)^(1/2)*Pi^(1/2)-Pi*erf(2^(1/2)*arcsinh(x*a)^(1/2))-Pi*erfi(2^(1/2)
*arcsinh(x*a)^(1/2)))/Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x\sqrt{\operatorname{asinh}\left(ax\right)}\,dx$$

input
```
integrate(x*asinh(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(x*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input

```
int(x*asinh(a*x)^(1/2),x)
```

output

```
int(x*asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{asinh\left(ax\right)}\,xdx$$

input

```
int(x*asinh(a*x)^(1/2),x)
```

output

```
int(sqrt(asinh(a*x))*x,x)
```

3.110 $\int \sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = x\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a}$$

output
```
x*arcsinh(a*x)^(1/2)+1/4*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a-1/4*Pi^(1/2)*e
rfi(arcsinh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = -\frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{3}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \Gamma\left(\frac{3}{2},\operatorname{arcsinh}(ax)\right)}{2a}$$

input
```
Integrate[Sqrt[ArcSinh[a*x]],x]
```

output

```
-1/2*((Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
 Gamma[3/2, ArcSinh[a*x]])/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6187, 6234, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6187}$$

$$x\sqrt{\text{arcsinh}(ax)}-\frac{1}{2}a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{6234}$$

$$x\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{ax}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{\text{arcsinh}(ax)}-\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a}$$

$$\Big\downarrow \text{26}$$

$$x\sqrt{\text{arcsinh}(ax)}+\frac{i\int \frac{\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a}$$

$$\Big\downarrow \text{3789}$$

$$x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{2a}$$

$$\Big\downarrow \text{2611}$$

$$x\sqrt{\operatorname{arcsinh}(ax)}+\frac{i\left(i\int e^{\operatorname{arcsinh}(ax)}d\sqrt{\operatorname{arcsinh}(ax)}-i\int e^{-\operatorname{arcsinh}(ax)}d\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a}$$

$$\Big\downarrow \text{2633}$$

$$x\sqrt{\operatorname{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-i\int e^{-\operatorname{arcsinh}(ax)}d\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a}$$

$$\Big\downarrow \text{2634}$$

$$x\sqrt{\operatorname{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)\right)}{2a}$$

input
```
Int[Sqrt[ArcSinh[a*x]],x]
```

output
```
x*Sqrt[ArcSinh[a*x]] + ((I/2)*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] +
 (I/2)*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]]))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{4\sqrt{\operatorname{arcsinh}\left(xa\right)}\,\sqrt{\pi}\,ax+\pi\,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}\left(xa\right)}\right)-\pi\,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}\left(xa\right)}\right)}{4\sqrt{\pi}\,a}$	42

input

```
int(arcsinh(x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4*(4*arcsinh(x*a)^(1/2)*Pi^(1/2)*a*x+Pi*erf(arcsinh(x*a)^(1/2))-Pi*erfi(
arcsinh(x*a)^(1/2)))/Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{asinh}{(ax)}}\,dx$$

input
```
integrate(asinh(a*x)**(1/2),x)
```

output
```
Integral(sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{arsinh}{(ax)}}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

input
```
int(asinh(a*x)^(1/2),x)
```

output
```
int(asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{asinh\left(ax\right)}dx$$

input
```
int(asinh(a*x)^(1/2),x)
```

output
```
int(sqrt(asinh(a*x)),x)
```

3.111 $\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \operatorname{Int}\left(\frac{\sqrt{\operatorname{arcsinh}(ax)}}{x},x\right)$$

output

```
Defer(Int)(arcsinh(a*x)^(1/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$$

input

```
Integrate[Sqrt[ArcSinh[a*x]]/x,x]
```

output

```
Integrate[Sqrt[ArcSinh[a*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arcsinh}(ax)}}{x} \, dx \Big\downarrow 6196 \int \frac{\sqrt{\text{arcsinh}(ax)}}{x}dx$$

input
```
Int[Sqrt[ArcSinh[a*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\text{arcsinh}\left(xa\right)}}{x}dx$$

input
```
int(arcsinh(x*a)^(1/2)/x,x)
```

output
```
int(arcsinh(x*a)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{asinh}{(ax)}}}{x}\,dx$$

input
```
integrate(asinh(a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(asinh(a*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(ax\right)}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(a*x))/x, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arsinh}(ax)}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(a*x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{x}\,dx$$

input
```
int(asinh(a*x)^(1/2)/x,x)
```

output
```
int(asinh(a*x)^(1/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{asinh\,(ax)}}{x}dx$$

input

```
int(asinh(a*x)^(1/2)/x,x)
```

output

```
int(sqrt(asinh(a*x))/x,x)
```

3.112 $\int x^4\text{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 330

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^{3/2}\,dx &= -\frac{4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^5} \\ &+ \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{50a} \\ &+ \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} + \frac{3\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{200a^5} \\ &- \frac{3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} \\ &+ \frac{3\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{200a^5} \\ &- \frac{3\sqrt{3\pi}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5}\end{aligned}$$

output
```
-4/25*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^5+2/25*x^2*(a^2*x^2+1)^(1/2)*
arcsinh(a*x)^(1/2)/a^3-3/50*x^4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a+1/5
*x^5*arcsinh(a*x)^(3/2)+3/64*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^5-1/384*3^
(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^5+3/16000*5^(1/2)*Pi^(1/2
)*erf(5^(1/2)*arcsinh(a*x)^(1/2))/a^5+3/64*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2
))/a^5-1/384*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*x)^(1/2))/a^5+3/16000
*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*arcsinh(a*x)^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.46

$$\int x^4 \operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{9\sqrt{5}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-5\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}+\frac{125\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+\frac{2250\sqrt{}}{}}{}$$

input
```
Integrate[x^4*ArcSinh[a*x]^(3/2),x]
```

output
```
((9*Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[
a*x]] + (125*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]])/Sqrt[
-ArcSinh[a*x]] + (2250*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt
[ArcSinh[a*x]] - 2250*Gamma[5/2, ArcSinh[a*x]] + 125*Sqrt[3]*Gamma[5/2, 3*
ArcSinh[a*x]] - 9*Sqrt[5]*Gamma[5/2, 5*ArcSinh[a*x]])/(36000*a^5)
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.28, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.417$, Rules used = {6192, 6227, 6195, 5971, 2009, 6227, 6195, 5971, 2009, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{arcsinh}(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-\frac{3}{10}a\int\frac{x^5\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-\frac{3}{10}a\left(-\frac{4\int\frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{5a^2}-\frac{\int\frac{x^4}{\sqrt{\text{arcsinh}(ax)}}dx}{10a}+\frac{x^4\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{5a^2}\right)$$

$$\Big\downarrow \text{6195}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-\frac{3}{10}a\left(-\frac{4\int\frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{5a^2}-\frac{\int\frac{a^4x^4\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^6}+\frac{x^4\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{5a^2}\right)$$

$$\Big\downarrow \text{5971}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-\frac{3}{10}a\left(-\frac{4\int\frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{5a^2}-\frac{\int\left(-\frac{3\cosh(3\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(5\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{\sqrt{a^2x^2+1}}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{10a^6}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-\frac{3}{10}a\left(-\frac{4\int\frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{5a^2}-\frac{\frac{1}{16}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{32}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\text{erf}\left(\sqrt{5}\sqrt{\text{arc}}\right.}{}\right.$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int\frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{6a}+\frac{x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{3a^2}\right)}{5a^2}-\frac{\frac{1}{16}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{32}\right.$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int\frac{a^2x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^4}+\frac{x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{3a^2}\right)}{5a^2}-\frac{\frac{1}{16}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(\right.\right.$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int\left(\frac{\cosh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}-\frac{\sqrt{a^2x^2+1}}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{6a^4}+\frac{x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{3a^2}\right)}{5a^2}-\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}}{6a^4}\right.}{5a^2}\right.$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}dx}{2a}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{}}{6a^4}\right.}{5a^2}\right.$$

$$\Big\downarrow \text{6189}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}}\right.}{}\right.}{5a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int\frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{a}}\right.}{}\right.}{5a^2}\right.$$

$$\Big\downarrow \; 3788$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}i\int-\frac{ie^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{ie^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{ar}}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}\int\frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int\frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsi}}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2611}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(a}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{10}a\left(-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{}\right.}{5}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} -$$

$$\frac{3}{10}a\Bigg(-\frac{\frac{1}{16}\sqrt{\pi}\text{erf}\Big(\sqrt{\text{arcsinh}(ax)}\Big) - \frac{1}{32}\sqrt{3\pi}\text{erf}\Big(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Big) + \frac{1}{32}\sqrt{\frac{\pi}{5}}\text{erf}\Big(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\Big) + \frac{1}{16}\sqrt{\pi}\text{erfi}\Big(}{10a^6}$$

input
```
Int[x^4*ArcSinh[a*x]^(3/2),x]
```

output
```
(x^5*ArcSinh[a*x]^(3/2))/5 - (3*a*((x^4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x
]])/(5*a^2) - (4*((x^2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(3*a^2) - (2*
((Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/a^2 - ((Sqrt[Pi]*Erf[Sqrt[ArcSinh[
a*x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/2)/(2*a^2)))/(3*a^2) - (-1
/8*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]]) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSi
nh[a*x]]])/8 - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[Pi/3]*Erfi[Sq
rt[3]*Sqrt[ArcSinh[a*x]]])/8)/(6*a^4)))/(5*a^2) - ((Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/16 - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[P
i/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x
]]])/16 - (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi/5]*E
rfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/32)/(10*a^6)))/10
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int x^4 \operatorname{arcsinh}(xa)^{\frac{3}{2}}\,dx$$

input
```
int(x^4*arcsinh(x*a)^(3/2),x)
```

output
```
int(x^4*arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \int x^4 \operatorname{asinh}^{\frac{3}{2}}(ax) \, dx$$

input
```
integrate(x**4*asinh(a*x)**(3/2),x)
```

output
```
Integral(x**4*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2}\,dx = \int x^4 \operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4*arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^{3/2}\,dx = \int x^4\,\text{asinh}(a\,x)^{3/2}\,dx$$

input
```
int(x^4*asinh(a*x)^(3/2),x)
```

output
```
int(x^4*asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{asinh\left(ax\right)}\,asinh(ax)\,x^4dx$$

input
```
int(x^4*asinh(a*x)^(3/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x)*x**4,x)
```

3.113 $\int x^3 \operatorname{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 199

$$
\begin{aligned}
\int x^3 \operatorname{arcsinh}(ax)^{3/2}\,dx &= \frac{9x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{64a^3} \\
&\quad - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{3/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{3/2} - \frac{3\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{128a^4} \\
&\quad + \frac{3\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} - \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{128a^4}
\end{aligned}
$$

output

```
9/64*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^3-3/32*x^3*(a^2*x^2+1)^(1/2)
*arcsinh(a*x)^(1/2)/a-3/32*arcsinh(a*x)^(3/2)/a^4+1/4*x^4*arcsinh(a*x)^(3/
2)-3/2048*Pi^(1/2)*erf(2*arcsinh(a*x)^(1/2))/a^4+3/256*2^(1/2)*Pi^(1/2)*er
f(2^(1/2)*arcsinh(a*x)^(1/2))/a^4+3/2048*Pi^(1/2)*erfi(2*arcsinh(a*x)^(1/2
))/a^4-3/256*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.50

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-4\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} + \frac{8\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}} - 8\sqrt{2}\Gamma\left(\frac{5}{2},}{512a^4}$$

input
```
Integrate[x^3*ArcSinh[a*x]^(3/2),x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + (8
*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x
]] - 8*Sqrt[2]*Gamma[5/2, 2*ArcSinh[a*x]] + Gamma[5/2, 4*ArcSinh[a*x]])/(5
12*a^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.40, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6192, 6227, 6195, 5971, 2009, 6227, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{4}x^4 \text{arcsinh}(ax)^{3/2} - \frac{3}{8}a\int \frac{x^4\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-\frac{3}{8}a\left(-\frac{3\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int\frac{x^3}{\sqrt{\text{arcsinh}(ax)}}dx}{8a}+\frac{x^3\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{4a^2}\right)$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-\frac{3}{8}a\left(-\frac{3\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{\int\frac{a^3x^3\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^5}+\frac{x^3\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{4a^2}\right)$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-\frac{3}{8}a\left(-\frac{\int\left(\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}-\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a^5}-\frac{3\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}\sqrt{\text{arcsin}}}{4a^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-\frac{3}{8}a\left(-\frac{3\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcs}}\right.}{8a^5}\right.$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-\frac{3}{8}a\left(-\frac{3\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{}}{}\right.$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{8}a\left(-\frac{3\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(a\right.}}{}\right.$$

$$\Big\downarrow \; 5971$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{8}a\left(-\frac{3\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsin}\right.}}{}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{8}a\left(-\frac{3\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsin}\right.}}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{8}a\left(-\frac{3\left(-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcs}\right.}}{}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{8}a\Bigg(-\frac{3\Bigg(\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\Bigg)}{4a^2}-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\Big(2\sqrt{\text{arcsinh}}}{}$$

$$\Big\downarrow\ 3789$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{8}a\Bigg(-\frac{3\Bigg(\frac{i\Big(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\Big)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(}}{2a^2}}{4a^2}$$

$$\Big\downarrow\ 2611$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{8}a\Bigg(-\frac{3\Bigg(\frac{i\Big(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\Big)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(a}}{2a^2}}{4a^2}$$

$$\Big\downarrow\ 2633$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{8}a\Bigg(-\frac{3\Bigg(\frac{i\Big(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\Big(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Big)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\Big)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax}}{2a^2}}{4a^2}$$

$$\Big\downarrow\ 2634$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{8}a\left(-\frac{3\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax}}{2a^2}\right.}{4a^2}\right.$$

$$\Big\downarrow 6198$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}-$$

$$\frac{3}{8}a\left(-\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\right.}{8a^5}\right.$$

input
```
Int[x^3*ArcSinh[a*x]^(3/2),x]
```

output
```
(x^4*ArcSinh[a*x]^(3/2))/4 - (3*a*((x^3*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x
]])/(4*a^2) - (-1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]]) + (Sqrt[Pi/2]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/3
2 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8)/(8*a^5) - (3*((x*Sqrt
[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a^2) - ArcSinh[a*x]^(3/2)/(3*a^3) + (
(I/8)*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi
/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]]))/a^3))/(4*a^2)))/8
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int x^{3} \operatorname{arcsinh}\left(x a\right)^{\frac{3}{2}}d x$$

input
```
int(x^3*arcsinh(x*a)^(3/2),x)
```

output
```
int(x^3*arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \operatorname{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \int x^3 \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(x**3*asinh(a*x)**(3/2),x)
```

output
```
Integral(x**3*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \int x^3 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arcsinh(a*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{arcsinh}(ax)^{3/2}\,dx=\int x^3\,\text{asinh}(a\,x)^{3/2}\,dx$$

input
```
int(x^3*asinh(a*x)^(3/2),x)
```

output
```
int(x^3*asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int x^3\text{arcsinh}(ax)^{3/2}\,dx=\int\sqrt{asinh\,(ax)}\,asinh(ax)\,x^3dx$$

input
```
int(x^3*asinh(a*x)^(3/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x)*x**3,x)
```

3.114 $\int x^2\text{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 179

$$\begin{aligned}\int x^2\text{arcsinh}(ax)^{3/2}\,dx &= \frac{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{3a^3}-\frac{x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{6a}\\&\quad+\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{3\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{96a^3}\\&\quad-\frac{3\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{96a^3}\end{aligned}$$

output

```
1/3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^3-1/6*x^2*(a^2*x^2+1)^(1/2)*arc
sinh(a*x)^(1/2)/a+1/3*x^3*arcsinh(a*x)^(3/2)-3/32*Pi^(1/2)*erf(arcsinh(a*x
)^(1/2))/a^3+1/288*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^3-3/
32*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a^3+1/288*3^(1/2)*Pi^(1/2)*erfi(3^(1/
2)*arcsinh(a*x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.55

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{3}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+\frac{27\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+27\Gamma\left(\frac{5}{2},\text{arc}\right.}{216a^3}$$

input
```
Integrate[x^2*ArcSinh[a*x]^(3/2),x]
```

output
```
((Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]])/Sqrt[ArcSinh[a*
x]] + (27*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]
] + 27*Gamma[5/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[5/2, 3*ArcSinh[a*x]])/(216
*a^3)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6192, 6227, 6195, 5971, 2009, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\int\frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int\frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{6a}+\frac{x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{3a^2}\right)$$

$$\Big\downarrow \text{6195}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int\frac{a^2x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^4}+\frac{x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{3a^2}\right)$$

$$\Big\downarrow \text{5971}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int\left(\frac{\cosh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}-\frac{\sqrt{a^2x^2+1}}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{6a^4}+\frac{x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(a}}{3a^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\int\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(a}\right.}{6a^4}\right.$$

$$\Big\downarrow \text{6213}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int\frac{1}{\sqrt{\text{arcsinh}(ax)}}dx}{2a}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{}\right.$$

$$\Big\downarrow \text{6189}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int\frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arc}}\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int\frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\Big(\sqrt{}}{}\right.$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}i\int-\frac{ie^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{ie^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\Big(\sqrt{a}}{}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}\int\frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int\frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\Big(\sqrt{\text{arcs}}}{}\right.$$

$$\Big\downarrow \text{2611}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\Big(\sqrt{\text{arcsinh}}}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-$$

$$\frac{1}{2}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{2a^2}\right)}{3a^2}-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(a\right.}{}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{3/2}-$$

$$\frac{1}{2}a\left(-\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{a}\right.}{6a^4}\right.$$

input
```
Int[x^2*ArcSinh[a*x]^(3/2),x]
```

output
```
(x^3*ArcSinh[a*x]^(3/2))/3 - (a*((x^2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]]
)/(3*a^2) - (2*((Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/a^2 - ((Sqrt[Pi]*Er
f[Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/2)/(2*a^2))
)/(3*a^2) - (-1/8*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]]) + (Sqrt[Pi/3]*Erf[Sqr
t[3]*Sqrt[ArcSinh[a*x]]])/8 - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/8 + (Sqr
t[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/8)/(6*a^4)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int x^2 \operatorname{arcsinh}(xa)^{\frac{3}{2}} dx$$

input
```
int(x^2*arcsinh(x*a)^(3/2),x)
```

output
```
int(x^2*arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \operatorname{arcsinh}(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \operatorname{arcsinh}(ax)^{3/2}\, dx = \int x^{2} \operatorname{asinh}^{\frac{3}{2}}{\left(ax \right)}\, dx$$

input

```
integrate(x**2*asinh(a*x)**(3/2),x)
```

output

```
Integral(x**2*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x^2 \operatorname{arcsinh}(ax)^{3/2}\, dx = \int x^{2} \operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}\, dx$$

input

```
integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output

```
integrate(x^2*arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int x^2 \text{arcsinh}(ax)^{3/2}\,dx = \int x^2 \operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}\,dx$$

input

```
integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output

```
integrate(x^2*arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(ax)^{3/2}\,dx = \int x^2 \operatorname{asinh}(a\,x)^{3/2}\,dx$$

input

```
int(x^2*asinh(a*x)^(3/2),x)
```

output

```
int(x^2*asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int x^2 \text{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{asinh\left(ax\right)}\,asinh(ax)\,x^2dx$$

input

```
int(x^2*asinh(a*x)^(3/2),x)
```

output

```
int(sqrt(asinh(a*x))*asinh(a*x)*x**2,x)
```

3.115 $\int x\text{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 122

$$\int x\text{arcsinh}(ax)^{3/2}\,dx = -\frac{3x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{8a}+\frac{\text{arcsinh}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a^2}$$

output

```
-3/8*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a+1/4*arcsinh(a*x)^(3/2)/a^2+1
/2*x^2*arcsinh(a*x)^(3/2)-3/128*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^
(1/2))/a^2+3/128*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int x\text{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+\Gamma\left(\frac{5}{2},2\text{arcsinh}(ax)\right)}{16\sqrt{2}a^2}$$

input

```
Integrate[x*ArcSinh[a*x]^(3/2),x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Ga
mma[5/2, 2*ArcSinh[a*x]])/(16*Sqrt[2]*a^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6192, 6227, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow \text{6195}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow \text{5971}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow\ 27$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow\ 3042$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow\ 26$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow\ 3789$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{}}{2a}\right.$$

$$\Big\downarrow\ 2611$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{4}a\left(\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{a}}{2a^2}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{a}}{2a^2}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{a}}{2a^2}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-$$
$$\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a^3}-\frac{\text{arcsinh}(ax)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsin}}}{2a^2}\right.$$

input
```
Int[x*ArcSinh[a*x]^(3/2),x]
```

output
```
(x^2*ArcSinh[a*x]^(3/2))/2 - (3*a*((x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]]
)/(2*a^2) - ArcSinh[a*x]^(3/2)/(3*a^3) + ((I/8)*((-1/2*I)*Sqrt[Pi/2]*Erf[S
qrt[2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*
x]]]))/a^3))/4
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{2}\,(-32\,\mathrm{arcsinh}(xa)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,a^2x^2+24\sqrt{2}\,\sqrt{\mathrm{arcsinh}(xa)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,ax-16\,\mathrm{arcsinh}(xa)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}+3\pi\,\mathrm{erf}(\sqrt{2}\,\sqrt{\mathrm{arcsinh}(xa)})}{128\sqrt{\pi}\,a^2}$

input
```
int(x*arcsinh(x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*2^(1/2)*(-32*arcsinh(x*a)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+24*2^(1/2)
*arcsinh(x*a)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-16*arcsinh(x*a)^(3/2)*2
^(1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arcsinh(x*a)^(1/2))-3*Pi*erfi(2^(1/2)*arc
sinh(x*a)^(1/2)))/Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\text{arcsinh}(ax)^{3/2}\,dx=\int x\,\text{asinh}^{\frac{3}{2}}\left(ax\right)dx$$

input
```
integrate(x*asinh(a*x)**(3/2),x)
```

output
```
Integral(x*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^{3/2}\,dx=\int x\operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int x\text{arcsinh}(ax)^{3/2}\,dx=\int x\operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^{3/2}\,dx=\int x\operatorname{asinh}(a\,x)^{3/2}\,dx$$

input
```
int(x*asinh(a*x)^(3/2),x)
```

output
```
int(x*asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int x\operatorname{arcsinh}(ax)^{3/2}\,dx=\int \sqrt{asinh\,(ax)}\,asinh(ax)\,xdx$$

input

```
int(x*asinh(a*x)^(3/2),x)
```

output

```
int(sqrt(asinh(a*x))*asinh(a*x)*x,x)
```

3.116 $\int \operatorname{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = -\frac{3\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{2a} + x\operatorname{arcsinh}(ax)^{3/2} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a}$$

output
```
-3/2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a+x*arcsinh(a*x)^(3/2)+3/8*Pi^(1
/2)*erf(arcsinh(a*x)^(1/2))/a+3/8*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.58

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{5}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} - \Gamma\left(\frac{5}{2},\operatorname{arcsinh}(ax)\right)}{2a}$$

input
```
Integrate[ArcSinh[a*x]^(3/2),x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamm
a[5/2, ArcSinh[a*x]])/(2*a)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{ 6187}$$

$$x\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{2}a\int \frac{x\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6213}$$

$$x\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}{a^2}-\frac{\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}dx}{2a}\right)$$

$$\Big\downarrow \text{ 6189}$$

$$x\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}{a^2}-\frac{\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{2a^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$x\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}{a^2}-\frac{\int \frac{\sin\left(i\operatorname{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{2a^2}\right)$$

$$\Big\downarrow \text{ 3788}$$

$$x\text{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}i\int -\frac{ie^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)$$

$$\Big\downarrow \; 26$$

$$x\text{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a^2}\right)$$

$$\Big\downarrow \; 2611$$

$$x\text{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)$$

$$\Big\downarrow \; 2633$$

$$x\text{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{2a^2}\right)$$

$$\Big\downarrow \; 2634$$

$$x\text{arcsinh}(ax)^{3/2}-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{a^2}-\frac{\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{2a^2}\right)$$

input

```
Int[ArcSinh[a*x]^(3/2),x]
```

output

```
x*ArcSinh[a*x]^(3/2) - (3*a*((Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/a^2 -
((Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]]
)/2)/(2*a^2)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{-8 \operatorname{arcsinh}(xa)^{\frac{3}{2}} \sqrt{\pi}\, ax+12 \sqrt{\operatorname{arcsinh}(xa)}\, \sqrt{\pi}\, \sqrt{a^{2} x^{2}+1}-3 \pi \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(xa)}\right)-3 \pi \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(xa)}\right)}{8 \sqrt{\pi}\, a}$	65

input
```
int(arcsinh(x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-8*arcsinh(x*a)^(3/2)*Pi^(1/2)*a*x+12*arcsinh(x*a)^(1/2)*Pi^(1/2)*(a
^2*x^2+1)^(1/2)-3*Pi*erf(arcsinh(x*a)^(1/2))-3*Pi*erfi(arcsinh(x*a)^(1/2))
)/Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

input

```
integrate(asinh(a*x)**(3/2),x)
```

output

```
Integral(asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

input

```
integrate(arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output

```
integrate(arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

input

```
integrate(arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output

```
integrate(arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{asinh}(a\,x)^{3/2}\,dx$$

input
```
int(asinh(a*x)^(3/2),x)
```

output
```
int(asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \sqrt{asinh\,(ax)}\,asinh(ax)\,dx$$

input
```
int(asinh(a*x)^(3/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x),x)
```

3.117 $\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x} \, dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)(arcsinh(a*x)^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x} \, dx = \int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x} \, dx$$

input
```
Integrate[ArcSinh[a*x]^(3/2)/x,x]
```

output
```
Integrate[ArcSinh[a*x]^(3/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{x}dx$$

input

```
Int[ArcSinh[a*x]^(3/2)/x,x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}\left(x a \right)^{\frac{3}{2}}}{x}d x$$

input

```
int(arcsinh(x*a)^(3/2)/x,x)
```

output

```
int(arcsinh(x*a)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{x}\,dx$$

input

```
integrate(asinh(a*x)**(3/2)/x,x)
```

output

```
Integral(asinh(a*x)**(3/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{x}\,dx$$

input

```
integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(3/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{3/2}}{x}\,dx$$

input
```
int(asinh(a*x)^(3/2)/x,x)
```

output
```
int(asinh(a*x)^(3/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\sqrt{asinh\,(ax)}\,asinh(ax)}{x}dx$$

input
```
int(asinh(a*x)^(3/2)/x,x)
```

output
```
int((sqrt(asinh(a*x))*asinh(a*x))/x,x)
```

3.118 $\int x^4 \operatorname{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 379

$$\int x^4 \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\operatorname{arcsinh}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2} + \frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{128a^5} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{240a^5} - \frac{\sqrt{3\pi}\operatorname{erf}\big(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}}{1280a^5}$$

output
```
2/5*x*arcsinh(a*x)^(1/2)/a^4-1/15*x^3*arcsinh(a*x)^(1/2)/a^2+3/100*x^5*arc
sinh(a*x)^(1/2)-4/15*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(3/2)/a^5+2/15*x^2*(a^
2*x^2+1)^(1/2)*arcsinh(a*x)^(3/2)/a^3-1/10*x^4*(a^2*x^2+1)^(1/2)*arcsinh(a
*x)^(3/2)/a+1/5*x^5*arcsinh(a*x)^(5/2)+15/128*Pi^(1/2)*erf(arcsinh(a*x)^(1
/2))/a^5-5/2304*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^5+3/320
00*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*arcsinh(a*x)^(1/2))/a^5-15/128*Pi^(1/2)*er
fi(arcsinh(a*x)^(1/2))/a^5+5/2304*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*
x)^(1/2))/a^5-3/32000*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*arcsinh(a*x)^(1/2))/a^
5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.40

$$\int x^4 \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{\frac{27\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-5\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} + \frac{625\sqrt{3}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + 3375}{}$$

input
```
Integrate[x^4*ArcSinh[a*x]^(5/2),x]
```

output
```
((27*Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -5*ArcSinh[a*x]])/Sqrt[-ArcSinh
[a*x]] + (625*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]])/Sqr
t[ArcSinh[a*x]] + (33750*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqr
t[-ArcSinh[a*x]] - 33750*Gamma[7/2, ArcSinh[a*x]] + 625*Sqrt[3]*Gamma[7/2,
 3*ArcSinh[a*x]] - 27*Sqrt[5]*Gamma[7/2, 5*ArcSinh[a*x]])/(540000*a^5)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.37, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6192, 6227, 6192, 6227, 6192, 6213, 6187, 6234, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{arcsinh}(ax)^{5/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{5}x^5 \operatorname{arcsinh}(ax)^{5/2} - \frac{1}{2}a\int \frac{x^5 \operatorname{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{4\int\frac{x^3\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{5a^2}-\frac{3\int x^4\sqrt{\text{arcsinh}(ax)}dx}{10a}+\frac{x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{5a^2}\right)$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{1}{10}a\int\frac{x^5}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{10a}-\frac{4\int\frac{x^3\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{5a^2}+\frac{x^4\sqrt{a^2x^2+1}\text{arcsinh}(}{5a^2}\right.$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{4\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2\sqrt{\text{arcsinh}(ax)}dx}{2a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{1}{10}a\right.}{10}\right.$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{1}{10}a\int\frac{x^5}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{10a}-\frac{4\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{1}{6}a}{}\right.}{}\right.$$

$$\Big\downarrow \text{6213}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{1}{10}a\int\frac{x^5}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{10a}-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\int\sqrt{\text{arcsinh}(ax)}dx}{2a}\right)}{3a^2}\right.}{}\right.$$

$$\Big\downarrow \text{6187}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{1}{10}a\int\frac{x^5}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{10a}-\frac{4\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{1}{2}\right.}{}\right.}{3a^2}\right.}{}\right.$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^5x^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^3x^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}-\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{i\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arc}}{6a^3}}{2a}\right.}{}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcs}}{6a^3}}{2a}\right.}{}\right.$$

↓ 3789

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2}-$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)}+\frac{i\int\frac{\sin(i\operatorname{arcsinh}(ax))^5}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\operatorname{arcsinh}(ax))^3}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcs}}{6a^3}}{2a}\right.}{}\right.$$

$$\Big\downarrow \text{2611}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcs}}{6a^3}}{2a}\right.}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcs}}{6a^3}}{2a}\right.}{}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^5}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcs}}{6a^3}}{2a}\right.}{}\right.$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{5}x^5\text{arcsinh}(ax)^{5/2}-$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\text{arcsinh}(ax)}+\frac{i\int\left(\frac{5iax}{8\sqrt{\text{arcsinh}(ax)}}-\frac{5i\sinh(3\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{i\sinh(5\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{10a^5}\right)}{10a}-\frac{4\left(-\frac{\frac{1}{3}x^3}{}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2}-$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)}+\frac{i\left(-\frac{5}{16}i\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+\frac{5}{32}i\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{32}i\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right.\right.}{}\right.}{10a}\right.$$

input
```
Int[x^4*ArcSinh[a*x]^(5/2),x]
```

output
```
(x^5*ArcSinh[a*x]^(5/2))/5 - (a*((x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2)
)/(5*a^2) - (4*((x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a^2) - (2*((
Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/a^2 - (3*(x*Sqrt[ArcSinh[a*x]] + ((I
/2)*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi]*Erfi[Sqrt[
ArcSinh[a*x]]]))/a))/(2*a)))/(3*a^2) - ((x^3*Sqrt[ArcSinh[a*x]])/3 - ((I/6
)*(((-3*I)/8)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] + (I/8)*Sqrt[Pi/3]*Erf[Sqrt
[3]*Sqrt[ArcSinh[a*x]]] + ((3*I)/8)*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]] - (I
/8)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]]))/a^3)/(2*a)))/(5*a^2) - (
3*((x^5*Sqrt[ArcSinh[a*x]])/5 + ((I/10)*(((-5*I)/16)*Sqrt[Pi]*Erf[Sqrt[Arc
Sinh[a*x]]] + ((5*I)/32)*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]] - (I/3
2)*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]] + ((5*I)/16)*Sqrt[Pi]*Erfi[S
qrt[ArcSinh[a*x]]] - ((5*I)/32)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]
] + (I/32)*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]]))/a^5))/(10*a)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6192

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6213

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^{4} \operatorname{arcsinh}\left(x a\right)^{\frac{5}{2}}d x$$

input
```
int(x^4*arcsinh(x*a)^(5/2),x)
```

output
```
int(x^4*arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**4*asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 \text{arcsinh}(ax)^{5/2}\,dx = \int x^4 \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^{5/2}\,dx = \int x^4 \operatorname{asinh}(a\,x)^{5/2}\,dx$$

input
```
int(x^4*asinh(a*x)^(5/2),x)
```

output
```
int(x^4*asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int x^4 \text{arcsinh}(ax)^{5/2}\, dx = \int \sqrt{asinh\,(ax)}\, asinh(ax)^2\, x^4 dx$$

input

```
int(x^4*asinh(a*x)^(5/2),x)
```

output

```
int(sqrt(asinh(a*x))*asinh(a*x)**2*x**4,x)
```

3.119 $\int x^3 \mathbf{arcsinh}(ax)^{5/2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 247

$$\int x^3 \text{arcsinh}(ax)^{5/2}\, dx = -\frac{225\sqrt{\text{arcsinh}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\text{arcsinh}(ax)}}{256a^2}$$

$$+ \frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)} + \frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a}$$

$$- \frac{3\text{arcsinh}(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{15\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{16384a^4} + \frac{15\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{512a^4} - 15$$

output
```
-225/2048*arcsinh(a*x)^(1/2)/a^4-45/256*x^2*arcsinh(a*x)^(1/2)/a^2+15/256*
x^4*arcsinh(a*x)^(1/2)+15/64*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(3/2)/a^3-5/
32*x^3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(3/2)/a-3/32*arcsinh(a*x)^(5/2)/a^4+
1/4*x^4*arcsinh(a*x)^(5/2)-15/16384*Pi^(1/2)*erf(2*arcsinh(a*x)^(1/2))/a^4
+15/1024*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^4-15/16384*Pi^
(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a^4+15/1024*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)
*arcsinh(a*x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx = \frac{\frac{\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-4\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}} + \frac{16\sqrt{2}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} - 16\sqrt{2}\Gamma(}{2048a^4}$$

input

```
Integrate[x^3*ArcSinh[a*x]^(5/2),x]
```

output

```
((Sqrt[ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + (1
6*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*
x]] - 16*Sqrt[2]*Gamma[7/2, 2*ArcSinh[a*x]] + Gamma[7/2, 4*ArcSinh[a*x]])/
(2048*a^4)
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6192, 6227, 6192, 6227, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{4}x^4 \text{arcsinh}(ax)^{5/2} - \frac{5}{8}a\int \frac{x^4 \text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{1}{4}x^4 \text{arcsinh}(ax)^{5/2} - \frac{5}{8}a\left(-\frac{3\int \frac{x^2\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{4a^2} - \frac{3\int x^3\sqrt{\text{arcsinh}(ax)}dx}{8a} + \frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{4a^2}\right)$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$
$$\frac{5}{8}a\Bigg(-\frac{3\int \frac{x^2\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{1}{8}a\int \frac{x^4}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{8a}+\frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(a}{4a^2}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$
$$\frac{5}{8}a\Bigg(-\frac{3\left(-\frac{\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\sqrt{\text{arcsinh}(ax)}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right)}{4a^2}-\frac{3\Big(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{1}{8}a\int -}{8a}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$
$$\frac{5}{8}a\Bigg(-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int \frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4a}-\frac{\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right)}{4a^2}-\frac{3\Big(}{}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$
$$\frac{5}{8}a\Bigg(-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{1}{8}a\int \frac{x^4}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{8a}-\frac{3\Bigg(-\frac{3\Big(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int \frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax}}}{4a}}{4}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^4x^4}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^4}\right)}{8a}-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^2x^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax}{4a^2}\right.}{4a}\right.}{4a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{\sin(i\text{arcsinh}(ax))^4}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^4}\right)}{8a}-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int-\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{ar}}{4a^2}\right.}{4a}\right.}{}\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{\sin(i\text{arcsinh}(ax))^4}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^4}\right)}{8a}-\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}+\frac{\int\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arc}}{4a^2}\right.}{4a}\right.}{}\right.$$

$$\Big\downarrow \textbf{3793}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{\int\left(-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}+\frac{3}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a^4}\right)}{8a}-\frac{3\left(-\frac{3\left(\int\left(\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}-\frac{\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{}}{8a^4}\right.}{8a}\right.$$

input
```
Int[x^3*ArcSinh[a*x]^(5/2),x]
```

output
```
(x^4*ArcSinh[a*x]^(5/2))/4 - (5*a*((x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/
2))/(4*a^2) - (3*((x^4*Sqrt[ArcSinh[a*x]])/4 - ((3*Sqrt[ArcSinh[a*x]])/4 +
 (Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/32 - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[Ar
cSinh[a*x]]])/4 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 - (Sqrt[Pi/2]*E
rfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4)/(8*a^4)))/(8*a) - (3*((x*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x]^(3/2))/(2*a^2) - ArcSinh[a*x]^(5/2)/(5*a^3) - (3*((x^2*S
qrt[ArcSinh[a*x]])/2 - (-Sqrt[ArcSinh[a*x]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt
[ArcSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4)/(4*a
^2)))/(4*a)))/(4*a^2)))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^3 \operatorname{arcsinh}(xa)^{\frac{5}{2}}\,dx$$

input
```
int(x^3*arcsinh(x*a)^(5/2),x)
```

output
```
int(x^3*arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \operatorname{arcsinh}(ax)^{5/2}\,dx = \int x^3 \operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

input
```
integrate(x**3*asinh(a*x)**(5/2),x)
```

output
```
Integral(x**3*asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int x^3 \operatorname{arcsinh}(ax)^{5/2}\,dx = \int x^3 \operatorname{arsinh}(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx = \int x^3\,\text{asinh}(a\,x)^{5/2}\,dx$$

input
```
int(x^3*asinh(a*x)^(5/2),x)
```

output
```
int(x^3*asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx = \int \sqrt{asinh\,(ax)}\,asinh(ax)^2\,x^3dx$$

input
```
int(x^3*asinh(a*x)^(5/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x)**2*x**3,x)
```

3.120 $\int x^2 \operatorname{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 210

$$\int x^2 \operatorname{arcsinh}(ax)^{5/2}\,dx = -\frac{5x\sqrt{\operatorname{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\operatorname{arcsinh}(ax)}$$
$$+ \frac{5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{18a}$$
$$+\frac{1}{3}x^3\operatorname{arcsinh}(ax)^{5/2} - \frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^3} + \frac{5\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{576a^3} + \frac{15\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^3}$$

output
```
-5/6*x*arcsinh(a*x)^(1/2)/a^2+5/36*x^3*arcsinh(a*x)^(1/2)+5/9*(a^2*x^2+1)^
(1/2)*arcsinh(a*x)^(3/2)/a^3-5/18*x^2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(3/2)
/a+1/3*x^3*arcsinh(a*x)^(5/2)-15/64*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^3+5
/1728*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^3+15/64*Pi^(1/2)*
erfi(arcsinh(a*x)^(1/2))/a^3-5/1728*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(
a*x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.47

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx = \frac{\frac{\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\frac{81\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+81\Gamma\left(\frac{7}{2},\text{arc}\right.}{648a^3}$$

input
```
Integrate[x^2*ArcSinh[a*x]^(5/2),x]
```

output
```
((Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*
x]] + (81*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]
] + 81*Gamma[7/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[7/2, 3*ArcSinh[a*x]])/(648
*a^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6192, 6227, 6192, 6213, 6187, 6234, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\int \frac{x^3\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\int x^2\sqrt{\text{arcsinh}(ax)}dx}{2a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{3a^2}\right)$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{2\int\frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{1}{6}a\int\frac{x^3}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx}{2a}+\frac{x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{3a^2}\right.$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\int\sqrt{\text{arcsinh}(ax)}dx}{2a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{1}{6}a\int\frac{x^3}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx}{2a}+\frac{x^2}{}\right.$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{1}{2}a\int\frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{2a}\right)}{3a^2}-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{1}{6}a\int\frac{}{\sqrt{}}}{2a}\right.$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{a^3x^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{ax}{\sqrt{\text{arcsin}}}}{}\right.}{2a}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{\int\frac{i\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{\int-\frac{i\,\text{si}}{}}{\sqrt{}}\right.}{}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\int\frac{\sin(}{\sqrt{}}}{}\right.}{}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{3789}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\int\right.}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2611}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(i\int e\right.}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{}\right.}{}\right.}{}\right.}{3a}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\frac{\sin(i\text{arcsinh}(ax))^3}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{}\right.}{}\right.}{}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{i\int\left(\frac{3iax}{4\sqrt{\text{arcsinh}(ax)}}-\frac{i\sinh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{6a^3}}{2a}-\frac{2\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} -$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)} - \frac{i\left(-\frac{3}{8}i\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}i\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)+\frac{3}{8}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}i\sqrt{\frac{\pi}{3}}}{6a^3}}{2a} \right.$$

input

```
Int[x^2*ArcSinh[a*x]^(5/2),x]
```

output

```
(x^3*ArcSinh[a*x]^(5/2))/3 - (5*a*((x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/
2))/(3*a^2) - (2*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/a^2 - (3*(x*Sqrt[
ArcSinh[a*x]] + ((I/2)*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] + (I/2)*
Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]]))/a))/(2*a)))/(3*a^2) - ((x^3*Sqrt[ArcSi
nh[a*x]])/3 - ((I/6)*(((-3*I)/8)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] + (I/8)*
Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]] + ((3*I)/8)*Sqrt[Pi]*Erfi[Sqrt[
ArcSinh[a*x]]] - (I/8)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]]))/a^3)/
(2*a)))/6
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6213

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \operatorname{arcsinh}(xa)^{\frac{5}{2}}\, dx$$

input

```
int(x^2*arcsinh(x*a)^(5/2),x)
```

output

```
int(x^2*arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \text{arcsinh}(ax)^{5/2}\,dx = \int x^2 \operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

input
```
integrate(x**2*asinh(a*x)**(5/2),x)
```

output
```
Integral(x**2*asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int x^2 \text{arcsinh}(ax)^{5/2}\,dx = \int x^2 \operatorname{arsinh}(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx=\int x^2\,\text{asinh}(a\,x)^{5/2}\,dx$$

input
```
int(x^2*asinh(a*x)^(5/2),x)
```

output
```
int(x^2*asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx=\int \sqrt{asinh\,(ax)}\,asinh(ax)^2\,x^2dx$$

input
```
int(x^2*asinh(a*x)^(5/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x)**2*x**2,x)
```

3.121 $\int x\text{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 152

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)}-\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a}+\frac{\text{arcsinh}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{15\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}$$

output

```
15/64*arcsinh(a*x)^(1/2)/a^2+15/32*x^2*arcsinh(a*x)^(1/2)-5/8*x*(a^2*x^2+1
)^(1/2)*arcsinh(a*x)^(3/2)/a+1/4*arcsinh(a*x)^(5/2)/a^2+1/2*x^2*arcsinh(a*
x)^(5/2)-15/512*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^2-15/51
2*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\frac{\frac{\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\Gamma\left(\frac{7}{2},2\text{arcsinh}(ax)\right)}{32\sqrt{2}a^2}$$

input
```
Integrate[x*ArcSinh[a*x]^(5/2),x]
```

output
```
((Sqrt[ArcSinh[a*x]]*Gamma[7/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Ga
mma[7/2, 2*ArcSinh[a*x]])/(32*Sqrt[2]*a^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6192, 6227, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)^{5/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{5}{4}a\int\frac{x^2\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{5}{4}a\left(-\frac{\int\frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{3\int x\sqrt{\text{arcsinh}(ax)}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right)$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int \frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4a}-\frac{\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/}}{2a^2}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{1}{4}a\int \frac{x^2}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{4a}-\frac{\text{arcsinh}(ax)^{5/2}}{5a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right)$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{a^2x^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4a}-\frac{\text{arcsinh}(ax)^{5/2}}{5a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\int -\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4a}-\frac{\text{arcsinh}(ax)^{5/2}}{5a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}+\frac{\int\frac{\sin(i\text{arcsinh}(ax))^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^2}\right)}{4a}-\frac{\text{arcsinh}(ax)^{5/2}}{5a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{2a^2}\right)$$

$$\Big\downarrow \ 3793$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{5}{4}a\left(-\frac{3\left(\frac{\int\left(\frac{1}{2\sqrt{\text{arcsinh}(ax)}}-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{4a^2}+\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}\right)}{4a}-\frac{\text{arcsinh}(ax)^{5/2}}{5a^3}+\frac{x\sqrt{a^2x^2+}}{}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{5}{4}a\left(-\frac{\text{arcsinh}(ax)^{5/2}}{5a^3}-\frac{3\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}(ax)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\sqrt{\text{arcsinh}}}{4a^2}\right.}{4a}\right.$$

input
```
Int[x*ArcSinh[a*x]^(5/2),x]
```

output
```
(x^2*ArcSinh[a*x]^(5/2))/2 - (5*a*((x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2)
)/(2*a^2) - ArcSinh[a*x]^(5/2)/(5*a^3) - (3*((x^2*Sqrt[ArcSinh[a*x]])/2 -
(-Sqrt[ArcSinh[a*x]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4 + (S
qrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4)/(4*a^2)))/(4*a)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{2}\,(-128\,\mathrm{arcsinh}(xa)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,a^{2}x^{2}+160\,\mathrm{arcsinh}(xa)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,ax-120\sqrt{2}\,\sqrt{\mathrm{arcsinh}(xa)}\,\sqrt{\pi}\,a^{2}x^{2}-64\,\mathrm{arcsinh}(xa)^{\frac{5}{2}}}{512\sqrt{\pi}\,a^{2}}$

input
```
int(x*arcsinh(x*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/512*2^(1/2)*(-128*arcsinh(x*a)^(5/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+160*arcsi
nh(x*a)^(3/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-120*2^(1/2)*arcsinh(x
*a)^(1/2)*Pi^(1/2)*a^2*x^2-64*arcsinh(x*a)^(5/2)*2^(1/2)*Pi^(1/2)-60*2^(1/
2)*arcsinh(x*a)^(1/2)*Pi^(1/2)+15*Pi*erf(2^(1/2)*arcsinh(x*a)^(1/2))+15*Pi
*erfi(2^(1/2)*arcsinh(x*a)^(1/2)))/Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx=\int x\,\mathrm{asinh}^{\frac{5}{2}}(ax)\,dx$$

input
```
integrate(x*asinh(a*x)**(5/2),x)
```

output
```
Integral(x*asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx=\int x\,\mathrm{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\int x\,\text{asinh}(a\,x)^{5/2}\,dx$$

input
```
int(x*asinh(a*x)^(5/2),x)
```

output
```
int(x*asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\int \sqrt{asinh\,(ax)}\,asinh(ax)^{2}\,xdx$$

input
```
int(x*asinh(a*x)^(5/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x)**2*x,x)
```

3.122 $\int \operatorname{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{15}{4}x\sqrt{\operatorname{arcsinh}(ax)} - \frac{5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{2a} + x\operatorname{arcsinh}(ax)^{5/2} + \frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a} - \frac{15\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a}$$

output

```
15/4*x*arcsinh(a*x)^(1/2)-5/2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(3/2)/a+x*arc
sinh(a*x)^(5/2)+15/16*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a-15/16*Pi^(1/2)*er
fi(arcsinh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = -\frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{7}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \Gamma\left(\frac{7}{2},\operatorname{arcsinh}(ax)\right)}{2a}$$

input

```
Integrate[ArcSinh[a*x]^(5/2),x]
```

output
```
-1/2*((Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
 Gamma[7/2, ArcSinh[a*x]])/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6187, 6213, 6187, 6234, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^{5/2}\,dx$$

$$\Big\downarrow \text{6187}$$

$$x\text{arcsinh}(ax)^{5/2}-\frac{5}{2}a\int \frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x\text{arcsinh}(ax)^{5/2}-\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\int \sqrt{\text{arcsinh}(ax)}dx}{2a}\right)$$

$$\Big\downarrow \text{6187}$$

$$x\text{arcsinh}(ax)^{5/2}-\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{1}{2}a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx\right)}{2a}\right)$$

$$\Big\downarrow \text{6234}$$

$$x\text{arcsinh}(ax)^{5/2}-\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{\int \frac{ax}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a}\right)}{2a}\right)$$

$$\Big\downarrow 3042$$

$$x\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}-\frac{\int -\frac{i\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a}\right)}{2a}\right)$$

$$\Big\downarrow 26$$

$$x\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\int \frac{\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{2a}\right)}{2a}\right)$$

$$\Big\downarrow 3789$$

$$x\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(\right.}{2a}\right.}{2a}\right.$$

$$\Big\downarrow 2611$$

$$x\text{arcsinh}(ax)^{5/2}-$$

$$\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(i\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{2a}\right.}{2a}\right.$$

$$\Big\downarrow 2633$$

$$x\text{arcsinh}(ax)^{5/2}-\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{2a}\right)}{2a}\right)$$

$$\Big\downarrow \textcolor{blue}{2634}$$

$$x\text{arcsinh}(ax)^{5/2}-\frac{5}{2}a\left(\frac{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{2a}\right)}{2a}\right)$$

input

```
Int[ArcSinh[a*x]^(5/2),x]
```

output

```
x*ArcSinh[a*x]^(5/2) - (5*a*((Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/a^2 -
(3*(x*Sqrt[ArcSinh[a*x]] + ((I/2)*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]
]] + (I/2)*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]]))/a))/(2*a)))/2
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result
default	$-\frac{-16\,\mathrm{arcsinh}(xa)^{\frac{5}{2}}\sqrt{\pi}\,ax+40\,\mathrm{arcsinh}(xa)^{\frac{3}{2}}\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}-60\sqrt{\mathrm{arcsinh}(xa)}\,\sqrt{\pi}\,ax-15\pi\,\mathrm{erf}\left(\sqrt{\mathrm{arcsinh}(xa)}\right)+15\pi\,\mathrm{erfi}\left(\sqrt{\mathrm{arcsin}}\right.}{16\sqrt{\pi}\,a}$

input
```
int(arcsinh(x*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-16*arcsinh(x*a)^(5/2)*Pi^(1/2)*a*x+40*arcsinh(x*a)^(3/2)*Pi^(1/2)*
(a^2*x^2+1)^(1/2)-60*arcsinh(x*a)^(1/2)*Pi^(1/2)*a*x-15*Pi*erf(arcsinh(x*a
)^(1/2))+15*Pi*erfi(arcsinh(x*a)^(1/2)))/Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \mathrm{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \mathrm{arcsinh}(ax)^{5/2}\,dx = \int \mathrm{asinh}^{\frac{5}{2}}(ax)\,dx$$

input
```
integrate(asinh(a*x)**(5/2),x)
```

output
```
Integral(asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \int \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \int \operatorname{asinh}(a\,x)^{5/2}\,dx$$

input
```
int(asinh(a*x)^(5/2),x)
```

output
```
int(asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \int \sqrt{asinh\left(ax\right)}\,asinh(ax)^{2}\,dx$$

input
```
int(asinh(a*x)^(5/2),x)
```

output
```
int(sqrt(asinh(a*x))*asinh(a*x)**2,x)
```

3.123 $\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^{5/2}}{x},x\right)$$

output
```
Defer(Int)(arcsinh(a*x)^(5/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[ArcSinh[a*x]^(5/2)/x,x]
```

output
```
Integrate[ArcSinh[a*x]^(5/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{\text{arcsinh}(ax)^{5/2}}{x} \, dx \\ \Big\downarrow \text{ 6196} \\ \int \frac{\text{arcsinh}(ax)^{5/2}}{x}dx \end{array}$$

input
```
Int[ArcSinh[a*x]^(5/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}\left(x a\right)^{\frac{5}{2}}}{x}dx$$

input
```
int(arcsinh(x*a)^(5/2)/x,x)
```

output
```
int(arcsinh(x*a)^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 18.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{asinh}^{\frac{5}{2}}(ax)}{x}\,dx$$

input
```
integrate(asinh(a*x)**(5/2)/x,x)
```

output
```
Integral(asinh(a*x)**(5/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{x}\,dx$$

input
```
int(asinh(a*x)^(5/2)/x,x)
```

output
```
int(asinh(a*x)^(5/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\sqrt{asinh\,(ax)}\,asinh(ax)^2}{x}dx$$

input
```
int(asinh(a*x)^(5/2)/x,x)
```

output
```
int((sqrt(asinh(a*x))*asinh(a*x)**2)/x,x)
```

3.124 $\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 163

$$\begin{aligned}\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx &= \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} \\ &\quad + \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} \\ &\quad - \frac{\sqrt{3\pi}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5}\end{aligned}$$

output

```
1/16*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^5-1/32*3^(1/2)*Pi^(1/2)*erf(3^(1/2
)*arcsinh(a*x)^(1/2))/a^5+1/160*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*arcsinh(a*x)^
(1/2))/a^5+1/16*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a^5-1/32*3^(1/2)*Pi^(1/2
)*erfi(3^(1/2)*arcsinh(a*x)^(1/2))/a^5+1/160*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)
*arcsinh(a*x)^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx$$
$$= \frac{\frac{\sqrt{5}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-5\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+\frac{5\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\frac{10\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-\text{arcsinh}(ax}{\sqrt{\text{arcsinh}(ax)}}}{160a^5}$$

input
```
Integrate[x^4/Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a*
x]] + (5*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[-Arc
Sinh[a*x]] + (10*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSi
nh[a*x]] - 10*Gamma[1/2, ArcSinh[a*x]] + 5*Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*
x]] - Sqrt[5]*Gamma[1/2, 5*ArcSinh[a*x]])/(160*a^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx$$
$$\Big\downarrow \; 6195$$
$$\frac{\int \frac{a^4x^4\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^5}$$
$$\Big\downarrow \; 5971$$

$$\frac{\int \left(-\frac{3\cosh(3\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{\cosh(5\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{\sqrt{a^2x^2+1}}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^5}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{\frac{1}{16}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{32}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{16}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsin}}\right.}{a^5}$$

input

```
Int[x^4/Sqrt[ArcSinh[a*x]],x]
```

output

```
((Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/16 - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcS
inh[a*x]]])/32 + (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[P
i]*Erfi[Sqrt[ArcSinh[a*x]]])/16 - (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*
x]]])/32 + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/32)/a^5
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))    Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(xa)}}dx$$

input
```
int(x^4/arcsinh(x*a)^(1/2),x)
```

output
```
int(x^4/arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(x**4/asinh(a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

input
```
int(x^4/asinh(a*x)^(1/2),x)
```

output
```
int(x^4/asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$
$$= \frac{2\sqrt{a^2x^2+1}\,\sqrt{asinh\,(ax)}\,a^4x^4 - \frac{8\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}}{3} + \frac{4\left(\int \frac{\sqrt{asinh(ax)}}{asinh(ax)a^2x^2+asinh(ax)}dx\right)a}{3} + \frac{4\left(\int \frac{\sqrt{asinh(ax)}\,x^2}{asinh(ax)a^2x^2+asinh(ax)}d\right.}{3}}{a^5}$$

input
```
int(x^4/asinh(a*x)^(1/2),x)
```

output
```
(2*(3*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*a**4*x**4 - 4*sqrt(a**2*x**2 +
1)*sqrt(asinh(a*x)) + 2*int(sqrt(asinh(a*x))/(asinh(a*x)*a**2*x**2 + asinh
(a*x)),x)*a + 2*int((sqrt(asinh(a*x))*x**2)/(asinh(a*x)*a**2*x**2 + asinh(
a*x)),x)*a**3 - 15*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**5)/(a**2*x
**2 + 1),x)*a**6 - 12*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**3)/(a**
2*x**2 + 1),x)*a**4 + 4*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x)/(a**2
*x**2 + 1),x)*a**2))/(3*a**5)
```

3.125 $\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^4}$$

output

```
-1/32*Pi^(1/2)*erf(2*arcsinh(a*x)^(1/2))/a^4+1/16*2^(1/2)*Pi^(1/2)*erf(2^(
1/2)*arcsinh(a*x)^(1/2))/a^4+1/32*Pi^(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a^4-
1/16*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-4\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} - 2\sqrt{2}\Gamma\left(\frac{1}{2},2\operatorname{arcsinh}(ax)\right) + \Gamma\left(\frac{1}{2},4\right.}{32a^4}$$

input
```
Integrate[x^3/Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + (2
*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x
]] - 2*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] + Gamma[1/2, 4*ArcSinh[a*x]])/(3
2*a^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \ 6195$$

$$\frac{\int \frac{a^3x^3\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \ 5971$$

$$\frac{\int \left(\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}-\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcs}}\right.}{a^4}$$

input
```
Int[x^3/Sqrt[ArcSinh[a*x]],x]
```

output
```
(-1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]]) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt
[ArcSinh[a*x]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32 - (Sqrt[Pi/2
]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8)/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(xa)}}dx$$

input
```
int(x^3/arcsinh(x*a)^(1/2),x)
```

output
```
int(x^3/arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(x**3/asinh(a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(x^3/asinh(a*x)^(1/2),x)
```

output
```
int(x^3/asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$
$$= \frac{2\sqrt{asinh\left(ax\right)}\,asinh(ax)+2\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh\left(ax\right)}\,a^{3}x^{3}-8\left(\int \frac{\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh(ax)}\,x^{4}}{a^{2}x^{2}+1}dx\right)a^{5}-6\Big(\int \sqrt{a^{2}x^{2}-}}{a^{4}}$$

input
```
int(x^3/asinh(a*x)^(1/2),x)
```

output

```
(2*sqrt(asinh(a*x))*asinh(a*x) + 2*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*a*
*3*x**3 - 8*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**4)/(a**2*x**2 + 1
),x)*a**5 - 6*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2)/(a**2*x**2 +
 1),x)*a**3 - 3*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/(a**2*x**2 + 1)
,x)*a)/a**4
```

3.126 $\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3}$$

output
```
-1/8*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^3+1/24*3^(1/2)*Pi^(1/2)*erf(3^(1/2
)*arcsinh(a*x)^(1/2))/a^3-1/8*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a^3+1/24*3
^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{3}\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}+\frac{3\sqrt{\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+3\Gamma\left(\frac{1}{2},\operatorname{arcsinh}(ax)\right)-\sqrt{3}\Gamma\bigl(\tfrac{1}{2},3\mathrm{ar}}{24a^3}$$

input
```
Integrate[x^2/Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[ArcSinh[a*
x]] + (3*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]]
 + 3*Gamma[1/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]])/(24*a^
3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \; 6195$$

$$\frac{\int \frac{a^2x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \; 5971$$

$$\frac{\int \left(\frac{\cosh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}-\frac{\sqrt{a^2x^2+1}}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(\right.}{a^3}$$

input
```
Int[x^2/Sqrt[ArcSinh[a*x]],x]
```

output
```
(-1/8*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]]) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[Ar
cSinh[a*x]]])/8 - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[Pi/3]*Erfi
[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/8)/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(xa)}}dx$$

input
```
int(x^2/arcsinh(x*a)^(1/2),x)
```

output
```
int(x^2/arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

input
```
integrate(x**2/asinh(a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(x^2/asinh(a*x)^(1/2),x)
```

output
```
int(x^2/asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{2\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}\,x^2-6\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^3}{a^2x^2+1}dx\right)a^2-4\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{a^2x^2+1}dx\right)}{a}$$

input
```
int(x^2/asinh(a*x)^(1/2),x)
```

output
```
(2*(sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2 - 3*int((sqrt(a**2*x**2 + 1)
*sqrt(asinh(a*x))*x**3)/(a**2*x**2 + 1),x)*a**2 - 2*int((sqrt(a**2*x**2 +
1)*sqrt(asinh(a*x))*x)/(a**2*x**2 + 1),x)))/a
```

3.127 $\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a^2}$$

output

```
-1/8*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^2+1/8*2^(1/2)*Pi^(
1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} + \Gamma\left(\frac{1}{2},2\text{arcsinh}(ax)\right)}{4\sqrt{2}a^2}$$

input

```
Integrate[x/Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Ga
mma[1/2, 2*ArcSinh[a*x]])/(4*Sqrt[2]*a^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \frac{\sinh(2\operatorname{arcsinh}(ax))}{2\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sinh(2\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i\sin(2i\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sin(2i\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \ 3789$$

$$-\frac{i\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{2a^2}$$

$$\Big\downarrow \ 2611$$

$$-\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{2a^2}$$

$$\Big\downarrow \ 2633$$

$$-\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{2a^2}$$

$$\Big\downarrow \ 2634$$

$$-\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{2a^2}$$

input

```
Int[x/Sqrt[ArcSinh[a*x]],x]
```

output

```
((-1/2*I)*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqr
t[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]]))/a^2
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{\sqrt{\pi}\,\sqrt{2}\,\left(\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(x a\right)}\right)-\operatorname{erfi}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(x a\right)}\right)\right)}{8 a^{2}}$	37

input
```
int(x/arcsinh(x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arcsinh(x*a)^(1/2))-erfi(2^(1/2)*arcsin
h(x*a)^(1/2)))/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(x/asinh(a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(x/asinh(a*x)^(1/2),x)
```

output
```
int(x/asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{-\frac{4\sqrt{asinh(ax)}\,asinh(ax)}{3}+2\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}\,ax-4\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^2}{a^2x^2+1}dx\right)a^3}{a^2}$$

input
```
int(x/asinh(a*x)^(1/2),x)
```

output

```
(2*( - 2*sqrt(asinh(a*x))*asinh(a*x) + 3*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*
x))*a*x - 6*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2)/(a**2*x**2 + 1
),x)*a**3))/(3*a**2)
```

3.128 $\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a}$$

output
```
1/2*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a+1/2*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2
))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} - \Gamma\left(\frac{1}{2},\operatorname{arcsinh}(ax)\right)}{2a}$$

input
```
Integrate[1/Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamm
a[1/2, ArcSinh[a*x]])/(2*a)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}}{a}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{a}$$
$$\Big\downarrow \text{ 2634}$$
$$\frac{\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{a}$$

input
```
Int[1/Sqrt[ArcSinh[a*x]],x]
```

output
```
((Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]]
)/2)/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(xa)}\right)+\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(xa)}\right)\right)}{2a}$	24

input
```
int(1/arcsinh(x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi^(1/2)*(erf(arcsinh(x*a)^(1/2))+erfi(arcsinh(x*a)^(1/2)))/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(1/asinh(a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(1/asinh(a*x)^(1/2),x)
```

output
```
int(1/asinh(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{2\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}-2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{a^2x^2+1}dx\right)a^2}{a}$$

input
```
int(1/asinh(a*x)^(1/2),x)
```

output
```
(2*(sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)) - int((sqrt(a**2*x**2 + 1)*sqrt(a
sinh(a*x))*x)/(a**2*x**2 + 1),x)*a**2))/a
```

3.129 $\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[ArcSinh[a*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[ArcSinh[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}dx$$

input
```
Int[1/(x*Sqrt[ArcSinh[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}\left(xa\right)}}dx$$

input
```
int(1/x/arcsinh(x*a)^(1/2),x)
```

output
```
int(1/x/arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(1/x/asinh(a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(asinh(a*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x*asinh(a*x)^(1/2)),x)
```

output
```
int(1/(x*asinh(a*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)x}dx$$

input

```
int(1/x/asinh(a*x)^(1/2),x)
```

output

```
int(sqrt(asinh(a*x))/(asinh(a*x)*x),x)
```

3.130 $\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

output
```
Defer(Int)(1/x^2/arcsinh(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\text{arcsinh}(ax)}}\,dx$$
$$\Big\downarrow \text{ 6196}$$
$$\int \frac{1}{x^2\sqrt{\text{arcsinh}(ax)}}dx$$

input
```
Int[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{\text{arcsinh}\left(xa\right)}}dx$$

input
```
int(1/x^2/arcsinh(x*a)^(1/2),x)
```

output
```
int(1/x^2/arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{asinh}(ax)}}\,dx$$

input
```
integrate(1/x**2/asinh(a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(asinh(a*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\,\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x^2*asinh(a*x)^(1/2)),x)
```

output
```
int(1/(x^2*asinh(a*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)x^{2}}dx$$

input

```
int(1/x^2/asinh(a*x)^(1/2),x)
```

output

```
int(sqrt(asinh(a*x))/(asinh(a*x)*x**2),x)
```

3.131 $\int \frac{x^4}{\textbf{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 188

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx &= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}\\ &\quad-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}\\ &\quad-\frac{\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}\\ &\quad-\frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}+\frac{\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}\end{aligned}$$

output

```
-2*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)-1/8*Pi^(1/2)*erf(arcsinh(a*x
)^(1/2))/a^5+3/16*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^5-1/1
6*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*arcsinh(a*x)^(1/2))/a^5+1/8*Pi^(1/2)*erfi(a
rcsinh(a*x)^(1/2))/a^5-3/16*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*x)^(1/
2))/a^5+1/16*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*arcsinh(a*x)^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{\text{arcsinh}(ax)^{3/2}}\,dx = \frac{e^{-5\text{arcsinh}(ax)}\left(-1+3e^{2\text{arcsinh}(ax)}-2e^{4\text{arcsinh}(ax)}-2e^{6\text{arcsinh}(ax)}+3e^{8\text{arcsinh}(ax)}-e^{10\text{ar}}\right.}{}$$

input
```
Integrate[x^4/ArcSinh[a*x]^(3/2),x]
```

output
```
(-1 + 3*E^(2*ArcSinh[a*x]) - 2*E^(4*ArcSinh[a*x]) - 2*E^(6*ArcSinh[a*x]) +
 3*E^(8*ArcSinh[a*x]) - E^(10*ArcSinh[a*x]) + Sqrt[5]*E^(5*ArcSinh[a*x])*S
qrt[-ArcSinh[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]] - 3*Sqrt[3]*E^(5*ArcSinh[a*
x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]] + 2*E^(5*ArcSinh[a*x])
*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] + 2*E^(5*ArcSinh[a*x])*Sqrt
[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]] - 3*Sqrt[3]*E^(5*ArcSinh[a*x])*Sqr
t[ArcSinh[a*x]]*Gamma[1/2, 3*ArcSinh[a*x]] + Sqrt[5]*E^(5*ArcSinh[a*x])*Sq
rt[ArcSinh[a*x]]*Gamma[1/2, 5*ArcSinh[a*x]])/(16*a^5*E^(5*ArcSinh[a*x])*Sq
rt[ArcSinh[a*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\text{arcsinh}(ax)^{3/2}}\,dx$$

$$\Big\downarrow \; 6193$$

$$\frac{2\int \left(\frac{ax}{8\sqrt{\text{arcsinh}(ax)}}-\frac{9\sinh(3\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}+\frac{5\sinh(5\text{arcsinh}(ax))}{16\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^5}-\frac{2x^4\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{1}{16}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{3}{32}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{32}\sqrt{5\pi}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{16}\sqrt{\pi}\text{erfi}\left(\sqrt{a}\right.\right.}{a^5}$$

$$\frac{2x^4\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

input
```
Int[x^4/ArcSinh[a*x]^(3/2),x]
```

output
```
(-2*x^4*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2*(-1/16*(Sqrt[Pi]*Er
f[Sqrt[ArcSinh[a*x]]]) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/32
 - (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/32 + (Sqrt[Pi]*Erfi[Sqrt[A
rcSinh[a*x]]])/16 - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/32 + (
Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/32))/a^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{x^4}{\text{arcsinh}\,(xa)^{\frac{3}{2}}}dx$$

input
```
int(x^4/arcsinh(x*a)^(3/2),x)
```

output
```
int(x^4/arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**4/asinh(a*x)**(3/2),x)
```

output
```
Integral(x**4/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x^4/asinh(a*x)^(3/2),x)
```

output
```
int(x^4/asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\frac{4 asinh(ax)\left(\int \frac{\sqrt{asinh(ax)}}{asinh(ax)^2 a^2 x^2 + asinh(ax)^2}dx\right)a}{3} + \frac{4 asinh(ax)\left(\int \frac{\sqrt{asinh(ax)}\,x^2}{asinh(ax)^2 a^2 x^2 + asinh(ax)^2}dx\right)a^3}{3} + 10 asinh}{}$$

input
```
int(x^4/asinh(a*x)^(3/2),x)
```

output

```
(2*(2*asinh(a*x)*int(sqrt(asinh(a*x))/(asinh(a*x)**2*a**2*x**2 + asinh(a*x
)**2),x)*a + 2*asinh(a*x)*int((sqrt(asinh(a*x))*x**2)/(asinh(a*x)**2*a**2*
x**2 + asinh(a*x)**2),x)*a**3 + 15*asinh(a*x)*int((sqrt(a**2*x**2 + 1)*sqr
t(asinh(a*x))*x**5)/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)*a**6 + 12*asinh
(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**3)/(asinh(a*x)*a**2*x**
2 + asinh(a*x)),x)*a**4 - 4*asinh(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh
(a*x))*x)/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)*a**2 - 3*sqrt(a**2*x**2 +
 1)*sqrt(asinh(a*x))*a**4*x**4 + 4*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))))/
(3*asinh(a*x)*a**5)
```

3.132 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 138

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4}$$

output

```
-2*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)+1/4*Pi^(1/2)*erf(2*arcsinh(a
*x)^(1/2))/a^4-1/4*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^4+1/
4*Pi^(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a^4-1/4*2^(1/2)*Pi^(1/2)*erfi(2^(1/2
)*arcsinh(a*x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-4\operatorname{arcsinh}(ax)\right)-\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)+}{}$$

input

```
Integrate[x^3/ArcSinh[a*x]^(3/2),x]
```

output

```
(Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[2]*Sqrt[-ArcSinh[a
*x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2
*ArcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + 2*Sinh[2*
ArcSinh[a*x]] - Sinh[4*ArcSinh[a*x]])/(4*a^4*Sqrt[ArcSinh[a*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{2\int \left(\frac{\cosh(4\operatorname{arcsinh}(ax))}{2\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\cosh(2\operatorname{arcsinh}(ax))}{2\sqrt{\operatorname{arcsinh}(ax)}}\right)d\operatorname{arcsinh}(ax)}{a^4}-\frac{2x^3\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{8}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsi}}\right.\right.}{a^4}$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

input

```
Int[x^3/ArcSinh[a*x]^(3/2),x]
```

output

```
(-2*x^3*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2*((Sqrt[Pi]*Erf[2*Sq
rt[ArcSinh[a*x]]])/8 - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4 + (S
qrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/8 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcS
inh[a*x]]])/4))/a^4
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{x^{3}}{\operatorname{arcsinh}\left(xa\right)^{\frac{3}{2}}}dx$$

input

```
int(x^3/arcsinh(x*a)^(3/2),x)
```

output

```
int(x^3/arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**3/asinh(a*x)**(3/2),x)
```

output
```
Integral(x**3/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/arcsinh(a*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^3}{{\mathrm{asinh}\left(a\,x\right)}^{3/2}} \,d x$$

input
```
int(x^3/asinh(a*x)^(3/2),x)
```

output
```
int(x^3/asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{-6\sqrt{asinh\left(ax\right)}\,asinh(ax)+8asinh(ax)\left(\int \frac{\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh(ax)}\,x^{4}}{asinh(ax)a^{2}x^{2}+asinh(ax)}d x\right)a^{5}+6asinh(a}{}$$

input
```
int(x^3/asinh(a*x)^(3/2),x)
```

output

```
( - 6*sqrt(asinh(a*x))*asinh(a*x) + 8*asinh(a*x)*int((sqrt(a**2*x**2 + 1)*
sqrt(asinh(a*x))*x**4)/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)*a**5 + 6*asi
nh(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2)/(asinh(a*x)*a**2*x
**2 + asinh(a*x)),x)*a**3 + 3*asinh(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(asi
nh(a*x)))/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)*a - 2*sqrt(a**2*x**2 + 1)
*sqrt(asinh(a*x))*a**3*x**3)/(asinh(a*x)*a**4)
```

3.133 $\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 130

$$\begin{aligned}\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx &= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} \\ &\quad+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3}-\frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} \\ &\quad-\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3}\end{aligned}$$

output

```
-2*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)+1/4*Pi^(1/2)*erf(arcsinh(a*x
)^(1/2))/a^3-1/4*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^3-1/4*
Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a^3+1/4*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*ar
csinh(a*x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{-e^{-3\operatorname{arcsinh}(ax)} + e^{-\operatorname{arcsinh}(ax)} + e^{\operatorname{arcsinh}(ax)} - e^{3\operatorname{arcsinh}(ax)} + \sqrt{3}\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2}, -\right.}{}$$

input
```
Integrate[x^2/ArcSinh[a*x]^(3/2),x]
```

output
```
(-E^(-3*ArcSinh[a*x]) + E^(-ArcSinh[a*x]) + E^ArcSinh[a*x] - E^(3*ArcSinh[
a*x]) + Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]] - Sqrt[-Ar
cSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcS
inh[a*x]] + Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 3*ArcSinh[a*x]])/(4*a^3*
Sqrt[ArcSinh[a*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{2\int \left(\frac{3\sinh(3\operatorname{arcsinh}(ax))}{4\sqrt{\operatorname{arcsinh}(ax)}} - \frac{ax}{4\sqrt{\operatorname{arcsinh}(ax)}}\right)d\operatorname{arcsinh}(ax)}{a^3} - \frac{2x^2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsin}}\right.}{a^3}$$

$$\frac{2x^2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

input
```
Int[x^2/ArcSinh[a*x]^(3/2),x]
```

output
```
(-2*x^2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2*((Sqrt[Pi]*Erf[Sqrt
[ArcSinh[a*x]]])/8 - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/8 - (Sqr
t[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[
a*x]]])/8))/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(xa)^{\frac{3}{2}}}dx$$

input
```
int(x^2/arcsinh(x*a)^(3/2),x)
```

output
```
int(x^2/arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x)**(3/2),x)
```

output
```
Integral(x**2/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x^2/asinh(a*x)^(3/2),x)
```

output
```
int(x^2/asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\frac{6asinh(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^3}{asinh(ax)a^2x^2+asinh(ax)}dx\right)a^2+4asinh(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)a^2x^2+asinh(ax)}dx\right)}{asinh\left(ax\right)a}$$

input
```
int(x^2/asinh(a*x)^(3/2),x)
```

output
```
(2*(3*asinh(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**3)/(asinh(a*
x)*a**2*x**2 + asinh(a*x)),x)*a**2 + 2*asinh(a*x)*int((sqrt(a**2*x**2 + 1)
*sqrt(asinh(a*x))*x)/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x) - sqrt(a**2*x*
*2 + 1)*sqrt(asinh(a*x))*x**2))/(asinh(a*x)*a)
```

3.134 $\int \frac{x}{\textbf{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2}$$

output
```
-2*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)+1/2*2^(1/2)*Pi^(1/2)*erf(2^(1/
2)*arcsinh(a*x)^(1/2))/a^2+1/2*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^
(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\Gamma\left(\frac{1}{2},2\operatorname{arcsinh}(ax)\right)}{\sqrt{2}} - \frac{\sinh(2\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}}{a^2}$$

input
```
Integrate[x/ArcSinh[a*x]^(3/2),x]
```

output
```
((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/(Sqrt[2]*Sqrt[ArcSinh[a
*x]]) - Gamma[1/2, 2*ArcSinh[a*x]]/Sqrt[2] - Sinh[2*ArcSinh[a*x]]/Sqrt[Arc
Sinh[a*x]])/a^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(ax)^{3/2}}dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{2\int \frac{\cosh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{2\left(\frac{1}{2}\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \; 2633$$

$$\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \; 2634$$

$$\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}$$

input
```
Int[x/ArcSinh[a*x]^(3/2),x]
```

output
```
(-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2*((Sqrt[Pi/2]*Erf[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])
/2))/a^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

method	result	siz
default	$-\frac{\sqrt{2}\left(2\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(x a\right)}\,\sqrt{\pi}\,\sqrt{a^{2} x^{2}+1}\,a x-\operatorname{arcsinh}\left(x a\right) \pi \,\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(x a\right)}\right)-\operatorname{arcsinh}\left(x a\right) \pi \,\operatorname{erfi}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(x a\right)}\right)\right)}{2 \sqrt{\pi}\, a^{2} \operatorname{arcsinh}\left(x a\right)}$	82

input
```
int(x/arcsinh(x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*(2*2^(1/2)*arcsinh(x*a)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-
arcsinh(x*a)*Pi*erf(2^(1/2)*arcsinh(x*a)^(1/2))-arcsinh(x*a)*Pi*erfi(2^(1/
2)*arcsinh(x*a)^(1/2)))/Pi^(1/2)/a^2/arcsinh(x*a)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/asinh(a*x)**(3/2),x)
```

output
```
Integral(x/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output

```
integrate(x/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

input

```
int(x/asinh(a*x)^(3/2),x)
```

output

```
int(x/asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{4\sqrt{asinh\left(ax\right)}\,asinh(ax) + 4asinh(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^2}{asinh(ax)a^2x^2+asinh(ax)}dx\right)a^3 - 2\sqrt{a^2x^2 +}{asinh\left(ax\right)a^2}$$

input

```
int(x/asinh(a*x)^(3/2),x)
```

output

```
(2*(2*sqrt(asinh(a*x))*asinh(a*x) + 2*asinh(a*x)*int((sqrt(a**2*x**2 + 1)*
sqrt(asinh(a*x))*x**2)/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)*a**3 - sqrt(
a**2*x**2 + 1)*sqrt(asinh(a*x))*a*x))/(asinh(a*x)*a**2)
```

3.135 $\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 64

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a}$$

output
```
-2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)-Pi^(1/2)*erf(arcsinh(a*x)^(1/2))
/a+Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{-e^{-\operatorname{arcsinh}(ax)} - e^{\operatorname{arcsinh}(ax)} + \sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2}, -\operatorname{arcsinh}(ax)\right) + \sqrt{\operatorname{arcsinh}(ax)}\,\Gamma}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

input
```
Integrate[ArcSinh[a*x]^(-3/2),x]
```

output
```
(-E^(-ArcSinh[a*x]) - E^ArcSinh[a*x] + Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -Arc
Sinh[a*x]] + Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]])/(a*Sqrt[ArcSinh[
a*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6188, 6234, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}dx$$

$$\Big\downarrow \text{6188}$$

$$2a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}dx-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{2\int \frac{ax}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\int -\frac{i\sin(i\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{2i\int \frac{\sin(i\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\int\frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a}$$

$$\Big\downarrow 2611$$

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(i\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a}$$

$$\Big\downarrow 2633$$

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a}$$

$$\Big\downarrow 2634$$

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a}$$

input

```
Int[ArcSinh[a*x]^(-3/2),x]
```

output

```
(-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - ((2*I)*((-1/2*I)*Sqrt[Pi]*
Erf[Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]]))/a
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\operatorname{arcsinh}\left(x a \right) \pi \,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}\left(x a \right)}\right)-\operatorname{arcsinh}\left(x a \right) \pi \,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}\left(x a \right)}\right)+2 \sqrt{\operatorname{arcsinh}\left(x a \right)}\, \sqrt{\pi}\, \sqrt{a^{2} x^{2}+1}}{\sqrt{\pi}\, a \,\operatorname{arcsinh}\left(x a \right)}$	65

input
```
int(1/arcsinh(x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(arcsinh(x*a)*Pi*erf(arcsinh(x*a)^(1/2))-arcsinh(x*a)*Pi*erfi(arcsinh(x*a
)^(1/2))+2*arcsinh(x*a)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsi
nh(x*a)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/asinh(a*x)**(3/2),x)
```

output
```
Integral(asinh(a*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/asinh(a*x)^(3/2),x)
```

output
```
int(1/asinh(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{2 asinh(ax) \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)a^2x^2+asinh(ax)}dx\right) a^2 - 2\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)a}$$

input

```
int(1/asinh(a*x)^(3/2),x)
```

output

```
(2*(asinh(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*x)*a*
*2*x**2 + asinh(a*x)),x)*a**2 - sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))))/(as
inh(a*x)*a)
```

3.136 $\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{6196}$$
$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}dx$$

input
```
Int[1/(x*ArcSinh[a*x]^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \operatorname{arcsinh}\left(x a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/arcsinh(x*a)^(3/2),x)
```

output
```
int(1/x/arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x/asinh(a*x)**(3/2),x)
```

output
```
Integral(1/(x*asinh(a*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)^{3/2}}\,dx$$

input
```
int(1/(x*asinh(a*x)^(3/2)),x)
```

output
```
int(1/(x*asinh(a*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)^{2}x}dx$$

input

```
int(1/x/asinh(a*x)^(3/2),x)
```

output

```
int(sqrt(asinh(a*x))/(asinh(a*x)**2*x),x)
```

3.137 $\int \frac{x^3}{\mathbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 167

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{2\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4}$$

output

```
-2/3*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-4*x^2/a^2/arcsinh(a*x)^(1/
2)-16/3*x^4/arcsinh(a*x)^(1/2)-2/3*Pi^(1/2)*erf(2*arcsinh(a*x)^(1/2))/a^4+
1/3*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^4+2/3*Pi^(1/2)*erfi
(2*arcsinh(a*x)^(1/2))/a^4-1/3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^
(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{\text{arcsinh}(ax)^{5/2}}\,dx = \frac{-4e^{-4\text{arcsinh}(ax)}\text{arcsinh}(ax)+4e^{-2\text{arcsinh}(ax)}\text{arcsinh}(ax)+4e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)-}{}$$

input
```
Integrate[x^3/ArcSinh[a*x]^(5/2),x]
```

output
```
((-4*ArcSinh[a*x])/E^(4*ArcSinh[a*x]) + (4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]
) + 4*E^(2*ArcSinh[a*x])*ArcSinh[a*x] - 4*E^(4*ArcSinh[a*x])*ArcSinh[a*x]
- 8*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] + 4*Sqrt[2]*(-ArcSin
h[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] - 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*G
amma[1/2, 2*ArcSinh[a*x]] + 8*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]
] + 2*Sinh[2*ArcSinh[a*x]] - Sinh[4*ArcSinh[a*x]])/(12*a^4*ArcSinh[a*x]^(3
/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6194, 6233, 6195, 5971, 27, 2009, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arcsinh}(ax)^{5/2}}\,dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{2\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}dx}{a}+\frac{8}{3}a\int \frac{x^4}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}dx-\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{2\left(\frac{4\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)}{a}+\frac{8}{3}a\left(\frac{8\int \frac{x^3}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^4}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 6195$$

$$\frac{2\left(\frac{4\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)}{a}+\frac{8}{3}a\left(\frac{8\int \frac{a^3x^3\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^5}-\frac{2x^4}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 5971$$

$$\frac{8}{3}a\left(\frac{8\int \left(\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}-\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^5}-\frac{2x^4}{a\sqrt{\text{arcsinh}(ax)}}\right)+\frac{2\left(\frac{4\int \frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)}{a}-\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{8}{3}a\left(\frac{8\int \left(\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}-\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^5}-\frac{2x^4}{a\sqrt{\text{arcsinh}(ax)}}\right)+\frac{2\left(\frac{2\int \frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)}{a}-\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(\frac{2\int \frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)}{a}+$$

$$\frac{8}{3}a\Bigg(\frac{8\Big(-\frac{1}{32}\sqrt{\pi}\text{erf}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\Big(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{32}\sqrt{\pi}\text{erfi}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\Big(\surd}{a^5}$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}\right)}{a}+$$

$$\frac{8}{3}a\Bigg(\frac{8\Big(-\frac{1}{32}\sqrt{\pi}\text{erf}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\Big(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{32}\sqrt{\pi}\text{erfi}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\Big(\surd}{a^5}$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 26$$

$$\frac{2\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\int \frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}\right)}{a}+$$

$$\frac{8}{3}a\Bigg(\frac{8\Big(-\frac{1}{32}\sqrt{\pi}\text{erf}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\Big(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{32}\sqrt{\pi}\text{erfi}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\Big(\surd}{a^5}$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 3789$$

$$\frac{2\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^3}\right)}{a}+$$

$$\frac{8}{3}a\Bigg(\frac{8\Big(-\frac{1}{32}\sqrt{\pi}\text{erf}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\Big(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{32}\sqrt{\pi}\text{erfi}\Big(2\sqrt{\text{arcsinh}(ax)}\Big)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\Big(\surd}{a^5}$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^3}\right)}{a}+$$

$$\frac{8}{3}a\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{\right.}\right.}{a^5}\right.$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^3}\right)}{a}+$$

$$\frac{8}{3}a\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{\right.}\right.}{a^5}\right.$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{8}{3}a\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{\right.}\right.}{a^5}\right.$$

$$\frac{2\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right)}{a}-$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

input

```
Int[x^3/ArcSinh[a*x]^(5/2),x]
```

output

```
(-2*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) + (8*a*((-2*x^4)/(a*Sq
rt[ArcSinh[a*x]]) + (8*(-1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]]) + (Sqrt
[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[
a*x]]])/32 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/8))/a^5))/3 + (
2*((-2*x^2)/(a*Sqrt[ArcSinh[a*x]]) - ((2*I)*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[
2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]]
))/a^3))/a
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(xa)^{\frac{5}{2}}}\,dx$$

input
```
int(x^3/arcsinh(x*a)^(5/2),x)
```

output
```
int(x^3/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**3/asinh(a*x)**(5/2),x)
```

output
```
Integral(x**3/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3/arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^3}{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int(x^3/asinh(a*x)^(5/2),x)
```

output
```
int(x^3/asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{8 asinh(ax)^2 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^4}{asinh(ax)^2a^2x^2+asinh(ax)^2}dx\right) a^5 + 6 asinh(ax)^2 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)^2a^2x^2+asinh(a}\right.}{}$$

input
```
int(x^3/asinh(a*x)^(5/2),x)
```

output
```
(8*asinh(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**4)/(asinh(a*
x)**2*a**2*x**2 + asinh(a*x)**2),x)*a**5 + 6*asinh(a*x)**2*int((sqrt(a**2*
x**2 + 1)*sqrt(asinh(a*x))*x**2)/(asinh(a*x)**2*a**2*x**2 + asinh(a*x)**2)
,x)*a**3 + 3*asinh(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/(asi
nh(a*x)**2*a**2*x**2 + asinh(a*x)**2),x)*a + 6*sqrt(asinh(a*x))*asinh(a*x)
 - 2*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*a**3*x**3)/(3*asinh(a*x)**2*a**4
)
```

3.138 $\int \frac{x^2}{\mathbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 161

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3}$$

output

```
-2/3*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-8/3*x/a^2/arcsinh(a*x)^(1/
2)-4*x^3/arcsinh(a*x)^(1/2)-1/6*Pi^(1/2)*erf(arcsinh(a*x)^(1/2))/a^3+1/2*3
^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a^3-1/6*Pi^(1/2)*erfi(arcs
inh(a*x)^(1/2))/a^3+1/2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*arcsinh(a*x)^(1/2))/
a^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{-\frac{e^{3\operatorname{arcsinh}(ax)}(1+6\operatorname{arcsinh}(ax))+6\sqrt{3}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2},-3\operatorname{arcsinh}(ax)\right)}{12\operatorname{arcsinh}(ax)^{3/2}}+\frac{e^{\operatorname{arcsinh}(ax)}(1+2\mathrm{ar}}{}}{}$$

input
```
Integrate[x^2/ArcSinh[a*x]^(5/2),x]
```

output
```
(-1/12*(E^(3*ArcSinh[a*x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x]
)^(3/2)*Gamma[1/2, -3*ArcSinh[a*x]])/ArcSinh[a*x]^(3/2) + (E^ArcSinh[a*x]*
(1 + 2*ArcSinh[a*x]) + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/
(12*ArcSinh[a*x]^(3/2)) + (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a
*x]^(3/2)*Gamma[1/2, ArcSinh[a*x]])/(12*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2))
 + (-(1/(E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2))) + 6/(E^(3*ArcSinh[a*x])*S
qrt[ArcSinh[a*x]]) - 6*Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]])/12)/a^3
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6194, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{4\int \frac{x}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{3/2}}dx}{3a}+2a\int \frac{x^3}{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{3/2}}dx-\frac{2x^2\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6233}$$

$$2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)+\frac{4\left(\frac{2\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \textbf{6189}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}+2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4\left(-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}\right)}{3a}+2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \textbf{3788}$$

$$\frac{4\left(-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}\right)}{3a}+2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{4\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}+2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 2611$$

$$\frac{4\left(\frac{2\left(\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}+2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 2633$$

$$\frac{4\left(\frac{2\left(\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}+2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 2634$$

$$2a\left(\frac{6\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 6195$$

$$2a\left(\frac{6\int \frac{a^2x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^4}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)+$$
$$\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \textbf{5971}$$

$$2a\left(\frac{6\int \left(\frac{\cosh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}-\frac{\sqrt{a^2x^2+1}}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^4}-\frac{2x^3}{a\sqrt{\text{arcsinh}(ax)}}\right)+$$
$$\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \textbf{2009}$$

$$2a\Bigg(\frac{6\Big(-\frac{1}{8}\sqrt{\pi}\text{erf}\Big(\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erf}\Big(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Big)-\frac{1}{8}\sqrt{\pi}\text{erfi}\Big(\sqrt{\text{arcsinh}(ax)}\Big)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\text{erfi}\Big(\sqrt{3}\sqrt{a}}{a^4}$$
$$\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

input
```
Int[x^2/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) + (4*((-2*x)/(a*Sqrt[A
rcSinh[a*x]]) + (2*((Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi]*Erfi[
Sqrt[ArcSinh[a*x]]])/2))/a^2))/(3*a) + 2*a*((-2*x^3)/(a*Sqrt[ArcSinh[a*x]]
) + (6*(-1/8*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]]) + (Sqrt[Pi/3]*Erf[Sqrt[3]*
Sqrt[ArcSinh[a*x]]])/8 - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[Pi/
3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/8))/a^4)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(xa)^{\frac{5}{2}}}dx$$

input
```
int(x^2/arcsinh(x*a)^(5/2),x)
```

output
```
int(x^2/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x)**(5/2),x)
```

output
```
Integral(x**2/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int(x^2/asinh(a*x)^(5/2),x)
```

output
```
int(x^2/asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{2 asinh(ax)^2 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^3}{asinh(ax)^2a^2x^2+asinh(ax)^2}dx\right) a^2 + \frac{4 asinh(ax)^2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)^2a^2x^2+asinh(ax)^2}dx\right)}{3} -}{asinh\left(ax\right)^2 a}$$

input
```
int(x^2/asinh(a*x)^(5/2),x)
```

output
```
(2*(3*asinh(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**3)/(asinh
(a*x)**2*a**2*x**2 + asinh(a*x)**2),x)*a**2 + 2*asinh(a*x)**2*int((sqrt(a*
*2*x**2 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*x)**2*a**2*x**2 + asinh(a*x)**2)
,x) - sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2))/(3*asinh(a*x)**2*a)
```

3.139 $\int \frac{x}{\textbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{8x^2}{3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2\sqrt{2\pi}\,\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^2} + \frac{2\sqrt{2\pi}\,\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^2}$$

output

```
-2/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-4/3/a^2/arcsinh(a*x)^(1/2)-8
/3*x^2/arcsinh(a*x)^(1/2)-2/3*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1
/2))/a^2+2/3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\operatorname{arcsinh}(ax)\Big(e^{-2\operatorname{arcsinh}(ax)} + e^{2\operatorname{arcsinh}(ax)} - \sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{1}{2}, -2\operatorname{arcsinh}(ax)\right) - \sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^2\operatorname{arcsinh}(ax)^{3/2}}$$

input

```
Integrate[x/ArcSinh[a*x]^(5/2),x]
```

output

```
-1/3*(2*ArcSinh[a*x]*(E^(-2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x]) - Sqrt[2]*S
qrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]
]*Gamma[1/2, 2*ArcSinh[a*x]]) + Sinh[2*ArcSinh[a*x]])/(a^2*ArcSinh[a*x]^(3
/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6194, 6198, 6233, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(ax)^{5/2}}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{2\int \frac{1}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}dx}{3a}+\frac{4}{3}a\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}dx-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{4}{3}a\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}dx-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{4}{3}a\left(\frac{4\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{6195}$$

$$\frac{4}{3}a\left(\frac{4\int \frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{4}{3}a\left(\frac{4\int \frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{3}a\left(\frac{2\int \frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int -\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{4}{3}a\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\int \frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^3}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{3789}$$

$$\frac{4}{3}a\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^3}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{4}{3}a\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^3}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \; 2633$$

$$\frac{4}{3}a\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^3}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

$$\Big\downarrow \; 2634$$

$$\frac{4}{3}a\left(-\frac{2x^2}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right)-\frac{2x\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}$$

input
```
Int[x/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcSinh[
a*x]]) + (4*a*((-2*x^2)/(a*Sqrt[ArcSinh[a*x]]) - ((2*I)*((-1/2*I)*Sqrt[Pi/
2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[Ar
cSinh[a*x]]]))/a^3))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

method	result
default	$-\frac{\sqrt{2}\left(4\operatorname{arcsinh}(xa)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,a^{2}x^{2}+\sqrt{2}\,\sqrt{\operatorname{arcsinh}(xa)}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,ax+2\operatorname{arcsinh}(xa)^{2}\pi\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(xa)}\right)-2\operatorname{arcsinh}(xa)^{2}\right.}{3\sqrt{\pi}\,a^{2}\operatorname{arcsinh}(xa)^{2}}$

input

```
int(x/arcsinh(x*a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*2^(1/2)*(4*arcsinh(x*a)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+2^(1/2)*arcsin
h(x*a)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x+2*arcsinh(x*a)^2*Pi*erf(2^(1/2
)*arcsinh(x*a)^(1/2))-2*arcsinh(x*a)^2*Pi*erfi(2^(1/2)*arcsinh(x*a)^(1/2))
+2*arcsinh(x*a)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(x*a)^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x/asinh(a*x)**(5/2),x)
```

output
```
Integral(x/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int(x/asinh(a*x)^(5/2),x)
```

output
```
int(x/asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{\frac{4 asinh(ax)^2 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^2}{asinh(ax)^2a^2x^2+asinh(ax)^2}dx\right)a^3}{3} - \frac{4\sqrt{asinh(ax)}\,asinh(ax)}{3} - \frac{2\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,ax}{3}}{asinh\left(ax\right)^2 a^2}$$

input
```
int(x/asinh(a*x)^(5/2),x)
```

output
```
(2*(2*asinh(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2)/(asinh
(a*x)**2*a**2*x**2 + asinh(a*x)**2),x)*a**3 - 2*sqrt(asinh(a*x))*asinh(a*x
) - sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*a*x))/(3*asinh(a*x)**2*a**2)
```

3.140 $\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 84

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-4/3*x/arcsinh(a*x)^(1/2)+2/3*P
i^(1/2)*erf(arcsinh(a*x)^(1/2))/a+2/3*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx=$$

$$-\frac{e^{-\operatorname{arcsinh}(ax)}\left(1+e^{2\operatorname{arcsinh}(ax)}-2\operatorname{arcsinh}(ax)+2e^{2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax)+2e^{\operatorname{arcsinh}(ax)}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma(\right.}{3a\operatorname{arcsinh}(ax)^{3/2}}$$

input
```
Integrate[ArcSinh[a*x]^(-5/2),x]
```

output
```
-1/3*(1 + E^(2*ArcSinh[a*x]) - 2*ArcSinh[a*x] + 2*E^(2*ArcSinh[a*x])*ArcSi
nh[a*x] + 2*E^ArcSinh[a*x]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]]
 + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, ArcSinh[a*x]])/(a*E^ArcS
inh[a*x]*ArcSinh[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6188, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{5/2}}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{2}{3}a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}dx-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}dx}{a}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6189}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2}{3}a\left(-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}\right)$$

$$\Big\downarrow \text{3788}$$

$$-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2}{3}a\left(-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{2}{3}a\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \; 2611$$

$$\frac{2}{3}a\left(\frac{2\left(\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \; 2633$$

$$\frac{2}{3}a\left(\frac{2\left(\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \; 2634$$

$$\frac{2}{3}a\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x}{a\sqrt{\text{arcsinh}(ax)}}\right)-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}}$$

input

```
Int[ArcSinh[a*x]^(-5/2),x]
```

output

```
(-2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) + (2*a*((-2*x)/(a*Sqrt[Arc
Sinh[a*x]]) + (2*((Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi]*Erfi[Sq
rt[ArcSinh[a*x]]])/2))/a^2))/3
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6188

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{-\frac{4 \operatorname{arcsinh}\left(x a \right)^{\frac{3}{2}} \sqrt{\pi}\, a x}{3}+\frac{2 \operatorname{arcsinh}\left(x a \right)^{2} \pi \,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}\left(x a \right)}\right)}{3}+\frac{2 \operatorname{arcsinh}\left(x a \right)^{2} \pi \,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}\left(x a \right)}\right)}{3}-\frac{2 \sqrt{\operatorname{arcsinh}\left(x a \right)}\, \sqrt{\pi}\, \sqrt{a^{2} x^{2}+1}}{3}}{\sqrt{\pi}\, a \operatorname{arcsinh}\left(x a \right)^{2}}$	81

input

```
int(1/arcsinh(x*a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*(-2*arcsinh(x*a)^(3/2)*Pi^(1/2)*a*x+arcsinh(x*a)^2*Pi*erf(arcsinh(x*a)
^(1/2))+arcsinh(x*a)^2*Pi*erfi(arcsinh(x*a)^(1/2))-arcsinh(x*a)^(1/2)*Pi^(
1/2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh(x*a)^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/asinh(a*x)**(5/2),x)
```

output
```
Integral(asinh(a*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output

```
integrate(arcsinh(a*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

input

```
int(1/asinh(a*x)^(5/2),x)
```

output

```
int(1/asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{\frac{2 asinh(ax)^2 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)^2a^2x^2+asinh(ax)^2}dx\right)a^2}{3} - \frac{2\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}}{3}}{asinh\left(ax\right)^2 a}$$

input

```
int(1/asinh(a*x)^(5/2),x)
```

output

```
(2*(asinh(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*x)
**2*a**2*x**2 + asinh(a*x)**2),x)*a**2 - sqrt(a**2*x**2 + 1)*sqrt(asinh(a*
x))))/(3*asinh(a*x)**2*a)
```

3.141 $\int \frac{1}{x\textbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx=\text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}dx$$

input
```
Int[1/(x*ArcSinh[a*x]^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \operatorname{arcsinh}\left(x a\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/arcsinh(x*a)^(5/2),x)
```

output
```
int(1/x/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 6.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{asinh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

input
```
integrate(1/x/asinh(a*x)**(5/2),x)
```

output
```
Integral(1/(x*asinh(a*x)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output

```
integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output

```
integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)^{5/2}}\,dx$$

input

```
int(1/(x*asinh(a*x)^(5/2)),x)
```

output

```
int(1/(x*asinh(a*x)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx=\int \frac{\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)^{3}x}dx$$

input
```
int(1/x/asinh(a*x)^(5/2),x)
```

output
```
int(sqrt(asinh(a*x))/(asinh(a*x)**3*x),x)
```

3.142 $\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 229

$$\begin{aligned}\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}\,dx = &-\frac{2x^3\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x^2}{5a^2\text{arcsinh}(ax)^{3/2}}\\ &-\frac{16x^4}{15\text{arcsinh}(ax)^{3/2}}-\frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}}-\frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\ &+\frac{16\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}\\ &+\frac{16\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}\end{aligned}$$

output

```
-2/5*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-4/5*x^2/a^2/arcsinh(a*x)^(
3/2)-16/15*x^4/arcsinh(a*x)^(3/2)-16/5*x*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x
)^(1/2)-128/15*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)+16/15*Pi^(1/2)*e
rf(2*arcsinh(a*x)^(1/2))/a^4-4/15*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x
)^(1/2))/a^4+16/15*Pi^(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a^4-4/15*2^(1/2)*Pi
^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}\,dx = 4\text{arcsinh}(ax)\left(e^{-2\text{arcsinh}(ax)}(1-4\text{arcsinh}(ax))+e^{2\text{arcsinh}(ax)}(1+4\text{arcsinh}(ax))+4$$

input
```
Integrate[x^3/ArcSinh[a*x]^(7/2),x]
```

output
```
(4*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSinh[a*
x])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*A
rcSinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]) -
4*ArcSinh[a*x]*((1 - 8*ArcSinh[a*x])/E^(4*ArcSinh[a*x]) + E^(4*ArcSinh[a*x
])*(1 + 8*ArcSinh[a*x]) + 16*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a
*x]] + 16*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]]) + 6*Sinh[2*ArcSin
h[a*x]] - 3*Sinh[4*ArcSinh[a*x]])/(60*a^4*ArcSinh[a*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6194, 6233, 6193, 2009, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{6\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx}{5a}+\frac{8}{5}a\int \frac{x^4}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx-\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{6\left(\frac{4\int \frac{x}{\text{arcsinh}(ax)^{3/2}}dx}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}+\frac{8}{5}a\left(\frac{8\int \frac{x^3}{\text{arcsinh}(ax)^{3/2}}dx}{3a}-\frac{2x^4}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \ 6193$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{\cosh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}+\frac{8}{5}a\left(\frac{8\left(\frac{2\int \left(\frac{\cosh(4\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}-\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^4}-\frac{2x^3\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^4}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{\cosh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{8}{5}a\Bigg(\frac{8\Bigg(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{a^4}}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{6\left(-\frac{2x^2}{3a\operatorname{arcsinh}(ax)^{3/2}}+\frac{4\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\int\frac{\sin\left(2i\operatorname{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a^2}\right)}{3a}\right)}{5a}-\frac{2x^3\sqrt{a^2x^2+1}}{5a\operatorname{arcsinh}(ax)^{5/2}}+\frac{8}{5}a\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)\right.}{a^4}\right.}{3a}\right.$$

$$\Big\downarrow \ 3788$$

$$\frac{6\left(-\frac{2x^2}{3a\operatorname{arcsinh}(ax)^{3/2}}+\frac{4\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int-\frac{ie^{2\operatorname{arcsinh}(ax)}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)-\frac{1}{2}i\int\frac{ie^{-2\operatorname{arcsinh}(ax)}}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)\right)}{a^2}\right)}{3a}\right)}{5a}-\frac{2x^3\sqrt{a^2x^2+1}}{5a\operatorname{arcsinh}(ax)^{5/2}}+\frac{8}{5}a\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)\right.}{a^4}\right.}{3a}\right.$$

$$\Big\downarrow \ 26$$

$$\frac{6\left(\frac{4\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{8}{5}a\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right.}{a^4}\right.}{3a}\right.$$

$$\Big\downarrow \; 2611$$

$$\frac{6\left(\frac{4\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-$$

$$\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{8}{5}a\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right.}{a^4}\right.}{3a}\right.$$

$$\Big\downarrow \; 2633$$

$$\frac{6\left(\frac{4\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{8}{5}a\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right.}{a^4}\right.}{3a}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{6\left(\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-\frac{2x^3\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{8}{5}a\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right.}{a^4}\right.}{3a}\right.$$

input
```
Int[x^3/ArcSinh[a*x]^(7/2),x]
```

output

```
(-2*x^3*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) + (8*a*((-2*x^4)/(3*a*
ArcSinh[a*x]^(3/2)) + (8*((-2*x^3*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]
) + (2*((Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/8 - (Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]])/4 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/8 - (Sqrt[Pi
/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4))/a^4))/(3*a)))/5 + (6*((-2*x^2)/(
3*a*ArcSinh[a*x]^(3/2)) + (4*((-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x
]]) + (2*((Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi/2]*Erf
i[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/2))/a^2))/(3*a)))/(5*a)
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(xa)^{\frac{7}{2}}}dx$$

input
```
int(x^3/arcsinh(x*a)^(7/2),x)
```

output
```
int(x^3/arcsinh(x*a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x**3/asinh(a*x)**(7/2),x)
```

output
```
Integral(x**3/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x^3/arcsinh(a*x)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)^{7/2}} \,d x$$

input
```
int(x^3/asinh(a*x)^(7/2),x)
```

output
```
int(x^3/asinh(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{8 asinh(ax)^3 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^4}{asinh(ax)^3a^2x^2+asinh(ax)^3}dx\right) a^5 + 6 asinh(ax)^3 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)^3a^2x^2+asinh(a}\right.}{}$$

input
```
int(x^3/asinh(a*x)^(7/2),x)
```

output

```
(8*asinh(a*x)**3*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**4)/(asinh(a*
x)**3*a**2*x**2 + asinh(a*x)**3),x)*a**5 + 6*asinh(a*x)**3*int((sqrt(a**2*
x**2 + 1)*sqrt(asinh(a*x))*x**2)/(asinh(a*x)**3*a**2*x**2 + asinh(a*x)**3)
,x)*a**3 + 3*asinh(a*x)**3*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/(asi
nh(a*x)**3*a**2*x**2 + asinh(a*x)**3),x)*a + 2*sqrt(asinh(a*x))*asinh(a*x)
 - 2*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*a**3*x**3)/(5*asinh(a*x)**3*a**4
)
```

3.143 $\int \frac{x^2}{\mathbf{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 222

$$
\begin{aligned}
\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = &-\frac{2x^2\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}}-\frac{8x}{15a^2\operatorname{arcsinh}(ax)^{3/2}} \\
&-\frac{4x^3}{5\operatorname{arcsinh}(ax)^{3/2}}-\frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\operatorname{arcsinh}(ax)}}-\frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\operatorname{arcsinh}(ax)}} \\
&+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^3}-\frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^3} \\
&-\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^3}
\end{aligned}
$$

output

```
-2/5*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-8/15*x/a^2/arcsinh(a*x)^(3
/2)-4/5*x^3/arcsinh(a*x)^(3/2)-16/15*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)^(1
/2)-24/5*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)+1/15*Pi^(1/2)*erf(arcs
inh(a*x)^(1/2))/a^3-3/5*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*arcsinh(a*x)^(1/2))/a
^3-1/15*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a^3+3/5*3^(1/2)*Pi^(1/2)*erfi(3^
(1/2)*arcsinh(a*x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\text{arcsinh}(ax)^{7/2}}\,dx = \frac{e^{\text{arcsinh}(ax)}(3+2\text{arcsinh}(ax)+4\text{arcsinh}(ax)^2)-3e^{3\text{arcsinh}(ax)}(1+2\text{arcsinh}(ax)+}{}$$

input
```
Integrate[x^2/ArcSinh[a*x]^(7/2),x]
```

output
```
(E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) - 3*E^(3*ArcSinh[a
*x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) + 36*Sqrt[3]*(-ArcSinh[a*x])
^(5/2)*Gamma[1/2, -3*ArcSinh[a*x]] - 4*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -A
rcSinh[a*x]] + (3 - 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2 - 4*E^ArcSinh[a*x]*A
rcSinh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (-3 + 6*ArcSi
nh[a*x] - 36*ArcSinh[a*x]^2 + 36*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(
5/2)*Gamma[1/2, 3*ArcSinh[a*x]])/E^(3*ArcSinh[a*x]))/(60*a^3*ArcSinh[a*x]^
(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6194, 6233, 6188, 6193, 2009, 6234, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arcsinh}(ax)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 6194}$$

$$\frac{4\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx}{5a}+\frac{6}{5}a\int \frac{x^3}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{ 6233}$$

$$\frac{6}{5}a\left(\frac{2\int \frac{x^2}{\text{arcsinh}(ax)^{3/2}}dx}{a}-\frac{2x^3}{3a\text{arcsinh}(ax)^{3/2}}\right)+$$
$$\frac{4\left(\frac{2\int \frac{1}{\text{arcsinh}(ax)^{3/2}}dx}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{ 6188}$$

$$\frac{4\left(\frac{2\left(2a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}+$$
$$\frac{6}{5}a\left(\frac{2\int \frac{x^2}{\text{arcsinh}(ax)^{3/2}}dx}{a}-\frac{2x^3}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{4\left(\frac{2\left(2a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}+$$
$$\frac{6}{5}a\left(\frac{2\left(\frac{2\int \left(\frac{3\sinh(3\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}-\frac{ax}{4\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^3}-\frac{2x^2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{a}-\frac{2x^3}{3a\text{arcsinh}(ax)^{3/2}}\right)-$$
$$\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\left(\frac{2\left(2a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$
$$\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \color{blue}{6234}$$

$$\frac{4\left(\frac{2\left(\frac{2\int \frac{ax}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{4\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int -\frac{i\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}\right)}{3a}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \color{blue}{26}$$

$$\frac{4\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\int\frac{\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}\right)}{3a}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \color{blue}{3789}$$

$$\frac{4\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\int\frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a}\right)}{3a}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \color{blue}{2611}$$

$$\frac{4\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(i\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a}\right)}{3a}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{4\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a}\right)}{3a}\right)}{5a}-\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{4\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a}\right)}{3a}\right)}{5a}-$$

$$\frac{2x^2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{6}{5}a\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{8}\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^3}\right.}{a}\right.$$

input

```
Int[x^2/ArcSinh[a*x]^(7/2),x]
```

output

```
(-2*x^2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) + (4*((-2*x)/(3*a*ArcS
inh[a*x]^(3/2)) + (2*((-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - ((2*
I)*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi]*Erfi[Sqrt[A
rcSinh[a*x]]]))/a))/(3*a)))/(5*a) + (6*a*((-2*x^3)/(3*a*ArcSinh[a*x]^(3/2)
) + (2*((-2*x^2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2*((Sqrt[Pi]*
Erf[Sqrt[ArcSinh[a*x]]])/8 - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/
8 - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/8 + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/8))/a^3))/a))/5
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(xa)^{\frac{7}{2}}}dx$$

input
```
int(x^2/arcsinh(x*a)^(7/2),x)
```

output
```
int(x^2/arcsinh(x*a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x**2/asinh(a*x)**(7/2),x)
```

output
```
Integral(x**2/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x^2/arcsinh(a*x)^(7/2), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(x^2/asinh(a*x)^(7/2),x)
```

output
```
int(x^2/asinh(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{\frac{6 asinh(ax)^3 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^3}{asinh(ax)^3a^2x^2+asinh(ax)^3}dx\right)a^2}{5} + \frac{4 asinh(ax)^3 \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)^3a^2x^2+asinh(ax)^3}dx\right)}{5} - 2\sqrt{a^2x^2-}}{asinh\left(ax\right)^3 a}$$

input
```
int(x^2/asinh(a*x)^(7/2),x)
```

output
```
(2*(3*asinh(a*x)**3*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**3)/(asinh
(a*x)**3*a**2*x**2 + asinh(a*x)**3),x)*a**2 + 2*asinh(a*x)**3*int((sqrt(a*
*2*x**2 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*x)**3*a**2*x**2 + asinh(a*x)**3)
,x) - sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2))/(5*asinh(a*x)**3*a)
```

3.144 $\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 147

$$\begin{aligned}\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx &= -\frac{2x\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} \\ &\quad -\frac{4}{15a^2\operatorname{arcsinh}(ax)^{3/2}}-\frac{8x^2}{15\operatorname{arcsinh}(ax)^{3/2}}-\frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}} \\ &\quad +\frac{8\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^2}\end{aligned}$$

output

```
-2/5*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-4/15/a^2/arcsinh(a*x)^(3/2)-
8/15*x^2/arcsinh(a*x)^(3/2)-32/15*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
+8/15*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a^2+8/15*2^(1/2)*Pi
^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{x}{\text{arcsinh}(ax)^{7/2}}\,dx =$$
$$-\frac{2\text{arcsinh}(ax)\left(e^{-2\text{arcsinh}(ax)}(1-4\text{arcsinh}(ax))+e^{2\text{arcsinh}(ax)}(1+4\text{arcsinh}(ax))+4\sqrt{2}(-\text{arcsinh}(ax))^{3/2}\right.}{15a^2\text{arcsinh}(ax)^{5/2}}$$

input
```
Integrate[x/ArcSinh[a*x]^(7/2),x]
```

output
```
-1/15*(2*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcS
inh[a*x])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2
, -2*ArcSinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x
]]) + 3*Sinh[2*ArcSinh[a*x]])/(a^2*ArcSinh[a*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6194, 6198, 6233, 6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(ax)^{7/2}}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{2\int \frac{1}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx}{5a}+\frac{4}{5}a\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{4}{5}a\int \frac{x^2}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{4}{5}a\left(\frac{4\int \frac{x}{\text{arcsinh}(ax)^{3/2}}dx}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\int \frac{\cosh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)-$$

$$\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{5}a\left(-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}+\frac{4\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}\right)}{3a}\right)-$$

$$\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{4}{5}a\left(-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}+\frac{4\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}\right.}{3a}\right.$$

$$\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(\frac{1}{2}\int\frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow 2611$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow 2633$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow 2634$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x^2}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}$$

input
```
Int[x/ArcSinh[a*x]^(7/2),x]
```

output
```
(-2*x*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - 4/(15*a^2*ArcSinh[a*x]
^(3/2)) + (4*a*((-2*x^2)/(3*a*ArcSinh[a*x]^(3/2)) + (4*((-2*x*Sqrt[1 + a^2
*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2*((Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/2))/a^2))/(3*a)))
/5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6198

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sqrt{2}\left(16\,\mathrm{arcsinh}(xa)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,ax+4\,\mathrm{arcsinh}(xa)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,a^{2}x^{2}+3\sqrt{2}\,\sqrt{\mathrm{arcsinh}(xa)}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,ax-8\,\mathrm{arcsinh}(xa)^{3}\pi\right.}{15\sqrt{\pi}\,a^{2}\,\mathrm{arcsinh}(xa)^{3}}$

input

```
int(x/arcsinh(x*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*2^(1/2)*(16*arcsinh(x*a)^(5/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*
x+4*arcsinh(x*a)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+3*2^(1/2)*arcsinh(x*a)^(1/
2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-8*arcsinh(x*a)^3*Pi*erf(2^(1/2)*arcsinh(
x*a)^(1/2))-8*arcsinh(x*a)^3*Pi*erfi(2^(1/2)*arcsinh(x*a)^(1/2))+2*arcsinh
(x*a)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(x*a)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x/asinh(a*x)**(7/2),x)
```

output
```
Integral(x/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x/arcsinh(a*x)^(7/2), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(x/arcsinh(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(x/asinh(a*x)^(7/2),x)
```

output
```
int(x/asinh(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{\frac{4 asinh(ax)^{3}\left(\int \frac{\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh(ax)}\,x^{2}}{asinh(ax)^{3}a^{2}x^{2}+asinh(ax)^{3}}dx\right)a^{3}}{5}-\frac{4\sqrt{asinh(ax)}\,asinh(ax)}{15}-\frac{2\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh(ax)}\,ax}{5}}{asinh\,(ax)^{3}\,a^{2}}$$

input

```
int(x/asinh(a*x)^(7/2),x)
```

output

```
(2*(6*asinh(a*x)**3*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2)/(asinh
(a*x)**3*a**2*x**2 + asinh(a*x)**3),x)*a**3 - 2*sqrt(asinh(a*x))*asinh(a*x
) - 3*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*a*x))/(15*asinh(a*x)**3*a**2)
```

3.145 $\int \frac{1}{\mathbf{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 112

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4x}{15\operatorname{arcsinh}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a}$$

output
```
-2/5*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-4/15*x/arcsinh(a*x)^(3/2)-8/15
*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)-4/15*Pi^(1/2)*erf(arcsinh(a*x)^(1/
2))/a+4/15*Pi^(1/2)*erfi(arcsinh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{-2e^{\operatorname{arcsinh}(ax)}(3+2\operatorname{arcsinh}(ax)+4\operatorname{arcsinh}(ax)^2)+8(-\operatorname{arcsinh}(ax))^{5/2}\Gamma\left(\frac{1}{2},-\mathrm{arcsi}\right.}{}$$

input
```
Integrate[ArcSinh[a*x]^(-7/2),x]
```

output
```
(-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 8*(-ArcSinh[a
*x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 + 4*ArcSinh[a*x] - 8*ArcSinh[a*
x]^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^Arc
Sinh[a*x])/(30*a*ArcSinh[a*x]^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6188, 6233, 6188, 6234, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{7/2}}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{2}{5}a\int \frac{x}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{5/2}}dx-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{2}{5}a\left(\frac{2\int \frac{1}{\text{arcsinh}(ax)^{3/2}}dx}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{6188}$$

$$\frac{2}{5}a\left(\frac{2\left(2a\int \frac{x}{\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}dx-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{2}{5}a\left(\frac{2\left(\frac{2\int \frac{ax}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3a}-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}\right)-$$

$$\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{2}{5}a\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int -\frac{i\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}\right)}{3a}\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{2}{5}a\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\int \frac{\sin(i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a}\right)}{3a}\right)$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+$$

$$\frac{2}{5}a\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\int \frac{e^{\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a}\right)}{3a}\right.$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{2}{5}a\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(i\int e^{\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a}\right)}{3a}\right)$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{2}{5}a\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a}\right)}{3a}\right)$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{2\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{2}{5}a\left(-\frac{2x}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\left(-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}-\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)\right)}{a}\right)}{3a}\right)$$

input
```
Int[ArcSinh[a*x]^(-7/2),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) + (2*a*((-2*x)/(3*a*ArcSin
h[a*x]^(3/2)) + (2*((-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - ((2*I)
*((-1/2*I)*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi]*Erfi[Sqrt[Arc
Sinh[a*x]]]))/a))/(3*a)))/5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2\left(2 \operatorname{arcsinh}(xa)^{3}\pi \,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(xa)}\right)-2 \operatorname{arcsinh}(xa)^{3}\pi \,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(xa)}\right)+4\sqrt{a^{2}x^{2}+1}\,\operatorname{arcsinh}(xa)^{\frac{5}{2}}\sqrt{\pi}+2 \operatorname{arcsinh}(xa)^{\frac{3}{2}}\right.}{15\sqrt{\pi}\,a \operatorname{arcsinh}(xa)^{3}}$

input
```
int(1/arcsinh(x*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(2*arcsinh(x*a)^3*Pi*erf(arcsinh(x*a)^(1/2))-2*arcsinh(x*a)^3*Pi*erf
i(arcsinh(x*a)^(1/2))+4*(a^2*x^2+1)^(1/2)*arcsinh(x*a)^(5/2)*Pi^(1/2)+2*ar
csinh(x*a)^(3/2)*Pi^(1/2)*a*x+3*arcsinh(x*a)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1
/2))/Pi^(1/2)/a/arcsinh(x*a)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(1/asinh(a*x)**(7/2),x)
```

output
```
Integral(asinh(a*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(1/asinh(a*x)^(7/2),x)
```

output
```
int(1/asinh(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{\frac{2 asinh(ax)^{3}\left(\int \frac{\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)^{3}a^{2}x^{2}+asinh(ax)^{3}}dx\right)a^{2}}{5}-\frac{2\sqrt{a^{2}x^{2}+1}\,\sqrt{asinh(ax)}}{5}}{asinh\left(ax\right)^{3}a}$$

input
```
int(1/asinh(a*x)^(7/2),x)
```

output
```
(2*(asinh(a*x)**3*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*x)
**3*a**2*x**2 + asinh(a*x)**3),x)*a**2 - sqrt(a**2*x**2 + 1)*sqrt(asinh(a*
x))))/(5*asinh(a*x)**3*a)
```

3.146 $\int \frac{1}{x\mathbf{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^{7/2}},x\right)$$

output
```
Defer(Int)(1/x/arcsinh(a*x)^(7/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a*x]^(7/2)),x]
```

output
```
Integrate[1/(x*ArcSinh[a*x]^(7/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}dx$$

input
```
Int[1/(x*ArcSinh[a*x]^(7/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\,\mathrm{arcsinh}\left(xa\right)^{\frac{7}{2}}}dx$$

input
```
int(1/x/arcsinh(x*a)^(7/2),x)
```

output
```
int(1/x/arcsinh(x*a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 63.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{x\,\text{asinh}^{\frac{7}{2}}\,(ax)}\,dx$$

input
```
integrate(1/x/asinh(a*x)**(7/2),x)
```

output
```
Integral(1/(x*asinh(a*x)**(7/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx=\int \frac{1}{x\,\text{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx=\int \frac{1}{x\,\text{asinh}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(1/(x*asinh(a*x)^(7/2)),x)
```

output
```
int(1/(x*asinh(a*x)^(7/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \int \frac{\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)^{4}x}dx$$

input
```
int(1/x/asinh(a*x)^(7/2),x)
```

output
```
int(sqrt(asinh(a*x))/(asinh(a*x)**4*x),x)
```

3.147 $\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\begin{aligned}\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx &= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}\\ &+ \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\\ &+ \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}\\ &- \frac{\sqrt{b}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\end{aligned}$$

output

```
1/3*x^3*(a+b*arcsinh(c*x))^(1/2)-1/16*b^(1/2)*exp(a/b)*Pi^(1/2)*erf((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))/c^3+1/144*b^(1/2)*exp(3*a/b)*3^(1/2)*Pi^(1/2)*
erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3+1/16*b^(1/2)*Pi^(1/2)*er
fi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3/exp(a/b)-1/144*b^(1/2)*3^(1/2)*Pi
^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.01

$$\int x^2\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$= \frac{e^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\Gamma\left(\frac{3}{2},-\frac{3(}{}\right.\right.}{72c^3\sqrt{}}$$

input
```
Integrate[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[a + b*ArcSinh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[3/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2
, (-3*(a + b*ArcSinh[c*x]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Ga
mma[3/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*Ar
cSinh[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/
b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6192, 6234, 25, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{3}x^3\sqrt{a+b\operatorname{arcsinh}(cx)}-\frac{1}{6}bc\int\frac{x^3}{\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}+\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int \frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int \left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}-\frac{i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{6c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\Bigg(\frac{3}{8}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}i\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{3}{8}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\Bigg(\frac{\sqrt{a+b\text{arcsinh}(}}{\sqrt{b}}}{6c^3}$$

input
```
Int[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
(x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + ((I/6)*(((3*I)/8)*Sqrt[b]*E^(a/b)*Sqrt[
Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - (I/8)*Sqrt[b]*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]] - (((3*I)/8)*Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b) + ((I/8)*Sqrt[b
]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/E^((3*a)/b)
))/c^3
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2\sqrt{a+b\operatorname{arcsinh}(xc)}dx$$

input
```
int(x^2*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(x^2*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int x^2\sqrt{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{b\,\text{arsinh}\,(cx)+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{b\,\text{arsinh}\,(cx)+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\int x^2\,\sqrt{a+b\operatorname{asinh}\left(c\,x\right)}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\int \sqrt{\mathit{asinh}\left(cx\right)b+a}\,x^2dx$$

input
```
int(x^2*(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)*x**2,x)
```

3.148 $\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 145

$$\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx=\frac{\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}$$

output

```
1/4*(a+b*arcsinh(c*x))^(1/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(1/2)-1/32*b^(
1/2)*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))/c^2-1/32*b^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/
2)/b^(1/2))/c^2/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{e^{-\frac{2a}{b}}\left(-b\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)+be^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\Gamma\left(\frac{3}{2},\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)\right)}{8\sqrt{2}c^2\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

input

```
Integrate[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
(-(b*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (-2*(a + b*ArcSinh[c*x]))/
b]) + b*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (2*(a + b*ArcSinh[
c*x]))/b])/(8*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6192, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2\sqrt{a+b\operatorname{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{2}x^2\sqrt{a+b\operatorname{arcsinh}(cx)}-\frac{\int\frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\operatorname{arcsinh}(cx)}}d(a+b\operatorname{arcsinh}(cx))}{4c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{\int \left(\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}$$

input

```
Int[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
(x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (-Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E
^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4 +
 (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*
E^((2*a)/b)))/(4*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\sqrt{a+b\,\mathrm{arcsinh}\,(xc)}dx$$

input
```
int(x*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(x*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\int x\sqrt{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int x\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\int\sqrt{b\operatorname{arsinh}(cx)+a}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int\sqrt{b\,\text{arsinh}\,(cx)+a}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int x\,\sqrt{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int\sqrt{asinh\,(cx)\,b+a}\,xdx$$

input
```
int(x*(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)*x,x)
```

3.149 $\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

output
```
x*(a+b*arcsinh(c*x))^(1/2)+1/4*b^(1/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(
c*x))^(1/2)/b^(1/2))/c-1/4*b^(1/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}$$

input

```
Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
(Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/
Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6187}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6234}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}+x\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{26}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{3789}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{2c}$$

$$\Big\downarrow \text{2611}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{2c}$$

$$\Big\downarrow \text{2633}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

$$\Big\downarrow \text{2634}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

input

```
Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(xc)}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\operatorname{arcsinh}(cx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \int \sqrt{a+b\operatorname{asinh}{(cx)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \int \sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \int \sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{asinh\left(cx\right)b+a}dx$$

input
```
int((a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)
```

3.150 $\int \frac{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{x}\,dx = \mathrm{Int}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{x},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{x}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/x,x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x}\,dx$$
$$\Big\downarrow \text{6196}$$
$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x}dx$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(xc)}}{x}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/x,x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\operatorname{asinh}{(cx)}}}{x}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{x}\,dx = \int \frac{\sqrt{b\operatorname{arsinh}{(cx)}+a}}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x}\,dx=\int \frac{\sqrt{a+b\,\text{asinh}\,(c\,x)}}{x}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/x,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}}{x}dx$$

input
```
int((a+b*asinh(c*x))^(1/2)/x,x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/x,x)
```

3.151 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2} \, dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2} \, dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2} \, dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/x^2,x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}dx \\ \Big\downarrow \text{6196} \\ \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}dx \end{array}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\left(xc\right)}}{x^{2}}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/x^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}\,dx=\int \frac{\sqrt{a+b\operatorname{asinh}{(cx)}}}{x^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{x^2}\,dx=\int \frac{\sqrt{a+b\,\text{asinh}\,(c\,x)}}{x^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/x^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{x^2}\,dx = \int \frac{\sqrt{asinh\,(cx)\,b + a}}{x^2}dx$$

input
```
int((a+b*asinh(c*x))^(1/2)/x^2,x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/x**2,x)
```

3.152 $\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 282

$$\begin{aligned}\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx &= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3}\\ &-\frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c}+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2}\\ &-\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{b^{3/2}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\\ &-\frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{b^{3/2}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\end{aligned}$$

output

```
1/3*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*x^2*(c^2*x^2+1)
^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+1/3*x^3*(a+b*arcsinh(c*x))^(3/2)-3/32*b^
(3/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3+1/288*b^
(3/2)*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))/c^3-3/32*b^(3/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3
/exp(a/b)+1/288*b^(3/2)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx =$$

$$-\frac{be^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{5}{2},\right.\right.}{216c}$$

input
```
Integrate[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
-1/216*(b*Sqrt[a + b*ArcSinh[c*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[
c*x])/b)]*Gamma[5/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]
]*Gamma[5/2, (-3*(a + b*ArcSinh[c*x]))/b] - 27*E^((2*a)/b)*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[5/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt
[-((a + b*ArcSinh[c*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c*x]))/b]))/(c^3*
E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.34, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6192, 6227, 6195, 5971, 2009, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\int\frac{x^3\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int\frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}+\frac{x^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}\right)$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^4}-\frac{2\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{3c^2}+x^2\sqrt{c^2}\right.$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{6c^4}-\frac{2\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{3c^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-}{6c}\right.$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int\frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)}{3c^2}-\frac{-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3}{}}}{}\right.$$

$$\Big\downarrow \text{ 6189}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)}{3c^2}-\frac{-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)}{3c^2}-\frac{-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\sqrt{a+b\text{ar}}\right.}{}\right.$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}i\int-\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)}{3c^2}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int\frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)}{3c^2}-\right.$$

$$\Big\downarrow 2611$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-$$

$$\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{2c^2}\right)}{3c^2}-\right.$$

$$\Big\downarrow 2633$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-$$

$$\frac{1}{2}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2c^2}\right)}{3c^2}-\right.$$

$$\Big\downarrow 2634$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}-$$

$$\frac{1}{2}bc\left(-\frac{-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\right.}{6c^4}\right.$$

input
```
Int[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

output

```
(x^3*(a + b*ArcSinh[c*x])^(3/2))/3 - (b*c*((x^2*Sqrt[1 + c^2*x^2]*Sqrt[a +
 b*ArcSinh[c*x]])/(3*c^2) - (2*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]
])/c^2 - ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b))
)/(2*c^2)))/(3*c^2) - (-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]]) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcSinh[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[
c*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((3*a)/b)))/(6*c^4)))/2
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int x^2 (a + b \operatorname{arcsinh}(xc))^{\frac{3}{2}} dx$$

input

```
int(x^2*(a+b*arcsinh(x*c))^(3/2),x)
```

output

```
int(x^2*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 (a + b \text{arcsinh}(cx))^{3/2} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int x^2(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)*x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int x^2\,(a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx=\left(\int\sqrt{\mathit{asinh}\,(cx)\,b+a}\;\mathit{asinh}(cx)\,x^2dx\right)b+\left(\int\sqrt{\mathit{asinh}\,(cx)\,b+a}\,x^2dx\right)a$$

input
```
int(x^2*(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2,x)*b + int(sqrt(asinh(c*x)*b +
a)*x**2,x)*a
```

3.153 $\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\begin{aligned}\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx&=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{8c}\\&+\frac{(a+b\text{arcsinh}(cx))^{3/2}}{4c^2}+\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}\\&-\frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}\end{aligned}$$

output

```
-3/8*b*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+1/4*(a+b*arcsinh(c*x
))^(3/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(3/2)-3/128*b^(3/2)*exp(2*a/b)*2^(
1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^2+3/128*b^(3
/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^2/ex
p(2*a/b)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{e^{-\frac{2a}{b}}\Big(b^2\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\big(\frac{5}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\big)+b^2e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\big(\frac{5}{2},}{16\sqrt{2}c^2\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Integrate[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(b^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[5/2, (-2*(a + b*ArcSinh[c*x]))/
b] + b^2*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[5/2, (2*(a + b*ArcSinh
[c*x]))/b])/(16*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6192, 6227, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\int\frac{x^2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{b\int\frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{4c}+\frac{x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c^2}\right)$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(-\frac{\int-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2}}{}\right.$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(\frac{\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}}{}\right.$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(\frac{\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(\frac{\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(c}}{2c^2}\right.$$

$$\Big\downarrow \ 26$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(-\frac{i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(c}}{2c^2}\right.$$

$$\Big\downarrow \ 3789$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(-\frac{i\left(\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{8c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{c^2x^2+}}}{2c^2}\right.$$

$$\Big\downarrow \ 2611$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(-\frac{i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c^3}-\frac{\int\frac{\sqrt{a+b\text{arc}}}{\sqrt{c^2}}}{2}\right.$$

$$\Big\downarrow \ 2633$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{4}bc\left(-\frac{i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c^3}-\frac{\int\frac{\sqrt{a+b\text{arcsi}}}{\sqrt{c^2x^2-}}}{2c^2}\right.$$

$$\Big\downarrow \ 2634$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-$$
$$\frac{3}{4}bc\left(-\frac{\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx}{\sqrt{b}}\right.\right.}{8c^3}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2}-$$
$$\frac{3}{4}bc\left(-\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c^3}-\frac{(a+b\text{arcsinh}(cx}{3bc^3}\right.$$

input
```
Int[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(x^2*(a + b*ArcSinh[c*x])^(3/2))/2 - (3*b*c*((x*Sqrt[1 + c^2*x^2]*Sqrt[a +
 b*ArcSinh[c*x]])/(2*c^2) - (a + b*ArcSinh[c*x])^(3/2)/(3*b*c^3) - ((I/8)*
((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
*x]])/Sqrt[b]])/E^((2*a)/b)))/c^3))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int x(a+b\,\mathrm{arcsinh}\,(xc))^{\frac{3}{2}}\,dx$$

input
```
int(x*(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(x*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\mathrm{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\int x(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\int (b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\int (b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int x\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^{3/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx=\left(\int \sqrt{asinh\left(cx\right)b+a}\,asinh(cx)\,xdx\right)b\\+\left(\int \sqrt{asinh\left(cx\right)b+a}\,xdx\right)a$$

input
```
int(x*(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x,x)*b + int(sqrt(asinh(c*x)*b + a)*
x,x)*a
```

3.154 $\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 135

$$\begin{aligned}\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = {} & -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{2c} \\ & + x(a+b\operatorname{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} \\ & + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}\end{aligned}$$

output

```
-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+x*(a+b*arcsinh(c*x))^(
3/2)+3/8*b^(3/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c
+3/8*b^(3/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\Bigl(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)(\cos}{8c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]]
)/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b
*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (
-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Si
nh[a/b])))/(8*c)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6187}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6213}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)$$

$$\Big\downarrow \text{ 6189}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$$\Big\downarrow \text{ 3788}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b}{2c^2}\right.$$

$$\Big\downarrow \text{ 26}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcs}}{2c^2}\right.$$

$$\Big\downarrow \text{ 2611}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-$$
$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsin}}{2c^2}\right.$$

$$\Big\downarrow \text{2633}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-$$
$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx}}{\sqrt{b}}\right.}{2c^2}\right.$$

$$\Big\downarrow \text{2634}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-$$
$$\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2c^2}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
x*(a + b*ArcSinh[c*x])^(3/2) - (3*b*c*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSi
nh[c*x]])/c^2 - ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E
^(a/b)))/(2*c^2)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [F]

$$\int (a+b\operatorname{arcsinh}(xc))^{\frac{3}{2}}\,dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\operatorname{asinh}(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \left(\int \sqrt{asinh\left(cx\right)b+a}dx\right)a + \left(\int \sqrt{asinh\left(cx\right)b+a}\,asinh(cx)\,dx\right)b$$

input

```
int((a+b*asinh(c*x))^(3/2),x)
```

output

```
int(sqrt(asinh(c*x)*b + a),x)*a + int(sqrt(asinh(c*x)*b + a)*asinh(c*x),x)
*b
```

3.155 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x} \, dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{x},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x} \, dx=\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x} \, dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/x,x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}{x}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/x,x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/x,x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{x}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/x,x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x}\,dx=\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}}{x}dx\right)a+\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}\;asinh(cx)}{x}dx\right)b$$

input

```
int((a+b*asinh(c*x))^(3/2)/x,x)
```

output

```
int(sqrt(asinh(c*x)*b + a)/x,x)*a + int((sqrt(asinh(c*x)*b + a)*asinh(c*x)
)/x,x)*b
```

3.156 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\,\text{arcsinh}(cx))^{3/2}}{x^2}\,dx=\text{Int}\left(\frac{(a+b\,\text{arcsinh}(cx))^{3/2}}{x^2},x\right)$$

output

```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\,\text{arcsinh}(cx))^{3/2}}{x^2}\,dx=\int \frac{(a+b\,\text{arcsinh}(cx))^{3/2}}{x^2}\,dx$$

input

```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/x^2,x]
```

output

```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x^2} \, dx$$
$$\Big\downarrow \text{ 6196}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x^2}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(x c\right)\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/x^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{x^2}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/x**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{x^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{x^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{x^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/x^2,x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{x^2}\,dx=\int \frac{(asinh(cx)\,b+a)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+b*asinh(c*x))^(3/2)/x^2,x)
```

output
```
int((a+b*asinh(c*x))^(3/2)/x^2,x)
```

3.157 $\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 327

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=-\frac{5b^2x\sqrt{a+b\text{arcsinh}(cx)}}{6c^2}+\frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)}$$
$$+\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3}-\frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c}$$
$$+\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{15b^{5/2}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^3}+\frac{5b^{5/2}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{576c^3}+$$

output

```
-5/6*b^2*x*(a+b*arcsinh(c*x))^(1/2)/c^2+5/36*b^2*x^3*(a+b*arcsinh(c*x))^(1
/2)+5/9*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(3/2)/c^3-5/18*b*x^2*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))^(3/2)/c+1/3*x^3*(a+b*arcsinh(c*x))^(5/2)-15
/64*b^(5/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3+5/
1728*b^(5/2)*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1
/2)/b^(1/2))/c^3+15/64*b^(5/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1
/2))/c^3/exp(a/b)-5/1728*b^(5/2)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsin
h(c*x))^(1/2)/b^(1/2))/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.61

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx =$$

$$-\frac{b^3e^{-\frac{3a}{b}}\left(-81e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{7}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{7}{2},-\frac{3(a+b\text{arcsinh}(cx))}{b}\right.\right.}{648c^3\sqrt{a+b}}$$

input
```
Integrate[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
-1/648*(b^3*(-81*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7/2, a/b + Arc
Sinh[c*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2, (-3*(a + b
*ArcSinh[c*x]))/b] - 81*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
7/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c
*x]]*Gamma[7/2, (3*(a + b*ArcSinh[c*x]))/b]))/(c^3*E^((3*a)/b)*Sqrt[a + b*
ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {6192, 6227, 6192, 6213, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{6}bc\int\frac{x^3(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{6}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\int x^2\sqrt{a+b\text{arcsinh}(cx)}dx}{2c}+\frac{x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{6}bc\left(-\frac{2\int\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{3c^2}-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{6}bc\int\frac{x^3}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{2c}+\frac{x^2\sqrt{c^2x^2}}{}\right.$$

$$\Big\downarrow \text{6213}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{6}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\int\sqrt{a+b\text{arcsinh}(cx)}dx}{2c}\right)}{3c^2}-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{6}bc\int\frac{x}{\sqrt{c^2x^2+1}\sqrt{a+}}\right.}{2c}\right.$$

$$\Big\downarrow \text{6187}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{6}bc\left(-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}bc\int\frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{2c}\right)}{3c^2}-\frac{b\left(\frac{1}{3}x^3\sqrt{a+ba}\right.}{}\right.$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/}}{c^2}\right.}{}\right.$$

$$\Big\downarrow 25$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{\int \frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}+\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}\right.}{}\right.$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{c^2}\right.}{}\right.$$

$\big\downarrow$ 26

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{c^2}\right.}{}\right.$$

$\big\downarrow$ 3789

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{c^2}\right.}{}\right.$$

$$\Big\downarrow \text{2611}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{c^2}\right.}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{c^2}\right.}{}\right.$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}-\frac{2\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{c^2}\right.}{}\right.$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\int\left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}-\frac{i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{6c^3}\right)}{2c}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}+\frac{i\left(\frac{3}{8}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}i\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{3}{8}i\sqrt{\pi}\sqrt{\right.}}{6c^3}\right.}{2c}\right.$$

input
```
Int[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(x^3*(a + b*ArcSinh[c*x])^(5/2))/3 - (5*b*c*((x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])^(3/2))/(3*c^2) - (2*((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)^(3/2))/c^2 - (3*b*(x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*((I/2)*Sqrt[b]*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt
[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c))/(2*c)))/(3*c^2)
 - (b*((x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + ((I/6)*(((3*I)/8)*Sqrt[b]*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - (I/8)*Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]] - (((3*I)/8)
*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b) + ((I/8)
*Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/E^((
3*a)/b)))/c^3))/(2*c)))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2(a+b\operatorname{arcsinh}(xc))^{\frac{5}{2}}\,dx$$

input
```
int(x^2*(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(x^2*(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\mathrm{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int x^2(a+b\operatorname{asinh}{(cx)})^{\frac{5}{2}}\,dx$$

input
```
integrate(x**2*(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral(x**2*(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int (b\operatorname{arsinh}{(cx)}+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2)*x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}\,dx$$

input
```
int(x^2*(a + b*asinh(c*x))^(5/2),x)
```

output
```
int(x^2*(a + b*asinh(c*x))^(5/2), x)
```

Reduce [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=2\left(\int \sqrt{\mathit{asinh}\left(cx\right)b+a}\,\mathit{asinh}(cx)\,x^2dx\right)ab+\left(\int \sqrt{\mathit{asinh}\left(cx\right)b+a}\,\mathit{asinh}(cx)^2\,x^2dx\right)b^2+\left(\int \sqrt{\mathit{asinh}\left(cx\right)b+a}\,x^2dx\right)a^2$$

input
```
int(x^2*(a+b*asinh(c*x))^(5/2),x)
```

output
```
2*int(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2,x)*a*b + int(sqrt(asinh(c*x)*
b + a)*asinh(c*x)**2*x**2,x)*b**2 + int(sqrt(asinh(c*x)*b + a)*x**2,x)*a**
2
```

3.158 $\int x(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 223

$$\int x(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = \frac{15b^2\sqrt{a+b\operatorname{arcsinh}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\operatorname{arcsinh}(cx)} - \frac{5bx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{8c}+\frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{5/2} - \frac{15b^{5/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2} - \frac{15b^{5/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right.}{256c^2}$$

output
```
15/64*b^2*(a+b*arcsinh(c*x))^(1/2)/c^2+15/32*b^2*x^2*(a+b*arcsinh(c*x))^(1
/2)-5/8*b*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(3/2)/c+1/4*(a+b*arcsinh(
c*x))^(5/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(5/2)-15/512*b^(5/2)*exp(2*a/b)
*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^2-15/512
*b^(5/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c
^2/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=\frac{e^{-\frac{2a}{b}}\left(-b^3\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{7}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\right.\right.}{32\sqrt{2}c^2\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Integrate[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(-(b^3*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh[c*x])
)/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7/2, (2*(a + b*ArcS
inh[c*x]))/b])/(32*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6192, 6227, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{4}bc\int\frac{x^2(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6227}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{4}bc\left(-\frac{\int\frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{2c^2}-\frac{3b\int x\sqrt{a+b\text{arcsinh}(cx)}dx}{4c}+\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{2c^2}\right)$$

$$\Big\downarrow \text{6192}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-$$
$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4c}-\frac{\int\frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2+1}}{}\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-$$
$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4c}-\frac{(a+b\text{arcsinh}(cx))^{5/2}}{5bc^3}+\frac{x\sqrt{c^2x^2+1}(}{}\right.$$

$$\Big\downarrow \text{6234}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-$$
$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int\frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4c}-\frac{(a+b\text{arcsinh}(cx))^{5/2}}{5bc^3}+\frac{x\sqrt{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-$$
$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int-\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4c}-\frac{(a+b\text{arcsinh}(cx))^{5/2}}{5bc^3}+\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4c}-\frac{(a+b\text{arcsinh}(cx))^{5/2}}{5bc^3}+\right.$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{4}bc\left(-\frac{3b\left(\frac{\int\left(\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}\right)}{4c}-\ (a+b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-\frac{5}{4}bc\left(-\frac{(a+b\text{arcsinh}(cx))^{5/2}}{5bc^3}-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\right.}{4c^2}\right.}{4c}\right.$$

input
```
Int[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(x^2*(a + b*ArcSinh[c*x])^(5/2))/2 - (5*b*c*((x*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x])^(3/2))/(2*c^2) - (a + b*ArcSinh[c*x])^(5/2)/(5*b*c^3) - (3*b*
((x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (-Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4
+ (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4
*E^((2*a)/b)))/(4*c^2)))/(4*c)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x(a+b\operatorname{arcsinh}(xc))^{\frac{5}{2}}dx$$

input
```
int(x*(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(x*(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx=\int x(a+b\operatorname{asinh}(cx))^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral(x*(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int x(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx=\int (b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2)*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int x\,\left(a+b\,\mathrm{asinh}(c\,x)\right)^{5/2}\,dx$$

input
```
int(x*(a + b*asinh(c*x))^(5/2),x)
```

output
```
int(x*(a + b*asinh(c*x))^(5/2), x)
```

Reduce [F]

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=2\left(\int\sqrt{asinh\left(cx\right)b+a}\,asinh(cx)\,xdx\right)ab+\left(\int\sqrt{asinh\left(cx\right)b+a}\,asinh(cx)^{2}\,xdx\right)b^{2}+\left(\int\sqrt{asinh\left(cx\right)b+a}\,xdx\right)a^{2}$$

input
```
int(x*(a+b*asinh(c*x))^(5/2),x)
```

output
```
2*int(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x,x)*a*b + int(sqrt(asinh(c*x)*b +
 a)*asinh(c*x)**2*x,x)*b**2 + int(sqrt(asinh(c*x)*b + a)*x,x)*a**2
```

3.159 $\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 155

$$\begin{aligned}\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx &= \frac{15}{4}b^2x\sqrt{a+b\operatorname{arcsinh}(cx)} \\ &\quad - \frac{5b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{2c} + x(a+b\operatorname{arcsinh}(cx))^{5/2} \\ &\quad + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c}\end{aligned}$$

output

```
15/4*b^2*x*(a+b*arcsinh(c*x))^(1/2)-5/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c
*x))^(3/2)/c+x*(a+b*arcsinh(c*x))^(5/2)+15/16*b^(5/2)*exp(a/b)*Pi^(1/2)*er
f((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c-15/16*b^(5/2)*Pi^(1/2)*erfi((a+b*arc
sinh(c*x))^(1/2)/b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.82

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = \frac{\sqrt{b}e^{-\frac{a}{b}}\Bigg(-\bigg((4a^2-15b^2)\,e^{\frac{2a}{b}}\sqrt{\pi}\operatorname{erf}\bigg(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\bigg)\bigg)+(4a^2-15b^2)\sqrt{\pi}\operatorname{erfi}\bigg(\frac{\sqrt{a}}{}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(Sqrt[b]*(-((4*a^2 - 15*b^2)*E^((2*a)/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
*x]]/Sqrt[b]]) + (4*a^2 - 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/S
qrt[b]] + (4*Sqrt[b]*(E^(a/b)*(a + b*ArcSinh[c*x])*(5*(3*b*c*x - 2*a*Sqrt[
1 + c^2*x^2]) + 2*(4*a*c*x - 5*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 4*b*c*x
*ArcSinh[c*x]^2) - 2*a^2*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, a
/b + ArcSinh[c*x]] - 2*a^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, -((a
 + b*ArcSinh[c*x])/b)]))/Sqrt[a + b*ArcSinh[c*x]]))/(16*c*E^(a/b))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6187, 6213, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{5}{2}bc\int \frac{x(a+b\operatorname{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\int\sqrt{a+b\operatorname{arcsinh}(cx)}dx}{2c}\right)$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\operatorname{arcsinh}(cx)}-\frac{1}{2}bc\int\frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}dx\right)}{2c}\right)$$

$$\Big\downarrow \text{6234}$$

$$x(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\operatorname{arcsinh}(cx)}-\frac{\int-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\operatorname{arcsinh}(cx)}}d(a+b\operatorname{arcsinh}(cx))}{2c}\right)}{2c}\right)$$

$$\Big\downarrow \text{25}$$

$$x(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(\frac{\int\frac{\sinh\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\operatorname{arcsinh}(cx)}}d(a+b\operatorname{arcsinh}(cx))}{2c}+x\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{2c}\right)$$

$$\Big\downarrow \text{3042}$$

$$x(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right.}{2c}\right.$$

$$\Big\downarrow 26$$

$$x(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right)}{2c}\right.$$

$$\Big\downarrow 3789$$

$$x(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int -\right.}{2c}\right.}{2c}\right.$$

$$\Big\downarrow 2611$$

$$x(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{2}bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+}{}}\right.}{2c}\right.}{2c}\right.$$

$$\Big\downarrow \text{2633}$$

$$x(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{2}bc\Bigg(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\Bigg(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\Big(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{}}{2c}}{2c}$$

$$\Big\downarrow \text{2634}$$

$$x(a+b\text{arcsinh}(cx))^{5/2}-$$

$$\frac{5}{2}bc\Bigg(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{c^2}-\frac{3b\Bigg(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\Big(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\Big(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\Big)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-}}{2c}}{2c}$$

input
```
Int[(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
x*(a + b*ArcSinh[c*x])^(5/2) - (5*b*c*((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^(3/2))/c^2 - (3*b*(x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*((I/2)*Sqrt[b]
*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c))/(2*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (a+b\,\text{arcsinh}\,(xc))^{\frac{5}{2}}\,dx$$

input
```
int((a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\text{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx=\int (a+b\operatorname{asinh}(cx))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx=\int (b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^{5/2}\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int (a+b\text{arcsinh}(cx))^{5/2}\,dx&=\left(\int \sqrt{asinh\left(cx\right)b+a}dx\right)a^{2}\\&\quad+2\left(\int \sqrt{asinh\left(cx\right)b+a}\,asinh(cx)\,dx\right)ab\\&\quad+\left(\int \sqrt{asinh\left(cx\right)b+a}\,asinh(cx)^{2}\,dx\right)b^{2}\end{aligned}$$

input
```
int((a+b*asinh(c*x))^(5/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)*a**2 + 2*int(sqrt(asinh(c*x)*b + a)*asinh(c*
x),x)*a*b + int(sqrt(asinh(c*x)*b + a)*asinh(c*x)**2,x)*b**2
```

3.160 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^{5/2}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x}\,dx = \operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(5/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(5/2)/x,x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(5/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x} \, dx$$
$$\Big\downarrow \text{ 6196}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(5/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^{\frac{5}{2}}}{x}dx$$

input
```
int((a+b*arcsinh(x*c))^(5/2)/x,x)
```

output
```
int((a+b*arcsinh(x*c))^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 21.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(5/2)/x,x)
```

output
```
Integral((a + b*asinh(c*x))**(5/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^{5/2}}{x}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^{5/2}}{x}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^{5/2}}{x}\,dx$$

input
```
int((a + b*asinh(c*x))^(5/2)/x,x)
```

output
```
int((a + b*asinh(c*x))^(5/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x}\,dx = \left(\int \frac{\sqrt{asinh\left(cx\right)b+a}}{x}dx\right)a^2 + 2\left(\int \frac{\sqrt{asinh\left(cx\right)b+a}\,asinh(cx)}{x}dx\right)ab + \left(\int \frac{\sqrt{asinh\left(cx\right)b+a}\,asinh(cx)^2}{x}dx\right)b^2$$

input

```
int((a+b*asinh(c*x))^(5/2)/x,x)
```

output

```
int(sqrt(asinh(c*x)*b + a)/x,x)*a**2 + 2*int((sqrt(asinh(c*x)*b + a)*asinh
(c*x))/x,x)*a*b + int((sqrt(asinh(c*x)*b + a)*asinh(c*x)**2)/x,x)*b**2
```

3.161 $\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(5/2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2} dx = \int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2} dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(5/2)/x^2,x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(5/2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(5/2)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a+b*arcsinh(x*c))^(5/2)/x^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(5/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 22.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x^2}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(5/2)/x**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(5/2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{x^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(5/2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{5/2}}{x^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(5/2)/x^2,x)
```

output
```
int((a + b*asinh(c*x))^(5/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{5/2}}{x^2}\,dx = \int \frac{(asinh(cx)\,b+a)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a+b*asinh(c*x))^(5/2)/x^2,x)
```

output
```
int((a+b*asinh(c*x))^(5/2)/x^2,x)
```

3.162 $\int \frac{x^2}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 194

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx = &-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}\\ &+\frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}\\ &-\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}\\ &+\frac{e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}\end{aligned}$$

output
```
-1/8*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^3+1
/24*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/
2))/b^(1/2)/c^3-1/8*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2
)/c^3/exp(a/b)+1/24*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))/b^(1/2)/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)-3\right.}{24c^3\sqrt{a+b\operatorname{arcs}}}$$

input
```
Integrate[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + S
qrt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x])
)/b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*A
rcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2,
 (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c*x]]
)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\operatorname{arcsinh}(cx)}}d(a+b\operatorname{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{bc^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right.}{bc^3}$$

input
```
Int[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) + (
Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E^(
a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(8*E^((3*a)/b)))/(b*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input

```
int(x^2/(a+b*arcsinh(x*c))^(1/2),x)
```

output

```
int(x^2/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input

```
integrate(x**2/(a+b*asinh(c*x))**(1/2),x)
```

output

```
Integral(x**2/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int(x^2/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(x^2/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}\,x^2}{asinh\left(cx\right)b+a}dx$$

input

```
int(x^2/(a+b*asinh(c*x))^(1/2),x)
```

output

```
int((sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*x)*b + a),x)
```

3.163 $\int \frac{x}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{x}{\sqrt{a+b\,\text{arcsinh}(cx)}}\,dx = -\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

output

```
-1/8*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))/b^(1/2)/c^2+1/8*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2
)/b^(1/2))/b^(1/2)/c^2/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$
$$= \frac{e^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{4\sqrt{2}c^2\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Integrate[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] +
 E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/
b])/(4*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$
$$\Big\downarrow \text{6195}$$
$$\frac{\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc^2}$$
$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cosh \left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right) \sinh \left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \; 5971$$

$$-\frac{\int \frac{\sinh \left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sinh \left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2bc^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int -\frac{i \sin \left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2bc^2}$$

$$\Big\downarrow \; 26$$

$$\frac{i \int \frac{\sin \left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2bc^2}$$

$$\Big\downarrow \; 3789$$

$$\frac{i \left(\frac{1}{2} i \int \frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2} i \int \frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{2bc^2}$$

$$\Big\downarrow \; 2611$$

$$\frac{i \left(i \int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i \int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{2bc^2}$$

$$\Big\downarrow \; 2633$$

$$\frac{i \left(i \int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2} i \sqrt{\frac{\pi}{2}} \sqrt{b} e^{-\frac{2a}{b}} \text{erfi}\left(\frac{\sqrt{2} \sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2bc^2}$$

$$\Big\downarrow \; 2634$$

$$\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2bc^2}$$

input

```
Int[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
((I/2)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]])/E^((2*a)/b)))/(b*c^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{a + b\,\mathrm{arcsinh}\,(xc)}}dx$$

input
```
int(x/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(x/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a + b\mathrm{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

input
```
integrate(x/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int(x/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(x/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}\,x}{asinh\left(cx\right)b+a}dx$$

input
```
int(x/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int((sqrt(asinh(c*x)*b + a)*x)/(asinh(c*x)*b + a),x)
```

3.164 $\int \frac{1}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

output

```
1/2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+1/2*
Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$

$$=\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\right)}{2c\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

input

```
Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) +
Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*
c*E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{bc}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

input

```
Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqr
t[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b)))/(b*c)
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b\,\mathrm{arcsinh}\,(xc)}}dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\mathrm{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\operatorname{asinh}{(cx)}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\,\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\,\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}} \,dx$$

input
```
int(1/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b+a}dx$$

input
```
int(1/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x)
```

3.165 $\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Int}\left(\frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \sqrt{a+b\text{arcsinh}(c x)}}dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x \sqrt{a+b\text{arcsinh}(c x)}}dx$$

input
```
Int[1/(x*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x \sqrt{a+b \operatorname{arcsinh}\left(x c\right)}}dx$$

input
```
int(1/x/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/x/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{x\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

input
```
integrate(1/x/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\operatorname{arsinh}(cx)+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*arcsinh(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\,\text{arsinh}\,(cx)+a}x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*arcsinh(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int(1/(x*(a + b*asinh(c*x))^(1/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{1}{x\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)bx+ax}dx$$

input

```
int(1/x/(a+b*asinh(c*x))^(1/2),x)
```

output

```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b*x + a*x),x)
```

3.166 $\int \frac{1}{x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx=\mathrm{Int}\left(\frac{1}{x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}},x\right)$$

output

```
Defer(Int)(1/x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx=\int \frac{1}{x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$$

input

```
Integrate[1/(x^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output

```
Integrate[1/(x^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{1}{x^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx \\ \Big\downarrow \text{6196} \\ \int \frac{1}{x^2\sqrt{a+b\text{arcsinh}(cx)}}dx \end{array}$$

input
```
Int[1/(x^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/x^2/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/x^2/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{1}{x^2\sqrt{a+b\operatorname{asinh}{(cx)}}}\,dx$$

input
```
integrate(1/x**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\operatorname{arsinh}(cx)+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*arcsinh(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\operatorname{arsinh}(cx)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*arcsinh(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{x^2\,\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b*x**2 + a*x**2),x)
```

3.167 $\int \frac{x^2}{(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 226

$$\int \frac{x^2}{(a+b\,\mathrm{arcsinh}(cx))^{3/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\,\mathrm{arcsinh}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{e^{-\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

output

```
-2*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)+1/4*exp(a/b)*Pi^(1/2
)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3-1/4*exp(3*a/b)*3^(1/2)
*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3-1/4*Pi
^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(a/b)+1/4*3^(
1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/e
xp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{3a}{b}}+e^{\frac{3a}{b}+2\text{arcsinh}(cx)}+e^{\frac{3a}{b}+4\text{arcsinh}(cx)}-e^{\frac{3a}{b}+6\text{arcsinh}(cx)}-\right.}{}$$

input
```
Integrate[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c*x]) + E^((3*a)/b + 4*ArcSinh[c*x]
) - E^((3*a)/b + 6*ArcSinh[c*x]) - E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b +
 ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh[c*x])
*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] -
 E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -
((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3*E^(3*(a/b
 + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{6193}$$

$$\frac{2\int \left(\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2x^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c}}{\sqrt{b}}\right.\right.}{b^2c^3}$$

$$\frac{2x^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*x^2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (2*((Sqrt[b]*E
^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/8 - (Sqrt[b]*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 - (Sqr
t[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[
b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((3*a
)/b))))/(b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{x^2}{(a+b\,\mathrm{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(x^2/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{(a+b\,\mathrm{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(x**2/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{\left(a+b\,\text{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int(x^2/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(x^2/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a + b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{asinh\,(cx)\,b + a}\,x^2}{asinh\,(cx)^2\,b^2 + 2asinh\,(cx)\,ab + a^2}dx$$

input

```
int(x^2/(a+b*asinh(c*x))^(3/2),x)
```

output

```
int((sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b +
 a**2),x)
```

3.168 $\int \frac{x}{(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 135

$$\int \frac{x}{(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\,\text{arcsinh}(cx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}$$

output
```
-2*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)+1/2*exp(2*a/b)*2^(1/2)
*Pi^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2+1/2*2^
(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2/
exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{x}{(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\,\text{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\,\text{arcsinh}(cx))}{b}\right)-\sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}}\right.}{2bc^2\sqrt{a+b\,\text{arcsinh}(c}}$$

input
```
Integrate[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x
]))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b
*ArcSinh[c*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c*x]])/(2*b*c^2*E^((2*a)
/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}$$

$$\Big\downarrow \text{ 3788}$$

$$-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2\left(\frac{1}{2}\int \frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 2611$$

$$\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 2633$$

$$\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \; 2634$$

$$\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input

```
Int[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

output

```
(-2*x*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (2*((Sqrt[b]*E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/2 + (
Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2*E^
((2*a)/b))))/(b^2*c^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{x}{\left(a+b \,\mathrm{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}}d x$$

input
```
int(x/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(x/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(x/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int(x/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(x/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}\,x}{asinh\left(cx\right)^{2}b^{2}+2asinh\left(cx\right)ab+a^{2}}dx$$

input
```
int(x/(a+b*asinh(c*x))^(3/2),x)
```

output
```
int((sqrt(asinh(c*x)*b + a)*x)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a*
*2),x)
```

3.169 $\int \frac{1}{(a+b\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

output

```
-2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-exp(a/b)*Pi^(1/2)*erf((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+Pi^(1/2)*erfi((a+b*arcsinh(c*x))
^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Bigg(-e^{a/b}\left(1+e^{2\operatorname{arcsinh}(cx)}\right)+e^{\frac{2a}{b}+\operatorname{arcsinh}(cx)}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma(\tfrac{1}{2}}{bc\sqrt{a+\mathrm{bar}}}$$

input

```
Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output

```
(-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a
 + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a +
 b*ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{2c\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \ 26$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \ 3789$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \ 2611$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c}$$

$$\Big\downarrow \ 2633$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

$$\Big\downarrow \ 2634$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

input

```
Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output

```
(-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + ((2*I)*((I/2)*Sqrt
[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/(b^2*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

input

```
int(1/(a + b*asinh(c*x))^(3/2),x)
```

output

```
int(1/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Too large to display}$$

input

```
int(1/(a+b*asinh(c*x))^(3/2),x)
```

output

```
(2*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x) - asinh(c*x)*int(
(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + a
sinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c
**2*x**2 + a**2),x)*b**2*c**3 - asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*asi
nh(c*x))/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)
*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c - 2*a
sinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*x)/(a
sinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x
**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**2 - 2*asinh(c*x)
*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)**2*x)/(asinh(c
*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 +
2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**2 - int((sqrt(asinh(c
*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*
b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a*
*2),x)*a*b*c**3 - int((sqrt(asinh(c*x)*b + a)*asinh(c*x))/(asinh(c*x)**2*b
**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(
c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c - 2*int((sqrt(c**2*x**2 + 1)*sq
rt(asinh(c*x)*b + a)*asinh(c*x)*x)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c
*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x*
*2 + a**2),x)*a**2*c**2 - 2*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b ...
```

3.170 $\int \frac{1}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{x(a+b\,\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{x(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\text{arcsinh}(cx))^{3/2}} \, dx$$
$$\Big\downarrow \; 6196$$
$$\int \frac{1}{x(a+b\text{arcsinh}(cx))^{3/2}}dx$$

input
```
Int[1/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x \left(a+b \operatorname{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/x/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{x\,(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

input
```
int(1/(x*(a + b*asinh(c*x))^(3/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)^2\,b^2x+2asinh\,(cx)\,abx+a^2x}dx$$

input
```
int(1/x/(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**2*b**2*x + 2*asinh(c*x)*a*b*x + a*
*2*x),x)
```

3.171 $\int \frac{1}{x^2(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \operatorname{Int}\left(\frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$
$$\Big\downarrow \text{6196}$$
$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}dx$$

input
```
Int[1/(x^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a+b\,\text{arcsinh}\left(xc\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/x^2/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsinh(c*x) + a)^(3/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsinh(c*x) + a)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)^{2}b^{2}x^{2}+2asinh\left(cx\right)ab\,x^{2}+a^{2}x^{2}}dx$$

input

```
int(1/x^2/(a+b*asinh(c*x))^(3/2),x)
```

output

```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**2*b**2*x**2 + 2*asinh(c*x)*a*b*x**
2 + a**2*x**2),x)
```

3.172 $\int \frac{x^2}{(a+b\,\mathbf{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 271

$$
\begin{aligned}
\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = {} & -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\operatorname{arcsinh}(cx))^{3/2}} \\
& -\frac{8x}{3b^2c^2\sqrt{a+b\operatorname{arcsinh}(cx)}}-\frac{4x^3}{b^2\sqrt{a+b\operatorname{arcsinh}(cx)}} \\
& -\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3} \\
& -\frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}
\end{aligned}
$$

output

```
-2/3*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(3/2)-8/3*x/b^2/c^2/(a+b
*arcsinh(c*x))^(1/2)-4*x^3/b^2/(a+b*arcsinh(c*x))^(1/2)-1/6*exp(a/b)*Pi^(1
/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c^3+1/2*exp(3*a/b)*3^(1/
2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c^3-1/6*
Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c^3/exp(a/b)+1/2*3
^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c^3
/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(2e^{\frac{4a}{b}+3\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))\Gamma\left(\frac{1}{2},\frac{a}{b}\right.\right.}{}$$

input
```
Integrate[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(2*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*
x])*Gamma[1/2, a/b + ArcSinh[c*x]] - 6*Sqrt[3]*b*E^(3*ArcSinh[c*x])*(-((a
+ b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 2*b*
E^((2*a)/b + 3*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2,
-((a + b*ArcSinh[c*x])/b)] - E^((3*a)/b)*((-1 + E^(2*ArcSinh[c*x]))*(b*(-1
 + E^(4*ArcSinh[c*x])) + a*(6 + 4*E^(2*ArcSinh[c*x]) + 6*E^(4*ArcSinh[c*x]
)) + 2*b*(3 + 2*E^(2*ArcSinh[c*x]) + 3*E^(4*ArcSinh[c*x]))*ArcSinh[c*x]) +
 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcS
inh[c*x])*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(12*b^2*c^3*E^(3*(a/b +
 ArcSinh[c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6194, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{4\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}dx}{3bc}+\frac{2c\int \frac{x^3}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}dx}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}+\frac{4\left(\frac{2\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 6189}$$

$$\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}+\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4\left(-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}\right)}{3bc}+\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{4\left(-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^2}\right)}{3bc}+\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{4\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}+\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{4\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}+\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{4\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}+\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{2c\left(\frac{6\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{6195}$$

$$\frac{2c\left(\frac{6\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{5971}$$

$$\frac{2c\left(\frac{6\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c^4}-\frac{2x^3}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{b}+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c\left(\frac{6\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\right.}{b^2c^4}\right.}{b}$$

$$\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3bc}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

input

```
Int[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

output

```
(-2*x^2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) + (4*((-2*x)
/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
 + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh
[c*x]]/Sqrt[b]])/(2*E^(a/b))))/(b^2*c^2)))/(3*b*c) + (2*c*((-2*x^3)/(b*c*S
qrt[a + b*ArcSinh[c*x]]) + (6*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]]) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sq
rt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/(b^2*c^4)))/b
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\,\mathrm{arcsinh}\left(xc\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(x^2/(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{5/2}}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral(x**2/(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))^{5/2}}\,dx$$

input
```
int(x^2/(a + b*asinh(c*x))^(5/2),x)
```

output
```
int(x^2/(a + b*asinh(c*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \int \frac{\sqrt{asinh\,(cx)\,b+a}\,x^2}{asinh\,(cx)^3\,b^3+3asinh\,(cx)^2\,a\,b^2+3asinh\,(cx)\,a^2b+a^3}dx$$

input

```
int(x^2/(a+b*asinh(c*x))^(5/2),x)
```

output

```
int((sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*x)**3*b**3 + 3*asinh(c*x)**2*a*
b**2 + 3*asinh(c*x)*a**2*b + a**3),x)
```

3.173 $\int \frac{x}{(a+b\textbf{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\begin{aligned}\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx &= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\operatorname{arcsinh}(cx))^{3/2}} \\ &\quad - \frac{4}{3b^2c^2\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\operatorname{arcsinh}(cx)}} \\ &\quad - \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} + \frac{2e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}\end{aligned}$$

output

```
-2/3*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(3/2)-4/3/b^2/c^2/(a+b*arc
sinh(c*x))^(1/2)-8/3*x^2/b^2/(a+b*arcsinh(c*x))^(1/2)-2/3*exp(2*a/b)*2^(1/
2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c^2+2/3*
2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c^
2/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \frac{e^{-2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-4\sqrt{2}be^{2\text{arcsinh}(cx)}\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsin}}{b}\right.\right.}{}$$

input
```
Integrate[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(-4*Sqrt[2]*b*E^(2*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1
/2, (-2*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*a*E^(4*ArcSin
h[c*x]) - b*E^(4*ArcSinh[c*x]) - 4*b*(1 + E^(4*ArcSinh[c*x]))*ArcSinh[c*x]
 + 4*Sqrt[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*Ar
cSinh[c*x])*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]))/(6*b^2*c^2*E^(2*(a/b
+ ArcSinh[c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6194, 6198, 6233, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}dx$$

$$\Big\downarrow \text{6194}$$

$$\frac{2\int \frac{1}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}dx}{3bc}+\frac{4c\int \frac{x^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}dx}{3b}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{4c\int \frac{x^2}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}dx}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 6233}$$

$$\frac{4c\left(\frac{4\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{4c\left(\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4c\left(-\frac{4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{4c\left(-\frac{4\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4c\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4c\left(-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{4c\left(-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{3789}$$

$$\frac{4c\left(-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{4c\left(-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \; 2633$$

$$\frac{4c\left(-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \; 2634$$

$$\frac{4c\left(-\frac{2x^2}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

input
```
Int[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

output
```
(-2*x*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - 4/(3*b^2*c^2
*Sqrt[a + b*ArcSinh[c*x]]) + (4*c*((-2*x^2)/(b*c*Sqrt[a + b*ArcSinh[c*x]])
 + ((2*I)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/E^((2*a)/b)))/(b^2*c^3)))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{x}{\left(a+b\,\operatorname{arcsinh}\left(xc\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(x/(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(x/(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral(x/(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{(a+b\,\text{asinh}\,(c\,x))^{5/2}}\,dx$$

input
```
int(x/(a + b*asinh(c*x))^(5/2),x)
```

output
```
int(x/(a + b*asinh(c*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}\,x}{asinh\left(cx\right)^{3}b^{3}+3asinh\left(cx\right)^{2}a\,b^{2}+3asinh\left(cx\right)a^{2}b+a^{3}}dx$$

input

```
int(x/(a+b*asinh(c*x))^(5/2),x)
```

output

```
int((sqrt(asinh(c*x)*b + a)*x)/(asinh(c*x)**3*b**3 + 3*asinh(c*x)**2*a*b**
2 + 3*asinh(c*x)*a**2*b + a**3),x)
```

3.174 $\int \frac{1}{(a+b\mathbf{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 143

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\text{arcsinh}(cx)}} + \frac{2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}$$

output
```
-2/3*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(3/2)-4/3*x/b^2/(a+b*arcsinh
(c*x))^(1/2)+2/3*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b
^(5/2)/c+2/3*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(5/2)/c/exp
(a/b)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\text{arcsinh}(cx)}\right)-2b\text{arcsinh}(cx)+be^{2\text{arcsinh}(}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-5/2),x]
```

output

```
(-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c*x])) - 2*b*ArcSinh[c*x] + b*E^(2*
ArcSinh[c*x])*(1 + 2*ArcSinh[c*x]))) - 2*E^((2*a)/b + ArcSinh[c*x])*Sqrt[a
/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*Gamma[1/2, a/b + ArcSinh[c*x]] - 2
*b*E^ArcSinh[c*x]*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*Ar
cSinh[c*x])/b)])/(3*b^2*c*E^((a + b*ArcSinh[c*x])/b)*(a + b*ArcSinh[c*x])^
(3/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6188, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{2c\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}dx}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{6233}$$

$$\frac{2c\left(\frac{2\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{bc}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{6189}$$

$$\frac{2c\left(\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}+\frac{2c\left(-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c^2}\right)}{3b}$$

$$\Big\downarrow \text{ 3788}$$

$$-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}+\frac{2c\left(-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2\left(\frac{1}{2}i\int-\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^2}\right)}{3b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2c\left(\frac{2\left(\frac{1}{2}\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int\frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{2c\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{2c\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

$$\Big\downarrow 2634$$

$$\frac{2c\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}-\frac{2x}{bc\sqrt{a+b\text{arcsinh}(cx)}}\right)}{3b}-\frac{2\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-5/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) + (2*c*((-2*x)/(
b*c*Sqrt[a + b*ArcSinh[c*x]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c
*x]]/Sqrt[b]])/(2*E^(a/b))))/(b^2*c^2)))/(3*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(xc))^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^(5/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\text{too large to display}$$

input
```
int(1/(a+b*asinh(c*x))^(5/2),x)
```

output

```
(asinh(c*x)**2*int((sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**3
*b**3*c**2*x**2 + asinh(c*x)**3*b**3 + 3*asinh(c*x)**2*a*b**2*c**2*x**2 +
3*asinh(c*x)**2*a*b**2 + 3*asinh(c*x)*a**2*b*c**2*x**2 + 3*asinh(c*x)*a**2
*b + a**3*c**2*x**2 + a**3),x)*b**3*c**3 + asinh(c*x)**2*int((sqrt(asinh(c
*x)*b + a)*asinh(c*x))/(asinh(c*x)**3*b**3*c**2*x**2 + asinh(c*x)**3*b**3
+ 3*asinh(c*x)**2*a*b**2*c**2*x**2 + 3*asinh(c*x)**2*a*b**2 + 3*asinh(c*x)
*a**2*b*c**2*x**2 + 3*asinh(c*x)*a**2*b + a**3*c**2*x**2 + a**3),x)*b**3*c
 - 2*asinh(c*x)**2*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c
*x)*x)/(asinh(c*x)**3*b**3*c**2*x**2 + asinh(c*x)**3*b**3 + 3*asinh(c*x)**
2*a*b**2*c**2*x**2 + 3*asinh(c*x)**2*a*b**2 + 3*asinh(c*x)*a**2*b*c**2*x**
2 + 3*asinh(c*x)*a**2*b + a**3*c**2*x**2 + a**3),x)*a*b**2*c**2 - 2*asinh(
c*x)**2*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)**2*x)/(
asinh(c*x)**3*b**3*c**2*x**2 + asinh(c*x)**3*b**3 + 3*asinh(c*x)**2*a*b**2
*c**2*x**2 + 3*asinh(c*x)**2*a*b**2 + 3*asinh(c*x)*a**2*b*c**2*x**2 + 3*as
inh(c*x)*a**2*b + a**3*c**2*x**2 + a**3),x)*b**3*c**2 + 2*sqrt(c**2*x**2 +
 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x) + 2*asinh(c*x)*int((sqrt(asinh(c*x)*
b + a)*asinh(c*x)*x**2)/(asinh(c*x)**3*b**3*c**2*x**2 + asinh(c*x)**3*b**3
 + 3*asinh(c*x)**2*a*b**2*c**2*x**2 + 3*asinh(c*x)**2*a*b**2 + 3*asinh(c*x
)*a**2*b*c**2*x**2 + 3*asinh(c*x)*a**2*b + a**3*c**2*x**2 + a**3),x)*a*b**
2*c**3 + 2*asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*asinh(c*x))/(asinh(c*...
```

3.175 $\int \frac{1}{x(a+b\,\textbf{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(a+b\,\mathrm{arcsinh}(cx))^{5/2}}\,dx = \mathrm{Int}\left(\frac{1}{x(a+b\,\mathrm{arcsinh}(cx))^{5/2}},x\right)$$

output

```
Defer(Int)(1/x/(a+b*arcsinh(c*x))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(a+b\,\mathrm{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{x(a+b\,\mathrm{arcsinh}(cx))^{5/2}}\,dx$$

input

```
Integrate[1/(x*(a + b*ArcSinh[c*x])^(5/2)),x]
```

output

```
Integrate[1/(x*(a + b*ArcSinh[c*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\text{arcsinh}(cx))^{5/2}}\,dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{1}{x(a+b\text{arcsinh}(cx))^{5/2}}dx$$

input
```
Int[1/(x*(a + b*ArcSinh[c*x])^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(a+b\ \text{arcsinh}\left(xc\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(1/x/(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 7.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral(1/(x*(a + b*asinh(c*x))**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsinh(c*x) + a)^(5/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}} \,d x$$

input
```
int(1/(x*(a + b*asinh(c*x))^(5/2)),x)
```

output
```
int(1/(x*(a + b*asinh(c*x))^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

$$\int \frac{1}{x(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)^{3}b^{3}x+3asinh\left(cx\right)^{2}a\,b^{2}x+3asinh\left(cx\right)a^{2}bx+a^{3}x}dx$$

input

```
int(1/x/(a+b*asinh(c*x))^(5/2),x)
```

output

```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**3*b**3*x + 3*asinh(c*x)**2*a*b**2*
x + 3*asinh(c*x)*a**2*b*x + a**3*x),x)
```

3.176 $\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcSinh[c*x])^(5/2)),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcSinh[c*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}} \, dx \\ \Big\downarrow \text{6196} \\ \displaystyle\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}dx \end{array}$$

input
```
Int[1/(x^2*(a + b*ArcSinh[c*x])^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{2} \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^2/(a+b*arcsinh(x*c))^(5/2),x)
```

output
```
int(1/x^2/(a+b*arcsinh(x*c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 13.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b*asinh(c*x))**(5/2),x)
```

output
```
Integral(1/(x**2*(a + b*asinh(c*x))**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsinh(c*x) + a)^(5/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsinh(c*x) + a)^(5/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{x^2\,\left(a+b\,\text{asinh}\,(c\,x)\right)^{5/2}}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(c*x))^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*asinh(c*x))^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^2(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)^{3}b^{3}x^{2}+3asinh\left(cx\right)^{2}a\,b^{2}x^{2}+3asinh\left(cx\right)a^{2}b\,x^{2}+a^{3}x^{2}}dx$$

input

```
int(1/x^2/(a+b*asinh(c*x))^(5/2),x)
```

output

```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**3*b**3*x**2 + 3*asinh(c*x)**2*a*b*
*2*x**2 + 3*asinh(c*x)*a**2*b*x**2 + a**3*x**2),x)
```

3.177 $\int x^m \mathbf{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size $= 12$, antiderivative size $= 12$

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \text{Int}\left(x^m \text{arcsinh}(ax)^{3/2}, x\right)$$

output

```
Defer(Int)(x^m*arcsinh(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time $= 0.28$ (sec) , antiderivative size $= 14$, normalized size of antiderivative $= 1.17$

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m \text{arcsinh}(ax)^{3/2}\,dx$$

input

```
Integrate[x^m*ArcSinh[a*x]^(3/2),x]
```

output

```
Integrate[x^m*ArcSinh[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \operatorname{arcsinh}(ax)^{3/2} \, dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int x^m \operatorname{arcsinh}(ax)^{3/2} dx$$

input
```
Int[x^m*ArcSinh[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x^m \operatorname{arcsinh}\left(xa\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*arcsinh(x*a)^(3/2),x)
```

output
```
int(x^m*arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 58.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(x**m*asinh(a*x)**(3/2),x)
```

output
```
Integral(x**m*asinh(a*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arcsinh(a*x)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m\,\mathrm{asinh}(a\,x)^{3/2}\,dx$$

input
```
int(x^m*asinh(a*x)^(3/2),x)
```

output
```
int(x^m*asinh(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m \sqrt{asinh\,(ax)}\,asinh(ax)\,dx$$

input
```
int(x^m*asinh(a*x)^(3/2),x)
```

output
```
int(x**m*sqrt(asinh(a*x))*asinh(a*x),x)
```

3.178 $\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(x^m \sqrt{\operatorname{arcsinh}(ax)},x\right)$$

output
```
Defer(Int)(x^m*arcsinh(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx$$

input
```
Integrate[x^m*Sqrt[ArcSinh[a*x]],x]
```

output
```
Integrate[x^m*Sqrt[ArcSinh[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx$$

$$\Big\downarrow \text{6196}$$

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}dx$$

input
```
Int[x^m*Sqrt[ArcSinh[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x^m \sqrt{\operatorname{arcsinh}\left(xa\right)}dx$$

input
```
int(x^m*arcsinh(x*a)^(1/2),x)
```

output
```
int(x^m*arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m \sqrt{\operatorname{asinh}{(ax)}}\,dx$$

input
```
integrate(x**m*asinh(a*x)**(1/2),x)
```

output
```
Integral(x**m*sqrt(asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m \sqrt{\operatorname{arsinh}{(ax)}}\,dx$$

input
```
integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sqrt(arcsinh(a*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m\sqrt{\operatorname{arcsinh}(ax)}\,dx=\text{Timed out}$$

input
```
integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^m\,\sqrt{\mathrm{asinh}\left(a\,x\right)}\,dx$$

input
```
int(x^m*asinh(a*x)^(1/2),x)
```

output
```
int(x^m*asinh(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{\operatorname{arcsinh}(ax)}\,dx=\int x^m\sqrt{\mathit{asinh}\left(ax\right)}dx$$

input
```
int(x^m*asinh(a*x)^(1/2),x)
```

output
```
int(x**m*sqrt(asinh(a*x)),x)
```

3.179 $\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\operatorname{Int}\left(\frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

output
```
Defer(Int)(x^m/arcsinh(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

input
```
Integrate[x^m/Sqrt[ArcSinh[a*x]],x]
```

output
```
Integrate[x^m/Sqrt[ArcSinh[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\text{arcsinh}(ax)}} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{x^m}{\sqrt{\text{arcsinh}(ax)}}dx$$

input
```
Int[x^m/Sqrt[ArcSinh[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^m}{\sqrt{\text{arcsinh}\left(xa\right)}}dx$$

input
```
int(x^m/arcsinh(x*a)^(1/2),x)
```

output
```
int(x^m/arcsinh(x*a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

input
```
integrate(x**m/asinh(a*x)**(1/2),x)
```

output
```
Integral(x**m/sqrt(asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

input
```
int(x^m/asinh(a*x)^(1/2),x)
```

output
```
int(x^m/asinh(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 11.42

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$
$$= \frac{2x^m\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}-2\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{a^2x^2+1}dx\right)a^2m-2\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{a^2x^2+1}dx\right)a^2-2}{a}$$

input
```
int(x^m/asinh(a*x)^(1/2),x)
```

output
```
(2*(x**m*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)) - int((x**m*sqrt(a**2*x**2 +
 1)*sqrt(asinh(a*x))*x)/(a**2*x**2 + 1),x)*a**2*m - int((x**m*sqrt(a**2*x*
*2 + 1)*sqrt(asinh(a*x))*x)/(a**2*x**2 + 1),x)*a**2 - int((x**m*sqrt(a**2*
x**2 + 1)*sqrt(asinh(a*x)))/(a**2*x**3 + x),x)*m))/a
```

3.180 $\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m/arcsinh(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/ArcSinh[a*x]^(3/2),x]
```

output
```
Integrate[x^m/ArcSinh[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}dx$$

input
```
Int[x^m/ArcSinh[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^m}{\operatorname{arcsinh}\left(x a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/arcsinh(x*a)^(3/2),x)
```

output
```
int(x^m/arcsinh(x*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**m/asinh(a*x)**(3/2),x)
```

output
```
Integral(x**m/asinh(a*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/arcsinh(a*x)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/arcsinh(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{x^m}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x^m/asinh(a*x)^(3/2),x)
```

output
```
int(x^m/asinh(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 14.67

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{2asinh(ax)\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)a^2x^2+asinh(ax)}dx\right)a^2m+2asinh(ax)\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x}{asinh(ax)a^2x^2+asinh(ax)}\right.}{asinh}$$

input

```
int(x^m/asinh(a*x)^(3/2),x)
```

output

```
(2*(asinh(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*
x)*a**2*x**2 + asinh(a*x)),x)*a**2*m + asinh(a*x)*int((x**m*sqrt(a**2*x**2
 + 1)*sqrt(asinh(a*x))*x)/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)*a**2 + as
inh(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/(asinh(a*x)*a**2*
x**3 + asinh(a*x)*x),x)*m - x**m*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))))/(a
sinh(a*x)*a)
```

3.181 $\int x^3\text{arcsinh}(ax)^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int x^3\text{arcsinh}(ax)^n\,dx = \frac{4^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-4\text{arcsinh}(ax))}{a^4} - \frac{2^{-4-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))}{a^4} - \frac{2^{-4-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^4} + \frac{4^{-3-n}\Gamma(1+n,4\text{arcsinh}(ax))}{a^4}$$

output
```
4^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-4*arcsinh(a*x))/a^4/((-arcsinh(a*x))^n)
-2^(-4-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcsinh(a*x))/a^4/((-arcsinh(a*x))^n
)-2^(-4-n)*GAMMA(1+n,2*arcsinh(a*x))/a^4+4^(-3-n)*GAMMA(1+n,4*arcsinh(a*x)
)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int x^3\text{arcsinh}(ax)^n\,dx = \frac{4^{-3-n}(-\text{arcsinh}(ax))^{-n}\left(\text{arcsinh}(ax)^n\Gamma(1+n,-4\text{arcsinh}(ax)) - 2^{2+n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(\right.}{a^4}$$

input
```
Integrate[x^3*ArcSinh[a*x]^n,x]
```

output
```
(4^(-3 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]] - 2^(2 + n)*ArcS
inh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(2^(2 + n)*
Gamma[1 + n, 2*ArcSinh[a*x]]) + Gamma[1 + n, 4*ArcSinh[a*x]])))/(a^4*(-Arc
Sinh[a*x])^n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}(ax)^n \, dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int a^3x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{1}{8}\text{arcsinh}(ax)^n \sinh(4\text{arcsinh}(ax)) - \frac{1}{4}\text{arcsinh}(ax)^n \sinh(2\text{arcsinh}(ax))\right) d\text{arcsinh}(ax)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2^{-2(n+3)}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-4\text{arcsinh}(ax)) - 2^{-n-4}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1}{a^4}$$

input
```
Int[x^3*ArcSinh[a*x]^n,x]
```

output
```
((ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]])/(2^(2*(3 + n))*(-ArcSinh[a
*x])^n) - (2^(-4 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(-ArcS
inh[a*x])^n - 2^(-4 - n)*Gamma[1 + n, 2*ArcSinh[a*x]] + Gamma[1 + n, 4*Arc
Sinh[a*x]]/2^(2*(3 + n)))/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int x^3 \operatorname{arcsinh}(xa)^n \, dx$$

input
```
int(x^3*arcsinh(x*a)^n,x)
```

output
```
int(x^3*arcsinh(x*a)^n,x)
```

Fricas [F]

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \int x^3 \operatorname{arsinh}(ax)^n \, dx$$

input
```
integrate(x^3*arcsinh(a*x)^n,x, algorithm="fricas")
```

output
```
integral(x^3*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \int x^3 \operatorname{asinh}^n(ax) \, dx$$

input
```
integrate(x**3*asinh(a*x)**n,x)
```

output
```
Integral(x**3*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \int x^3 \operatorname{arsinh}(ax)^n \, dx$$

input
```
integrate(x^3*arcsinh(a*x)^n,x, algorithm="maxima")
```

output
```
integrate(x^3*arcsinh(a*x)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \operatorname{arcsinh}(ax)^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arcsinh(a*x)^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arcsinh}(ax)^n\,dx = \int x^3\,\operatorname{asinh}(a\,x)^n\,dx$$

input
```
int(x^3*asinh(a*x)^n,x)
```

output
```
int(x^3*asinh(a*x)^n, x)
```

Reduce [F]

$$\int x^3 \operatorname{arcsinh}(ax)^n\,dx = \int asinh(ax)^n\,x^3dx$$

input
```
int(x^3*asinh(a*x)^n,x)
```

output
```
int(asinh(a*x)**n*x**3,x)
```

3.182 $\int x^2 \text{arcsinh}(ax)^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\begin{aligned}\int x^2 \text{arcsinh}(ax)^n \, dx &= \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax))}{8a^3} \\ &\quad - \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{8a^3} \\ &\quad + \frac{\Gamma(1+n,\text{arcsinh}(ax))}{8a^3} - \frac{3^{-1-n}\Gamma(1+n,3\text{arcsinh}(ax))}{8a^3}\end{aligned}$$

output
```
1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^3/((-arcsinh(a*x)
)^n)-1/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^3/((-arcsinh(a*x))^n)+1
/8*GAMMA(1+n,arcsinh(a*x))/a^3-1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\begin{aligned}&\int x^2 \text{arcsinh}(ax)^n \, dx \\ &= \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax)) - (-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,}{8a^3}\end{aligned}$$

input
```
Integrate[x^2*ArcSinh[a*x]^n,x]
```

output
```
((3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(-ArcSinh[a*x])
^n - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamm
a[1 + n, ArcSinh[a*x]] - 3^(-1 - n)*Gamma[1 + n, 3*ArcSinh[a*x]])/(8*a^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}(ax)^n\,dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int a^2x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^nd\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{1}{4}\text{arcsinh}(ax)^n\cosh(3\text{arcsinh}(ax))-\frac{1}{4}\sqrt{a^2x^2+1}\text{arcsinh}(ax)^n\right)d\text{arcsinh}(ax)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}3^{-n-1}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-3\text{arcsinh}(ax))-\frac{1}{8}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\text{arc}}{a^3}$$

input
```
Int[x^2*ArcSinh[a*x]^n,x]
```

output
```
((3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*(-ArcSinh[a*
x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*(-ArcSinh[a*x])^n
) + Gamma[1 + n, ArcSinh[a*x]]/8 - (3^(-1 - n)*Gamma[1 + n, 3*ArcSinh[a*x]
])/8)/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \operatorname{arcsinh}(xa)^n\,dx$$

input
```
int(x^2*arcsinh(x*a)^n,x)
```

output
```
int(x^2*arcsinh(x*a)^n,x)
```

Fricas [F]

$$\int x^2 \operatorname{arcsinh}(ax)^n\,dx = \int x^2 \operatorname{arsinh}(ax)^n\,dx$$

input
```
integrate(x^2*arcsinh(a*x)^n,x, algorithm="fricas")
```

output
```
integral(x^2*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx=\int x^2\,\text{asinh}^n\,(ax)\,dx$$

input
```
integrate(x**2*asinh(a*x)**n,x)
```

output
```
Integral(x**2*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx=\int x^2\,\text{arsinh}\,(ax)^n\;dx$$

input
```
integrate(x^2*arcsinh(a*x)^n,x, algorithm="maxima")
```

output
```
integrate(x^2*arcsinh(a*x)^n, x)
```

Giac [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx=\int x^2\,\text{arsinh}\,(ax)^n\;dx$$

input
```
integrate(x^2*arcsinh(a*x)^n,x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)^n\,dx=\int x^2\,\mathrm{asinh}(a\,x)^n\,dx$$

input
```
int(x^2*asinh(a*x)^n,x)
```

output
```
int(x^2*asinh(a*x)^n, x)
```

Reduce [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx=\int asinh(ax)^n\,x^2dx$$

input
```
int(x^2*asinh(a*x)^n,x)
```

output
```
int(asinh(a*x)**n*x**2,x)
```

3.183 $\int x\text{arcsinh}(ax)^n\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x\text{arcsinh}(ax)^n\,dx = \frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))}{a^2} + \frac{2^{-3-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^2}$$

output
```
2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)
+2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x\text{arcsinh}(ax)^n\,dx = \frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax)) + 2^{-3-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^2}$$

input
```
Integrate[x*ArcSinh[a*x]^n,x]
```

output
```
((2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(-ArcSinh[a*x])
^n + 2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6195, 5971, 27, 3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax)^n\,dx$$

$$\Big\downarrow \text{6195}$$

$$\frac{\int ax\sqrt{a^2x^2+1}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \frac{1}{2}\text{arcsinh}(ax)^n\sinh(2\text{arcsinh}(ax))d\text{arcsinh}(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \text{arcsinh}(ax)^n\sinh(2\text{arcsinh}(ax))d\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\text{arcsinh}(ax)^n\sin(2i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \text{arcsinh}(ax)^n\sin(2i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{2a^2}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{i\left(\frac{1}{2}i\int e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)-\frac{1}{2}i\int e^{-2\text{arcsinh}(ax)}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)\right)}{2a^2}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{i\left(i2^{-n-2}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-2\text{arcsinh}(ax))+i2^{-n-2}\Gamma(n+1,2\text{arcsinh}(ax))\right)}{2a^2}$$

input

```
Int[x*ArcSinh[a*x]^n,x]
```

output

```
((-1/2*I)*((I*2^(-2 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(-A
rcSinh[a*x])^n + I*2^(-2 - n)*Gamma[1 + n, 2*ArcSinh[a*x]]))/a^2
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\operatorname{arcsinh}(xa)^{2+n} \operatorname{hypergeom}\left(\left[1+\frac{n}{2}\right],\left[\frac{3}{2},2+\frac{n}{2}\right],\operatorname{arcsinh}(xa)^{2}\right)}{a^{2}(2+n)}$	38

input
```
int(x*arcsinh(x*a)^n,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/(2+n)*arcsinh(x*a)^(2+n)*hypergeom([1+1/2*n],[3/2,2+1/2*n],arcsinh(x
*a)^2)
```

Fricas [F]

$$\int x\mathrm{arcsinh}(ax)^n\,dx=\int x\,\mathrm{arsinh}\,(ax)^n\,dx$$

input
```
integrate(x*arcsinh(a*x)^n,x, algorithm="fricas")
```

output
```
integral(x*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\,\text{asinh}^n\,(ax)\,dx$$

input
```
integrate(x*asinh(a*x)**n,x)
```

output
```
Integral(x*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\,\text{arsinh}\,(ax)^n\,dx$$

input
```
integrate(x*arcsinh(a*x)^n,x, algorithm="maxima")
```

output
```
integrate(x*arcsinh(a*x)^n, x)
```

Giac [F]

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\,\text{arsinh}\,(ax)^n\,dx$$

input
```
integrate(x*arcsinh(a*x)^n,x, algorithm="giac")
```

output
```
integrate(x*arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{arcsinh}(ax)^n\,dx=\int x\,\mathrm{asinh}(a\,x)^n\,dx$$

input
```
int(x*asinh(a*x)^n,x)
```

output
```
int(x*asinh(a*x)^n, x)
```

Reduce [F]

$$\int x\mathrm{arcsinh}(ax)^n\,dx=\int asinh(ax)^n\,xdx$$

input
```
int(x*asinh(a*x)^n,x)
```

output
```
int(asinh(a*x)**n*x,x)
```

3.184 $\int \operatorname{arcsinh}(ax)^n\,dx$

Optimal result

Integrand size = 6, antiderivative size = 49

$$\int \operatorname{arcsinh}(ax)^n\,dx=\frac{(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-\operatorname{arcsinh}(ax))}{2a}-\frac{\Gamma(1+n,\operatorname{arcsinh}(ax))}{2a}$$

output
```
1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a/((-arcsinh(a*x))^n)-1/2*GAMM
A(1+n,arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \operatorname{arcsinh}(ax)^n\,dx=\frac{(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-\operatorname{arcsinh}(ax))-\Gamma(1+n,\operatorname{arcsinh}(ax))}{2a}$$

input
```
Integrate[ArcSinh[a*x]^n,x]
```

output
```
((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - Gamma[1
+ n, ArcSinh[a*x]])/(2*a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6189, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^n\,dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \sqrt{a^2x^2+1}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \text{arcsinh}(ax)^n \sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right) d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -ie^{\text{arcsinh}(ax)}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)-\frac{1}{2}i\int ie^{-\text{arcsinh}(ax)}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int e^{-\text{arcsinh}(ax)}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)+\frac{1}{2}\int e^{\text{arcsinh}(ax)}\text{arcsinh}(ax)^n d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{2612}$$

$$\frac{\frac{1}{2}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(n+1,-\text{arcsinh}(ax))-\frac{1}{2}\Gamma(n+1,\text{arcsinh}(ax))}{a}$$

input
```
Int[ArcSinh[a*x]^n,x]
```

output
```
((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*(-ArcSinh[a*x])^n) - Gamm
a[1 + n, ArcSinh[a*x]]/2)/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\operatorname{arcsinh}(xa)^{1+n} \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{n}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{n}{2}\right],\frac{\operatorname{arcsinh}(xa)^{2}}{4}\right)}{a(1+n)}$	40

input

```
int(arcsinh(x*a)^n,x,method=_RETURNVERBOSE)
```

output

```
1/a/(1+n)*arcsinh(x*a)^(1+n)*hypergeom([1/2+1/2*n],[1/2,3/2+1/2*n],1/4*arc
sinh(x*a)^2)
```

Fricas [F]

$$\int \operatorname{arcsinh}(ax)^n \, dx = \int \operatorname{arsinh}\left(ax\right)^n \, dx$$

input

```
integrate(arcsinh(a*x)^n,x, algorithm="fricas")
```

output

```
integral(arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int \operatorname{arcsinh}(ax)^n \, dx = \int \operatorname{asinh}^{n}\left(ax\right) dx$$

input

```
integrate(asinh(a*x)**n,x)
```

output

```
Integral(asinh(a*x)**n, x)
```

Maxima [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{arsinh}\left(ax\right)^n\,dx$$

input
```
integrate(arcsinh(a*x)^n,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^n, x)
```

Giac [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{arsinh}\left(ax\right)^n\,dx$$

input
```
integrate(arcsinh(a*x)^n,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{asinh}(a\,x)^n\,dx$$

input
```
int(asinh(a*x)^n,x)
```

output
```
int(asinh(a*x)^n, x)
```

Reduce [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int asinh(ax)^n\,dx$$

input
```
int(asinh(a*x)^n,x)
```

output
```
int(asinh(a*x)**n,x)
```

3.185 $\int \frac{\text{arcsinh}(ax)^n}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{arcsinh}(ax)^n}{x}\,dx = \text{Int}\left(\frac{\text{arcsinh}(ax)^n}{x},x\right)$$

output
```
Defer(Int)(arcsinh(a*x)^n/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{arcsinh}(ax)^n}{x}\,dx = \int \frac{\text{arcsinh}(ax)^n}{x}\,dx$$

input
```
Integrate[ArcSinh[a*x]^n/x,x]
```

output
```
Integrate[ArcSinh[a*x]^n/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x} \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}dx$$

input
```
Int[ArcSinh[a*x]^n/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(xa\right)^{n}}{x}dx$$

input
```
int(arcsinh(x*a)^n/x,x)
```

output
```
int(arcsinh(x*a)^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{n}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \int \frac{\operatorname{asinh}^{n}\left(ax\right)}{x}\,dx$$

input
```
integrate(asinh(a*x)**n/x,x)
```

output
```
Integral(asinh(a*x)**n/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{n}}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^n/x, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{x}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^n/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^n}{x}\,dx$$

input
```
int(asinh(a*x)^n/x,x)
```

output
```
int(asinh(a*x)^n/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{asinh(ax)^n}{x}dx$$

input
```
int(asinh(a*x)^n/x,x)
```

output
```
int(asinh(a*x)**n/x,x)
```

3.186 $\int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^n}{x^2}, x\right)$$

output
```
Defer(Int)(arcsinh(a*x)^n/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx = \int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx$$

input
```
Integrate[ArcSinh[a*x]^n/x^2,x]
```

output
```
Integrate[ArcSinh[a*x]^n/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^n}{x^2} \, dx$$
$$\Big\downarrow \text{ 6196}$$
$$\int \frac{\text{arcsinh}(ax)^n}{x^2}dx$$

input
```
Int[ArcSinh[a*x]^n/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(xa\right)^{n}}{x^{2}}dx$$

input
```
int(arcsinh(x*a)^n/x^2,x)
```

output
```
int(arcsinh(x*a)^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{n}}{x^{2}}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{asinh}^{n}\left(ax\right)}{x^{2}}\,dx$$

input
```
integrate(asinh(a*x)**n/x**2,x)
```

output
```
Integral(asinh(a*x)**n/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{n}}{x^{2}}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x^2,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^n/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x)^n/x^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^n/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)^n}{x^2}\,dx$$

input
```
int(asinh(a*x)^n/x^2,x)
```

output
```
int(asinh(a*x)^n/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{asinh(ax)^n}{x^2}dx$$

input
```
int(asinh(a*x)^n/x^2,x)
```

output
```
int(asinh(a*x)**n/x**2,x)
```

3.187 $\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx=\text{Int}\left((dx)^{3/2}\text{arcsinh}(cx)^n,x\right)$$

output
```
Defer(Int)((d*x)^(3/2)*arcsinh(c*x)^n,x)
```

Mathematica [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx=\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx$$

input
```
Integrate[(d*x)^(3/2)*ArcSinh[c*x]^n,x]
```

output
```
Integrate[(d*x)^(3/2)*ArcSinh[c*x]^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n dx$$

input
```
Int[(d*x)^(3/2)*ArcSinh[c*x]^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (dx)^{\frac{3}{2}} \operatorname{arcsinh}\left(xc\right)^{n} dx$$

input
```
int((d*x)^(3/2)*arcsinh(x*c)^n,x)
```

output
```
int((d*x)^(3/2)*arcsinh(x*c)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx = \int (dx)^{\frac{3}{2}}\,\text{arsinh}\,(cx)^n\,\,dx$$

input
```
integrate((d*x)^(3/2)*arcsinh(c*x)^n,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x*arcsinh(c*x)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(3/2)*asinh(c*x)**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx = \int (dx)^{\frac{3}{2}}\,\text{arsinh}\,(cx)^n\,\,dx$$

input
```
integrate((d*x)^(3/2)*arcsinh(c*x)^n,x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)*arcsinh(c*x)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx=\text{Timed out}$$

input
```
integrate((d*x)^(3/2)*arcsinh(c*x)^n,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx=\int \text{asinh}(c\,x)^n\,{(d\,x)}^{3/2}\,dx$$

input
```
int(asinh(c*x)^n*(d*x)^(3/2),x)
```

output
```
int(asinh(c*x)^n*(d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (dx)^{3/2}\text{arcsinh}(cx)^n\,dx=\sqrt{d}\left(\int \sqrt{x}\,asinh(cx)^n\,xdx\right)d$$

input
```
int((d*x)^(3/2)*asinh(c*x)^n,x)
```

output
```
sqrt(d)*int(sqrt(x)*asinh(c*x)**n*x,x)*d
```

3.188 $\int \sqrt{dx}\,\mathbf{arcsinh}(cx)^n\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt{dx}\,\text{arcsinh}(cx)^n\,dx = \text{Int}\left(\sqrt{dx}\,\text{arcsinh}(cx)^n, x\right)$$

output
```
Defer(Int)((d*x)^(1/2)*arcsinh(c*x)^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt{dx}\,\text{arcsinh}(cx)^n\,dx = \int \sqrt{dx}\,\text{arcsinh}(cx)^n\,dx$$

input
```
Integrate[Sqrt[d*x]*ArcSinh[c*x]^n,x]
```

output
```
Integrate[Sqrt[d*x]*ArcSinh[c*x]^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\text{arcsinh}(cx)^n\,dx$$

$$\Big\downarrow \text{6196}$$

$$\int \sqrt{dx}\text{arcsinh}(cx)^n dx$$

input
```
Int[Sqrt[d*x]*ArcSinh[c*x]^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{dx}\,\operatorname{arcsinh}\left(xc\right)^{n}dx$$

input
```
int((d*x)^(1/2)*arcsinh(x*c)^n,x)
```

output
```
int((d*x)^(1/2)*arcsinh(x*c)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}\text{arcsinh}(cx)^n\,dx = \int \sqrt{dx}\,\text{arsinh}\left(cx\right)^n\,dx$$

input
```
integrate((d*x)^(1/2)*arcsinh(c*x)^n,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*arcsinh(c*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}\text{arcsinh}(cx)^n\,dx = \int \sqrt{dx}\,\text{asinh}^n\left(cx\right)dx$$

input
```
integrate((d*x)**(1/2)*asinh(c*x)**n,x)
```

output
```
Integral(sqrt(d*x)*asinh(c*x)**n, x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}\text{arcsinh}(cx)^n\,dx = \int \sqrt{dx}\,\text{arsinh}\left(cx\right)^n\,dx$$

input
```
integrate((d*x)^(1/2)*arcsinh(c*x)^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)*arcsinh(c*x)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{dx}\operatorname{arcsinh}(cx)^n\,dx = \text{Timed out}$$

input
```
integrate((d*x)^(1/2)*arcsinh(c*x)^n,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}\operatorname{arcsinh}(cx)^n\,dx = \int \operatorname{asinh}(c\,x)^n\,\sqrt{d\,x}\,dx$$

input
```
int(asinh(c*x)^n*(d*x)^(1/2),x)
```

output
```
int(asinh(c*x)^n*(d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}\operatorname{arcsinh}(cx)^n\,dx = \sqrt{d}\left(\int \sqrt{x}\,asinh(cx)^n\,dx\right)$$

input
```
int((d*x)^(1/2)*asinh(c*x)^n,x)
```

output

```
sqrt(d)*int(sqrt(x)*asinh(c*x)**n,x)
```

3.189 $\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}} \, dx = \text{Int}\left(\frac{\text{arcsinh}(cx)^n}{\sqrt{dx}}, x\right)$$

output

```
Defer(Int)(arcsinh(c*x)^n/(d*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}} \, dx = \int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}} \, dx$$

input

```
Integrate[ArcSinh[c*x]^n/Sqrt[d*x],x]
```

output

```
Integrate[ArcSinh[c*x]^n/Sqrt[d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}} \, dx$$
$$\Big\downarrow \textcolor{blue}{6196}$$
$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}}dx$$

input
```
Int[ArcSinh[c*x]^n/Sqrt[d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{arcsinh}\left(xc\right)^{n}}{\sqrt{dx}}dx$$

input
```
int(arcsinh(x*c)^n/(d*x)^(1/2),x)
```

output
```
int(arcsinh(x*c)^n/(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}}\,dx = \int \frac{\text{arsinh}\left(cx\right)^n}{\sqrt{dx}}\,dx$$

input
```
integrate(arcsinh(c*x)^n/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*arcsinh(c*x)^n/(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}}\,dx = \int \frac{\text{asinh}^n\left(cx\right)}{\sqrt{dx}}\,dx$$

input
```
integrate(asinh(c*x)**n/(d*x)**(1/2),x)
```

output
```
Integral(asinh(c*x)**n/sqrt(d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(cx)^n}{\sqrt{dx}}\,dx = \int \frac{\text{arsinh}\left(cx\right)^n}{\sqrt{dx}}\,dx$$

input
```
integrate(arcsinh(c*x)^n/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)^n/sqrt(d*x), x)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(cx)^n}{\sqrt{dx}}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^n}{\sqrt{dx}}\,dx$$

input
```
integrate(arcsinh(c*x)^n/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)^n/sqrt(d*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(cx)^n}{\sqrt{dx}}\,dx = \int \frac{\operatorname{asinh}(c\,x)^n}{\sqrt{d\,x}}\,dx$$

input
```
int(asinh(c*x)^n/(d*x)^(1/2),x)
```

output
```
int(asinh(c*x)^n/(d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{arcsinh}(cx)^n}{\sqrt{dx}}\,dx = \frac{\int \frac{asinh(cx)^n}{\sqrt{x}}dx}{\sqrt{d}}$$

input
```
int(asinh(c*x)^n/(d*x)^(1/2),x)
```

output
```
int(asinh(c*x)**n/sqrt(x),x)/sqrt(d)
```

3.190 $\int \frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \text{Int}\left(\frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}},x\right)$$

output
```
Defer(Int)(arcsinh(c*x)^n/(d*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \int \frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx$$

input
```
Integrate[ArcSinh[c*x]^n/(d*x)^(3/2),x]
```

output
```
Integrate[ArcSinh[c*x]^n/(d*x)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6196}$$

$$\int \frac{\text{arcsinh}(cx)^n}{(dx)^{3/2}}dx$$

input
```
Int[ArcSinh[c*x]^n/(d*x)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{arcsinh}\left(xc\right)^{n}}{\left(dx\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(x*c)^n/(d*x)^(3/2),x)
```

output
```
int(arcsinh(x*c)^n/(d*x)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^n}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(c*x)^n/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*arcsinh(c*x)^n/(d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 12.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^n\left(cx\right)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(c*x)**n/(d*x)**(3/2),x)
```

output
```
Integral(asinh(c*x)**n/(d*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^n}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(c*x)^n/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)^n/(d*x)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^n}{\left(dx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(c*x)^n/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)^n/(d*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \int \frac{{\mathrm{asinh}\left(c\,x\right)}^n}{{\left(d\,x\right)}^{3/2}}\,dx$$

input
```
int(asinh(c*x)^n/(d*x)^(3/2),x)
```

output
```
int(asinh(c*x)^n/(d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{arcsinh}(cx)^n}{(dx)^{3/2}}\,dx = \frac{\int \frac{asinh(cx)^n}{\sqrt{x}\,x}dx}{\sqrt{d}\,d}$$

input
```
int(asinh(c*x)^n/(d*x)^(3/2),x)
```

output
```
int(asinh(c*x)**n/(sqrt(x)*x),x)/(sqrt(d)*d)
```

3.191 $\int (bx)^m \text{arcsinh}(ax)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \text{Int}((bx)^m \text{arcsinh}(ax)^n, x)$$

output
```
Defer(Int)((b*x)^m*arcsinh(a*x)^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \int (bx)^m \text{arcsinh}(ax)^n \, dx$$

input
```
Integrate[(b*x)^m*ArcSinh[a*x]^n,x]
```

output
```
Integrate[(b*x)^m*ArcSinh[a*x]^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx$$

$$\Big\downarrow \text{6196}$$

$$\int (bx)^m \text{arcsinh}(ax)^n dx$$

input
```
Int[(b*x)^m*ArcSinh[a*x]^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \operatorname{arcsinh}(xa)^n \, dx$$

input
```
int((b*x)^m*arcsinh(x*a)^n,x)
```

output
```
int((b*x)^m*arcsinh(x*a)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n\,dx = \int (bx)^m \operatorname{arsinh}\left(ax\right)^n\,dx$$

input
```
integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="fricas")
```

output
```
integral((b*x)^m*arcsinh(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \text{arcsinh}(ax)^n\,dx = \int (bx)^m \operatorname{asinh}^n\left(ax\right)dx$$

input
```
integrate((b*x)**m*asinh(a*x)**n,x)
```

output
```
Integral((b*x)**m*asinh(a*x)**n, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n\,dx = \int (bx)^m \operatorname{arsinh}\left(ax\right)^n\,dx$$

input
```
integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="maxima")
```

output
```
integrate((b*x)^m*arcsinh(a*x)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int (bx)^m \operatorname{arcsinh}(ax)^n \, dx = \text{Timed out}$$

input
```
integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \operatorname{arcsinh}(ax)^n \, dx = \int \operatorname{asinh}(a\,x)^n\,(b\,x)^m \, dx$$

input
```
int(asinh(a*x)^n*(b*x)^m,x)
```

output
```
int(asinh(a*x)^n*(b*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int (bx)^m \operatorname{arcsinh}(ax)^n \, dx = b^m \left(\int x^m asinh(ax)^n \, dx \right)$$

input
```
int((b*x)^m*asinh(a*x)^n,x)
```

output

```
b**m*int(x**m*asinh(a*x)**n,x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:



ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file