Computer Algebra
Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/327-7.1.3

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**272**]. This is test number [327].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (272)	0.00 (0)
Rubi	99.63 (271)	0.37 (1)
Maple	81.25 (221)	18.75 (51)
Maxima	34.93 (95)	65.07 (177)
Reduce	31.62 (86)	68.38 (186)
Sympy	28.68 (78)	71.32 (194)
Giac	23.90 (65)	76.10 (207)
Fricas	23.53 (64)	76.47 (208)
Mupad	23.53 (64)	76.47 (208)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

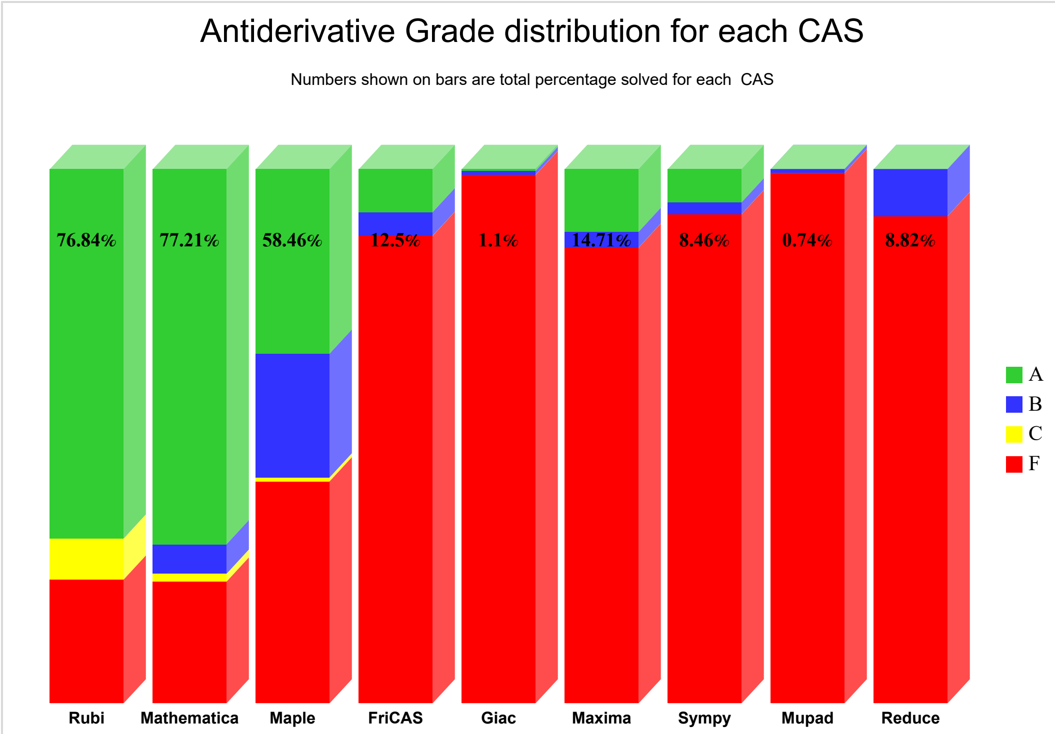
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

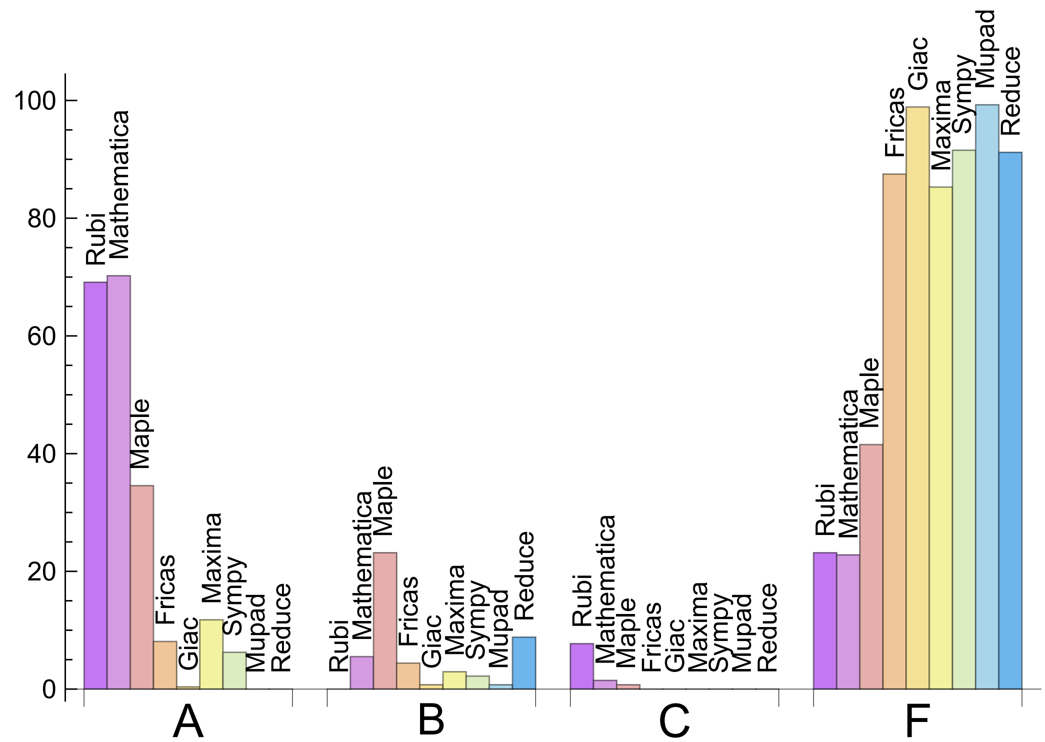
System	% A grade	% B grade	% C grade	% F grade
Mathematica	70.221	5.515	1.471	22.794
Rubi	69.118	0.000	7.721	23.162
Maple	34.559	23.162	0.735	41.544
Maxima	11.765	2.941	0.000	85.294
Fricas	8.088	4.412	0.000	87.500
Sympy	6.250	2.206	0.000	91.544
Giac	0.368	0.735	0.000	98.897
Mupad	0.000	0.735	0.000	99.265
Reduce	0.000	8.824	0.000	91.176

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	51	100.00	0.00	0.00
Maxima	177	66.67	4.52	28.81
Fricas	208	62.02	0.00	37.98
Reduce	186	100.00	0.00	0.00
Sympy	194	76.29	23.71	0.00
Giac	207	60.39	0.00	39.61
Mupad	208	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Maxima	0.21
Giac	0.43
Rubi	0.88
Mupad	2.88
Maple	3.47
Mathematica	3.65
Sympy	11.44
Reduce	18.99

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	22.73	0.99	22.00	1.00
Giac	24.48	1.04	22.00	1.00
Reduce	89.45	3.29	59.50	1.92
Sympy	117.92	1.49	26.00	1.00
Rubi	172.31	0.86	140.00	1.00
Maxima	188.40	4.74	30.00	1.00
Fricas	190.91	2.38	90.50	1.75
Mathematica	293.43	1.15	151.00	1.08
Maple	413.14	1.63	219.00	1.13

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

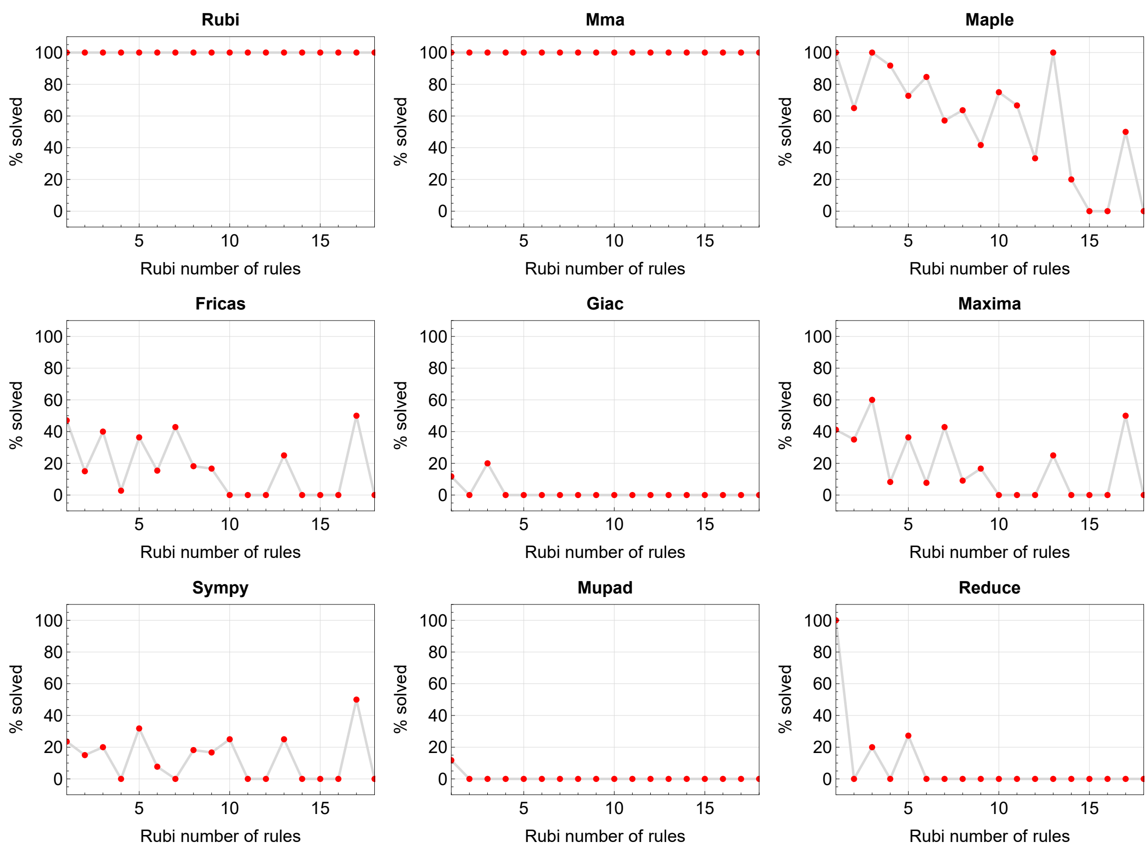


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

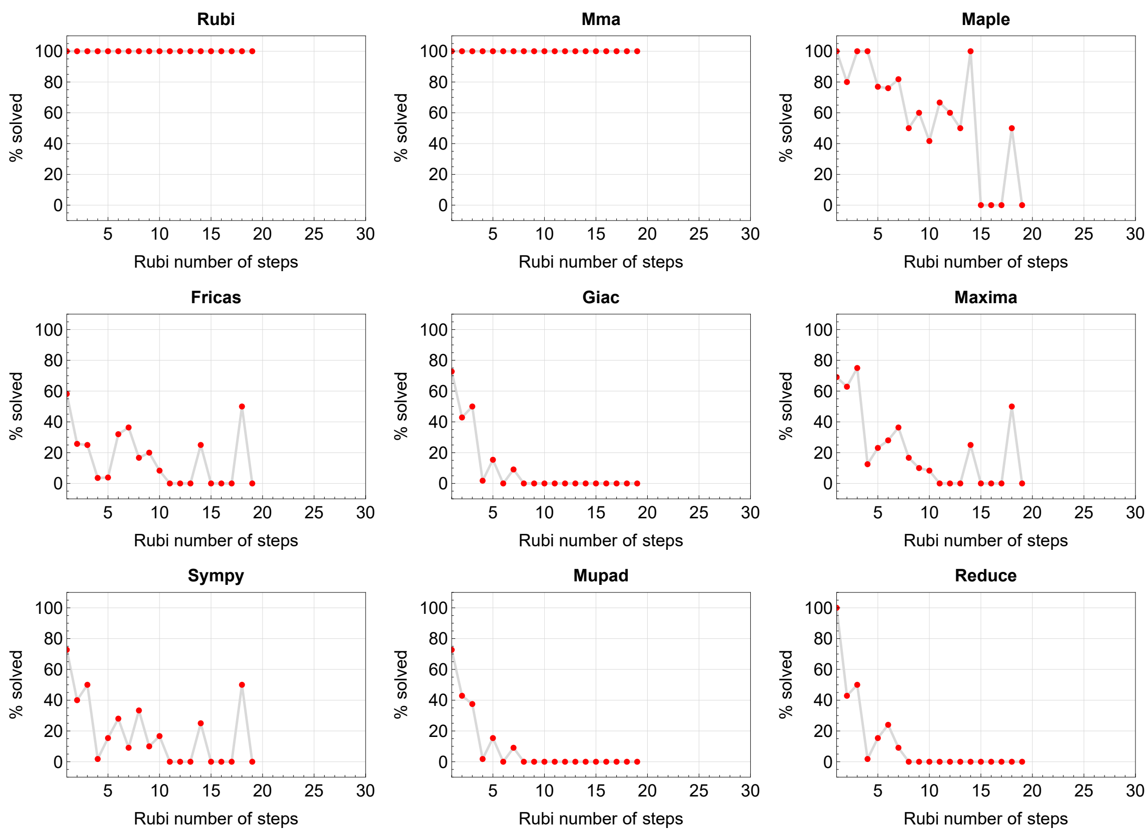


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

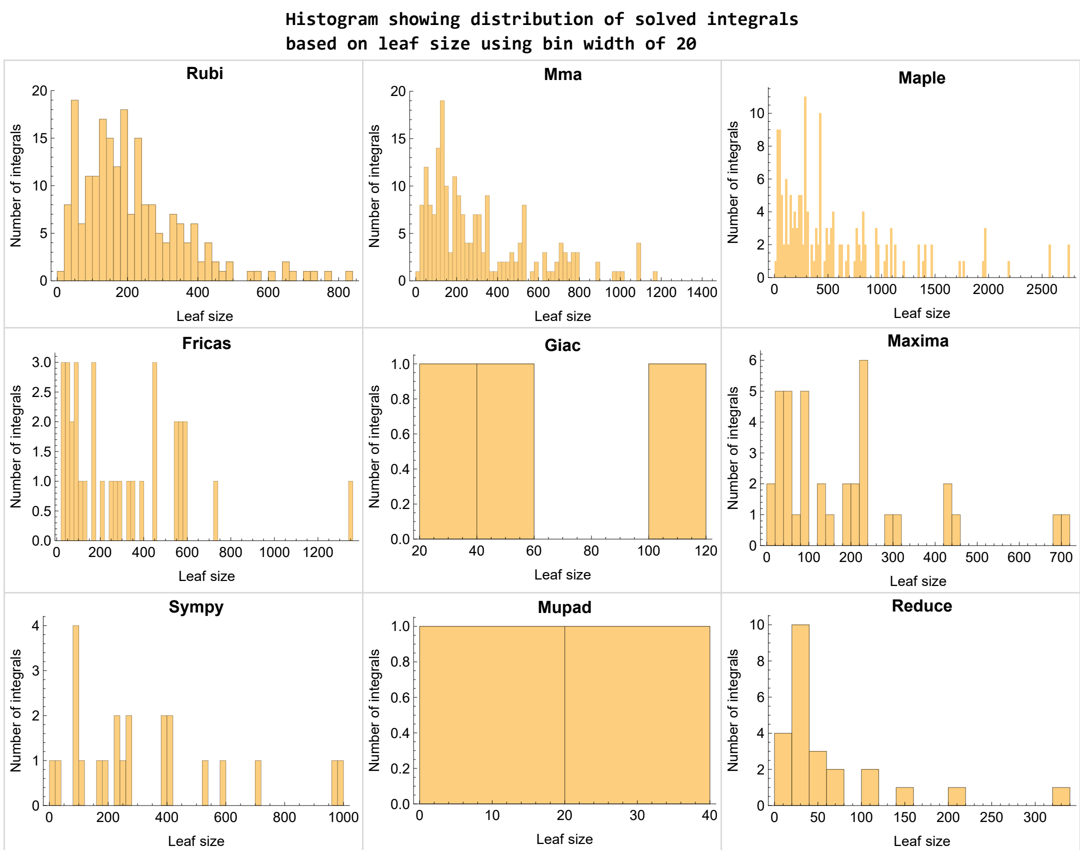


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

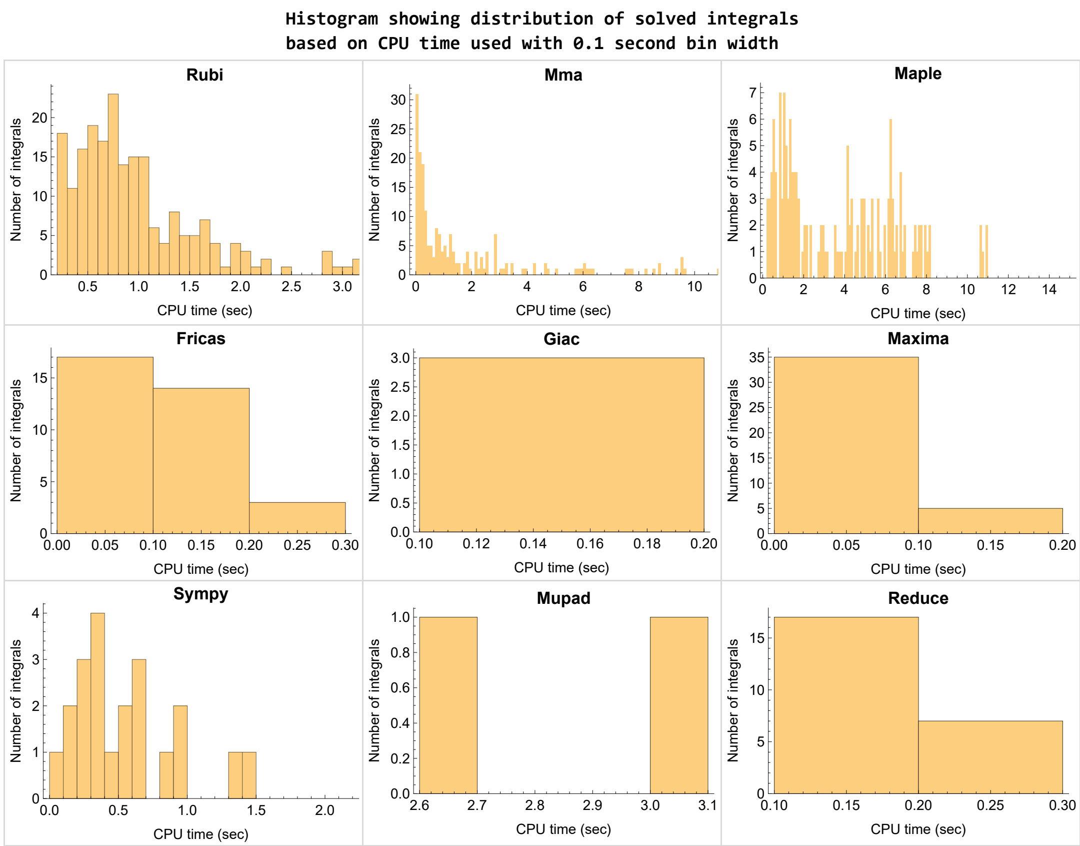


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

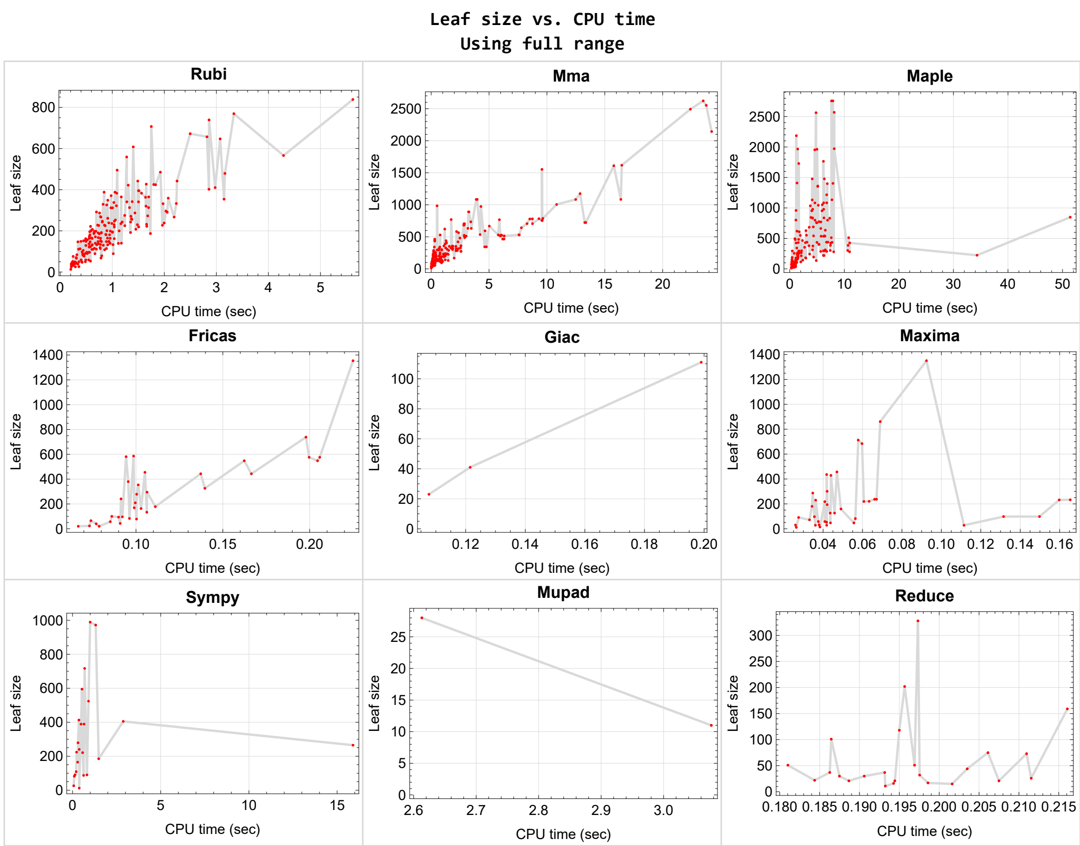


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{20, 21, 25, 26, 30, 31, 35, 36, 71, 72, 77, 78, 83, 84, 88, 89, 94, 95, 100, 101, 106, 107, 111, 112, 116, 117, 121, 122, 126, 127, 132, 133, 138, 139, 143, 144, 159, 160, 164, 165, 166, 167, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 190, 191, 195, 196, 199, 200}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{102, 169, 170, 183, 188, 242, 248, 254, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272\}$

Maple $\{150\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

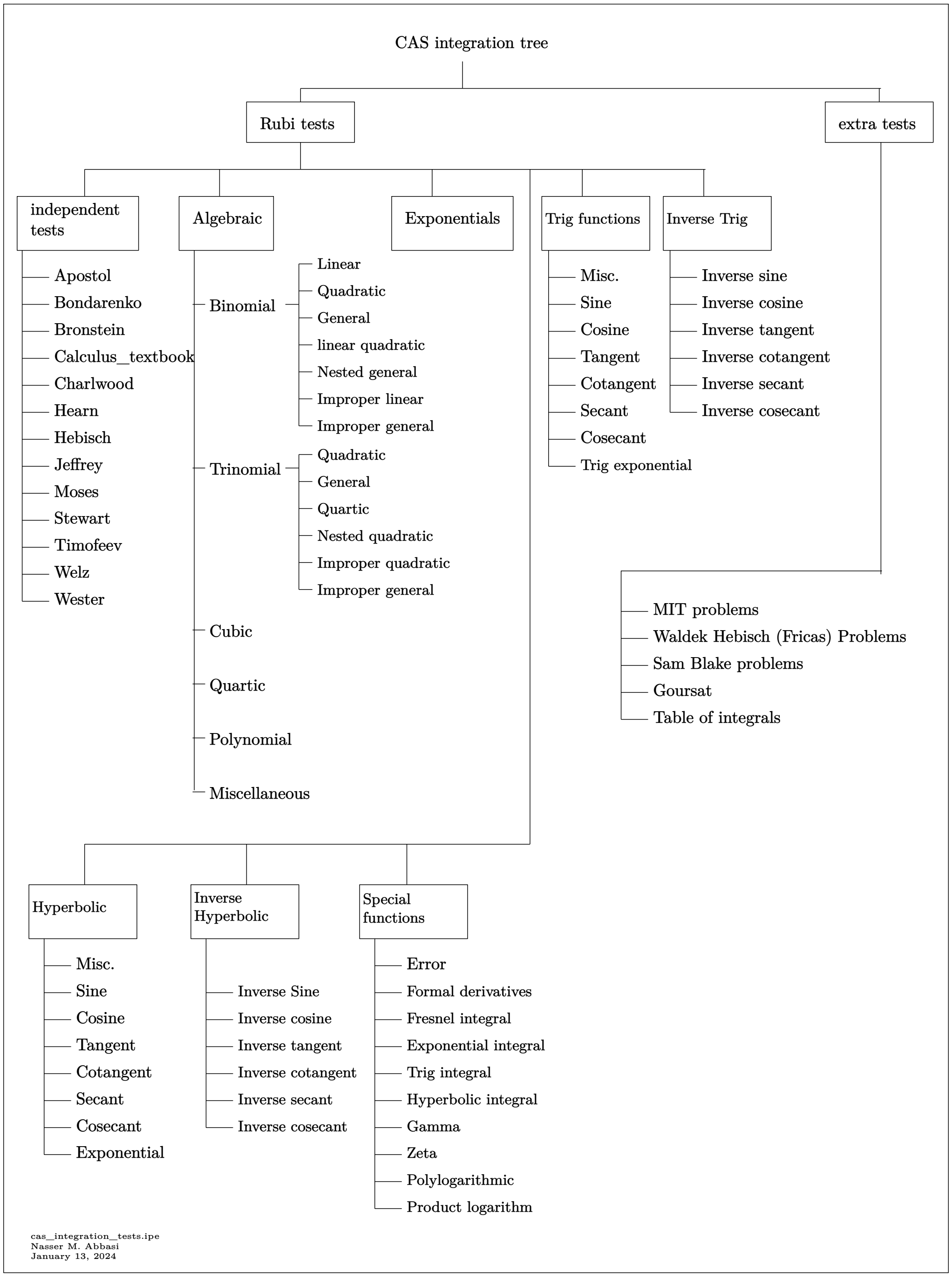
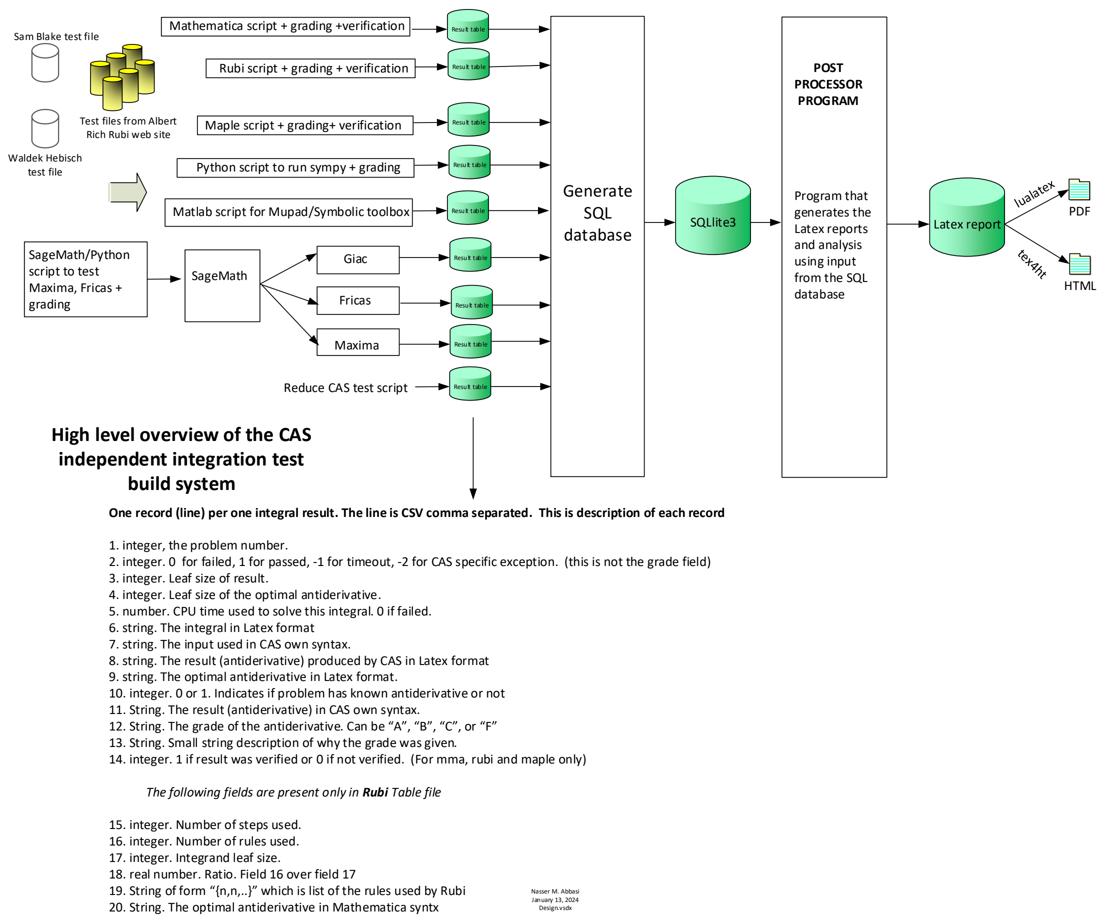


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 67, 68, 69, 70, 73, 74, 76, 79, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 105, 108, 109, 110, 115, 120, 123, 124, 125, 128, 129, 130, 131, 134, 135, 137, 140, 141, 142, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 168, 169, 170, 183, 184, 188, 189, 192, 193, 194, 197, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272 }

B grade { }

C grade { 47, 48, 60, 61, 65, 66, 75, 80, 81, 82, 99, 102, 103, 104, 114, 118, 119, 136, 163, 185, 198 }

F normal fail { 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 79, 80, 81, 82, 87, 90, 91, 92, 93, 96, 97, 98, 99, 102, 103, 104, 105, 108, 109, 110, 113, 114, 115, 118, 119, 120, 123, 124, 125, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 183, 184, 185, 188, 189, 192, 193, 194, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 255, 256, 257, 259, 260, 262, 263, 264, 266, 269, 270, 271, 272 }

B grade { 15, 16, 59, 85, 86, 218, 231, 248, 253, 254, 258, 261, 265, 267, 268 }

C grade { 150, 168, 169, 170 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 17, 18, 19, 22, 23, 24, 27, 28, 29, 37, 38, 39, 44, 45, 49, 53, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 79, 105, 110, 115, 120, 125, 131, 137, 142, 145, 146, 147, 148, 151, 152, 153, 154, 156, 157, 158, 162, 163, 205, 211, 212, 217, 218, 224, 225, 226, 227, 230, 231, 232, 234, 235, 241, 247, 248, 253, 254, 259, 260, 261, 262, 263, 264, 266, 267, 268, 270, 271 }

B grade { 32, 33, 34, 40, 41, 42, 43, 46, 47, 48, 50, 51, 52, 54, 55, 56, 57, 58, 59, 61, 161, 201, 202, 203, 204, 206, 207, 208, 209, 210, 213, 214, 215, 216, 219, 220, 221, 222, 223, 228, 229, 233, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 249, 250, 251, 252, 255, 256, 257, 258, 265, 269, 272 }

C grade { 149, 150 }

F normal fail { 10, 11, 15, 16, 80, 81, 82, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 99, 102, 103, 104, 108, 109, 113, 114, 118, 119, 123, 124, 128, 129, 130, 134, 135, 136, 140, 141, 155, 168, 169, 170, 183, 184, 185, 188, 189, 192, 193, 194, 197, 198 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 7, 8, 9, 12, 13, 14, 49, 70, 76, 120, 125, 142, 145, 146, 147, 148, 151, 152, 153 }

B grade { 79, 137, 154, 168, 169, 170, 206, 223, 224, 228, 233, 234 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 149, 150, 155, 156, 157, 158, 161, 162, 163, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 225, 226, 227, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272 }

F(-1) timedout fail { }

F(-2) exception fail { 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200 }

Maxima

A grade { 2, 3, 40, 41, 42, 49, 53, 54, 55, 59, 64, 79, 145, 146, 147, 148, 151, 152, 153, 154, 206, 222, 223, 224, 228, 229, 230, 233, 234, 235, 236, 258 }

B grade { 1, 7, 8, 9, 12, 13, 14, 46 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 43, 47, 48, 56, 60, 61, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 80, 81, 82, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 99, 102, 103, 104, 105, 108, 109, 110, 113, 114, 115, 118, 119, 120, 123, 124, 125, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 156, 157, 158, 161, 162, 163, 169, 170, 183, 184, 185, 188, 189, 192, 193, 194, 197, 198, 204, 205, 211, 212, 217, 218, 221, 225, 226, 227, 231, 232, 240, 241, 247, 253, 257, 259, 261, 262, 263, 264, 265, 272 }

F(-1) timedout fail { 242, 248, 254, 260, 267, 268, 269, 270 }

F(-2) exception fail { 37, 38, 39, 44, 45, 50, 51, 52, 57, 58, 62, 63, 149, 150, 155, 166, 167, 168, 171, 172, 173, 186, 190, 201, 202, 203, 207, 208, 209, 210, 213, 214, 215, 216, 219, 220, 237, 238, 239, 243, 244, 245, 246, 249, 250, 251, 252, 255, 256, 266, 271 }

Giac

A grade { 148 }

B grade { 79, 154 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 40, 41, 42, 43, 46, 47, 48, 49, 53, 54, 55, 56, 59, 60, 61, 64, 65, 67, 68, 69, 70, 73, 74, 75, 76, 82, 87, 90, 91, 92, 93, 96, 97, 98, 99, 105, 110, 115, 118, 119, 120, 123, 124, 125, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 149, 150, 155, 156, 157, 158, 161, 162, 163, 168, 169, 170, 183, 184, 185, 189, 192, 193, 194, 197, 198, 204, 205, 206, 210, 212, 220, 221, 222, 223, 224, 227, 228, 229, 232, 233, 234, 240, 241, 242, 246, 256, 257, 258, 259, 260, 263, 264, 265, 269, 270 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 7, 8, 9, 12, 13, 14, 37, 38, 39, 44, 45, 50, 51, 52, 57, 58, 62, 63, 66, 80, 81, 85, 86, 102, 103, 104, 108, 109, 113, 114, 145, 146, 147, 151, 152, 153, 188, 201, 202, 203, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 225, 226, 230, 231, 235, 236, 237, 238, 239, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 261, 262, 266, 267, 268, 271, 272 }

Mupad

A grade { }

B grade { 79, 148 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 80, 81, 82, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 99, 102, 103, 104, 105, 108, 109, 110, 113, 114, 115, 118, 119, 120, 123, 124, 125, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 168, 169, 170, 183, 184, 185, 188, 189, 192, 193, 194, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 7, 8, 9, 37, 38, 79, 145, 146, 147, 148, 151, 152, 153, 154 }

B grade { 12, 13, 14, 40, 44, 46 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 39, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 80, 81, 82, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 99, 103, 104, 105, 109, 110, 118, 119, 120, 124, 125, 129, 130, 131, 135, 136, 137, 140, 141, 142, 149, 150, 155, 156, 157, 158, 161, 162, 163, 168, 169, 183, 184, 185, 188, 189, 192, 193, 194, 197, 198, 202, 203, 204, 205, 206, 209, 210, 211, 212, 220, 221, 222, 223, 224, 226, 227, 228, 229, 232, 233, 234, 238, 239, 240, 241, 242, 245, 246, 247, 248, 256, 257, 258, 259, 260, 262, 263, 264, 265, 268, 269, 270 }

F(-1) timedout fail { 102, 108, 113, 114, 115, 116, 117, 123, 128, 134, 139, 143, 144, 165, 170, 200, 201, 207, 208, 213, 214, 215, 216, 217, 218, 219, 225, 230, 231, 235, 236, 237, 243, 244, 249, 250, 251, 252, 253, 254, 255, 261, 266, 267, 271, 272 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 40, 46, 53, 59, 64, 70, 76, 79, 105, 110, 115, 120, 125, 131, 137, 142, 145, 146, 147, 148, 154 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63,

65, 66, 67, 68, 69, 73, 74, 75, 80, 81, 82, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 99, 102, 103, 104, 108, 109, 113, 114, 118, 119, 123, 124, 128, 129, 130, 134, 135, 136, 140, 141, 149, 150, 151, 152, 153, 155, 156, 157, 158, 161, 162, 163, 168, 169, 170, 183, 184, 185, 188, 189, 192, 193, 194, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272
}

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	162	119	153	301	169	221	0	159	0
N.S.	1	0.95	0.70	0.90	1.77	0.99	1.30	0.00	0.94	0.00
time (sec)	N/A	0.489	0.087	0.391	0.042	0.099	0.572	0.000	0.216	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	124	95	116	194	133	165	0	118	0
N.S.	1	0.97	0.74	0.91	1.52	1.04	1.29	0.00	0.92	0.00
time (sec)	N/A	0.561	0.064	0.357	0.042	0.106	0.290	0.000	0.195	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	86	73	97	83	90	0	75	0
N.S.	1	1.03	1.15	0.97	1.29	1.11	1.20	0.00	1.00	0.00
time (sec)	N/A	0.458	0.032	0.289	0.036	0.096	0.139	0.000	0.206	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	59	135	143	0	0	0	0	35	0
N.S.	1	0.84	1.93	2.04	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.547	0.099	2.375	0.000	0.000	0.000	0.000	0.213	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	114	216	195	0	0	0	0	107	0
N.S.	1	0.92	1.74	1.57	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.770	0.157	1.524	0.000	0.000	0.000	0.000	0.209	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	164	341	248	0	0	0	0	199	0
N.S.	1	0.92	1.92	1.39	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.776	0.277	1.565	0.000	0.000	0.000	0.000	0.201	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	342	239	354	712	354	524	0	274	0
N.S.	1	1.18	0.82	1.22	2.45	1.22	1.80	0.00	0.94	0.00
time (sec)	N/A	1.383	1.209	2.714	0.058	0.101	0.905	0.000	0.195	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	239	191	264	457	278	389	0	209	0
N.S.	1	1.12	0.89	1.23	2.14	1.30	1.82	0.00	0.98	0.00
time (sec)	N/A	0.934	0.891	2.105	0.047	0.100	0.481	0.000	0.212	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	137	135	166	230	178	224	0	142	0
N.S.	1	1.10	1.08	1.33	1.84	1.42	1.79	0.00	1.14	0.00
time (sec)	N/A	0.555	0.109	1.471	0.036	0.111	0.229	0.000	0.186	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	111	274	0	0	0	0	0	64	0
N.S.	1	0.80	1.99	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.696	0.200	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	187	403	0	0	0	0	0	188	0
N.S.	1	0.89	1.92	0.00	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	1.734	1.303	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	647	407	628	1350	581	972	0	416	0
N.S.	1	1.49	0.94	1.44	3.10	1.34	2.23	0.00	0.96	0.00
time (sec)	N/A	3.073	1.327	2.960	0.092	0.094	1.312	0.000	0.232	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	424	327	466	861	455	717	0	326	0
N.S.	1	1.32	1.02	1.45	2.67	1.41	2.23	0.00	1.01	0.00
time (sec)	N/A	1.832	1.258	2.324	0.069	0.105	0.686	0.000	0.221	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	230	235	294	436	295	413	0	233	0
N.S.	1	1.20	1.23	1.54	2.28	1.54	2.16	0.00	1.22	0.00
time (sec)	N/A	1.483	0.171	1.701	0.042	0.106	0.361	0.000	0.195	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	167	464	0	0	0	0	0	93	0
N.S.	1	0.80	2.23	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.732	0.310	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	296	974	0	0	0	0	0	267	0
N.S.	1	0.85	2.78	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	2.032	4.298	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	50	43	42	0	0	0	0	63	0
N.S.	1	0.75	0.64	0.63	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.360	0.123	0.966	0.000	0.000	0.000	0.000	0.201	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	39	34	33	0	0	0	0	46	0
N.S.	1	0.78	0.68	0.66	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.338	0.064	0.592	0.000	0.000	0.000	0.000	0.208	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	26	23	22	0	0	0	0	27	0
N.S.	1	0.90	0.79	0.76	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.360	0.023	0.582	0.000	0.000	0.000	0.000	0.184	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	20	21	24	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.05	1.11	1.26	1.11
time (sec)	N/A	0.340	0.387	1.106	0.088	0.073	0.520	0.129	0.202	2.631

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	36	36	21	36	21
N.S.	1	1.00	1.11	1.00	1.11	1.89	1.89	1.11	1.89	1.11
time (sec)	N/A	0.340	1.090	0.382	0.099	0.086	1.003	0.131	0.200	2.688

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	79	82	106	0	0	0	0	63	0
N.S.	1	0.84	0.87	1.13	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.799	0.284	1.302	0.000	0.000	0.000	0.000	0.194	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	68	69	84	0	0	0	0	46	0
N.S.	1	0.88	0.90	1.09	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.784	0.214	0.884	0.000	0.000	0.000	0.000	0.199	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	54	60	0	0	0	0	27	0
N.S.	1	0.98	1.00	1.11	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.582	0.329	0.831	0.000	0.000	0.000	0.000	0.197	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	226	21	24	21	28	21
N.S.	1	1.00	1.11	1.00	11.89	1.11	1.26	1.11	1.47	1.11
time (sec)	N/A	0.374	1.318	1.120	0.193	0.079	0.642	0.132	0.203	2.670

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	325	36	41	21	42	21
N.S.	1	1.00	1.11	1.00	17.11	1.89	2.16	1.11	2.21	1.11
time (sec)	N/A	0.348	3.257	0.386	0.226	0.082	1.508	0.135	0.210	2.629

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	207	183	242	0	0	0	0	79	0
N.S.	1	0.77	0.68	0.90	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.620	0.813	3.635	0.000	0.000	0.000	0.000	0.215	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	157	139	182	0	0	0	0	58	0
N.S.	1	0.78	0.69	0.91	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.525	0.413	2.891	0.000	0.000	0.000	0.000	0.200	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	105	93	114	0	0	0	0	35	0
N.S.	1	0.84	0.74	0.91	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.442	0.150	2.014	0.000	0.000	0.000	0.000	0.190	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	35	34	25	36	25
N.S.	1	1.00	1.09	1.00	1.09	1.52	1.48	1.09	1.57	1.09
time (sec)	N/A	0.226	0.619	0.651	0.099	0.079	1.064	0.130	0.190	2.653

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	67	61	25	58	25
N.S.	1	1.00	1.09	1.00	1.09	2.91	2.65	1.09	2.52	1.09
time (sec)	N/A	0.228	32.784	0.585	0.102	0.083	2.884	0.137	0.195	2.525

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	243	306	844	0	0	0	0	135	0
N.S.	1	0.80	1.01	2.79	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.809	1.384	3.875	0.000	0.000	0.000	0.000	0.197	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	193	230	552	0	0	0	0	100	0
N.S.	1	0.82	0.98	2.35	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.003	0.760	3.069	0.000	0.000	0.000	0.000	0.219	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	139	121	298	0	0	0	0	63	0
N.S.	1	0.89	0.77	1.90	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.960	0.292	2.160	0.000	0.000	0.000	0.000	0.202	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	405	66	71	25	70	25
N.S.	1	1.00	1.09	1.00	17.61	2.87	3.09	1.09	3.04	1.09
time (sec)	N/A	0.665	5.795	0.650	0.243	0.084	2.805	0.144	0.183	2.443

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	609	119	121	25	112	25
N.S.	1	1.00	1.09	1.00	26.48	5.17	5.26	1.09	4.87	1.09
time (sec)	N/A	0.535	145.140	0.590	0.280	0.084	22.056	0.142	0.195	2.454

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	186	153	202	0	0	265	0	155	0
N.S.	1	1.08	0.88	1.17	0.00	0.00	1.53	0.00	0.90	0.00
time (sec)	N/A	0.752	0.451	1.450	0.000	0.000	15.901	0.000	0.219	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	129	111	152	0	0	185	0	108	0
N.S.	1	1.08	0.93	1.28	0.00	0.00	1.55	0.00	0.91	0.00
time (sec)	N/A	0.494	0.271	1.339	0.000	0.000	1.483	0.000	0.189	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	100	0	0	0	0	60	0
N.S.	1	1.00	1.03	1.49	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.312	0.167	0.888	0.000	0.000	0.000	0.000	0.187	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	53	28	0	87	0	37	0
N.S.	1	1.00	1.00	2.12	1.12	0.00	3.48	0.00	1.48	0.00
time (sec)	N/A	0.215	0.018	0.836	0.036	0.000	0.625	0.000	0.193	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	110	58	0	0	0	131	0
N.S.	1	1.00	1.29	2.16	1.14	0.00	0.00	0.00	2.57	0.00
time (sec)	N/A	0.242	0.135	1.182	0.038	0.000	0.000	0.000	0.213	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	100	619	126	0	0	0	274	0
N.S.	1	1.05	0.93	5.73	1.17	0.00	0.00	0.00	2.54	0.00
time (sec)	N/A	0.384	0.172	1.315	0.046	0.000	0.000	0.000	0.199	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	175	140	1409	0	0	0	0	446	0
N.S.	1	1.08	0.86	8.70	0.00	0.00	0.00	0.00	2.75	0.00
time (sec)	N/A	0.564	0.207	1.327	0.000	0.000	0.000	0.000	0.208	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	253	202	289	0	0	405	0	171	0
N.S.	1	1.20	0.96	1.38	0.00	0.00	1.93	0.00	0.81	0.00
time (sec)	N/A	1.061	1.096	1.621	0.000	0.000	2.874	0.000	0.211	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	119	124	179	0	0	0	0	90	0
N.S.	1	0.98	1.02	1.47	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.765	0.570	1.122	0.000	0.000	0.000	0.000	0.197	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	72	47	0	90	0	51	0
N.S.	1	1.00	1.00	2.88	1.88	0.00	3.60	0.00	2.04	0.00
time (sec)	N/A	0.292	0.022	1.062	0.044	0.000	0.817	0.000	0.181	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	153	243	0	0	0	0	234	0
N.S.	1	1.04	1.47	2.34	0.00	0.00	0.00	0.00	2.25	0.00
time (sec)	N/A	0.853	0.705	1.500	0.000	0.000	0.000	0.000	0.184	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	208	293	1729	0	0	0	0	486	0
N.S.	1	1.02	1.44	8.48	0.00	0.00	0.00	0.00	2.38	0.00
time (sec)	N/A	1.453	0.909	1.648	0.000	0.000	0.000	0.000	0.181	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	11	0
N.S.	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	0.34	0.00
time (sec)	N/A	0.250	0.011	0.874	0.111	0.077	0.000	0.000	0.191	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	267	317	801	0	0	0	0	155	0
N.S.	1	1.06	1.26	3.19	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.783	0.682	1.125	0.000	0.000	0.000	0.000	0.210	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	196	200	496	0	0	0	0	108	0
N.S.	1	1.11	1.13	2.80	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.568	0.677	0.932	0.000	0.000	0.000	0.000	0.197	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	120	256	0	0	0	0	60	0
N.S.	1	1.00	1.08	2.31	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.353	0.380	0.807	0.000	0.000	0.000	0.000	0.200	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	76	77	28	0	0	0	37	0
N.S.	1	1.00	1.62	1.64	0.60	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.227	0.088	0.478	0.042	0.000	0.000	0.000	0.186	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	100	143	58	0	0	0	131	0
N.S.	1	1.00	1.32	1.88	0.76	0.00	0.00	0.00	1.72	0.00
time (sec)	N/A	0.249	0.199	1.058	0.041	0.000	0.000	0.000	0.179	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	152	143	425	126	0	0	0	274	0
N.S.	1	1.03	0.97	2.89	0.86	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.449	0.209	1.165	0.044	0.000	0.000	0.000	0.191	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	228	185	2186	0	0	0	0	446	0
N.S.	1	1.06	0.86	10.17	0.00	0.00	0.00	0.00	2.07	0.00
time (sec)	N/A	0.850	0.250	1.204	0.000	0.000	0.000	0.000	0.196	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	320	329	959	0	0	0	0	171	0
N.S.	1	1.13	1.16	3.38	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	1.648	2.532	1.148	0.000	0.000	0.000	0.000	0.211	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	163	200	480	0	0	0	0	90	0
N.S.	1	0.89	1.09	2.61	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.613	1.069	1.010	0.000	0.000	0.000	0.000	0.196	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	115	120	47	0	0	0	51	0
N.S.	1	1.00	2.45	2.55	1.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.281	0.160	0.616	0.056	0.000	0.000	0.000	0.197	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	133	152	343	0	0	0	0	234	0
N.S.	1	0.74	0.85	1.92	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.616	0.733	1.286	0.000	0.000	0.000	0.000	0.204	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	258	236	1967	0	0	0	0	486	0
N.S.	1	0.88	0.81	6.74	0.00	0.00	0.00	0.00	1.66	0.00
time (sec)	N/A	1.369	1.364	1.453	0.000	0.000	0.000	0.000	0.220	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	354	136	484	0	0	0	0	50	0
N.S.	1	1.06	0.41	1.44	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	3.148	0.269	1.619	0.000	0.000	0.000	0.000	0.210	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	164	86	231	0	0	0	0	22	0
N.S.	1	0.80	0.42	1.13	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.961	0.164	1.376	0.000	0.000	0.000	0.000	0.194	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	39	14	0	0	0	16	0
N.S.	1	1.00	1.02	0.98	0.35	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.220	0.042	0.856	0.038	0.000	0.000	0.000	0.194	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	138	133	262	0	0	0	0	47	0
N.S.	1	0.63	0.61	1.20	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.733	0.269	1.727	0.000	0.000	0.000	0.000	0.181	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	273	195	550	0	0	0	0	65	0
N.S.	1	0.75	0.54	1.52	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.682	0.445	1.930	0.000	0.000	0.000	0.000	0.205	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	201	183	292	0	0	0	0	91	0
N.S.	1	0.48	0.44	0.70	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.594	0.838	1.647	0.000	0.000	0.000	0.000	0.192	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	149	137	224	0	0	0	0	58	0
N.S.	1	0.52	0.48	0.79	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.495	0.504	1.498	0.000	0.000	0.000	0.000	0.196	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	98	91	157	0	0	0	0	26	0
N.S.	1	0.60	0.56	0.96	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.422	0.351	1.293	0.000	0.000	0.000	0.000	0.197	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	44	0	65	0	0	21	0
N.S.	1	1.00	1.00	1.02	0.00	1.51	0.00	0.00	0.49	0.00
time (sec)	N/A	0.312	0.177	1.097	0.000	0.074	0.000	0.000	0.207	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	80	24	25	81	25
N.S.	1	1.00	1.08	0.92	1.00	3.20	0.96	1.00	3.24	1.00
time (sec)	N/A	0.359	2.834	0.871	0.143	0.079	1.875	0.171	0.199	2.891

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	104	24	25	123	25
N.S.	1	1.00	1.08	0.92	1.00	4.16	0.96	1.00	4.92	1.00
time (sec)	N/A	0.396	3.714	0.965	0.114	0.084	8.116	0.192	0.195	2.962

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	235	341	609	0	0	0	0	133	0
N.S.	1	0.57	0.82	1.47	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.165	0.743	1.732	0.000	0.000	0.000	0.000	0.196	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	183	150	432	0	0	0	0	86	0
N.S.	1	0.66	0.54	1.55	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.876	0.576	1.556	0.000	0.000	0.000	0.000	0.195	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	134	115	258	0	0	0	0	40	0
N.S.	1	0.85	0.73	1.64	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.773	0.329	1.355	0.000	0.000	0.000	0.000	0.198	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	46	0	78	0	0	26	0
N.S.	1	1.00	1.02	1.02	0.00	1.73	0.00	0.00	0.58	0.00
time (sec)	N/A	0.244	0.068	0.535	0.000	0.100	0.000	0.000	0.212	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	540	132	26	25	135	25
N.S.	1	1.00	1.08	0.92	21.60	5.28	1.04	1.00	5.40	1.00
time (sec)	N/A	0.427	7.488	0.809	0.334	0.092	5.391	0.173	0.215	2.686

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	678	173	26	25	207	25
N.S.	1	1.00	1.08	0.92	27.12	6.92	1.04	1.00	8.28	1.00
time (sec)	N/A	0.424	9.009	0.957	0.389	0.098	28.711	0.194	0.203	3.250

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	12	23	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.77	0.92	1.77	0.85	0.85
time (sec)	N/A	0.203	0.008	0.288	0.026	0.073	0.379	0.108	0.193	3.076

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	506	657	422	0	0	0	0	0	55	0
N.S.	1	1.30	0.83	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.820	2.315	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	333	302	0	0	0	0	0	33	0
N.S.	1	1.37	1.24	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.231	1.840	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	101	0	0	0	0	0	11	0
N.S.	1	1.08	0.99	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.625	0.087	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	22	25	27	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.88	1.00	1.08	1.00
time (sec)	N/A	0.227	1.435	0.807	0.245	0.000	0.377	0.836	0.208	2.914

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	32	25	35	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	1.28	1.00	1.40	1.00
time (sec)	N/A	0.692	12.067	0.994	0.249	0.000	1.585	0.855	0.207	2.801

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	632	769	1553	0	0	0	0	0	126	0
N.S.	1	1.22	2.46	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	3.336	9.570	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	402	768	0	0	0	0	0	78	0
N.S.	1	1.26	2.42	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	2.858	5.893	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	140	251	0	0	0	0	0	32	0
N.S.	1	1.04	1.86	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.674	0.371	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	53	25	59	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	2.12	1.00	2.36	1.00
time (sec)	N/A	0.241	1.882	0.825	0.277	0.000	2.165	1.318	0.225	3.831

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	71	25	75	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	2.84	1.00	3.00	1.00
time (sec)	N/A	0.857	17.358	1.171	0.266	0.000	8.431	1.318	0.232	4.475

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	426	388	390	0	0	0	0	0	116	0
N.S.	1	0.91	0.92	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.843	1.203	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	292	295	0	0	0	0	0	86	0
N.S.	1	0.92	0.93	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.697	0.670	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	190	197	0	0	0	0	0	54	0
N.S.	1	0.96	0.99	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.730	0.273	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	101	0	0	0	0	0	22	0
N.S.	1	1.01	1.15	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.668	0.063	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	34	25	46	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	1.36	1.00	1.84	1.00
time (sec)	N/A	0.384	0.557	0.805	0.240	0.000	0.881	0.878	0.198	3.042

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	56	25	68	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	2.24	1.00	2.72	1.00
time (sec)	N/A	0.384	1.063	0.898	0.259	0.000	3.597	0.858	0.210	3.062

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	454	422	583	0	0	0	0	0	1180	0
N.S.	1	0.93	1.28	0.00	0.00	0.00	0.00	0.00	2.60	0.00
time (sec)	N/A	1.307	2.191	0.000	0.000	0.000	0.000	0.000	0.432	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	326	440	0	0	0	0	0	1003	0
N.S.	1	0.94	1.27	0.00	0.00	0.00	0.00	0.00	2.90	0.00
time (sec)	N/A	0.893	1.283	0.000	0.000	0.000	0.000	0.000	0.385	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	226	295	0	0	0	0	0	824	0
N.S.	1	0.99	1.29	0.00	0.00	0.00	0.00	0.00	3.61	0.00
time (sec)	N/A	0.741	0.515	0.000	0.000	0.000	0.000	0.000	0.334	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	130	137	0	0	0	0	0	810	0
N.S.	1	1.12	1.18	0.00	0.00	0.00	0.00	0.00	6.98	0.00
time (sec)	N/A	0.609	0.166	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	82	25	80	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	3.28	1.00	3.20	1.00
time (sec)	N/A	0.419	0.637	0.799	0.256	0.000	2.468	0.240	0.226	2.637

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	133	25	122	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	5.32	1.00	4.88	1.00
time (sec)	N/A	0.433	1.163	1.125	0.268	0.000	15.001	0.235	0.426	2.731

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	632	838	667	0	0	0	0	0	88	0
N.S.	1	1.33	1.06	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	5.622	5.024	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	422	479	288	0	0	0	0	0	56	0
N.S.	1	1.14	0.68	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	3.165	1.514	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	222	156	0	0	0	0	0	25	0
N.S.	1	0.92	0.65	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.663	0.823	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	43	0	0	0	0	30	0
N.S.	1	1.00	1.02	0.88	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.291	0.063	0.591	0.000	0.000	0.000	0.000	0.187	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	26	25	140	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	0.96	0.93	5.19	0.93
time (sec)	N/A	0.471	9.977	1.775	0.269	0.000	1.514	0.873	0.220	2.727

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	26	25	518	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	0.96	0.93	19.19	0.93
time (sec)	N/A	0.877	10.552	2.211	0.285	0.000	22.707	0.897	0.245	2.725

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	589	566	632	0	0	0	0	0	121	0
N.S.	1	0.96	1.07	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	4.290	3.466	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	256	355	0	0	0	0	0	56	0
N.S.	1	0.76	1.06	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.380	1.880	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	43	0	0	0	0	44	0
N.S.	1	1.00	1.02	0.88	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.268	0.086	0.589	0.000	0.000	0.000	0.000	0.204	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	26	25	130	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	0.96	0.93	4.81	0.93
time (sec)	N/A	0.462	9.841	1.839	0.268	0.000	12.290	1.358	0.243	2.709

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	26	25	381	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	0.96	0.93	14.11	0.93
time (sec)	N/A	1.379	14.993	2.482	0.294	0.000	78.992	1.378	0.269	2.717

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	514	0	201	0	0	0	0	0	60	0
N.S.	1	0.00	0.39	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.328	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	238	135	0	0	0	0	0	27	0
N.S.	1	0.80	0.45	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.998	0.269	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	43	36	0	0	0	0	21	0
N.S.	1	1.00	1.02	0.86	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.279	0.047	0.603	0.000	0.000	0.000	0.000	0.189	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	102	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	4.43	0.91
time (sec)	N/A	0.459	1.601	1.765	0.253	0.000	0.000	0.481	0.209	2.938

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	318	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	13.83	0.91
time (sec)	N/A	1.124	1.657	1.766	0.265	0.000	0.000	0.487	0.222	3.037

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	332	156	0	0	0	0	0	44	0
N.S.	1	1.07	0.50	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.977	0.130	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	162	110	0	0	0	0	0	18	0
N.S.	1	0.92	0.62	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.039	0.067	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	15	0
N.S.	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	0.38	0.00
time (sec)	N/A	0.275	0.022	1.020	0.000	0.067	0.000	0.000	0.202	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	35	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	1.59	0.91
time (sec)	N/A	0.474	1.174	2.007	0.234	0.000	1.399	0.358	0.196	2.656

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	43	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	1.95	0.91
time (sec)	N/A	0.955	1.249	2.154	0.244	0.000	24.289	0.375	0.200	2.660

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	433	410	210	0	0	0	0	0	56	0
N.S.	1	0.95	0.48	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.977	0.256	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	195	133	0	0	0	0	0	24	0
N.S.	1	0.75	0.51	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.005	0.215	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	17	0
N.S.	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	0.44	0.00
time (sec)	N/A	0.211	0.024	1.045	0.000	0.079	0.000	0.000	0.199	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	89	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	4.05	0.91
time (sec)	N/A	0.334	1.238	1.936	0.228	0.000	9.351	0.601	0.211	2.781

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	20	0	19	20	289	20
N.S.	1	1.00	1.09	0.82	0.91	0.00	0.86	0.91	13.14	0.91
time (sec)	N/A	1.117	1.061	2.147	0.230	0.000	54.886	0.618	0.213	2.608

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	535	338	359	0	0	0	0	0	118	0
N.S.	1	0.63	0.67	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.031	1.099	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	359	236	252	0	0	0	0	0	76	0
N.S.	1	0.66	0.70	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.993	0.765	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	147	165	0	0	0	0	0	35	0
N.S.	1	0.71	0.80	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.576	0.407	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	43	0	0	0	0	22	0
N.S.	1	1.00	1.02	0.91	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.258	0.075	0.586	0.000	0.000	0.000	0.000	0.184	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	27	25	80	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	1.00	0.93	2.96	0.93
time (sec)	N/A	0.242	0.281	1.852	0.282	0.000	3.931	0.886	0.215	2.689

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	27	25	102	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	1.00	0.93	3.78	0.93
time (sec)	N/A	0.250	0.291	2.086	0.289	0.000	56.980	0.911	0.300	2.824

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	533	371	633	0	0	0	0	0	160	0
N.S.	1	0.70	1.19	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.979	3.167	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	352	269	324	0	0	0	0	0	104	0
N.S.	1	0.76	0.92	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.775	1.718	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	190	189	0	0	0	0	0	49	0
N.S.	1	0.94	0.93	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.791	1.111	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	43	0	100	0	0	32	0
N.S.	1	1.00	1.02	0.91	0.00	2.13	0.00	0.00	0.68	0.00
time (sec)	N/A	0.421	0.084	0.685	0.000	0.086	0.000	0.000	0.198	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	27	25	134	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	1.00	0.93	4.96	0.93
time (sec)	N/A	0.680	0.324	1.872	0.263	0.000	27.014	0.269	0.366	2.799

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	25	0	0	25	176	25
N.S.	1	1.00	1.07	0.85	0.93	0.00	0.00	0.93	6.52	0.93
time (sec)	N/A	0.762	0.321	2.839	0.275	0.000	0.000	0.306	0.478	3.196

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	232	262	0	0	0	0	0	60	0
N.S.	1	0.81	0.91	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.660	0.322	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	165	122	0	0	0	0	0	27	0
N.S.	1	0.91	0.67	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.607	0.140	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	43	36	0	57	0	0	21	0
N.S.	1	1.00	1.02	0.86	0.00	1.36	0.00	0.00	0.50	0.00
time (sec)	N/A	0.242	0.050	0.612	0.000	0.085	0.000	0.000	0.194	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	60	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	2.61	0.91
time (sec)	N/A	0.376	1.594	1.710	0.250	0.000	0.000	0.173	0.204	2.586

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	21	0	0	21	74	21
N.S.	1	1.00	1.09	0.83	0.91	0.00	0.00	0.91	3.22	0.91
time (sec)	N/A	0.371	1.677	1.807	0.237	0.000	0.000	0.199	0.213	2.575

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	222	187	295	287	241	389	0	328	0
N.S.	1	1.00	0.85	1.33	1.30	1.09	1.76	0.00	1.48	0.00
time (sec)	N/A	0.583	0.170	0.455	0.035	0.091	0.652	0.000	0.197	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	149	125	188	180	163	240	0	202	0
N.S.	1	1.01	0.85	1.28	1.22	1.11	1.63	0.00	1.37	0.00
time (sec)	N/A	0.399	0.102	0.431	0.034	0.103	0.379	0.000	0.196	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	84	71	97	91	94	109	0	101	0
N.S.	1	1.04	0.88	1.20	1.12	1.16	1.35	0.00	1.25	0.00
time (sec)	N/A	0.321	0.046	0.362	0.028	0.090	0.208	0.000	0.186	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	30	43	26	41	30	28
N.S.	1	1.00	1.00	0.97	1.00	1.43	0.87	1.37	1.00	0.93
time (sec)	N/A	0.216	0.006	0.289	0.026	0.091	0.066	0.122	0.191	2.613

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	434	224	0	0	0	0	46	0
N.S.	1	1.00	0.89	0.46	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.925	0.314	34.329	0.000	0.000	0.000	0.000	0.218	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	707	707	622	848	0	0	0	0	137	0
N.S.	1	1.00	0.88	1.20	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.750	1.172	51.416	0.000	0.000	0.000	0.000	0.200	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	443	683	684	586	989	0	476	0
N.S.	1	1.00	0.79	1.22	1.22	1.05	1.77	0.00	0.85	0.00
time (sec)	N/A	1.277	0.296	3.560	0.060	0.099	0.999	0.000	0.216	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	289	431	429	380	595	0	317	0
N.S.	1	1.00	0.88	1.31	1.30	1.16	1.81	0.00	0.96	0.00
time (sec)	N/A	0.824	0.214	2.852	0.044	0.096	0.535	0.000	0.206	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	164	215	218	209	279	0	180	0
N.S.	1	1.00	1.07	1.41	1.42	1.37	1.82	0.00	1.18	0.00
time (sec)	N/A	0.492	0.123	2.098	0.041	0.100	0.309	0.000	0.206	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	74	72	72	96	82	111	73	0
N.S.	1	1.09	1.61	1.57	1.57	2.09	1.78	2.41	1.59	0.00
time (sec)	N/A	0.484	0.030	0.481	0.033	0.092	0.100	0.199	0.211	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	985	0	0	0	0	0	74	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.860	0.510	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	253	380	0	0	0	0	57	0
N.S.	1	1.00	0.65	0.98	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.052	0.378	3.520	0.000	0.000	0.000	0.000	0.196	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	126	178	0	0	0	0	33	0
N.S.	1	1.00	0.70	0.99	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.570	0.161	2.954	0.000	0.000	0.000	0.000	0.200	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	45	56	0	0	0	0	12	0
N.S.	1	0.91	0.83	1.04	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.416	0.039	0.910	0.000	0.000	0.000	0.000	0.195	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	29	17	22	31	22
N.S.	1	1.00	1.10	1.00	1.10	1.45	0.85	1.10	1.55	1.10
time (sec)	N/A	0.214	0.596	0.993	0.092	0.076	3.255	0.125	0.216	2.636

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	53	19	22	59	22
N.S.	1	1.00	1.10	1.00	1.10	2.65	0.95	1.10	2.95	1.10
time (sec)	N/A	0.214	2.549	0.961	0.095	0.096	101.944	0.132	0.250	2.702

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	356	1036	0	0	0	0	99	0
N.S.	1	1.00	0.72	2.09	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.094	1.817	3.767	0.000	0.000	0.000	0.000	0.205	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	190	438	0	0	0	0	61	0
N.S.	1	1.00	0.77	1.77	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.811	0.773	3.156	0.000	0.000	0.000	0.000	0.195	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	71	118	0	0	0	0	26	0
N.S.	1	1.05	0.84	1.39	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.020	0.139	1.038	0.000	0.000	0.000	0.000	0.193	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	766	57	19	22	64	22
N.S.	1	1.00	1.10	1.00	38.30	2.85	0.95	1.10	3.20	1.10
time (sec)	N/A	0.346	10.464	0.983	0.787	0.107	45.920	0.135	0.224	2.953

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1027	98	0	22	115	22
N.S.	1	1.00	1.10	1.00	51.35	4.90	0.00	1.10	5.75	1.10
time (sec)	N/A	0.352	22.276	1.013	1.077	0.090	0.000	0.154	0.272	2.920

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	61	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	3.05	1.00
time (sec)	N/A	0.315	3.386	3.814	0.000	0.097	1.468	0.157	0.220	3.031

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	48	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	2.40	1.00
time (sec)	N/A	0.200	2.604	2.432	0.000	0.107	0.908	0.134	0.248	3.134

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	75	0	0	326	0	0	123	0
N.S.	1	1.00	1.07	0.00	0.00	4.66	0.00	0.00	1.76	0.00
time (sec)	N/A	0.305	0.088	0.000	0.000	0.140	0.000	0.000	0.246	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	147	139	0	0	738	0	0	276	0
N.S.	1	1.01	0.95	0.00	0.00	5.05	0.00	0.00	1.89	0.00
time (sec)	N/A	0.341	0.237	0.000	0.000	0.198	0.000	0.000	0.273	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	247	191	0	0	1354	0	0	461	0
N.S.	1	1.09	0.84	0.00	0.00	5.96	0.00	0.00	2.03	0.00
time (sec)	N/A	1.042	0.287	0.000	0.000	0.225	0.000	0.000	0.295	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	22	89	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	1.00	4.05	1.00
time (sec)	N/A	0.219	11.966	1.594	0.000	0.089	1.072	0.220	0.245	3.016

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	22	76	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	1.00	3.45	1.00
time (sec)	N/A	0.222	7.407	1.196	0.000	0.103	0.889	0.158	0.273	2.982

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	54	20	22	222	22
N.S.	1	1.00	1.09	0.91	0.00	2.45	0.91	1.00	10.09	1.00
time (sec)	N/A	0.218	2.478	2.383	0.000	0.098	4.526	0.163	0.272	2.923

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	99	65	20	22	488	22
N.S.	1	1.00	1.09	0.91	4.50	2.95	0.91	1.00	22.18	1.00
time (sec)	N/A	0.226	4.387	2.392	0.341	0.109	61.666	0.167	0.310	3.074

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	19	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.219	0.905	3.905	0.088	0.080	0.382	0.137	200.026	2.928

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	39	20	22	29	22
N.S.	1	1.00	1.09	0.91	1.00	1.77	0.91	1.00	1.32	1.00
time (sec)	N/A	0.224	0.733	2.911	0.095	0.079	0.642	0.138	0.604	2.980

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	63	20	22	63	22
N.S.	1	1.00	1.09	0.91	1.00	2.86	0.91	1.00	2.86	1.00
time (sec)	N/A	0.227	1.250	3.514	0.103	0.094	1.755	0.155	0.764	3.025

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	87	20	22	107	22
N.S.	1	1.00	1.09	0.91	1.00	3.95	0.91	1.00	4.86	1.00
time (sec)	N/A	0.342	2.423	3.665	0.095	0.083	8.005	0.161	0.950	2.949

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	575	36	20	22	22	22
N.S.	1	1.00	1.09	0.91	26.14	1.64	0.91	1.00	1.00	1.00
time (sec)	N/A	0.293	3.094	3.933	0.521	0.095	0.680	0.147	200.014	2.745

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	591	67	22	22	22	22
N.S.	1	1.00	1.09	0.91	26.86	3.05	1.00	1.00	1.00	1.00
time (sec)	N/A	0.308	6.155	2.923	0.626	0.096	1.241	0.145	200.026	2.877

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	864	108	22	22	22	22
N.S.	1	1.00	1.09	0.91	39.27	4.91	1.00	1.00	1.00	1.00
time (sec)	N/A	0.279	12.850	3.595	1.264	0.094	4.986	0.156	200.016	2.877

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	1123	149	22	22	187	22
N.S.	1	1.00	1.09	0.91	51.05	6.77	1.00	1.00	8.50	1.00
time (sec)	N/A	0.285	20.542	3.623	1.767	0.099	28.153	0.175	0.727	2.816

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	672	672	535	0	0	0	0	0	54	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.497	4.229	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	319	0	0	0	0	0	31	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.081	1.868	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	101	0	0	0	0	0	11	0
N.S.	1	1.08	0.99	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.610	0.090	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	19	22	21	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.86	1.00	0.95	1.00
time (sec)	N/A	0.224	2.643	1.325	0.000	0.000	0.832	0.838	13.389	2.810

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00	1.00
time (sec)	N/A	0.228	14.924	1.757	0.259	0.000	15.421	0.837	200.032	2.856

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	770	0	0	0	0	0	74	0
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.654	1.727	0.000	0.000	0.000	0.000	0.000	0.299	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	140	251	0	0	0	0	0	32	0
N.S.	1	1.04	1.86	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.176	0.366	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	19	22	22	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.370	0.498	1.329	0.000	0.000	10.088	1.332	200.020	2.749

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00	1.00
time (sec)	N/A	0.360	7.243	1.757	0.353	0.000	106.356	1.330	200.021	2.802

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0	85	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.404	0.772	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	218	0	0	0	0	0	52	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.756	0.412	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	101	0	0	0	0	0	22	0
N.S.	1	1.01	1.15	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.432	0.065	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	41	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.86	1.00
time (sec)	N/A	0.228	0.163	1.355	0.239	0.000	2.193	0.859	0.248	2.856

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	22	22	69	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	1.00	1.00	3.14	1.00
time (sec)	N/A	0.226	0.218	1.647	0.266	0.000	45.970	0.849	0.377	3.263

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	303	0	0	0	0	0	1183	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	3.39	0.00
time (sec)	N/A	0.914	0.901	0.000	0.000	0.000	0.000	0.000	0.347	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	130	137	0	0	0	0	0	810	0
N.S.	1	1.12	1.18	0.00	0.00	0.00	0.00	0.00	6.98	0.00
time (sec)	N/A	0.828	0.166	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	74	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	3.36	1.00
time (sec)	N/A	0.381	0.165	1.293	0.253	0.000	9.498	0.230	0.269	3.439

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	0	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.390	0.209	1.628	0.296	0.000	0.000	0.244	200.024	2.870

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	192	361	1053	0	0	0	0	206	0
N.S.	1	0.46	0.87	2.53	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.351	2.579	5.951	0.000	0.000	0.000	0.000	0.328	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	148	273	760	0	0	0	0	149	0
N.S.	1	0.49	0.90	2.50	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.727	2.565	4.882	0.000	0.000	0.000	0.000	0.277	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	96	233	376	0	0	0	0	73	0
N.S.	1	0.65	1.59	2.56	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.509	1.050	4.238	0.000	0.000	0.000	0.000	0.235	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	96	227	313	0	0	0	0	75	0
N.S.	1	0.61	1.44	1.98	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.547	1.298	4.167	0.000	0.000	0.000	0.000	0.228	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	115	283	280	0	0	0	0	106	0
N.S.	1	0.64	1.56	1.55	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.658	2.141	8.049	0.000	0.000	0.000	0.000	0.228	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	123	141	435	219	548	0	0	208	0
N.S.	1	0.64	0.73	2.27	1.14	2.85	0.00	0.00	1.08	0.00
time (sec)	N/A	0.578	1.214	4.984	0.061	0.205	0.000	0.000	0.242	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	198	683	1359	0	0	0	0	264	0
N.S.	1	0.41	1.41	2.81	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.685	2.813	5.143	0.000	0.000	0.000	0.000	0.370	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	151	352	689	0	0	0	0	131	0
N.S.	1	0.60	1.40	2.73	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.775	2.135	4.309	0.000	0.000	0.000	0.000	0.273	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	148	273	756	0	0	0	0	149	0
N.S.	1	0.49	0.90	2.49	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.887	2.579	4.964	0.000	0.000	0.000	0.000	0.273	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	133	344	542	0	0	0	0	130	0
N.S.	1	0.50	1.29	2.04	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.014	4.623	4.358	0.000	0.000	0.000	0.000	0.253	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	149	514	302	0	0	0	0	171	0
N.S.	1	0.52	1.78	1.05	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.938	5.975	7.422	0.000	0.000	0.000	0.000	0.274	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	189	706	426	0	0	0	0	429	0
N.S.	1	0.51	1.89	1.14	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.649	8.279	7.526	0.000	0.000	0.000	0.000	0.312	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	201	481	1083	0	0	0	0	190	0
N.S.	1	0.53	1.26	2.84	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.896	2.483	4.615	0.000	0.000	0.000	0.000	0.338	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	198	683	1353	0	0	0	0	264	0
N.S.	1	0.41	1.41	2.80	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.658	2.844	5.114	0.000	0.000	0.000	0.000	0.400	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	192	565	1044	0	0	0	0	206	0
N.S.	1	0.46	1.36	2.51	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.791	2.369	5.227	0.000	0.000	0.000	0.000	0.313	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	181	465	799	0	0	0	0	190	0
N.S.	1	0.48	1.22	2.10	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.949	6.180	4.625	0.000	0.000	0.000	0.000	0.298	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	200	779	386	0	0	0	0	244	0
N.S.	1	0.44	1.72	0.85	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.133	8.518	7.461	0.000	0.000	0.000	0.000	0.308	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	230	1005	512	0	0	0	0	596	0
N.S.	1	0.47	2.07	1.06	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	1.037	10.837	7.612	0.000	0.000	0.000	0.000	0.345	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	181	465	799	0	0	0	0	190	0
N.S.	1	0.48	1.22	2.10	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.881	6.293	6.926	0.000	0.000	0.000	0.000	0.285	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	133	344	542	0	0	0	0	130	0
N.S.	1	0.50	1.29	2.04	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.701	4.775	6.362	0.000	0.000	0.000	0.000	0.258	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	96	227	313	0	0	0	0	74	0
N.S.	1	0.61	1.44	1.98	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.565	1.481	6.511	0.000	0.000	0.000	0.000	0.221	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	113	136	32	0	0	0	60	0
N.S.	1	1.00	1.92	2.31	0.54	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.402	0.984	4.857	0.038	0.000	0.000	0.000	0.216	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	89	113	220	98	443	0	0	105	0
N.S.	1	0.80	1.02	1.98	0.88	3.99	0.00	0.00	0.95	0.00
time (sec)	N/A	0.478	0.768	6.162	0.150	0.166	0.000	0.000	0.221	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	170	143	297	233	576	0	0	245	0
N.S.	1	0.58	0.48	1.01	0.79	1.95	0.00	0.00	0.83	0.00
time (sec)	N/A	0.561	0.886	6.158	0.165	0.206	0.000	0.000	0.236	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	199	781	386	0	0	0	0	256	0
N.S.	1	0.44	1.72	0.85	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.964	8.745	10.687	0.000	0.000	0.000	0.000	0.297	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	148	515	302	0	0	0	0	180	0
N.S.	1	0.52	1.79	1.05	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.934	6.311	10.614	0.000	0.000	0.000	0.000	0.262	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	114	285	280	0	0	0	0	112	0
N.S.	1	0.63	1.58	1.56	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.889	2.262	10.960	0.000	0.000	0.000	0.000	0.244	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	90	94	220	98	443	0	0	108	0
N.S.	1	0.80	0.84	1.96	0.88	3.96	0.00	0.00	0.96	0.00
time (sec)	N/A	0.521	0.894	6.213	0.132	0.137	0.000	0.000	0.225	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	82	118	219	82	0	0	0	103	0
N.S.	1	0.80	1.15	2.13	0.80	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.436	0.978	6.289	0.056	0.000	0.000	0.000	0.235	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	164	201	438	237	0	0	0	290	0
N.S.	1	0.55	0.67	1.47	0.80	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.561	1.184	6.725	0.067	0.000	0.000	0.000	0.255	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	228	1083	512	0	0	0	0	608	0
N.S.	1	0.47	2.24	1.06	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	0.719	12.484	10.799	0.000	0.000	0.000	0.000	0.353	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	187	706	426	0	0	0	0	437	0
N.S.	1	0.50	1.90	1.15	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.648	8.754	10.980	0.000	0.000	0.000	0.000	0.292	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	121	131	435	220	548	0	0	212	0
N.S.	1	0.64	0.69	2.29	1.16	2.88	0.00	0.00	1.12	0.00
time (sec)	N/A	0.558	1.471	7.757	0.063	0.162	0.000	0.000	0.227	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	171	139	297	232	576	0	0	245	0
N.S.	1	0.58	0.47	1.01	0.79	1.96	0.00	0.00	0.83	0.00
time (sec)	N/A	0.676	0.899	6.375	0.160	0.200	0.000	0.000	0.247	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	165	202	438	237	0	0	0	295	0
N.S.	1	0.55	0.68	1.47	0.80	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.908	1.214	6.994	0.066	0.000	0.000	0.000	0.245	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	132	193	537	159	0	0	0	244	0
N.S.	1	0.61	0.89	2.46	0.73	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.914	1.068	6.448	0.049	0.000	0.000	0.000	0.226	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	341	890	1972	0	0	0	0	318	0
N.S.	1	0.50	1.31	2.90	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.504	3.228	8.145	0.000	0.000	0.000	0.000	0.453	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	261	705	1403	0	0	0	0	223	0
N.S.	1	0.51	1.39	2.76	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.914	2.884	7.918	0.000	0.000	0.000	0.000	0.354	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	148	352	635	0	0	0	0	108	0
N.S.	1	0.61	1.44	2.60	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.794	1.571	6.515	0.000	0.000	0.000	0.000	0.264	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	134	315	536	0	0	0	0	112	0
N.S.	1	0.63	1.47	2.50	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.712	2.157	5.744	0.000	0.000	0.000	0.000	0.238	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	243	594	286	0	0	0	0	162	0
N.S.	1	0.45	1.09	0.53	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.353	4.656	6.774	0.000	0.000	0.000	0.000	0.250	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	267	783	2757	0	0	0	0	372	0
N.S.	1	0.51	1.49	5.25	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	2.189	9.313	7.904	0.000	0.000	0.000	0.000	0.302	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	383	1084	2569	0	0	0	0	415	0
N.S.	1	0.47	1.33	3.14	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	1.086	3.892	8.139	0.000	0.000	0.000	0.000	0.566	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	275	524	1216	0	0	0	0	204	0
N.S.	1	0.67	1.28	2.98	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.239	2.855	6.767	0.000	0.000	0.000	0.000	0.325	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	261	705	1399	0	0	0	0	223	0
N.S.	1	0.51	1.39	2.75	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.914	2.833	6.875	0.000	0.000	0.000	0.000	0.340	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	225	532	978	0	0	0	0	206	0
N.S.	1	0.52	1.22	2.24	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.242	7.578	4.201	0.000	0.000	0.000	0.000	0.321	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	313	1174	832	0	0	0	0	285	0
N.S.	1	0.44	1.63	1.16	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	1.698	12.861	4.185	0.000	0.000	0.000	0.000	0.325	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-1)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	287	1609	837	0	0	0	0	759	0
N.S.	1	0.48	2.66	1.39	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	1.436	15.778	4.160	0.000	0.000	0.000	0.000	0.369	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	425	735	1954	0	0	0	0	301	0
N.S.	1	0.70	1.20	3.20	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.796	3.424	4.579	0.000	0.000	0.000	0.000	0.431	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	383	1084	2563	0	0	0	0	415	0
N.S.	1	0.47	1.33	3.14	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	1.562	3.978	4.812	0.000	0.000	0.000	0.000	0.575	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	341	890	1963	0	0	0	0	318	0
N.S.	1	0.50	1.31	2.89	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.281	3.261	4.985	0.000	0.000	0.000	0.000	0.471	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	312	723	1477	0	0	0	0	306	0
N.S.	1	0.51	1.18	2.40	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.005	13.358	4.113	0.000	0.000	0.000	0.000	0.356	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	947	395	2492	953	0	0	0	0	417	0
N.S.	1	0.42	2.63	1.01	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.504	22.395	4.034	0.000	0.000	0.000	0.000	0.437	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-1)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	772	359	2622	1132	0	0	0	0	1094	0
N.S.	1	0.47	3.40	1.47	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	2.076	23.498	4.120	0.000	0.000	0.000	0.000	0.486	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	312	723	1477	0	0	0	0	306	0
N.S.	1	0.51	1.18	2.40	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.975	13.269	6.335	0.000	0.000	0.000	0.000	0.360	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	225	529	978	0	0	0	0	206	0
N.S.	1	0.52	1.21	2.24	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.865	7.615	6.209	0.000	0.000	0.000	0.000	0.279	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	134	315	536	0	0	0	0	111	0
N.S.	1	0.63	1.47	2.50	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.712	2.353	6.211	0.000	0.000	0.000	0.000	0.219	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	168	193	53	0	0	0	97	0
N.S.	1	1.00	2.85	3.27	0.90	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	0.491	1.989	4.372	0.042	0.000	0.000	0.000	0.209	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	219	508	403	0	0	0	0	178	0
N.S.	1	0.47	1.09	0.87	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.040	2.871	5.370	0.000	0.000	0.000	0.000	0.248	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	942	442	524	775	0	0	0	0	442	0
N.S.	1	0.47	0.56	0.82	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	2.245	6.092	5.371	0.000	0.000	0.000	0.000	0.271	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	947	395	2143	953	0	0	0	0	441	0
N.S.	1	0.42	2.26	1.01	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.513	24.228	5.906	0.000	0.000	0.000	0.000	0.404	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	313	1084	832	0	0	0	0	302	0
N.S.	1	0.44	1.51	1.16	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.331	16.386	7.316	0.000	0.000	0.000	0.000	0.327	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	243	530	286	0	0	0	0	172	0
N.S.	1	0.45	0.97	0.53	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.191	5.796	5.664	0.000	0.000	0.000	0.000	0.243	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	219	511	401	0	0	0	0	182	0
N.S.	1	0.47	1.10	0.86	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	1.508	3.027	5.326	0.000	0.000	0.000	0.000	0.240	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	139	488	663	0	0	0	0	176	0
N.S.	1	0.58	2.04	2.77	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	1.118	2.898	5.683	0.000	0.000	0.000	0.000	0.245	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	779	365	754	1084	0	0	0	0	529	0
N.S.	1	0.47	0.97	1.39	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	1.174	9.595	6.213	0.000	0.000	0.000	0.000	0.274	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-1)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	776	363	2552	1134	0	0	0	0	1118	0
N.S.	1	0.47	3.29	1.46	0.00	0.00	0.00	0.00	1.44	0.00
time (sec)	N/A	1.640	23.762	7.729	0.000	0.000	0.000	0.000	0.487	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-1)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	291	1617	837	0	0	0	0	775	0
N.S.	1	0.48	2.66	1.38	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	2.056	16.464	6.263	0.000	0.000	0.000	0.000	0.377	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	271	788	2757	0	0	0	0	380	0
N.S.	1	0.51	1.49	5.21	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.359	9.647	7.647	0.000	0.000	0.000	0.000	0.261	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	942	442	528	771	0	0	0	0	442	0
N.S.	1	0.47	0.56	0.82	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.490	6.037	5.690	0.000	0.000	0.000	0.000	0.261	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	779	365	757	1084	0	0	0	0	538	0
N.S.	1	0.47	0.97	1.39	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	1.701	9.560	6.767	0.000	0.000	0.000	0.000	0.266	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	227	642	1765	0	0	0	0	442	0
N.S.	1	0.56	1.57	4.33	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	1.964	7.794	6.169	0.000	0.000	0.000	0.000	0.244	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [163] had the largest ratio of [1.10000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.95	21	0.238
2	A	6	5	0.97	21	0.238
3	A	6	5	1.03	19	0.263
4	A	6	5	0.84	21	0.238
5	A	9	8	0.92	21	0.381
6	A	11	10	0.92	21	0.476
7	A	9	9	1.18	23	0.391
8	A	8	8	1.12	23	0.348
9	A	5	5	1.10	21	0.238
10	A	7	6	0.80	23	0.261
11	A	11	10	0.89	23	0.435
12	A	18	17	1.49	23	0.739
13	A	14	13	1.32	23	0.565
14	A	10	9	1.20	21	0.429
15	A	8	7	0.80	23	0.304
16	A	16	15	0.85	23	0.652
17	A	5	4	0.75	19	0.211
18	A	5	4	0.78	19	0.211
19	A	5	4	0.90	17	0.235
20	N/A	1	0	1.00	19	0.000
21	N/A	1	0	1.00	19	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	0.84	19	0.211
23	A	5	4	0.88	19	0.211
24	A	5	4	0.98	17	0.235
25	N/A	2	0	1.00	19	0.000
26	N/A	2	0	1.00	19	0.000
27	A	5	4	0.77	23	0.174
28	A	5	4	0.78	23	0.174
29	A	5	4	0.84	21	0.190
30	N/A	1	0	1.00	23	0.000
31	N/A	1	0	1.00	23	0.000
32	A	6	5	0.80	23	0.217
33	A	6	5	0.82	23	0.217
34	A	6	5	0.89	21	0.238
35	N/A	2	0	1.00	23	0.000
36	N/A	2	0	1.00	23	0.000
37	A	8	8	1.08	23	0.348
38	A	6	6	1.08	23	0.261
39	A	3	3	1.00	23	0.130
40	A	1	1	1.00	23	0.043
41	A	2	2	1.00	23	0.087
42	A	4	4	1.05	23	0.174
43	A	6	6	1.08	23	0.261
44	A	10	10	1.20	25	0.400
45	A	5	5	0.98	25	0.200
46	A	1	1	1.00	25	0.040
47	C	9	8	1.04	25	0.320
48	C	12	11	1.02	25	0.440
49	A	3	3	1.00	12	0.250
50	A	8	8	1.06	23	0.348
51	A	6	6	1.11	23	0.261
52	A	3	3	1.00	23	0.130
53	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	23	0.087
55	A	4	4	1.03	23	0.174
56	A	6	6	1.06	23	0.261
57	A	10	10	1.13	25	0.400
58	A	5	5	0.89	25	0.200
59	A	1	1	1.00	25	0.040
60	C	9	8	0.74	25	0.320
61	C	12	11	0.88	25	0.440
62	A	14	14	1.06	21	0.667
63	A	6	6	0.80	21	0.286
64	A	1	1	1.00	21	0.048
65	C	10	9	0.63	21	0.429
66	C	14	13	0.75	21	0.619
67	A	5	4	0.48	25	0.160
68	A	5	4	0.52	25	0.160
69	A	5	4	0.60	25	0.160
70	A	1	1	1.00	25	0.040
71	N/A	1	0	1.00	25	0.000
72	N/A	1	0	1.00	25	0.000
73	A	6	5	0.57	25	0.200
74	A	6	5	0.66	25	0.200
75	C	14	13	0.85	25	0.520
76	A	1	1	1.00	25	0.040
77	N/A	2	0	1.00	25	0.000
78	N/A	2	0	1.00	25	0.000
79	A	1	1	1.00	20	0.050
80	C	15	14	1.30	25	0.560
81	C	13	12	1.37	23	0.522
82	C	10	9	1.08	12	0.750
83	N/A	1	0	1.00	25	0.000
84	N/A	4	0	1.00	25	0.000
85	A	17	16	1.22	25	0.640

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	15	14	1.26	23	0.609
87	A	10	9	1.04	12	0.750
88	N/A	1	0	1.00	25	0.000
89	N/A	5	0	1.00	25	0.000
90	A	5	4	0.91	25	0.160
91	A	5	4	0.92	25	0.160
92	A	5	4	0.96	23	0.174
93	A	8	7	1.01	12	0.583
94	N/A	1	0	1.00	25	0.000
95	N/A	1	0	1.00	25	0.000
96	A	6	5	0.93	25	0.200
97	A	6	5	0.94	25	0.200
98	A	6	5	0.99	23	0.217
99	C	10	9	1.12	12	0.750
100	N/A	2	0	1.00	25	0.000
101	N/A	2	0	1.00	25	0.000
102	C	19	18	1.33	27	0.667
103	C	18	17	1.14	27	0.630
104	C	13	12	0.92	27	0.444
105	A	1	1	1.00	27	0.037
106	N/A	2	0	1.00	27	0.000
107	N/A	3	0	1.00	27	0.000
108	A	15	14	0.96	27	0.519
109	A	9	8	0.76	27	0.296
110	A	1	1	1.00	27	0.037
111	N/A	2	0	1.00	27	0.000
112	N/A	5	0	1.00	27	0.000
113	F	0	0	N/A	0.000	N/A
114	C	15	14	0.80	23	0.609
115	A	1	1	1.00	23	0.043
116	N/A	2	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	N/A	5	0	1.00	23	0.000
118	C	17	16	1.07	22	0.727
119	C	12	11	0.92	22	0.500
120	A	1	1	1.00	22	0.045
121	N/A	3	0	1.00	22	0.000
122	N/A	5	0	1.00	22	0.000
123	A	16	15	0.95	22	0.682
124	A	9	8	0.75	22	0.364
125	A	1	1	1.00	22	0.045
126	N/A	3	0	1.00	22	0.000
127	N/A	7	0	1.00	22	0.000
128	A	5	4	0.63	27	0.148
129	A	5	4	0.66	27	0.148
130	A	5	4	0.71	27	0.148
131	A	1	1	1.00	27	0.037
132	N/A	1	0	1.00	27	0.000
133	N/A	1	0	1.00	27	0.000
134	A	6	5	0.70	27	0.185
135	A	6	5	0.76	27	0.185
136	C	12	11	0.94	27	0.407
137	A	1	1	1.00	27	0.037
138	N/A	2	0	1.00	27	0.000
139	N/A	2	0	1.00	27	0.000
140	A	10	9	0.81	23	0.391
141	A	9	8	0.91	23	0.348
142	A	1	1	1.00	23	0.043
143	N/A	2	0	1.00	23	0.000
144	N/A	2	0	1.00	23	0.000
145	A	6	5	1.00	18	0.278
146	A	6	5	1.01	18	0.278
147	A	6	5	1.04	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	1	1	1.00	8	0.125
149	A	2	2	1.00	18	0.111
150	A	2	2	1.00	18	0.111
151	A	2	2	1.00	20	0.100
152	A	2	2	1.00	20	0.100
153	A	2	2	1.00	18	0.111
154	A	3	3	1.09	10	0.300
155	A	2	2	1.00	20	0.100
156	A	2	2	1.00	20	0.100
157	A	2	2	1.00	18	0.111
158	A	9	8	0.91	10	0.800
159	N/A	1	0	1.00	20	0.000
160	N/A	1	0	1.00	20	0.000
161	A	2	2	1.00	20	0.100
162	A	2	2	1.00	18	0.111
163	C	12	11	1.05	10	1.100
164	N/A	1	0	1.00	20	0.000
165	N/A	1	0	1.00	20	0.000
166	N/A	1	0	1.00	20	0.000
167	N/A	1	0	1.00	20	0.000
168	A	6	5	1.00	20	0.250
169	A	7	6	1.01	20	0.300
170	A	9	8	1.09	20	0.400
171	N/A	1	0	1.00	22	0.000
172	N/A	1	0	1.00	22	0.000
173	N/A	1	0	1.00	22	0.000
174	N/A	1	0	1.00	22	0.000
175	N/A	1	0	1.00	22	0.000
176	N/A	1	0	1.00	22	0.000
177	N/A	1	0	1.00	22	0.000
178	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	N/A	1	0	1.00	22	0.000
180	N/A	1	0	1.00	22	0.000
181	N/A	1	0	1.00	22	0.000
182	N/A	1	0	1.00	22	0.000
183	A	2	2	1.00	22	0.091
184	A	2	2	1.00	20	0.100
185	C	10	9	1.08	12	0.750
186	N/A	1	0	1.00	22	0.000
187	N/A	1	0	1.00	22	0.000
188	A	2	2	1.00	20	0.100
189	A	10	9	1.04	12	0.750
190	N/A	1	0	1.00	22	0.000
191	N/A	1	0	1.00	22	0.000
192	A	2	2	1.00	22	0.091
193	A	2	2	1.00	20	0.100
194	A	8	7	1.01	12	0.583
195	N/A	1	0	1.00	22	0.000
196	N/A	1	0	1.00	22	0.000
197	A	2	2	1.00	20	0.100
198	C	10	9	1.12	12	0.750
199	N/A	1	0	1.00	22	0.000
200	N/A	1	0	1.00	22	0.000
201	A	4	4	0.46	35	0.114
202	A	4	4	0.49	35	0.114
203	A	4	4	0.65	35	0.114
204	A	4	4	0.61	35	0.114
205	A	4	4	0.64	35	0.114
206	A	7	7	0.64	35	0.200
207	A	4	4	0.41	35	0.114
208	A	7	7	0.60	35	0.200
209	A	4	4	0.49	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
210	A	4	4	0.50	35	0.114
211	A	4	4	0.52	35	0.114
212	A	4	4	0.51	35	0.114
213	A	9	9	0.53	35	0.257
214	A	4	4	0.41	35	0.114
215	A	4	4	0.46	35	0.114
216	A	4	4	0.48	35	0.114
217	A	4	4	0.44	35	0.114
218	A	4	4	0.47	35	0.114
219	A	4	4	0.48	35	0.114
220	A	4	4	0.50	35	0.114
221	A	4	4	0.61	35	0.114
222	A	2	2	1.00	35	0.057
223	A	6	6	0.80	35	0.171
224	A	4	4	0.58	35	0.114
225	A	4	4	0.44	35	0.114
226	A	4	4	0.52	35	0.114
227	A	4	4	0.63	35	0.114
228	A	7	7	0.80	35	0.200
229	A	3	3	0.80	35	0.086
230	A	4	4	0.55	35	0.114
231	A	4	4	0.47	35	0.114
232	A	4	4	0.50	35	0.114
233	A	7	7	0.64	35	0.200
234	A	4	4	0.58	35	0.114
235	A	4	4	0.55	35	0.114
236	A	5	5	0.61	35	0.143
237	A	4	4	0.50	37	0.108
238	A	4	4	0.51	37	0.108
239	A	6	6	0.61	37	0.162
240	A	4	4	0.63	37	0.108
241	A	4	4	0.45	37	0.108

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
242	A	4	4	0.51	37	0.108
243	A	4	4	0.47	37	0.108
244	A	11	11	0.67	37	0.297
245	A	4	4	0.51	37	0.108
246	A	7	6	0.52	37	0.162
247	A	4	4	0.44	37	0.108
248	A	4	4	0.48	37	0.108
249	A	13	13	0.70	37	0.351
250	A	4	4	0.47	37	0.108
251	A	4	4	0.50	37	0.108
252	A	7	6	0.51	37	0.162
253	A	4	4	0.42	37	0.108
254	A	4	4	0.47	37	0.108
255	A	7	6	0.51	37	0.162
256	A	7	6	0.52	37	0.162
257	A	4	4	0.63	37	0.108
258	A	2	2	1.00	37	0.054
259	A	4	4	0.47	37	0.108
260	A	4	4	0.47	37	0.108
261	A	4	4	0.42	37	0.108
262	A	4	4	0.44	37	0.108
263	A	4	4	0.45	37	0.108
264	A	4	4	0.47	37	0.108
265	A	10	9	0.58	37	0.243
266	A	4	4	0.47	37	0.108
267	A	4	4	0.47	37	0.108
268	A	4	4	0.48	37	0.108
269	A	4	4	0.51	37	0.108
270	A	4	4	0.47	37	0.108
271	A	4	4	0.47	37	0.108
272	A	13	12	0.56	37	0.324

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{35c} - \frac{8bd^3(1+c^2x^2)^{3/2}}{105c} - \frac{6bd^3(1+c^2x^2)^{5/2}}{175c} - \frac{bd^3(1+c^2x^2)^{7/2}}{49c}$$
$$+d^3x(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx$$

output
```
-16/35*b*d^3*(c^2*x^2+1)^(1/2)/c-8/105*b*d^3*(c^2*x^2+1)^(3/2)/c-6/175*b*d
^3*(c^2*x^2+1)^(5/2)/c-1/49*b*d^3*(c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcsinh(c
*x))+c^2*d^3*x^3*(a+b*arcsinh(c*x))+3/5*c^4*d^3*x^5*(a+b*arcsinh(c*x))+1/7
*c^6*d^3*x^7*(a+b*arcsinh(c*x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{d^3\left(105acx(35+35c^2x^2+21c^4x^4+5c^6x^6)-b\sqrt{1+c^2x^2}(2161+757c^2x^2+351c^4x^4+75c^6x^6)+105bcx\right.}{3675c}$$

input
```
Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - b*Sqrt[1 + c^
2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(35 + 3
5*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]))/(3675*c)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6199, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^3(a+b\text{arcsinh}(cx))dx$$

$$\Big\downarrow \text{6199}$$

$$-bc\int \frac{d^3x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}bcd^3\int \frac{x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{70}bcd^3\int \frac{5c^6x^6+21c^4x^4+35c^2x^2+35}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2389}$$

$$-\frac{1}{70}bcd^3\int \left(5\left(c^2x^2+1\right)^{5/2}+6\left(c^2x^2+1\right)^{3/2}+8\sqrt{c^2x^2+1}+\frac{16}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}c^6d^3x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4d^3x^5(a+b\text{arcsinh}(cx))+c^2d^3x^3(a+b\text{arcsinh}(cx))+d^3x(a+b\text{arcsinh}(cx))-\frac{1}{70}bcd^3\left(\frac{10(c^2x^2+1)^{7/2}}{7c^2}+\frac{12(c^2x^2+1)^{5/2}}{5c^2}+\frac{16(c^2x^2+1)^{3/2}}{3c^2}+\frac{32\sqrt{c^2x^2+1}}{c^2}\right)$$

input
```
Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/70*(b*c*d^3*((32*Sqrt[1 + c^2*x^2])/c^2 + (16*(1 + c^2*x^2)^(3/2))/(3*c
^2) + (12*(1 + c^2*x^2)^(5/2))/(5*c^2) + (10*(1 + c^2*x^2)^(7/2))/(7*c^2))
) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^4
*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.90

method	result
parts	$d^3a\left(\frac{1}{7}c^6x^7+\frac{3}{5}c^4x^5+x^3c^2+x\right)+\frac{d^3b\left(\frac{\operatorname{arcsinh}(xc)x^7c^7}{7}+\frac{3\operatorname{arcsinh}(xc)x^5c^5}{5}+\operatorname{arcsinh}(xc)x^3c^3+xc\operatorname{arcsinh}(\right.}{}$
derivativedivides	$\frac{d^3a\left(\frac{1}{7}x^7c^7+\frac{3}{5}x^5c^5+x^3c^3+xc\right)+d^3b\left(\frac{\operatorname{arcsinh}(xc)x^7c^7}{7}+\frac{3\operatorname{arcsinh}(xc)x^5c^5}{5}+\operatorname{arcsinh}(xc)x^3c^3+xc\operatorname{arcsinh}(xc)-\frac{2161\sqrt{c^2}}{3675}\right.}{c}$
default	$\frac{d^3a\left(\frac{1}{7}x^7c^7+\frac{3}{5}x^5c^5+x^3c^3+xc\right)+d^3b\left(\frac{\operatorname{arcsinh}(xc)x^7c^7}{7}+\frac{3\operatorname{arcsinh}(xc)x^5c^5}{5}+\operatorname{arcsinh}(xc)x^3c^3+xc\operatorname{arcsinh}(xc)-\frac{2161\sqrt{c^2}}{3675}\right.}{c}$
orering	$\frac{x\left(325c^6x^6+1437c^4x^4+2739c^2x^2+5547\right)\left(c^2d\,x^2+d\right)^3\left(a+b\operatorname{arcsinh}(xc)\right)}{1225\left(c^2x^2+1\right)^3}-\frac{\left(75c^6x^6+351c^4x^4+757c^2x^2+2161\right)\left(6(c\right.}{367}$

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/7*c^6*x^7+3/5*c^4*x^5+x^3*c^2+x)+d^3*b/c*(1/7*arcsinh(x*c)*x^7*c^
7+3/5*arcsinh(x*c)*x^5*c^5+arcsinh(x*c)*x^3*c^3+x*c*arcsinh(x*c)-2161/3675
*(c^2*x^2+1)^(1/2)-757/3675*x^2*c^2*(c^2*x^2+1)^(1/2)-117/1225*x^4*c^4*(c^
2*x^2+1)^(1/2)-1/49*x^6*c^6*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \left(d+c^2dx^2\right)^3\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{525\,ac^7d^3x^7+2205\,ac^5d^3x^5+3675\,ac^3d^3x^3+3675\,acd^3x+105\left(5\,bc^7d^3x^7+21\,bc^5d^3x^5+35\,bc^3d^3x^3+3\right.}{3675\,c}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*d^3*x^7 + 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 + 3675
*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 + 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 +
35*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d^3*x^6 + 351*b*c^4
*d^3*x^4 + 757*b*c^2*d^3*x^2 + 2161*b*d^3)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.30

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^6d^3x^7}{7} + \frac{3ac^4d^3x^5}{5} + ac^2d^3x^3 + ad^3x + \frac{bc^6d^3x^7\operatorname{asinh}(cx)}{7} - \frac{bc^5d^3x^6\sqrt{c^2x^2+1}}{49} + \frac{3bc^4d^3x^5\operatorname{asinh}(cx)}{5} - \frac{117bc^3d^3x^4\sqrt{c^2x^2}}{1225} \\ ad^3x \end{cases}$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 + a*c**2*d**3*x**3 +
a*d**3*x + b*c**6*d**3*x**7*asinh(c*x)/7 - b*c**5*d**3*x**6*sqrt(c**2*x**2
 + 1)/49 + 3*b*c**4*d**3*x**5*asinh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**
2*x**2 + 1)/1225 + b*c**2*d**3*x**3*asinh(c*x) - 757*b*c*d**3*x**2*sqrt(c*
*2*x**2 + 1)/3675 + b*d**3*x*asinh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 + 1)/
(3675*c), Ne(c, 0)), (a*d**3*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(150) = 300.

Time = 0.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.77

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}\,ac^6d^3x^7 + \frac{3}{5}\,ac^4d^3x^5$$
$$+ \frac{1}{245}\left(35\,x^7\operatorname{arsinh}(cx) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^6$$
$$+ \frac{1}{25}\left(15\,x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^3$$
$$+ ac^2d^3x^3 + \frac{1}{3}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^3$$
$$+ ad^3x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sq
rt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1
)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsinh(
c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x)
- c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d
^3*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{d^3 \left(525 asinh(cx)\, b\, c^7x^7 + 2205 asinh(cx)\, b\, c^5x^5 + 3675 asinh(cx)\, b\, c^3x^3 + 3675 asinh(cx)\, bcx - 75\sqrt{c^2x^2 -}\right.}{}$$

input
```
int((c^2*d*x^2+d)^3*(a+b*asinh(c*x)),x)
```

output
```
(d**3*(525*asinh(c*x)*b*c**7*x**7 + 2205*asinh(c*x)*b*c**5*x**5 + 3675*asi
nh(c*x)*b*c**3*x**3 + 3675*asinh(c*x)*b*c*x - 75*sqrt(c**2*x**2 + 1)*b*c**
6*x**6 - 351*sqrt(c**2*x**2 + 1)*b*c**4*x**4 - 757*sqrt(c**2*x**2 + 1)*b*c
**2*x**2 - 2161*sqrt(c**2*x**2 + 1)*b + 525*a*c**7*x**7 + 2205*a*c**5*x**5
 + 3675*a*c**3*x**3 + 3675*a*c*x))/(3675*c)
```

3.2 $\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{8bd^2\sqrt{1+c^2x^2}}{15c}-\frac{4bd^2(1+c^2x^2)^{3/2}}{45c} -\frac{bd^2(1+c^2x^2)^{5/2}}{25c}+d^2x(a+b\text{arcsinh}(cx)) +\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx)) +\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))$$

output

```
-8/15*b*d^2*(c^2*x^2+1)^(1/2)/c-4/45*b*d^2*(c^2*x^2+1)^(3/2)/c-1/25*b*d^2*
(c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*arcsinh(c*x))+2/3*c^2*d^2*x^3*(a+b*arcsinh(
c*x))+1/5*c^4*d^2*x^5*(a+b*arcsinh(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{d^2\left(15acx(15+10c^2x^2+3c^4x^4)-b\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4)+15bcx(15+10c^2x^2+3c^4x^4)\,\text{arcs}\right.}{225c}$$

input
```
Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(149 +
38*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c
*x]))/(225*c)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6199, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6199}$$

$$-bc\int \frac{d^2x\left(3c^4x^4+10c^2x^2+15\right)}{15\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bcd^2\int \frac{x\left(3c^4x^4+10c^2x^2+15\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}bcd^2\int \frac{3c^4x^4+10c^2x^2+15}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}bcd^2\int \left(3\left(c^2x^2+1\right)^{3/2}+4\sqrt{c^2x^2+1}+\frac{8}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^4d^2x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2d^2x^3(a+b\text{arcsinh}(cx))+d^2x(a+b\text{arcsinh}(cx))-\frac{1}{30}bcd^2\left(\frac{6\left(c^2x^2+1\right)^{5/2}}{5c^2}+\frac{8\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{16\sqrt{c^2x^2+1}}{c^2}\right)$$

input
```
Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/30*(b*c*d^2*((16*Sqrt[1 + c^2*x^2])/c^2 + (8*(1 + c^2*x^2)^(3/2))/(3*c^
2) + (6*(1 + c^2*x^2)^(5/2))/(5*c^2))) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c
^2*d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

method	result
parts	$d^2a\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{d^2b\left(\frac{\operatorname{arcsinh}(xc)x^5c^5}{5}+\frac{2\operatorname{arcsinh}(xc)x^3c^3}{3}+xc\operatorname{arcsinh}(xc)-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{38x^2c^2\sqrt{}}{22}\right.}{c}$
derivativedivides	$\frac{d^2a\left(\frac{1}{5}x^5c^5+\frac{2}{3}x^3c^3+xc\right)+d^2b\left(\frac{\operatorname{arcsinh}(xc)x^5c^5}{5}+\frac{2\operatorname{arcsinh}(xc)x^3c^3}{3}+xc\operatorname{arcsinh}(xc)-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{38x^2c^2\sqrt{c^2x^2+1}}{225}-x\right.}{c}$
default	$\frac{d^2a\left(\frac{1}{5}x^5c^5+\frac{2}{3}x^3c^3+xc\right)+d^2b\left(\frac{\operatorname{arcsinh}(xc)x^5c^5}{5}+\frac{2\operatorname{arcsinh}(xc)x^3c^3}{3}+xc\operatorname{arcsinh}(xc)-\frac{149\sqrt{c^2x^2+1}}{225}-\frac{38x^2c^2\sqrt{c^2x^2+1}}{225}-x\right.}{c}$
orering	$\frac{x(81c^4x^4+302c^2x^2+821)(c^2dx^2+d)^2(a+b\operatorname{arcsinh}(xc))}{225(c^2x^2+1)^2}-\frac{(9c^4x^4+38c^2x^2+149)\left(4(c^2dx^2+d)(a+b\operatorname{arcsinh}(xc))c\right.}{225c^2(c^2x^2+1)}$

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/5*c^4*x^5+2/3*x^3*c^2+x)+d^2*b/c*(1/5*arcsinh(x*c)*x^5*c^5+2/3*ar
csinh(x*c)*x^3*c^3+x*c*arcsinh(x*c)-149/225*(c^2*x^2+1)^(1/2)-38/225*x^2*c
^2*(c^2*x^2+1)^(1/2)-1/25*x^4*c^4*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int (d + c^2dx^2)^2 (a + b\text{arcsinh}(cx)) \, dx$$
$$= \frac{45\,ac^5d^2x^5 + 150\,ac^3d^2x^3 + 225\,acd^2x + 15\,(3\,bc^5d^2x^5 + 10\,bc^3d^2x^3 + 15\,bcd^2x)\log\left(cx + \sqrt{c^2x^2 + 1}\right) - (}{225\,c}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*d^2*x^5 + 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*
d^2*x^5 + 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)) -
(9*b*c^4*d^2*x^4 + 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int (d + c^2dx^2)^2 (a + b\text{arcsinh}(cx)) \, dx$$
$$= \begin{cases} \frac{ac^4d^2x^5}{5} + \frac{2ac^2d^2x^3}{3} + ad^2x + \frac{bc^4d^2x^5\operatorname{asinh}(cx)}{5} - \frac{bc^3d^2x^4\sqrt{c^2x^2+1}}{25} + \frac{2bc^2d^2x^3\operatorname{asinh}(cx)}{3} - \frac{38bcd^2x^2\sqrt{c^2x^2+1}}{225} + bd^2x\,\text{as} \\ ad^2x \end{cases}$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**5/5 + 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d
**2*x**5*asinh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/25 + 2*b*c**2
*d**2*x**3*asinh(c*x)/3 - 38*b*c*d**2*x**2*sqrt(c**2*x**2 + 1)/225 + b*d**
2*x*asinh(c*x) - 149*b*d**2*sqrt(c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**
2*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.52

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{5}\,ac^4d^2x^5 + \frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^2 + \frac{2}{3}\,ac^2d^2x^3 + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d^2 + ad^2x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 +
 2/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{d^2\left(45asinh(cx)\,b\,c^5x^5+150asinh(cx)\,b\,c^3x^3+225asinh(cx)\,bcx-9\sqrt{c^2x^2+1}\,b\,c^4x^4-38\sqrt{c^2x^2+1}\,b\,c^2}{225c}$$

input
```
int((c^2*d*x^2+d)^2*(a+b*asinh(c*x)),x)
```

output
```
(d**2*(45*asinh(c*x)*b*c**5*x**5 + 150*asinh(c*x)*b*c**3*x**3 + 225*asinh(
c*x)*b*c*x - 9*sqrt(c**2*x**2 + 1)*b*c**4*x**4 - 38*sqrt(c**2*x**2 + 1)*b*
c**2*x**2 - 149*sqrt(c**2*x**2 + 1)*b + 45*a*c**5*x**5 + 150*a*c**3*x**3 +
 225*a*c*x))/(225*c)
```

3.3 $\int (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = -\frac{2bd\sqrt{1 + c^2x^2}}{3c} - \frac{bd(1 + c^2x^2)^{3/2}}{9c} + dx(a + b\text{arcsinh}(cx)) + \frac{1}{3}c^2dx^3(a + b\text{arcsinh}(cx))$$

output
```
-2/3*b*d*(c^2*x^2+1)^(1/2)/c-1/9*b*d*(c^2*x^2+1)^(3/2)/c+d*x*(a+b*arcsinh(
c*x))+1/3*c^2*d*x^3*(a+b*arcsinh(c*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = adx + \frac{1}{3}ac^2dx^3 - \frac{7bd\sqrt{1 + c^2x^2}}{9c} - \frac{1}{9}bcdx^2\sqrt{1 + c^2x^2} + bdx\text{arcsinh}(cx) + \frac{1}{3}bc^2dx^3\text{arcsinh}(cx)$$

input
```
Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
a*d*x + (a*c^2*d*x^3)/3 - (7*b*d*Sqrt[1 + c^2*x^2])/(9*c) - (b*c*d*x^2*Sqr
t[1 + c^2*x^2])/9 + b*d*x*ArcSinh[c*x] + (b*c^2*d*x^3*ArcSinh[c*x])/3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6199, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6199}$$

$$-bc\int \frac{dx\left(c^2x^2+3\right)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}bcd\int \frac{x\left(c^2x^2+3\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{1}{6}bcd\int \frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{6}bcd\int \left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}c^2dx^3(a+b\text{arcsinh}(cx))+dx(a+b\text{arcsinh}(cx))-\frac{1}{6}bcd\left(\frac{2\left(c^2x^2+1\right)^{3/2}}{3c^2}+\frac{4\sqrt{c^2x^2+1}}{c^2}\right)$$

input
```
Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/6*(b*c*d*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)^(3/2))/(3*c^2)))
 + d*x*(a + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result	size
parts	$ad\left(\frac{1}{3}x^3c^2+x\right)+\frac{bd\left(\frac{\operatorname{arcsinh}(xc)x^3c^3}{3}+xc\operatorname{arcsinh}(xc)-\frac{x^2c^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	73
derivativedivides	$\frac{ad\left(\frac{1}{3}x^3c^3+xc\right)+bd\left(\frac{\operatorname{arcsinh}(xc)x^3c^3}{3}+xc\operatorname{arcsinh}(xc)-\frac{x^2c^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	76
default	$\frac{ad\left(\frac{1}{3}x^3c^3+xc\right)+bd\left(\frac{\operatorname{arcsinh}(xc)x^3c^3}{3}+xc\operatorname{arcsinh}(xc)-\frac{x^2c^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$	76
orering	$\frac{x(5c^2x^2+23)(c^2dx^2+d)(a+b\operatorname{arcsinh}(xc))}{9c^2x^2+9}-\frac{(c^2x^2+7)\left(2c^2dx(a+b\operatorname{arcsinh}(xc))+\frac{(c^2dx^2+d)bc}{\sqrt{c^2x^2+1}}\right)}{9c^2}$	98

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/3*x^3*c^2+x)+b*d/c*(1/3*arcsinh(x*c)*x^3*c^3+x*c*arcsinh(x*c)-1/9*x
^2*c^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int (d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{3\,ac^3dx^3+9\,acdx+3\,(bc^3dx^3+3\,bcdx)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^2dx^2+7\,bd)\sqrt{c^2x^2+1}}{9\,c}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*d*x^3 + 9*a*c*d*x + 3*(b*c^3*d*x^3 + 3*b*c*d*x)*log(c*x + sqr
t(c^2*x^2 + 1)) - (b*c^2*d*x^2 + 7*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = \begin{cases} \frac{ac^2dx^3}{3} + adx + \frac{bc^2dx^3 \operatorname{asinh}(cx)}{3} - \frac{bcdx^2\sqrt{c^2x^2+1}}{9} + bdx \operatorname{asinh}(cx) - \frac{7bd\sqrt{c^2x^2+1}}{9c} & \text{for } c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*c**2*d*x**3/3 + a*d*x + b*c**2*d*x**3*asinh(c*x)/3 - b*c*d*x*
*2*sqrt(c**2*x**2 + 1)/9 + b*d*x*asinh(c*x) - 7*b*d*sqrt(c**2*x**2 + 1)/(9
*c), Ne(c, 0)), (a*d*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx)) \, dx = \frac{1}{3} ac^2dx^3 + \frac{1}{9} \left(3 x^3 \operatorname{arsinh}(cx) - c \left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right) bc^2d + adx + \frac{\left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*c^2*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*
x^2 + 1))*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))\,dx = \frac{d\left(3asinh(cx)\,b\,c^3x^3+9asinh(cx)\,bcx-\sqrt{c^2x^2+1}\,b\,c^2x^2-7\sqrt{c^2x^2+1}\,b+3a\,c^3x^3+9acx\right)}{9c}$$

input
```
int((c^2*d*x^2+d)*(a+b*asinh(c*x)),x)
```

output

```
(d*(3*asinh(c*x)*b*c**3*x**3 + 9*asinh(c*x)*b*c*x - sqrt(c**2*x**2 + 1)*b*
c**2*x**2 - 7*sqrt(c**2*x**2 + 1)*b + 3*a*c**3*x**3 + 9*a*c*x))/(9*c)
```

3.4 $\int \frac{a+b\,\mathrm{arcsinh}(cx)}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{a+b\,\mathrm{arcsinh}(cx)}{d+c^2dx^2}\,dx = \frac{2(a+b\,\mathrm{arcsinh}(cx))\arctan\left(e^{\mathrm{arcsinh}(cx)}\right)}{cd} - \frac{ib\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(cx)}\right)}{cd} + \frac{ib\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arcsinh}(cx)}\right)}{cd}$$

output
```
2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-I*b*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/d+I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{a+b\,\mathrm{arcsinh}(cx)}{d+c^2dx^2}\,dx = -\frac{c\left(a\sqrt{-c^2}\arctan(cx) - b\,\mathrm{arcsinh}(cx)\log\left(1+\frac{ce^{\mathrm{arcsinh}(cx)}}{\sqrt{-c^2}}\right) + b\,\mathrm{arcsinh}(cx)\log\left(1+\frac{\sqrt{-c^2}e^{\mathrm{arcsinh}(cx)}}{c}\right) + \cdots}{(-c^2)^{3/2}\,d}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

output
```
-((c*(a*Sqrt[-c^2]*ArcTan[c*x] - b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
])/Sqrt[-c^2]] + b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] +
 b*c*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*c*PolyLog[2, (Sqrt[-c^2
]*E^ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{c^2dx^2+d}dx$$

$$\Big\downarrow \text{6204}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+ba}{cd}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\text{arctar}}{cd}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

output
```
(2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^Arc
Sinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

method	result
derivativedivides	$\frac{\frac{a\arctan(xc)}{d}+\frac{b\left(\arctan(xc)\operatorname{arcsinh}(xc)+\arctan(xc)\ln\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(xc)\ln\left(1-\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)+i\,\mathrm{d}}{d}}{c}$
default	$\frac{\frac{a\arctan(xc)}{d}+\frac{b\left(\arctan(xc)\operatorname{arcsinh}(xc)+\arctan(xc)\ln\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(xc)\ln\left(1-\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)+i\,\mathrm{d}}{d}}{c}$
parts	$\frac{a\arctan(xc)}{dc}+\frac{b\left(\arctan(xc)\operatorname{arcsinh}(xc)+\arctan(xc)\ln\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)-\arctan(xc)\ln\left(1-\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\right.\right.}{dc}$

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d*arctan(x*c)+b/d*(arctan(x*c)*arcsinh(x*c)+arctan(x*c)*ln(1+I*(1+I
*x*c)/(c^2*x^2+1)^(1/2))-arctan(x*c)*ln(1-I*(1+I*x*c)/(c^2*x^2+1)^(1/2))-I
*dilog(1+I*(1+I*x*c)/(c^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*x*c)/(c^2*x^2+1)^
(1/2))))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx = \frac{\int \frac{a}{c^2x^2+1}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

output
```
(Integral(a/(c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**2*x**2 + 1), x
))/d
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x) + a*arctan(c*
x)/(c*d)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+c^2dx^2}\,dx=\frac{atan(cx)\,a+\left(\int \frac{asinh(cx)}{c^2x^2+1}dx\right)bc}{cd}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d),x)
```

output
```
(atan(c*x)*a + int(asinh(c*x)/(c**2*x**2 + 1),x)*b*c)/(c*d)
```

3.5 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x(a+b\text{arcsinh}(cx))}{2d^2(1+c^2x^2)} + \frac{(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2cd^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2}$$

output

```
1/2*b/c/d^2/(c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+(a+
b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^2-1/2*I*b*polylog(2,-I*(
c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))
/c/d^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{acx+b\sqrt{1+c^2x^2}+bcx\operatorname{arcsinh}(cx)+a\arctan(cx)+ac^2x^2\arctan(cx)+ib\operatorname{arcsinh}(cx)\log\left(1-ie^{\operatorname{arcsinh}(cx)}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

output
```
(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] + a*ArcTan[c*x] + a*c^2*
x^2*ArcTan[c*x] + I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*ArcSinh[c*x]*Log[1 + I*E^Arc
Sinh[c*x]] - I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*b*(1 +
 c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*(1 + c^2*x^2)*PolyLog[2, I
*E^ArcSinh[c*x]])/(2*d^2*(c + c^3*x^2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6203, 27, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(c^2dx^2+d)^2}\,dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{d(c^2x^2+1)}dx}{2d} - \frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2d^2} + \frac{x(a+b\operatorname{arcsinh}(cx))}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\operatorname{arcsinh}(cx)}{c^2x^2+1}dx}{2d^2} - \frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx}{2d^2} + \frac{x(a+b\operatorname{arcsinh}(cx))}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6204}$$

$$\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+ba}{2cd^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan}{2cd^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2}+$$

$$\frac{x(a+b\text{arcsinh}(cx))}{2d^2\left(c^2x^2+1\right)}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

output
```
b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x
^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{\frac{a\left(\frac{xc}{2c^2x^2+2}+\frac{\arctan(xc)}{2}\right)}{d^2}+\frac{b\left(\frac{xc\,\mathrm{arcsinh}(xc)}{2c^2x^2+2}+\frac{\arctan(xc)\,\mathrm{arcsinh}(xc)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(xc)\ln\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(xc)}{}\right.}{d^2}}{c}$
default	$\frac{\frac{a\left(\frac{xc}{2c^2x^2+2}+\frac{\arctan(xc)}{2}\right)}{d^2}+\frac{b\left(\frac{xc\,\mathrm{arcsinh}(xc)}{2c^2x^2+2}+\frac{\arctan(xc)\,\mathrm{arcsinh}(xc)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(xc)\ln\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan(xc)}{}\right.}{d^2}}{c}$
parts	$\frac{a\left(\frac{x}{2c^2x^2+2}+\frac{\arctan(xc)}{2c}\right)}{d^2}+\frac{b\left(\frac{xc\,\mathrm{arcsinh}(xc)}{2c^2x^2+2}+\frac{\arctan(xc)\,\mathrm{arcsinh}(xc)}{2}+\frac{1}{2\sqrt{c^2x^2+1}}+\frac{\arctan(xc)\ln\left(1+\frac{i(ixc+1)}{\sqrt{c^2x^2+1}}\right)}{2}-\frac{\arctan}{}\right.}{d^2c}$

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^2*(1/2*x*c/(c^2*x^2+1)+1/2*arctan(x*c))+b/d^2*(1/2*arcsinh(x*c)*x
*c/(c^2*x^2+1)+1/2*arctan(x*c)*arcsinh(x*c)+1/2/(c^2*x^2+1)^(1/2)+1/2*arct
an(x*c)*ln(1+I*(1+I*x*c)/(c^2*x^2+1)^(1/2))-1/2*arctan(x*c)*ln(1-I*(1+I*x*
c)/(c^2*x^2+1)^(1/2))-1/2*I*dilog(1+I*(1+I*x*c)/(c^2*x^2+1)^(1/2))+1/2*I*d
ilog(1-I*(1+I*x*c)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2}\, dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^2}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2}\, dx = \frac{\int \frac{a}{c^4x^4+2c^2x^2+1}\, dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\, dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**
4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^2}\, dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)^2}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + b*integrate(log(c*x
+ sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^2}\,dx = \frac{atan(cx)\,a\,c^2x^2+atan(cx)\,a+2\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)b\,c^3x^2+2\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)bc+acx}{2c\,d^2\,(c^2x^2+1)}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^2,x)
```

output
```
(atan(c*x)*a*c**2*x**2 + atan(c*x)*a + 2*int(asinh(c*x)/(c**4*x**4 + 2*c**
2*x**2 + 1),x)*b*c**3*x**2 + 2*int(asinh(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1
),x)*b*c + a*c*x)/(2*c*d**2*(c**2*x**2 + 1))
```

3.6 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx = \frac{b}{12cd^3(1+c^2x^2)^{3/2}}+\frac{3b}{8cd^3\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3(1+c^2x^2)^2}+\frac{3x(a+b\text{arcsinh}(cx))}{8d^3(1+c^2x^2)}+\frac{3(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{4cd^3}-\frac{3ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{8cd^3}+\frac{3ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{8cd^3}$$

output
```
1/12*b/c/d^3/(c^2*x^2+1)^(3/2)+3/8*b/c/d^3/(c^2*x^2+1)^(1/2)+1/4*x*(a+b*ar
csinh(c*x))/d^3/(c^2*x^2+1)^2+3/8*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+3/4
*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^3-3/8*I*b*polylog(2,
-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/
2)))/c/d^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.92

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx$$
$$= \frac{15acx+9ac^3x^3+11b\sqrt{1+c^2x^2}+9bc^2x^2\sqrt{1+c^2x^2}+15bcx\operatorname{arcsinh}(cx)+9bc^3x^3\operatorname{arcsinh}(cx)+9a\operatorname{arcta}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

output
```
(15*a*c*x + 9*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] + 9*b*c^2*x^2*Sqrt[1 + c^
2*x^2] + 15*b*c*x*ArcSinh[c*x] + 9*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x
] + 18*a*c^2*x^2*ArcTan[c*x] + 9*a*c^4*x^4*ArcTan[c*x] + (9*I)*b*ArcSinh[c
*x]*Log[1 - I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^
ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (
9*I)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c
*x]*Log[1 + I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (
9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c*d^3*(1 + c^2*x^
2)^2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6203, 27, 241, 6203, 241, 6204, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(c^2dx^2+d)^3}\,dx$$
$$\Big\downarrow \text{6203}$$
$$\frac{3\int \frac{a+b\operatorname{arcsinh}(cx)}{d^2(c^2x^2+1)^2}dx}{4d}-\frac{bc\int \frac{x}{(c^2x^2+1)^{5/2}}dx}{4d^3}+\frac{x(a+b\operatorname{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{4d^3}-\frac{bc\int \frac{x}{(c^2x^2+1)^{5/2}}dx}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^2}dx}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{x}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{6204}$$

$$\frac{3\left(\frac{\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{b}{2c\sqrt{c^2x^2+1}}\right)}{4d^3}+\frac{x(a+b\text{arcsinh}(cx))}{4d^3\left(c^2x^2+1\right)^2}+\frac{b}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{3\left(\frac{-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{2c}+\frac{x(a+\ldots}{\ldots}\right.}{4d^3}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$\Big\downarrow$ 2715

$$\frac{3\left(\frac{-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+\ldots}{2c}\right.}{4d^3}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

$\Big\downarrow$ 2838

$$\frac{3\left(\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2c}+\frac{x(a+b\text{arcsinh}(cx))}{2(c^2x^2+1)}+\frac{\ldots}{2c\sqrt{c\ldots}}\right.}{4d^3}$$
$$\frac{x(a+b\text{arcsinh}(cx))}{4d^3(c^2x^2+1)^2}+\frac{b}{12cd^3(c^2x^2+1)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

output
```
b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^
2*x^2)^2) + (3*(b/(2*c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*(1
 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]] - I*b*PolyLo
g[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c)))/(4*d
^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a\left(\frac{xc}{4(c^2x^2+1)^2}+\frac{3xc}{8(c^2x^2+1)}+\frac{3\arctan(xc)}{8}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)xc}{4(c^2x^2+1)^2}+\frac{3\operatorname{arcsinh}(xc)xc}{8(c^2x^2+1)}+\frac{3\arctan(xc)\operatorname{arcsinh}(xc)}{8}+\frac{11}{24(c^2x^2+1)^{\frac{3}{2}}}+\right.}{}$
default	$\frac{a\left(\frac{xc}{4(c^2x^2+1)^2}+\frac{3xc}{8(c^2x^2+1)}+\frac{3\arctan(xc)}{8}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)xc}{4(c^2x^2+1)^2}+\frac{3\operatorname{arcsinh}(xc)xc}{8(c^2x^2+1)}+\frac{3\arctan(xc)\operatorname{arcsinh}(xc)}{8}+\frac{11}{24(c^2x^2+1)^{\frac{3}{2}}}+\right.}{}$
parts	$\frac{a\left(\frac{x}{4(c^2x^2+1)^2}+\frac{3x}{8(c^2x^2+1)}+\frac{3\arctan(xc)}{8c}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)xc}{4(c^2x^2+1)^2}+\frac{3\operatorname{arcsinh}(xc)xc}{8(c^2x^2+1)}+\frac{3\arctan(xc)\operatorname{arcsinh}(xc)}{8}+\frac{11}{24(c^2x^2+1)}\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^3*(1/4*x*c/(c^2*x^2+1)^2+3/8*x*c/(c^2*x^2+1)+3/8*arctan(x*c))+b/d
^3*(1/4*arcsinh(x*c)*x*c/(c^2*x^2+1)^2+3/8*arcsinh(x*c)*x*c/(c^2*x^2+1)+3/
8*arctan(x*c)*arcsinh(x*c)+11/24/(c^2*x^2+1)^(3/2)+3/8*x^2*c^2/(c^2*x^2+1)
^(3/2)+3/8*arctan(x*c)*ln(1+I*(1+I*x*c)/(c^2*x^2+1)^(1/2))-3/8*arctan(x*c)
*ln(1-I*(1+I*x*c)/(c^2*x^2+1)^(1/2))-3/8*I*dilog(1+I*(1+I*x*c)/(c^2*x^2+1)
^(1/2))+3/8*I*dilog(1-I*(1+I*x*c)/(c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^3}\, dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^3}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 + d^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^3}\, dx = \frac{\int \frac{a}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx + \int \frac{b\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx}{d^3}$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

output
```
(Integral(a/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*a
sinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^3}\, dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(c^2dx^2 + d)^3}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/8*a*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c*
x)/(c*d^3)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^
4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2+d)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^3}\,dx$$
$$=\frac{3atan(cx)\,a\,c^4x^4+6atan(cx)\,a\,c^2x^2+3atan(cx)\,a+8\left(\int \frac{asinh(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}dx\right)b\,c^5x^4+16\left(\int \frac{asinh(\ldots}{c^6x^6+3c^4x^4+\ldots}\right.}{8c\,d^3\,(c^4x^4+2c^2x^2+1)\ldots}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^3,x)
```

output
```
(3*atan(c*x)*a*c**4*x**4 + 6*atan(c*x)*a*c**2*x**2 + 3*atan(c*x)*a + 8*int
(asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1),x)*b*c**5*x**4 + 1
6*int(asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1),x)*b*c**3*x**
2 + 8*int(asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1),x)*b*c +
3*a*c**3*x**3 + 5*a*c*x)/(8*c*d**3*(c**4*x**4 + 2*c**2*x**2 + 1))
```

3.7 $\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 291

$$\begin{aligned}
&\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx \\
&= \frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 \\
&\quad - \frac{32bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{35c} - \frac{16bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{105c} \\
&\quad - \frac{12bd^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))}{175c} - \frac{2bd^3(1+c^2x^2)^{7/2}(a+b\text{arcsinh}(cx))}{49c} \\
&\quad + \frac{16}{35}d^3x(a+b\text{arcsinh}(cx))^2 + \frac{8}{35}d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2 + \frac{6}{35}d^3x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))^2 +
\end{aligned}$$

output
```
4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3+234/6125*b^2*c^4*d^3*x^5+2/
343*b^2*c^6*d^3*x^7-32/35*b*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-16/
105*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-12/175*b*d^3*(c^2*x^2+1)^
(5/2)*(a+b*arcsinh(c*x))/c-2/49*b*d^3*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))
/c+16/35*d^3*x*(a+b*arcsinh(c*x))^2+8/35*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*
x))^2+6/35*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/7*d^3*x*(c^2*x^2+1)^
3*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{d^3\left(11025a^2cx(35+35c^2x^2+21c^4x^4+5c^6x^6)-210ab\sqrt{1+c^2x^2}(2161+757c^2x^2+351c^4x^4+75c^6x^6)+\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 210*a*b*S
qrt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 2*b^2*c
*x*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 210*b*(-105*a*
c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(2161
 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c*x*(
35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]^2))/(385875*c)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6201, 27, 6201, 6201, 6187, 6213, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{6}{7}d\int d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{6}{7}d^3\int \left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 6201

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{6}{7}d^3\left(-\frac{2}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}x\left(c^2x^2+1\right)^2(a\right.$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 6201

$$-\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{6}{7}d^3\left(-\frac{2}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int (a+b\right.\right.$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 6187

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\right.\right.$$
$$\frac{2}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 6213

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+}{3c^2}\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\int \left(c^2x^2+1\right)^3dx}{7c}\right)+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$\Big\downarrow$ 24

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\int \left(c^2x^2+1\right)^3dx}{7c}\right)+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{210}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\int\left(c^6x^6+3c^4x^4+3c^2x^2+1\right)dx}{7c}\right)+$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^2+$$
$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))}{7c^2}-\frac{b\left(\frac{c^6x^7}{7}+\frac{3c^4x^5}{5}+c^2x^3+x\right)}{7c}\right)$$

input

```
Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output

```
(d^3*x*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/7 - (2*b*c*d^3*(-1/7*(b*(x
+ c^2*x^3 + (3*c^4*x^5)/5 + (c^6*x^7)/7))/c + ((1 + c^2*x^2)^(7/2)*(a + b*
ArcSinh[c*x]))/(7*c^2)))/7 + (6*d^3*((x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
])^2)/5 - (2*b*c*(-1/5*(b*(x + (2*c^2*x^3)/3 + (c^4*x^5)/5))/c + ((1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2)))/5 + (4*((x*(1 + c^2*x^2)*(a +
b*ArcSinh[c*x])^2)/3 - (2*b*c*(-1/3*(b*(x + (c^2*x^3)/3))/c + ((1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSinh[c*x])^2
- 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2)))/3))/
5))/7
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{d^3a^2\left(\frac{1}{7}x^7c^7+\frac{3}{5}x^5c^5+x^3c^3+xc\right)+d^3b^2\left(\frac{16\operatorname{arcsinh}(xc)^2xc}{35}+\frac{\left(c^2x^2+1\right)^3\operatorname{arcsinh}(xc)^2xc}{7}+\frac{6\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)^2}{35}+\frac{8\operatorname{arc}}{}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{7}x^7c^7+\frac{3}{5}x^5c^5+x^3c^3+xc\right)+d^3b^2\left(\frac{16\operatorname{arcsinh}(xc)^2xc}{35}+\frac{\left(c^2x^2+1\right)^3\operatorname{arcsinh}(xc)^2xc}{7}+\frac{6\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)^2}{35}+\frac{8\operatorname{arc}}{}\right.}{}$
parts	$d^3a^2\left(\frac{1}{7}c^6x^7+\frac{3}{5}c^4x^5+x^3c^2+x\right)+\frac{d^3b^2\left(\frac{16\operatorname{arcsinh}(xc)^2xc}{35}+\frac{\left(c^2x^2+1\right)^3\operatorname{arcsinh}(xc)^2xc}{7}+\frac{6\operatorname{arcsinh}(xc)^2xc\left(c\right.}{35}\right.}{}$
orering	$\frac{x\left(47625c^8x^8+271212c^6x^6+741678c^4x^4+3539900c^2x^2+128625\right)\left(c^2d\,x^2+d\right)^3\left(a+b\operatorname{arcsinh}(xc)\right)^2}{128625\left(c^2x^2+1\right)^4}-\frac{\left(20250c^8x^8+12\right.}{}$

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^3*a^2*(1/7*x^7*c^7+3/5*x^5*c^5+x^3*c^3+x*c)+d^3*b^2*(16/35*arcsinh(
x*c)^2*x*c+1/7*(c^2*x^2+1)^3*arcsinh(x*c)^2*x*c+6/35*arcsinh(x*c)^2*x*c*(c
^2*x^2+1)^2+8/35*arcsinh(x*c)^2*x*c*(c^2*x^2+1)-32/35*arcsinh(x*c)*(c^2*x^
2+1)^(1/2)+413312/385875*x*c-2/49*(c^2*x^2+1)^(7/2)*arcsinh(x*c)+2/343*x*c
*(c^2*x^2+1)^3+888/42875*x*c*(c^2*x^2+1)^2+30256/385875*x*c*(c^2*x^2+1)-12
/175*arcsinh(x*c)*(c^2*x^2+1)^(5/2)-16/105*arcsinh(x*c)*(c^2*x^2+1)^(3/2))
+2*d^3*a*b*(1/7*arcsinh(x*c)*x^7*c^7+3/5*arcsinh(x*c)*x^5*c^5+arcsinh(x*c)
*x^3*c^3+x*c*arcsinh(x*c)-2161/3675*(c^2*x^2+1)^(1/2)-757/3675*x^2*c^2*(c^
2*x^2+1)^(1/2)-117/1225*x^4*c^4*(c^2*x^2+1)^(1/2)-1/49*x^6*c^6*(c^2*x^2+1)
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.22

$$\int \left(d+c^2dx^2\right)^3\left(a+b\operatorname{arcsinh}(cx)\right)^2\,dx$$

$$=\frac{1125\left(49\,a^2+2\,b^2\right)c^7d^3x^7+189\left(1225\,a^2+78\,b^2\right)c^5d^3x^5+35\left(11025\,a^2+1514\,b^2\right)c^3d^3x^3+105\left(3675\,a^2\right.}{}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d^3*x^7 + 189*(1225*a^2 + 78*b^2)*c^5*
d^3*x^5 + 35*(11025*a^2 + 1514*b^2)*c^3*d^3*x^3 + 105*(3675*a^2 + 4322*b^2
)*c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 + 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^3
*x^3 + 35*b^2*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d
^3*x^7 + 2205*a*b*c^5*d^3*x^5 + 3675*a*b*c^3*d^3*x^3 + 3675*a*b*c*d^3*x -
(75*b^2*c^6*d^3*x^6 + 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 + 2161*b^2
*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d^
3*x^6 + 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 + 2161*a*b*d^3)*sqrt(c^2
*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.80

$$\int \left(d+c^2dx^2\right)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} \frac{a^2c^6d^3x^7}{7}+\frac{3a^2c^4d^3x^5}{5}+a^2c^2d^3x^3+a^2d^3x+\frac{2abc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{2abc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{6abc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{234abc\ldots}{} \\ a^2d^3x \end{cases}$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 + a**2*c**2*d**
3*x**3 + a**2*d**3*x + 2*a*b*c**6*d**3*x**7*asinh(c*x)/7 - 2*a*b*c**5*d**3
*x**6*sqrt(c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asinh(c*x)/5 - 234*a*b
*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 2*a*b*c**2*d**3*x**3*asinh(c*x)
 - 1514*a*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asinh(c*x)
 - 4322*a*b*d**3*sqrt(c**2*x**2 + 1)/(3675*c) + b**2*c**6*d**3*x**7*asinh(
c*x)**2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/49 + 3*b**2*c**4*d**3*x**5*asinh(c*x)**2/5 + 234*b**2*c
**4*d**3*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x
)/1225 + b**2*c**2*d**3*x**3*asinh(c*x)**2 + 1514*b**2*c**2*d**3*x**3/1102
5 - 1514*b**2*c*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3675 + b**2*d**3*
x*asinh(c*x)**2 + 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(c**2*x**2 +
1)*asinh(c*x)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(259) = 518$.

Time = 0.06 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.45

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*c^6*d^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3
*x^5*arcsinh(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (
5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2
 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*(5
*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2
+ 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 12
6*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^6*d^3 + b^2*c^2*d^3*x^3*arcsi
nh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*s
qrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 - 2/375
*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c
^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^
2*c^4*d^3 + a^2*c^2*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sqrt(c^2*x^
2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c
^2)*b^2*c^2*d^3 + b^2*d^3*x*arcsinh(c*x)^2 + 2*b^2*d^3*(x - sqrt(c^2*x^2 +
 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))
*a*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx = \frac{d^3\left(3675asinh(cx)^2\,b^2cx-7350\sqrt{c^2x^2+1}\,asinh(cx)\,b^2+1050asinh(cx)\,ab\,c^7x^7+4410asinh(cx)\,ab\,c^5x^5\right.}{}$$

input
```
int((c^2*d*x^2+d)^3*(a+b*asinh(c*x))^2,x)
```

output
```
(d**3*(3675*asinh(c*x)**2*b**2*c*x - 7350*sqrt(c**2*x**2 + 1)*asinh(c*x)*b
**2 + 1050*asinh(c*x)*a*b*c**7*x**7 + 4410*asinh(c*x)*a*b*c**5*x**5 + 7350
*asinh(c*x)*a*b*c**3*x**3 + 7350*asinh(c*x)*a*b*c*x - 150*sqrt(c**2*x**2 +
 1)*a*b*c**6*x**6 - 702*sqrt(c**2*x**2 + 1)*a*b*c**4*x**4 - 1514*sqrt(c**2
*x**2 + 1)*a*b*c**2*x**2 - 4322*sqrt(c**2*x**2 + 1)*a*b + 3675*int(asinh(c
*x)**2*x**6,x)*b**2*c**7 + 11025*int(asinh(c*x)**2*x**4,x)*b**2*c**5 + 110
25*int(asinh(c*x)**2*x**2,x)*b**2*c**3 + 525*a**2*c**7*x**7 + 2205*a**2*c*
*5*x**5 + 3675*a**2*c**3*x**3 + 3675*a**2*c*x + 7350*b**2*c*x))/(3675*c)
```

3.8 $\int (d+c^2dx^2)^2 (a+b\,\mathrm{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int (d+c^2dx^2)^2 (a+b\,\mathrm{arcsinh}(cx))^2\,dx = {} & \frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5 \\ & -\frac{16bd^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}{15c} \\ & -\frac{8bd^2(1+c^2x^2)^{3/2}(a+b\,\mathrm{arcsinh}(cx))}{45c} \\ & -\frac{2bd^2(1+c^2x^2)^{5/2}(a+b\,\mathrm{arcsinh}(cx))}{25c} \\ & +\frac{8}{15}d^2x(a+b\,\mathrm{arcsinh}(cx))^2 \\ & +\frac{4}{15}d^2x(1+c^2x^2)(a+b\,\mathrm{arcsinh}(cx))^2 \\ & +\frac{1}{5}d^2x(1+c^2x^2)^2(a+b\,\mathrm{arcsinh}(cx))^2\end{aligned}$$

output
```
298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3+2/125*b^2*c^4*d^2*x^5-16/15*b*d^2
*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-8/45*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))/c-2/25*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c+8/15*d^2
*x*(a+b*arcsinh(c*x))^2+4/15*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/5*d^
2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.89

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d^2\left(225a^2cx(15+10c^2x^2+3c^4x^4)-30ab\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4)+2b^2cx(2235+190c^2x^2+2\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x
^4) - 30*b*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*
(149 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*b^2*c*x*(15 + 10*c^2*x^
2 + 3*c^4*x^4)*ArcSinh[c*x]^2))/(3375*c)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6201, 27, 6201, 6187, 6213, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \text{6201}$$
$$-\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}d\int d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$
$$\Big\downarrow \text{27}$$
$$-\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{4}{5}d^2\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6201}$$

$$-\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+$$
$$\frac{4}{5}d^2\left(-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}\int(a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6187}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\int\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}x\right.$$
$$\frac{2}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6213}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{ar}}{3c^2}\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\int\left(c^2x^2+1\right)^2dx}{5c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$
$$b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{24}$$

$$\frac{4}{5}d^2\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\int\left(c^2x^2+1\right)^2dx}{5c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$
$$b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{210}$$

$$\frac{4}{5}d^2\left(-\frac{2}{3}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int\left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\int\left(c^4x^4+2c^2x^2+1\right)dx}{5c}\right)+$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^2+$$
$$\frac{4}{5}d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right.\right.$$
$$\frac{2}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2}-\frac{b\left(\frac{c^4x^5}{5}+\frac{2c^2x^3}{3}+x\right)}{5c}\right)$$

input
```
Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/5 - (2*b*c*d^2*(-1/5*(b*(x
+ (2*c^2*x^3)/3 + (c^4*x^5)/5))/c + ((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x]))/(5*c^2)))/5 + (4*d^2*((x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2
*b*c*(-1/3*(b*(x + (c^2*x^3)/3))/c + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2)))/3))/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{d^2a^2\left(\frac{1}{5}x^5c^5+\frac{2}{3}x^3c^3+xc\right)+b^2d^2\left(\frac{8\operatorname{arcsinh}(xc)^2xc}{15}+\frac{\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)^2}{5}+\frac{4\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)}{15}-\frac{16\operatorname{arcsinh}(xc)}{15}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{5}x^5c^5+\frac{2}{3}x^3c^3+xc\right)+b^2d^2\left(\frac{8\operatorname{arcsinh}(xc)^2xc}{15}+\frac{\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)^2}{5}+\frac{4\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)}{15}-\frac{16\operatorname{arcsinh}(xc)}{15}\right.}{}$
parts	$d^2a^2\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{b^2d^2\left(\frac{8\operatorname{arcsinh}(xc)^2xc}{15}+\frac{\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)^2}{5}+\frac{4\operatorname{arcsinh}(xc)^2xc\left(c^2x^2+1\right)}{15}-\frac{16}{}\right.}{}$
orering	$\frac{x\left(1647c^6x^6+8677c^4x^4+51845c^2x^2+3375\right)\left(c^2d\,x^2+d\right)^2\left(a+b\operatorname{arcsinh}(xc)\right)^2}{3375\left(c^2x^2+1\right)^3}-\frac{\left(324c^6x^6+2035c^4x^4+18450c^2x^2+22\right.}{}$

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^2*a^2*(1/5*x^5*c^5+2/3*x^3*c^3+x*c)+b^2*d^2*(8/15*arcsinh(x*c)^2*x*
c+1/5*arcsinh(x*c)^2*x*c*(c^2*x^2+1)^2+4/15*arcsinh(x*c)^2*x*c*(c^2*x^2+1)
-16/15*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+4144/3375*x*c-2/25*arcsinh(x*c)*(c^2
*x^2+1)^(5/2)+2/125*x*c*(c^2*x^2+1)^2+272/3375*x*c*(c^2*x^2+1)-8/45*arcsin
h(x*c)*(c^2*x^2+1)^(3/2))+2*d^2*a*b*(1/5*arcsinh(x*c)*x^5*c^5+2/3*arcsinh(
x*c)*x^3*c^3+x*c*arcsinh(x*c)-149/225*(c^2*x^2+1)^(1/2)-38/225*x^2*c^2*(c^
2*x^2+1)^(1/2)-1/25*x^4*c^4*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.30

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$

$$= \frac{27\,(25\,a^2+2\,b^2)c^5d^2x^5+10\,(225\,a^2+38\,b^2)c^3d^2x^3+15\,(225\,a^2+298\,b^2)cd^2x+225\,(3\,b^2c^5d^2x^5+10\,b^2c}{}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 + 2*b^2)*c^5*d^2*x^5 + 10*(225*a^2 + 38*b^2)*c^3*d^2*x^
3 + 15*(225*a^2 + 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 + 10*b^2*c^3*d
^2*x^3 + 15*b^2*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^5*d
^2*x^5 + 150*a*b*c^3*d^2*x^3 + 225*a*b*c*d^2*x - (9*b^2*c^4*d^2*x^4 + 38*b
^2*c^2*d^2*x^2 + 149*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
1)) - 30*(9*a*b*c^4*d^2*x^4 + 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2)*sqrt(c^2*x
^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.82

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} \frac{a^2c^4d^2x^5}{5}+\frac{2a^2c^2d^2x^3}{3}+a^2d^2x+\frac{2abc^4d^2x^5\,\text{asinh}\,(cx)}{5}-\frac{2abc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{4abc^2d^2x^3\,\text{asinh}\,(cx)}{3}-\frac{76abcd^2x^2\sqrt{c^2x^2+1}}{225}+ \\ a^2d^2x \end{cases}$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**5/5 + 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
 2*a*b*c**4*d**2*x**5*asinh(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(c**2*x**2 +
 1)/25 + 4*a*b*c**2*d**2*x**3*asinh(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(c**2*
x**2 + 1)/225 + 2*a*b*d**2*x*asinh(c*x) - 298*a*b*d**2*sqrt(c**2*x**2 + 1)
/(225*c) + b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 2*b**2*c**4*d**2*x**5/125
 - 2*b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + 2*b**2*c**2*d
**2*x**3*asinh(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**
2*sqrt(c**2*x**2 + 1)*asinh(c*x)/225 + b**2*d**2*x*asinh(c*x)**2 + 298*b**
2*d**2*x/225 - 298*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c), Ne(c,
 0)), (a**2*d**2*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(190) = 380$.

Time = 0.05 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.14

$$
\begin{aligned}
&\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{5}\,b^2c^4d^2x^5\,\text{arsinh}\left(cx\right)^2 + \frac{1}{5}\,a^2c^4d^2x^5 + \frac{2}{3}\,b^2c^2d^2x^3\,\text{arsinh}\left(cx\right)^2 \\
&\quad + \frac{2}{75}\left(15\,x^5\,\text{arsinh}\left(cx\right) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^4d^2 \\
&\quad - \frac{2}{1125}\left(15\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\left(cx\right) - \frac{9\,c^4x^5 - 20\,c^2x^3 + 120\,x}{c^4}\right. \\
&\quad + \frac{2}{3}\,a^2c^2d^2x^3 + \frac{4}{9}\left(3\,x^3\,\text{arsinh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d^2 \\
&\quad - \frac{4}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\left(cx\right) - \frac{c^2x^3 - 6\,x}{c^2}\right)b^2c^2d^2 \\
&\quad + b^2d^2x\,\text{arsinh}\left(cx\right)^2 + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\left(cx\right)}{c}\right) \\
&\quad + a^2d^2x + \frac{2\left(cx\,\text{arsinh}\left(cx\right) - \sqrt{c^2x^2+1}\right)abd^2}{c}
\end{aligned}
$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*c^4*d^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 + 2/3*b^2*c^2*d^2
*x^3*arcsinh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4
/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d
^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4
 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120
*x)/c^4)*b^2*c^4*d^2 + 2/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 - 4/27*(
3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (
c^2*x^3 - 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2*b^2*d^2*(x
- sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*a*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2 \, dx = \int (a+b\,\mathrm{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2 \,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \left(d + c^2dx^2\right)^2 (a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{d^2\left(225asinh(cx)^2 b^2cx - 450\sqrt{c^2x^2+1}\, asinh(cx)\, b^2 + 90asinh(cx)\, ab\, c^5x^5 + 300asinh(cx)\, ab\, c^3x^3 + 450\right.}{}$$

input
```
int((c^2*d*x^2+d)^2*(a+b*asinh(c*x))^2,x)
```

output
```
(d**2*(225*asinh(c*x)**2*b**2*c*x - 450*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**
2 + 90*asinh(c*x)*a*b*c**5*x**5 + 300*asinh(c*x)*a*b*c**3*x**3 + 450*asinh
(c*x)*a*b*c*x - 18*sqrt(c**2*x**2 + 1)*a*b*c**4*x**4 - 76*sqrt(c**2*x**2 +
 1)*a*b*c**2*x**2 - 298*sqrt(c**2*x**2 + 1)*a*b + 225*int(asinh(c*x)**2*x*
*4,x)*b**2*c**5 + 450*int(asinh(c*x)**2*x**2,x)*b**2*c**3 + 45*a**2*c**5*x
**5 + 150*a**2*c**3*x**3 + 225*a**2*c*x + 450*b**2*c*x))/(225*c)
```

3.9 $\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3 - \frac{4bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c} - \frac{2bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{9c} + \frac{2}{3}dx(a+b\text{arcsinh}(cx))^2 + \frac{1}{3}dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^2$$

output
```
14/9*b^2*d*x+2/27*b^2*c^2*d*x^3-4/3*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x
))/c-2/9*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c+2/3*d*x*(a+b*arcsinh(c
*x))^2+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d\left(9a^2cx(3+c^2x^2)-6ab\sqrt{1+c^2x^2}(7+c^2x^2)+2b^2cx(21+c^2x^2)-6b\left(-3acx(3+c^2x^2)+b\sqrt{1+c^2x^2}(\right.\right.}{27c}$$

input
```
Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(9*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^
2*c*x*(21 + c^2*x^2) - 6*b*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(
7 + c^2*x^2))*ArcSinh[c*x] + 9*b^2*c*x*(3 + c^2*x^2)*ArcSinh[c*x]^2))/(27*
c)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6201, 6187, 6213, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{2}{3}bcd\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{2}{3}d\int (a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6187}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx\right)-\frac{2}{3}bcd\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6213}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)\right)-\frac{2}{3}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{24}$$

$$-\frac{2}{3}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\int \left(c^2x^2+1\right)dx}{3c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^2+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)\right)-\frac{2}{3}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}-\frac{b\left(\frac{c^2x^3}{3}+x\right)}{3c}\right)$$

input
```
Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*b*c*d*(-1/3*(b*(x + (c^2
*x^3)/3))/c + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2)))/3 + (2*
d*(x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/c^2)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{a^2d(\frac{1}{3}x^3c^3+xc)+b^2d\left(\frac{2\operatorname{arcsinh}(xc)^2xc}{3}+\frac{\operatorname{arcsinh}(xc)^2xc(c^2x^2+1)}{3}-\frac{4\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}}{3}+\frac{40xc}{27}-\frac{2\operatorname{arcsinh}(xc)(c^2x^2+1)}{9}\right.}{c}$
default	$\frac{a^2d(\frac{1}{3}x^3c^3+xc)+b^2d\left(\frac{2\operatorname{arcsinh}(xc)^2xc}{3}+\frac{\operatorname{arcsinh}(xc)^2xc(c^2x^2+1)}{3}-\frac{4\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}}{3}+\frac{40xc}{27}-\frac{2\operatorname{arcsinh}(xc)(c^2x^2+1)}{9}\right.}{c}$
parts	$a^2d\left(\frac{1}{3}x^3c^2+x\right)+\frac{b^2d\left(\frac{2\operatorname{arcsinh}(xc)^2xc}{3}+\frac{\operatorname{arcsinh}(xc)^2xc(c^2x^2+1)}{3}-\frac{4\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}}{3}+\frac{40xc}{27}-\frac{2\operatorname{arcsinh}(xc)(c}{9}\right.}{c}$
orering	$\frac{x(19c^4x^4+166c^2x^2+27)(c^2dx^2+d)(a+b\operatorname{arcsinh}(xc))^2}{27(c^2x^2+1)^2}-\frac{(2c^4x^4+29c^2x^2+7)\left(2c^2dx(a+b\operatorname{arcsinh}(xc))^2+\frac{2(c^2dx^2}{}\right.}{9c^2(c^2x^2+1)}$

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*d*(1/3*x^3*c^3+x*c)+b^2*d*(2/3*arcsinh(x*c)^2*x*c+1/3*arcsinh(x*c
)^2*x*c*(c^2*x^2+1)-4/3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+40/27*x*c-2/9*arcsi
nh(x*c)*(c^2*x^2+1)^(3/2)+2/27*x*c*(c^2*x^2+1))+2*a*b*d*(1/3*arcsinh(x*c)*
x^3*c^3+x*c*arcsinh(x*c)-1/9*x^2*c^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\int (d+c^2dx^2)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{(9a^2+2b^2)c^3dx^3+3(9a^2+14b^2)cdx+9(b^2c^3dx^3+3b^2cdx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3abc^3dx^3+9\right.}{27c}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 + 2*b^2)*c^3*d*x^3 + 3*(9*a^2 + 14*b^2)*c*d*x + 9*(b^2*c^3*d*
x^3 + 3*b^2*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^3*d*x^3 + 9
*a*b*c*d*x - (b^2*c^2*d*x^2 + 7*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c
^2*x^2 + 1)) - 6*(a*b*c^2*d*x^2 + 7*a*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.79

$$\int \left(d + c^2dx^2\right)\left(a + b\text{arcsinh}(cx)\right)^2 dx$$

$$= \begin{cases} \frac{a^2c^2dx^3}{3} + a^2dx + \frac{2abc^2dx^3\operatorname{asinh}(cx)}{3} - \frac{2abcdx^2\sqrt{c^2x^2+1}}{9} + 2abdx\operatorname{asinh}(cx) - \frac{14abd\sqrt{c^2x^2+1}}{9c} + \frac{b^2c^2dx^3\operatorname{asinh}^2(cx)}{3} + \\ a^2dx \end{cases}$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*c**2*d*x**3/3 + a**2*d*x + 2*a*b*c**2*d*x**3*asinh(c*x)/3
- 2*a*b*c*d*x**2*sqrt(c**2*x**2 + 1)/9 + 2*a*b*d*x*asinh(c*x) - 14*a*b*d*s
qrt(c**2*x**2 + 1)/(9*c) + b**2*c**2*d*x**3*asinh(c*x)**2/3 + 2*b**2*c**2*
d*x**3/27 - 2*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/9 + b**2*d*x*as
inh(c*x)**2 + 14*b**2*d*x/9 - 14*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*
c), Ne(c, 0)), (a**2*d*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(109) = 218$.

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.84

$$\begin{aligned}\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{3}\,b^2c^2dx^3\,\text{arsinh}\,(cx)^2 + \frac{1}{3}\,a^2c^2dx^3 \\ &+ \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d \\ &- \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2c^2d \\ &+ b^2dx\,\text{arsinh}\,(cx)^2 + 2\,b^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) \\ &+ a^2dx + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd}{c}\end{aligned}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*arcsinh(
c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d
- 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(
c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsinh(c*x)^2 + 2*b^2*d*(
x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*a*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{d\left(9asinh(cx)^2b^2cx-18\sqrt{c^2x^2+1}\,asinh(cx)\,b^2+6asinh(cx)\,ab\,c^3x^3+18asinh(cx)\,abcx-2\sqrt{c^2x^2+1}\,a\right.}{9c}$$

input
```
int((c^2*d*x^2+d)*(a+b*asinh(c*x))^2,x)
```

output
```
(d*(9*asinh(c*x)**2*b**2*c*x - 18*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2 + 6*
asinh(c*x)*a*b*c**3*x**3 + 18*asinh(c*x)*a*b*c*x - 2*sqrt(c**2*x**2 + 1)*a
*b*c**2*x**2 - 14*sqrt(c**2*x**2 + 1)*a*b + 9*int(asinh(c*x)**2*x**2,x)*b*
*2*c**3 + 3*a**2*c**3*x**3 + 9*a**2*c*x + 18*b**2*c*x))/(9*c)
```

3.10 $\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2} dx = \frac{2(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib(a+b\text{arcsinh}(cx))\operatorname{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{2ib^2\operatorname{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{2ib^2\operatorname{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

output
```
2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-2*I*b*(a+b*arcsin
h(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b*(a+b*arcsinh(c*x))
*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b^2*polylog(3,-I*(c*x+(c^2*x
^2+1)^(1/2)))/c/d-2*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.99

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx = -\frac{c\left(a^2\sqrt{-c^2}\arctan(cx)-2abc\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)-b^2c\text{arcsinh}(cx)^2\log\left(1+\frac{ce^{\text{arcsinh}(cx)}}{\sqrt{-c^2}}\right)\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

output
```
-((c*(a^2*Sqrt[-c^2]*ArcTan[c*x] - 2*a*b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSi
nh[c*x])/Sqrt[-c^2]] - b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 2*a*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b
^2*c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b*c*(a + b*
ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 2*b*c*(a + b*Arc
Sinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b^2*c*PolyLog[3,
(c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 2*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[
c*x])/c]))/((-c^2)^(3/2)*d))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6204, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2dx^2+d}dx$$

$$\Big\downarrow \text{ 6204}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (a+b\text{arcsinh}(cx))^2 \csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{cd}$$

$\Big\downarrow$ 4668

$$\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcs}}{cd}$$

$\Big\downarrow$ 3011

$$\frac{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyL}\right.}{}$$

$\Big\downarrow$ 2720

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$

$\Big\downarrow$ 7143

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+ba\right.}{cd}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

output
```
(2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSi
nh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*
x]]) - (2*I)*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*P
olyLog[3, I*E^ArcSinh[c*x]]))/(c*d)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(xc))^2}{c^2 d x^2 + d} dx$$

input
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{c^2dx^2 + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^2 + d),
x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = \frac{\int \frac{a^2}{c^2x^2+1}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**2/(c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**
2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{c^2dx^2 + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
a^2*arctan(c*x)/(c*d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*
d*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2}{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+c^2dx^2}\,dx=\frac{atan(cx)\,a^2+2\left(\int \frac{asinh(cx)}{c^2x^2+1}dx\right)abc+\left(\int \frac{asinh(cx)^2}{c^2x^2+1}dx\right)b^2c}{cd}$$

input
```
int((a+b*asinh(c*x))^2/(c^2*d*x^2+d),x)
```

output
```
(atan(c*x)*a**2 + 2*int(asinh(c*x)/(c**2*x**2 + 1),x)*a*b*c + int(asinh(c*
x)**2/(c**2*x**2 + 1),x)*b**2*c)/(c*d)
```

3.11 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 210

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx &= \frac{b(a+b\text{arcsinh}(cx))}{cd^2\sqrt{1+c^2x^2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(1+c^2x^2)}\\&\quad+\frac{(a+b\text{arcsinh}(cx))^2\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2}-\frac{b^2\arctan(cx)}{cd^2}\\&\quad-\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\&\quad+\frac{ib(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\&\quad+\frac{ib^2\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\&\quad-\frac{ib^2\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\end{aligned}$$

output
```
b*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2/d^
2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^2-b^2
*arctan(c*x)/c/d^2-I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1
/2)))/c/d^2+I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/
d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2-I*b^2*polylog(3,I*(c
*x+(c^2*x^2+1)^(1/2)))/c/d^2
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$

$$= \frac{\frac{a^2x}{1+c^2x^2}+\frac{a^2\arctan(cx)}{c}+\frac{2ab\left(\sqrt{1+c^2x^2}+cx\text{arcsinh}(cx)+i\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(cx)}\right)+ic^2x^2\text{arcsinh}(cx)\log\left(1-ie^{\text{arcsinh}(\ldots}\right.\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

output
```
((a^2*x)/(1 + c^2*x^2) + (a^2*ArcTan[c*x])/c + (2*a*b*(Sqrt[1 + c^2*x^2] +
 c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*c^2*x^2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I*E^ArcSinh
[c*x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1 + c^2*x^2
)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh
[c*x]]))/(c + c^3*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x^2] + (c*x*Arc
Sinh[c*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
+ ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^Ar
cSinh[c*x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c
*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*P
olyLog[3, I/E^ArcSinh[c*x]])))/c)/(2*d^2)
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6203, 27, 6204, 3042, 4668, 3011, 2720, 6213, 216, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^2}\,dx$$

$$\Big\downarrow \; 6203$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{d(c^2x^2+1)}dx}{2d}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6204}$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{\int (a+b\text{arcsinh}(cx))^2\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-2ib\int (a+b\text{arcsinh}(cx))\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2ib\int (a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcs}}{2cd^2}$$

$$\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib\left(b\int \text{PolyL}\right.}{}$$

$$\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$

$$\frac{bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6213}$$

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$

$$\frac{bc\left(\frac{b\int\frac{1}{c^2x^2+1}dx}{c}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$\big\downarrow$ 216

$$\frac{2ib\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))\right)-2ib}{}$$

$$\frac{bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{d^2}+\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

$\big\downarrow$ 7143

$$-\frac{bc\left(\frac{b\arctan(cx)}{c^2}-\frac{a+b\text{arcsinh}(cx)}{c^2\sqrt{c^2x^2+1}}\right)}{d^2}+$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+ba\right.}{2cd^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{2d^2(c^2x^2+1)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*x^2)) - (b*c*(-((a + b*ArcSinh[
c*x])/(c^2*Sqrt[1 + c^2*x^2])) + (b*ArcTan[c*x])/c^2))/d^2 + (2*(a + b*Arc
Sinh[c*x])^2*ArcTan[E^ArcSinh[c*x]] + (2*I)*b*(-((a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]]) + b*PolyLog[3, (-I)*E^ArcSinh[c*x]]) - (2*I)*
b*(-((a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]]) + b*PolyLog[3, I*E
^ArcSinh[c*x]]))/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(xc))^2}{(c^2 d x^2 + d)^2} dx$$

input
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2\,\text{asinh}^2\,(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2ab\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x
)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**4*x
**4 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^2*log(
c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 2*a*b*log
(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^2}\,dx$$
$$=\frac{atan(cx)\,a^2c^2x^2+atan(cx)\,a^2+4\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)ab\,c^3x^2+4\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)abc+2\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2}\right.}{2c\,d^2\,(c^2x^2+1)}$$

input
```
int((a+b*asinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

output
```
(atan(c*x)*a**2*c**2*x**2 + atan(c*x)*a**2 + 4*int(asinh(c*x)/(c**4*x**4 +
 2*c**2*x**2 + 1),x)*a*b*c**3*x**2 + 4*int(asinh(c*x)/(c**4*x**4 + 2*c**2*
x**2 + 1),x)*a*b*c + 2*int(asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1),x)*
b**2*c**3*x**2 + 2*int(asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b**2
*c + a**2*c*x)/(2*c*d**2*(c**2*x**2 + 1))
```

3.12 $\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 435

$$\int (d+c^2dx^2)^3 (a+b\text{arcsinh}(cx))^3 \, dx = -\frac{413312b^3d^3\sqrt{1+c^2x^2}}{128625c} - \frac{30256b^3d^3(1+c^2x^2)^{3/2}}{385875c} - \frac{2664b^3d^3(1+c^2x^2)^{5/2}}{214375c} - \frac{6b^3d^3(1+c^2x^2)^{7/2}}{2401c} + \frac{4322b^2d^3x(a+b\text{arcsinh}(cx))}{1225} + \frac{1514b^2c^2d^3x^3(a+b\text{arcsinh}(cx))}{3675} + \frac{702b^2c^4d^3x^5(a+b\text{arcsinh}(cx))}{6125} + \frac{6}{343}b^2c^6d^3x^7(a+b\text{arcsinh}(cx)) - \frac{48bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{35c} - \frac{8bd^3(1+c^2x^2)^{3/2}(a+b\text{arcsinh}}{35c}$$

output
```
-413312/128625*b^3*d^3*(c^2*x^2+1)^(1/2)/c-30256/385875*b^3*d^3*(c^2*x^2+1
)^(3/2)/c-2664/214375*b^3*d^3*(c^2*x^2+1)^(5/2)/c-6/2401*b^3*d^3*(c^2*x^2+
1)^(7/2)/c+4322/1225*b^2*d^3*x*(a+b*arcsinh(c*x))+1514/3675*b^2*c^2*d^3*x^
3*(a+b*arcsinh(c*x))+702/6125*b^2*c^4*d^3*x^5*(a+b*arcsinh(c*x))+6/343*b^2
*c^6*d^3*x^7*(a+b*arcsinh(c*x))-48/35*b*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh
(c*x))^2/c-8/35*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c-18/175*b*d^
3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c-3/49*b*d^3*(c^2*x^2+1)^(7/2)*(a
+b*arcsinh(c*x))^2/c+16/35*d^3*x*(a+b*arcsinh(c*x))^3+8/35*d^3*x*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^3+6/35*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^3+1/7*
d^3*x*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^3
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.94

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^3\,dx$$
$$=\frac{d^3\left(385875a^3cx(35+35c^2x^2+21c^4x^4+5c^6x^6)-11025a^2b\sqrt{1+c^2x^2}(2161+757c^2x^2+351c^4x^4+75c^6\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^3,x]
```

output
```
(d^3*(385875*a^3*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 11025*a^
2*b*Sqrt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 21
0*a*b^2*c*x*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 2*b^3
*Sqrt[1 + c^2*x^2]*(22329151 + 747937*c^2*x^2 + 134541*c^4*x^4 + 16875*c^6
*x^6) + 105*b*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) -
210*a*b*Sqrt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6)
+ 2*b^2*c*x*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6))*ArcSin
h[c*x] - 11025*b^2*(-105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)
+ b*Sqrt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*Arc
Sinh[c*x]^2 + 385875*b^3*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*Ar
cSinh[c*x]^3))/(13505625*c)
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {6201, 27, 6201, 6201, 6187, 6213, 2009, 6199, 27, 353, 53, 1576, 1140, 2009, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^3 (a+b\text{arcsinh}(cx))^3\,dx$$
$$\Big\downarrow \text{6201}$$

$$-\frac{3}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx+\frac{6}{7}d\int d^2\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx+\frac{6}{7}d^3\int\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx+\frac{6}{7}d^3\left(-\frac{3}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{4}{5}\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3dx+\frac{1}{5}x\left(c^2x^2+1\right)^2(a\right.$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx+\frac{6}{7}d^3\left(-\frac{3}{5}bc\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{4}{5}\left(-bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\frac{2}{3}\int(a+b\right.\right.$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{6187}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3bc\int\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx\right)-bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\right.\right.$$
$$\frac{3}{7}bcd^3\int x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{6213}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arcsinh}(cx))dx}{c}\right)\right)-bc\left(\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\int\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))dx}{7c}\right)+$$
$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\int(c^2x^2+1)(a+b\text{arcsinh}(cx))dx}{3c}\right)+\frac{1}{3}x(c^2x^2+1)(a+\right.\right.$$
$$\frac{3}{7}bcd^3\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\int(c^2x^2+1)^3(a+b\text{arcsinh}(cx))dx}{7c}\right)+$$
$$\frac{1}{7}d^3x(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 6199

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-bc\int\frac{x(c^2x^2+3)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+ba\right.}{3c}\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-bc\int\frac{x(5c^6x^6+21c^4x^4+35c^2x^2+35)}{35\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))+\right.}{}\right.$$
$$\frac{1}{7}d^3x(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 27

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{3}bc\int\frac{x(c^2x^2+3)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+\right.}{3c}\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{35}bc\int\frac{x(5c^6x^6+21c^4x^4+35c^2x^2+35)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))\right.}{}\right.$$
$$\frac{1}{7}d^3x(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 353

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+\right.}{3c}\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(\frac{(c^2x^2+1)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{35}bc\int\frac{x(5c^6x^6+21c^4x^4+35c^2x^2+35)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))\right.}{}\right.$$
$$\frac{1}{7}d^3x(c^2x^2+1)^3(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 53

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsin}\right.}{3c}\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{35}bc\int\frac{x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))\right.}{}\right.$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{1576}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsin}\right.}{3c}\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{35}bc\int\frac{x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))\right.}{}\right.$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{1140}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsin}\right.}{3c}\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{35}bc\int\frac{x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))\right.}{}\right.$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{35}bc\int\frac{x\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx\right.}{}\right.$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3\right.\right.\right.$$

$$\Big\downarrow \text{2331}$$

$$-\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{70}bc\int\frac{5c^6x^6+21c^4x^4+35c^2x^2+35}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))\right.}{}\right.$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3\right.\right.\right.$$

$\downarrow$ 2389

$$-\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{70}bc\int\left(5\left(c^2x^2+1\right)^{5/2}+6\left(c^2x^2+1\right)^{3/2}+8\sqrt{c^2x^2+1}+\frac{}{\sqrt{c}}\right.\right.}{}\right.$$

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3\right.\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(\frac{\left(c^2x^2+1\right)^{7/2}(a+b\text{arcsinh}(cx))^2}{7c^2}-\frac{2b\left(\frac{1}{7}c^6x^7(a+b\text{arcsinh}(cx))+\frac{3}{5}c^4x^5(a+b\text{arcsinh}(cx))+c^2x^3(a+\right.}{}\right.$$

input
```
Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^3,x]
```

output

```
(d^3*x*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^3)/7 - (3*b*c*d^3*(((1 + c^2*x
^2)^(7/2)*(a + b*ArcSinh[c*x])^2)/(7*c^2) - (2*b*(-1/70*(b*c*((32*Sqrt[1 +
 c^2*x^2])/c^2 + (16*(1 + c^2*x^2)^(3/2))/(3*c^2) + (12*(1 + c^2*x^2)^(5/2
))/(5*c^2) + (10*(1 + c^2*x^2)^(7/2))/(7*c^2))) + x*(a + b*ArcSinh[c*x]) +
 c^2*x^3*(a + b*ArcSinh[c*x]) + (3*c^4*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*
x^7*(a + b*ArcSinh[c*x]))/7))/(7*c)))/7 + (6*d^3*((x*(1 + c^2*x^2)^2*(a +
b*ArcSinh[c*x])^3)/5 - (3*b*c*(((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2
)/(5*c^2) - (2*b*(-1/30*(b*c*((16*Sqrt[1 + c^2*x^2])/c^2 + (8*(1 + c^2*x^2
)^(3/2))/(3*c^2) + (6*(1 + c^2*x^2)^(5/2))/(5*c^2))) + x*(a + b*ArcSinh[c*
x]) + (2*c^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*x^5*(a + b*ArcSinh[c*x]))/
5))/(5*c)))/5 + (4*((x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^3)/3 - b*c*(((1
+ c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2) - (2*b*(-1/6*(b*c*((4*Sqr
t[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSinh
[c*x]) + (c^2*x^3*(a + b*ArcSinh[c*x]))/3))/(3*c)) + (2*(x*(a + b*ArcSinh[
c*x])^3 - 3*b*c*((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c^2 - (2*b*(a*
x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]))/c)))/3))/5))/7
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{d^3a^3\left(\frac{1}{7}x^7c^7+\frac{3}{5}x^5c^5+x^3c^3+xc\right)+d^3b^3\left(\frac{16\operatorname{arcsinh}(xc)^3xc}{35}+\frac{\left(c^2x^2+1\right)^3\operatorname{arcsinh}(xc)^3xc}{7}+\frac{6\operatorname{arcsinh}(xc)^3xc\left(c^2x^2+1\right)^2}{35}+\frac{8\operatorname{arc}}{}\right.}{}$
default	$\frac{d^3a^3\left(\frac{1}{7}x^7c^7+\frac{3}{5}x^5c^5+x^3c^3+xc\right)+d^3b^3\left(\frac{16\operatorname{arcsinh}(xc)^3xc}{35}+\frac{\left(c^2x^2+1\right)^3\operatorname{arcsinh}(xc)^3xc}{7}+\frac{6\operatorname{arcsinh}(xc)^3xc\left(c^2x^2+1\right)^2}{35}+\frac{8\operatorname{arc}}{}\right.}{}$
parts	$d^3a^3\left(\frac{1}{7}c^6x^7+\frac{3}{5}c^4x^5+x^3c^2+x\right)+\frac{d^3b^3\left(\frac{16\operatorname{arcsinh}(xc)^3xc}{35}+\frac{\left(c^2x^2+1\right)^3\operatorname{arcsinh}(xc)^3xc}{7}+\frac{6\operatorname{arcsinh}(xc)^3xc\left(c^{}\right.}{35}\right.}{}$
orering	$\frac{x\left(6215625c^8x^8+37489212c^6x^6+126346014c^4x^4+1949470892c^2x^2-879660415\right)\left(c^2d\,x^2+d\right)^3\left(a+b\operatorname{arcsinh}(xc)\right)^3}{13505625\left(c^2x^2+1\right)^4}-$

input
```
int((c^2*d*x^2+d)^3*(a+b*arcsinh(x*c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^3*a^3*(1/7*x^7*c^7+3/5*x^5*c^5+x^3*c^3+x*c)+d^3*b^3*(16/35*arcsinh(
x*c)^3*x*c+1/7*(c^2*x^2+1)^3*arcsinh(x*c)^3*x*c+6/35*arcsinh(x*c)^3*x*c*(c
^2*x^2+1)^2+8/35*arcsinh(x*c)^3*x*c*(c^2*x^2+1)-48/35*arcsinh(x*c)^2*(c^2*
x^2+1)^(1/2)+413312/128625*x*c*arcsinh(x*c)-413312/128625*(c^2*x^2+1)^(1/2
)-3/49*(c^2*x^2+1)^(7/2)*arcsinh(x*c)^2+6/343*arcsinh(x*c)*x*c*(c^2*x^2+1)
^3+2664/42875*arcsinh(x*c)*x*c*(c^2*x^2+1)^2+30256/128625*arcsinh(x*c)*x*c
*(c^2*x^2+1)-6/2401*(c^2*x^2+1)^(7/2)-2664/214375*(c^2*x^2+1)^(5/2)-30256/
385875*(c^2*x^2+1)^(3/2)-18/175*arcsinh(x*c)^2*(c^2*x^2+1)^(5/2)-8/35*arcs
inh(x*c)^2*(c^2*x^2+1)^(3/2))+3*d^3*a*b^2*(16/35*arcsinh(x*c)^2*x*c+1/7*(c
^2*x^2+1)^3*arcsinh(x*c)^2*x*c+6/35*arcsinh(x*c)^2*x*c*(c^2*x^2+1)^2+8/35*
arcsinh(x*c)^2*x*c*(c^2*x^2+1)-32/35*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+413312
/385875*x*c-2/49*(c^2*x^2+1)^(7/2)*arcsinh(x*c)+2/343*x*c*(c^2*x^2+1)^3+88
8/42875*x*c*(c^2*x^2+1)^2+30256/385875*x*c*(c^2*x^2+1)-12/175*arcsinh(x*c)
*(c^2*x^2+1)^(5/2)-16/105*arcsinh(x*c)*(c^2*x^2+1)^(3/2))+3*d^3*a^2*b*(1/7
*arcsinh(x*c)*x^7*c^7+3/5*arcsinh(x*c)*x^5*c^5+arcsinh(x*c)*x^3*c^3+x*c*ar
csinh(x*c)-2161/3675*(c^2*x^2+1)^(1/2)-757/3675*x^2*c^2*(c^2*x^2+1)^(1/2)-
117/1225*x^4*c^4*(c^2*x^2+1)^(1/2)-1/49*x^6*c^6*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.34

$$\int (d + c^2dx^2)^3 (a + b\text{arcsinh}(cx))^3\,dx$$
$$= \frac{39375\,(49\,a^3 + 6\,ab^2)c^7d^3x^7 + 6615\,(1225\,a^3 + 234\,ab^2)c^5d^3x^5 + 3675\,(3675\,a^3 + 1514\,ab^2)c^3d^3x^3 + 1102}{}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^3,x, algorithm="fricas")
```

output
```
1/13505625*(39375*(49*a^3 + 6*a*b^2)*c^7*d^3*x^7 + 6615*(1225*a^3 + 234*a*
b^2)*c^5*d^3*x^5 + 3675*(3675*a^3 + 1514*a*b^2)*c^3*d^3*x^3 + 11025*(1225*
a^3 + 4322*a*b^2)*c*d^3*x + 385875*(5*b^3*c^7*d^3*x^7 + 21*b^3*c^5*d^3*x^5
 + 35*b^3*c^3*d^3*x^3 + 35*b^3*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^3 + 1
1025*(525*a*b^2*c^7*d^3*x^7 + 2205*a*b^2*c^5*d^3*x^5 + 3675*a*b^2*c^3*d^3*
x^3 + 3675*a*b^2*c*d^3*x - (75*b^3*c^6*d^3*x^6 + 351*b^3*c^4*d^3*x^4 + 757
*b^3*c^2*d^3*x^2 + 2161*b^3*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 105*(1125*(49*a^2*b + 2*b^3)*c^7*d^3*x^7 + 189*(1225*a^2*b + 78
*b^3)*c^5*d^3*x^5 + 35*(11025*a^2*b + 1514*b^3)*c^3*d^3*x^3 + 105*(3675*a^
2*b + 4322*b^3)*c*d^3*x - 210*(75*a*b^2*c^6*d^3*x^6 + 351*a*b^2*c^4*d^3*x^
4 + 757*a*b^2*c^2*d^3*x^2 + 2161*a*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) - (16875*(49*a^2*b + 2*b^3)*c^6*d^3*x^6 + 81*(47775*a^2*
b + 3322*b^3)*c^4*d^3*x^4 + (8345925*a^2*b + 1495874*b^3)*c^2*d^3*x^2 + (2
3825025*a^2*b + 44658302*b^3)*d^3)*sqrt(c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(422) = 844$.

Time = 1.31 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.23

$$\int \left(d + c^2dx^2\right)^3 (a + b\text{arcsinh}(cx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**3,x)
```

output

```
Piecewise((a**3*c**6*d**3*x**7/7 + 3*a**3*c**4*d**3*x**5/5 + a**3*c**2*d**
3*x**3 + a**3*d**3*x + 3*a**2*b*c**6*d**3*x**7*asinh(c*x)/7 - 3*a**2*b*c**
5*d**3*x**6*sqrt(c**2*x**2 + 1)/49 + 9*a**2*b*c**4*d**3*x**5*asinh(c*x)/5
- 351*a**2*b*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 3*a**2*b*c**2*d**3*
x**3*asinh(c*x) - 757*a**2*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/1225 + 3*a**2
*b*d**3*x*asinh(c*x) - 2161*a**2*b*d**3*sqrt(c**2*x**2 + 1)/(1225*c) + 3*a
*b**2*c**6*d**3*x**7*asinh(c*x)**2/7 + 6*a*b**2*c**6*d**3*x**7/343 - 6*a*b
**2*c**5*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/49 + 9*a*b**2*c**4*d**3*
x**5*asinh(c*x)**2/5 + 702*a*b**2*c**4*d**3*x**5/6125 - 702*a*b**2*c**3*d*
*3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/1225 + 3*a*b**2*c**2*d**3*x**3*asin
h(c*x)**2 + 1514*a*b**2*c**2*d**3*x**3/3675 - 1514*a*b**2*c*d**3*x**2*sqrt
(c**2*x**2 + 1)*asinh(c*x)/1225 + 3*a*b**2*d**3*x*asinh(c*x)**2 + 4322*a*b
**2*d**3*x/1225 - 4322*a*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1225*c)
 + b**3*c**6*d**3*x**7*asinh(c*x)**3/7 + 6*b**3*c**6*d**3*x**7*asinh(c*x)/
343 - 3*b**3*c**5*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/49 - 6*b**3*
c**5*d**3*x**6*sqrt(c**2*x**2 + 1)/2401 + 3*b**3*c**4*d**3*x**5*asinh(c*x)
**3/5 + 702*b**3*c**4*d**3*x**5*asinh(c*x)/6125 - 351*b**3*c**3*d**3*x**4*
sqrt(c**2*x**2 + 1)*asinh(c*x)**2/1225 - 29898*b**3*c**3*d**3*x**4*sqrt(c*
*2*x**2 + 1)/1500625 + b**3*c**2*d**3*x**3*asinh(c*x)**3 + 1514*b**3*c**2*
d**3*x**3*asinh(c*x)/3675 - 757*b**3*c*d**3*x**2*sqrt(c**2*x**2 + 1)*as...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(387) = 774$.

Time = 0.09 (sec) , antiderivative size = 1350, normalized size of antiderivative = 3.10

$$\int \left(d + c^2dx^2\right)^3 (a + b\text{arcsinh}(cx))^3\, dx = \text{Too large to display}$$

input

```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^3,x, algorithm="maxima")
```

output
```
1/7*b^3*c^6*d^3*x^7*arcsinh(c*x)^3 + 3/7*a*b^2*c^6*d^3*x^7*arcsinh(c*x)^2
+ 1/7*a^3*c^6*d^3*x^7 + 3/5*b^3*c^4*d^3*x^5*arcsinh(c*x)^3 + 9/5*a*b^2*c^4
*d^3*x^5*arcsinh(c*x)^2 + 3/5*a^3*c^4*d^3*x^5 + b^3*c^2*d^3*x^3*arcsinh(c*
x)^3 + 3/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(
c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/
c^8)*c)*a^2*b*c^6*d^3 - 2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(
c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/
c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^
6)*a*b^2*c^6*d^3 - 1/900375*(11025*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c
^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c
^8)*c*arcsinh(c*x)^2 + 2*c*((1125*sqrt(c^2*x^2 + 1)*c^4*x^6 - 3996*sqrt(c^
2*x^2 + 1)*c^2*x^4 + 15128*sqrt(c^2*x^2 + 1)*x^2 - 206656*sqrt(c^2*x^2 + 1
)/c^2)/c^6 - 105*(75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)*arcsinh
(c*x)/c^7))*b^3*c^6*d^3 + 3*a*b^2*c^2*d^3*x^3*arcsinh(c*x)^2 + 3/25*(15*x^
5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^
4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a^2*b*c^4*d^3 - 2/125*(15*(3*sqrt(c^2*x^2
+ 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*ar
csinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*a*b^2*c^4*d^3 - 1/1875*
(225*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c
^2*x^2 + 1)/c^6)*c*arcsinh(c*x)^2 + 2*c*((27*sqrt(c^2*x^2 + 1)*c^2*x^4 ...
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^3\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)^3\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))^3*(d + c^2*d*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))^3*(d + c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^3 (a+b\text{arcsinh}(cx))^3\,dx$$
$$=\frac{d^3\left(525asinh(cx)\,a^2b\,c^7x^7+2205asinh(cx)\,a^2b\,c^5x^5+3675asinh(cx)\,a^2b\,c^3x^3-75\sqrt{c^2x^2+1}\,a^2b\,c^6x^6-\cdots\right.}{}$$

input
```
int((c^2*d*x^2+d)^3*(a+b*asinh(c*x))^3,x)
```

output
```
(d**3*(1225*asinh(c*x)**3*b**3*c*x - 3675*sqrt(c**2*x**2 + 1)*asinh(c*x)**
2*b**3 + 3675*asinh(c*x)**2*a*b**2*c*x - 7350*sqrt(c**2*x**2 + 1)*asinh(c*
x)*a*b**2 + 525*asinh(c*x)*a**2*b*c**7*x**7 + 2205*asinh(c*x)*a**2*b*c**5*
x**5 + 3675*asinh(c*x)*a**2*b*c**3*x**3 + 3675*asinh(c*x)*a**2*b*c*x + 735
0*asinh(c*x)*b**3*c*x - 75*sqrt(c**2*x**2 + 1)*a**2*b*c**6*x**6 - 351*sqrt
(c**2*x**2 + 1)*a**2*b*c**4*x**4 - 757*sqrt(c**2*x**2 + 1)*a**2*b*c**2*x**
2 - 2161*sqrt(c**2*x**2 + 1)*a**2*b - 7350*sqrt(c**2*x**2 + 1)*b**3 + 1225
*int(asinh(c*x)**3*x**6,x)*b**3*c**7 + 3675*int(asinh(c*x)**3*x**4,x)*b**3
*c**5 + 3675*int(asinh(c*x)**3*x**2,x)*b**3*c**3 + 3675*int(asinh(c*x)**2*
x**6,x)*a*b**2*c**7 + 11025*int(asinh(c*x)**2*x**4,x)*a*b**2*c**5 + 11025*
int(asinh(c*x)**2*x**2,x)*a*b**2*c**3 + 175*a**3*c**7*x**7 + 735*a**3*c**5
*x**5 + 1225*a**3*c**3*x**3 + 1225*a**3*c*x + 7350*a*b**2*c*x))/(1225*c)
```

3.13 $\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\begin{aligned}\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^3 \, dx &= -\frac{4144b^3d^2\sqrt{1+c^2x^2}}{1125c}-\frac{272b^3d^2(1+c^2x^2)^{3/2}}{3375c}\\&-\frac{6b^3d^2(1+c^2x^2)^{5/2}}{625c}+\frac{298}{75}b^2d^2x(a+b\text{arcsinh}(cx))+\frac{76}{225}b^2c^2d^2x^3(a+b\text{arcsinh}(cx))\\&+\frac{6}{125}b^2c^4d^2x^5(a+b\text{arcsinh}(cx))-\frac{8bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{5c}\\&-\frac{4bd^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{15c}-\frac{3bd^2\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{25c}\\&+\frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^3+\frac{4}{15}d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^3+\frac{1}{5}d^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^3\end{aligned}$$

output
```
-4144/1125*b^3*d^2*(c^2*x^2+1)^(1/2)/c-272/3375*b^3*d^2*(c^2*x^2+1)^(3/2)/
c-6/625*b^3*d^2*(c^2*x^2+1)^(5/2)/c+298/75*b^2*d^2*x*(a+b*arcsinh(c*x))+76
/225*b^2*c^2*d^2*x^3*(a+b*arcsinh(c*x))+6/125*b^2*c^4*d^2*x^5*(a+b*arcsinh
(c*x))-8/5*b*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c-4/15*b*d^2*(c^2*
x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c-3/25*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcs
inh(c*x))^2/c+8/15*d^2*x*(a+b*arcsinh(c*x))^3+4/15*d^2*x*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^3+1/5*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^3
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.02

$$\int \left(d+c^2dx^2\right)^2 (a+b\operatorname{arcsinh}(cx))^3\,dx$$
$$= \frac{d^2\left(1125a^3cx(15+10c^2x^2+3c^4x^4)-225a^2b\sqrt{1+c^2x^2}(149+38c^2x^2+9c^4x^4)+30ab^2cx(2235+190c^2x^2\cdots\right.}{\cdots}$$

input
```
Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^3,x]
```

output
```
(d^2*(1125*a^3*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 225*a^2*b*Sqrt[1 + c^2*
x^2]*(149 + 38*c^2*x^2 + 9*c^4*x^4) + 30*a*b^2*c*x*(2235 + 190*c^2*x^2 + 2
7*c^4*x^4) - 2*b^3*Sqrt[1 + c^2*x^2]*(31841 + 842*c^2*x^2 + 81*c^4*x^4) +
15*b*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x
^4))*ArcSinh[c*x] - 225*b^2*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*S
qrt[1 + c^2*x^2]*(149 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x]^2 + 1125*b^3
*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]^3))/(16875*c)
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.32, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6201, 27, 6201, 6187, 6213, 2009, 6199, 27, 353, 53, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 (a+b\operatorname{arcsinh}(cx))^3\,dx$$

$$\Big\downarrow \; 6201$$

$$-\frac{3}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2dx+\frac{4}{5}d\int d\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))^3dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\operatorname{arcsinh}(cx))^3$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{4}{5}d^2\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 6201

$$-\frac{3}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{4}{5}d^2\left(-bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\frac{2}{3}\int (a+b\text{arcsinh}(cx))^3dx+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 6187

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx\right)-bc\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}\right.$$
$$\frac{3}{5}bcd^2\int x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 6213

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\int (a+b\text{arcsinh}(cx))dx}{c}\right)\right)-bc\left(\frac{(c^2x\right.\right.$$
$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\int \left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{5c}\right)+$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 2009

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3c}\right)+\frac{1}{3}x\left(c^2x^2+1\right)(a+bar\right.$$
$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\int \left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx}{5c}\right)+$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 6199

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-bc\int\frac{x(c^2x^2+3)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcs}\right.}{3c}\right.\right.$$
$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(-bc\int\frac{x(3c^4x^4+10c^2x^2+15)}{15\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(\right.}{5c}\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 27

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{3}bc\int\frac{x(c^2x^2+3)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arc}\right.}{3c}\right.\right.$$
$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(-\frac{1}{15}bc\int\frac{x(3c^4x^4+10c^2x^2+15)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x\right.}{5c}\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 353

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arc}\right.}{3c}\right.\right.$$
$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(-\frac{1}{15}bc\int\frac{x(3c^4x^4+10c^2x^2+15)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x\right.}{5c}\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 53

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(c\right.}{3c}\right.\right.$$
$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(-\frac{1}{15}bc\int\frac{x(3c^4x^4+10c^2x^2+15)}{\sqrt{c^2x^2+1}}dx+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x\right.}{5c}\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$\downarrow$ 1576

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(c\right.}{3c}\right.\right.$$

$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(-\frac{1}{30}bc\int\frac{3c^4x^4+10c^2x^2+15}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{1140}$$

$$\frac{4}{5}d^2\left(-bc\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(c\right.}{3c}\right.\right.$$

$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(-\frac{1}{30}bc\int\left(3\left(c^2x^2+1\right)^{3/2}+4\sqrt{c^2x^2+1}+\frac{8}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{5}c^4x^5\right.}{}\right.$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^3+$$

$$\frac{4}{5}d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(\right.}{}\right.\right.\right.$$

$$\frac{3}{5}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2}{5c^2}-\frac{2b\left(\frac{1}{5}c^4x^5(a+b\text{arcsinh}(cx))+\frac{2}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{a}\right.}{5c}\right.$$

input
```
Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^3,x]
```

output

```
(d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^3)/5 - (3*b*c*d^2*(((1 + c^2*x
^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2) - (2*b*(-1/30*(b*c*((16*Sqrt[1 +
 c^2*x^2])/c^2 + (8*(1 + c^2*x^2)^(3/2))/(3*c^2) + (6*(1 + c^2*x^2)^(5/2))
/(5*c^2))) + x*(a + b*ArcSinh[c*x]) + (2*c^2*x^3*(a + b*ArcSinh[c*x]))/3 +
 (c^4*x^5*(a + b*ArcSinh[c*x]))/5))/(5*c)))/5 + (4*d^2*((x*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x])^3)/3 - b*c*(((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^
2)/(3*c^2) - (2*b*(-1/6*(b*c*((4*Sqrt[1 + c^2*x^2])/c^2 + (2*(1 + c^2*x^2)
^(3/2))/(3*c^2))) + x*(a + b*ArcSinh[c*x]) + (c^2*x^3*(a + b*ArcSinh[c*x])
)/3))/(3*c)) + (2*(x*(a + b*ArcSinh[c*x])^3 - 3*b*c*((Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x])^2)/c^2 - (2*b*(a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcS
inh[c*x]))/c)))/3))/5
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{a^3d^2\left(\frac{1}{5}x^5c^5+\frac{2}{3}x^3c^3+xc\right)+d^2b^3\left(\frac{8\,\mathrm{arcsinh}(xc)^3xc}{15}+\frac{\mathrm{arcsinh}(xc)^3xc\left(c^2x^2+1\right)^2}{5}+\frac{4\,\mathrm{arcsinh}(xc)^3xc\left(c^2x^2+1\right)}{15}-\frac{8\,\mathrm{arcsinh}(xc)^2}{5}\right.}{}$
default	$\frac{a^3d^2\left(\frac{1}{5}x^5c^5+\frac{2}{3}x^3c^3+xc\right)+d^2b^3\left(\frac{8\,\mathrm{arcsinh}(xc)^3xc}{15}+\frac{\mathrm{arcsinh}(xc)^3xc\left(c^2x^2+1\right)^2}{5}+\frac{4\,\mathrm{arcsinh}(xc)^3xc\left(c^2x^2+1\right)}{15}-\frac{8\,\mathrm{arcsinh}(xc)^2}{5}\right.}{}$
parts	$a^3d^2\left(\frac{1}{5}c^4x^5+\frac{2}{3}x^3c^2+x\right)+\frac{d^2b^3\left(\frac{8\,\mathrm{arcsinh}(xc)^3xc}{15}+\frac{\mathrm{arcsinh}(xc)^3xc\left(c^2x^2+1\right)^2}{5}+\frac{4\,\mathrm{arcsinh}(xc)^3xc\left(c^2x^2+1\right)}{15}-\right.}{}$
orering	$\frac{x\left(29889c^6x^6+179507c^4x^4+2768347c^2x^2-1732471\right)\left(c^2d\,x^2+d\right)^2\left(a+b\,\mathrm{arcsinh}(xc)\right)^3}{50625\left(c^2x^2+1\right)^3}-\frac{\left(7857c^6x^6+60788c^4x^4+144\right.}{}$

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^3*d^2*(1/5*x^5*c^5+2/3*x^3*c^3+x*c)+d^2*b^3*(8/15*arcsinh(x*c)^3*x*
c+1/5*arcsinh(x*c)^3*x*c*(c^2*x^2+1)^2+4/15*arcsinh(x*c)^3*x*c*(c^2*x^2+1)
-8/5*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)+4144/1125*x*c*arcsinh(x*c)-4144/1125
*(c^2*x^2+1)^(1/2)-3/25*arcsinh(x*c)^2*(c^2*x^2+1)^(5/2)+6/125*arcsinh(x*c
)*x*c*(c^2*x^2+1)^2+272/1125*arcsinh(x*c)*x*c*(c^2*x^2+1)-6/625*(c^2*x^2+1
)^(5/2)-272/3375*(c^2*x^2+1)^(3/2)-4/15*arcsinh(x*c)^2*(c^2*x^2+1)^(3/2))+
3*d^2*a*b^2*(8/15*arcsinh(x*c)^2*x*c+1/5*arcsinh(x*c)^2*x*c*(c^2*x^2+1)^2+
4/15*arcsinh(x*c)^2*x*c*(c^2*x^2+1)-16/15*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+4
144/3375*x*c-2/25*arcsinh(x*c)*(c^2*x^2+1)^(5/2)+2/125*x*c*(c^2*x^2+1)^2+2
72/3375*x*c*(c^2*x^2+1)-8/45*arcsinh(x*c)*(c^2*x^2+1)^(3/2))+3*a^2*b*d^2*(
1/5*arcsinh(x*c)*x^5*c^5+2/3*arcsinh(x*c)*x^3*c^3+x*c*arcsinh(x*c)-149/225
*(c^2*x^2+1)^(1/2)-38/225*x^2*c^2*(c^2*x^2+1)^(1/2)-1/25*x^4*c^4*(c^2*x^2+
1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.41

$$\int \left(d+c^2dx^2\right)^2\left(a+b\mathrm{arcsinh}(cx)\right)^3dx$$

$$=\frac{135\left(25\,a^3+6\,ab^2\right)c^5d^2x^5+150\left(75\,a^3+38\,ab^2\right)c^3d^2x^3+225\left(75\,a^3+298\,ab^2\right)cd^2x+1125\left(3\,b^3c^5d^2x^5+\right.}{}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^3,x, algorithm="fricas")
```

output
```
1/16875*(135*(25*a^3 + 6*a*b^2)*c^5*d^2*x^5 + 150*(75*a^3 + 38*a*b^2)*c^3*
d^2*x^3 + 225*(75*a^3 + 298*a*b^2)*c*d^2*x + 1125*(3*b^3*c^5*d^2*x^5 + 10*
b^3*c^3*d^2*x^3 + 15*b^3*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^3 + 225*(45
*a*b^2*c^5*d^2*x^5 + 150*a*b^2*c^3*d^2*x^3 + 225*a*b^2*c*d^2*x - (9*b^3*c^
4*d^2*x^4 + 38*b^3*c^2*d^2*x^2 + 149*b^3*d^2)*sqrt(c^2*x^2 + 1))*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 15*(27*(25*a^2*b + 2*b^3)*c^5*d^2*x^5 + 10*(225*a^
2*b + 38*b^3)*c^3*d^2*x^3 + 15*(225*a^2*b + 298*b^3)*c*d^2*x - 30*(9*a*b^2
*c^4*d^2*x^4 + 38*a*b^2*c^2*d^2*x^2 + 149*a*b^2*d^2)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) - (81*(25*a^2*b + 2*b^3)*c^4*d^2*x^4 + 2*(4275*
a^2*b + 842*b^3)*c^2*d^2*x^2 + (33525*a^2*b + 63682*b^3)*d^2)*sqrt(c^2*x^2
 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(311) = 622$.

Time = 0.69 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.23

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^3\,dx=\text{Too large to display}$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**3,x)
```

output
```
Piecewise((a**3*c**4*d**2*x**5/5 + 2*a**3*c**2*d**2*x**3/3 + a**3*d**2*x +
 3*a**2*b*c**4*d**2*x**5*asinh(c*x)/5 - 3*a**2*b*c**3*d**2*x**4*sqrt(c**2*
x**2 + 1)/25 + 2*a**2*b*c**2*d**2*x**3*asinh(c*x) - 38*a**2*b*c*d**2*x**2*
sqrt(c**2*x**2 + 1)/75 + 3*a**2*b*d**2*x*asinh(c*x) - 149*a**2*b*d**2*sqrt
(c**2*x**2 + 1)/(75*c) + 3*a*b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 6*a*b**
2*c**4*d**2*x**5/125 - 6*a*b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c
*x)/25 + 2*a*b**2*c**2*d**2*x**3*asinh(c*x)**2 + 76*a*b**2*c**2*d**2*x**3/
225 - 76*a*b**2*c*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/75 + 3*a*b**2*d
**2*x*asinh(c*x)**2 + 298*a*b**2*d**2*x/75 - 298*a*b**2*d**2*sqrt(c**2*x**
2 + 1)*asinh(c*x)/(75*c) + b**3*c**4*d**2*x**5*asinh(c*x)**3/5 + 6*b**3*c*
*4*d**2*x**5*asinh(c*x)/125 - 3*b**3*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*as
inh(c*x)**2/25 - 6*b**3*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/625 + 2*b**3*c*
*2*d**2*x**3*asinh(c*x)**3/3 + 76*b**3*c**2*d**2*x**3*asinh(c*x)/225 - 38*
b**3*c*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/75 - 1684*b**3*c*d**2*x
**2*sqrt(c**2*x**2 + 1)/16875 + b**3*d**2*x*asinh(c*x)**3 + 298*b**3*d**2*
x*asinh(c*x)/75 - 149*b**3*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/(75*c) -
 63682*b**3*d**2*sqrt(c**2*x**2 + 1)/(16875*c), Ne(c, 0)), (a**3*d**2*x, T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(286) = 572.

Time = 0.07 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.67

$$\int \left(d+c^2dx^2\right)^2(a+b\text{arcsinh}(cx))^3\,dx=\text{Too large to display}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^3,x, algorithm="maxima")
```

output
```
1/5*b^3*c^4*d^2*x^5*arcsinh(c*x)^3 + 3/5*a*b^2*c^4*d^2*x^5*arcsinh(c*x)^2
+ 1/5*a^3*c^4*d^2*x^5 + 2/3*b^3*c^2*d^2*x^3*arcsinh(c*x)^3 + 2*a*b^2*c^2*d
^2*x^3*arcsinh(c*x)^2 + 1/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x
^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a^2*b*c
^4*d^2 - 2/375*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 +
120*x)/c^4)*a*b^2*c^4*d^2 - 1/5625*(225*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*s
qrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x)^2 + 2*c
*((27*sqrt(c^2*x^2 + 1)*c^2*x^4 - 136*sqrt(c^2*x^2 + 1)*x^2 + 2072*sqrt(c^
2*x^2 + 1)/c^2)/c^4 - 15*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*arcsinh(c*x)/c^5
))*b^3*c^4*d^2 + 2/3*a^3*c^2*d^2*x^3 + b^3*d^2*x*arcsinh(c*x)^3 + 2/3*(3*x
^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))
*a^2*b*c^2*d^2 - 4/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)
/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*a*b^2*c^2*d^2 - 2/27*(9*c*(sqrt(
c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x)^2 + 2*c*((sqr
t(c^2*x^2 + 1)*x^2 - 20*sqrt(c^2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 - 6*x)*arc
sinh(c*x)/c^3))*b^3*c^2*d^2 + 3*a*b^2*d^2*x*arcsinh(c*x)^2 - 3*(sqrt(c^2*x
^2 + 1)*arcsinh(c*x)^2/c - 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))/c)*b^3
*d^2 + 6*a*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^3*d^2*x + 3*
(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a^2*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^3\,dx=\int (a+b\,\text{asinh}(c\,x))^3\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))^3*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^3*(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^3\,dx$$
$$=\frac{d^2\left(75asinh(cx)^3\,b^3cx-225\sqrt{c^2x^2+1}\,asinh(cx)^2\,b^3+225asinh(cx)^2\,a\,b^2cx-450\sqrt{c^2x^2+1}\,asinh(cx)\,a}{}$$

input
```
int((c^2*d*x^2+d)^2*(a+b*asinh(c*x))^3,x)
```

output
```
(d**2*(75*asinh(c*x)**3*b**3*c*x - 225*sqrt(c**2*x**2 + 1)*asinh(c*x)**2*b
**3 + 225*asinh(c*x)**2*a*b**2*c*x - 450*sqrt(c**2*x**2 + 1)*asinh(c*x)*a*
b**2 + 45*asinh(c*x)*a**2*b*c**5*x**5 + 150*asinh(c*x)*a**2*b*c**3*x**3 +
225*asinh(c*x)*a**2*b*c*x + 450*asinh(c*x)*b**3*c*x - 9*sqrt(c**2*x**2 + 1
)*a**2*b*c**4*x**4 - 38*sqrt(c**2*x**2 + 1)*a**2*b*c**2*x**2 - 149*sqrt(c*
*2*x**2 + 1)*a**2*b - 450*sqrt(c**2*x**2 + 1)*b**3 + 75*int(asinh(c*x)**3*
x**4,x)*b**3*c**5 + 150*int(asinh(c*x)**3*x**2,x)*b**3*c**3 + 225*int(asin
h(c*x)**2*x**4,x)*a*b**2*c**5 + 450*int(asinh(c*x)**2*x**2,x)*a*b**2*c**3
+ 15*a**3*c**5*x**5 + 50*a**3*c**3*x**3 + 75*a**3*c*x + 450*a*b**2*c*x))/(
75*c)
```

3.14 $\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 191

$$\int (d+c^2dx^2)(a+b\text{arcsinh}(cx))^3\,dx = -\frac{40b^3d\sqrt{1+c^2x^2}}{9c}-\frac{2b^3d(1+c^2x^2)^{3/2}}{27c}+\frac{14}{3}b^2dx(a+b\text{arcsinh}(cx))+\frac{2}{9}b^2c^2dx^3(a+b\text{arcsinh}(cx))-\frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{c}-\frac{bd(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c}+\frac{2}{3}dx(a+b\text{arcsinh}(cx))^3+\frac{1}{3}dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^3$$

output
```
-40/9*b^3*d*(c^2*x^2+1)^(1/2)/c-2/27*b^3*d*(c^2*x^2+1)^(3/2)/c+14/3*b^2*d*
x*(a+b*arcsinh(c*x))+2/9*b^2*c^2*d*x^3*(a+b*arcsinh(c*x))-2*b*d*(c^2*x^2+1
)^(1/2)*(a+b*arcsinh(c*x))^2/c-1/3*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x)
)^2/c+2/3*d*x*(a+b*arcsinh(c*x))^3+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^
3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.23

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^3 dx$$
$$= \frac{d(9a^3cx(3 + c^2x^2) - 9a^2b\sqrt{1 + c^2x^2}(7 + c^2x^2) + 6ab^2cx(21 + c^2x^2) - 2b^3\sqrt{1 + c^2x^2}(61 + c^2x^2) + 3b(9a$$

input
```
Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^3,x]
```

output
```
(d*(9*a^3*c*x*(3 + c^2*x^2) - 9*a^2*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 6*
a*b^2*c*x*(21 + c^2*x^2) - 2*b^3*Sqrt[1 + c^2*x^2]*(61 + c^2*x^2) + 3*b*(9
*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^2*c*x
*(21 + c^2*x^2))*ArcSinh[c*x] - 9*b^2*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 +
 c^2*x^2]*(7 + c^2*x^2))*ArcSinh[c*x]^2 + 9*b^3*c*x*(3 + c^2*x^2)*ArcSinh[
c*x]^3))/(27*c)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6201, 6187, 6213, 2009, 6199, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c^2dx^2 + d) (a + b\text{arcsinh}(cx))^3 dx$$

$$\Big\downarrow \text{ 6201}$$

$$-bcd \int x\sqrt{c^2x^2 + 1}(a + b\text{arcsinh}(cx))^2dx + \frac{2}{3}d \int (a + b\text{arcsinh}(cx))^3dx + \frac{1}{3}dx(c^2x^2 + 1) (a + b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx\right)-bcd\int x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{6213}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\int(a+b\text{arcsinh}(cx))dx}{c}\right)\right)-bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3$$

$$\Big\downarrow \text{2009}$$

$$-bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\int\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{3c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)\right)$$

$$\Big\downarrow \text{6199}$$

$$-bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-bc\int\frac{x\left(c^2x^2+3\right)}{3\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(c\right.}{3c}\right.$$
$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{3}bc\int\frac{x\left(c^2x^2+3\right)}{\sqrt{c^2x^2+1}}dx+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}\right.}{3c}\right.$$
$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^3+\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)\right)$$

$$\Big\downarrow \ 353$$

$$-bcd\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\frac{c^2x^2+3}{\sqrt{c^2x^2+1}}dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(\right.}{3c}\right.$$
$$\frac{1}{3}dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^3+$$
$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)\right)$$

$$\Big\downarrow \ 53$$

$$-bcd\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{6}bc\int\left(\sqrt{c^2x^2+1}+\frac{2}{\sqrt{c^2x^2+1}}\right)dx^2+\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+\right.}{3c}\right.$$
$$\frac{1}{3}dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^3+$$
$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}dx(c^2x^2+1)(a+b\text{arcsinh}(cx))^3+$$
$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^3-3bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c^2}-\frac{2b\left(ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}\right)}{c}\right)\right)-$$
$$bcd\left(\frac{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}c^2x^3(a+b\text{arcsinh}(cx))+x(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2(c^2x^2+1)^{3/2}}{3c^2}\right.\right.}{3c}\right.$$

input
```
Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^3,x]
```

output
```
(d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^3)/3 - b*c*d*(((1 + c^2*x^2)^(3/2)
*(a + b*ArcSinh[c*x])^2)/(3*c^2) - (2*b*(-1/6*(b*c*((4*Sqrt[1 + c^2*x^2])/
c^2 + (2*(1 + c^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSinh[c*x]) + (c^2*x^
3*(a + b*ArcSinh[c*x]))/3))/(3*c)) + (2*d*(x*(a + b*ArcSinh[c*x])^3 - 3*b*
c*((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c^2 - (2*b*(a*x - (b*Sqrt[1
+ c^2*x^2])/c + b*x*ArcSinh[c*x]))/c)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6199
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{d a^3 \left(\frac{1}{3}x^3c^3+xc\right)+d b^3 \left(\frac{2 \operatorname{arcsinh}(xc)^3 xc}{3}+\frac{\operatorname{arcsinh}(xc)^3 xc \left(c^2x^2+1\right)}{3}-2 \operatorname{arcsinh}(xc)^2 \sqrt{c^2x^2+1}+\frac{40xc \operatorname{arcsinh}(xc)}{9}-\frac{40\sqrt{c^2x}}{9}\right.}{}$
default	$\frac{d a^3 \left(\frac{1}{3}x^3c^3+xc\right)+d b^3 \left(\frac{2 \operatorname{arcsinh}(xc)^3 xc}{3}+\frac{\operatorname{arcsinh}(xc)^3 xc \left(c^2x^2+1\right)}{3}-2 \operatorname{arcsinh}(xc)^2 \sqrt{c^2x^2+1}+\frac{40xc \operatorname{arcsinh}(xc)}{9}-\frac{40\sqrt{c^2x}}{9}\right.}{}$
parts	$d a^3 \left(\frac{1}{3}x^3c^2+x\right)+\frac{d b^3 \left(\frac{2 \operatorname{arcsinh}(xc)^3 xc}{3}+\frac{\operatorname{arcsinh}(xc)^3 xc \left(c^2x^2+1\right)}{3}-2 \operatorname{arcsinh}(xc)^2 \sqrt{c^2x^2+1}+\frac{40xc \operatorname{arcsinh}(xc)}{9}-\right.}{c}$
orering	$\frac{5x\left(13c^4x^4+194c^2x^2-179\right)\left(c^2d\,x^2+d\right)\left(a+b \operatorname{arcsinh}(xc)\right)^3}{81\left(c^2x^2+1\right)^2}-\frac{\left(25c^4x^4+683c^2x^2-242\right)\left(2c^2dx\left(a+b \operatorname{arcsinh}(xc)\right)^3+\right.}{81c^2\left(c^2x^2+1\right)}$

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d*a^3*(1/3*x^3*c^3+x*c)+d*b^3*(2/3*arcsinh(x*c)^3*x*c+1/3*arcsinh(x*c
)^3*x*c*(c^2*x^2+1)-2*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)+40/9*x*c*arcsinh(x*
c)-40/9*(c^2*x^2+1)^(1/2)-1/3*arcsinh(x*c)^2*(c^2*x^2+1)^(3/2)+2/9*arcsinh
(x*c)*x*c*(c^2*x^2+1)-2/27*(c^2*x^2+1)^(3/2))+3*d*a*b^2*(2/3*arcsinh(x*c)^
2*x*c+1/3*arcsinh(x*c)^2*x*c*(c^2*x^2+1)-4/3*arcsinh(x*c)*(c^2*x^2+1)^(1/2
)+40/27*x*c-2/9*arcsinh(x*c)*(c^2*x^2+1)^(3/2)+2/27*x*c*(c^2*x^2+1))+3*d*a
^2*b*(1/3*arcsinh(x*c)*x^3*c^3+x*c*arcsinh(x*c)-1/9*x^2*c^2*(c^2*x^2+1)^(1
/2)-7/9*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.54

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^3 dx$$
$$= \frac{3(3a^3 + 2ab^2)c^3dx^3 + 9(3a^3 + 14ab^2)cdx + 9(b^3c^3dx^3 + 3b^3cdx)\log\left(cx + \sqrt{c^2x^2 + 1}\right)^3 + 9\left(3ab^2c^3dx\right.}{}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^3,x, algorithm="fricas")
```

output
```
1/27*(3*(3*a^3 + 2*a*b^2)*c^3*d*x^3 + 9*(3*a^3 + 14*a*b^2)*c*d*x + 9*(b^3*
c^3*d*x^3 + 3*b^3*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^3 + 9*(3*a*b^2*c^3*d
*x^3 + 9*a*b^2*c*d*x - (b^3*c^2*d*x^2 + 7*b^3*d)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 3*((9*a^2*b + 2*b^3)*c^3*d*x^3 + 3*(9*a^2*b + 1
4*b^3)*c*d*x - 6*(a*b^2*c^2*d*x^2 + 7*a*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1)) - ((9*a^2*b + 2*b^3)*c^2*d*x^2 + (63*a^2*b + 122*b^3)
*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(184) = 368.

Time = 0.36 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.16

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^3 dx$$
$$= \begin{cases} \frac{a^3c^2dx^3}{3} + a^3dx + a^2bc^2dx^3\operatorname{asinh}(cx) - \frac{a^2bcdx^2\sqrt{c^2x^2+1}}{3} + 3a^2bdx\operatorname{asinh}(cx) - \frac{7a^2bd\sqrt{c^2x^2+1}}{3c} + ab^2c^2dx^3\operatorname{as} \\ a^3dx \end{cases}$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**3,x)
```

output
```
Piecewise((a**3*c**2*d*x**3/3 + a**3*d*x + a**2*b*c**2*d*x**3*asinh(c*x) -
 a**2*b*c*d*x**2*sqrt(c**2*x**2 + 1)/3 + 3*a**2*b*d*x*asinh(c*x) - 7*a**2*
b*d*sqrt(c**2*x**2 + 1)/(3*c) + a*b**2*c**2*d*x**3*asinh(c*x)**2 + 2*a*b**
2*c**2*d*x**3/9 - 2*a*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3 + 3*a
*b**2*d*x*asinh(c*x)**2 + 14*a*b**2*d*x/3 - 14*a*b**2*d*sqrt(c**2*x**2 + 1
)*asinh(c*x)/(3*c) + b**3*c**2*d*x**3*asinh(c*x)**3/3 + 2*b**3*c**2*d*x**3
*asinh(c*x)/9 - b**3*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/3 - 2*b**3
*c*d*x**2*sqrt(c**2*x**2 + 1)/27 + b**3*d*x*asinh(c*x)**3 + 14*b**3*d*x*as
inh(c*x)/3 - 7*b**3*d*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/(3*c) - 122*b**3*d
*sqrt(c**2*x**2 + 1)/(27*c), Ne(c, 0)), (a**3*d*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(169) = 338$.

Time = 0.04 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.28

$$\int \left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^3\,dx$$

$$\begin{aligned}
&= \frac{1}{3}b^3c^2dx^3\operatorname{arsinh}(cx)^3 + ab^2c^2dx^3\operatorname{arsinh}(cx)^2 + \frac{1}{3}a^3c^2dx^3 + b^3dx\operatorname{arsinh}(cx)^3 \\
&\quad + \frac{1}{3}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)a^2bc^2d \\
&\quad - \frac{2}{9}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6x}{c^2}\right)ab^2c^2d \\
&\quad - \frac{1}{27}\left(9c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx)^2 + 2c\left(\frac{\sqrt{c^2x^2+1}x^2 - \frac{20\sqrt{c^2x^2+1}}{c^2}}{c^2} - \frac{3(c^2x^3-6x)}{c^3}\right.\right. \\
&\quad + 3ab^2dx\operatorname{arsinh}(cx)^2 \\
&\quad - 3\left(\frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)^2}{c} - \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)}{c}\right)b^3d \\
&\quad + 6ab^2d\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^3dx + \frac{3\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)a^2bd}{c}
\end{aligned}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*c^2*d*x^3*arcsinh(c*x)^3 + a*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^
3*c^2*d*x^3 + b^3*d*x*arcsinh(c*x)^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a^2*b*c^2*d - 2/9*(3*c*(sq
rt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3
 - 6*x)/c^2)*a*b^2*c^2*d - 1/27*(9*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c
^2*x^2 + 1)/c^4)*arcsinh(c*x)^2 + 2*c*((sqrt(c^2*x^2 + 1)*x^2 - 20*sqrt(c^
2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 - 6*x)*arcsinh(c*x)/c^3))*b^3*c^2*d + 3*a
*b^2*d*x*arcsinh(c*x)^2 - 3*(sqrt(c^2*x^2 + 1)*arcsinh(c*x)^2/c - 2*(c*x*a
rcsinh(c*x) - sqrt(c^2*x^2 + 1))/c)*b^3*d + 6*a*b^2*d*(x - sqrt(c^2*x^2 +
1)*arcsinh(c*x)/c) + a^3*d*x + 3*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a^
2*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^3 dx = \int (a + b\,\text{asinh}(c\,x))^3 \left(d\,c^2\,x^2 + d\right) dx$$

input
```
int((a + b*asinh(c*x))^3*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^3*(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^3 \, dx$$
$$= \frac{d\left(3asinh(cx)^3 b^3cx - 9\sqrt{c^2x^2+1}\, asinh(cx)^2 b^3 + 9asinh(cx)^2 a b^2cx - 18\sqrt{c^2x^2+1}\, asinh(cx)\, a b^2 + 3a\right.}{}$$

input
```
int((c^2*d*x^2+d)*(a+b*asinh(c*x))^3,x)
```

output
```
(d*(3*asinh(c*x)**3*b**3*c*x - 9*sqrt(c**2*x**2 + 1)*asinh(c*x)**2*b**3 +
9*asinh(c*x)**2*a*b**2*c*x - 18*sqrt(c**2*x**2 + 1)*asinh(c*x)*a*b**2 + 3*
asinh(c*x)*a**2*b*c**3*x**3 + 9*asinh(c*x)*a**2*b*c*x + 18*asinh(c*x)*b**3
*c*x - sqrt(c**2*x**2 + 1)*a**2*b*c**2*x**2 - 7*sqrt(c**2*x**2 + 1)*a**2*b
 - 18*sqrt(c**2*x**2 + 1)*b**3 + 3*int(asinh(c*x)**3*x**2,x)*b**3*c**3 + 9
*int(asinh(c*x)**2*x**2,x)*a*b**2*c**3 + a**3*c**3*x**3 + 3*a**3*c*x + 18*
a*b**2*c*x))/(3*c)
```

3.15 $\int \frac{(a+b\text{arcsinh}(cx))^3}{d+c^2dx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 208

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{d+c^2dx^2}\,dx = \frac{2(a+b\text{arcsinh}(cx))^3\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd} - \frac{3ib(a+b\text{arcsinh}(cx))^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{3ib(a+b\text{arcsinh}(cx))^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{6ib^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{6ib^2(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd} - \frac{6ib^3\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(cx)}\right)}{cd} + \frac{6ib^3\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(cx)}\right)}{cd}$$

output

```
2*(a+b*arcsinh(c*x))^3*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-3*I*b*(a+b*arcsin
h(c*x))^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+3*I*b*(a+b*arcsinh(c*x
))^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+6*I*b^2*(a+b*arcsinh(c*x))*p
olylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d-6*I*b^2*(a+b*arcsinh(c*x))*polylo
g(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d-6*I*b^3*polylog(4,-I*(c*x+(c^2*x^2+1)^(
1/2)))/c/d+6*I*b^3*polylog(4,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 464 vs. $2(208) = 416$.

Time = 0.31 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.23

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^3}{d+c^2dx^2}\,dx = -\frac{c\Big(a^3\sqrt{-c^2}\arctan(cx)-3a^2bc\operatorname{arcsinh}(cx)\log\Big(1+\frac{ce^{\operatorname{arcsinh}(cx)}}{\sqrt{-c^2}}\Big)-3ab^2c\operatorname{arcsinh}(cx)^2\log\Big(1+\frac{ce^{\operatorname{arcsinh}(c}}{\sqrt{-c^2}}}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^3/(d + c^2*d*x^2),x]
```

output
```
-((c*(a^3*Sqrt[-c^2]*ArcTan[c*x] - 3*a^2*b*c*ArcSinh[c*x]*Log[1 + (c*E^Arc
Sinh[c*x])/Sqrt[-c^2]] - 3*a*b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x
])/Sqrt[-c^2]] - b^3*c*ArcSinh[c*x]^3*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2
]] + 3*a^2*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*a*b
^2*c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b^3*c*ArcSinh
[c*x]^3*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*b*c*(a + b*ArcSinh[c*x]
)^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 3*b*c*(a + b*ArcSinh[c*x])
^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a*b^2*c*PolyLog[3, (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] - 6*b^3*c*ArcSinh[c*x]*PolyLog[3, (c*E^ArcSinh[c*
x])/Sqrt[-c^2]] + 6*a*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 6*
b^3*c*ArcSinh[c*x]*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 6*b^3*c*Pol
yLog[4, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 6*b^3*c*PolyLog[4, (Sqrt[-c^2]*E^
ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6204, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{c^2dx^2+d}dx$$

$\Big\downarrow$ 6204

$$\frac{\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd}$$

$\Big\downarrow$ 3042

$$\frac{\int (a+b\text{arcsinh}(cx))^3\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{cd}$$

$\Big\downarrow$ 4668

$$\frac{-3ib\int (a+b\text{arcsinh}(cx))^2\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+3ib\int (a+b\text{arcsinh}(cx))^2\log\left(1+ie^{\text{arcsinh}(cx)}\right)da}{cd}$$

$\Big\downarrow$ 3011

$$\frac{3ib\big(2b\int (a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx}{}$$

$\Big\downarrow$ 7163

$$\frac{3ib\big(2b\big(\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-b\int \text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\big)-\text{PolyLog}\,(}{}$$

$\Big\downarrow$ 2720

$$\frac{3ib\big(2b\big(\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}\big)-}{}$$

$\Big\downarrow$ 7143

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^3+3ib\big(2b\big(\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-b\,\text{PolyLog}\,(4,}{}$$

input
```
Int[(a + b*ArcSinh[c*x])^3/(d + c^2*d*x^2),x]
```

output
```
(2*(a + b*ArcSinh[c*x])^3*ArcTan[E^ArcSinh[c*x]] + (3*I)*b*(-((a + b*ArcSi
nh[c*x])^2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + 2*b*((a + b*ArcSinh[c*x])*Po
lyLog[3, (-I)*E^ArcSinh[c*x]] - b*PolyLog[4, (-I)*E^ArcSinh[c*x]])) - (3*I
)*b*(-((a + b*ArcSinh[c*x])^2*PolyLog[2, I*E^ArcSinh[c*x]]) + 2*b*((a + b*
ArcSinh[c*x])*PolyLog[3, I*E^ArcSinh[c*x]] - b*PolyLog[4, I*E^ArcSinh[c*x]
])))/(c*d)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(xc\right)\right)^{3}}{c^{2}d\,x^{2}+d}dx$$

input
```
int((a+b*arcsinh(x*c))^3/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^3/(c^2*d*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{d+c^2dx^2}\,dx=\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{3}}{c^{2}dx^{2}+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(c*x)^3 + 3*a*b^2*arcsinh(c*x)^2 + 3*a^2*b*arcsinh(c*
x) + a^3)/(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{d+c^2dx^2}\,dx = \frac{\int \frac{a^3}{c^2x^2+1}\,dx + \int \frac{b^3\operatorname{asinh}^3(cx)}{c^2x^2+1}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{3a^2b\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**3/(c**2*d*x**2+d),x)
```

output
```
(Integral(a**3/(c**2*x**2 + 1), x) + Integral(b**3*asinh(c*x)**3/(c**2*x**
2 + 1), x) + Integral(3*a*b**2*asinh(c*x)**2/(c**2*x**2 + 1), x) + Integra
l(3*a**2*b*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{d+c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^3}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
a^3*arctan(c*x)/(c*d) + integrate(b^3*log(c*x + sqrt(c^2*x^2 + 1))^3/(c^2*
d*x^2 + d) + 3*a*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 3*a^
2*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{d+c^2dx^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^3}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^3/(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^3}{d + c^2dx^2}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^3}{d\,c^2\,x^2 + d}\,dx$$

input
```
int((a + b*asinh(c*x))^3/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^3/(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^3}{d + c^2dx^2}\,dx = \frac{atan(cx)\,a^3 + 3\left(\int \frac{asinh(cx)}{c^2x^2+1}dx\right)a^2bc + \left(\int \frac{asinh(cx)^3}{c^2x^2+1}dx\right)b^3c + 3\left(\int \frac{asinh(cx)^2}{c^2x^2+1}dx\right)a\,b^2c}{cd}$$

input
```
int((a+b*asinh(c*x))^3/(c^2*d*x^2+d),x)
```

output
```
(atan(c*x)*a**3 + 3*int(asinh(c*x)/(c**2*x**2 + 1),x)*a**2*b*c + int(asinh
(c*x)**3/(c**2*x**2 + 1),x)*b**3*c + 3*int(asinh(c*x)**2/(c**2*x**2 + 1),x
)*a*b**2*c)/(c*d)
```

3.16 $\int \frac{(a+b\,\text{arcsinh}(cx))^3}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 350

$$\begin{aligned}\int \frac{(a+b\,\text{arcsinh}(cx))^3}{(d+c^2dx^2)^2}\,dx &= \frac{3b(a+b\,\text{arcsinh}(cx))^2}{2cd^2\sqrt{1+c^2x^2}}+\frac{x(a+b\,\text{arcsinh}(cx))^3}{2d^2(1+c^2x^2)}\\
&-\frac{6b^2(a+b\,\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&+\frac{(a+b\,\text{arcsinh}(cx))^3\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&+\frac{3ib^3\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&-\frac{3ib(a+b\,\text{arcsinh}(cx))^2\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{2cd^2}\\
&-\frac{3ib^3\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&+\frac{3ib(a+b\,\text{arcsinh}(cx))^2\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{2cd^2}\\
&+\frac{3ib^2(a+b\,\text{arcsinh}(cx))\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&-\frac{3ib^2(a+b\,\text{arcsinh}(cx))\,\text{PolyLog}\left(3,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&-\frac{3ib^3\,\text{PolyLog}\left(4,-ie^{\text{arcsinh}(cx)}\right)}{cd^2}\\
&+\frac{3ib^3\,\text{PolyLog}\left(4,ie^{\text{arcsinh}(cx)}\right)}{cd^2}\end{aligned}$$

output
```
3/2*b*(a+b*arcsinh(c*x))^2/c/d^2/(c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsinh(c*x)
)^3/d^2/(c^2*x^2+1)-6*b^2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))
/c/d^2+(a+b*arcsinh(c*x))^3*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^2+3*I*b^3*po
lylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2-3/2*I*b*(a+b*arcsinh(c*x))^2*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2-3*I*b^3*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c/d^2+3/2*I*b*(a+b*arcsinh(c*x))^2*polylog(2,I*(c*x+(c^2*x^2+1
)^(1/2)))/c/d^2+3*I*b^2*(a+b*arcsinh(c*x))*polylog(3,-I*(c*x+(c^2*x^2+1)^(
1/2)))/c/d^2-3*I*b^2*(a+b*arcsinh(c*x))*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)
))/c/d^2-3*I*b^3*polylog(4,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+3*I*b^3*polyl
og(4,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 974 vs. $2(350) = 700$.

Time = 4.30 (sec) , antiderivative size = 974, normalized size of antiderivative = 2.78

$$\int \frac{(a + b\text{arcsinh}(cx))^3}{(d + c^2dx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^3/(d + c^2*d*x^2)^2,x]
```

output
```
((64*a^3*x)/(1 + c^2*x^2) + (64*a^3*ArcTan[c*x])/c + (192*a^2*b*(Sqrt[1 +
c^2*x^2] + c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I
*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I
*E^ArcSinh[c*x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1
 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I
*E^ArcSinh[c*x]]))/(c + c^3*x^2) + (384*a*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x
^2] + (c*x*ArcSinh[c*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[Arc
Sinh[c*x]/2]] + ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*
Log[1 + I/E^ArcSinh[c*x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]]
 - 2*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSi
nh[c*x]] - 2*PolyLog[3, I/E^ArcSinh[c*x]])))/c - (I*b^3*(7*Pi^4 + (8*I)*Pi
^3*ArcSinh[c*x] + 24*Pi^2*ArcSinh[c*x]^2 + ((192*I)*ArcSinh[c*x]^2)/Sqrt[1
 + c^2*x^2] - (32*I)*Pi*ArcSinh[c*x]^3 + ((64*I)*c*x*ArcSinh[c*x]^3)/(1 +
c^2*x^2) - 16*ArcSinh[c*x]^4 - 384*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]]
+ (8*I)*Pi^3*Log[1 + I/E^ArcSinh[c*x]] + 384*ArcSinh[c*x]*Log[1 + I/E^ArcS
inh[c*x]] + 48*Pi^2*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - (96*I)*Pi*Arc
Sinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 64*ArcSinh[c*x]^3*Log[1 + I/E^ArcS
inh[c*x]] - 48*Pi^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (96*I)*Pi*Arc
Sinh[c*x]^2*Log[1 - I*E^ArcSinh[c*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcSinh[c*x
]] + 64*ArcSinh[c*x]^3*Log[1 + I*E^ArcSinh[c*x]] + (8*I)*Pi^3*Log[Tan[(...
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6203, 27, 6204, 3042, 4668, 3011, 6213, 6204, 3042, 4668, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^3}{d(c^2x^2+1)}dx}{2d}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^3}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$\downarrow$ 6204

$$-\frac{3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^3}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$\downarrow$ 3042

$$-\frac{3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{\int (a+b\text{arcsinh}(cx))^3\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{2cd^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$\downarrow$ 4668

$$\frac{-3ib\int (a+b\text{arcsinh}(cx))^2\log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+3ib\int (a+b\text{arcsinh}(cx))^2\log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{ar}}{2cd^2}$$

$$\frac{3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$\downarrow$ 3011

$$3ib\left(2b\int (a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(c\right.$$

$$\frac{3bc\int \frac{x(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$\downarrow$ 6213

$$3ib\left(2b\int (a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(c\right.$$

$$\frac{3bc\left(\frac{2b\int \frac{a+b\text{arcsinh}(cx)}{c^2x^2+1}dx}{c}-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}\right)}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2(c^2x^2+1)}$$

$\downarrow$ 6204

$$\frac{3ib\left(2b\int (a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(ca\right.}{}$$

$$\frac{3bc\left(\frac{2b\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c^2}-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}\right)}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3ib\left(2b\int (a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(ca\right.}{}$$

$$\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\int (a+b\text{arcsinh}(cx))\csc\left(i\text{arcsinh}(cx)+\frac{\pi}{2}\right)d\text{arcsinh}(cx)}{c^2}\right)}{2d^2}+$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\left(-ib\int \log\left(1-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+ib\int \log\left(1+ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)+2\arctan\left(e^{\text{arcsin}}\right.\right.}{c^2}\right.}{2d^2}$$

$$\frac{3ib\left(2b\int (a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(ca\right.}{}$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\left(-ib\int e^{-\text{arcsinh}(cx)}\log\left(1-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}+ib\int e^{-\text{arcsinh}(cx)}\log\left(1+ie^{\text{arcsinh}(cx)}\right)de^{\text{arcs}}\right.}{c^2}\right.}{2d^2}$$

$$\frac{3ib\left(2b\int (a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(ca\right.}{}$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$$\Big\downarrow \text{2838}$$

$$\frac{3ib\left(2b\int (a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)-\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(ca\right.}{}$$

$$\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right)}{c^2}\right.}{2d^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$\Big\downarrow$ 7163

$$\frac{3ib\left(2b\left(\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-b\int\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\text{PolyLog}\left(\right.\right.}{}$$

$$\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right)}{c^2}\right.}{2d^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$\Big\downarrow$ 2720

$$\frac{3ib\left(2b\left(\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)de^{\text{arcsinh}(cx)}\right)-\right.}{}$$

$$\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)\right)}{c^2}\right.}{2d^2}$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

$\Big\downarrow$ 7143

$$-\frac{3bc\left(-\frac{(a+b\text{arcsinh}(cx))^2}{c^2\sqrt{c^2x^2+1}}+\frac{2b\left(2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right.\right.}{c^2}\right.}{2d^2}$$

$$\frac{2\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))^3+3ib\left(2b\left(\text{PolyLog}\left(3,-ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))-b\,\text{PolyLog}\left(4,\right.\right.\right.}{}$$

$$\frac{x(a+b\text{arcsinh}(cx))^3}{2d^2\left(c^2x^2+1\right)}$$

input
```
Int[(a + b*ArcSinh[c*x])^3/(d + c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcSinh[c*x])^3)/(2*d^2*(1 + c^2*x^2)) - (3*b*c*(-((a + b*ArcSin
h[c*x])^2/(c^2*Sqrt[1 + c^2*x^2])) + (2*b*(2*(a + b*ArcSinh[c*x])*ArcTan[E
^ArcSinh[c*x]] - I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*PolyLog[2, I*E^
ArcSinh[c*x]]))/c^2))/(2*d^2) + (2*(a + b*ArcSinh[c*x])^3*ArcTan[E^ArcSinh
[c*x]] + (3*I)*b*(-((a + b*ArcSinh[c*x])^2*PolyLog[2, (-I)*E^ArcSinh[c*x]]
) + 2*b*((a + b*ArcSinh[c*x])*PolyLog[3, (-I)*E^ArcSinh[c*x]] - b*PolyLog[
4, (-I)*E^ArcSinh[c*x]])) - (3*I)*b*(-((a + b*ArcSinh[c*x])^2*PolyLog[2, I
*E^ArcSinh[c*x]]) + 2*b*((a + b*ArcSinh[c*x])*PolyLog[3, I*E^ArcSinh[c*x]]
 - b*PolyLog[4, I*E^ArcSinh[c*x]])))/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6204
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d)   Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(xc))^3}{(c^2d\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(x*c))^3/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(x*c))^3/(c^2*d*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^3}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^3}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(c*x)^3 + 3*a*b^2*arcsinh(c*x)^2 + 3*a^2*b*arcsinh(c*
x) + a^3)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^3}{(d + c^2dx^2)^2}\,dx = \frac{\int \frac{a^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^3\operatorname{asinh}^3(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{3a^2b\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**3/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**3*asinh(c*x
)**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(3*a*b**2*asinh(c*x)**2/(
c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(3*a**2*b*asinh(c*x)/(c**4*x**4
 + 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^3}{(d + c^2dx^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^3}{(c^2dx^2 + d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^3*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^3*log(
c*x + sqrt(c^2*x^2 + 1))^3/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 3*a*b^2*l
og(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 3*a^2*
b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^3}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^3/(c^2*d*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(d+c^2dx^2)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^3}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^3/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^3/(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^3}{(d+c^2dx^2)^2}\,dx$$
$$=\frac{atan(cx)\,a^3c^2x^2+atan(cx)\,a^3+6\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)a^2b\,c^3x^2+6\left(\int \frac{asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)a^2bc+2\left(\int \frac{asinh(}{c^4x^4+2c^}\right.}{2c\,d^2}$$

input
```
int((a+b*asinh(c*x))^3/(c^2*d*x^2+d)^2,x)
```

output
```
(atan(c*x)*a**3*c**2*x**2 + atan(c*x)*a**3 + 6*int(asinh(c*x)/(c**4*x**4 +
 2*c**2*x**2 + 1),x)*a**2*b*c**3*x**2 + 6*int(asinh(c*x)/(c**4*x**4 + 2*c*
*2*x**2 + 1),x)*a**2*b*c + 2*int(asinh(c*x)**3/(c**4*x**4 + 2*c**2*x**2 +
1),x)*b**3*c**3*x**2 + 2*int(asinh(c*x)**3/(c**4*x**4 + 2*c**2*x**2 + 1),x
)*b**3*c + 6*int(asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1),x)*a*b**2*c**
3*x**2 + 6*int(asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1),x)*a*b**2*c + a
**3*c*x)/(2*c*d**2*(c**2*x**2 + 1))
```

3.17 $\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \frac{35c^3\text{Chi}(\text{arcsinh}(ax))}{64a} + \frac{21c^3\text{Chi}(3\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Chi}(5\text{arcsinh}(ax))}{64a} + \frac{c^3\text{Chi}(7\text{arcsinh}(ax))}{64a}$$

output
```
35/64*c^3*Chi(arcsinh(a*x))/a+21/64*c^3*Chi(3*arcsinh(a*x))/a+7/64*c^3*Chi
(5*arcsinh(a*x))/a+1/64*c^3*Chi(7*arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \frac{c^3(35\text{Chi}(\text{arcsinh}(ax)) + 21\text{Chi}(3\text{arcsinh}(ax)) + 7\text{Chi}(5\text{arcsinh}(ax)) + \text{Chi}(7\text{arcsinh}(ax)))}{64a}$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

output
```
(c^3*(35*CoshIntegral[ArcSinh[a*x]] + 21*CoshIntegral[3*ArcSinh[a*x]] + 7*
CoshIntegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]]))/(64*a)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{c^3\int \frac{(a^2x^2+1)^{7/2}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^3\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^7}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{c^3\int \left(\frac{21\cosh(3\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{7\cosh(5\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{\cosh(7\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{35\sqrt{a^2x^2+1}}{64\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(\frac{35}{64}\text{Chi}(\text{arcsinh}(ax))+\frac{21}{64}\text{Chi}(3\text{arcsinh}(ax))+\frac{7}{64}\text{Chi}(5\text{arcsinh}(ax))+\frac{1}{64}\text{Chi}(7\text{arcsinh}(ax))\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

output
```
(c^3*((35*CoshIntegral[ArcSinh[a*x]])/64 + (21*CoshIntegral[3*ArcSinh[a*x]
])/64 + (7*CoshIntegral[5*ArcSinh[a*x]])/64 + CoshIntegral[7*ArcSinh[a*x]]
/64))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{c^3(35\ \mathrm{Chi}(\mathrm{arcsinh}(xa))+21\ \mathrm{Chi}(3\ \mathrm{arcsinh}(xa))+7\ \mathrm{Chi}(5\ \mathrm{arcsinh}(xa))+\mathrm{Chi}(7\ \mathrm{arcsinh}(xa)))}{64a}$	42
default	$\frac{c^3(35\ \mathrm{Chi}(\mathrm{arcsinh}(xa))+21\ \mathrm{Chi}(3\ \mathrm{arcsinh}(xa))+7\ \mathrm{Chi}(5\ \mathrm{arcsinh}(xa))+\mathrm{Chi}(7\ \mathrm{arcsinh}(xa)))}{64a}$	42

input
```
int((a^2*c*x^2+c)^3/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(35*Chi(arcsinh(x*a))+21*Chi(3*arcsinh(x*a))+7*Chi(5*arcsinh(x*
a))+Chi(7*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\text{arsinh}\,(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x),
 x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = c^3\left(\int \frac{3a^2x^2}{\text{asinh}\,(ax)}\,dx + \int \frac{3a^4x^4}{\text{asinh}\,(ax)}\,dx + \int \frac{a^6x^6}{\text{asinh}\,(ax)}\,dx + \int \frac{1}{\text{asinh}\,(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/asinh(a*x),x)
```

output
```
c**3*(Integral(3*a**2*x**2/asinh(a*x), x) + Integral(3*a**4*x**4/asinh(a*x
), x) + Integral(a**6*x**6/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\text{arsinh}\,(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\operatorname{asinh}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^3/asinh(a*x),x)
```

output
```
int((c + a^2*c*x^2)^3/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)}\,dx = c^3\left(\left(\int \frac{x^6}{asinh(ax)}dx\right)a^6 + 3\left(\int \frac{x^4}{asinh(ax)}dx\right)a^4 + 3\left(\int \frac{x^2}{asinh(ax)}dx\right)a^2 + \int \frac{1}{asinh(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^3/asinh(a*x),x)
```

output
```
c**3*(int(x**6/asinh(a*x),x)*a**6 + 3*int(x**4/asinh(a*x),x)*a**4 + 3*int(
x**2/asinh(a*x),x)*a**2 + int(1/asinh(a*x),x))
```

3.18 $\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx = \frac{5c^2\text{Chi}(\text{arcsinh}(ax))}{8a} + \frac{5c^2\text{Chi}(3\text{arcsinh}(ax))}{16a} + \frac{c^2\text{Chi}(5\text{arcsinh}(ax))}{16a}$$

output
```
5/8*c^2*Chi(arcsinh(a*x))/a+5/16*c^2*Chi(3*arcsinh(a*x))/a+1/16*c^2*Chi(5*
arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{(c+a^2cx^2)^2}{\text{arcsinh}(ax)}\,dx = \frac{c^2(10\text{Chi}(\text{arcsinh}(ax)) + 5\text{Chi}(3\text{arcsinh}(ax)) + \text{Chi}(5\text{arcsinh}(ax)))}{16a}$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

output
```
(c^2*(10*CoshIntegral[ArcSinh[a*x]] + 5*CoshIntegral[3*ArcSinh[a*x]] + Cos
hIntegral[5*ArcSinh[a*x]]))/(16*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{c^2\int \frac{(a^2x^2+1)^{5/2}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^5}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{c^2\int \left(\frac{5\cosh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\cosh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{5\sqrt{a^2x^2+1}}{8\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(\frac{5}{8}\text{Chi}(\text{arcsinh}(ax))+\frac{5}{16}\text{Chi}(3\text{arcsinh}(ax))+\frac{1}{16}\text{Chi}(5\text{arcsinh}(ax))\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

output
```
(c^2*((5*CoshIntegral[ArcSinh[a*x]])/8 + (5*CoshIntegral[3*ArcSinh[a*x]])/
16 + CoshIntegral[5*ArcSinh[a*x]]/16))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{c^2(10\,\mathrm{Chi}(\mathrm{arcsinh}(xa))+5\,\mathrm{Chi}(3\,\mathrm{arcsinh}(xa))+\mathrm{Chi}(5\,\mathrm{arcsinh}(xa)))}{16a}$	33
default	$\frac{c^2(10\,\mathrm{Chi}(\mathrm{arcsinh}(xa))+5\,\mathrm{Chi}(3\,\mathrm{arcsinh}(xa))+\mathrm{Chi}(5\,\mathrm{arcsinh}(xa)))}{16a}$	33

input
```
int((a^2*c*x^2+c)^2/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(10*Chi(arcsinh(x*a))+5*Chi(3*arcsinh(x*a))+Chi(5*arcsinh(x*a))
)
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/asinh(a*x),x)
```

output
```
c**2*(Integral(2*a**2*x**2/asinh(a*x), x) + Integral(a**4*x**4/asinh(a*x),
 x) + Integral(1/asinh(a*x), x))
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\operatorname{asinh}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^2/asinh(a*x),x)
```

output
```
int((c + a^2*c*x^2)^2/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)}\,dx = c^2\left(\left(\int \frac{x^4}{asinh(ax)}dx\right)a^4 + 2\left(\int \frac{x^2}{asinh(ax)}dx\right)a^2 + \int \frac{1}{asinh(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/asinh(a*x),x)
```

output
```
c**2*(int(x**4/asinh(a*x),x)*a**4 + 2*int(x**2/asinh(a*x),x)*a**2 + int(1/
asinh(a*x),x))
```

3.19 $\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx = \frac{3c\operatorname{Chi}(\operatorname{arcsinh}(ax))}{4a} + \frac{c\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{4a}$$

output
```
3/4*c*Chi(arcsinh(a*x))/a+1/4*c*Chi(3*arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx = \frac{c(3\operatorname{Chi}(\operatorname{arcsinh}(ax)) + \operatorname{Chi}(3\operatorname{arcsinh}(ax)))}{4a}$$

input
```
Integrate[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

output
```
(c*(3*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\text{arcsinh}(ax)}dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{c\int \frac{(a^2x^2+1)^{3/2}}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{c\int \left(\frac{\cosh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}+\frac{3\sqrt{a^2x^2+1}}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(\frac{3}{4}\text{Chi}(\text{arcsinh}(ax))+\frac{1}{4}\text{Chi}(3\text{arcsinh}(ax))\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

output
```
(c*((3*CoshIntegral[ArcSinh[a*x]])/4 + CoshIntegral[3*ArcSinh[a*x]]/4))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{c(3\ \mathrm{Chi}(\mathrm{arcsinh}(xa))+\mathrm{Chi}(3\ \mathrm{arcsinh}(xa)))}{4a}$	22
default	$\frac{c(3\ \mathrm{Chi}(\mathrm{arcsinh}(xa))+\mathrm{Chi}(3\ \mathrm{arcsinh}(xa)))}{4a}$	22

input
```
int((a^2*c*x^2+c)/arcsinh(x*a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(3*Chi(arcsinh(x*a))+Chi(3*arcsinh(x*a)))
```

Fricas [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx=c\left(\int \frac{a^2x^2}{\operatorname{asinh}(ax)}\,dx+\int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/asinh(a*x),x)
```

output
```
c*(Integral(a**2*x**2/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Maxima [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{c\,a^2\,x^2+c}{\operatorname{asinh}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)/asinh(a*x),x)
```

output
```
int((c + a^2*c*x^2)/asinh(a*x), x)
```

Reduce [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)}\,dx = c\left(\left(\int \frac{x^2}{asinh(ax)}dx\right)a^2+\int \frac{1}{asinh(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)/asinh(a*x),x)
```

output
```
c*(int(x**2/asinh(a*x),x)*a**2 + int(1/asinh(a*x),x))
```

3.20 $\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(\frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)\left(a^2cx^2+c\right)}\,dx$$

$$\Big\downarrow \; 6209$$

$$\int \frac{1}{\text{arcsinh}(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)\text{arcsinh}\left(xa\right)}dx$$

input
```
int(1/(a^2*c*x^2+c)/arcsinh(x*a),x)
```

output
```
int(1/(a^2*c*x^2+c)/arcsinh(x*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \frac{\int \frac{1}{a^2x^2\,\text{asinh}\,(ax)+\text{asinh}\,(ax)}\,dx}{c}$$

input
```
integrate(1/(a**2*c*x**2+c)/asinh(a*x),x)
```

output
```
Integral(1/(a**2*x**2*asinh(a*x) + asinh(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\,\text{arsinh}\,(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)}\,dx = \frac{\int \frac{1}{asinh(ax)a^2x^2+asinh(ax)}dx}{c}$$

input
```
int(1/(a^2*c*x^2+c)/asinh(a*x),x)
```

output
```
int(1/(asinh(a*x)*a**2*x**2 + asinh(a*x)),x)/c
```

3.21 $\int \frac{1}{(c+a^2cx^2)^2\text{arcsinh}(ax)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} \, dx = \text{Int}\left(\frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)}, x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} \, dx = \int \frac{1}{(c+a^2cx^2)^2 \text{arcsinh}(ax)} \, dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{\text{arcsinh}(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^2 \text{arcsinh}\left(xa\right)}dx$$

input
```
int(1/(a^2*c*x^2+c)^2/arcsinh(x*a),x)
```

output
```
int(1/(a^2*c*x^2+c)^2/arcsinh(x*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{asinh}(ax)+2a^2x^2 \operatorname{asinh}(ax)+\operatorname{asinh}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/asinh(a*x),x)
```

output
```
Integral(1/(a**4*x**4*asinh(a*x) + 2*a**2*x**2*asinh(a*x) + asinh(a*x)), x
)/c**2
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)} \, dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx = \int \frac{1}{\operatorname{asinh}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(asinh(a*x)*(c + a^2*c*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)} \, dx = \frac{\int \frac{1}{asinh(ax)a^4x^4+2asinh(ax)a^2x^2+asinh(ax)}dx}{c^2}$$

input
```
int(1/(a^2*c*x^2+c)^2/asinh(a*x),x)
```

output
```
int(1/(asinh(a*x)*a**4*x**4 + 2*asinh(a*x)*a**2*x**2 + asinh(a*x)),x)/c**2
```

3.22 $\int \frac{(c+a^2cx^2)^3}{\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)^2}\,dx = -\frac{c^3(1+a^2x^2)^{7/2}}{a\text{arcsinh}(ax)} + \frac{35c^3\text{Shi}(\text{arcsinh}(ax))}{64a} + \frac{63c^3\text{Shi}(3\text{arcsinh}(ax))}{64a} + \frac{35c^3\text{Shi}(5\text{arcsinh}(ax))}{64a} + \frac{7c^3\text{Shi}(7\text{arcsinh}(ax))}{64a}$$

output
```
-c^3*(a^2*x^2+1)^(7/2)/a/arcsinh(a*x)+35/64*c^3*Shi(arcsinh(a*x))/a+63/64*
c^3*Shi(3*arcsinh(a*x))/a+35/64*c^3*Shi(5*arcsinh(a*x))/a+7/64*c^3*Shi(7*a
rcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{(c+a^2cx^2)^3}{\text{arcsinh}(ax)^2}\,dx = \frac{c^3\left(-64(1+a^2x^2)^{7/2} + 35\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax)) + 63\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax)) + 35\text{arcsinh}(\right.}{64a\text{arcsinh}(ax)}$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

output
```
(c^3*(-64*(1 + a^2*x^2)^(7/2) + 35*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
 + 63*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 35*ArcSinh[a*x]*SinhInte
gral[5*ArcSinh[a*x]] + 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]]))/(64*a
*ArcSinh[a*x])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6205}$$

$$7ac^3\int \frac{x(a^2x^2+1)^{5/2}}{\text{arcsinh}(ax)}dx-\frac{c^3(a^2x^2+1)^{7/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6234}$$

$$\frac{7c^3\int \frac{ax(a^2x^2+1)^3}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{c^3(a^2x^2+1)^{7/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{7c^3\int \left(\frac{5ax}{64\text{arcsinh}(ax)}+\frac{9\sinh(3\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{5\sinh(5\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}+\frac{\sinh(7\text{arcsinh}(ax))}{64\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}-\frac{c^3(a^2x^2+1)^{7/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7c^3\left(\frac{5}{64}\text{Shi}(\text{arcsinh}(ax))+\frac{9}{64}\text{Shi}(3\text{arcsinh}(ax))+\frac{5}{64}\text{Shi}(5\text{arcsinh}(ax))+\frac{1}{64}\text{Shi}(7\text{arcsinh}(ax))\right)}{a}-\frac{c^3(a^2x^2+1)^{7/2}}{a\text{arcsinh}(ax)}$$

input
```
Int[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

output
```
-((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (7*c^3*((5*SinhIntegral[Ar
cSinh[a*x]])/64 + (9*SinhIntegral[3*ArcSinh[a*x]])/64 + (5*SinhIntegral[5*
ArcSinh[a*x]])/64 + SinhIntegral[7*ArcSinh[a*x]]/64))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{c^3\left(35 \operatorname{Shi}(\operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)+63 \operatorname{Shi}(3 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)+35 \operatorname{Shi}(5 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)+7 \operatorname{Shi}(7\right.}{64a \operatorname{arcsinh}}$
default	$\frac{c^3\left(35 \operatorname{Shi}(\operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)+63 \operatorname{Shi}(3 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)+35 \operatorname{Shi}(5 \operatorname{arcsinh}(xa)) \operatorname{arcsinh}(xa)+7 \operatorname{Shi}(7\right.}{64a \operatorname{arcsinh}}$

input
```
int((a^2*c*x^2+c)^3/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(35*Shi(arcsinh(x*a))*arcsinh(x*a)+63*Shi(3*arcsinh(x*a))*arcsi
nh(x*a)+35*Shi(5*arcsinh(x*a))*arcsinh(x*a)+7*Shi(7*arcsinh(x*a))*arcsinh(
x*a)-21*cosh(3*arcsinh(x*a))-7*cosh(5*arcsinh(x*a))-cosh(7*arcsinh(x*a))-3
5*(a^2*x^2+1)^(1/2))/arcsinh(x*a)
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x)^
2, x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/asinh(a*x)**2,x)
```

output
```
c**3*(Integral(3*a**2*x**2/asinh(a*x)**2, x) + Integral(3*a**4*x**4/asinh(
a*x)**2, x) + Integral(a**6*x**6/asinh(a*x)**2, x) + Integral(asinh(a*x)**
(-2), x))
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x +
(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*sqrt(a
^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x
^2 + 1))) + integrate((7*a^10*c^3*x^10 + 29*a^8*c^3*x^8 + 46*a^6*c^3*x^6 +
 34*a^4*c^3*x^4 + 11*a^2*c^3*x^2 + c^3 + (7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 +
 18*a^4*c^3*x^4 + 4*a^2*c^3*x^2 - c^3)*(a^2*x^2 + 1) + 7*(2*a^9*c^3*x^9 +
7*a^7*c^3*x^7 + 9*a^5*c^3*x^5 + 5*a^3*c^3*x^3 + a*c^3*x)*sqrt(a^2*x^2 + 1)
)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a
^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\operatorname{asinh}(a\,x)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^3/asinh(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^3/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{(c+a^2cx^2)^3}{\operatorname{arcsinh}(ax)^2}\,dx = c^3\left(\left(\int \frac{x^6}{asinh(ax)^2}dx\right)a^6+3\left(\int \frac{x^4}{asinh(ax)^2}dx\right)a^4 +3\left(\int \frac{x^2}{asinh(ax)^2}dx\right)a^2+\int \frac{1}{asinh(ax)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^3/asinh(a*x)^2,x)
```

output
```
c**3*(int(x**6/asinh(a*x)**2,x)*a**6 + 3*int(x**4/asinh(a*x)**2,x)*a**4 +
3*int(x**2/asinh(a*x)**2,x)*a**2 + int(1/asinh(a*x)**2,x))
```

3.23 $\int \frac{(c+a^2cx^2)^2}{\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{c^2(1+a^2x^2)^{5/2}}{a\operatorname{arcsinh}(ax)} + \frac{5c^2\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8a} + \frac{15c^2\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{16a} + \frac{5c^2\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{16a}$$

output
```
-c^2*(a^2*x^2+1)^(5/2)/a/arcsinh(a*x)+5/8*c^2*Shi(arcsinh(a*x))/a+15/16*c^
2*Shi(3*arcsinh(a*x))/a+5/16*c^2*Shi(5*arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = \frac{c^2\left(-16(1+a^2x^2)^{5/2} + 10\operatorname{arcsinh}(ax)\operatorname{Shi}(\operatorname{arcsinh}(ax)) + 15\operatorname{arcsinh}(ax)\operatorname{Shi}(3\operatorname{arcsinh}(ax)) + 5\operatorname{arcsinh}(a\right.}{16a\operatorname{arcsinh}(ax)}$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

output
```
(c^2*(-16*(1 + a^2*x^2)^(5/2) + 10*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
 + 15*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 5*ArcSinh[a*x]*SinhInteg
ral[5*ArcSinh[a*x]]))/(16*a*ArcSinh[a*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$5ac^2\int \frac{x(a^2x^2+1)^{3/2}}{\text{arcsinh}(ax)}dx-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{5c^2\int \frac{ax(a^2x^2+1)^2}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{5c^2\int \left(\frac{ax}{8\text{arcsinh}(ax)}+\frac{3\sinh(3\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}+\frac{\sinh(5\text{arcsinh}(ax))}{16\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5c^2\left(\frac{1}{8}\text{Shi}(\text{arcsinh}(ax))+\frac{3}{16}\text{Shi}(3\text{arcsinh}(ax))+\frac{1}{16}\text{Shi}(5\text{arcsinh}(ax))\right)}{a}-\frac{c^2(a^2x^2+1)^{5/2}}{a\text{arcsinh}(ax)}$$

input
```
Int[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

output
```
-((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*(SinhIntegral[ArcSi
nh[a*x]]/8 + (3*SinhIntegral[3*ArcSinh[a*x]])/16 + SinhIntegral[5*ArcSinh[
a*x]]/16))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{c^2\left(10\,\mathrm{Shi}(\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)+15\,\mathrm{Shi}(3\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)+5\,\mathrm{Shi}(5\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)-5\cosh(3\right.}{16a\operatorname{arcsinh}(xa)}$
default	$\frac{c^2\left(10\,\mathrm{Shi}(\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)+15\,\mathrm{Shi}(3\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)+5\,\mathrm{Shi}(5\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)-5\cosh(3\right.}{16a\operatorname{arcsinh}(xa)}$

input
```
int((a^2*c*x^2+c)^2/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(10*Shi(arcsinh(x*a))*arcsinh(x*a)+15*Shi(3*arcsinh(x*a))*arcsi
nh(x*a)+5*Shi(5*arcsinh(x*a))*arcsinh(x*a)-5*cosh(3*arcsinh(x*a))-cosh(5*a
rcsinh(x*a))-10*(a^2*x^2+1)^(1/2))/arcsinh(x*a)
```

Fricas [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2/asinh(a*x)**2, x) + Integral(a**4*x**4/asinh(a*
x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Maxima [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x + (a^6*c^2*x^6 + 3
*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^
2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^8*c^2
*x^8 + 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 + 8*a^2*c^2*x^2 + (5*a^6*c^2*x^6 +
9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*(a^2*x^2 + 1) + c^2 + 5*(2*a^7*c^2*x^
7 + 5*a^5*c^2*x^5 + 4*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2*x^2 + 1))/((a^4*x^4
+ (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1)
+ 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^2}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)^2/asinh(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^2/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{(c+a^2cx^2)^2}{\operatorname{arcsinh}(ax)^2}\,dx=c^2\left(\left(\int \frac{x^4}{asinh(ax)^2}dx\right)a^4+2\left(\int \frac{x^2}{asinh(ax)^2}dx\right)a^2+\int \frac{1}{asinh(ax)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/asinh(a*x)^2,x)
```

output
```
c**2*(int(x**4/asinh(a*x)**2,x)*a**4 + 2*int(x**2/asinh(a*x)**2,x)*a**2 +
int(1/asinh(a*x)**2,x))
```

3.24 $\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2}\,dx = -\frac{c(1+a^2x^2)^{3/2}}{a\text{arcsinh}(ax)} + \frac{3c\text{Shi}(\text{arcsinh}(ax))}{4a} + \frac{3c\text{Shi}(3\text{arcsinh}(ax))}{4a}$$

output
```
-c*(a^2*x^2+1)^(3/2)/a/arcsinh(a*x)+3/4*c*Shi(arcsinh(a*x))/a+3/4*c*Shi(3*
arcsinh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\text{arcsinh}(ax)^2}\,dx = \frac{c\left(-4(1+a^2x^2)^{3/2}+3\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax))+3\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax))\right)}{4a\text{arcsinh}(ax)}$$

input
```
Integrate[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

output
```
(c*(-4*(1 + a^2*x^2)^(3/2) + 3*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]] + 3
*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]]))/(4*a*ArcSinh[a*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\text{arcsinh}(ax)^2}dx$$

$$\Big\downarrow \text{6205}$$

$$3ac\int \frac{x\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}dx-\frac{c(a^2x^2+1)^{3/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6234}$$

$$\frac{3c\int \frac{ax(a^2x^2+1)}{\text{arcsinh}(ax)}d\text{arcsinh}(ax)}{a}-\frac{c(a^2x^2+1)^{3/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{3c\int \left(\frac{ax}{4\text{arcsinh}(ax)}+\frac{\sinh(3\text{arcsinh}(ax))}{4\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)}{a}-\frac{c(a^2x^2+1)^{3/2}}{a\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3c\left(\frac{1}{4}\text{Shi}(\text{arcsinh}(ax))+\frac{1}{4}\text{Shi}(3\text{arcsinh}(ax))\right)}{a}-\frac{c(a^2x^2+1)^{3/2}}{a\text{arcsinh}(ax)}$$

input
```
Int[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

output
```
-((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*(SinhIntegral[ArcSinh[a
*x]]/4 + SinhIntegral[3*ArcSinh[a*x]]/4))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{c\left(3\,\mathrm{Shi}(\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)+3\,\mathrm{Shi}(3\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)-\cosh(3\operatorname{arcsinh}(xa))-3\sqrt{a^2x^2+1}\right)}{4a\operatorname{arcsinh}(xa)}$	60
default	$\frac{c\left(3\,\mathrm{Shi}(\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)+3\,\mathrm{Shi}(3\operatorname{arcsinh}(xa))\operatorname{arcsinh}(xa)-\cosh(3\operatorname{arcsinh}(xa))-3\sqrt{a^2x^2+1}\right)}{4a\operatorname{arcsinh}(xa)}$	60

input
```
int((a^2*c*x^2+c)/arcsinh(x*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(3*Shi(arcsinh(x*a))*arcsinh(x*a)+3*Shi(3*arcsinh(x*a))*arcsinh(x*
a)-cosh(3*arcsinh(x*a))-3*(a^2*x^2+1)^(1/2))/arcsinh(x*a)
```

Fricas [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{a^2cx^2+c}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx=c\left(\int \frac{a^2x^2}{\operatorname{asinh}^2\left(ax\right)}\,dx+\int \frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/asinh(a*x)**2,x)
```

output
```
c*(Integral(a**2*x**2/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Maxima [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{a^2cx^2+c}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 + 2*a^2*c*x^2 + c)*sqrt(a^2
*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2
 + 1))) + integrate((3*a^6*c*x^6 + 7*a^4*c*x^4 + 5*a^2*c*x^2 + (3*a^4*c*x^
4 + 2*a^2*c*x^2 - c)*(a^2*x^2 + 1) + 3*(2*a^5*c*x^5 + 3*a^3*c*x^3 + a*c*x)
*sqrt(a^2*x^2 + 1) + c)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*
(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{c\,a^2\,x^2+c}{\operatorname{asinh}(a\,x)^2}\,dx$$

input
```
int((c + a^2*c*x^2)/asinh(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)/asinh(a*x)^2, x)
```

Reduce [F]

$$\int \frac{c+a^2cx^2}{\operatorname{arcsinh}(ax)^2}\,dx = c\left(\left(\int \frac{x^2}{asinh\,(ax)^2}dx\right)a^2 + \int \frac{1}{asinh\,(ax)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)/asinh(a*x)^2,x)
```

output
```
c*(int(x**2/asinh(a*x)**2,x)*a**2 + int(1/asinh(a*x)**2,x))
```

3.25 $\int \frac{1}{(c+a^2cx^2)\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \text{Int}\left(\frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^2 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{a\int \frac{x}{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}dx}{c} - \frac{1}{ac\sqrt{a^2x^2+1}\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{a\int \frac{x}{(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}dx}{c} - \frac{1}{ac\sqrt{a^2x^2+1}\text{arcsinh}(ax)}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2c\,x^2+c)\,\text{arcsinh}\,(xa)^2} dx$$

input
```
int(1/(a^2*c*x^2+c)/arcsinh(x*a)^2,x)
```

output
```
int(1/(a^2*c*x^2+c)/arcsinh(x*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\mathrm{arcsinh}(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)\,\mathrm{arsinh}\,(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+a^2cx^2)\,\mathrm{arcsinh}(ax)^2}\,dx=\frac{\int \frac{1}{a^2x^2\,\mathrm{asinh}^2\,(ax)+\mathrm{asinh}^2\,(ax)}\,dx}{c}$$

input
```
integrate(1/(a**2*c*x**2+c)/asinh(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**2*asinh(a*x)**2 + asinh(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.89

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*x + sqrt(a^2*x^2 + 1))/((a^3*c*x^2 + sqrt(a^2*x^2 + 1)*a^2*c*x + a*c)*
log(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^4*x^4 + (a^2*x^2 + 1)^2 + (2*
a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^6*c*x^6 + 3*a^4*c*x^4 + 3*a^2*c*
x^2 + (a^4*c*x^4 + a^2*c*x^2)*(a^2*x^2 + 1) + 2*(a^5*c*x^5 + 2*a^3*c*x^3 +
 a*c*x)*sqrt(a^2*x^2 + 1) + c)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{\text{asinh}\,(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{(c+a^2cx^2)\,\text{arcsinh}(ax)^2}\,dx = \frac{\int \frac{1}{asinh(ax)^2a^2x^2+asinh(ax)^2}dx}{c}$$

input
```
int(1/(a^2*c*x^2+c)/asinh(a*x)^2,x)
```

output
```
int(1/(asinh(a*x)**2*a**2*x**2 + asinh(a*x)**2),x)/c
```

3.26 $\int \frac{1}{(c+a^2cx^2)^2\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{1}{(c+a^2cx^2)^2\operatorname{arcsinh}(ax)^2}\,dx = \operatorname{Int}\left(\frac{1}{(c+a^2cx^2)^2\operatorname{arcsinh}(ax)^2},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{(c+a^2cx^2)^2\operatorname{arcsinh}(ax)^2}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{3a\int \frac{x}{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)}dx}{c^2}-\frac{1}{ac^2(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{3a\int \frac{x}{(a^2x^2+1)^{5/2}\text{arcsinh}(ax)}dx}{c^2}-\frac{1}{ac^2(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2c\,x^2+c)^2 \operatorname{arcsinh}(xa)^2}dx$$

input
```
int(1/(a^2*c*x^2+c)^2/arcsinh(x*a)^2,x)
```

output
```
int(1/(a^2*c*x^2+c)^2/arcsinh(x*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{asinh}^2(ax)+2a^2x^2 \operatorname{asinh}^2(ax)+\operatorname{asinh}^2(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**4*asinh(a*x)**2 + 2*a**2*x**2*asinh(a*x)**2 + asinh(a*
x)**2), x)/c**2
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 325, normalized size of antiderivative = 17.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*x + sqrt(a^2*x^2 + 1))/((a^5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2 + (a^4*c^
2*x^3 + a^2*c^2*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))) - inte
grate((3*a^4*x^4 + 2*a^2*x^2 + (3*a^2*x^2 + 1)*(a^2*x^2 + 1) + 3*(2*a^3*x^
3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^8*c^2*x^8 + 4*a^6*c^2*x^6 + 6*a^4*c^2*
x^4 + 4*a^2*c^2*x^2 + (a^6*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a^2*x^2
 + 1) + c^2 + 2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*sq
rt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2} dx = \int \frac{1}{(a^2cx^2+c)^2 \operatorname{arsinh}(ax)^2} dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^2\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{1}{(c+a^2cx^2)^2 \operatorname{arcsinh}(ax)^2}\,dx = \frac{\int \frac{1}{asinh(ax)^2a^4x^4+2asinh(ax)^2a^2x^2+asinh(ax)^2}dx}{c^2}$$

input
```
int(1/(a^2*c*x^2+c)^2/asinh(a*x)^2,x)
```

output
```
int(1/(asinh(a*x)**2*a**4*x**4 + 2*asinh(a*x)**2*a**2*x**2 + asinh(a*x)**2
),x)/c**2
```

3.27 $\int \frac{(d+c^2dx^2)^3}{a+b\operatorname{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 269

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3}{a+b\operatorname{arcsinh}(cx)}\,dx &= \frac{35d^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc} \\ &+ \frac{21d^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc} \\ &+ \frac{7d^3\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc} \\ &+ \frac{d^3\cosh\left(\frac{7a}{b}\right)\operatorname{Chi}\left(\frac{7(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc} \\ &- \frac{35d^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{64bc} \\ &- \frac{21d^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc} \\ &- \frac{7d^3\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc} \\ &- \frac{d^3\sinh\left(\frac{7a}{b}\right)\operatorname{Shi}\left(\frac{7(a+b\operatorname{arcsinh}(cx))}{b}\right)}{64bc}\end{aligned}$$

output
```
35/64*d^3*cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c+21/64*d^3*cosh(3*a/b)*Ch
i(3*(a+b*arcsinh(c*x))/b)/b/c+7/64*d^3*cosh(5*a/b)*Chi(5*(a+b*arcsinh(c*x)
)/b)/b/c+1/64*d^3*cosh(7*a/b)*Chi(7*(a+b*arcsinh(c*x))/b)/b/c-35/64*d^3*si
nh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c-21/64*d^3*sinh(3*a/b)*Shi(3*(a+b*arc
sinh(c*x))/b)/b/c-7/64*d^3*sinh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c-1/6
4*d^3*sinh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.68

$$\int \frac{(d + c^2dx^2)^3}{a + b\text{arcsinh}(cx)} dx$$
$$= \frac{d^3\left(35 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right) + 21 \cosh\left(\frac{3a}{b}\right) \text{Chi}\left(3\left(\frac{a}{b} + \text{arcsinh}(cx)\right)\right) + 7 \cosh\left(\frac{5a}{b}\right) \text{Chi}\left(5\left(\frac{a}{b} + \text{a}\right.\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(35*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 21*Cosh[(3*a)/b]*Cos
hIntegral[3*(a/b + ArcSinh[c*x])] + 7*Cosh[(5*a)/b]*CoshIntegral[5*(a/b +
ArcSinh[c*x])] + Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 35*S
inh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 21*Sinh[(3*a)/b]*SinhIntegral[
3*(a/b + ArcSinh[c*x])] - 7*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*
x])] - Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])]))/(64*b*c)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^3}{a+b\text{arcsinh}(cx)}\,dx$$

$\Big\downarrow$ 6206

$$\frac{d^3\int \frac{\cosh^7\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$\Big\downarrow$ 3042

$$\frac{d^3\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^7}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$\Big\downarrow$ 3793

$$\frac{d^3\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{7\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{21\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{35\cosh\left(\frac{a}{b}-\frac{a+b\text{arc}}{}\right.}{64(a+b\text{arcsin}}\right.}{bc}$$

$\Big\downarrow$ 2009

$$d^3\left(\frac{35}{64}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{21}{64}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{7}{64}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right.\right.$$

input
```
Int[(d + c^2*d*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*((35*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/64 + (21*Cosh[(3
*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 + (7*Cosh[(5*a)/b]*Cos
hIntegral[(5*(a + b*ArcSinh[c*x]))/b])/64 + (Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/64 - (35*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/64 - (21*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])
/64 - (7*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/64 - (Sin
h[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSinh[c*x]))/b])/64))/(b*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.90

method	result
derivativedivides	$-\frac{d^3 e^{\frac{7a}{b}} \operatorname{expIntegral}_1\left(7 \operatorname{arcsinh}(xc)+\frac{7a}{b}\right)}{128b}-\frac{7d^3 e^{\frac{5a}{b}} \operatorname{expIntegral}_1\left(5 \operatorname{arcsinh}(xc)+\frac{5a}{b}\right)}{128b}-\frac{21d^3 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3 \operatorname{arcsinh}(xc)+\frac{3a}{b}\right.}{128b}$
default	$-\frac{d^3 e^{\frac{7a}{b}} \operatorname{expIntegral}_1\left(7 \operatorname{arcsinh}(xc)+\frac{7a}{b}\right)}{128b}-\frac{7d^3 e^{\frac{5a}{b}} \operatorname{expIntegral}_1\left(5 \operatorname{arcsinh}(xc)+\frac{5a}{b}\right)}{128b}-\frac{21d^3 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3 \operatorname{arcsinh}(xc)+\frac{3a}{b}\right.}{128b}$

input
```
int((c^2*d*x^2+d)^3/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/128*d^3/b*exp(7*a/b)*Ei(1,7*arcsinh(x*c)+7*a/b)-7/128*d^3/b*exp(5*
a/b)*Ei(1,5*arcsinh(x*c)+5*a/b)-21/128*d^3/b*exp(3*a/b)*Ei(1,3*arcsinh(x*c
)+3*a/b)-35/128*d^3/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-35/128*d^3/b*exp(-a/
b)*Ei(1,-arcsinh(x*c)-a/b)-21/128*d^3/b*exp(-3*a/b)*Ei(1,-3*arcsinh(x*c)-3
*a/b)-7/128*d^3/b*exp(-5*a/b)*Ei(1,-5*arcsinh(x*c)-5*a/b)-1/128*d^3/b*exp(
-7*a/b)*Ei(1,-7*arcsinh(x*c)-7*a/b))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3)/(b*arcsinh(c*
x) + a), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3}{a+b\text{arcsinh}(cx)}\,dx=d^3\left(\int \frac{3c^2x^2}{a+b\,\text{asinh}\,(cx)}\,dx+\int \frac{3c^4x^4}{a+b\,\text{asinh}\,(cx)}\,dx+\int \frac{c^6x^6}{a+b\,\text{asinh}\,(cx)}\,dx+\int \frac{1}{a+b\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3/(a+b*asinh(c*x)),x)
```

output
```
d**3*(Integral(3*c**2*x**2/(a + b*asinh(c*x)), x) + Integral(3*c**4*x**4/(
a + b*asinh(c*x)), x) + Integral(c**6*x**6/(a + b*asinh(c*x)), x) + Integr
al(1/(a + b*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d\,c^2\,x^2+d)^3}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x)),x)
```

output
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^3}{a+b\text{arcsinh}(cx)}\,dx=d^3\left(\left(\int \frac{x^6}{asinh\,(cx)\,b+a}dx\right)c^6+3\left(\int \frac{x^4}{asinh\,(cx)\,b+a}dx\right)c^4+3\left(\int \frac{x^2}{asinh\,(cx)\,b+a}dx\right)c^2+\int \frac{1}{asinh\,(cx)\,b+a}dx\right)$$

input
```
int((c^2*d*x^2+d)^3/(a+b*asinh(c*x)),x)
```

output
```
d**3*(int(x**6/(asinh(c*x)*b + a),x)*c**6 + 3*int(x**4/(asinh(c*x)*b + a),
x)*c**4 + 3*int(x**2/(asinh(c*x)*b + a),x)*c**2 + int(1/(asinh(c*x)*b + a)
,x))
```

3.28 $\int \frac{(d+c^2dx^2)^2}{a+b\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\begin{aligned}\int \frac{(d+c^2dx^2)^2}{a+b\text{arcsinh}(cx)}\,dx &= \frac{5d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc} \\ &+ \frac{5d^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} \\ &+ \frac{d^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} \\ &- \frac{5d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc} \\ &- \frac{5d^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc} \\ &- \frac{d^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc}\end{aligned}$$

output
```
5/8*d^2*cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c+5/16*d^2*cosh(3*a/b)*Chi(3
*(a+b*arcsinh(c*x))/b)/b/c+1/16*d^2*cosh(5*a/b)*Chi(5*(a+b*arcsinh(c*x))/b
)/b/c-5/8*d^2*sinh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c-5/16*d^2*sinh(3*a/b)
*Shi(3*(a+b*arcsinh(c*x))/b)/b/c-1/16*d^2*sinh(5*a/b)*Shi(5*(a+b*arcsinh(c
*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int \frac{(d+c^2dx^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{d^2\left(10\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+5\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\text{arcs}\right.\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(10*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 5*Cosh[(3*a)/b]*Cosh
Integral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Arc
Sinh[c*x])] - 10*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 5*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])]))/(16*b*c)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{d^2\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$\Big\downarrow$ 3042

$$\frac{d^2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^5}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$\Big\downarrow$ 3793

$$\frac{d^2\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{5\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc}$$

$\Big\downarrow$ 2009

$$\frac{d^2\left(\frac{5}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{5}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\right.}{bc}$$

input
```
Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*((5*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/8 + (5*Cosh[(3*a)
/b]*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh[(5*a)/b]*CoshInte
gral[(5*(a + b*ArcSinh[c*x]))/b])/16 - (5*Sinh[a/b]*SinhIntegral[(a + b*Ar
cSinh[c*x])/b])/8 - (5*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))
/b])/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16))/(b
*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{d^2 e^{\frac{5a}{b}} \operatorname{expIntegral}_1\left(5 \operatorname{arcsinh}(xc)+\frac{5a}{b}\right)}{32b}-\frac{5d^2 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3 \operatorname{arcsinh}(xc)+\frac{3a}{b}\right)}{32b}-\frac{5d^2 e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)}{16b}-5$
default	$-\frac{d^2 e^{\frac{5a}{b}} \operatorname{expIntegral}_1\left(5 \operatorname{arcsinh}(xc)+\frac{5a}{b}\right)}{32b}-\frac{5d^2 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3 \operatorname{arcsinh}(xc)+\frac{3a}{b}\right)}{32b}-\frac{5d^2 e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)}{16b}-5$

input
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/32*d^2/b*exp(5*a/b)*Ei(1,5*arcsinh(x*c)+5*a/b)-5/32*d^2/b*exp(3*a/
b)*Ei(1,3*arcsinh(x*c)+3*a/b)-5/16*d^2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-5
/16*d^2/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)-5/32*d^2/b*exp(-3*a/b)*Ei(1,-3
*arcsinh(x*c)-3*a/b)-1/32*d^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(x*c)-5*a/b))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2}{a+b\text{arcsinh}(cx)}\,dx = d^2\left(\int \frac{2c^2x^2}{a+b\,\text{asinh}\,(cx)}\,dx+\int \frac{c^4x^4}{a+b\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{a+b\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x)),x)
```

output
```
d**2*(Integral(2*c**2*x**2/(a + b*asinh(c*x)), x) + Integral(c**4*x**4/(a
+ b*asinh(c*x)), x) + Integral(1/(a + b*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(c^2dx^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(c^2dx^2+d)^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(d\,c^2\,x^2+d)^2}{a+b\operatorname{asinh}(c\,x)}\,dx$$

input
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x)),x)
```

output
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = d^2\left(\left(\int \frac{x^4}{asinh(cx)\,b+a}dx\right)c^4+2\left(\int \frac{x^2}{asinh(cx)\,b+a}dx\right)c^2 + \int \frac{1}{asinh(cx)\,b+a}dx\right)$$

input
```
int((c^2*d*x^2+d)^2/(a+b*asinh(c*x)),x)
```

output
```
d**2*(int(x**4/(asinh(c*x)*b + a),x)*c**4 + 2*int(x**2/(asinh(c*x)*b + a),
x)*c**2 + int(1/(asinh(c*x)*b + a),x))
```

3.29 $\int \frac{d+c^2dx^2}{a+b\,\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{d+c^2dx^2}{a+b\,\text{arcsinh}(cx)}\,dx=\frac{3d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc}+\frac{d\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc}-\frac{3d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc}-\frac{d\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc}$$

output
```
3/4*d*cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c+1/4*d*cosh(3*a/b)*Chi(3*(a+b
*arcsinh(c*x))/b)/b/c-3/4*d*sinh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c-1/4*d*
sinh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{d + c^2dx^2}{a + b\text{arcsinh}(cx)}\,dx$$
$$= \frac{d\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b} + \text{arcsinh}(cx)\right)\right) - 3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b} + \text{arcsinh}(c\right.\right.}{4bc}$$

input
```
Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcSinh[c*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]]
 - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])]))/(4*b*c)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2dx^2 + d}{a + b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{d\int \frac{\cosh^3\left(\frac{a}{b} - \frac{a + b\text{arcsinh}(cx)}{b}\right)}{a + b\text{arcsinh}(cx)}d(a + b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{\sin\left(\frac{ia}{b} - \frac{i(a + b\text{arcsinh}(cx))}{b} + \frac{\pi}{2}\right)^3}{a + b\text{arcsinh}(cx)}d(a + b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$\frac{d\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(\frac{3}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{3}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\text{si}\right.}{bc}$$

input
```
Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d*((3*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/4 + (Cosh[(3*a)/b]*
CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b])/4 - (3*Sinh[a/b]*SinhIntegral[(a
 + b*ArcSinh[c*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/4))/(b*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{d\,e^{\frac{3a}{b}}\operatorname{expIntegral}_1\left(3\,\operatorname{arcsinh}(xc)+\frac{3a}{b}\right)}{8b}-\frac{3d\,e^{\frac{a}{b}}\operatorname{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)}{8b}-\frac{3d\,e^{-\frac{a}{b}}\operatorname{expIntegral}_1\left(-\operatorname{arcsinh}(xc)-\frac{a}{b}\right)}{8b}-\frac{d\,e^{-}}{}}{c}$
default	$\frac{-\frac{d\,e^{\frac{3a}{b}}\operatorname{expIntegral}_1\left(3\,\operatorname{arcsinh}(xc)+\frac{3a}{b}\right)}{8b}-\frac{3d\,e^{\frac{a}{b}}\operatorname{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)}{8b}-\frac{3d\,e^{-\frac{a}{b}}\operatorname{expIntegral}_1\left(-\operatorname{arcsinh}(xc)-\frac{a}{b}\right)}{8b}-\frac{d\,e^{-}}{}}{c}$

input
```
int((c^2*d*x^2+d)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/8*d/b*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)-3/8*d/b*exp(a/b)*Ei(1,
arcsinh(x*c)+a/b)-3/8*d/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)-1/8*d/b*exp(-3
*a/b)*Ei(1,-3*arcsinh(x*c)-3*a/b))
```

Fricas [F]

$$\int \frac{d+c^2dx^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{c^2dx^2+d}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d+c^2dx^2}{a+b\text{arcsinh}(cx)}\,dx = d\left(\int \frac{c^2x^2}{a+b\operatorname{asinh}(cx)}\,dx+\int \frac{1}{a+b\operatorname{asinh}(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)/(a+b*asinh(c*x)),x)
```

output
```
d*(Integral(c**2*x**2/(a + b*asinh(c*x)), x) + Integral(1/(a + b*asinh(c*x
)), x))
```

Maxima [F]

$$\int \frac{d+c^2dx^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{c^2dx^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+c^2dx^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{c^2dx^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+c^2dx^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{d\,c^2\,x^2+d}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{d + c^2dx^2}{a + b\text{arcsinh}(cx)} \, dx = d\left(\left(\int \frac{x^2}{asinh\left(cx\right)b + a}dx\right)c^2 + \int \frac{1}{asinh\left(cx\right)b + a}dx\right)$$

input
```
int((c^2*d*x^2+d)/(a+b*asinh(c*x)),x)
```

output
```
d*(int(x**2/(asinh(c*x)*b + a),x)*c**2 + int(1/(asinh(c*x)*b + a),x))
```

3.30 $\int \frac{1}{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(c^2dx^2+d)(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c^2d\,x^2+d)\,(a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}\,dx = \frac{\int \frac{1}{ac^2x^2+a+bc^2x^2\,\text{asinh}\,(cx)+b\,\text{asinh}\,(cx)}\,dx}{d}$$

input
```
integrate(1/(c**2*d*x**2+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(a*c**2*x**2 + a + b*c**2*x**2*asinh(c*x) + b*asinh(c*x)), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,(d\,c^2\,x^2+d)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\mathrm{arcsinh}(cx))}\,dx = \frac{\int \frac{1}{asinh(cx)b\,c^2x^2+asinh(cx)b+a\,c^2x^2+a}dx}{d}$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b*c**2*x**2 + asinh(c*x)*b + a*c**2*x**2 + a),x)/d
```

3.31 $\int \frac{1}{(d+c^2dx^2)^2(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 32.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^2 (a+b\text{arcsinh}(cx))} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(c^2dx^2+d)^2 (a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c^2d\,x^2+d)^2 (a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b
*c^2*d^2*x^2 + b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \frac{\int \frac{1}{ac^4x^4+2ac^2x^2+a+bc^4x^4\,\text{asinh}\,(cx)+2bc^2x^2\,\text{asinh}\,(cx)+b\,\text{asinh}\,(cx)}\, dx}{d^2}$$

input
```
integrate(1/(c**2*d*x**2+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(a*c**4*x**4 + 2*a*c**2*x**2 + a + b*c**4*x**4*asinh(c*x) + 2*b
*c**2*x**2*asinh(c*x) + b*asinh(c*x)), x)/d**2
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^2} \, dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{1}{\left(d+c^2dx^2\right)^2\left(a+b\text{arcsinh}(cx)\right)}\,dx = \frac{\int \frac{1}{asinh(cx)b\,c^4x^4+2asinh(cx)b\,c^2x^2+asinh(cx)b+a\,c^4x^4+2a\,c^2x^2+a}dx}{d^2}$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asinh(c*x)*b +
a*c**4*x**4 + 2*a*c**2*x**2 + a),x)/d**2
```

3.32 $\int \frac{(d+c^2dx^2)^3}{(a+b\,\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 303

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{d^3(1+c^2x^2)^{7/2}}{bc(a+b\text{arcsinh}(cx))} - \frac{35d^3\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64b^2c} \\
&- \frac{63d^3\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{64b^2c} \\
&- \frac{35d^3\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{64b^2c} \\
&- \frac{7d^3\text{Chi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{7a}{b}\right)}{64b^2c} \\
&+ \frac{35d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{64b^2c} \\
&+ \frac{63d^3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c} \\
&+ \frac{35d^3\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c} \\
&+ \frac{7d^3\cosh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64b^2c}\end{aligned}$$

output
```
-d^3*(c^2*x^2+1)^(7/2)/b/c/(a+b*arcsinh(c*x))-35/64*d^3*Chi((a+b*arcsinh(c
*x))/b)*sinh(a/b)/b^2/c-63/64*d^3*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/
b^2/c-35/64*d^3*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c-7/64*d^3*Chi
(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b^2/c+35/64*d^3*cosh(a/b)*Shi((a+b*ar
csinh(c*x))/b)/b^2/c+63/64*d^3*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2
/c+35/64*d^3*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b^2/c+7/64*d^3*cosh(7
*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.01

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$= \frac{d^3\left(-\frac{64b\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \frac{192bc^2x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \frac{192bc^4x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \frac{64bc^6x^6\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - 35\text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)\sinh\left(\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3/(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^3*((-64*b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) - (192*b*c^2*x^2*Sqrt
[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) - (192*b*c^4*x^4*Sqrt[1 + c^2*x^2])/(a
 + b*ArcSinh[c*x]) - (64*b*c^6*x^6*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])
 - 35*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 63*CoshIntegral[3*(a/b
+ ArcSinh[c*x])]*Sinh[(3*a)/b] - 35*CoshIntegral[5*(a/b + ArcSinh[c*x])]*S
inh[(5*a)/b] - 7*CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] + 35*C
osh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 63*Cosh[(3*a)/b]*SinhIntegral[
3*(a/b + ArcSinh[c*x])] + 35*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c
*x])] + 7*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])]))/(64*b^2*c)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^3}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{7cd^3\int \frac{x\left(c^2x^2+1\right)^{5/2}}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{d^3\left(c^2x^2+1\right)^{7/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{7d^3\int -\frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d^3\left(c^2x^2+1\right)^{7/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{7d^3\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d^3\left(c^2x^2+1\right)^{7/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{7d^3\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{a}{b}-\frac{a+b\text{arc}}{}\right.}{64(a+b\text{arcsi}}\right.}{b^2c}$$

$$\frac{d^3\left(c^2x^2+1\right)^{7/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{7d^3\left(-\frac{5}{64}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{9}{64}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{5}{64}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx}{b}\right.\right.}{}$$

$$\frac{d^3\left(c^2x^2+1\right)^{7/2}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + c^2*d*x^2)^3/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d^3*(1 + c^2*x^2)^(7/2))/(b*c*(a + b*ArcSinh[c*x]))) + (7*d^3*((-5*Cosh
Integral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/64 - (9*CoshIntegral[(3*(a + b
*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/64 - (5*CoshIntegral[(5*(a + b*ArcSinh[c
*x]))/b]*Sinh[(5*a)/b])/64 - (CoshIntegral[(7*(a + b*ArcSinh[c*x]))/b]*Sin
h[(7*a)/b])/64 + (5*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/64 + (
9*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/64 + (5*Cosh[(5*
a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/64 + (Cosh[(7*a)/b]*SinhIn
tegral[(7*(a + b*ArcSinh[c*x]))/b])/64))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(285) = 570$.

Time = 3.88 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.79

method	result
derivativedivides	$\frac{\left(-64x^6c^6\sqrt{c^2x^2+1}+64x^7c^7-80x^4c^4\sqrt{c^2x^2+1}+112x^5c^5-24x^2c^2\sqrt{c^2x^2+1}+56x^3c^3-\sqrt{c^2x^2+1}+7xc\right)d^3}{128b(a+b\ \mathrm{arcsinh}(xc))}+\frac{7d^3\mathrm{e}^{\frac{7a}{b}}\ \mathrm{expIntegral}_1}{128}$
default	$\frac{\left(-64x^6c^6\sqrt{c^2x^2+1}+64x^7c^7-80x^4c^4\sqrt{c^2x^2+1}+112x^5c^5-24x^2c^2\sqrt{c^2x^2+1}+56x^3c^3-\sqrt{c^2x^2+1}+7xc\right)d^3}{128b(a+b\ \mathrm{arcsinh}(xc))}+\frac{7d^3\mathrm{e}^{\frac{7a}{b}}\ \mathrm{expIntegral}_1}{128}$

input
```
int((c^2*d*x^2+d)^3/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/128*(-64*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x^7*c^7-80*x^4*c^4*(c^2*x^2+1
)^(1/2)+112*x^5*c^5-24*x^2*c^2*(c^2*x^2+1)^(1/2)+56*x^3*c^3-(c^2*x^2+1)^(1
/2)+7*x*c)*d^3/b/(a+b*arcsinh(x*c))+7/128*d^3/b^2*exp(7*a/b)*Ei(1,7*arcsin
h(x*c)+7*a/b)+7/128*(-16*x^4*c^4*(c^2*x^2+1)^(1/2)+16*x^5*c^5-12*x^2*c^2*(
c^2*x^2+1)^(1/2)+20*x^3*c^3-(c^2*x^2+1)^(1/2)+5*x*c)*d^3/b/(a+b*arcsinh(x*
c))+35/128*d^3/b^2*exp(5*a/b)*Ei(1,5*arcsinh(x*c)+5*a/b)+21/128*(-4*x^2*c^
2*(c^2*x^2+1)^(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3*x*c)*d^3/b/(a+b*arcsinh(
x*c))+63/128*d^3/b^2*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)+35/128*(x*c-(c^
2*x^2+1)^(1/2))*d^3/b/(a+b*arcsinh(x*c))+35/128*d^3/b^2*exp(a/b)*Ei(1,arcs
inh(x*c)+a/b)-35/128/b*d^3*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-35/1
28/b^2*d^3*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)-21/128/b*d^3*(4*x^3*c^3+3*x*c
+4*x^2*c^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-63/128/
b^2*d^3*exp(-3*a/b)*Ei(1,-3*arcsinh(x*c)-3*a/b)-7/128/b*d^3*(16*x^5*c^5+20
*x^3*c^3+16*x^4*c^4*(c^2*x^2+1)^(1/2)+5*x*c+12*x^2*c^2*(c^2*x^2+1)^(1/2)+(
c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-35/128/b^2*d^3*exp(-5*a/b)*Ei(1,-5*ar
csinh(x*c)-5*a/b)-1/128/b*d^3*(64*x^7*c^7+112*x^5*c^5+64*x^6*c^6*(c^2*x^2+
1)^(1/2)+56*x^3*c^3+80*x^4*c^4*(c^2*x^2+1)^(1/2)+7*x*c+24*x^2*c^2*(c^2*x^2
+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-7/128/b^2*d^3*exp(-7*a/b)*
Ei(1,-7*arcsinh(x*c)-7*a/b))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3)/(b^2*arcsinh(
c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx = d^3\left(\int \frac{3c^2x^2}{a^2+2ab\,\text{asinh}\,(cx)+b^2\,\text{asinh}^2\,(cx)}\,dx + \int \frac{3c^4x^4}{a^2+2ab\,\text{asinh}\,(cx)+b^2\,\text{asinh}^2\,(cx)}\,dx + \int \frac{c^6x^6}{a^2+2ab\,\text{asinh}\,(cx)+b^2\,\text{asinh}^2\,(cx)}\,dx + \int \frac{1}{a^2+2ab\,\text{asinh}\,(cx)+b^2\,\text{asinh}^2\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3/(a+b*asinh(c*x))**2,x)
```

output
```
d**3*(Integral(3*c**2*x**2/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2),
 x) + Integral(3*c**4*x**4/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2),
 x) + Integral(c**6*x**6/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2), x
) + Integral(1/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^9*d^3*x^9 + 4*c^7*d^3*x^7 + 6*c^5*d^3*x^5 + 4*c^3*d^3*x^3 + c*d^3*x +
(c^8*d^3*x^8 + 4*c^6*d^3*x^6 + 6*c^4*d^3*x^4 + 4*c^2*d^3*x^2 + d^3)*sqrt(c
^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3
*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
+ integrate((7*c^10*d^3*x^10 + 29*c^8*d^3*x^8 + 46*c^6*d^3*x^6 + 34*c^4*d^
3*x^4 + 11*c^2*d^3*x^2 + d^3 + (7*c^8*d^3*x^8 + 20*c^6*d^3*x^6 + 18*c^4*d^
3*x^4 + 4*c^2*d^3*x^2 - d^3)*(c^2*x^2 + 1) + 7*(2*c^9*d^3*x^9 + 7*c^7*d^3*
x^7 + 9*c^5*d^3*x^5 + 5*c^3*d^3*x^3 + c*d^3*x)*sqrt(c^2*x^2 + 1))/(a*b*c^4
*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c
^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x
)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2dx^2+d)^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^3/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(d\,c^2\,x^2+d)^3}{(a+b\,\mathrm{asinh}(c\,x))^2}\,dx$$

input
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = d^3\left(\left(\int \frac{x^6}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2}dx\right)c^6 + 3\left(\int \frac{x^4}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2}dx\right)c^4 + 3\left(\int \frac{x^2}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2}dx\right)c^2 + \int \frac{1}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2}dx\right)$$

input
```
int((c^2*d*x^2+d)^3/(a+b*asinh(c*x))^2,x)
```

output
```
d**3*(int(x**6/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**6 + 3*
int(x**4/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**4 + 3*int(x*
*2/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**2 + int(1/(asinh(c
*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x))
```

3.33 $\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 235

$$
\begin{aligned}
\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = & -\frac{d^2(1+c^2x^2)^{5/2}}{bc(a+b\text{arcsinh}(cx))} - \frac{5d^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c} \\
& - \frac{15d^2\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c} \\
& - \frac{5d^2\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c} \\
& + \frac{5d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c} \\
& + \frac{15d^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c} \\
& + \frac{5d^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c}
\end{aligned}
$$

output
```
-d^2*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))-5/8*d^2*Chi((a+b*arcsinh(c*x
))/b)*sinh(a/b)/b^2/c-15/16*d^2*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^
2/c-5/16*d^2*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c+5/8*d^2*cosh(a/
b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c+15/16*d^2*cosh(3*a/b)*Shi(3*(a+b*arcsin
h(c*x))/b)/b^2/c+5/16*d^2*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

$$\int \frac{(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \frac{d^2\left(-\frac{16b\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}-\frac{32bc^2x^2\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}-\frac{16bc^4x^4\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}-10\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-15\operatorname{Chi}\left(3\left(\frac{a}{b}\right.\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*((-16*b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) - (32*b*c^2*x^2*Sqrt[
1 + c^2*x^2])/(a + b*ArcSinh[c*x]) - (16*b*c^4*x^4*Sqrt[1 + c^2*x^2])/(a +
 b*ArcSinh[c*x]) - 10*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 15*Cosh
Integral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 5*CoshIntegral[5*(a/b + A
rcSinh[c*x])]*Sinh[(5*a)/b] + 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]
] + 15*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Cosh[(5*a)/b
]*SinhIntegral[5*(a/b + ArcSinh[c*x])]))/(16*b^2*c)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^2}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \ \textcolor{blue}{6205}$$

$$\frac{5cd^2\int \frac{x\left(c^2x^2+1\right)^{3/2}}{a+b\text{arcsinh}(cx)}dx}{b}-\frac{d^2\left(c^2x^2+1\right)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ \textcolor{blue}{6234}$$

$$\frac{5d^2\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d^2\left(c^2x^2+1\right)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ \textcolor{blue}{25}$$

$$-\frac{5d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d^2\left(c^2x^2+1\right)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ \textcolor{blue}{5971}$$

$$-\frac{5d^2\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx)}{b^2c}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \ \textcolor{blue}{2009}$$

$$\frac{5d^2\left(-\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{3}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx)}{b}\right.\right.}{}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d^2*(1 + c^2*x^2)^(5/2))/(b*c*(a + b*ArcSinh[c*x]))) + (5*d^2*(-1/8*(Co
shIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b]) - (3*CoshIntegral[(3*(a + b*
ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegral[(5*(a + b*ArcSinh[c*x]
))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])
/8 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/16 + (Cosh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/16))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(221) = 442.

Time = 3.07 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.35

method	result
derivativedivides	$\frac{\left(-16x^4c^4\sqrt{c^2x^2+1}+16x^5c^5-12x^2c^2\sqrt{c^2x^2+1}+20x^3c^3-\sqrt{c^2x^2+1}+5xc\right)d^2}{32b(a+b\,\mathrm{arcsinh}(xc))}+\frac{5d^2\mathrm{e}^{\frac{5a}{b}}\,\mathrm{expIntegral}_1\left(5\,\mathrm{arcsinh}(xc)+\frac{5a}{b}\right)}{32b^2}+\frac{5(-4}{}$
default	$\frac{\left(-16x^4c^4\sqrt{c^2x^2+1}+16x^5c^5-12x^2c^2\sqrt{c^2x^2+1}+20x^3c^3-\sqrt{c^2x^2+1}+5xc\right)d^2}{32b(a+b\,\mathrm{arcsinh}(xc))}+\frac{5d^2\mathrm{e}^{\frac{5a}{b}}\,\mathrm{expIntegral}_1\left(5\,\mathrm{arcsinh}(xc)+\frac{5a}{b}\right)}{32b^2}+\frac{5(-4}{}$

input
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/32*(-16*x^4*c^4*(c^2*x^2+1)^(1/2)+16*x^5*c^5-12*x^2*c^2*(c^2*x^2+1)
^(1/2)+20*x^3*c^3-(c^2*x^2+1)^(1/2)+5*x*c)*d^2/b/(a+b*arcsinh(x*c))+5/32*d
^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(x*c)+5*a/b)+5/32*(-4*x^2*c^2*(c^2*x^2+1)^
(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3*x*c)*d^2/b/(a+b*arcsinh(x*c))+15/32*d^
2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)+5/16*(x*c-(c^2*x^2+1)^(1/2))*d
^2/b/(a+b*arcsinh(x*c))+5/16*d^2/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-5/16/
b*d^2*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-5/16/b^2*d^2*exp(-a/b)*Ei
(1,-arcsinh(x*c)-a/b)-5/32/b*d^2*(4*x^3*c^3+3*x*c+4*x^2*c^2*(c^2*x^2+1)^(1
/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-15/32/b^2*d^2*exp(-3*a/b)*Ei(1,-
3*arcsinh(x*c)-3*a/b)-1/32/b*d^2*(16*x^5*c^5+20*x^3*c^3+16*x^4*c^4*(c^2*x^
2+1)^(1/2)+5*x*c+12*x^2*c^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcs
inh(x*c))-5/32/b^2*d^2*exp(-5*a/b)*Ei(1,-5*arcsinh(x*c)-5*a/b))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^2}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*a
rcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = d^2\left(\int \frac{2c^2x^2}{a^2+2ab\operatorname{asinh}(cx)+b^2\operatorname{asinh}^2(cx)}\,dx + \int \frac{c^4x^4}{a^2+2ab\operatorname{asinh}(cx)+b^2\operatorname{asinh}^2(cx)}\,dx + \int \frac{1}{a^2+2ab\operatorname{asinh}(cx)+b^2\operatorname{asinh}^2(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
d**2*(Integral(2*c**2*x**2/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2),
 x) + Integral(c**4*x**4/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2), x
) + Integral(1/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^7*d^2*x^7 + 3*c^5*d^2*x^5 + 3*c^3*d^2*x^3 + c*d^2*x + (c^6*d^2*x^6 + 3
*c^4*d^2*x^4 + 3*c^2*d^2*x^2 + d^2)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt
(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2
*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((5*c^8*d^2*x^8 + 16*
c^6*d^2*x^6 + 18*c^4*d^2*x^4 + 8*c^2*d^2*x^2 + (5*c^6*d^2*x^6 + 9*c^4*d^2*
x^4 + 3*c^2*d^2*x^2 - d^2)*(c^2*x^2 + 1) + d^2 + 5*(2*c^7*d^2*x^7 + 5*c^5*
d^2*x^5 + 4*c^3*d^2*x^3 + c*d^2*x)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*
b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x
^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(d\,c^2\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = d^2\left(\left(\int \frac{x^4}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)c^4 + 2\left(\int \frac{x^2}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{1}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)$$

input
```
int((c^2*d*x^2+d)^2/(a+b*asinh(c*x))^2,x)
```

output
```
d**2*(int(x**4/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**4 + 2*
int(x**2/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**2 + int(1/(a
sinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x))
```

3.34 $\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\begin{aligned}\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx = & -\frac{d(1+c^2x^2)^{3/2}}{bc(a+b\text{arcsinh}(cx))}-\frac{3d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c}\\ & -\frac{3d\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c}\\ & +\frac{3d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c}\\ & +\frac{3d\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c}\end{aligned}$$

output
```
-d*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))-3/4*d*Chi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c-3/4*d*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c+3/4*
d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c+3/4*d*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.77

$$\int \frac{d+c^2dx^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$
$$= \frac{d\left(-\frac{4b(1+c^2x^2)^{3/2}}{a+b\operatorname{arcsinh}(cx)}+3\left(-\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)-\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right.\right.\right.}{4b^2c}$$

input
```
Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*((-4*b*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]) + 3*(-(CoshIntegral[a/
b + ArcSinh[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(
3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])])))/(4*b^2*c)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2dx^2+d}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \; 6205$$

$$\frac{3cd\int \frac{x\sqrt{c^2x^2+1}}{a+b\operatorname{arcsinh}(cx)}dx}{b}-\frac{d(c^2x^2+1)^{3/2}}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \; 6234$$

$$\frac{3d\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d(c^2x^2+1)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d(c^2x^2+1)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{3d\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{d(c^2x^2+1)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\frac{1}{4}\right.}{b^2c} - \frac{d(c^2x^2+1)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

input

```
Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

output

```
-((d*(1 + c^2*x^2)^(3/2))/(b*c*(a + b*ArcSinh[c*x]))) + (3*d*(-1/4*(CoshIn
tegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b]) - (CoshIntegral[(3*(a + b*ArcSin
h[c*x]))/b]*Sinh[(3*a)/b])/4 + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x]
)/b])/4 + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/4))/(b^
2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(147) = 294$.

Time = 2.16 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{\frac{\left(-4x^2c^2\sqrt{c^2x^2+1}+4x^3c^3-\sqrt{c^2x^2+1}+3xc\right)d}{8b(a+b\ \mathrm{arcsinh}(xc))}+\frac{3d\,\mathrm{e}^{\frac{3a}{b}}\ \mathrm{expIntegral}_1\left(3\ \mathrm{arcsinh}(xc)+\frac{3a}{b}\right)}{8b^2}+\frac{3\left(xc-\sqrt{c^2x^2+1}\right)d}{8b(a+b\ \mathrm{arcsinh}(xc))}+\frac{3d\,\mathrm{e}^{\frac{a}{b}}\ \mathrm{expInteg}}{}}{}$
default	$\frac{\frac{\left(-4x^2c^2\sqrt{c^2x^2+1}+4x^3c^3-\sqrt{c^2x^2+1}+3xc\right)d}{8b(a+b\ \mathrm{arcsinh}(xc))}+\frac{3d\,\mathrm{e}^{\frac{3a}{b}}\ \mathrm{expIntegral}_1\left(3\ \mathrm{arcsinh}(xc)+\frac{3a}{b}\right)}{8b^2}+\frac{3\left(xc-\sqrt{c^2x^2+1}\right)d}{8b(a+b\ \mathrm{arcsinh}(xc))}+\frac{3d\,\mathrm{e}^{\frac{a}{b}}\ \mathrm{expInteg}}{}}{}$

input
```
int((c^2*d*x^2+d)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/8*(-4*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3*x*c)*
d/b/(a+b*arcsinh(x*c))+3/8*d/b^2*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)+3/8
*(x*c-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(x*c))+3/8*d/b^2*exp(a/b)*Ei(1,ar
csinh(x*c)+a/b)-3/8*d/b*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-3/8*d/b
^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)-1/8*d/b*(4*x^3*c^3+3*x*c+4*x^2*c^2*(c
^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-3/8*d/b^2*exp(-3*a/b
)*Ei(1,-3*arcsinh(x*c)-3*a/b))
```

Fricas [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{c^2dx^2+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^2*d*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
x)
```

Sympy [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx = d\left(\int \frac{c^2x^2}{a^2+2ab\operatorname{asinh}(cx)+b^2\operatorname{asinh}^2(cx)}\,dx + \int \frac{1}{a^2+2ab\operatorname{asinh}(cx)+b^2\operatorname{asinh}^2(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**2,x)
```

output
```
d*(Integral(c**2*x**2/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2), x) +
 Integral(1/(a**2 + 2*a*b*asinh(c*x) + b**2*asinh(c*x)**2), x))
```

Maxima [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{c^2dx^2+d}{\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^5*d*x^5 + 2*c^3*d*x^3 + c*d*x + (c^4*d*x^4 + 2*c^2*d*x^2 + d)*sqrt(c^2
*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x
^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) +
integrate((3*c^6*d*x^6 + 7*c^4*d*x^4 + 5*c^2*d*x^2 + (3*c^4*d*x^4 + 2*c^2*
d*x^2 - d)*(c^2*x^2 + 1) + 3*(2*c^5*d*x^5 + 3*c^3*d*x^3 + c*d*x)*sqrt(c^2*
x^2 + 1) + d)/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a
*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b
^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*
(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{c^2dx^2+d}{\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{d\,c^2\,x^2+d}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^2}\,dx=d\left(\left(\int \frac{x^2}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2}dx\right)c^2+\int \frac{1}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2}dx\right)$$

input
```
int((c^2*d*x^2+d)/(a+b*asinh(c*x))^2,x)
```

output
```
d*(int(x**2/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**2 + int(1
/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x))
```

3.35 $\int \frac{1}{(d+c^2dx^2)(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)\,(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{c\int \frac{x}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx}{bd}-\frac{1}{bcd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{c\int \frac{x}{(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}dx}{bd}-\frac{1}{bcd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c^2d\,x^2+d)\,(a+b\,\text{arcsinh}\,(xc))^2}dx$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \frac{1}{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2
 + 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{1}{(d+c^2dx^2)(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{a^2c^2x^2+a^2+2abc^2x^2\,\text{asinh}\,(cx)+2ab\,\text{asinh}\,(cx)+b^2c^2x^2\,\text{asinh}^2\,(cx)+b^2\,\text{asinh}^2\,(cx)}\,dx}{d}$$

input
```
integrate(1/(c**2*d*x**2+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(a**2*c**2*x**2 + a**2 + 2*a*b*c**2*x**2*asinh(c*x) + 2*a*b*asi
nh(c*x) + b**2*c**2*x**2*asinh(c*x)**2 + b**2*asinh(c*x)**2), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 405, normalized size of antiderivative = 17.61

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/(a*b*c^3*d*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*d*x
+ a*b*c*d + (b^2*c^3*d*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*d*x + b^2*c*d)*log(
c*x + sqrt(c^2*x^2 + 1))) - integrate((c^4*x^4 + (c^2*x^2 + 1)^2 + (2*c^3*
x^3 + c*x)*sqrt(c^2*x^2 + 1) - 1)/(a*b*c^6*d*x^6 + 3*a*b*c^4*d*x^4 + 3*a*b
*c^2*d*x^2 + a*b*d + (a*b*c^4*d*x^4 + a*b*c^2*d*x^2)*(c^2*x^2 + 1) + (b^2*
c^6*d*x^6 + 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 + b^2*d + (b^2*c^4*d*x^4 + b
^2*c^2*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^5*d*x^5 + 2*b^2*c^3*d*x^3 + b^2*c*d
*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^5*d*x^5 + 2
*a*b*c^3*d*x^3 + a*b*c*d*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\text{asinh}\,(c\,x)\right)^2\,\left(d\,c^2\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{asinh(cx)^2b^2c^2x^2+asinh(cx)^2b^2+2asinh(cx)ab\,c^2x^2+2asinh(cx)ab+a^2c^2x^2+a^2}dx}{d}$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*
b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)/d
```

3.36 $\int \frac{1}{(d+c^2dx^2)^2(a+b\,\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 145.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(d+c^2dx^2)^2\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^2 (a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{3c\int \frac{x}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}dx}{bd^2}-\frac{1}{bcd^2 (c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{3c\int \frac{x}{(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))}dx}{bd^2}-\frac{1}{bcd^2 (c^2x^2+1)^{3/2} (a+b\text{arcsinh}(cx))}$$

input
```
Int[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c^2d\,x^2+d)^2 (a+b\,\text{arcsinh}\,(xc))^2}dx$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 5.17

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^2 (b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x
^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*
a*b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 22.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 5.26

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{a^2c^4x^4+2a^2c^2x^2+a^2+2abc^4x^4\,\text{asinh}\,(cx)+4abc^2x^2\,\text{asinh}\,(cx)+2ab\,\text{asinh}\,(cx)+b^2c^4x^4\,\text{asinh}^2\,(cx)+2b^2c^2x^2\,\text{asinh}^2\,(cx)+b^2\,\text{asinh}^2\,(cx)}\,dx}{d^2}$$

input
```
integrate(1/(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(a**2*c**4*x**4 + 2*a**2*c**2*x**2 + a**2 + 2*a*b*c**4*x**4*asi
nh(c*x) + 4*a*b*c**2*x**2*asinh(c*x) + 2*a*b*asinh(c*x) + b**2*c**4*x**4*a
sinh(c*x)**2 + 2*b**2*c**2*x**2*asinh(c*x)**2 + b**2*asinh(c*x)**2), x)/d*
*2
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 609, normalized size of antiderivative = 26.48

$$\int \frac{1}{(d + c^2 d x^2)^2 (a + b \text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(c^2 d x^2 + d)^2 (b \operatorname{arsinh}(cx) + a)^2} \, dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/(a*b*c^5*d^2*x^4 + 2*a*b*c^3*d^2*x^2 + a*b*c*d^
2 + (b^2*c^5*d^2*x^4 + 2*b^2*c^3*d^2*x^2 + b^2*c*d^2 + (b^2*c^4*d^2*x^3 +
b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^4*
d^2*x^3 + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^4*x^4 + 2*c^2
*x^2 + (3*c^2*x^2 + 1)*(c^2*x^2 + 1) + 3*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 +
1) - 1)/(a*b*c^8*d^2*x^8 + 4*a*b*c^6*d^2*x^6 + 6*a*b*c^4*d^2*x^4 + 4*a*b*c
^2*d^2*x^2 + a*b*d^2 + (a*b*c^6*d^2*x^6 + 2*a*b*c^4*d^2*x^4 + a*b*c^2*d^2*
x^2)*(c^2*x^2 + 1) + (b^2*c^8*d^2*x^8 + 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*
x^4 + 4*b^2*c^2*d^2*x^2 + b^2*d^2 + (b^2*c^6*d^2*x^6 + 2*b^2*c^4*d^2*x^4 +
 b^2*c^2*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^7*d^2*x^7 + 3*b^2*c^5*d^2*x^5 +
 3*b^2*c^3*d^2*x^3 + b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^
2 + 1)) + 2*(a*b*c^7*d^2*x^7 + 3*a*b*c^5*d^2*x^5 + 3*a*b*c^3*d^2*x^3 + a*b
*c*d^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d + c^2 d x^2)^2 (a + b \text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(c^2 d x^2 + d)^2 (b \operatorname{arsinh}(cx) + a)^2} \, dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.87

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{asinh(cx)^2 b^2 c^4 x^4+2asinh(cx)^2 b^2 c^2 x^2+asinh(cx)^2 b^2+2asinh(cx)ab\,c^4 x^4+4asinh(cx)ab\,c^2 x^2+2asinh(cx)ab+a^2 c^4 x^4+2a^2 c^2 x^2+a^2}dx}{d^2}$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2*c**4*x**4 + 2*asinh(c*x)**2*b**2*c**2*x**2 + asi
nh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**4*x**4 + 4*asinh(c*x)*a*b*c**2*x**2
+ 2*asinh(c*x)*a*b + a**2*c**4*x**4 + 2*a**2*c**2*x**2 + a**2),x)/d**2
```

3.37 $\int (\pi + c^2\pi x^2)^{5/2} (a + b\mathbf{arcsinh}(cx))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 173

$$\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{5}{32}bc\pi^{5/2}x^2 - \frac{5b\pi^{5/2}(1+c^2x^2)^2}{96c} - \frac{b\pi^{5/2}(1+c^2x^2)^3}{36c} + \frac{5}{16}\pi^2 x\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{5}{24}\pi x(\pi + c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{1}{6}x(\pi + c^2\pi x^2)^{5/2}(a+b\text{arcs}$$

output
```
-5/32*b*c*Pi^(5/2)*x^2-5/96*b*Pi^(5/2)*(c^2*x^2+1)^2/c-1/36*b*Pi^(5/2)*(c^
2*x^2+1)^3/c+5/16*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))+5/24*Pi*
x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*(a+
b*arcsinh(c*x))+5/32*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int (\pi + c^2\pi x^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \frac{\pi^{5/2}\left(1584acx\sqrt{1+c^2x^2} + 1248ac^3x^3\sqrt{1+c^2x^2} + 384ac^5x^5\sqrt{1+c^2x^2} + 360b\text{arcsinh}(\right.}{}$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(5/2)*(1584*a*c*x*Sqrt[1 + c^2*x^2] + 1248*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 360*b*ArcSinh[c*x]^2 - 270*b*Cosh[2*A
rcSinh[c*x]] - 27*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] + 12*A
rcSinh[c*x]*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] +
 b*Sinh[6*ArcSinh[c*x]])))/(2304*c)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6201}$$

$$\frac{5}{6}\pi\int \left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}\pi^{5/2}bc\int x\left(c^2x^2+1\right)^2dx+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{241}$$

$$\frac{5}{6}\pi\int \left(c^2\pi x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$$\Big\downarrow \text{6201}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx\right)\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$$\Big\downarrow \text{244}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 2009

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\right)-$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 6200

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3}\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 15

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a\right.$$
$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

$\big\downarrow$ 6198

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{5}{6}\pi\left(\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\right.\right.$$
$$\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3}{36c}$$

input
```
Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/36*(b*Pi^(5/2)*(1 + c^2*x^2)^3)/c + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/6 + (5*Pi*(-1/4*(b*c*Pi^(3/2)*(x^2/2 + (c^2*x^4)/4)) + (x*(P
i + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*Pi*(-1/4*(b*c*Sqrt[Pi]*
x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi]*(a + b
*ArcSinh[c*x])^2)/(4*b*c)))/4))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.17

method	result
default	$\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6}+\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\pi^{\frac{5}{2}}\left(48\operatorname{arcsinh}(xc)\sqrt{c^2x^2}\right.}{}$
parts	$\frac{ax(\pi c^2x^2+\pi)^{\frac{5}{2}}}{6}+\frac{5a\pi x(\pi c^2x^2+\pi)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\pi^{\frac{5}{2}}\left(48\operatorname{arcsinh}(xc)\sqrt{c^2x^2}\right.}{}$

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(Pi*c^2*x^2+Pi)^(5/2)+5/24*a*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a*Pi^
2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a*Pi^3*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/288*b*Pi^(5/2)*(48*arcsinh(x*c)*(c^2*x^2+1)
^(1/2)*x^5*c^5-8*c^6*x^6+156*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-39*c^4
*x^4+198*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c-99*c^2*x^2+45*arcsinh(x*c)^2-6
8)/c
```

Fricas [F]

$$\int \left(\pi+c^2\pi x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int \left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 15.90 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.53

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \begin{cases} \frac{\pi^{\frac{5}{2}} a c^4 x^5 \sqrt{c^2x^2+1}}{6} + \frac{13\pi^{\frac{5}{2}} a c^2 x^3 \sqrt{c^2x^2+1}}{24} + \frac{11\pi^{\frac{5}{2}} a x \sqrt{c^2x^2+1}}{16} + \frac{5\pi^{\frac{5}{2}} a \operatorname{asinh}(cx)}{16c} - \frac{\pi^{\frac{5}{2}} b c^5 x^6}{36} + \frac{\pi^{\frac{5}{2}} b c^4 x^5}{} \\ \pi^{\frac{5}{2}} a x \end{cases}$$

input
```
integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(5/2)*a*c**4*x**5*sqrt(c**2*x**2 + 1)/6 + 13*pi**(5/2)*a*c*
*2*x**3*sqrt(c**2*x**2 + 1)/24 + 11*pi**(5/2)*a*x*sqrt(c**2*x**2 + 1)/16 +
 5*pi**(5/2)*a*asinh(c*x)/(16*c) - pi**(5/2)*b*c**5*x**6/36 + pi**(5/2)*b*
c**4*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/6 - 13*pi**(5/2)*b*c**3*x**4/96 +
 13*pi**(5/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/24 - 11*pi**(5/2)
*b*c*x**2/32 + 11*pi**(5/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/16 + 5*pi**
(5/2)*b*asinh(c*x)**2/(32*c), Ne(c, 0)), (pi**(5/2)*a*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\,\pi^2\big(8\sqrt{c^2x^2+1}\,a\,c^5x^5 + 26\sqrt{c^2x^2+1}\,a\,c^3x^3 + 33\sqrt{c^2x^2+1}\,acx + 48\big(\int \sqrt{c^2x^2+1}}{}$$

input
```
int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(pi)*pi**2*(8*sqrt(c**2*x**2 + 1)*a*c**5*x**5 + 26*sqrt(c**2*x**2 + 1
)*a*c**3*x**3 + 33*sqrt(c**2*x**2 + 1)*a*c*x + 48*int(sqrt(c**2*x**2 + 1)*
asinh(c*x)*x**4,x)*b*c**5 + 96*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*
b*c**3 + 48*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c + 15*log(sqrt(c**2*x
**2 + 1) + c*x)*a))/(48*c)
```

3.38 $\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = -\frac{3}{16} bc\pi^{3/2}x^2 - \frac{b\pi^{3/2}(1 + c^2x^2)^2}{16c} + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x(\pi+c^2\pi x^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3\pi^{3/2}(a + b\text{arcsinh}(cx))^2}{16bc}$$

output
```
-3/16*b*c*Pi^(3/2)*x^2-1/16*b*Pi^(3/2)*(c^2*x^2+1)^2/c+3/8*Pi*x*(Pi*c^2*x^
2+Pi)^(1/2)*(a+b*arcsinh(c*x))+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*
x))+3/16*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int (\pi + c^2\pi x^2)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \frac{\pi^{3/2}(80acx\sqrt{1 + c^2x^2} + 32ac^3x^3\sqrt{1 + c^2x^2} + 24b\text{arcsinh}(cx)^2 - 16b\cosh(2\text{arcsinh}(cx$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(Pi^(3/2)*(80*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 2
4*b*ArcSinh[c*x]^2 - 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] +
4*ArcSinh[c*x]*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))
)/(128*c)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 244}$$

$$\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}\pi^{3/2}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{4}\pi\int\sqrt{c^2\pi x^2+\pi}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{ 6200}$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2\right)+$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2\right)-$$
$$\frac{1}{4}\pi^{3/2}bc\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)$$

input
```
Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*Pi^(3/2)*(x^2/2 + (c^2*x^4)/4)) + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/4 + (3*Pi*(-1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*P
i*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.28

method	result
default	$\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4}+\frac{3a\pi x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{3a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}}+\frac{b\pi^{\frac{3}{2}}\left(4\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4+10\operatorname{arcsin}\right.}{16c}$
parts	$\frac{ax(\pi c^2x^2+\pi)^{\frac{3}{2}}}{4}+\frac{3a\pi x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{3a\pi^2\ln\left(\frac{\pi c^2x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}}+\frac{b\pi^{\frac{3}{2}}\left(4\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^3c^3-c^4x^4+10\operatorname{arcsin}\right.}{16c}$

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(Pi*c^2*x^2+Pi)^(3/2)+3/8*a*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a*Pi^2*
ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/16*b*Pi
^(3/2)*(4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-c^4*x^4+10*arcsinh(x*c)*(
c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+3*arcsinh(x*c)^2-4)/c
```

Fricas [F]

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b
)*arcsinh(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \begin{cases} \frac{\pi^{\frac{3}{2}} a c^2 x^3 \sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a x \sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a \operatorname{asinh}(cx)}{8c} - \frac{\pi^{\frac{3}{2}} b c^3 x^4}{16} + \frac{\pi^{\frac{3}{2}} b c^2 x^3 \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{4} - 5\pi \\ \pi^{\frac{3}{2}} a x \end{cases}$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((pi**(3/2)*a*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x*s
qrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a*asinh(c*x)/(8*c) - pi**(3/2)*b*c**3*x
**4/16 + pi**(3/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/4 - 5*pi**(3
/2)*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/8 + 3*pi*
*(3/2)*b*asinh(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*a*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int (a + b\,\text{asinh}(c\,x))\, \left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2} dx$$

input
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int (\pi + c^2\pi x^2)^{3/2}(a + b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\,\pi\left(2\sqrt{c^2x^2+1}\,a\,c^3x^3 + 5\sqrt{c^2x^2+1}\,acx + 8\left(\int \sqrt{c^2x^2+1}\,asinh(cx)\,x^2dx\right)b\,c^3 + 8(\right.}{8c}$$

input

```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*asinh(c*x)),x)
```

output

```
(sqrt(pi)*pi*(2*sqrt(c**2*x**2 + 1)*a*c**3*x**3 + 5*sqrt(c**2*x**2 + 1)*a*
c*x + 8*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*c**3 + 8*int(sqrt(c**
2*x**2 + 1)*asinh(c*x),x)*b*c + 3*log(sqrt(c**2*x**2 + 1) + c*x)*a))/(8*c)
```

3.39 $\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = -\frac{1}{4}bc\sqrt{\pi}x^2 + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx)) + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^2}{4bc}$$

output
```
-1/4*b*c*Pi^(1/2)*x^2+1/2*x*(Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))+1/4*P
i^(1/2)*(a+b*arcsinh(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\left(4acx\sqrt{1 + c^2x^2} + 2b\text{arcsinh}(cx)^2 - b\cosh(2\text{arcsinh}(cx)) + 2\text{arcsinh}(cx)(2a + b\sinh(2\text{arcsinh}(cx)))\right)}{8c}$$

input
```
Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[Pi]*(4*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSi
nh[c*x]] + 2*ArcSinh[c*x]*(2*a + b*Sinh[2*ArcSinh[c*x]])))/(8*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6200}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))-\frac{1}{4}\sqrt{\pi}bcx^2$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2$$

input
```
Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*Sqrt[Pi]*x^2) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2
 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{ax\sqrt{\pi\, c^{2} x^{2}+\pi}}{2}+\frac{a\pi \ln\left(\frac{\pi\, c^{2} x}{\sqrt{\pi\, c^{2}}}+\sqrt{\pi\, c^{2} x^{2}+\pi}\right)}{2\sqrt{\pi\, c^{2}}}+\frac{b\sqrt{\pi}\left(2\,\operatorname{arcsinh}\left(xc\right)\sqrt{c^{2} x^{2}+1}\, xc-c^{2} x^{2}+\operatorname{arcsinh}\left(xc\right)^{2}-1\right)}{4c}$	100
parts	$\frac{ax\sqrt{\pi\, c^{2} x^{2}+\pi}}{2}+\frac{a\pi \ln\left(\frac{\pi\, c^{2} x}{\sqrt{\pi\, c^{2}}}+\sqrt{\pi\, c^{2} x^{2}+\pi}\right)}{2\sqrt{\pi\, c^{2}}}+\frac{b\sqrt{\pi}\left(2\,\operatorname{arcsinh}\left(xc\right)\sqrt{c^{2} x^{2}+1}\, xc-c^{2} x^{2}+\operatorname{arcsinh}\left(xc\right)^{2}-1\right)}{4c}$	100

input
```
int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*
x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(1/2)*(2*arcsinh(x*c)*(c^2*x^2+1)^(
1/2)*x*c-c^2*x^2+arcsinh(x*c)^2-1)/c
```

Fricas [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\, dx = \int \sqrt{\pi + \pi c^2x^2}(b \operatorname{arsinh}(cx) + a)\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\, dx = \sqrt{\pi}\left(\int a\sqrt{c^2x^2 + 1}\, dx + \int b\sqrt{c^2x^2 + 1} \operatorname{asinh}(cx)\, dx\right)$$

input
```
integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 +
 1)*asinh(c*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

input
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))\,dx = \frac{\sqrt{\pi}\left(\sqrt{c^2x^2+1}\,acx + 2\left(\int \sqrt{c^2x^2+1}\,asinh(cx)\,dx\right)bc + \log\left(\sqrt{c^2x^2+1} + cx\right)a\right)}{2c}$$

input
```
int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(pi)*(sqrt(c**2*x**2 + 1)*a*c*x + 2*int(sqrt(c**2*x**2 + 1)*asinh(c*x
),x)*b*c + log(sqrt(c**2*x**2 + 1) + c*x)*a))/(2*c)
```

3.40 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2/b/c/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi c^2x^2+\pi}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2\sqrt{\pi}bc}$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

Time = 0.84 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

method	result	size
default	$\frac{a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{\sqrt{\pi\,c^{2}}}+\frac{b\,\mathrm{arcsinh}(xc)^{2}}{2\sqrt{\pi}\,c}$	53
parts	$\frac{a\ln\left(\frac{\pi\,c^{2}x}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{\sqrt{\pi\,c^{2}}}+\frac{b\,\mathrm{arcsinh}(xc)^{2}}{2\sqrt{\pi}\,c}$	53

input
```
int((a+b*arcsinh(x*c))/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/P
i^(1/2)/c*arcsinh(x*c)^2
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{b\operatorname{arsinh}\left(cx\right)+a}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(19) = 38$.

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.48

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\begin{cases} a\left(\begin{cases} \frac{\log\left(2\pi c^2x+2\sqrt{\pi}\sqrt{\pi c^2x^2+\pi}\sqrt{c^2}\right)}{\sqrt{\pi}\sqrt{c^2}} & \text{for}\ \pi c^2\neq 0\\ \frac{x}{\sqrt{\pi}} & \text{otherwise} \end{cases}\right) & \text{for}\ b=0\\ \frac{ax}{\sqrt{\pi}} & \text{for}\ c=0\\ \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{2\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((a*Piecewise((log(2*pi*c**2*x + 2*sqrt(pi)*sqrt(pi*c**2*x**2 + p
i)*sqrt(c**2))/(sqrt(pi)*sqrt(c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), True))
, Eq(b, 0)), (a*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**2/(2*sqrt(pi)*
b*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{\pi + c^2\pi x^2}}\, dx = \frac{b \operatorname{arsinh}(cx)^2}{2\sqrt{\pi}c} + \frac{a \operatorname{arsinh}(cx)}{\sqrt{\pi}c}$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a*arcsinh(c*x)/(sqrt(pi)*c)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{\pi + c^2\pi x^2}}\, dx = \int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2x^2}}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\sqrt{\pi}\left(asinh(cx)^2 b+2\log\left(\sqrt{c^2x^2+1}+cx\right)a\right)}{2c\pi}$$

input
```
int((a+b*asinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

output
```
(sqrt(pi)*(asinh(c*x)**2*b + 2*log(sqrt(c**2*x**2 + 1) + c*x)*a))/(2*c*pi)
```

3.41 $\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{x(a+b\,\text{arcsinh}(cx))}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\log(1+c^2x^2)}{2c\pi^{3/2}}$$

output
```
x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/2*b*ln(c^2*x^2+1)/c/Pi^(3/
2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{2acx+2bcx\,\text{arcsinh}(cx)-b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2c\pi^{3/2}\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2
*c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{bc\int\frac{x}{c^2x^2+1}dx}{\pi^{3/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\operatorname{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Log[1 + c^2*x^2])
/(2*c*Pi^(3/2))
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(45) = 90.

Time = 1.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

method	result	size
default	$\frac{ax}{\pi\sqrt{\pi c^{2} x^{2}+\pi}}+b\left(\frac{2\,\mathrm{arcsinh}(xc)}{\pi^{\frac{3}{2}} c}-\frac{\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1}\, xc+1\right) \mathrm{arcsinh}(xc)}{\pi^{\frac{3}{2}} c \left(c^{2} x^{2}+1\right)}-\frac{\ln\left(1+\left(xc+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{\pi^{\frac{3}{2}} c}\right)$	110
parts	$\frac{ax}{\pi\sqrt{\pi c^{2} x^{2}+\pi}}+b\left(\frac{2\,\mathrm{arcsinh}(xc)}{\pi^{\frac{3}{2}} c}-\frac{\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1}\, xc+1\right) \mathrm{arcsinh}(xc)}{\pi^{\frac{3}{2}} c \left(c^{2} x^{2}+1\right)}-\frac{\ln\left(1+\left(xc+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{\pi^{\frac{3}{2}} c}\right)$	110

input
```
int((a+b*arcsinh(x*c))/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+b*(2/Pi^(3/2)/c*arcsinh(x*c)-1/Pi^(3/2)*(c^2*
x^2-(c^2*x^2+1)^(1/2)*x*c+1)*arcsinh(x*c)/c/(c^2*x^2+1)-1/Pi^(3/2)/c*ln(1+
(x*c+(c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^4 + 2*pi^2
*c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + In
tegral(b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)),
 x))/pi**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \frac{bx\operatorname{arsinh}(cx)}{\pi\sqrt{\pi + \pi c^2x^2}} + \frac{ax}{\pi\sqrt{\pi + \pi c^2x^2}} - \frac{b\log\left(x^2 + \frac{1}{c^2}\right)}{2\,\pi^{\frac{3}{2}}c}$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a*x/(pi*sqrt(pi + pi*c^2*x^2
)) - 1/2*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(\pi + c^2\pi x^2)^{3/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx) + a}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\sqrt{c^2x^2+1}\,acx+\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)b\,c^3x^2+\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)}{\sqrt{\pi}\,c\pi\,(c^2x^2+1)}$$

input
```
int((a+b*asinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

output
```
(sqrt(c**2*x**2 + 1)*a*c*x + int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**2*x**2
 + sqrt(c**2*x**2 + 1)),x)*b*c**3*x**2 + int(asinh(c*x)/(sqrt(c**2*x**2 +
1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c + a*c**2*x**2 + a)/(sqrt(pi)*c*
pi*(c**2*x**2 + 1))
```

3.42 $\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{b}{6c\pi^{5/2}(1+c^2x^2)} + \frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi+c^2\pi x^2)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{b\log(1+c^2x^2)}{3c\pi^{5/2}}$$

output
```
1/6*b/c/Pi^(5/2)/(c^2*x^2+1)+1/3*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(
3/2)+2/3*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)-1/3*b*ln(c^2*x^2+
1)/c/Pi^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\,\text{arcsinh}(cx)-2b(1+c^2x^2)^{3/2}\log}{6c\pi^{5/2}(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*c*x*(3 + 2*c^2*x^2)*Arc
Sinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c*Pi^(5/2)*(1 + c
^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi}-\frac{bc\int \frac{x}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}$$

$$\Big\downarrow \text{6202}$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{bc\int \frac{x}{c^2x^2+1}dx}{\pi^{3/2}}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log(c^2x^2+1)}{2\pi^{3/2}c}\right)}{3\pi}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
b/(6*c*Pi^(5/2)*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*
Pi*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) -
 (b*Log[1 + c^2*x^2])/(2*c*Pi^(3/2))))/(3*Pi)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(92) = 184$.

Time = 1.32 (sec) , antiderivative size = 619, normalized size of antiderivative = 5.73

method	result
default	$a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\,\text{arcsinh}(xc)}{3\pi^{\frac{5}{2}}c}+\frac{2b\,c^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2b\,c^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-$
parts	$a\left(\frac{x}{3\pi(\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}\right)+\frac{4b\,\text{arcsinh}(xc)}{3\pi^{\frac{5}{2}}c}+\frac{2b\,c^7x^8}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)^2}-\frac{2b\,c^5x^6}{3\pi^{\frac{5}{2}}(3c^2x^2+4)(c^2x^2+1)}-$

input
```
int((a+b*arcsinh(x*c))/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2))+4/3*b/
Pi^(5/2)/c*arcsinh(x*c)+2/3*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8
-2/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-2*b/Pi^(5/2)*c^5/(3*c^2*
x^2+4)/(c^2*x^2+1)^2*arcsinh(x*c)*x^6+2*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*
x^2+1)^(3/2)*arcsinh(x*c)*x^5+8/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*x^6-2*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-20/3*b/Pi^(5/2)*c^3/
(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(x*c)*x^4+17/3*b/Pi^(5/2)*c^2/(3*c^2*x^
2+4)/(c^2*x^2+1)^(3/2)*arcsinh(x*c)*x^3+4*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^
2*x^2+1)^2*x^4-3/2*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-22/3*b/Pi^(5
/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(x*c)*x^2+4*b/Pi^(5/2)/(3*c^2*x^2
+4)/(c^2*x^2+1)^(3/2)*arcsinh(x*c)*x+8/3*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x
^2+1)^2*x^2-8/3*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(x*c)+2/3*
b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-2/3*b/Pi^(5/2)/c*ln(1+(x*c+(c^2*x
^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^6 + 3*pi^3
*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 +
1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**4*sqrt(c**2
*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/p
i**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{1}{6}bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2} - \frac{2\log(c^2x^2+1)}{\pi^{\frac{5}{2}}c^2}\right) + \frac{1}{3}b\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}} + \frac{2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\operatorname{arsinh}(cx) + \frac{1}{3}a\left(\frac{x}{\pi(\pi+\pi c^2x^2)^{\frac{3}{2}}} + \frac{2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

output
```
1/6*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/2
)*c^2)) + 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi*c
^2*x^2)))*arcsinh(c*x) + 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2
*sqrt(pi + pi*c^2*x^2)))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{2\sqrt{c^2x^2+1}\,a\,c^3x^3+3\sqrt{c^2x^2+1}\,acx+3\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^4x^4+2\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)$$

input
```
int((a+b*asinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

output
```
(2*sqrt(c**2*x**2 + 1)*a*c**3*x**3 + 3*sqrt(c**2*x**2 + 1)*a*c*x + 3*int(a
sinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2
 + sqrt(c**2*x**2 + 1)),x)*b*c**5*x**4 + 6*int(asinh(c*x)/(sqrt(c**2*x**2
+ 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)
*b*c**3*x**2 + 3*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c*
*2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c - 2*a*c**4*x**4 - 4*a
*c**2*x**2 - 2*a)/(3*sqrt(pi)*c*pi**2*(c**4*x**4 + 2*c**2*x**2 + 1))
```

3.43 $\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx = \frac{b}{20c\pi^{7/2}(1+c^2x^2)^2} + \frac{2b}{15c\pi^{7/2}(1+c^2x^2)} + \frac{x(a+b\,\text{arcsinh}(cx))}{5\pi(\pi+c^2\pi x^2)^{5/2}} + \frac{4x(a+b\,\text{arcsinh}(cx))}{15\pi^2(\pi+c^2\pi x^2)^{3/2}} + \frac{8x(a+b\,\text{arcsinh}(cx))}{15\pi^3\sqrt{\pi+c^2\pi x^2}} - \frac{4b\log(1+c^2x^2)}{15c\pi^{7/2}}$$

output
```
1/20*b/c/Pi^(7/2)/(c^2*x^2+1)^2+2/15*b/c/Pi^(7/2)/(c^2*x^2+1)+1/5*x*(a+b*a
rcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(5/2)+4/15*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c
^2*x^2+Pi)^(3/2)+8/15*x*(a+b*arcsinh(c*x))/Pi^3/(Pi*c^2*x^2+Pi)^(1/2)-4/15
*b*ln(c^2*x^2+1)/c/Pi^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx = \frac{60acx+80ac^3x^3+32ac^5x^5+11b\sqrt{1+c^2x^2}+8bc^2x^2\sqrt{1+c^2x^2}+4bcx(15+20c}{60c\pi^{7/2}\left(1+c^2x^2\right)^{5/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(7/2),x]
```

output
```
(60*a*c*x + 80*a*c^3*x^3 + 32*a*c^5*x^5 + 11*b*Sqrt[1 + c^2*x^2] + 8*b*c^2
*x^2*Sqrt[1 + c^2*x^2] + 4*b*c*x*(15 + 20*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x
] - 16*b*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(60*c*Pi^(7/2)*(1 + c^2*x^2
)^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6203, 241, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi c^2x^2+\pi)^{7/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{4\int \frac{a+b\text{arcsinh}(cx)}{(c^2\pi x^2+\pi)^{5/2}}dx}{5\pi}-\frac{bc\int \frac{x}{(c^2x^2+1)^3}dx}{5\pi^{7/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5\pi\left(\pi c^2x^2+\pi\right)^{5/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\int \frac{a+b\text{arcsinh}(cx)}{(c^2\pi x^2+\pi)^{5/2}}dx}{5\pi}+\frac{x(a+b\text{arcsinh}(cx))}{5\pi\left(\pi c^2x^2+\pi\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(c^2x^2+1\right)^2}$$

$$\Big\downarrow \text{6203}$$

$$\frac{4\left(\frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2\pi x^2+\pi\right)^{3/2}}dx}{3\pi}-\frac{bc\int \frac{x}{\left(c^2x^2+1\right)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}\right)}{5\pi}+\frac{x(a+b\text{arcsinh}(cx))}{5\pi\left(\pi c^2x^2+\pi\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 241

$$\frac{4\left(\frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2\pi x^2+\pi\right)^{3/2}}dx}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}\right)}{5\pi}+\frac{x(a+b\text{arcsinh}(cx))}{5\pi\left(\pi c^2x^2+\pi\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 6202

$$\frac{4\left(\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{bc\int \frac{x}{c^2x^2+1}dx}{\pi^{3/2}}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}\right)}{5\pi}+\frac{x(a+b\text{arcsinh}(cx))}{5\pi\left(\pi c^2x^2+\pi\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(c^2x^2+1\right)^2}$$

$\big\downarrow$ 240

$$\frac{x(a+b\text{arcsinh}(cx))}{5\pi\left(\pi c^2x^2+\pi\right)^{5/2}}+\frac{4\left(\frac{x(a+b\text{arcsinh}(cx))}{3\pi(\pi c^2x^2+\pi)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c}\right)}{3\pi}+\frac{b}{6\pi^{5/2}c(c^2x^2+1)}\right)}{5\pi}+\frac{b}{20\pi^{7/2}c\left(c^2x^2+1\right)^2}$$

input

```
Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(7/2),x]
```

output
```
b/(20*c*Pi^(7/2)*(1 + c^2*x^2)^2) + (x*(a + b*ArcSinh[c*x]))/(5*Pi*(Pi + c
^2*Pi*x^2)^(5/2)) + (4*(b/(6*c*Pi^(5/2)*(1 + c^2*x^2)) + (x*(a + b*ArcSinh
[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/(Pi*
Sqrt[Pi + c^2*Pi*x^2]) - (b*Log[1 + c^2*x^2])/(2*c*Pi^(3/2))))/(3*Pi)))/(5
*Pi)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. $2(138) = 276$.

Time = 1.33 (sec) , antiderivative size = 1409, normalized size of antiderivative = 8.70

method	result	size
default	Expression too large to display	1409
parts	Expression too large to display	1409

input
```
int((a+b*arcsinh(x*c))/(Pi*c^2*x^2+Pi)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
176/15*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)/c/(c^2*x^2+1)^2+
a*(1/5/Pi*x/(Pi*c^2*x^2+Pi)^(5/2)+4/5/Pi*(1/3/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2
/3/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)))-4136/15*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x
^4+159*c^2*x^2+64)*c^3/(c^2*x^2+1)^2*arcsinh(x*c)*x^4-2296/15*b/Pi^(7/2)/(
40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c/(c^2*x^2+1)^2*arcsinh(x*c)*x^2-64
/3*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^9/(c^2*x^2+1)^2*ar
csinh(x*c)*x^10-344/3*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c
^7/(c^2*x^2+1)^2*arcsinh(x*c)*x^8-3752/15*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x
^4+159*c^2*x^2+64)*c^5/(c^2*x^2+1)^2*arcsinh(x*c)*x^6+541/3*b/Pi^(7/2)/(40
*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^2/(c^2*x^2+1)^(3/2)*arcsinh(x*c)*x^
3+64/3*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^8/(c^2*x^2+1)^
(3/2)*arcsinh(x*c)*x^9+104*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+
64)*c^6/(c^2*x^2+1)^(3/2)*arcsinh(x*c)*x^7+1004/5*b/Pi^(7/2)/(40*c^6*x^6+1
35*c^4*x^4+159*c^2*x^2+64)*c^4/(c^2*x^2+1)^(3/2)*arcsinh(x*c)*x^5+944/15*b
/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^11/(c^2*x^2+1)^2*x^12+
992/5*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^9/(c^2*x^2+1)^2
*x^10+1040/3*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^7/(c^2*x
^2+1)^2*x^8+1088/3*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^2*x^2+64)*c^5/
(c^2*x^2+1)^2*x^6-8/15*b/Pi^(7/2)/c*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)+16/15*
b/Pi^(7/2)/c*arcsinh(x*c)-140*b/Pi^(7/2)/(40*c^6*x^6+135*c^4*x^4+159*c^...
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^4*c^8*x^8 + 4*pi^4
*c^6*x^6 + 6*pi^4*c^4*x^4 + 4*pi^4*c^2*x^2 + pi^4), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx=\frac{\int \frac{a}{c^6x^6\sqrt{c^2x^2+1}+3c^4x^4\sqrt{c^2x^2+1}+3c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^6x^6\sqrt{c^2x^2+1}+3c^4x^4\sqrt{c^2x^2+1}+3c}}{\pi^{\frac{7}{2}}}$$

input
```
integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(7/2),x)
```

output
```
(Integral(a/(c**6*x**6*sqrt(c**2*x**2 + 1) + 3*c**4*x**4*sqrt(c**2*x**2 +
1) + 3*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral
(b*asinh(c*x)/(c**6*x**6*sqrt(c**2*x**2 + 1) + 3*c**4*x**4*sqrt(c**2*x**2
+ 1) + 3*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(7/
2)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(3*x/(pi*(pi + pi*c^2*x^2)^(5/2)) + 4*x/(pi^2*(pi + pi*c^2*x^2)^(3/
2)) + 8*x/(pi^3*sqrt(pi + pi*c^2*x^2))) + b*integrate(log(c*x + sqrt(c^2*x
^2 + 1))/(pi + pi*c^2*x^2)^(7/2), x)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(\pi+\pi c^2x^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(\Pi\,c^2\,x^2+\Pi)^{7/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(7/2),x)
```

output
```
int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(\pi+c^2\pi x^2)^{7/2}}\,dx=\frac{8\sqrt{c^2x^2+1}\,a\,c^5x^5+20\sqrt{c^2x^2+1}\,a\,c^3x^3+15\sqrt{c^2x^2+1}\,acx+15\left(\int \frac{}{\sqrt{c^2x^2+1}\,c^6x^6+}\right.}{}$$

input
```
int((a+b*asinh(c*x))/(Pi*c^2*x^2+Pi)^(7/2),x)
```

output
```
(8*sqrt(c**2*x**2 + 1)*a*c**5*x**5 + 20*sqrt(c**2*x**2 + 1)*a*c**3*x**3 +
15*sqrt(c**2*x**2 + 1)*a*c*x + 15*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**6
*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqrt(c**2*x**2 + 1)*c**2*x**2
+ sqrt(c**2*x**2 + 1)),x)*b*c**7*x**6 + 45*int(asinh(c*x)/(sqrt(c**2*x**2
+ 1)*c**6*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqrt(c**2*x**2 + 1)*c
**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c**5*x**4 + 45*int(asinh(c*x)/(sqrt(c
**2*x**2 + 1)*c**6*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqrt(c**2*x*
*2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c**3*x**2 + 15*int(asinh(c*x
)/(sqrt(c**2*x**2 + 1)*c**6*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqr
t(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c - 8*a*c**6*x**6 -
 24*a*c**4*x**4 - 24*a*c**2*x**2 - 8*a)/(15*sqrt(pi)*c*pi**3*(c**6*x**6 +
3*c**4*x**4 + 3*c**2*x**2 + 1))
```

3.44 $\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{15}{64} b^2 \pi^{3/2} x \sqrt{1 + c^2x^2} + \frac{1}{32} b^2 \pi^{3/2} x \left(1 + c^2x^2\right)^{3/2} - \frac{9b^2\pi^{3/2}\text{arcsinh}(cx)}{64c} - \frac{3}{8} bc\pi^{3/2} x^2 (a + b\text{arcsinh}(cx)) - \frac{b\pi^{3/2}\left(1 + c^2x^2\right)^2 (a + b\text{arcsi}$$

output

```
15/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)+1/32*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(3/
2)-9/64*b^2*Pi^(3/2)*arcsinh(c*x)/c-3/8*b*c*Pi^(3/2)*x^2*(a+b*arcsinh(c*x)
)-1/8*b*Pi^(3/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c+3/8*Pi*x*(Pi*c^2*x^2+P
i)^(1/2)*(a+b*arcsinh(c*x))^2+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x
))^2+1/8*Pi^(3/2)*(a+b*arcsinh(c*x))^3/b/c
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \frac{\pi^{3/2}\left(160a^2cx\sqrt{1 + c^2x^2} + 64a^2c^3x^3\sqrt{1 + c^2x^2} + 32b^2\text{arcsinh}(cx)^3 - 64ab\cosh(2\text{arcsi}\right.}{}$$

input
```
Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Pi^(3/2)*(160*a^2*c*x*Sqrt[1 + c^2*x^2] + 64*a^2*c^3*x^3*Sqrt[1 + c^2*x^2
] + 32*b^2*ArcSinh[c*x]^3 - 64*a*b*Cosh[2*ArcSinh[c*x]] - 4*a*b*Cosh[4*Arc
Sinh[c*x]] + 32*b^2*Sinh[2*ArcSinh[c*x]] + b^2*Sinh[4*ArcSinh[c*x]] + 8*b*
ArcSinh[c*x]^2*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]])
+ 4*ArcSinh[c*x]*(-16*b^2*Cosh[2*ArcSinh[c*x]] - b^2*Cosh[4*ArcSinh[c*x]]
+ 4*a*(6*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))))/(256*c)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi c^2x^2+\pi\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))dx+\frac{3}{4}\pi\int \sqrt{c^2\pi x^2+\pi}(a+b\operatorname{arcsinh}(cx))^2dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6200}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\operatorname{arcsinh}(cx))dx+\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-\sqrt{\pi}bc\int x(a+b\operatorname{arcsinh}(cx))dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\operatorname{arcsinh}(cx))^2\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+$$
$$\frac{3}{4}\pi\left(-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\right.$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 262

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+$$
$$\frac{3}{4}\pi\left(-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx\right.$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 222

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}b\right.\right.$$
$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$\downarrow$ 6198

$$-\frac{1}{2}\pi^{3/2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)\right)$$

$\downarrow$ 6213

$$-\frac{1}{2}\pi^{3/2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int\left(c^2x^2+1\right)^{3/2}dx}{4c}\right)+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}(a+$$
$$b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)\right)$$

$\downarrow$ 211

$$-\frac{1}{2}\pi^{3/2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)+\frac{1}{4}x(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)\right)$$

$\Big\downarrow$ 211

$$-\frac{1}{2}\pi^{3/2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)+\frac{1}{4}x(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))^2+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)\right)$$

$\Big\downarrow$ 222

$$\frac{1}{4}x(\pi c^2x^2+\pi)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{1}{2}\pi^{3/2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)\right)$$

input
```
Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*Pi*((x*Sqrt[Pi +
 c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^3)
/(6*b*c) - b*c*Sqrt[Pi]*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 +
c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)))/4 - (b*c*Pi^(3/2)*(((1 + c
^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 +
(3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c))/4))/(4*c)))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

method	result
default	$\frac{a^2 x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{4}+\frac{3a^2\pi x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{3a^2\pi^2\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}}+ b^2\pi^{\frac{3}{2}}\left(16\,\mathrm{arcsinh}(xc)^2\sqrt{c^2x^2+1}\,x^3c^3-8\,\mathrm{arcsinh}(\right.$
parts	$\frac{a^2 x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{4}+\frac{3a^2\pi x\sqrt{\pi c^2x^2+\pi}}{8}+\frac{3a^2\pi^2\ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8\sqrt{\pi c^2}}+ b^2\pi^{\frac{3}{2}}\left(16\,\mathrm{arcsinh}(xc)^2\sqrt{c^2x^2+1}\,x^3c^3-8\,\mathrm{arcsinh}(\right.$

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/8*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a^2
*Pi^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6
4*b^2*Pi^(3/2)*(16*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*x^3*c^3-8*arcsinh(x*c)
*c^4*x^4+2*(c^2*x^2+1)^(1/2)*c^3*x^3+40*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*x
*c-40*arcsinh(x*c)*c^2*x^2+17*(c^2*x^2+1)^(1/2)*x*c+8*arcsinh(x*c)^3-17*ar
csinh(x*c))/c+1/8*a*b*Pi^(3/2)*(4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-c
^4*x^4+10*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+3*arcsinh(x*c)^2-4)
/c
```

Fricas [F]

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2\, dx = \int \left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} (b\,\text{arsinh}\,(cx) + a)^2\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(pi*a^2*c^2*x^2 + pi*a^2 + (pi*b^2*c^2*x^2
+ pi*b^2)*arcsinh(c*x)^2 + 2*(pi*a*b*c^2*x^2 + pi*a*b)*arcsinh(c*x)), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(197) = 394$.

Time = 2.87 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.93

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2\, dx = \begin{cases} \frac{\pi^{\frac{3}{2}} a^2c^2x^3\sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a^2x\sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a^2\,\text{asinh}\,(cx)}{8c} - \frac{\pi^{\frac{3}{2}} abc^3x^4}{8} + \frac{\pi^{\frac{3}{2}} abc^2x^3\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{2} \\ \pi^{\frac{3}{2}} a^2x \end{cases}$$

input
```
integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((pi**(3/2)*a**2*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*
*2*x*sqrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a**2*asinh(c*x)/(8*c) - pi**(3/2)
*a*b*c**3*x**4/8 + pi**(3/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/
2 - 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(c**2*x**2 + 1)*asinh
(c*x)/4 + 3*pi**(3/2)*a*b*asinh(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*a
sinh(c*x)/8 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/4
 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/32 - 5*pi**(3/2)*b**2*c*x*
*2*asinh(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/8 +
 17*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)/64 + pi**(3/2)*b**2*asinh(c*x)**3
/(8*c) - 17*pi**(3/2)*b**2*asinh(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*a**2*x
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b\,\text{asinh}(c\,x))^2 \left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2} \, dx$$

input
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} (a + b\text{arcsinh}(cx))^2\, dx = \frac{\sqrt{\pi}\,\pi\left(2\sqrt{c^2x^2+1}\,a^2c^3x^3 + 5\sqrt{c^2x^2+1}\,a^2cx + 16\left(\int \sqrt{c^2x^2+1}\,asinh(cx)\,x^2dx\right)ab\,c^3$$

input
```
int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(pi)*pi*(2*sqrt(c**2*x**2 + 1)*a**2*c**3*x**3 + 5*sqrt(c**2*x**2 + 1)
*a**2*c*x + 16*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*a*b*c**3 + 16*in
t(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*a*b*c + 8*int(sqrt(c**2*x**2 + 1)*asin
h(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(c**2*x**2 + 1)*asinh(c*x)**2,x)*b
**2*c + 3*log(sqrt(c**2*x**2 + 1) + c*x)*a**2))/(8*c)
```

3.45 $\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \frac{1}{4}b^2\sqrt{\pi}x\sqrt{1 + c^2x^2} - \frac{b^2\sqrt{\pi}\text{arcsinh}(cx)}{4c} - \frac{1}{2}bc\sqrt{\pi}x^2(a + b\text{arcsinh}(cx)) + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 + \frac{\sqrt{\pi}(a + b\text{arcsinh}(cx))^3}{6bc}$$

output
```
1/4*b^2*Pi^(1/2)*x*(c^2*x^2+1)^(1/2)-1/4*b^2*Pi^(1/2)*arcsinh(c*x)/c-1/2*b
*c*Pi^(1/2)*x^2*(a+b*arcsinh(c*x))+1/2*x*(Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsin
h(c*x))^2+1/6*Pi^(1/2)*(a+b*arcsinh(c*x))^3/b/c
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{\sqrt{\pi}\left(4b^2\text{arcsinh}(cx)^3 + 6b\text{arcsinh}(cx)^2(2a + b\sinh(2\text{arcsinh}(cx))) + 3\left(4a^2cx\sqrt{1 + c^2x^2} - 2ab\cosh(2\text{arcs}\right.\right.}{}$$

input
```
Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Sqrt[Pi]*(4*b^2*ArcSinh[c*x]^3 + 6*b*ArcSinh[c*x]^2*(2*a + b*Sinh[2*ArcSi
nh[c*x]]) + 3*(4*a^2*c*x*Sqrt[1 + c^2*x^2] - 2*a*b*Cosh[2*ArcSinh[c*x]] +
b^2*Sinh[2*ArcSinh[c*x]]) + 6*ArcSinh[c*x]*(-(b^2*Cosh[2*ArcSinh[c*x]]) +
2*a*(a + b*Sinh[2*ArcSinh[c*x]])))))/(24*c)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6200}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2 + 1}}dx - \sqrt{\pi}bc\int x(a + b\text{arcsinh}(cx))dx + \frac{1}{2}x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a + b\text{arcsinh}(cx)) - \frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2 + 1}}dx\right) + \frac{1}{2}\sqrt{\pi}\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2 + 1}}dx + \frac{1}{2}x\sqrt{\pi c^2x^2 + \pi}(a + b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}(a+b\text{arcsinh}(cx))^2-\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)+\frac{\sqrt{\pi}(a+b\text{arcsinh}(cx))^3}{6bc}$$

input
```
Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcS
inh[c*x])^3)/(6*b*c) - b*c*Sqrt[Pi]*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*(
(x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

method	result
default	$\frac{a^{2} x \sqrt{\pi c^{2} x^{2}+\pi}}{2}+\frac{a^{2} \pi \ln\left(\frac{\pi c^{2} x}{\sqrt{\pi c^{2}}}+\sqrt{\pi c^{2} x^{2}+\pi}\right)}{2 \sqrt{\pi c^{2}}}+\frac{b^{2} \sqrt{\pi}\,\left(6 \operatorname{arcsinh}(x c)^{2} \sqrt{c^{2} x^{2}+1}\, x c-6 \operatorname{arcsinh}(x c) c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1}\, x c+2 \operatorname{arc}\right.}{12 c}$
parts	$\frac{a^{2} x \sqrt{\pi c^{2} x^{2}+\pi}}{2}+\frac{a^{2} \pi \ln\left(\frac{\pi c^{2} x}{\sqrt{\pi c^{2}}}+\sqrt{\pi c^{2} x^{2}+\pi}\right)}{2 \sqrt{\pi c^{2}}}+\frac{b^{2} \sqrt{\pi}\,\left(6 \operatorname{arcsinh}(x c)^{2} \sqrt{c^{2} x^{2}+1}\, x c-6 \operatorname{arcsinh}(x c) c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1}\, x c+2 \operatorname{arc}\right.}{12 c}$

input
```
int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*
c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/12*b^2*Pi^(1/2)*(6*arcsinh(x*c)^2*(c^2
*x^2+1)^(1/2)*x*c-6*arcsinh(x*c)*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+2*arcsinh
(x*c)^3-3*arcsinh(x*c))/c+1/2*a*b*Pi^(1/2)*(2*arcsinh(x*c)*(c^2*x^2+1)^(1/
2)*x*c-c^2*x^2+arcsinh(x*c)^2-1)/c
```

Fricas [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \int \sqrt{\pi + \pi c^2x^2}(b \operatorname{arsinh}(cx) + a)^2\, dx$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2), x)
```

Sympy [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\, dx = \sqrt{\pi}\left(\int a^2\sqrt{c^2x^2+1}\, dx + \int b^2\sqrt{c^2x^2+1}\operatorname{asinh}^2(cx)\, dx + \int 2ab\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\, dx\right)$$

input
```
integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
sqrt(pi)*(Integral(a**2*sqrt(c**2*x**2 + 1), x) + Integral(b**2*sqrt(c**2*
x**2 + 1)*asinh(c*x)**2, x) + Integral(2*a*b*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2\,dx = \int (a + b\,\text{asinh}(c\,x))^2\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\pi + c^2\pi x^2}(a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{\sqrt{\pi}\left(\sqrt{c^2x^2+1}\,a^2cx + 4\left(\int \sqrt{c^2x^2+1}\,asinh(cx)\,dx\right)abc + 2\left(\int \sqrt{c^2x^2+1}\,asinh(cx)^2\,dx\right)b^2c + \log\left(\sqrt{c^2x\right.\right.}{2c}$$

input
```
int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(pi)*(sqrt(c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt(c**2*x**2 + 1)*asinh(
c*x),x)*a*b*c + 2*int(sqrt(c**2*x**2 + 1)*asinh(c*x)**2,x)*b**2*c + log(sq
rt(c**2*x**2 + 1) + c*x)*a**2))/(2*c)
```

3.46 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\,\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\,\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

output
```
1/3*(a+b*arcsinh(c*x))^3/b/c/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{(a+b\,\text{arcsinh}(cx))^3}{3bc\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi c^2x^2+\pi}}dx$$
$$\Big\downarrow \text{6198}$$
$$\frac{(a+b\text{arcsinh}(cx))^3}{3\sqrt{\pi}bc}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

output
```
(a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(21) = 42$.

Time = 1.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

method	result	size
default	$\frac{a^2 \ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b^2 \operatorname{arcsinh}(xc)^3}{3\sqrt{\pi}\, c}+\frac{ab \operatorname{arcsinh}(xc)^2}{\sqrt{\pi}\, c}$	72
parts	$\frac{a^2 \ln\left(\frac{\pi c^2 x}{\sqrt{\pi c^2}}+\sqrt{\pi c^2 x^2+\pi}\right)}{\sqrt{\pi c^2}}+\frac{b^2 \operatorname{arcsinh}(xc)^3}{3\sqrt{\pi}\, c}+\frac{ab \operatorname{arcsinh}(xc)^2}{\sqrt{\pi}\, c}$	72

input
```
int((a+b*arcsinh(x*c))^2/(Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(Pi*c^2*x/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3*b
^2/Pi^(1/2)/c*arcsinh(x*c)^3+a*b/Pi^(1/2)/c*arcsinh(x*c)^2
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(pi + pi*c^2*
x^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.

Time = 0.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \begin{cases} a^2\left(\begin{cases} \frac{\log\left(2\pi c^2x+2\sqrt{\pi}\sqrt{\pi c^2x^2+\pi}\sqrt{c^2}\right)}{\sqrt{\pi}\sqrt{c^2}} & \text{for}\ \pi c^2\neq 0 \\ \frac{x}{\sqrt{\pi}} & \text{otherwise} \end{cases}\right) & \text{for}\ b=0 \\ \frac{a^2x}{\sqrt{\pi}} & \text{for}\ c=0 \\ \frac{(a+b\,\text{asinh}\,(cx))^3}{3\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((a**2*Piecewise((log(2*pi*c**2*x + 2*sqrt(pi)*sqrt(pi*c**2*x**2
+ pi)*sqrt(c**2))/(sqrt(pi)*sqrt(c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), Tru
e)), Eq(b, 0)), (a**2*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**3/(3*sqr
t(pi)*b*c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{b^2\,\text{arsinh}\,(cx)^3}{3\,\sqrt{\pi}c} + \frac{ab\,\text{arsinh}\,(cx)^2}{\sqrt{\pi}c} + \frac{a^2\,\text{arsinh}\,(cx)}{\sqrt{\pi}c}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima"
)
```

output
```
1/3*b^2*arcsinh(c*x)^3/(sqrt(pi)*c) + a*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a^
2*arcsinh(c*x)/(sqrt(pi)*c)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/sqrt(pi + pi*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\sqrt{\pi}\left(asinh(cx)^3\,b^2+3asinh(cx)^2\,ab+3\log\left(\sqrt{c^2x^2+1}+cx\right)a^2\right)}{3c\pi}$$

input
```
int((a+b*asinh(c*x))^2/(Pi*c^2*x^2+Pi)^(1/2),x)
```

output
```
(sqrt(pi)*(asinh(c*x)**3*b**2 + 3*asinh(c*x)**2*a*b + 3*log(sqrt(c**2*x**2
 + 1) + c*x)*a**2))/(3*c*pi)
```

3.47 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{(a+b\operatorname{arcsinh}(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))^2}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2b(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c\pi^{3/2}}-\frac{b^2\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c\pi^{3/2}}$$

output
```
(a+b*arcsinh(c*x))^2/c/Pi^(3/2)+x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^
(1/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(3/2)-b^
2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(3/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\operatorname{arcsinh}(cx)^2+2b\operatorname{arcsinh}(cx)\left(acx-b\sqrt{1+c^2x^2}\log\left(\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*
c*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2
*ArcSinh[c*x])])/(c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi c^2x^2+\pi)^{3/2}}dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{\pi^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{26}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{4201}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + ((2*I)*b*(((-1/2*I
)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*Ar
cSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*Pi^(3/2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(114) = 228.

Time = 1.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

method	result
default	$\frac{a^2 x}{\pi\sqrt{\pi c^2x^2+\pi}}+b^2\left(-\frac{\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\operatorname{arcsinh}(xc)^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)}+\frac{2\operatorname{arcsinh}(xc)^2}{\pi^{\frac{3}{2}}c}-\frac{2\operatorname{arcsinh}(xc)\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)}{\pi^{\frac{3}{2}}c}-\right.$
parts	$\frac{a^2 x}{\pi\sqrt{\pi c^2x^2+\pi}}+b^2\left(-\frac{\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\operatorname{arcsinh}(xc)^2}{\pi^{\frac{3}{2}}c(c^2x^2+1)}+\frac{2\operatorname{arcsinh}(xc)^2}{\pi^{\frac{3}{2}}c}-\frac{2\operatorname{arcsinh}(xc)\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)}{\pi^{\frac{3}{2}}c}-\right.$

input
```
int((a+b*arcsinh(x*c))^2/(Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+b^2*(-1/Pi^(3/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)
*x*c+1)*arcsinh(x*c)^2/c/(c^2*x^2+1)+2/Pi^(3/2)/c*arcsinh(x*c)^2-2/Pi^(3/2
)/c*arcsinh(x*c)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)-1/Pi^(3/2)/c*polylog(2,-(
x*c+(c^2*x^2+1)^(1/2))^2))+2*a*b*(2/Pi^(3/2)/c*arcsinh(x*c)-1/Pi^(3/2)*(c^
2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*arcsinh(x*c)/c/(c^2*x^2+1)-1/Pi^(3/2)/c*ln(
1+(x*c+(c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2)/(pi^2*c^4*x^4 + 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
 Integral(b**2*asinh(c*x)**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x*
*2 + 1)), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) +
sqrt(c**2*x**2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima"
)
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(pi + pi*c^2*x^2)^(3/2), x) +
 2*a*b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a^2*x/(pi*sqrt(pi + pi*
c^2*x^2)) - a*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2+\Pi)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{3/2}}\,dx=\frac{\sqrt{c^2x^2+1}\,a^2cx+2\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)ab\,c^3x^2+2\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x}}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(Pi*c^2*x^2+Pi)^(3/2),x)
```

output
```
(sqrt(c**2*x**2 + 1)*a**2*c*x + 2*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**2
*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c**3*x**2 + 2*int(asinh(c*x)/(sqrt(c**
2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c + int(asinh(c*x)**2/
(sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b**2*c**3*x**2 +
int(asinh(c*x)**2/(sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)
*b**2*c + a**2*c**2*x**2 + a**2)/(sqrt(pi)*c*pi*(c**2*x**2 + 1))
```

3.48 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{(a+b\,\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = -\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b(a+b\,\text{arcsinh}(cx))}{3c\pi^{5/2}(1+c^2x^2)} + \frac{2(a+b\,\text{arcsinh}(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\,\text{arcsinh}(cx))^2}{3\pi(\pi+c^2\pi x^2)^{3/2}}+\frac{2x(a+b\,\text{arcsinh}(cx))^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{4b(a+b\,\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}-\frac{2b^2\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3c\pi^{5/2}}$$

output
```
-1/3*b^2*x/Pi^(5/2)/(c^2*x^2+1)^(1/2)+1/3*b*(a+b*arcsinh(c*x))/c/Pi^(5/2)/
(c^2*x^2+1)+2/3*(a+b*arcsinh(c*x))^2/c/Pi^(5/2)+1/3*x*(a+b*arcsinh(c*x))^2
/Pi/(Pi*c^2*x^2+Pi)^(3/2)+2/3*x*(a+b*arcsinh(c*x))^2/Pi^2/(Pi*c^2*x^2+Pi)^
(1/2)-4/3*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-
2/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{3a^2cx-b^2cx+2a^2c^3x^3-b^2c^3x^3+ab\sqrt{1+c^2x^2}-b^2\left(-3cx-2c^3x^3+2\sqrt{1+\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 - b^2*c^3*x^3 + a*b*Sqrt[1 + c^2*x^2]
 - b^2*(-3*c*x - 2*c^3*x^3 + 2*Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*
x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-6*a*c*x - 4*a*c^3*x^3 - b*Sqrt[1 +
 c^2*x^2] + 4*b*(1 + c^2*x^2)^(3/2)*Log[1 + E^(-2*ArcSinh[c*x])]) - 2*a*b*
Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1
 + c^2*x^2] + 2*b^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])])/
(3*c*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi c^2x^2+\pi)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2\pi x^2+\pi)^{3/2}}dx}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi(\pi c^2x^2+\pi)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{\pi^{3/2}}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 2715

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx)}\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 2838

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\int\frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 6213

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}-\frac{2bc\left(\frac{b\int\frac{1}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3\pi^{5/2}}+\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}$$

$\Big\downarrow$ 208

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2bc\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3\pi^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (2*b*c*((b*x)/
(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 + c^2*x^2))))/(3*
Pi^(5/2)) + (2*((x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + ((
2*I)*b*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])
*Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))
/(c*Pi^(3/2))))/(3*Pi)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. 2(192) = 384.

Time = 1.65 (sec) , antiderivative size = 1729, normalized size of antiderivative = 8.48

method	result	size
default	Expression too large to display	1729
parts	Expression too large to display	1729

input
```
int((a+b*arcsinh(x*c))^2/(Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
10/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^6+6*b^2/Pi^(5/2)*c^3/(
3*c^2*x^2+4)/(c^2*x^2+1)^2*x^4-7/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2
+1)^(3/2)*x^3-b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2+4*b^2/Pi^(5/2)/
(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(x*c)^2*x-2/3*b^2/Pi^(5/2)*c^5/(3*c
^2*x^2+4)/(c^2*x^2+1)*x^6+4/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-4
/3*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x-4/3*b^2/Pi^(5/2)/c*arcsi
nh(x*c)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)+4/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(
c^2*x^2+1)^2-2/3*b^2*polylog(2,-(x*c+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-4*b^
2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(x*c)*x^4+17/3*b^2/Pi^(5/2
)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(x*c)^2*x^3-4/3*b^2/Pi^(5/2)*
c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(x*c)*x^6+2*b^2/Pi^(5/2)*c^4/(3*c^2*x
^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(x*c)^2*x^5-3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/
(c^2*x^2+1)*arcsinh(x*c)*x^2-22/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*arcsinh(x*c)^2*x^2-b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x^5
-5/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4+2/3*b^2/Pi^(5/2)*c^7/(
3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-8/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1
)^2*arcsinh(x*c)^2+4/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(
x*c)+14/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2+4*a*b/Pi^(5/2)*c^
4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(x*c)*x^5+8/3*a*b/Pi^(5/2)/c*arcs
inh(x*c)-4/3*a*b/Pi^(5/2)/c*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)+34/3*a*b/Pi...
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2)/(pi^3*c^6*x^6 + 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{a^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b**2*asinh(c*x)**2/(c**4*x**4*
sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1
)), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2
*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima"
)
```

output
```
1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5
/2)*c^2)) + 2/3*a*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi +
pi*c^2*x^2)))*arcsinh(c*x) + 1/3*a^2*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x
/(pi^2*sqrt(pi + pi*c^2*x^2))) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1)
)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(\pi+\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2+\Pi)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(\pi+c^2\pi x^2)^{5/2}}\,dx = \frac{2\sqrt{c^2x^2+1}\,a^2c^3x^3+3\sqrt{c^2x^2+1}\,a^2cx+6\left(\int\frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^4x^4+2\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(Pi*c^2*x^2+Pi)^(5/2),x)
```

output
```
(2*sqrt(c**2*x**2 + 1)*a**2*c**3*x**3 + 3*sqrt(c**2*x**2 + 1)*a**2*c*x + 6
*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**
2*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c**5*x**4 + 12*int(asinh(c*x)/(sqrt(c
**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2
 + 1)),x)*a*b*c**3*x**2 + 6*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4
+ 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c + 3*int(
asinh(c*x)**2/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*
x**2 + sqrt(c**2*x**2 + 1)),x)*b**2*c**5*x**4 + 6*int(asinh(c*x)**2/(sqrt(
c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**
2 + 1)),x)*b**2*c**3*x**2 + 3*int(asinh(c*x)**2/(sqrt(c**2*x**2 + 1)*c**4*
x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b**2*c -
2*a**2*c**4*x**4 - 4*a**2*c**2*x**2 - 2*a**2)/(3*sqrt(pi)*c*pi**2*(c**4*x*
*4 + 2*c**2*x**2 + 1))
```

3.49 $\int \sqrt{1+x^2}\,\text{arcsinh}(x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{1+x^2}\,\text{arcsinh}(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\,\text{arcsinh}(x) + \frac{\text{arcsinh}(x)^2}{4}$$

output
```
-1/4*x^2+1/2*x*(x^2+1)^(1/2)*arcsinh(x)+1/4*arcsinh(x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\,\text{arcsinh}(x)\,dx = \frac{1}{4}\left(-x^2 + 2x\sqrt{1+x^2}\,\text{arcsinh}(x) + \text{arcsinh}(x)^2\right)$$

input
```
Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

output
```
(-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+1}\text{arcsinh}(x)\,dx$$

$$\Big\downarrow \text{6200}$$

$$\frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{x^2+1}}dx-\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{x^2+1}\text{arcsinh}(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\text{arcsinh}(x)}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)-\frac{x^2}{4}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}\sqrt{x^2+1}x\text{arcsinh}(x)+\frac{\text{arcsinh}(x)^2}{4}-\frac{x^2}{4}$$

input
```
Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

output
```
-1/4*x^2 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{x^2+1}\,\operatorname{arcsinh}(x)}{2}+\frac{\operatorname{arcsinh}(x)^2}{4}-\frac{x^2}{4}-\frac{1}{4}$	26

input
```
int((x^2+1)^(1/2)*arcsinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2+1)^(1/2)*arcsinh(x)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx = \frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2+\frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

input
```
integrate((x^2+1)^(1/2)*arcsinh(x),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^
2 + 1))^2
```

Sympy [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{asinh}\,(x)\,dx$$

input
```
integrate((x**2+1)**(1/2)*asinh(x),x)
```

output
```
Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = -\frac{1}{4}\,x^2+\frac{1}{2}\left(\sqrt{x^2+1}x+\text{arsinh}\,(x)\right)\text{arsinh}\,(x)-\frac{1}{4}\,\text{arsinh}\,(x)^2$$

input
```
integrate((x^2+1)^(1/2)*arcsinh(x),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^
2
```

Giac [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,\text{arsinh}\,(x)\,dx$$

input
```
integrate((x^2+1)^(1/2)*arcsinh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \text{asinh}(x)\,\sqrt{x^2+1}\,dx$$

input
```
int(asinh(x)*(x^2 + 1)^(1/2),x)
```

output
```
int(asinh(x)*(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1+x^2}\text{arcsinh}(x)\,dx = \int \sqrt{x^2+1}\,asinh(x)\,dx$$

input
```
int((x^2+1)^(1/2)*asinh(x),x)
```

output
```
int(sqrt(x**2 + 1)*asinh(x),x)
```

3.50 $\int (d+c^2dx^2)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 251

$$\int (d+c^2dx^2)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = -\frac{5bcd^2x^2\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}} - \frac{5bd^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{96c} - \frac{bd^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}(a+\text{barcsi}$$

output
```
-5/32*b*c*d^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*d^2*(c^2*x^
2+1)^(3/2)*(c^2*d*x^2+d)^(1/2)/c-1/36*b*d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d
)^(1/2)/c+5/16*d^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+5/24*d*x*(c^2*
d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c
*x))+5/32*d^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.26

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{d^2\left(1584acx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+1248ac^3x^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+384ac^5x^5\sqrt{1-$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*(1584*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 1248*a*c^3*x^3*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqrt
[d + c^2*d*x^2] + 360*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 - 270*b*Sqrt[d
+ c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 27*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSin
h[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 720*a*Sqrt[d]*Sqr
t[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 12*b*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] +
Sinh[6*ArcSinh[c*x]])))/(2304*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{5}{6}d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{bcd^2\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)^2dx}{6\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{241}$$

$$\frac{5}{6}d\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$$\Big\downarrow \text{6201}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsin}\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$$\Big\downarrow \text{244}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^3+x\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+}}{4\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$$\Big\downarrow \text{6200}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(c^2\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$$\Big\downarrow \text{15}$$

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)+\frac{1}{4}x(c^2dx^2\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

$$\downarrow \text{6198}$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))+$$
$$\frac{5}{6}d\left(\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(}{4bc\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

input
```
Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/36*(b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/c + (x*(d + c^2*d*x^
2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d*(-1/4*(b*c*d*Sqrt[d + c^2*d*x^2]*(
x^2/2 + (c^2*x^4)/4))/Sqrt[1 + c^2*x^2] + (x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x
^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])))/4))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(215) = 430$.

Time = 1.12 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.19

method	result
default	$\frac{ax(c^2d\,x^2+d)^{\frac{5}{2}}}{6}+\frac{5adx(c^2d\,x^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5a\,d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16\sqrt{c^2d}}+b\Bigg(\frac{5\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc}{32\sqrt{c^2x^2+1}\,c}$
parts	$\frac{ax(c^2d\,x^2+d)^{\frac{5}{2}}}{6}+\frac{5adx(c^2d\,x^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5a\,d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16\sqrt{c^2d}}+b\Bigg(\frac{5\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc}{32\sqrt{c^2x^2+1}\,c}$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(c
^2*d*x^2+d)^(1/2)+5/16*a*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))
/(c^2*d)^(1/2)+b*(5/32*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x
*c)^2*d^2+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^
(1/2)+64*x^5*c^5+48*x^4*c^4*(c^2*x^2+1)^(1/2)+38*x^3*c^3+18*x^2*c^2*(c^2*x
^2+1)^(1/2)+6*x*c+(c^2*x^2+1)^(1/2))*(-1+6*arcsinh(x*c))*d^2/c/(c^2*x^2+1)
+3/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3
*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*
c))*d^2/c/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c
^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c))*d^2/c/(c^2*x^
2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2
*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(x*c))*d^2/c/(c^2*x^2+1)+3/512*(d*(c^2
*x^2+1))^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3-8*x^2*c^2
*(c^2*x^2+1)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(x*c))*d^2/c/(c^2*
x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7-32*x^6*c^6*(c^2*x^2+1)^(1/
2)+64*x^5*c^5-48*x^4*c^4*(c^2*x^2+1)^(1/2)+38*x^3*c^3-18*x^2*c^2*(c^2*x^2+
1)^(1/2)+6*x*c-(c^2*x^2+1)^(1/2))*(1+6*arcsinh(x*c))*d^2/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \left(d + c^2dx^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int \left(c^2dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)\, dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d + c^2 d x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int \left(d\left(c^2 x^2 + 1\right)\right)^{\frac{5}{2}} (a + b\operatorname{asinh}(cx))\, dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d + c^2 d x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d + c^2 d x^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{d}\,d^2\left(8\sqrt{c^2x^2+1}\,a\,c^5x^5+26\sqrt{c^2x^2+1}\,a\,c^3x^3+33\sqrt{c^2x^2+1}\,acx+48\left(\int\sqrt{c^2x^2+1}\right.\right.}{}$$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(8*sqrt(c**2*x**2 + 1)*a*c**5*x**5 + 26*sqrt(c**2*x**2 + 1)*
a*c**3*x**3 + 33*sqrt(c**2*x**2 + 1)*a*c*x + 48*int(sqrt(c**2*x**2 + 1)*as
inh(c*x)*x**4,x)*b*c**5 + 96*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*
c**3 + 48*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c + 15*log(sqrt(c**2*x**
2 + 1) + c*x)*a))/(48*c)
```

3.51 $\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 177

$$\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx)) \, dx = -\frac{3bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{16c} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

output
```
-3/16*b*c*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*d*(c^2*x^2+1)
^(3/2)*(c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)
)+1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+3/16*d*(c^2*d*x^2+d)^(1/2)*
(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\frac{1}{8}adx\left(5+2c^2x^2\right)\sqrt{d+c^2dx^2}$$
$$+\frac{3ad^{3/2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{8c}$$
$$+\frac{bd\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{8c\sqrt{1+c^2x^2}}$$
$$-\frac{bd\sqrt{d+c^2dx^2}(8\text{arcsinh}(cx)^2+\cosh(4\text{arcsinh}(cx))-4\text{arcsinh}(cx)\sinh(4\text{arcsinh}(cx)))}{128c\sqrt{1+c^2x^2}}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(a*d*x*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/8 + (3*a*d^(3/2)*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c) + (b*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*Arc
Sinh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*S
qrt[1 + c^2*x^2]) - (b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*Ar
cSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^
2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6201}$$

$$\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx-\frac{bcd\sqrt{c^2dx^2+d}\int \left(c^2x^3+x\right)dx}{4\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6200}$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}\right)-\frac{bcd\left(\frac{c^2x^4}{4}+\frac{x^2}{2}\right)\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*d*Sqrt[d + c^2*d*x^2]*(x^2/2 + (c^2*x^4)/4))/Sqrt[1 + c^2*x^2] +
 (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sq
rt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
 c^2*x^2])))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(151) = 302$.

Time = 0.93 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.80

method	result
default	$\frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{c^2dx^2+d}}{8}+\frac{3a\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b\Bigg(\frac{3\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(xc)^2d}{16\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(8x}{}$
parts	$\frac{ax(c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{c^2dx^2+d}}{8}+\frac{3a\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}}+b\Bigg(\frac{3\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(xc)^2d}{16\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(8x}{}$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*d^2*ln(x*c
^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(3/16*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)
*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(
1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*c))*d/(c^2*x^2+1)/c+1/16*(d*
(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1
)^(1/2))*(-1+2*arcsinh(x*c))*d/(c^2*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1/2)*(2
*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh
(x*c))*d/(c^2*x^2+1)/c+1/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c
^2*x^2+1)^(1/2)+12*x^3*c^3-8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c-(c^2*x^2+1)^(
1/2))*(1+4*arcsinh(x*c))*d/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d), x)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{asinh}\left(cx\right)\right)\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{c^2x^2+1}\,a\,c^3x^3+5\sqrt{c^2x^2+1}\,acx+8\left(\int\sqrt{c^2x^2+1}\,asinh(cx)\,x^2dx\right)b\,c^3+8(\right.}{8c}$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*d*(2*sqrt(c**2*x**2 + 1)*a*c**3*x**3 + 5*sqrt(c**2*x**2 + 1)*a*c*
x + 8*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*c**3 + 8*int(sqrt(c**2*
x**2 + 1)*asinh(c*x),x)*b*c + 3*log(sqrt(c**2*x**2 + 1) + c*x)*a))/(8*c)
```

3.52 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
-1/4*b*c*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*x*(c^2*d*x^2+d)^(1/
2)*(a+b*arcsinh(c*x))+1/4*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^
2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{8}\left(4ax\sqrt{d+c^2dx^2}+\frac{4a\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c}\right.$$
$$\left.+\frac{b\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{c\sqrt{1+c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(4*a*x*Sqrt[d + c^2*d*x^2] + (4*a*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2
*d*x^2]])/c + (b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*
x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2]))/8
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{6200}$$
$$\frac{\sqrt{c^2dx^2+d}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2dx^2+d}\int xdx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))$$
$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/4*(b*c*x^2*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (x*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)
/(4*b*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(95) = 190$.

Time = 0.81 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.31

method	result
default	$\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,\left(2x^3c^3+2x^2c^2\sqrt{c^2x^2+1}+2xc+\right.}{16(c^2x^2+1)c}\right.$
parts	$\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,\left(2x^3c^3+2x^2c^2\sqrt{c^2x^2+1}+2xc+\right.}{16(c^2x^2+1)c}\right.$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(c^2*d*x^2+d)^(1/2)+1/2*a*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)
^(1/2))/(c^2*d)^(1/2)+b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arc
sinh(x*c)^2+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1
/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c))/(c^2*x^2+1)/c+1/16*(d*(c^
2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(
1/2))*(1+2*arcsinh(x*c))/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = \int \sqrt{d\,(c^2x^2+1)}(a+b\operatorname{asinh}(cx))\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(\sqrt{c^2x^2+1}\,acx+2\left(\int \sqrt{c^2x^2+1}\,asinh(cx)\,dx\right)bc+\log\left(\sqrt{c^2x^2+1}+cx\right)a\right)}{2c}$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*(sqrt(c**2*x**2 + 1)*a*c*x + 2*int(sqrt(c**2*x**2 + 1)*asinh(c*x)
,x)*b*c + log(sqrt(c**2*x**2 + 1) + c*x)*a))/(2*c)
```

3.53 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

output
```
1/2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d(1+c^2x^2)}}+\frac{a\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{c\sqrt{d}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d*(1 + c^2*x^2)]) + (a*ArcT
anh[(c*Sqrt[d]*x)/Sqrt[d + c^2*d*x^2]])/(c*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(xc)^2}{2\sqrt{c^2x^2+1}\,dc}$	77
parts	$\frac{a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(xc)^2}{2\sqrt{c^2x^2+1}\,dc}$	77

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d/c*arcsinh(x*c)^2
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\left(cx\right) + a}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}\left(cx\right)}{\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \frac{b \operatorname{arsinh}(cx)^2}{2\,c\sqrt{d}} + \frac{a \operatorname{arsinh}(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b \operatorname{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{d}\,\left(asinh(cx)^2\,b+2\log\left(\sqrt{c^2x^2+1}+cx\right)a\right)}{2cd}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*(asinh(c*x)**2*b + 2*log(sqrt(c**2*x**2 + 1) + c*x)*a))/(2*c*d)
```

3.54 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{2cd\sqrt{d+c^2dx^2}}$$

output
```
x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-1/2*b*(c^2*x^2+1)^(1/2)*ln(c^2*
x^2+1)/c/d/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d+c^2dx^2}\left(2acx\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}\text{arcsinh}(cx)-(b+bc^2x^2)\log(1+\right.}{2cd^2(1+c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(2*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x] - (b + b*c^2*x^2)*Log[1 + c^2*x^2]))/(2*c*d^2*(1 + c^2*x^2)
^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\operatorname{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Lo
g[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(68) = 136$.

Time = 1.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{ax}{d\sqrt{c^{2}d\,x^{2}+d}}+\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsinh}\left(xc\right)}{\sqrt{c^{2}x^{2}+1}\,d^{2}c}+\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsinh}\left(xc\right)x}{d^{2}\left(c^{2}x^{2}+1\right)}-\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\ln\left(1+\left(xc+\sqrt{c^{2}x^{2}+1}\right)^{2}\right)}{\sqrt{c^{2}x^{2}+1}\,d^{2}c}$	143
parts	$\frac{ax}{d\sqrt{c^{2}d\,x^{2}+d}}+\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsinh}\left(xc\right)}{\sqrt{c^{2}x^{2}+1}\,d^{2}c}+\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsinh}\left(xc\right)x}{d^{2}\left(c^{2}x^{2}+1\right)}-\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\ln\left(1+\left(xc+\sqrt{c^{2}x^{2}+1}\right)^{2}\right)}{\sqrt{c^{2}x^{2}+1}\,d^{2}c}$	143

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*x/d/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2/c*
arcsinh(x*c)+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d^2/(c^2*x^2+1)*x-b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2/c*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{b\,\operatorname{arsinh}\left(cx\right)+a}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2
*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \frac{bx\,\text{arsinh}(cx)}{\sqrt{c^2dx^2+d}d} + \frac{ax}{\sqrt{c^2dx^2+d}d} - \frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a*x/(sqrt(c^2*d*x^2 + d)*d) - 1
/2*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}(cx)+a}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\frac{\sqrt{c^2x^2+1}\,acx+\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)b\,c^3x^2+\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)}{\sqrt{d}\,cd\,(c^2x^2+1)}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

output
```
(sqrt(c**2*x**2 + 1)*a*c*x + int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**2*x**2
 + sqrt(c**2*x**2 + 1)),x)*b*c**3*x**2 + int(asinh(c*x)/(sqrt(c**2*x**2 +
1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c + a*c**2*x**2 + a)/(sqrt(d)*c*d
*(c**2*x**2 + 1))
```

3.55 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+c^2dx^2)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3cd^2\sqrt{d+c^2dx^2}}$$

output
```
1/6*b/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arcsinh(c*x))
/d/(c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/
3*b*(c^2*x^2+1)^(1/2)*ln(c^2*x^2+1)/c/d^2/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(b+bc^2x^2+6acx\sqrt{1+c^2x^2}+4ac^3x^3\sqrt{1+c^2x^2}+2bcx\sqrt{1+c^2x^2}(3\right.}{6cd^3(1+c^2x^2)^{5/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 + 6*a*c*x*Sqrt[1 + c^2*x^2] + 4*a*c^3*
x^3*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2)*ArcSinh[
c*x] - 2*b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]))/(6*c*d^3*(1 + c^2*x^2)^(5/2)
)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{3/2}}dx}{3d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{3d\,(c^2dx^2+d)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log(c^2x^2+1)}{2cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

output
```
b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x]
))/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d
*x^2])))/(3*d)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(127) = 254$.

Time = 1.16 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.89

method	result
default	$a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(2x^3c^3+2x^2c^2\sqrt{c^2x^2+1}+3xc+2\sqrt{c^2x^2+1}\right)\left(-8\ln\left(1+\left(xc+\sqrt{c^2x^2+\right.\right.\right.}{}$
parts	$a\left(\frac{x}{3d(c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{c^2dx^2+d}}\right)+\frac{b\sqrt{d(c^2x^2+1)}\left(2x^3c^3+2x^2c^2\sqrt{c^2x^2+1}+3xc+2\sqrt{c^2x^2+1}\right)\left(-8\ln\left(1+\left(xc+\sqrt{c^2x^2+\right.\right.\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))+1/6*b*(d*(c^
2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+3*x*c+2*(c^2*x^2+1)
^(1/2))*(-8*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^6*c^6+8*(c^2*x^2+1)^(1/2)*ln
(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^5*c^5-24*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x
^4*c^4+20*(c^2*x^2+1)^(1/2)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+2*c^4*
x^4-2*(c^2*x^2+1)^(1/2)*c^3*x^3+6*arcsinh(x*c)*c^2*x^2-24*ln(1+(x*c+(c^2*x
^2+1)^(1/2))^2)*x^2*c^2+12*(c^2*x^2+1)^(1/2)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^
2)*x*c+4*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+8*arcsinh(x*c)-8*ln(1+(x*c+(c^2*x
^2+1)^(1/2))^2)+2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3/c
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(d\,(c^2x^2 + 1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{5/2}}\,dx = \frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2 + c^2d^{\frac{5}{2}}} - \frac{2\,\log\left(c^2x^2 + 1\right)}{c^2d^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{c^2dx^2 + d}d^2} + \frac{x}{(c^2dx^2 + d)^{\frac{3}{2}}d}\right)\text{arsinh}\,(cx) + \frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{c^2dx^2 + d}d^2} + \frac{x}{(c^2dx^2 + d)^{\frac{3}{2}}d}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/6*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/
2))) + 1/3*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))
*arcsinh(c*x) + 1/3*a*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^
(3/2)*d))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\frac{2\sqrt{c^2x^2+1}\,a\,c^3x^3+3\sqrt{c^2x^2+1}\,acx+3\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^4x^4+2\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)}{}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

output
```
(2*sqrt(c**2*x**2 + 1)*a*c**3*x**3 + 3*sqrt(c**2*x**2 + 1)*a*c*x + 3*int(a
sinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2
 + sqrt(c**2*x**2 + 1)),x)*b*c**5*x**4 + 6*int(asinh(c*x)/(sqrt(c**2*x**2
+ 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)
*b*c**3*x**2 + 3*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c*
*2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c - 2*a*c**4*x**4 - 4*a
*c**2*x**2 - 2*a)/(3*sqrt(d)*c*d**2*(c**4*x**4 + 2*c**2*x**2 + 1))
```

3.56 $\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{7/2}}\,dx = \frac{b}{20cd^3(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}} + \frac{2b}{15cd^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d(d+c^2dx^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2(d+c^2dx^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{15cd^3\sqrt{d+c^2dx^2}}$$

output
```
1/20*b/c/d^3/(c^2*x^2+1)^(3/2)/(c^2*d*x^2+d)^(1/2)+2/15*b/c/d^3/(c^2*x^2+1
)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/5*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(5/2)
+4/15*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(3/2)+8/15*x*(a+b*arcsinh(c*x
))/d^3/(c^2*d*x^2+d)^(1/2)-4/15*b*(c^2*x^2+1)^(1/2)*ln(c^2*x^2+1)/c/d^3/(c
^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{7/2}}\,dx = \frac{\sqrt{d+c^2dx^2}\Big(11b+19bc^2x^2+8bc^4x^4+60acx\sqrt{1+c^2x^2}+80ac^3x^3\sqrt{1+c^2x^2}+3}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(7/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(11*b + 19*b*c^2*x^2 + 8*b*c^4*x^4 + 60*a*c*x*Sqrt[1
+ c^2*x^2] + 80*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 32*a*c^5*x^5*Sqrt[1 + c^2*x^
2] + 4*b*c*x*Sqrt[1 + c^2*x^2]*(15 + 20*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]
- 16*b*(1 + c^2*x^2)^3*Log[1 + c^2*x^2]))/(60*c*d^4*(1 + c^2*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6203, 241, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{7/2}}dx$$

$$\Big\downarrow \text{ 6203}$$

$$\frac{4\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{5/2}}dx}{5d} - \frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^3}dx}{5d^3\sqrt{c^2dx^2+d}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(c^2dx^2+d)^{5/2}}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{4\int \frac{a+b\text{arcsinh}(cx)}{(c^2dx^2+d)^{5/2}}dx}{5d} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(c^2dx^2+d)^{5/2}} + \frac{b}{20cd^3\,(c^2x^2+1)^{3/2}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6203}$$

$$\frac{4\left(\frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2dx^2+d\right)^{3/2}}dx}{3d}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{\left(c^2x^2+1\right)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}\right)}{5d}+\frac{x(a+b\text{arcsinh}(cx))}{5d\left(c^2dx^2+d\right)^{5/2}}+\frac{b}{20cd^3\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\left(\frac{2\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2dx^2+d\right)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}\right)}{5d}+\frac{x(a+b\text{arcsinh}(cx))}{5d\left(c^2dx^2+d\right)^{5/2}}+\frac{b}{20cd^3\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{4\left(\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}\right)}{5d}+\frac{x(a+b\text{arcsinh}(cx))}{5d\left(c^2dx^2+d\right)^{5/2}}+\frac{b}{20cd^3\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{240}$$

$$\frac{4\left(\frac{x(a+b\text{arcsinh}(cx))}{3d(c^2dx^2+d)^{3/2}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}\right)}{5d}+\frac{x(a+b\text{arcsinh}(cx))}{5d\left(c^2dx^2+d\right)^{5/2}}+\frac{b}{20cd^3\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(7/2),x]
```

output
```
b/(20*c*d^3*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c
*x]))/(5*d*(d + c^2*d*x^2)^(5/2)) + (4*(b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[
d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + (
2*((x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]
*Log[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])))/(3*d)))/(5*d)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2185 vs. $2(185) = 370$.

Time = 1.20 (sec) , antiderivative size = 2186, normalized size of antiderivative = 10.17

method	result	size
default	Expression too large to display	2186
parts	Expression too large to display	2186

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-3526/15*b*(d*(c^2*x^2+1))^(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517
*c^4*x^4+287*c^2*x^2+64)/d^4*c^4*x^5-334/3*b*(d*(c^2*x^2+1))^(1/2)/(40*c^1
0*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^4*c^2*x^3+176
/15*b*(d*(c^2*x^2+1))^(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*
x^4+287*c^2*x^2+64)/d^4/c*(c^2*x^2+1)^(1/2)-8/15*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/d^4/c*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)+16/15*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^4/c*arcsinh(x*c)+22*b*(d*(c^2*x^2+1))^(1/2)
/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^4*(c^
2*x^2+1)*x+64*b*(d*(c^2*x^2+1))^(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^
6+517*c^4*x^4+287*c^2*x^2+64)/d^4*arcsinh(x*c)*x-128/15*b*(d*(c^2*x^2+1))^
(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^
4*c^12*x^13-176/3*b*(d*(c^2*x^2+1))^(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^
6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^4*c^10*x^11-2552/15*b*(d*(c^2*x^2+1))^
(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^
4*c^8*x^9-3986/15*b*(d*(c^2*x^2+1))^(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^
6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^4*c^6*x^7-22*b*(d*(c^2*x^2+1))^(1/2)/(
40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^4*x+541
/3*b*(d*(c^2*x^2+1))^(1/2)/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x
^4+287*c^2*x^2+64)/d^4*c^2*arcsinh(x*c)*x^3+519/20*b*(d*(c^2*x^2+1))^(1/2)
/(40*c^10*x^10+215*c^8*x^8+469*c^6*x^6+517*c^4*x^4+287*c^2*x^2+64)/d^4*...
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+c^2dx^2)^{7/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(c^2dx^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^8*d^4*x^8 + 4*c^6*d^4
*x^6 + 6*c^4*d^4*x^4 + 4*c^2*d^4*x^2 + d^4), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{7/2}}\,dx = \int \frac{a + b\operatorname{asinh}(cx)}{(d(c^2x^2 + 1))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(7/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(7/2), x)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{7/2}}\,dx = \int \frac{b\,\text{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(8*x/(sqrt(c^2*d*x^2 + d)*d^3) + 4*x/((c^2*d*x^2 + d)^(3/2)*d^2) +
3*x/((c^2*d*x^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/
(c^2*d*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{7/2}}\,dx = \int \frac{b\,\text{arsinh}(cx) + a}{(c^2dx^2 + d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{7/2}} \, dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(d\,c^2\,x^2 + d)^{7/2}} \, dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(7/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + c^2dx^2)^{7/2}} \, dx = \frac{8\sqrt{c^2x^2+1}\,a\,c^5x^5 + 20\sqrt{c^2x^2+1}\,a\,c^3x^3 + 15\sqrt{c^2x^2+1}\,acx + 15\Big(\int \frac{}{\sqrt{c^2x^2+1}\,c^6x^6+}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^(7/2),x)
```

output
```
(8*sqrt(c**2*x**2 + 1)*a*c**5*x**5 + 20*sqrt(c**2*x**2 + 1)*a*c**3*x**3 +
15*sqrt(c**2*x**2 + 1)*a*c*x + 15*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**6
*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqrt(c**2*x**2 + 1)*c**2*x**2
+ sqrt(c**2*x**2 + 1)),x)*b*c**7*x**6 + 45*int(asinh(c*x)/(sqrt(c**2*x**2
+ 1)*c**6*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqrt(c**2*x**2 + 1)*c
**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c**5*x**4 + 45*int(asinh(c*x)/(sqrt(c
**2*x**2 + 1)*c**6*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqrt(c**2*x*
*2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c**3*x**2 + 15*int(asinh(c*x
)/(sqrt(c**2*x**2 + 1)*c**6*x**6 + 3*sqrt(c**2*x**2 + 1)*c**4*x**4 + 3*sqr
t(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b*c - 8*a*c**6*x**6 -
 24*a*c**4*x**4 - 24*a*c**2*x**2 - 8*a)/(15*sqrt(d)*c*d**3*(c**6*x**6 + 3*
c**4*x**4 + 3*c**2*x**2 + 1))
```

3.57 $\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 284

$$\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^2 \, dx = \frac{15}{64}b^2dx\sqrt{d+c^2dx^2}$$
$$+ \frac{1}{32}b^2x(d+c^2dx^2)^{3/2} - \frac{9b^2d\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}}$$
$$- \frac{3bcdx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{bd(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c}$$
$$+ \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2 + \frac{1}{4}x(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2 + \frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx}{8bc\sqrt{1+c^2x^2}}$$

output
```
15/64*b^2*d*x*(c^2*d*x^2+d)^(1/2)+1/32*b^2*x*(c^2*d*x^2+d)^(3/2)-9/64*b^2*
d*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(1/2)-3/8*b*c*d*x^2*(c^2*
d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-1/8*b*d*(c^2*x^2+1)^(3
/2)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c+3/8*d*x*(c^2*d*x^2+d)^(1/2)*(
a+b*arcsinh(c*x))^2+1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*d*(
c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.16

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{96a^2cdx\sqrt{1+c^2x^2}(5+2c^2x^2)\sqrt{d+c^2dx^2}+288a^2d^{3/2}\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d-\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(96*a^2*c*d*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 288*
a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 3
2*b^2*d*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcS
inh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 192*a*b*d*Sqrt[
d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh
[2*ArcSinh[c*x]])) - 12*a*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh
[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - b^2*d*Sqrt[d + c
^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1
 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/(768*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6200}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsin}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}}{2\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(\right.}{\sqrt{c}}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6198}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$
$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$$\Big\downarrow \text{6213}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int (c^2x^2+1)^{3/2}dx}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+ b\text{arcsinh}(cx))^2+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$$\Big\downarrow \text{211}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int \sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$$\Big\downarrow \text{211}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsinh}(cx)}{2c}+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)}{2\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(c\right.}{\sqrt{c^2}}\right.$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*((x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^3)/(6*b*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]*((x^2*(a + b*A
rcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^
3)))/2))/Sqrt[1 + c^2*x^2]))/4 - (b*c*d*Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x
*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 + c^2*x
^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(244) = 488$.

Time $= 1.15$ (sec) , antiderivative size $= 959$, normalized size of antiderivative $= 3.38$

method	result
default	$\frac{a^2x(c^2d\,x^2+d)^{\frac{3}{2}}}{4}+\frac{3a^2dx\sqrt{c^2d\,x^2+d}}{8}+\frac{3a^2d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(xc)^3d}{8\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}}{}\right.$
parts	$\frac{a^2x(c^2d\,x^2+d)^{\frac{3}{2}}}{4}+\frac{3a^2dx\sqrt{c^2d\,x^2+d}}{8}+\frac{3a^2d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\mathrm{arcsinh}(xc)^3d}{8\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}}{}\right.$

input

```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/4*a^2*x*(c^2*d*x^2+d)^(3/2)+3/8*a^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2*
ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/8*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3*d+1/512*(d*(c^2*x^2+1)
)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x
^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(8*arcsinh(x*c)^2-4*arcsinh(x*c)+1)*d
/(c^2*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)
^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)*d/(c^2
*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2
)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)*d/(c^2*x^2+
1)/c+1/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12
*x^3*c^3-8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*(8*arcsinh(x
*c)^2+4*arcsinh(x*c)+1)*d/(c^2*x^2+1)/c)+2*a*b*(3/16*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*d+1/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c
^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*
c+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*c))*d/(c^2*x^2+1)/c+1/16*(d*(c^2*x^2+
1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*
(-1+2*arcsinh(x*c))*d/(c^2*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-
2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(x*c))*d/
(c^2*x^2+1)/c+1/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)
^(1/2)+12*x^3*c^3-8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*...
```

Fricas [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
 2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))^2\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{c^2x^2+1}\,a^2c^3x^3+5\sqrt{c^2x^2+1}\,a^2cx+16\left(\int\sqrt{c^2x^2+1}\,asinh(cx)\,x^2dx\right)ab\,c^3\right.}{}$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x))^2,x)
```

output

```
(sqrt(d)*d*(2*sqrt(c**2*x**2 + 1)*a**2*c**3*x**3 + 5*sqrt(c**2*x**2 + 1)*a
**2*c*x + 16*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*a*b*c**3 + 16*int(
sqrt(c**2*x**2 + 1)*asinh(c*x),x)*a*b*c + 8*int(sqrt(c**2*x**2 + 1)*asinh(
c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(c**2*x**2 + 1)*asinh(c*x)**2,x)*b**
2*c + 3*log(sqrt(c**2*x**2 + 1) + c*x)*a**2))/(8*c)
```

3.58 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\ &-\frac{bcx^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\ &+\frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\\ &+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
1/4*b^2*x*(c^2*d*x^2+d)^(1/2)-1/4*b^2*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)/c/(
c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x
^2+1)^(1/2)+1/2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2+1/6*(c^2*d*x^2+
d)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{1}{24}\left(12a^2x\sqrt{d+c^2dx^2}+\frac{12a^2\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{c}\right.$$
$$+\frac{b^2\sqrt{d+c^2dx^2}(4\text{arcsinh}(cx)^3-6\text{arcsinh}(cx)\cosh(2\text{arcsinh}(cx))+(3+6\text{arcsinh}(cx)^2)\sinh(2\text{arcsinh}(cx)))}{c\sqrt{1+c^2x^2}}$$
$$\left.+\frac{6ab\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{c\sqrt{1+c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(12*a^2*x*Sqrt[d + c^2*d*x^2] + (12*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d
 + c^2*d*x^2]])/c + (b^2*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh
[c*x]*Cosh[2*ArcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))
/(c*Sqrt[1 + c^2*x^2]) + (6*a*b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]]
 + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*
x^2]))/24
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \; 6200$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int x(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{6191}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int \frac{x^2}{\sqrt{c^2x^2+1}}dx\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{262}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)}{\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2-\frac{bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(cx)}{2c^3}\right)\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[d + c^2*d*x^2]
*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - Arc
Sinh[c*x]/(2*c^3)))/2))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(158) = 316$.

Time = 1.01 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.61

method	result
default	$\frac{a^2x\sqrt{c^2d\,x^2+d}}{2}+\frac{a^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\left(2x^3c^3+2x^2c^2\sqrt{c^2x^2+1}+2\right.}{16}\right.$
parts	$\frac{a^2x\sqrt{c^2d\,x^2+d}}{2}+\frac{a^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\left(2x^3c^3+2x^2c^2\sqrt{c^2x^2+1}+2\right.}{16}\right.$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2)+b^2*(1/6*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c*arcsinh(x*c)^3+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2
+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)/(c^
2*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/
2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)/(c^2*x^2+1
)/c)+2*a*b*(1/4*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2+1
/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^
2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c))/(c^2*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1/
2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*ar
csinh(x*c))/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2), x)
```

Sympy [F]

$$\int \sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^2 \, dx = \int \sqrt{d (c^2 x^2 + 1)} (a + b \operatorname{asinh}(cx))^2 \, dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d + c^2 d x^2} (a + b \text{arcsinh}(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{\sqrt{d}\,\left(\sqrt{c^2x^2+1}\,a^2cx+4\left(\int \sqrt{c^2x^2+1}\,asinh(cx)\,dx\right)abc+2\left(\int \sqrt{c^2x^2+1}\,asinh(cx)^2\,dx\right)b^2c+\log\left(\sqrt{c^2x\right.\right.}{2c}$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(d)*(sqrt(c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt(c**2*x**2 + 1)*asinh(c
*x),x)*a*b*c + 2*int(sqrt(c**2*x**2 + 1)*asinh(c*x)**2,x)*b**2*c + log(sqr
t(c**2*x**2 + 1) + c*x)*a**2))/(2*c)
```

3.59 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+c^2dx^2}}$$

output
```
1/3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\frac{3ab\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{\sqrt{d+c^2dx^2}}+\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^3}{\sqrt{d+c^2dx^2}}+\frac{3a^2\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{\sqrt{d}}}{3c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

output
```
((3*a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]^3)/Sqrt[d + c^2*d*x^2] + (3*a^2*ArcTanh[(c*Sqrt[
d]*x)/Sqrt[d + c^2*d*x^2]])/Sqrt[d])/(3*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{6198\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{c^2dx^2 + d}}\,dx$$

$$\Big\downarrow \; 6198$$

$$\frac{\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))^3}{3bc\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(41) = 82$.

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.55

method	result	size
default	$\frac{a^{2} \ln\left(\frac{x\,c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(x c\right)^{3}}{3 \sqrt{c^{2} x^{2}+1}\, d c}+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(x c\right)^{2}}{\sqrt{c^{2} x^{2}+1}\, d c}$	120
parts	$\frac{a^{2} \ln\left(\frac{x\,c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(x c\right)^{3}}{3 \sqrt{c^{2} x^{2}+1}\, d c}+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(x c\right)^{2}}{\sqrt{c^{2} x^{2}+1}\, d c}$	120

input
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d/c*arcsinh(x*c)^3+a*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/d/c*arcsinh(x*c)^2
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(c^2*d*x^2 +
d), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{b^2\,\text{arsinh}\,(cx)^3}{3\,c\sqrt{d}} + \frac{ab\,\text{arsinh}\,(cx)^2}{c\sqrt{d}} + \frac{a^2\,\text{arsinh}\,(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*b^2*arcsinh(c*x)^3/(c*sqrt(d)) + a*b*arcsinh(c*x)^2/(c*sqrt(d)) + a^2*
arcsinh(c*x)/(c*sqrt(d))
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d}\left(asinh(cx)^3 b^2+3asinh(cx)^2 ab+3\log\left(\sqrt{c^2x^2+1}+cx\right)a^2\right)}{3cd}$$

input
```
int((a+b*asinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*(asinh(c*x)**3*b**2 + 3*asinh(c*x)**2*a*b + 3*log(sqrt(c**2*x**2
+ 1) + c*x)*a**2))/(3*c*d)
```

3.60 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+c^2dx^2}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}-\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+c^2dx^2}}$$

output

```
x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)+(c^2*x^2+1)^(1/2)*(a+b*arcsin
h(c*x))^2/c/d/(c^2*d*x^2+d)^(1/2)-2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/d/(c^2*d*x^2+d)^(1/2)-b^2*(c^2*x^2+1)^(
1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/d/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \frac{-b^2\left(-cx+\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)^2+2b\text{arcsinh}(cx)\left(acx-b\sqrt{1+c^2x^2}\log\left(\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*
c*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2
*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6212}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{26}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+b\text{arc}}{}\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcsinh}(cx}\right.\right.}{cd\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right.}{cd\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2
*x^2]*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*
Log[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/
(c*d*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.92

method	result
default	$\frac{a^2 x}{d\sqrt{c^2 d\,x^2+d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^2 x}{d^2(c^2x^2+1)}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^2}{d^2 c\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)\ln\left(1+\left(xc+\sqrt{c\right.\right.}}{\sqrt{c^2x^2+1}\,d^2 c}$
parts	$\frac{a^2 x}{d\sqrt{c^2 d\,x^2+d}}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^2 x}{d^2(c^2x^2+1)}+\frac{b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)^2}{d^2 c\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d(c^2x^2+1)}\,\operatorname{arcsinh}(xc)\ln\left(1+\left(xc+\sqrt{c\right.\right.}}{\sqrt{c^2x^2+1}\,d^2 c}$

input
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*x/d/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)^2/d^2/(
c^2*x^2+1)*x+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)^2/d^2/c/(c^2*x^2+1)^(1
/2)-2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^2/c*arcsinh(x*c)*ln(1+
(x*c+(c^2*x^2+1)^(1/2))^2)-b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^2
/c*polylog(2,-(x*c+(c^2*x^2+1)^(1/2))^2)+2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x
^2+1))^(1/2)/d^2/c*arcsinh(x*c)+2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d
^2/(c^2*x^2+1)*x-2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^2/c*ln(1+
(x*c+(c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{(d(c^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2), x) + 2
*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a^2*x/(sqrt(c^2*d*x^2 + d)*d
) - a*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{c^2x^2+1}\,a^2cx + 2\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}dx\right)ab\,c^3x^2 + 2\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x}}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

output
```
(sqrt(c**2*x**2 + 1)*a**2*c*x + 2*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**2
*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c**3*x**2 + 2*int(asinh(c*x)/(sqrt(c**
2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c + int(asinh(c*x)**2/
(sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b**2*c**3*x**2 +
int(asinh(c*x)**2/(sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)
*b**2*c + a**2*c**2*x**2 + a**2)/(sqrt(d)*c*d*(c**2*x**2 + 1))
```

3.61 $\int \frac{(a+b\textbf{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 292

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx &= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} \\ &+ \frac{b(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(d+c^2dx^2)^{3/2}} \\ &+ \frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2\sqrt{d+c^2dx^2}} + \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+c^2dx^2}} \\ &- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}} \\ &- \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+c^2dx^2}}\end{aligned}$$

output

```
-1/3*b^2*x/d^2/(c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2
+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(
3/2)+2/3*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)+2/3*(c^2*x^2+1)^(1
/2)*(a+b*arcsinh(c*x))^2/c/d^2/(c^2*d*x^2+d)^(1/2)-4/3*b*(c^2*x^2+1)^(1/2)
*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/d^2/(c^2*d*x^2+d)^(1
/2)-2/3*b^2*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/d^2/
(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \frac{a^2cx(3+2c^2x^2)+ab\big((6cx+4c^3x^3)\,\text{arcsinh}(cx)+\sqrt{1+c^2x^2}(1-2(1+c^2x^2)$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(a^2*c*x*(3 + 2*c^2*x^2) + a*b*((6*c*x + 4*c^3*x^3)*ArcSinh[c*x] + Sqrt[1
+ c^2*x^2]*(1 - 2*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - b^2*(c*x + c^3*x^3 -
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]^2 - 2*c*x*(1 + c^2*x^2)*
ArcSinh[c*x]^2 + 2*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[
1 + E^(-2*ArcSinh[c*x])]) - 2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSin
h[c*x])]))/(3*d^2*(c + c^3*x^2)*Sqrt[d + c^2*d*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2dx^2+d)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx}{d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \; 2620$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)-\frac{i(a+}{}\right.}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)de^{2\text{arcs}}\right.\right.}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2bc\sqrt{c^2x^2+1}\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(}{2b}\right.}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 6213$$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{b\int \frac{1}{\left(c^2x^2+1\right)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(}{2b}\right.}{cd\sqrt{c^2dx^2+d}}\right.}{3d}$$

$$\frac{x(a+b\text{arcsinh}(cx))^2}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$-\frac{2bc\sqrt{c^2x^2+1}\left(\frac{bx}{2c\sqrt{c^2x^2+1}}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^2}{d\sqrt{c^2dx^2+d}}+\frac{2ib\sqrt{c^2x^2+1}\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{cd\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^2}{3d\left(c^2dx^2+d\right)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 + c
^2*x^2]*((b*x)/(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 +
c^2*x^2))))/(3*d^2*Sqrt[d + c^2*d*x^2]) + (2*((x*(a + b*ArcSinh[c*x])^2)/(
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b*Sqrt[1 + c^2*x^2]*(((-1/2*I)*(a + b*ArcS
inh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])
/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/(c*d*Sqrt[d + c^2*d*x^2])))/
(3*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(274) = 548$.

Time = 1.45 (sec) , antiderivative size = 1967, normalized size of antiderivative = 6.74

method	result	size
default	Expression too large to display	1967
parts	Expression too large to display	1967

input
```
int((a+b*arcsinh(x*c))^2/(c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*(c^2
*x^2+1)*x+4*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/
d^3*arcsinh(x*c)^2*x-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*
c^2*x^2+4)/d^3*arcsinh(x*c)*x+4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1
/2)/d^3/c*arcsinh(x*c)^2-2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d
^3/c*polylog(2,-(x*c+(c^2*x^2+1)^(1/2))^2)-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*c^6*x^7-3*b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*c^4*x^5-13/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*c^2*x^3+4/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3/c*(c^2*x^2+1)^(1/2)+a^2*
(1/3*x/d/(c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(c^2*d*x^2+d)^(1/2))-1/3*a*b*(c^2*x
^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)*(4*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^4*c
^4-4*arcsinh(x*c)*c^4*x^4-4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3+8*ln(1+
(x*c+(c^2*x^2+1)^(1/2))^2)*x^2*c^2-8*arcsinh(x*c)*c^2*x^2-6*arcsinh(x*c)*(
c^2*x^2+1)^(1/2)*x*c-c^2*x^2+4*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)-4*arcsinh(x
*c)-1)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/d^3/c+10/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*c^2*(c^2*x^2+1)*arcsinh(x*c)*x^3
-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*c*
(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^
6+10*c^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*arcsinh(x*c)*x+2/3*b^2*(d*(c...
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(
5/2))) + 2/3*a*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)
*d))*arcsinh(c*x) + 1/3*a^2*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2
 + d)^(3/2)*d)) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2
+ d)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+c^2dx^2)^{5/2}}\,dx=\frac{2\sqrt{c^2x^2+1}\,a^2c^3x^3+3\sqrt{c^2x^2+1}\,a^2cx+6\left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,c^4x^4+2\sqrt{c^2x^2+1}\,c^2x^2+\sqrt{c^2x^2+1}}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

output
```
(2*sqrt(c**2*x**2 + 1)*a**2*c**3*x**3 + 3*sqrt(c**2*x**2 + 1)*a**2*c*x + 6
*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**
2*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c**5*x**4 + 12*int(asinh(c*x)/(sqrt(c
**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2
 + 1)),x)*a*b*c**3*x**2 + 6*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*c**4*x**4
+ 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*a*b*c + 3*int(
asinh(c*x)**2/(sqrt(c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*
x**2 + sqrt(c**2*x**2 + 1)),x)*b**2*c**5*x**4 + 6*int(asinh(c*x)**2/(sqrt(
c**2*x**2 + 1)*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**
2 + 1)),x)*b**2*c**3*x**2 + 3*int(asinh(c*x)**2/(sqrt(c**2*x**2 + 1)*c**4*
x**4 + 2*sqrt(c**2*x**2 + 1)*c**2*x**2 + sqrt(c**2*x**2 + 1)),x)*b**2*c -
2*a**2*c**4*x**4 - 4*a**2*c**2*x**2 - 2*a**2)/(3*sqrt(d)*c*d**2*(c**4*x**4
 + 2*c**2*x**2 + 1))
```

3.62 $\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 335

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = -\frac{45acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} - \frac{3c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}{128a} + \frac{45}{64}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax) + \frac{3}{32}x\left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax) - \frac{27c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{128a\sqrt{1+a^2x^2}} - \frac{9acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^2}{16\sqrt{1+a^2x^2}} - \frac{3c(1+a^2}{}$$

output
```
-45/128*a*c*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/128*c*(a^2*x^2+1)^
(3/2)*(a^2*c*x^2+c)^(1/2)/a+45/64*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)+3/3
2*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)-27/128*c*(a^2*c*x^2+c)^(1/2)*arcsinh(
a*x)^2/a/(a^2*x^2+1)^(1/2)-9/16*a*c*x^2*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^2
/(a^2*x^2+1)^(1/2)-3/16*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)*arcsinh(a*
x)^2/a+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^3+1/4*x*(a^2*c*x^2+c)^(3/2
)*arcsinh(a*x)^3+3/32*c*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^4/a/(a^2*x^2+1)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.41

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^3 \, dx = \frac{c\sqrt{c+a^2cx^2}(96\operatorname{arcsinh}(ax)^4 - 24\operatorname{arcsinh}(ax)^2(16\cosh(2\operatorname{arcsinh}(ax)) + \cosh$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(96*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*(16*Cosh[2*A
rcSinh[a*x]] + Cosh[4*ArcSinh[a*x]]) - 3*(64*Cosh[2*ArcSinh[a*x]] + Cosh[4
*ArcSinh[a*x]]) + 32*ArcSinh[a*x]^3*(8*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSi
nh[a*x]]) + 12*ArcSinh[a*x]*(32*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]
]))/(1024*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6201, 6200, 6191, 6198, 6213, 6201, 244, 2009, 6200, 15, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x(a^2x^2+1)\operatorname{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}} + \frac{3}{4}c\int \sqrt{a^2cx^2+c}\operatorname{arcsinh}(ax)^3dx + \frac{1}{4}x\operatorname{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6200}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}\right)+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6191}$$

$$-\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+$$
$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)\right.$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^2dx}{4\sqrt{a^2x^2+1}}+\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{\left(a^2x^2+1\right)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\int\left(a^2x^2+1\right)^{3/2}\text{arcsinh}(ax)dx}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{c}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx-\frac{1}{4}a\int x(a^2x^2+1)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 244$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{c}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx-\frac{1}{4}a\int(a^2x^3+x)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{c}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\int\sqrt{a^2x^2+1}\text{arcsinh}(ax)dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}+$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 6200$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{c}\right.$$
$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx-\frac{a\int xdx}{2}+\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)\right)+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}}{2a}\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 15$$

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a\right.$$

$$\frac{3ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx+\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)-\frac{ax^2}{4}\right)+\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax}{2a}\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6198

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-$$

$$\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$\downarrow$ 6227

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)}{8a\sqrt{a^2\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-$$

$$\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$\downarrow$ 15

$$\frac{3}{4}c\left(-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{\ldots}}{8a\sqrt{a^2x^2\ldots}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$\downarrow$ 6198

$$\frac{1}{4}x\text{arcsinh}(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{3ac\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^2}{4a^2}-\frac{\frac{1}{4}x(a^2x^2+1)^{3/2}\text{arcsinh}(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\text{arcsinh}(ax)+\frac{\text{arcsinh}(ax)^2}{4a}-\frac{ax^2}{4}\right)-\frac{1}{4}a\left(\frac{a^2x^4}{4}+\frac{x^2}{2}\right)}{2a}\right)}{4\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}-\frac{3a\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2\ldots}}{\ldots}\right.\right.}{2\sqrt{a^2x^2+1}}\right.$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

output
```
(x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/4 + (3*c*((x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(8*a*Sqrt[1 + a^2
*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^2*ArcSinh[a*x]^2)/2 - a*(-1/4*x^2/a
+ (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4*a^3))))/(
2*Sqrt[1 + a^2*x^2])))/4 - (3*a*c*Sqrt[c + a^2*c*x^2]*(((1 + a^2*x^2)^2*Ar
cSinh[a*x]^2)/(4*a^2) - (-1/4*(a*(x^2/2 + (a^2*x^4)/4)) + (x*(1 + a^2*x^2)
^(3/2)*ArcSinh[a*x])/4 + (3*(-1/4*(a*x^2) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a
*x])/2 + ArcSinh[a*x]^2/(4*a)))/4)/(2*a)))/(4*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.44

method	result
default	$\frac{3\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(xa)^4 c}{32\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\left(8x^5a^5+8x^4a^4\sqrt{a^2x^2+1}+12x^3a^3+8x^2a^2\sqrt{a^2x^2+1}+4xa+\sqrt{a^2x^2+1}\right)\left(32\operatorname{arcsinh}(\right.}{2048a(a^2x^2+1)}$

input
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(x*a)^4*c+1/2048*(c*
(a^2*x^2+1))^(1/2)*(8*x^5*a^5+8*x^4*a^4*(a^2*x^2+1)^(1/2)+12*x^3*a^3+8*x^2
*a^2*(a^2*x^2+1)^(1/2)+4*x*a+(a^2*x^2+1)^(1/2))*(32*arcsinh(x*a)^3-24*arcs
inh(x*a)^2+12*arcsinh(x*a)-3)*c/a/(a^2*x^2+1)+1/32*(c*(a^2*x^2+1))^(1/2)*(
2*x^3*a^3+2*x^2*a^2*(a^2*x^2+1)^(1/2)+2*x*a+(a^2*x^2+1)^(1/2))*(4*arcsinh(
x*a)^3-6*arcsinh(x*a)^2+6*arcsinh(x*a)-3)*c/a/(a^2*x^2+1)+1/32*(c*(a^2*x^2
+1))^(1/2)*(2*x^3*a^3-2*x^2*a^2*(a^2*x^2+1)^(1/2)+2*x*a-(a^2*x^2+1)^(1/2))
*(4*arcsinh(x*a)^3+6*arcsinh(x*a)^2+6*arcsinh(x*a)+3)*c/a/(a^2*x^2+1)+1/20
48*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*x^4*a^4*(a^2*x^2+1)^(1/2)+12*x^3*a^3
-8*x^2*a^2*(a^2*x^2+1)^(1/2)+4*x*a-(a^2*x^2+1)^(1/2))*(32*arcsinh(x*a)^3+2
4*arcsinh(x*a)^2+12*arcsinh(x*a)+3)*c/a/(a^2*x^2+1)
```

Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{asinh}^3\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx = \int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\operatorname{arcsinh}(ax)^3\,dx = \sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,asinh(ax)^3\,x^2dx\right)a^2 + \int \sqrt{a^2x^2+1}\,asinh(ax)^3\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*asinh(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*asinh(a*x)**3*x**2,x)*a**2 + int(sqrt(a
**2*x**2 + 1)*asinh(a*x)**3,x))
```

3.63 $\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^3\,dx &= -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}}+\frac{3}{4}x\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)\\ &\quad-\frac{3\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^2}{8a\sqrt{1+a^2x^2}}\\ &\quad-\frac{3ax^2\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^2}{4\sqrt{1+a^2x^2}}\\ &\quad+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^3\\ &\quad+\frac{\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^4}{8a\sqrt{1+a^2x^2}}\end{aligned}$$

output
```
-3/8*a*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/4*x*(a^2*c*x^2+c)^(1/2)
*arcsinh(a*x)-3/8*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^2/a/(a^2*x^2+1)^(1/2)-3
/4*a*x^2*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2)+1/2*x*(a^2*c
*x^2+c)^(1/2)*arcsinh(a*x)^3+1/8*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^4/a/(a^2
*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx$$
$$=\frac{\sqrt{c(1+a^2x^2)}(-3(1+2\text{arcsinh}(ax)^2)\cosh(2\text{arcsinh}(ax))+2\text{arcsinh}(ax)\,(\text{arcsinh}(ax)^3+(3+2\text{arcsinh}}{16a\sqrt{1+a^2x^2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-3*(1 + 2*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]] + 2
*ArcSinh[a*x]*(ArcSinh[a*x]^3 + (3 + 2*ArcSinh[a*x]^2)*Sinh[2*ArcSinh[a*x]
])))/(16*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6200, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{6200}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^2dx}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{6191}$$

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$\downarrow$ 6198

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\int\frac{x^2\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$\downarrow$ 6227

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int xdx}{2a}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$\downarrow$ 15

$$-\frac{3a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\int\frac{\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)}{2\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}$$

$\downarrow$ 6198

$$\frac{\text{arcsinh}(ax)^4\sqrt{a^2cx^2+c}}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^3\sqrt{a^2cx^2+c}-\frac{3a\left(\frac{1}{2}x^2\text{arcsinh}(ax)^2-a\left(-\frac{\text{arcsinh}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a^2}-\frac{x^2}{4a}\right)\right)\sqrt{a^2cx^2+c}}{2\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x]^4)/(8*a*Sqrt[1 + a^2*x^2]) - (3*a*Sqrt[c + a^2*c*x^2]*((x^2*ArcSinh[a*x
]^2)/2 - a*(-1/4*x^2/a + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcS
inh[a*x]^2/(4*a^3))))/(2*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\,\text{arcsinh}(xa)^4}{8\sqrt{a^2x^2+1}\,a}+\frac{\sqrt{c(a^2x^2+1)}\left(2x^3a^3+2x^2a^2\sqrt{a^2x^2+1}+2xa+\sqrt{a^2x^2+1}\right)\left(4\,\text{arcsinh}(xa)^3-6\,\text{arcsinh}(xa)^2+6\,\text{arcsi}\right.}{32(a^2x^2+1)a}$

input
```
int((a^2*c*x^2+c)^(1/2)*arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(x*a)^4+1/32*(c*(a^2*
x^2+1))^(1/2)*(2*x^3*a^3+2*x^2*a^2*(a^2*x^2+1)^(1/2)+2*x*a+(a^2*x^2+1)^(1/
2))*(4*arcsinh(x*a)^3-6*arcsinh(x*a)^2+6*arcsinh(x*a)-3)/(a^2*x^2+1)/a+1/3
2*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*x^2*a^2*(a^2*x^2+1)^(1/2)+2*x*a-(a^2*
x^2+1)^(1/2))*(4*arcsinh(x*a)^3+6*arcsinh(x*a)^2+6*arcsinh(x*a)+3)/(a^2*x^
2+1)/a
```

Fricas [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx=\int \sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx=\int \sqrt{c\left(a^2x^2+1\right)}\,\text{asinh}^3\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx=\int \text{asinh}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^3\,dx=\sqrt{c}\left(\int \sqrt{a^2x^2+1}\,asinh(ax)^3\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*asinh(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*asinh(a*x)**3,x)
```

3.64 $\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

output
```
1/4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^4/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\text{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^4}{4a\sqrt{c\,(1+a^2x^2)}}$$

input
```
Integrate[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c*(1 + a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\sqrt{c(a^2x^2+1)}\operatorname{arcsinh}(xa)^4}{4\sqrt{a^2x^2+1}\,ca}$	39

input
```
int(arcsinh(x*a)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/c/a*arcsinh(x*a)^4
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(asinh(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.35

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\operatorname{arsinh}(ax)^4}{4\,a\sqrt{c}}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*arcsinh(a*x)^4/(a*sqrt(c))
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.40

$$\int \frac{\operatorname{arcsinh}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\,asinh(ax)^4}{4ac}$$

input
```
int(asinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*asinh(a*x)**4)/(4*a*c)
```

3.65 $\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\operatorname{arcsinh}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\log\left(1+e^{2\operatorname{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(1/2)+(a^2*x^2+1)^(1/2)*arcsinh(a*x)^3/a/
c/(a^2*c*x^2+c)^(1/2)-3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^
2+1)^(1/2))^2)/a/c/(a^2*c*x^2+c)^(1/2)-3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*po
lylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)/a/c/(a^2*c*x^2+c)^(1/2)+3/2*(a^2*x^2+1
)^(1/2)*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.61

$$\int \frac{\text{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2ax\text{arcsinh}(ax)^3 - 2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2\left(\text{arcsinh}(ax) + 3\log\left(1+e^{-2\text{arcsinh}(ax)}\right)\right.}{2a}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*a*x*ArcSinh[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*(ArcSinh[a*x] +
 3*Log[1 + E^(-2*ArcSinh[a*x])]) + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLo
g[2, -E^(-2*ArcSinh[a*x])] + 3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*ArcSinh
[a*x])])/(2*a*c*Sqrt[c*(1 + a^2*x^2)])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6202, 6212, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^3}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{6202}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}} - \frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}} - \frac{3\sqrt{a^2x^2+1}\int \frac{ax\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int -i\text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{26}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\int \text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1+e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\int \text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\frac{1}{3}i\text{ar}\right.}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arc}\right.\right.}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsin}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*((-1
/3*I)*ArcSinh[a*x]^3 + (2*I)*((ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])
/2 + (ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/2 - PolyLog[3, -E^(2*A
rcSinh[a*x])]/4)))/(a*c*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\left(xa-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(xa)^3}{a\,c^2(a^2x^2+1)}+\frac{2\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(xa)^3}{\sqrt{a^2x^2+1}\,a\,c^2}-\frac{3\sqrt{c(a^2x^2+1)}\,\operatorname{arcsinh}(xa)^2\ln\left(1+\left(xa+\sqrt{a^2x^2+\right.\right.}{\sqrt{a^2x^2+1}\,a\,c^2}$

input
```
int(arcsinh(x*a)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2+1))^(1/2)*(x*a-(a^2*x^2+1)^(1/2))*arcsinh(x*a)^3/a/c^2/(a^2*x^
2+1)+2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(x*a)^3-3/(a^2
*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(x*a)^2*ln(1+(x*a+(a^2*x^
2+1)^(1/2))^2)-3/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(x*a
)*polylog(2,-(x*a+(a^2*x^2+1)^(1/2))^2)+3/2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+
1))^(1/2)/a/c^2*polylog(3,-(x*a+(a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{asinh(ax)^3}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(asinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(asinh(a*x)**3/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)
/(sqrt(c)*c)
```

3.66 $\int \frac{\textbf{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 363

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{x\operatorname{arcsinh}(ax)}{c^2\sqrt{c+a^2cx^2}}+\frac{\operatorname{arcsinh}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}+\frac{x\operatorname{arcsinh}(ax)^3}{3c(c+a^2cx^2)^{3/2}}+\frac{2x\operatorname{arcsinh}(ax)^3}{3c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2\log\left(1+e^{2\operatorname{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\log(1+a^2x^2)}{2ac^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

output
```
-x*arcsinh(a*x)/c^2/(a^2*c*x^2+c)^(1/2)+1/2*arcsinh(a*x)^2/a/c^2/(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/3*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(3/2)+2/
3*x*arcsinh(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)+2/3*(a^2*x^2+1)^(1/2)*arcsinh(a
*x)^3/a/c^2/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^2*ln(1+(a
*x+(a^2*x^2+1)^(1/2))^2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/2*(a^2*x^2+1)^(1/2)*l
n(a^2*x^2+1)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*po
lylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)/a/c^2/(a^2*c*x^2+c)^(1/2)+(a^2*x^2+1)^
(1/2)*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.54

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(1+a^2x^2)^{3/2}\left(-\frac{6ax\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}+\frac{3\operatorname{arcsinh}(ax)^2}{1+a^2x^2}-4\operatorname{arcsinh}(ax)^3+\frac{2ax\operatorname{arcsinh}(ax)^3}{(1+a^2x^2)^{3/2}}+\right.}{}$$

input
```
Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
((1 + a^2*x^2)^(3/2)*((-6*a*x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2] + (3*ArcSinh
[a*x]^2)/(1 + a^2*x^2) - 4*ArcSinh[a*x]^3 + (2*a*x*ArcSinh[a*x]^3)/(1 + a^
2*x^2)^(3/2) + (4*a*x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2] - 12*ArcSinh[a*x]^
2*Log[1 + E^(-2*ArcSinh[a*x])] + 3*Log[1 + a^2*x^2] + 12*ArcSinh[a*x]*Poly
Log[2, -E^(-2*ArcSinh[a*x])] + 6*PolyLog[3, -E^(-2*ArcSinh[a*x])]))/(6*a*c
*(c + a^2*c*x^2)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6203, 6202, 6212, 3042, 26, 4201, 2620, 3011, 2720, 6213, 6202, 240, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 6203$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\operatorname{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\operatorname{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 6202$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{a^2x^2+1}dx}{c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 6212

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int \frac{ax\text{arcsinh}(ax)^2}{\sqrt{a^2x^2+1}}d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\int -i\text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 26

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\int \text{arcsinh}(ax)^2\tan(i\text{arcsinh}(ax))d\text{arcsinh}(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 4201

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\int \frac{e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)^2}{1+e^{2\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{3}i\text{arcsinh}(ax)^3\right)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 2620

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2\text{arcsinh}(ax)}+1\right)-\int \text{arcsinh}(ax)\log\left(1+e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)\right)-\frac{1}{3}i\text{ar}\ldots}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 3011

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)d\text{arcsinh}(ax)+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arc}\ldots}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 2720

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^2}{(a^2x^2+1)^2}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsi}\ldots}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 6213

$$-\frac{a\sqrt{a^2x^2+1}\left(\frac{\int \frac{\text{arcsinh}(ax)}{(a^2x^2+1)^{3/2}}dx}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{2\text{arcsi}\ldots}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 6202

$$-\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-a\int\frac{x}{a^2x^2+1}dx}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsi}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(-\frac{1}{4}\int e^{-2\text{arcsinh}(ax)}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)de^{2\text{arcsinh}(ax)}+\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsi}}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{a\sqrt{a^2x^2+1}\left(\frac{\frac{x\text{arcsinh}(ax)}{\sqrt{a^2x^2+1}}-\frac{\log\left(a^2x^2+1\right)}{2a}}{a}-\frac{\text{arcsinh}(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2\sqrt{a^2cx^2+c}}+$$

$$\frac{2\left(\frac{x\text{arcsinh}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\left(2i\left(\frac{1}{2}\text{arcsinh}(ax)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2\text{arcsinh}(ax)}\right)+\frac{1}{2}\text{arcsinh}(ax)^2\log\left(e^{2a}\right.\right.\right.}{ac\sqrt{a^2cx^2+c}}\right.}{3c}$$

$$\frac{x\text{arcsinh}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - (a*Sqrt[1 + a^2*x^2]*(-1/
2*ArcSinh[a*x]^2/(a^2*(1 + a^2*x^2)) + ((x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2]
 - Log[1 + a^2*x^2]/(2*a))/a))/(c^2*Sqrt[c + a^2*c*x^2]) + (2*((x*ArcSinh[
a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*((-1/3*I)*ArcSi
nh[a*x]^3 + (2*I)*((ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/2 + (ArcSi
nh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/2 - PolyLog[3, -E^(2*ArcSinh[a*x]
)]/4)))/(a*c*Sqrt[c + a^2*c*x^2])))/(3*c)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{c(a^2x^2+1)}\left(2x^3a^3-2x^2a^2\sqrt{a^2x^2+1}+3xa-2\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(xa)\left(-6x^4a^4\operatorname{arcsinh}(xa)-6x^3a^3\operatorname{arcsinh}(xa)\sqrt{a^2x^2+1}-6a^4x^4\right.}{6(3a^6x}$

input
```
int(arcsinh(x*a)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*x^2*a^2*(a^2*x^2+1)^(1/2)+3*x*a-2*(
a^2*x^2+1)^(1/2))*arcsinh(x*a)*(-6*x^4*a^4*arcsinh(x*a)-6*x^3*a^3*arcsinh(
x*a)*(a^2*x^2+1)^(1/2)-6*a^4*x^4-6*x^3*a^3*(a^2*x^2+1)^(1/2)+6*a^2*x^2*arc
sinh(x*a)^2-12*arcsinh(x*a)*x^2*a^2-9*arcsinh(x*a)*(a^2*x^2+1)^(1/2)*x*a-1
8*a^2*x^2-6*x*a*(a^2*x^2+1)^(1/2)+8*arcsinh(x*a)^2-6*arcsinh(x*a)-12)/(3*a
^6*x^6+10*a^4*x^4+11*a^2*x^2+4)/a/c^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^
(1/2)/a/c^3*ln(x*a+(a^2*x^2+1)^(1/2))+1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^
(1/2)/a/c^3*ln(1+(x*a+(a^2*x^2+1)^(1/2))^2)+4/3/(a^2*x^2+1)^(1/2)*(c*(a^2*
x^2+1))^(1/2)/a/c^3*arcsinh(x*a)^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/
2)/a/c^3*arcsinh(x*a)^2*ln(1+(x*a+(a^2*x^2+1)^(1/2))^2)-2/(a^2*x^2+1)^(1/2
)*(c*(a^2*x^2+1))^(1/2)/a/c^3*arcsinh(x*a)*polylog(2,-(x*a+(a^2*x^2+1)^(1/
2))^2)+1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^3*polylog(3,-(x*a+(a^
2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{\operatorname{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx=\frac{\int \frac{asinh(ax)^3}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(asinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(asinh(a*x)**3/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*a
**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.67 $\int \frac{(d+c^2dx^2)^{5/2}}{a+b\operatorname{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 416

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{5/2}}{a+b\operatorname{arcsinh}(cx)}\,dx &= \frac{15d^2\sqrt{d+c^2dx^2}\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc\sqrt{1+c^2x^2}} \\ &+ \frac{3d^2\sqrt{d+c^2dx^2}\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc\sqrt{1+c^2x^2}} \\ &+ \frac{d^2\sqrt{d+c^2dx^2}\cosh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc\sqrt{1+c^2x^2}} \\ &+ \frac{5d^2\sqrt{d+c^2dx^2}\log(a+b\operatorname{arcsinh}(cx))}{16bc\sqrt{1+c^2x^2}} \\ &- \frac{15d^2\sqrt{d+c^2dx^2}\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc\sqrt{1+c^2x^2}} \\ &- \frac{3d^2\sqrt{d+c^2dx^2}\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc\sqrt{1+c^2x^2}} \\ &- \frac{d^2\sqrt{d+c^2dx^2}\sinh\left(\frac{6a}{b}\right)\operatorname{Shi}\left(\frac{6(a+b\operatorname{arcsinh}(cx))}{b}\right)}{32bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
15/32*d^2*(c^2*d*x^2+d)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arcsinh(c*x))/b)/b/c/
(c^2*x^2+1)^(1/2)+3/16*d^2*(c^2*d*x^2+d)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arcs
inh(c*x))/b)/b/c/(c^2*x^2+1)^(1/2)+1/32*d^2*(c^2*d*x^2+d)^(1/2)*cosh(6*a/b
)*Chi(6*(a+b*arcsinh(c*x))/b)/b/c/(c^2*x^2+1)^(1/2)+5/16*d^2*(c^2*d*x^2+d)
^(1/2)*ln(a+b*arcsinh(c*x))/b/c/(c^2*x^2+1)^(1/2)-15/32*d^2*(c^2*d*x^2+d)^
(1/2)*sinh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b/c/(c^2*x^2+1)^(1/2)-3/16*d
^2*(c^2*d*x^2+d)^(1/2)*sinh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b/c/(c^2*x^
2+1)^(1/2)-1/32*d^2*(c^2*d*x^2+d)^(1/2)*sinh(6*a/b)*Shi(6*(a+b*arcsinh(c*x
))/b)/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.44

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{d^2\sqrt{d+c^2dx^2}\left(15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{ar}\right.\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c
*x])] + 6*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/
b]*CoshIntegral[6*(a/b + ArcSinh[c*x])] + 10*Log[a + b*ArcSinh[c*x]] - 15*
Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 6*Sinh[(4*a)/b]*SinhI
ntegral[4*(a/b + ArcSinh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcS
inh[c*x])]))/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$\Big\downarrow$ 6206

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 3042

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^6}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 3793

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{3\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16(a+b\text{arcsinh}(cx))}+\frac{15\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{}{16(a-}\right.}{bc\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2009

$$d^2\sqrt{c^2dx^2+d}\left(\frac{15}{32}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{3}{16}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{32}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(}{}\right.\right.$$

input
```
Int[(d + c^2*d*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((15*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/32 + (3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/16 + (Cosh[(6*a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32 + (5*Lo
g[a + b*ArcSinh[c*x]])/16 - (15*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSi
nh[c*x]))/b])/32 - (3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/
b])/16 - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/32))/(b*
c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\left(-20\ln(a+b\ \mathrm{arcsinh}(xc))cx-20\sqrt{c^2x^2+1}\ \ln(a+b\ \mathrm{arcsinh}(xc))+\mathrm{expIntegral}_1\left(6\ \mathrm{arcsinh}(\right.\right.}{}$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(-20*ln(a+b*
arcsinh(x*c))*c*x-20*(c^2*x^2+1)^(1/2)*ln(a+b*arcsinh(x*c))+Ei(1,6*arcsinh
(x*c)+6*a/b)*exp((b*arcsinh(x*c)+6*a)/b)+Ei(1,-6*arcsinh(x*c)-6*a/b)*exp(-
(-b*arcsinh(x*c)+6*a)/b)+6*Ei(1,4*arcsinh(x*c)+4*a/b)*exp((b*arcsinh(x*c)+
4*a)/b)+15*Ei(1,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c)+2*a)/b)+15*Ei(1,
-2*arcsinh(x*c)-2*a/b)*exp(-(-b*arcsinh(x*c)+2*a)/b)+6*Ei(1,-4*arcsinh(x*c
)-4*a/b)*exp(-(-b*arcsinh(x*c)+4*a)/b))*d^2/c/(c^2*x^2+1)/b
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)/(b*arcsin
h(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d\,c^2\,x^2+d)^{5/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{a+b\text{arcsinh}(cx)}\,dx=\sqrt{d}\,d^2\left(\int \frac{\sqrt{c^2x^2+1}}{asinh\,(cx)\,b+a}dx+\left(\int \frac{\sqrt{c^2x^2+1}\,x^4}{asinh\,(cx)\,b+a}dx\right)c^4+2\left(\int \frac{\sqrt{c^2x^2+1}\,x^2}{asinh\,(cx)\,b+a}dx\right)c^2\right)$$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x)),x)
```

output
```
sqrt(d)*d**2*(int(sqrt(c**2*x**2 + 1)/(asinh(c*x)*b + a),x) + int((sqrt(c*
*2*x**2 + 1)*x**4)/(asinh(c*x)*b + a),x)*c**4 + 2*int((sqrt(c**2*x**2 + 1)
*x**2)/(asinh(c*x)*b + a),x)*c**2)
```

3.68 $\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 284

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{d\sqrt{d+c^2dx^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc\sqrt{1+c^2x^2}} + \frac{d\sqrt{d+c^2dx^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc\sqrt{1+c^2x^2}} + \frac{3d\sqrt{d+c^2dx^2}\log(a+b\text{arcsinh}(cx))}{8bc\sqrt{1+c^2x^2}} - \frac{d\sqrt{d+c^2dx^2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc\sqrt{1+c^2x^2}} - \frac{d\sqrt{d+c^2dx^2}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8bc\sqrt{1+c^2x^2}}$$

output
```
1/2*d*(c^2*d*x^2+d)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arcsinh(c*x))/b)/b/c/(c^2
*x^2+1)^(1/2)+1/8*d*(c^2*d*x^2+d)^(1/2)*cosh(4*a/b)*Chi(4*(a+b*arcsinh(c*x
))/b)/b/c/(c^2*x^2+1)^(1/2)+3/8*d*(c^2*d*x^2+d)^(1/2)*ln(a+b*arcsinh(c*x))
/b/c/(c^2*x^2+1)^(1/2)-1/2*d*(c^2*d*x^2+d)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*ar
csinh(c*x))/b)/b/c/(c^2*x^2+1)^(1/2)-1/8*d*(c^2*d*x^2+d)^(1/2)*sinh(4*a/b)
*Shi(4*(a+b*arcsinh(c*x))/b)/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.48

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{d\sqrt{d+c^2dx^2}\left(4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsin}\right.\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x]
)] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] + 3*Log[a + b*ArcS
inh[c*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - Sinh[(4
*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])]))/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \; 6206$$

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 3793$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}+\frac{3}{8(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2dx^2+d}\left(\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{8}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+ba}{}\right.\right.}{bc\sqrt{c^2x^2+1}}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]
))/b])/2 + (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 + (3
*Log[a + b*ArcSinh[c*x]])/8 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSin
h[c*x]))/b])/2 - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/
8))/(b*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\left(-6\ln(a+b\ \mathrm{arcsinh}(xc))cx-6\sqrt{c^2x^2+1}\ \ln(a+b\ \mathrm{arcsinh}(xc))+\mathrm{expIntegral}_1\left(4\ \mathrm{arcsinh}(xc\right.\right.}{}$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(-6*ln(a+b*a
rcsinh(x*c))*c*x-6*(c^2*x^2+1)^(1/2)*ln(a+b*arcsinh(x*c))+Ei(1,4*arcsinh(x
*c)+4*a/b)*exp((b*arcsinh(x*c)+4*a)/b)+Ei(1,-4*arcsinh(x*c)-4*a/b)*exp(-(-
b*arcsinh(x*c)+4*a)/b)+4*Ei(1,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c)+2*
a)/b)+4*Ei(1,-2*arcsinh(x*c)-2*a/b)*exp(-(-b*arcsinh(x*c)+2*a)/b))*d/(c^2*
x^2+1)/c/b
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\mathrm{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((c^2*d*x^2 + d)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d\,c^2\,x^2+d)^{3/2}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{a+b\text{arcsinh}(cx)}\,dx = \sqrt{d}\,d\left(\int \frac{\sqrt{c^2x^2+1}}{asinh\,(cx)\,b+a}dx+\left(\int \frac{\sqrt{c^2x^2+1}\,x^2}{asinh\,(cx)\,b+a}dx\right)c^2\right)$$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x)),x)
```

output
```
sqrt(d)*d*(int(sqrt(c**2*x**2 + 1)/(asinh(c*x)*b + a),x) + int((sqrt(c**2*
x**2 + 1)*x**2)/(asinh(c*x)*b + a),x)*c**2)
```

3.69 $\int \frac{\sqrt{d+c^2dx^2}}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\text{arcsinh}(cx)}\,dx = \frac{\sqrt{d+c^2dx^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\log(a+b\text{arcsinh}(cx))}{2bc\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2bc\sqrt{1+c^2x^2}}$$

output
```
1/2*(c^2*d*x^2+d)^(1/2)*cosh(2*a/b)*Chi(2*(a+b*arcsinh(c*x))/b)/b/c/(c^2*x
^2+1)^(1/2)+1/2*(c^2*d*x^2+d)^(1/2)*ln(a+b*arcsinh(c*x))/b/c/(c^2*x^2+1)^(
1/2)-1/2*(c^2*d*x^2+d)^(1/2)*sinh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b/c/(
c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{\sqrt{d\left(1+c^2x^2\right)}\left(\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)+\log(a+b\operatorname{arcsinh}(cx))-\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\left(\frac{a}{b}+\operatorname{arcsin}\right.\right.\right.}{2bc\sqrt{1+c^2x^2}}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d*(1 + c^2*x^2)]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]
 + Log[a + b*ArcSinh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c
*x])]))/(2*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}}{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\operatorname{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^2}{a+b\operatorname{arcsinh}(cx)}d(a+b\operatorname{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 3793$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}+\frac{1}{2(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{2}\log(a+b\text{arcsinh}(cx))\right)}{bc\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*((Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))
/b])/2 + Log[a + b*ArcSinh[c*x]]/2 - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b
*ArcSinh[c*x]))/b])/2))/(b*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

method	result
default	$-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\left(-2\ln(a+b\ \mathrm{arcsinh}(xc))cx-2\sqrt{c^2x^2+1}\ \ln(a+b\ \mathrm{arcsinh}(xc))+\mathrm{expIntegral}_1\left(2\ \mathrm{arcsinh}(xc\right.\right.}{4(c^2x^2+1)cb}$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(-2*ln(a+b*ar
csinh(x*c))*c*x-2*(c^2*x^2+1)^(1/2)*ln(a+b*arcsinh(x*c))+Ei(1,2*arcsinh(x*
c)+2*a/b)*exp((b*arcsinh(x*c)+2*a)/b)+Ei(1,-2*arcsinh(x*c)-2*a/b)*exp(-(-b
*arcsinh(x*c)+2*a)/b))/(c^2*x^2+1)/c/b
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\mathrm{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2dx^2+d}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\mathrm{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2dx^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{c^2dx^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d\,c^2\,x^2+d}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d + c^2 d x^2}}{a + b\text{arcsinh}(cx)}\,dx = \sqrt{d}\left(\int \frac{\sqrt{c^2 x^2 + 1}}{asinh\,(cx)\,b + a}dx\right)$$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x)),x)
```

output
```
sqrt(d)*int(sqrt(c**2*x**2 + 1)/(asinh(c*x)*b + a),x)
```

3.70 $\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 43

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\sqrt{1+c^2x^2}\log(a+b\text{arcsinh}(cx))}{bc\sqrt{d+c^2dx^2}}$$

output
```
(c^2*x^2+1)^(1/2)*ln(a+b*arcsinh(c*x))/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\sqrt{1+c^2x^2}\log(a+b\text{arcsinh}(cx))}{bc\sqrt{d+c^2dx^2}}$$

input
```
Integrate[1/(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(Sqrt[1 + c^2*x^2]*Log[a + b*ArcSinh[c*x]])/(b*c*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6197}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \; 6197$$

$$\frac{\sqrt{c^2x^2+1}\log(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2dx^2+d}}$$

input
```
Int[1/(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
(Sqrt[1 + c^2*x^2]*Log[a + b*ArcSinh[c*x]])/(b*c*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 6197
```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{d(c^2x^2+1)}\,\ln(a+b\,\text{arcsinh}(xc))}{\sqrt{c^2x^2+1}\,cdb}$	44

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*ln(a+b*arcsinh(x*c))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx = \frac{\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}\log\left(\frac{b\log\left(cx+\sqrt{c^2x^2+1}\right)+a}{b}\right)}{bc^3dx^2+bcd}$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*log((b*log(c*x + sqrt(c^2*x^2 + 1))
+ a)/b)/(b*c^3*d*x^2 + b*c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}\left(cx\right)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}\,dx=\frac{\sqrt{d}\log(asinh(cx)\,b+a)}{bcd}$$

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*log(asinh(c*x)*b + a))/(b*c*d)
```

3.71 $\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \ 6209$$
$$\int \frac{1}{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2d\,x^2+d)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.20

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)/(a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b
*c^4*d^2*x^4 + 2*b*c^2*d^2*x^2 + b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(d\,(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\,(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))}\,dx = \frac{\int \frac{1}{\sqrt{c^2x^2+1}\,asinh(cx)b\,c^2x^2+\sqrt{c^2x^2+1}\,asinh(cx)b+\sqrt{c^2x^2+1}\,a\,c^2x^2+\sqrt{c^2x^2+1}\,a}dx}{\sqrt{d}\,d}$$

input

```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x)),x)
```

output

```
int(1/(sqrt(c**2*x**2 + 1)*asinh(c*x)*b*c**2*x**2 + sqrt(c**2*x**2 + 1)*as
inh(c*x)*b + sqrt(c**2*x**2 + 1)*a*c**2*x**2 + sqrt(c**2*x**2 + 1)*a),x)/(
sqrt(d)*d)
```

3.72 $\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c^2d\,x^2+d\right)^{\frac{5}{2}}\left(a+b\,\text{arcsinh}\left(xc\right)\right)}dx$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.16

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)/(a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^
3*x^2 + a*d^3 + (b*c^6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3
)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d\,(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.92

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \frac{\int \frac{1}{\sqrt{c^2x^2+1}\,asinh(cx)b\,c^4x^4+2\sqrt{c^2x^2+1}\,asinh(cx)b\,c^2x^2+\sqrt{c^2x^2+1}\,asinh(cx)b+\sqrt{c^2x^2}}}{\sqrt{d}\,d^2}$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(sqrt(c**2*x**2 + 1)*asinh(c*x)*b*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*
asinh(c*x)*b*c**2*x**2 + sqrt(c**2*x**2 + 1)*asinh(c*x)*b + sqrt(c**2*x**2
 + 1)*a*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*a*c**2*x**2 + sqrt(c**2*x**2 + 1
)*a),x)/(sqrt(d)*d**2)
```

3.73 $\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 415

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\,\text{arcsinh}(cx))^2}\,dx &= -\frac{d^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{bc(a+b\,\text{arcsinh}(cx))} \\ &- \frac{15d^2\sqrt{d+c^2dx^2}\,\text{Chi}\left(\frac{2(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c\sqrt{1+c^2x^2}} \\ &- \frac{3d^2\sqrt{d+c^2dx^2}\,\text{Chi}\left(\frac{4(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{4b^2c\sqrt{1+c^2x^2}} \\ &- \frac{3d^2\sqrt{d+c^2dx^2}\,\text{Chi}\left(\frac{6(a+b\,\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{6a}{b}\right)}{16b^2c\sqrt{1+c^2x^2}} \\ &+ \frac{15d^2\sqrt{d+c^2dx^2}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\,\text{arcsinh}(cx))}{b}\right)}{16b^2c\sqrt{1+c^2x^2}} \\ &+ \frac{3d^2\sqrt{d+c^2dx^2}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\,\text{arcsinh}(cx))}{b}\right)}{4b^2c\sqrt{1+c^2x^2}} \\ &+ \frac{3d^2\sqrt{d+c^2dx^2}\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\,\text{arcsinh}(cx))}{b}\right)}{16b^2c\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/b/c/(a+b*arcsinh(c*x))-15/16*d^
2*(c^2*d*x^2+d)^(1/2)*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c/(c^2*x
^2+1)^(1/2)-3/4*d^2*(c^2*d*x^2+d)^(1/2)*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4
*a/b)/b^2/c/(c^2*x^2+1)^(1/2)-3/16*d^2*(c^2*d*x^2+d)^(1/2)*Chi(6*(a+b*arcs
inh(c*x))/b)*sinh(6*a/b)/b^2/c/(c^2*x^2+1)^(1/2)+15/16*d^2*(c^2*d*x^2+d)^(
1/2)*cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c/(c^2*x^2+1)^(1/2)+3/4*d
^2*(c^2*d*x^2+d)^(1/2)*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c/(c^2*
x^2+1)^(1/2)+3/16*d^2*(c^2*d*x^2+d)^(1/2)*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c
*x))/b)/b^2/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.82

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx =$$
$$-\frac{d^2\sqrt{d+c^2dx^2}\left(16b+48bc^2x^2+48bc^4x^4+16bc^6x^6+15(a+b\text{arcsinh}(cx))\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*(d^2*Sqrt[d + c^2*d*x^2]*(16*b + 48*b*c^2*x^2 + 48*b*c^4*x^4 + 16*b*
c^6*x^6 + 15*(a + b*ArcSinh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sin
h[(2*a)/b] + 12*(a + b*ArcSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*
Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3
*b*ArcSinh[c*x]*CoshIntegral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] - 15*a*
Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 15*b*ArcSinh[c*x]*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 12*a*Cosh[(4*a)/b]*SinhI
ntegral[4*(a/b + ArcSinh[c*x])] - 12*b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhInte
gral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + Arc
Sinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh
[c*x])]))/(b^2*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{6cd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{a+b\text{arcsinh}(cx)}dx}{b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{6d^2\sqrt{c^2dx^2+d}\int -\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{6d^2\sqrt{c^2dx^2+d}\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{6d^2\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32(a+b\text{arcsinh}(cx))}\right)d(a\ \cdots}{b^2c\sqrt{c^2x^2+1}}$$
$$\frac{\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6d^2\sqrt{c^2dx^2+d}\left(-\frac{5}{32}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{32}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\right.\right.}{\frac{\sqrt{c^2x^2+1}\left(c^2dx^2+d\right)^{5/2}}{bc(a+b\text{arcsinh}(cx))}}$$

input
```
Int[(d + c^2*d*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((Sqrt[1 + c^2*x^2]*(d + c^2*d*x^2)^(5/2))/(b*c*(a + b*ArcSinh[c*x]))) +
(6*d^2*Sqrt[d + c^2*d*x^2]*((-5*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]*S
inh[(2*a)/b])/32 - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sinh[(4*a)/b]
)/8 - (CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b]*Sinh[(6*a)/b])/32 + (5*Cos
h[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/32 + (Cosh[(4*a)/b]*S
inhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8 + (Cosh[(6*a)/b]*SinhIntegral[(
6*(a + b*ArcSinh[c*x]))/b])/32))/(b^2*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\left(32b\,c^7x^7+32\sqrt{c^2x^2+1}\,b\,c^6x^6+96b\,c^5x^5+96\sqrt{c^2x^2+1}\,b\,c^4x^4+96b\,c^3x^3+96\sqrt{c^2x^2+1}\,b\,c^2\right.}{}$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(32*b*c^7*x^
7+32*(c^2*x^2+1)^(1/2)*b*c^6*x^6+96*b*c^5*x^5+96*(c^2*x^2+1)^(1/2)*b*c^4*x
^4+96*b*c^3*x^3+96*(c^2*x^2+1)^(1/2)*b*c^2*x^2+12*arcsinh(x*c)*b*Ei(1,-4*a
rcsinh(x*c)-4*a/b)*exp(-(-b*arcsinh(x*c)+4*a)/b)+3*arcsinh(x*c)*b*Ei(1,-6*
arcsinh(x*c)-6*a/b)*exp(-(-b*arcsinh(x*c)+6*a)/b)+15*arcsinh(x*c)*b*Ei(1,-
2*arcsinh(x*c)-2*a/b)*exp(-(-b*arcsinh(x*c)+2*a)/b)-12*exp((b*arcsinh(x*c)
+4*a)/b)*Ei(1,4*arcsinh(x*c)+4*a/b)*b*arcsinh(x*c)-15*Ei(1,2*arcsinh(x*c)+
2*a/b)*exp((b*arcsinh(x*c)+2*a)/b)*b*arcsinh(x*c)-3*Ei(1,6*arcsinh(x*c)+6*
a/b)*exp((b*arcsinh(x*c)+6*a)/b)*b*arcsinh(x*c)+12*a*Ei(1,-4*arcsinh(x*c)-
4*a/b)*exp(-(-b*arcsinh(x*c)+4*a)/b)+3*a*Ei(1,-6*arcsinh(x*c)-6*a/b)*exp(-
(-b*arcsinh(x*c)+6*a)/b)+15*a*Ei(1,-2*arcsinh(x*c)-2*a/b)*exp(-(-b*arcsinh
(x*c)+2*a)/b)-12*exp((b*arcsinh(x*c)+4*a)/b)*Ei(1,4*arcsinh(x*c)+4*a/b)*a-
15*Ei(1,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c)+2*a)/b)*a-3*Ei(1,6*arcsi
nh(x*c)+6*a/b)*exp((b*arcsinh(x*c)+6*a)/b)*a+32*b*x*c+32*(c^2*x^2+1)^(1/2)
*b)*d^2/(c^2*x^2+1)/c/b^2/(a+b*arcsinh(x*c))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)/(b^2*arcs
inh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*d^(5/2)*x^6 + 3*c^4*d^(5/2)*x^4 + 3*c^2*d^(5/2)*x^2 + d^(5/2))*(c^2
*x^2 + 1) + (c^7*d^(5/2)*x^7 + 3*c^5*d^(5/2)*x^5 + 3*c^3*d^(5/2)*x^3 + c*d
^(5/2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x +
a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt
(c^2*x^2 + 1))) + integrate(((6*c^6*d^(5/2)*x^6 + 11*c^4*d^(5/2)*x^4 + 4*c
^2*d^(5/2)*x^2 - d^(5/2))*(c^2*x^2 + 1)^(3/2) + 6*(2*c^7*d^(5/2)*x^7 + 5*c
^5*d^(5/2)*x^5 + 4*c^3*d^(5/2)*x^3 + c*d^(5/2)*x)*(c^2*x^2 + 1) + (6*c^8*d
^(5/2)*x^8 + 19*c^6*d^(5/2)*x^6 + 21*c^4*d^(5/2)*x^4 + 9*c^2*d^(5/2)*x^2 +
 d^(5/2))*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*
a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x
^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{(d\,c^2\,x^2+d)^{5/2}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \sqrt{d}\,d^2\left(\int \frac{\sqrt{c^2x^2+1}}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx + \left(\int \frac{\sqrt{c^2x^2+1}\,x^4}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)c^4 + 2\left(\int \frac{\sqrt{c^2x^2+1}\,x^2}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)c^2\right)$$

input

```
int((c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x))^2,x)
```

output

```
sqrt(d)*d**2*(int(sqrt(c**2*x**2 + 1)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a
*b + a**2),x) + int((sqrt(c**2*x**2 + 1)*x**4)/(asinh(c*x)**2*b**2 + 2*asi
nh(c*x)*a*b + a**2),x)*c**4 + 2*int((sqrt(c**2*x**2 + 1)*x**2)/(asinh(c*x)
**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**2)
```

3.74 $\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 278

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = &-\frac{d(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{bc(a+b\text{arcsinh}(cx))} \\ &-\frac{d\sqrt{d+c^2dx^2}\,\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{1+c^2x^2}} \\ &-\frac{d\sqrt{d+c^2dx^2}\,\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c\sqrt{1+c^2x^2}} \\ &+\frac{d\sqrt{d+c^2dx^2}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c\sqrt{1+c^2x^2}} \\ &+\frac{d\sqrt{d+c^2dx^2}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-d*(c^2*x^2+1)^(3/2)*(c^2*d*x^2+d)^(1/2)/b/c/(a+b*arcsinh(c*x))-d*(c^2*d*x
^2+d)^(1/2)*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c/(c^2*x^2+1)^(1/2
)-1/2*d*(c^2*d*x^2+d)^(1/2)*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c/
(c^2*x^2+1)^(1/2)+d*(c^2*d*x^2+d)^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x
))/b)/b^2/c/(c^2*x^2+1)^(1/2)+1/2*d*(c^2*d*x^2+d)^(1/2)*cosh(4*a/b)*Shi(4*
(a+b*arcsinh(c*x))/b)/b^2/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.54

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{d\sqrt{d+c^2dx^2}\left(-\frac{2b(1+c^2x^2)^2}{a+b\text{arcsinh}(cx)} - 2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \text{Chi}\left(4\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((-2*b*(1 + c^2*x^2)^2)/(a + b*ArcSinh[c*x]) - 2*Co
shIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + A
rcSinh[c*x])]*Sinh[(4*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])]))/(2*b^2*c*S
qrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{4cd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{a+b\text{arcsinh}(cx)}dx}{b\sqrt{c^2x^2+1}} - \frac{\sqrt{c^2x^2+1}(c^2dx^2+d)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{4d\sqrt{c^2dx^2+d}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\left(c^2dx^2+d\right)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 25

$$-\frac{4d\sqrt{c^2dx^2+d}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\left(c^2dx^2+d\right)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 5971

$$-\frac{4d\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8(a+b\text{arcsinh}(cx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4(a+b\text{arcsinh}(cx))}\right)d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\left(c^2dx^2+d\right)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

$\Big\downarrow$ 2009

$$\frac{4d\sqrt{c^2dx^2+d}\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(cx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+\right.\right.}{b^2c\sqrt{c^2x^2+1}}$$

$$\frac{\sqrt{c^2x^2+1}\left(c^2dx^2+d\right)^{3/2}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((Sqrt[1 + c^2*x^2]*(d + c^2*d*x^2)^(3/2))/(b*c*(a + b*ArcSinh[c*x]))) +
(4*d*Sqrt[d + c^2*d*x^2]*(-1/4*(CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]*S
inh[(2*a)/b]) - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/8
 + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/4 + (Cosh[(4*a
)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/8))/(b^2*c*Sqrt[1 + c^2*x^2
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\left(4b\,c^5x^5+4\sqrt{c^2x^2+1}\,b\,c^4x^4+8b\,c^3x^3+8\sqrt{c^2x^2+1}\,b\,c^2x^2+\operatorname{arcsinh}(xc)b\ \operatorname{expIntegral}_1\left(-4\right.\right.}{}$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(4*b*c^5*x^5+
4*(c^2*x^2+1)^(1/2)*b*c^4*x^4+8*b*c^3*x^3+8*(c^2*x^2+1)^(1/2)*b*c^2*x^2+ar
csinh(x*c)*b*Ei(1,-4*arcsinh(x*c)-4*a/b)*exp(-(-b*arcsinh(x*c)+4*a)/b)+2*a
rcsinh(x*c)*b*Ei(1,-2*arcsinh(x*c)-2*a/b)*exp(-(-b*arcsinh(x*c)+2*a)/b)-ex
p((b*arcsinh(x*c)+4*a)/b)*Ei(1,4*arcsinh(x*c)+4*a/b)*b*arcsinh(x*c)-2*Ei(1
,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c)+2*a)/b)*b*arcsinh(x*c)+a*Ei(1,-
4*arcsinh(x*c)-4*a/b)*exp(-(-b*arcsinh(x*c)+4*a)/b)+2*a*Ei(1,-2*arcsinh(x*
c)-2*a/b)*exp(-(-b*arcsinh(x*c)+2*a)/b)-exp((b*arcsinh(x*c)+4*a)/b)*Ei(1,4
*arcsinh(x*c)+4*a/b)*a-2*Ei(1,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c)+2*
a)/b)*a+4*b*x*c+4*(c^2*x^2+1)^(1/2)*b)*d/(c^2*x^2+1)/c/b^2/(a+b*arcsinh(x*
c))
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^2*d*x^2 + d)^(3/2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
a^2), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^4*d^(3/2)*x^4 + 2*c^2*d^(3/2)*x^2 + d^(3/2))*(c^2*x^2 + 1) + (c^5*d^(
3/2)*x^5 + 2*c^3*d^(3/2)*x^3 + c*d^(3/2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^
2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)
*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^4*d^(3
/2)*x^4 + 3*c^2*d^(3/2)*x^2 - d^(3/2))*(c^2*x^2 + 1)^(3/2) + 4*(2*c^5*d^(3
/2)*x^5 + 3*c^3*d^(3/2)*x^3 + c*d^(3/2)*x)*(c^2*x^2 + 1) + (4*c^6*d^(3/2)*
x^6 + 9*c^4*d^(3/2)*x^4 + 6*c^2*d^(3/2)*x^2 + d^(3/2))*sqrt(c^2*x^2 + 1))/
(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*
x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b
^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 +
 a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{3/2}}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{(d\,c^2\,x^2 + d)^{3/2}}{(a + b\,\text{asinh}\,(c\,x))^2} \, dx$$

input
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d + c^2dx^2)^{3/2}}{(a + b\text{arcsinh}(cx))^2} \, dx = \sqrt{d}\,d\left(\int \frac{\sqrt{c^2x^2+1}}{asinh\,(cx)^2\,b^2 + 2asinh\,(cx)\,ab + a^2}dx + \left(\int \frac{\sqrt{c^2x^2+1}\,x^2}{asinh\,(cx)^2\,b^2 + 2asinh\,(cx)\,ab + a^2}dx\right)c^2\right)$$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x))^2,x)
```

output
```
sqrt(d)*d*(int(sqrt(c**2*x**2 + 1)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b
+ a**2),x) + int((sqrt(c**2*x**2 + 1)*x**2)/(asinh(c*x)**2*b**2 + 2*asinh(
c*x)*a*b + a**2),x)*c**2)
```

3.75 $\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{\sqrt{d+c^2dx^2}\,\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c\sqrt{1+c^2x^2}}$$

output
```
-(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)/b/c/(a+b*arcsinh(c*x))-(c^2*d*x^2+d
)^(1/2)*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c/(c^2*x^2+1)^(1/2)+(c
^2*d*x^2+d)^(1/2)*cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c/(c^2*x^2+1
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =$$
$$-\frac{d\sqrt{1+c^2x^2}\left(b+bc^2x^2+(a+b\operatorname{arcsinh}(cx))\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-(a+b\operatorname{arcsinh}(cx))\,\mathrm{c}\right.}{b^2c\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d*Sqrt[1 + c^2*x^2]*(b + b*c^2*x^2 + (a + b*ArcSinh[c*x])*CoshIntegral[
2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcSinh[c*x])*Cosh[(2*a)/b]
*SinhIntegral[2*(a/b + ArcSinh[c*x])]))/(b^2*c*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {6205, 6195, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}}{(a+b\operatorname{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{2c\sqrt{c^2dx^2+d}\int \frac{x}{a+b\operatorname{arcsinh}(cx)}dx}{b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\operatorname{arcsinh}(cx))}$$

$$\Big\downarrow \text{6195}$$

$$\frac{2\sqrt{c^2dx^2+d}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\sqrt{c^2dx^2+d}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{2\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2(a+b\text{arcsinh}(cx))}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}-\frac{\sqrt{c^2dx^2+d}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+$$

$$\frac{i\sqrt{c^2dx^2+d}\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a\right.}{b^2c\sqrt{c^2x^2+1}}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sqrt{c^2dx^2+d}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+\right.}{b^2c\sqrt{c^2x^2+1}}$$

$\downarrow$ 3042

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sqrt{c^2dx^2+d}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}\right.}{b^2c\sqrt{c^2x^2+1}}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sqrt{c^2dx^2+d}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+\right.}{b^2c\sqrt{c^2x^2+1}}$$

$\downarrow$ 3779

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sqrt{c^2dx^2+d}\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c\sqrt{c^2x^2+1}}$$

$\downarrow$ 3782

$$-\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\sqrt{c^2dx^2+d}\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{b^2c\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])/(b*c*(a + b*ArcSinh[c*x]))) + (I
*Sqrt[d + c^2*d*x^2]*(I*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b]*Sinh[(2*a
)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b]))/(b^2*c*S
qrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

method	result
default	$-\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2-\sqrt{c^2x^2+1}\,xc+1\right)\left(2b\,c^3x^3+2\sqrt{c^2x^2+1}\,b\,c^2x^2+\operatorname{arcsinh}(xc)b\ \operatorname{expIntegral}_1\left(-2\ \operatorname{arcsinh}(xc)-\frac{2a}{b}\right)\mathrm{e}^{-\frac{-b\ \operatorname{arcsin}}{}}$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(2*b*c^3*x^3+
2*(c^2*x^2+1)^(1/2)*b*c^2*x^2+arcsinh(x*c)*b*Ei(1,-2*arcsinh(x*c)-2*a/b)*e
xp(-(-b*arcsinh(x*c)+2*a)/b)-Ei(1,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c
)+2*a)/b)*b*arcsinh(x*c)+a*Ei(1,-2*arcsinh(x*c)-2*a/b)*exp(-(-b*arcsinh(x*
c)+2*a)/b)-Ei(1,2*arcsinh(x*c)+2*a/b)*exp((b*arcsinh(x*c)+2*a)/b)*a+2*b*x*
c+2*(c^2*x^2+1)^(1/2)*b)/(c^2*x^2+1)/c/b^2/(a+b*arcsinh(x*c))
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^
2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*sqrt(d)*x^2 + sqrt(d))*(c^2*x^2 + 1) + (c^3*sqrt(d)*x^3 + c*sqrt(d)
*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x
^2 + 1))) + integrate(((2*c^2*sqrt(d)*x^2 - sqrt(d))*(c^2*x^2 + 1)^(3/2) +
 2*(2*c^3*sqrt(d)*x^3 + c*sqrt(d)*x)*(c^2*x^2 + 1) + (2*c^4*sqrt(d)*x^4 +
3*c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 +
1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^
2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1
)), x)
```

Giac [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{d\,c^2\,x^2+d}}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{c^2x^2+1}}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)$$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x))^2,x)
```

output
```
sqrt(d)*int(sqrt(c**2*x**2 + 1)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a
**2),x)
```

3.76 $\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}$$

output
```
-(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc\sqrt{d(1+c^2x^2)}(a+b\text{arcsinh}(cx))}$$

input
```
Integrate[1/(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(Sqrt[1 + c^2*x^2]/(b*c*Sqrt[d*(1 + c^2*x^2)]*(a + b*ArcSinh[c*x])))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{ 6198}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
-(Sqrt[1 + c^2*x^2]/(b*c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])))
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\sqrt{d(c^2x^2+1)}}{\sqrt{c^2x^2+1}\,dc(a+b\,\text{arcsinh}(xc))b}$	46

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d/c/(a+b*arcsinh(x*c))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}}{abc^3dx^2+abcd+(b^2c^3dx^2+b^2cd)\log\left(cx+\sqrt{c^2x^2+1}\right)}$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
-sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)/(a*b*c^3*d*x^2 + a*b*c*d + (b^2*c^3
*d*x^2 + b^2*c*d)*log(c*x + sqrt(c^2*x^2 + 1)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{d(c^2x^2+1)}\,(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*sqrt(d)*x +
((c^2*x^2 + 1)*b^2*c^2*sqrt(d)*x + (b^2*c^3*sqrt(d)*x^2 + b^2*c*sqrt(d))*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*sqrt(d)*x^2 + a*
b*c*sqrt(d))*sqrt(c^2*x^2 + 1)) - integrate(-(c^2*sqrt(d)*x^2 - (c^2*x^2 +
 1)*sqrt(d) + sqrt(d))/((c^2*x^2 + 1)^(3/2)*a*b*c^2*d*x^2 + 2*(a*b*c^3*d*x
^3 + a*b*c*d*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^2*d*x^2 + 2*(b^
2*c^3*d*x^3 + b^2*c*d*x)*(c^2*x^2 + 1) + (b^2*c^4*d*x^4 + 2*b^2*c^2*d*x^2
+ b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^4*d*x^4
+ 2*a*b*c^2*d*x^2 + a*b*d)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}\,dx=\frac{\sqrt{d}\,asinh(cx)}{acd\left(asinh\left(cx\right)b+a\right)}$$

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(d)*asinh(c*x))/(a*c*d*(asinh(c*x)*b + a))
```

3.77 $\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{2c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}dx}{bd\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}}{bc\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{2c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}dx}{bd\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}}{bc\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsinh}\left(xc\right)\right)^2}dx$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 5.28

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)/(a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^
2 + (b^2*c^4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*
b*c^4*d^2*x^4 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d\,(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 540, normalized size of antiderivative = 21.60

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*d^(3/2)*x + ((c^2*x^2 +
1)*b^2*c^2*d^(3/2)*x + (b^2*c^3*d^(3/2)*x^2 + b^2*c*d^(3/2))*sqrt(c^2*x^2
+ 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*d^(3/2)*x^2 + a*b*c*d^(3/2))
*sqrt(c^2*x^2 + 1)) - integrate((2*c^4*sqrt(d)*x^4 + c^2*sqrt(d)*x^2 + (2*
c^2*sqrt(d)*x^2 + sqrt(d))*(c^2*x^2 + 1) + 2*(2*c^3*sqrt(d)*x^3 + c*sqrt(d
)*x)*sqrt(c^2*x^2 + 1) - sqrt(d))/((a*b*c^4*d^2*x^4 + a*b*c^2*d^2*x^2)*(c^
2*x^2 + 1)^(3/2) + 2*(a*b*c^5*d^2*x^5 + 2*a*b*c^3*d^2*x^3 + a*b*c*d^2*x)*(
c^2*x^2 + 1) + ((b^2*c^4*d^2*x^4 + b^2*c^2*d^2*x^2)*(c^2*x^2 + 1)^(3/2) +
2*(b^2*c^5*d^2*x^5 + 2*b^2*c^3*d^2*x^3 + b^2*c*d^2*x)*(c^2*x^2 + 1) + (b^2
*c^6*d^2*x^6 + 3*b^2*c^4*d^2*x^4 + 3*b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(c^2*x
^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^6*d^2*x^6 + 3*a*b*c^4*d^2*x
^4 + 3*a*b*c^2*d^2*x^2 + a*b*d^2)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.40

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{1}{\sqrt{c^2x^2+1}\,asinh(cx)^2b^2c^2x^2+\sqrt{c^2x^2+1}\,asinh(cx)^2b^2+2\sqrt{c^2x^2+1}\,asinh(cx)ab\,c^2x^2-}}{\sqrt{d}\,d}$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(sqrt(c**2*x**2 + 1)*asinh(c*x)**2*b**2*c**2*x**2 + sqrt(c**2*x**2 +
 1)*asinh(c*x)**2*b**2 + 2*sqrt(c**2*x**2 + 1)*asinh(c*x)*a*b*c**2*x**2 +
2*sqrt(c**2*x**2 + 1)*asinh(c*x)*a*b + sqrt(c**2*x**2 + 1)*a**2*c**2*x**2
+ sqrt(c**2*x**2 + 1)*a**2),x)/(sqrt(d)*d)
```

3.78 $\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{4c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}dx}{bd^2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}}{bc(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{4c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}dx}{bd^2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}}{bc(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))}$$

input
```
Int[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2d\,x^2+d)^{\frac{5}{2}}(a+b\,\text{arcsinh}\,(xc))^2}dx$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 6.92

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(c^2*d*x^2 + d)/(a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*
c^2*d^3*x^2 + a^2*d^3 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d
^3*x^2 + b^2*d^3)*arcsinh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4
+ 3*a*b*c^2*d^3*x^2 + a*b*d^3)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 28.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d\,(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 678, normalized size of antiderivative = 27.12

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*d^(5/2)*x^3 + a*b*c^2*d^(5/2)*x)*(c^2
*x^2 + 1) + ((b^2*c^4*d^(5/2)*x^3 + b^2*c^2*d^(5/2)*x)*(c^2*x^2 + 1) + (b^
2*c^5*d^(5/2)*x^4 + 2*b^2*c^3*d^(5/2)*x^2 + b^2*c*d^(5/2))*sqrt(c^2*x^2 +
1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*d^(5/2)*x^4 + 2*a*b*c^3*d^(5/2
)*x^2 + a*b*c*d^(5/2))*sqrt(c^2*x^2 + 1)) - integrate((4*c^4*sqrt(d)*x^4 +
 3*c^2*sqrt(d)*x^2 + (4*c^2*sqrt(d)*x^2 + sqrt(d))*(c^2*x^2 + 1) + 4*(2*c^
3*sqrt(d)*x^3 + c*sqrt(d)*x)*sqrt(c^2*x^2 + 1) - sqrt(d))/((a*b*c^6*d^3*x^
6 + 2*a*b*c^4*d^3*x^4 + a*b*c^2*d^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^7*
d^3*x^7 + 3*a*b*c^5*d^3*x^5 + 3*a*b*c^3*d^3*x^3 + a*b*c*d^3*x)*(c^2*x^2 +
1) + ((b^2*c^6*d^3*x^6 + 2*b^2*c^4*d^3*x^4 + b^2*c^2*d^3*x^2)*(c^2*x^2 + 1
)^(3/2) + 2*(b^2*c^7*d^3*x^7 + 3*b^2*c^5*d^3*x^5 + 3*b^2*c^3*d^3*x^3 + b^2
*c*d^3*x)*(c^2*x^2 + 1) + (b^2*c^8*d^3*x^8 + 4*b^2*c^6*d^3*x^6 + 6*b^2*c^4
*d^3*x^4 + 4*b^2*c^2*d^3*x^2 + b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + (a*b*c^8*d^3*x^8 + 4*a*b*c^6*d^3*x^6 + 6*a*b*c^4*d^3*x^4 +
 4*a*b*c^2*d^3*x^2 + a*b*d^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 8.28

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \frac{\int \frac{}{\sqrt{c^2x^2+1}\,asinh(cx)^2b^2c^4x^4+2\sqrt{c^2x^2+1}\,asinh(cx)^2b^2c^2x^2+\sqrt{c^2x^2+1}\,asinh(cx)^2b^2}}{}$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(sqrt(c**2*x**2 + 1)*asinh(c*x)**2*b**2*c**4*x**4 + 2*sqrt(c**2*x**2
 + 1)*asinh(c*x)**2*b**2*c**2*x**2 + sqrt(c**2*x**2 + 1)*asinh(c*x)**2*b**
2 + 2*sqrt(c**2*x**2 + 1)*asinh(c*x)*a*b*c**4*x**4 + 4*sqrt(c**2*x**2 + 1)
*asinh(c*x)*a*b*c**2*x**2 + 2*sqrt(c**2*x**2 + 1)*asinh(c*x)*a*b + sqrt(c*
*2*x**2 + 1)*a**2*c**4*x**4 + 2*sqrt(c**2*x**2 + 1)*a**2*c**2*x**2 + sqrt(
c**2*x**2 + 1)*a**2),x)/(sqrt(d)*d**2)
```

3.79 $\int \frac{1}{\sqrt{1+a^2x^2}\,\mathbf{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

output
```
-1/2/a/arcsinh(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arcsinh}(ax)^2}$$

input
```
Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

output
```
-1/2*1/(a*ArcSinh[a*x]^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}dx$$

$$\Big\downarrow \text{ 6198}$$

$$-\frac{1}{2a\text{arcsinh}(ax)^2}$$

input
```
Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

output
```
-1/2*1/(a*ArcSinh[a*x]^2)
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2a\,\text{arcsinh}(xa)^2}$	12
default	$-\frac{1}{2a\,\text{arcsinh}(xa)^2}$	12

input
```
int(1/(a^2*x^2+1)^(1/2)/arcsinh(x*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/arcsinh(x*a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

input
```
integrate(1/(a^2*x^2+1)^(1/2)/arcsinh(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2a\,\text{asinh}^2(ax)}$$

input
```
integrate(1/(a**2*x**2+1)**(1/2)/asinh(a*x)**3,x)
```

output
```
-1/(2*a*asinh(a*x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\,\text{arsinh}\,(ax)^2}$$

input
```
integrate(1/(a^2*x^2+1)^(1/2)/arcsinh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2/(a*arcsinh(a*x)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

input
```
integrate(1/(a^2*x^2+1)^(1/2)/arcsinh(a*x)^3,x, algorithm="giac")
```

output
```
-1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx = -\frac{1}{2\,a\,\text{asinh}\,(a\,x)^2}$$

input
```
int(1/(asinh(a*x)^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
-1/(2*a*asinh(a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx=-\frac{1}{2asinh\left(ax\right)^2a}$$

input
```
int(1/(a^2*x^2+1)^(1/2)/asinh(a*x)^3,x)
```

output
```
( - 1)/(2*asinh(a*x)**2*a)
```

3.80 $\int \left(d+c^2dx^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 506

$$\int (d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{8}{15}d^2x\sqrt{a+b\text{arcsinh}(cx)} + \frac{4}{15}d^2x(1+c^2x^2)\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{5}d^2x(1+c^2x^2)^2\sqrt{a+b\text{arcsinh}(cx)} + \frac{5\sqrt{b}d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c} + \frac{\sqrt{b}d^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{60c} + \frac{\sqrt{b}d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c} + \frac{\sqrt{b}d^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c} - \frac{5\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c} - \frac{\sqrt{b}d^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{60c} - \frac{\sqrt{b}d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c} - \frac{\sqrt{b}d^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c}$$

output
```
8/15*d^2*x*(a+b*arcsinh(c*x))^(1/2)+4/15*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*
x))^(1/2)+1/5*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^(1/2)+5/32*b^(1/2)*d^
2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c+5/576*b^(1/2)*
d^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/
2))/c+1/1600*b^(1/2)*d^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))/c-5/32*b^(1/2)*d^2*Pi^(1/2)*erfi((a+b*arcsinh(c*x
))^(1/2)/b^(1/2))/c/exp(a/b)-5/576*b^(1/2)*d^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/
2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(3*a/b)-1/1600*b^(1/2)*d^2*5^(1/
2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(5*a/b)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.83

$$\int \left(d+c^2dx^2\right)^2\sqrt{a+b\text{arcsinh}(cx)}\,dx =$$

$$-\frac{bd^2e^{-\frac{5a}{b}}\left(-450e^{\frac{6a}{b}}\left(3a\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+3b\text{arcsinh}(cx)\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+8b\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\sqrt{\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/7200*(b*d^2*(-450*E^((6*a)/b)*(3*a*Sqrt[a/b + ArcSinh[c*x]] + 3*b*ArcSi
nh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + 8*b*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqr
t[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b + ArcSinh[c*x]] - 9*Sqrt[
5]*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[3/2, (-5*(a + b*ArcSinh[c*x])
)/b] - 125*Sqrt[3]*b*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[3
/2, (-3*(a + b*ArcSinh[c*x]))/b] - 450*E^((4*a)/b)*(3*a*Sqrt[-((a + b*ArcS
inh[c*x])/b)] + 3*b*ArcSinh[c*x]*Sqrt[-((a + b*ArcSinh[c*x])/b)] - 8*b*Sqr
t[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a
 + b*ArcSinh[c*x])/b)] + E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSi
nh[c*x])*(125*Sqrt[3]*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b] + 9*Sqrt[5]*E
^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b])))/(c*E^((5*a)/b)*(a + b
*ArcSinh[c*x])^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {6201, 27, 6201, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \ 6201$$

$$-\frac{1}{10}bcd^2\int \frac{x\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{4}{5}d\int d\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{10}bcd^2\int \frac{x\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{4}{5}d^2\int \left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \ 6201$$

$$-\frac{1}{10}bcd^2\int \frac{x\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{4}{5}d^2\left(-\frac{1}{6}bc\int \frac{x\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{2}{3}\int \sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{3}x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \ 6187$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)-\frac{1}{6}bc\int \frac{x\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{1}{3}x(\right.$$
$$\frac{1}{10}bcd^2\int \frac{x\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \ 6234$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right)-\frac{\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arc}}}\right.$$

$$\frac{d^2\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 25$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}+x\sqrt{a+b\text{arcsinh}(cx)}\right)+\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh}{\sqrt{a+b\text{arcsinh}}}\right.$$

$$\frac{d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right)+\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right.}{\sqrt{a+b\text{arc}}}\right.$$

$$\frac{d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 26$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right)+\frac{\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\text{s}}{\sqrt{a+b\text{arcs}}}}{}\right.$$

$$\frac{d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3789}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsi}\right.}{2c}\right.\right.$$

$$\frac{d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2611}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinl}}\right.}{2c}\right.\right.$$

$$\frac{d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right.\right.}{2c}\right.\right.$$

$$\frac{d^2\int\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2634}$$

$$\frac{4}{5}d^2\left(\frac{\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+\frac{1}{3}x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}+\right.$$

$$\frac{d^2\int\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{4}{5}d^2\left(\frac{\int\left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{6c}+\frac{1}{3}x\left(c^2x^2+1\right)\sqrt{a+b\text{arcs}\right.$$

$$\frac{d^2\int\left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{10c}+$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}d^2\left(\frac{1}{3}x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}+\frac{2}{3}\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\right.}{2c}\right.\right.$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\sqrt{a+b\text{arcsinh}(cx)}-$$

$$\frac{d^2\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{ar}}}{\sqrt{}}\right.\right.}{}$$

input
```
Int[(d + c^2*d*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^2*x*(1 + c^2*x^2)^2*Sqrt[a + b*ArcSinh[c*x]])/5 + (4*d^2*((x*(1 + c^2*x
^2)*Sqrt[a + b*ArcSinh[c*x]])/3 + (2*(x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*
((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((
I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c)
)/3 - (-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(8*E^((3*a)/b)))/(6*c)))/5 - (d^2*(-1/16*(Sqrt[b]*E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi
]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/
b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[
b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*E^(a/b)) + (Sqrt[b
]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((3*a
)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(32*E^((5*a)/b))))/(10*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(c^{2} d\,x^{2}+d\right)^{2} \sqrt{a +b \,\operatorname{arcsinh}\left(x c \right)}d x$$

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = d^2\left(\int 2c^2x^2\sqrt{a+b\,\text{asinh}\,(cx)}\,dx + \int c^4x^4\sqrt{a+b\,\text{asinh}\,(cx)}\,dx + \int \sqrt{a+b\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
```

output
```
d**2*(Integral(2*c**2*x**2*sqrt(a + b*asinh(c*x)), x) + Integral(c**4*x**4
*sqrt(a + b*asinh(c*x)), x) + Integral(sqrt(a + b*asinh(c*x)), x))
```

Maxima [F]

$$\int (d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int (c^2dx^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int (d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = d^2\left(\int \sqrt{asinh\left(cx\right)b+a}dx + \left(\int \sqrt{asinh\left(cx\right)b+a}\,x^4dx\right)c^4 + 2\left(\int \sqrt{asinh\left(cx\right)b+a}\,x^2dx\right)c^2\right)$$

input
```
int((c^2*d*x^2+d)^2*(a+b*asinh(c*x))^(1/2),x)
```

output

```
d**2*(int(sqrt(asinh(c*x)*b + a),x) + int(sqrt(asinh(c*x)*b + a)*x**4,x)*c
**4 + 2*int(sqrt(asinh(c*x)*b + a)*x**2,x)*c**2)
```

3.81 $\int (d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 243

$$\int (d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{2}{3}dx\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{3}dx(1+c^2x^2)\sqrt{a+b\text{arcsinh}(cx)} + \frac{3\sqrt{b}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} + \frac{\sqrt{b}de^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c} - \frac{3\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} - \frac{\sqrt{b}de^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c}$$

output
```
2/3*d*x*(a+b*arcsinh(c*x))^(1/2)+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^(1
/2)+3/16*b^(1/2)*d*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))
/c+1/144*b^(1/2)*d*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*
x))^(1/2)/b^(1/2))/c-3/16*b^(1/2)*d*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)
/b^(1/2))/c/exp(a/b)-1/144*b^(1/2)*d*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*ar
csinh(c*x))^(1/2)/b^(1/2))/c/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.24

$$\int \left(d+c^2dx^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$= \frac{de^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(36e^{\frac{2a}{b}}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)+\frac{9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(}{b}}}{}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*Sqrt[a + b*ArcSinh[c*x]]*(36*E^((2*a)/b)*(-((E^((2*a)/b)*Gamma[3/2, a/b
 + ArcSinh[c*x]])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[
c*x])/b)]/Sqrt[-((a + b*ArcSinh[c*x])/b)]) + (9*E^((4*a)/b)*Sqrt[-((a + b*
ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] - 9*E^((2*a)/b)*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/
b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b])
/Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)]))/(72*c*E^((3*a)/b))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6201, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2 d x^2+d\right) \sqrt{a+b \text{arcsinh}(c x)} \, dx$$

$$\Big\downarrow \ 6201$$

$$-\frac{1}{6} b c d \int \frac{x \sqrt{c^2 x^2+1}}{\sqrt{a+b \text{arcsinh}(c x)}}dx+\frac{2}{3} d \int \sqrt{a+b \text{arcsinh}(c x)}dx+\frac{1}{3} d x \left(c^2 x^2+1\right) \sqrt{a+b \text{arcsinh}(c x)}$$

$$\Big\downarrow \ 6187$$

$$\frac{2}{3} d \left(x \sqrt{a+b \text{arcsinh}(c x)}-\frac{1}{2} b c \int \frac{x}{\sqrt{c^2 x^2+1} \sqrt{a+b \text{arcsinh}(c x)}}dx\right)-\frac{1}{6} b c d \int \frac{x \sqrt{c^2 x^2+1}}{\sqrt{a+b \text{arcsinh}(c x)}}dx+\frac{1}{3} d x \left(c^2 x^2+1\right) \sqrt{a+b \text{arcsinh}(c x)}$$

$$\Big\downarrow \ 6234$$

$$\frac{2}{3} d \left(x \sqrt{a+b \text{arcsinh}(c x)}-\frac{\int -\frac{\sinh \left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)}{\sqrt{a+b \text{arcsinh}(c x)}}d(a+b \text{arcsinh}(c x))}{2 c}\right)-\frac{d \int -\frac{\cosh ^2\left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}-\frac{a+b \text{arcsinh}(c x)}{b}\right)}{\sqrt{a+b \text{arcsinh}(c x)}}d(a+b \text{arcsinh}(c x))}{6 c}+\frac{1}{3} d x \left(c^2 x^2+1\right) \sqrt{a+b \text{arcsinh}(c x)}$$

$$\Big\downarrow \ 25$$

$$\frac{2}{3}d\left(\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}+x\sqrt{a+b\text{arcsinh}(cx)}\right)+\frac{d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right)+\frac{d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{26}$$

$$\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right)+\frac{d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3789}$$

$$\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(c\right.}{2c}\right.$$

$$\frac{d\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)\right.}{2c}\right.$$

$$\frac{d\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}\right)$$

$$\frac{d\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2634}$$

$$\frac{d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c}+\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}+i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}\right)$$

$$\Big\downarrow \text{5971}$$

$$\frac{d\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{6c}+\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}+i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}d\left(x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{3}dx\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}+i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}\right)-\frac{d\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(}}{\sqrt{b}}\right.\right.}{6c}$$

input
```
Int[(d + c^2*d*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*x*(1 + c^2*x^2)*Sqrt[a + b*ArcSinh[c*x]])/3 + (2*d*(x*Sqrt[a + b*ArcSin
h[c*x]] - ((I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[
b]])/E^(a/b)))/c))/3 - (d*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcS
inh[c*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/(6*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(c^2 d\,x^2 + d\right) \sqrt{a + b\,\mathrm{arcsinh}\left(xc\right)}dx$$

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx=d\left(\int c^2x^2\sqrt{a+b\operatorname{asinh}(cx)}\,dx+\int \sqrt{a+b\operatorname{asinh}(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

output
```
d*(Integral(c**2*x**2*sqrt(a + b*asinh(c*x)), x) + Integral(sqrt(a + b*asi
nh(c*x)), x))
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(c^2dx^2+d\right)\sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d + c^2dx^2) \sqrt{a + b\text{arcsinh}(cx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + c^2dx^2) \sqrt{a + b\text{arcsinh}(cx)}\, dx = \int \sqrt{a + b\,\text{asinh}(c\,x)}\, \left(d\,c^2\,x^2 + d\right)\, dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int (d + c^2dx^2) \sqrt{a + b\text{arcsinh}(cx)}\, dx = d\left(\int \sqrt{asinh(cx)\, b + a}dx + \left(\int \sqrt{asinh(cx)\, b + a}\, x^2dx\right) c^2\right)$$

input
```
int((c^2*d*x^2+d)*(a+b*asinh(c*x))^(1/2),x)
```

output
```
d*(int(sqrt(asinh(c*x)*b + a),x) + int(sqrt(asinh(c*x)*b + a)*x**2,x)*c**2
)
```

3.82 $\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

output
```
x*(a+b*arcsinh(c*x))^(1/2)+1/4*b^(1/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(
c*x))^(1/2)/b^(1/2))/c-1/4*b^(1/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/
Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6187}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6234}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}+x\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{26}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{3789}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{2c}$$

$$\Big\downarrow \text{2611}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{2c}$$

$$\Big\downarrow \text{2633}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

$$\Big\downarrow \text{2634}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(xc)}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\operatorname{arcsinh}(cx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(cx)}\,dx=\int \sqrt{asinh\left(cx\right)b+a}dx$$

input
```
int((a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)
```

3.83 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2} \, dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2} \, dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2} \, dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2),x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2dx^2+d}dx$$

$$\Big\downarrow \text{6209}$$

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2dx^2+d}dx$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(xc)}}{c^2d\,x^2+d}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2}\,dx = \frac{\int \frac{\sqrt{a+b\operatorname{asinh}(cx)}}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(c**2*d*x**2+d),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(c**2*x**2 + 1), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2}\,dx = \int \frac{\sqrt{b\operatorname{arsinh}(cx)+a}}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{d + c^2dx^2}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx) + a}}{c^2dx^2 + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{d + c^2dx^2}\,dx = \int \frac{\sqrt{a + b\,\text{asinh}\,(c\,x)}}{d\,c^2\,x^2 + d}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+c^2dx^2}\,dx = \frac{\int \frac{\sqrt{asinh(cx)b+a}}{c^2x^2+1}dx}{d}$$

input
```
int((a+b*asinh(c*x))^(1/2)/(c^2*d*x^2+d),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(c**2*x**2 + 1),x)/d
```

3.84 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\,\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx = \text{Int}\left(\frac{\sqrt{a+b\,\text{arcsinh}(cx)}}{(d+c^2dx^2)^2},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\,\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx = \int \frac{\sqrt{a+b\,\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^2,x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4d^2}+\frac{\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d(c^2x^2+1)}dx}{2d}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4d^2}+\frac{\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d^2}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6209}$$

$$-\frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4d^2}+\frac{\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d^2}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{bc\int \frac{x}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4d^2}+\frac{\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d^2}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{2d^2(c^2x^2+1)}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b \operatorname{arcsinh}(xc)}}{(c^2 d x^2+d)^2} dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{\sqrt{a+b\operatorname{asinh}(cx)}}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(c**2*d*x**2+d)**2,x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(c**4*x**4 + 2*c**2*x**2 + 1), x)/d**2
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + c^2dx^2)^2}\, dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx) + a}}{(c^2dx^2 + d)^2}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + c^2dx^2)^2}\, dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx) + a}}{(c^2dx^2 + d)^2}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx = \int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{\sqrt{asinh(cx)b+a}}{c^4x^4+2c^2x^2+1}dx}{d^2}$$

input
```
int((a+b*asinh(c*x))^(1/2)/(c^2*d*x^2+d)^2,x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(c**4*x**4 + 2*c**2*x**2 + 1),x)/d**2
```

3.85 $\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 632

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^{3/2}\,dx = -\frac{4bd^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{5c}$$
$$+\frac{8}{15}d^2x(a+b\text{arcsinh}(cx))^{3/2}+\frac{4}{15}d^2x\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))^{3/2}$$
$$+\frac{1}{5}d^2x\left(1+c^2x^2\right)^2(a+b\text{arcsinh}(cx))^{3/2}-\frac{2bd^2\sqrt{a+b\text{arcsinh}(cx)}\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{15c}-\frac{3bd^2\sqrt{a+b\text{a}}}{}$$

output
```
-4/5*b*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+8/15*d^2*x*(a+b*ar
csinh(c*x))^(3/2)+4/15*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^(3/2)+1/5*d^2*
x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^(3/2)-2/15*b*d^2*(a+b*arcsinh(c*x))^(1/
2)*cosh(a/b-(a+b*arcsinh(c*x))/b)^3/c-3/50*b*d^2*(a+b*arcsinh(c*x))^(1/2)*
cosh(a/b-(a+b*arcsinh(c*x))/b)^5/c+15/64*b^(3/2)*d^2*exp(a/b)*Pi^(1/2)*erf
((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c+5/1152*b^(3/2)*d^2*exp(3*a/b)*3^(1/2)
*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c+3/16000*b^(3/2)*
d^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/
2))/c+15/64*b^(3/2)*d^2*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/
exp(a/b)+5/1152*b^(3/2)*d^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x
))^(1/2)/b^(1/2))/c/exp(3*a/b)+3/16000*b^(3/2)*d^2*5^(1/2)*Pi^(1/2)*erfi(5
^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(5*a/b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1553 vs. $2(632) = 1264$.

Time = 9.57 (sec) , antiderivative size = 1553, normalized size of antiderivative = 2.46

$$\int \left(d + c^2 d x^2\right)^2 (a + b\text{arcsinh}(cx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*d^2*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
*x]])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sq
rt[-((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*d^2*Sqrt[a + b*ArcSinh[
c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcS
inh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh
[c*x]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*Ar
cSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamm
a[3/2, (3*(a + b*ArcSinh[c*x]))/b]))/(36*c*E^((3*a)/b)*Sqrt[-((a + b*ArcSi
nh[c*x])^2/b^2)]) + (a*d^2*Sqrt[a + b*ArcSinh[c*x]]*(-150*E^((6*a)/b)*Sqrt
[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] - 25*Sqrt[3]
*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))
/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh
[c*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
3/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E^((10*a)/b)*Sqrt[-((a + b*Ar
cSinh[c*x])/b)]*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b]))/(2400*c*E^((5*a)/
b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)]) + (Sqrt[b]*d^2*(4*Sqrt[b]*Sqrt[a +
 b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)
*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
 (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b]...
```

Rubi [A] (verified)

Time = 3.34 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.22, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {6201, 27, 6201, 6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634, 6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^2 (a+b\text{arcsinh}(cx))^{3/2} \, dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{10}bcd^2\int x\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{4}{5}d\int d\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{10}bcd^2\int x\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{4}{5}d^2\int \left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6201}$$

$$-\frac{3}{10}bcd^2\int x\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{4}{5}d^2\left(-\frac{1}{2}bc\int x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{2}{3}\int (a+b\text{arcsinh}(cx))^{3/2}dx+\frac{1}{3}x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.$$
$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6187}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{2}bc\int x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}dx+\right.$$
$$\frac{3}{10}bcd^2\int x\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6213}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int\frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)\right)-\frac{1}{2}bc\left(\frac{(c^2\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 6189

$$-\frac{3}{10}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+$$

$$\frac{4}{5}d^2\left(-\frac{1}{2}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\right.\right.\right.$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3042

$$-\frac{3}{10}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+$$

$$\frac{4}{5}d^2\left(-\frac{1}{2}bc\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\right.\right.\right.$$

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3788

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}i\int-\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(\right.\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{26}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx\right.\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2611}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{}\right.\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{\left(c^2x^2+1\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+\frac{1}{5}d^2x\left(c^2x^2+1\right)^2(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2633}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{2c\right.\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+\frac{1}{5}d^2x(c^2x^2+1)^2(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{4}{5}d^2\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\right.\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{b\int\frac{(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{10c}\right)+\frac{1}{5}d^2x(c^2x^2+1)^2(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{4}{5}d^2\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\int\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^2}\right)+\frac{2}{3}\left(x(a+b\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{\int\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c^2}\right)+$$

$$\frac{1}{5}d^2x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{5}d^2\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^2}\right)+\frac{2}{3}\left(x(a\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^5}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{10c^2}\right)+$$

$$\frac{1}{5}d^2x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{4}{5}d^2\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(\right.}{6c^2}\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{\int\left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\cos}{}\right.}{10c^2}\right.$$

$$\frac{1}{5}d^2x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}}{2c^2}\right.\right.\right.$$

$$\frac{3}{10}bcd^2\left(\frac{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}{5c^2}-\frac{\frac{5}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{5}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcs}}}{\sqrt{b}}\right.}{}\right.$$

$$\frac{1}{5}d^2x(c^2x^2+1)^2(a+b\text{arcsinh}(cx))^{3/2}$$

input
```
Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^(3/2))/5 + (4*d^2*((x*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^(3/2))/3 + (2*(x*(a + b*ArcSinh[c*x])^(3/2) - (
3*b*c*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/c^2 - ((Sqrt[b]*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b)))/(2*c^2)))/2))/3 - (b*c*
(((1 + c^2*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]])/(3*c^2) - ((3*Sqrt[b]*E^(a
/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/8 + (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 + (3*Sqrt
[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b
]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((3*a)
/b)))/(6*c^2)))/2))/5 - (3*b*c*d^2*(((1 + c^2*x^2)^(5/2)*Sqrt[a + b*ArcSin
h[c*x]])/(5*c^2) - ((5*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x
]]/Sqrt[b]])/16 + (5*Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
b*ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[
5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/32 + (5*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*E^(a/b)) + (5*Sqrt[b]*Sqrt[Pi/3]*Erfi[(S
qrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqr
t[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))
)/(10*c^2)))/10
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [F]

$$\int \left(c^{2} d \,x^{2}+d \right)^{2} \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}d x$$

input
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^2*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^2 (a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2}\,dx = d^2\left(\int a\sqrt{a+b\,\text{asinh}\,(cx)}\,dx + \int b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)\,dx + \int 2ac^2x^2\sqrt{a+b\,\text{asinh}\,(cx)}\,dx + \int ac^4x^4\sqrt{a+b\,\text{asinh}\,(cx)}\,dx + \int 2bc^2x^2\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)\,dx + \int bc^4x^4\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(a*sqrt(a + b*asinh(c*x)), x) + Integral(b*sqrt(a + b*asinh(
c*x))*asinh(c*x), x) + Integral(2*a*c**2*x**2*sqrt(a + b*asinh(c*x)), x) +
 Integral(a*c**4*x**4*sqrt(a + b*asinh(c*x)), x) + Integral(2*b*c**2*x**2*
sqrt(a + b*asinh(c*x))*asinh(c*x), x) + Integral(b*c**4*x**4*sqrt(a + b*as
inh(c*x))*asinh(c*x), x))
```

Maxima [F]

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2}\,dx = \int (c^2dx^2+d)^2(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2} dx = \int (a+b\,\text{asinh}(c\,x))^{3/2} \left(d\,c^2\,x^2+d\right)^2 dx$$

input
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int (d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2} dx = d^2\left(\left(\int \sqrt{asinh(cx)\,b+a}dx\right)a + \left(\int \sqrt{asinh(cx)\,b+a}\,asinh(cx)\,x^4dx\right)b\,c^4 + 2\left(\int \sqrt{asinh(cx)\,b+a}\,asinh(cx)\,x^2dx\right)b\,c^2 + \left(\int \sqrt{asinh(cx)\,b+a}\,asinh(cx)\,dx\right)b + \left(\int \sqrt{asinh(cx)\,b+a}\,x^4dx\right)a\,c^4 + 2\left(\int \sqrt{asinh(cx)\,b+a}\,x^2dx\right)a\,c^2\right)$$

input
```
int((c^2*d*x^2+d)^2*(a+b*asinh(c*x))^(3/2),x)
```

output
```
d**2*(int(sqrt(asinh(c*x)*b + a),x)*a + int(sqrt(asinh(c*x)*b + a)*asinh(c
*x)*x**4,x)*b*c**4 + 2*int(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2,x)*b*c**
2 + int(sqrt(asinh(c*x)*b + a)*asinh(c*x),x)*b + int(sqrt(asinh(c*x)*b + a
)*x**4,x)*a*c**4 + 2*int(sqrt(asinh(c*x)*b + a)*x**2,x)*a*c**2)
```

3.86 $\int (d+c^2dx^2) (a+b\text{arcsinh}(cx))^{3/2} dx$

Optimal result

Integrand size = 23, antiderivative size = 318

$$\int (d+c^2dx^2) (a+b\text{arcsinh}(cx))^{3/2} dx = -\frac{bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{c} + \frac{2}{3}dx(a+b\text{arcsinh}(cx))^{3/2} + \frac{1}{3}dx(1+c^2x^2)(a+b\text{arcsinh}(cx))^{3/2} - \frac{bd\sqrt{a+b\text{arcsinh}(cx)}\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{6c} + \frac{9b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c} + \frac{b^{3/2}de^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c} + \frac{9b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c} + \frac{b^{3/2}de^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c}$$

output
```
-b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+2/3*d*x*(a+b*arcsinh(c*x
))^(3/2)+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^(3/2)-1/6*b*d*(a+b*arcsinh
(c*x))^(1/2)*cosh(a/b-(a+b*arcsinh(c*x))/b)^3/c+9/32*b^(3/2)*d*exp(a/b)*Pi
^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c+1/288*b^(3/2)*d*exp(3*a/b)*
3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c+9/32*b^(3
/2)*d*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(a/b)+1/288*b^(
3/2)*d*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/e
xp(3*a/b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 768 vs. $2(318) = 636$.

Time = 5.89 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.42

$$\int (d + c^2dx^2) (a + b\text{arcsinh}(cx))^{3/2} dx = \text{Too large to display}$$

input
```
Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
d*((a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*
x]])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqr
t[-((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*Sqrt[a + b*ArcSinh[c*x]]
*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c
*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]
))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh
[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2
, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*
x])^2/b^2)]) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c
^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt[b]*(
-9*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSi
nh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Co
sh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (-2*a +
b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/
b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*ArcSinh
[c*x]] + 2*ArcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c))
```

Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {6201, 6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634, 6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)(a+b\text{arcsinh}(cx))^{3/2}dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{1}{2}bcd\int x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{2}{3}d\int (a+b\text{arcsinh}(cx))^{3/2}dx+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6187}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx\right)-\frac{1}{2}bcd\int x\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6213}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)\right)-\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int \frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6189}$$

$$-\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3042

$$-\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinl}}{2c^2}\right.\right.$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3788

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}i\int-\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}\right.\right.$$

$$\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{1}{3}dx\left(c^2x^2+1\right)(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 26

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\int\frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+}{2c^2}\right.\right.$$

$$\frac{1}{2}bcd\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{1}{3}dx(c^2x^2+1)(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int}{2c^2}\right.\right.$$

$$\frac{1}{2}bcd\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{1}{3}dx(c^2x^2+1)(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}}{2c^2}\right.\right.$$

$$\frac{1}{2}bcd\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+\frac{1}{3}dx(c^2x^2+1)(a+$$

$$b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{b\int\frac{c^2x^2+1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{6c}\right)+$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{}}{2c^2}\right.\right.$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6206}$$

$$-\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\int\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^2}\right)+$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{}}{2c^2}\right.\right.$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}bcd\left(\frac{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{6c^2}\right)+$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{}}{2c^2}\right.\right.$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{1}{2}bcd\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b}{6c^2}\right.$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}}{2c^2}\right.\right.$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}d\left(x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}}{2c^2}\right.\right.$$

$$\frac{1}{2}bcd\left(\frac{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^2}-\frac{\frac{3}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}}}{\sqrt{b}}\right.}{}\right.$$

$$\frac{1}{3}dx\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))^{3/2}$$

input
```
Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^(3/2))/3 + (2*d*(x*(a + b*ArcSinh[
c*x])^(3/2) - (3*b*c*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/c^2 - (
(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqrt
[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b)))/(2*c^2))
)/2))/3 - (b*c*d*(((1 + c^2*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]])/(3*c^2) -
 ((3*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/8 + (
Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/8 + (3*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E
^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(8*E^((3*a)/b)))/(6*c^2)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [F]

$$\int \left(c^{2} d\,x^{2}+d\right) \left(a +b\,\operatorname{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}d x$$

input
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d + c^2 d x^2) (a + b \text{arcsinh}(cx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d + c^2 d x^2) (a + b \text{arcsinh}(cx))^{3/2} \, dx = d \left(\int a \sqrt{a + b \operatorname{asinh}(cx)} \, dx + \int b \sqrt{a + b \operatorname{asinh}(cx)} \operatorname{asinh}(cx) \, dx + \int a c^2 x^2 \sqrt{a + b \operatorname{asinh}(cx)} \, dx + \int b c^2 x^2 \sqrt{a + b \operatorname{asinh}(cx)} \operatorname{asinh}(cx) \, dx \right)$$

input
```
integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(a*sqrt(a + b*asinh(c*x)), x) + Integral(b*sqrt(a + b*asinh(c*x
))*asinh(c*x), x) + Integral(a*c**2*x**2*sqrt(a + b*asinh(c*x)), x) + Inte
gral(b*c**2*x**2*sqrt(a + b*asinh(c*x))*asinh(c*x), x))
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)^{3/2}\,dx=\int \left(c^2dx^2+d\right)\left(b\,\text{arsinh}\,(cx)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)\left(a+b\text{arcsinh}(cx)\right)^{3/2}\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}\,\left(d\,c^2\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2), x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)(a+b\text{arcsinh}(cx))^{3/2}\,dx = d\left(\left(\int \sqrt{asinh\left(cx\right)b+a}dx\right)a + \left(\int \sqrt{asinh\left(cx\right)b+a}\,asinh(cx)\,x^2dx\right)b\,c^2 + \left(\int \sqrt{asinh\left(cx\right)b+a}\,asinh(cx)\,dx\right)b + \left(\int \sqrt{asinh\left(cx\right)b+a}\,x^2dx\right)a\,c^2\right)$$

input
```
int((c^2*d*x^2+d)*(a+b*asinh(c*x))^(3/2),x)
```

output
```
d*(int(sqrt(asinh(c*x)*b + a),x)*a + int(sqrt(asinh(c*x)*b + a)*asinh(c*x)
*x**2,x)*b*c**2 + int(sqrt(asinh(c*x)*b + a)*asinh(c*x),x)*b + int(sqrt(as
inh(c*x)*b + a)*x**2,x)*a*c**2)
```

3.87 $\int (a + b\text{arcsinh}(cx))^{3/2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 135

$$\int (a + b\text{arcsinh}(cx))^{3/2}\, dx = -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c} + x(a+b\text{arcsinh}(cx))^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}$$

output
```
-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+x*(a+b*arcsinh(c*x))^(
3/2)+3/8*b^(3/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c
+3/8*b^(3/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)(\cos$$

$$8c$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]]
)/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b
*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (
-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Si
nh[a/b])))/(8*c)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.750$, Rules used = {6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6213}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)$$

$$\Big\downarrow \text{ 6189}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$$\Big\downarrow \text{ 3788}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsin}}}{2c^2}\right.$$

$\downarrow$ 2633

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx}}{\sqrt{b}}\right.}{2c^2}\right.$$

$\downarrow$ 2634

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2c^2}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
x*(a + b*ArcSinh[c*x])^(3/2) - (3*b*c*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSi
nh[c*x]])/c^2 - ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E
^(a/b)))/(2*c^2)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [F]

$$\int (a + b \operatorname{arcsinh}(xc))^{\frac{3}{2}} dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a + b\operatorname{arcsinh}(cx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a + b\operatorname{arcsinh}(cx))^{3/2} dx = \int (a + b \operatorname{asinh}(cx))^{\frac{3}{2}} dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int (a + b\text{arcsinh}(cx))^{3/2}\,dx = \left(\int \sqrt{asinh\,(cx)\,b + a}dx\right)a + \left(\int \sqrt{asinh\,(cx)\,b + a}\,asinh(cx)\,dx\right)b$$

input
```
int((a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)*a + int(sqrt(asinh(c*x)*b + a)*asinh(c*x),x)
*b
```

3.88 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2} dx = \int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2} dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{c^2dx^2+d}\,dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{c^2dx^2+d}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\,\operatorname{arcsinh}\left(xc\right)\right)^{\frac{3}{2}}}{c^2d\,x^2+d}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2}\,dx = \frac{\int \frac{a\sqrt{a+b\,\text{asinh}\,(cx)}}{c^2x^2+1}\,dx + \int \frac{b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(c**2*d*x**2+d),x)
```

output
```
(Integral(a*sqrt(a + b*asinh(c*x))/(c**2*x**2 + 1), x) + Integral(b*sqrt(a
 + b*asinh(c*x))*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{c^2dx^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+c^2dx^2}\,dx=\frac{\left(\int \frac{\sqrt{asinh(cx)b+a}}{c^2x^2+1}dx\right)a+\left(\int \frac{\sqrt{asinh(cx)b+a}\,asinh(cx)}{c^2x^2+1}dx\right)b}{d}$$

input
```
int((a+b*asinh(c*x))^(3/2)/(c^2*d*x^2+d),x)
```

output
```
(int(sqrt(asinh(c*x)*b + a)/(c**2*x**2 + 1),x)*a + int((sqrt(asinh(c*x)*b
+ a)*asinh(c*x))/(c**2*x**2 + 1),x)*b)/d
```

3.89 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^2,x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(c^2dx^2+d)^2}dx$$

$$\Big\downarrow \text{ 6203}$$

$$-\frac{3bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{(c^2x^2+1)^{3/2}}dx}{4d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d(c^2x^2+1)}dx}{2d}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{(c^2x^2+1)^{3/2}}dx}{4d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 6209}$$

$$-\frac{3bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{(c^2x^2+1)^{3/2}}dx}{4d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 6213}$$

$$-\frac{3bc\left(\frac{b\int \frac{1}{(c^2x^2+1)\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}-\frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2\sqrt{c^2x^2+1}}\right)}{4d^2}+\frac{\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{2d^2(c^2x^2+1)}$$

$$\Big\downarrow \text{ 6209}$$

$$-\frac{3bc\left(\frac{b\int\frac{1}{(c^2x^2+1)\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}-\frac{\sqrt{a+b\text{arcsinh}(cx)}}{c^2\sqrt{c^2x^2+1}}\right)}{4d^2}+\frac{\int\frac{(a+b\text{arcsinh}(cx))^{3/2}}{c^2x^2+1}dx}{2d^2}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{2d^2\left(c^2x^2+1\right)}$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\,\text{arcsinh}\left(xc\right)\right)^{\frac{3}{2}}}{\left(c^{2}d\,x^{2}+d\right)^{2}}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\left(d+c^2dx^2\right)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 8.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx = \frac{\int \frac{a\sqrt{a+b\,\text{asinh}\,(cx)}}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*sqrt(a + b*asinh(c*x))/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Int
egral(b*sqrt(a + b*asinh(c*x))*asinh(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x
))/d**2
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}{(c^2dx^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))^{3/2}}{(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^2}\,dx=\frac{\left(\int \frac{\sqrt{asinh(cx)b+a}}{c^4x^4+2c^2x^2+1}dx\right)a+\left(\int \frac{\sqrt{asinh(cx)b+a}\,asinh(cx)}{c^4x^4+2c^2x^2+1}dx\right)b}{d^2}$$

input
```
int((a+b*asinh(c*x))^(3/2)/(c^2*d*x^2+d)^2,x)
```

output
```
(int(sqrt(asinh(c*x)*b + a)/(c**4*x**4 + 2*c**2*x**2 + 1),x)*a + int((sqrt
(asinh(c*x)*b + a)*asinh(c*x))/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b)/d**2
```

3.90 $\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 426

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{35d^3e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{7d^3e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{7d^3e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{d^3e^{\frac{7a}{b}}\sqrt{\frac{\pi}{7}}\,\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{35d^3e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{7d^3e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{7d^3e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c} + \frac{d^3e^{-\frac{7a}{b}}\sqrt{\frac{\pi}{7}}\,\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{128\sqrt{b}c}$$

output
```
35/128*d^3*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)
/c+7/128*d^3*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1
/2)/b^(1/2))/b^(1/2)/c+7/640*d^3*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(
a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+1/896*d^3*exp(7*a/b)*7^(1/2)*Pi
^(1/2)*erf(7^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+35/128*d^3*
Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(a/b)+7/128*d
^3*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)
/c/exp(3*a/b)+7/640*d^3*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))/b^(1/2)/c/exp(5*a/b)+1/896*d^3*7^(1/2)*Pi^(1/2)*erfi(7^(1/2)
*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(7*a/b)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.92

$$\int \frac{(d + c^2dx^2)^3}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx =$$

$$-\frac{d^3e^{-\frac{7a}{b}}\left(1225e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)-5\sqrt{7}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{7(a+b\text{arcsinh}}{b}\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/4480*(d^3*(1225*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + A
rcSinh[c*x]] - 5*Sqrt[7]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a
 + b*ArcSinh[c*x]))/b] - 49*Sqrt[5]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x]
)/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] - 245*Sqrt[3]*E^((4*a)/b)*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 12
25*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh
[c*x])/b)] + 245*Sqrt[3]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2,
(3*(a + b*ArcSinh[c*x]))/b] + 49*Sqrt[5]*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c
*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b] + 5*Sqrt[7]*E^((14*a)/b)*Sqrt[
a/b + ArcSinh[c*x]]*Gamma[1/2, (7*(a + b*ArcSinh[c*x]))/b]))/(c*E^((7*a)/b
)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{d^3\int \frac{\cosh^7\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^3\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^7}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$\frac{d^3\int \left(\frac{\cosh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{7\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{21\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{35\cosh\left(\frac{a}{b}-\frac{a+b\text{arc}}{}\right.}{64\sqrt{a+b\text{arcsi}}}\right.}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\left(\frac{35}{128}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{7}{128}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{7}{128}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+ba}}{}\right.\right.}{}$$

input
```
Int[(d + c^2*d*x^2)^3/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^3*((35*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
128 + (7*Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/128 + (7*Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/128 + (Sqrt[b]*E^((7*a)/b)*Sqrt[Pi/7]*Erf[(S
qrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/128 + (35*Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(128*E^(a/b)) + (7*Sqrt[b]*Sqrt[3*Pi]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^((3*a)/b)) + (7*S
qrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E
^((5*a)/b)) + (Sqrt[b]*Sqrt[Pi/7]*Erfi[(Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(128*E^((7*a)/b))))/(b*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{(c^2d\,x^2+d)^3}{\sqrt{a+b\,\operatorname{arcsinh}(xc)}}dx$$

input
```
int((c^2*d*x^2+d)^3/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^3/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = d^3\left(\int \frac{3c^2x^2}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx + \int \frac{3c^4x^4}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx + \int \frac{c^6x^6}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx + \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3/(a+b*asinh(c*x))**(1/2),x)
```

output
```
d**3*(Integral(3*c**2*x**2/sqrt(a + b*asinh(c*x)), x) + Integral(3*c**4*x**
*4/sqrt(a + b*asinh(c*x)), x) + Integral(c**6*x**6/sqrt(a + b*asinh(c*x)),
 x) + Integral(1/sqrt(a + b*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(c^2dx^2+d)^3}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^3/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(c^2dx^2+d)^3}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^3/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(d\,c^2\,x^2+d)^3}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^3}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = d^3\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)\,b+a}dx + \left(\int \frac{\sqrt{asinh\,(cx)\,b+a}\,x^6}{asinh\,(cx)\,b+a}dx\right)c^6 + 3\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}\,x^4}{asinh\,(cx)\,b+a}dx\right)c^4 + 3\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}\,x^2}{asinh\,(cx)\,b+a}dx\right)c^2\right)$$

input

```
int((c^2*d*x^2+d)^3/(a+b*asinh(c*x))^(1/2),x)
```

output

```
d**3*(int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x) + int((sqrt(asinh(c
*x)*b + a)*x**6)/(asinh(c*x)*b + a),x)*c**6 + 3*int((sqrt(asinh(c*x)*b + a
)*x**4)/(asinh(c*x)*b + a),x)*c**4 + 3*int((sqrt(asinh(c*x)*b + a)*x**2)/(
asinh(c*x)*b + a),x)*c**2)
```

3.91 $\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 318

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{5d^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c} + \frac{5d^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c} + \frac{d^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c} + \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c} + \frac{5d^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c} + \frac{d^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c}$$

output
```
5/16*d^2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c
+5/96*d^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))/b^(1/2)/c+1/160*d^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+5/16*d^2*Pi^(1/2)*erfi((a+b*arcsin
h(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(a/b)+5/96*d^2*3^(1/2)*Pi^(1/2)*erfi(3
^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(3*a/b)+1/160*d^2*5^
(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/ex
p(5*a/b)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.93

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = -\frac{d^2e^{-\frac{5a}{b}}\left(150e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)-3\sqrt{5}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{5(a+b\text{arcsinh}(c\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/480*(d^2*(150*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Arc
Sinh[c*x]] - 3*Sqrt[5]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a +
 b*ArcSinh[c*x]))/b] - 25*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/
b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 150*E^((4*a)/b)*Sqrt[-((a +
b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 25*Sqrt[3]*E^(
(8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] +
 3*Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcS
inh[c*x]))/b]))/(c*E^((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{d^2\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^5}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{d^2\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 2009}$$

$$d^2\left(\frac{5}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{5}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsi}}}{\sqrt{b}}\right.\right.$$

input
```
Int[(d + c^2*d*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^2*((5*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/1
6 + (5*Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/32 + (5*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[
c*x]]/Sqrt[b]])/(16*E^(a/b)) + (5*Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(
Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/(b*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^2}{\sqrt{a + b\,\mathrm{arcsinh}\,(xc)}} dx$$

input
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = d^2\left(\int \frac{2c^2x^2}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx + \int \frac{c^4x^4}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx + \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
d**2*(Integral(2*c**2*x**2/sqrt(a + b*asinh(c*x)), x) + Integral(c**4*x**4
/sqrt(a + b*asinh(c*x)), x) + Integral(1/sqrt(a + b*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(c^2dx^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(c^2dx^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(d\,c^2\,x^2+d)^2}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = d^2\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)\,b+a}dx + \left(\int \frac{\sqrt{asinh\,(cx)\,b+a}\,x^4}{asinh\,(cx)\,b+a}dx\right)c^4 + 2\left(\int \frac{\sqrt{asinh\,(cx)\,b+a}\,x^2}{asinh\,(cx)\,b+a}dx\right)c^2\right)$$

input
```
int((c^2*d*x^2+d)^2/(a+b*asinh(c*x))^(1/2),x)
```

output
```
d**2*(int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x) + int((sqrt(asinh(c
*x)*b + a)*x**4)/(asinh(c*x)*b + a),x)*c**4 + 2*int((sqrt(asinh(c*x)*b + a
)*x**2)/(asinh(c*x)*b + a),x)*c**2)
```

3.92 $\int \frac{d+c^2dx^2}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{3de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c} + \frac{de^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c} + \frac{3de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c} + \frac{de^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c}$$

output
```
3/8*d*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+1/
24*d*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))/b^(1/2)/c+3/8*d*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/
2)/c/exp(a/b)+1/24*d*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2
)/b^(1/2))/b^(1/2)/c/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{de^{-\frac{3a}{b}}\left(-9e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\ldots\right)}{24c\sqrt{a+b\text{arc}\ldots}}$$

input
```
Integrate[(d + c^2*d*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*(-9*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]
 + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c
*x]))/b] + 9*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a +
 b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(24*c*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c*
x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2dx^2+d}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{d\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$\frac{d\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\frac{3}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{3}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx}}{\sqrt{b}}\right.\right.}{bc}$$

input
```
Int[(d + c^2*d*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*((3*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/8 +
 (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/8 + (3*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8
*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(8*E^((3*a)/b))))/(b*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{c^2 d\, x^2 + d}{\sqrt{a + b\, \mathrm{arcsinh}\,(xc)}} dx$$

input
```
int((c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d + c^2 d x^2}{\sqrt{a + b \mathrm{arcsinh}(cx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=d\left(\int \frac{c^2x^2}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx+\int \frac{1}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
d*(Integral(c**2*x**2/sqrt(a + b*asinh(c*x)), x) + Integral(1/sqrt(a + b*a
sinh(c*x)), x))
```

Maxima [F]

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{c^2dx^2+d}{\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{c^2dx^2+d}{\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{d\,c^2\,x^2+d}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{d+c^2dx^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = d\left(\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b+a}dx + \left(\int \frac{\sqrt{asinh\left(cx\right)b+a}\,x^2}{asinh\left(cx\right)b+a}dx\right)c^2\right)$$

input
```
int((c^2*d*x^2+d)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
d*(int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x) + int((sqrt(asinh(c*x)
*b + a)*x**2)/(asinh(c*x)*b + a),x)*c**2)
```

3.93 $\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

output
```
1/2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+1/2*
Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$
$$=\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\right)}{2c\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) +
Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*
c*E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{bc}$$

$$\Big\downarrow \; 2633$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

$$\Big\downarrow \; 2634$$

$$\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

input
```
Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqr
t[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b)))/(b*c)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b \operatorname{arcsinh}(xc)}} dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a + b\operatorname{asinh}{(cx)}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\operatorname{arsinh}(cx) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\operatorname{arsinh}(cx) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b+a}dx$$

input
```
int(1/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x)
```

3.94 $\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(c^2dx^2+d)\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + c^2*d*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2d\,x^2+d)\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{\int \frac{1}{c^2x^2\sqrt{a+b\operatorname{asinh}(cx)}+\sqrt{a+b\operatorname{asinh}(cx)}}\,dx}{d}$$

input
```
integrate(1/(c**2*d*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(c**2*x**2*sqrt(a + b*asinh(c*x)) + sqrt(a + b*asinh(c*x))), x)
/d
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)\sqrt{b\,\text{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,\left(d\,c^2\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{(d+c^2dx^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{\int \frac{\sqrt{asinh(cx)b+a}}{asinh(cx)b\,c^2x^2+asinh(cx)b+a\,c^2x^2+a}dx}{d}$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b*c**2*x**2 + asinh(c*x)*b + a*c**2
*x**2 + a),x)/d
```

3.95 $\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\textbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Int}\left(\frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^2\sqrt{a+b\text{arcsinh}(cx)}}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(c^2dx^2+d)^2\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + c^2*d*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2d\,x^2+d)^2\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{1}{(d+c^2dx^2)^2 \sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{\int \frac{1}{c^4x^4\sqrt{a+b\,\text{asinh}\,(cx)}+2c^2x^2\sqrt{a+b\,\text{asinh}\,(cx)}+\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx}{d^2}$$

input
```
integrate(1/(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(c**4*x**4*sqrt(a + b*asinh(c*x)) + 2*c**2*x**2*sqrt(a + b*asin
h(c*x)) + sqrt(a + b*asinh(c*x))), x)/d**2
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.72

$$\int \frac{1}{(d+c^2dx^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{\int \frac{\sqrt{asinh(cx)b+a}}{asinh(cx)b\,c^4x^4+2asinh(cx)b\,c^2x^2+asinh(cx)b+a\,c^4x^4+2a\,c^2x^2+a}dx}{d^2}$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x
**2 + asinh(c*x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)/d**2
```

3.96 $\int \frac{(d+c^2dx^2)^3}{(a+b\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 454

$$\begin{aligned}\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d^3(1+c^2x^2)^{7/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\ &\quad -\frac{35d^3e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c}-\frac{21d^3e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c} \\ &\quad -\frac{7d^3e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c}-\frac{d^3e^{\frac{7a}{b}}\sqrt{7\pi}\,\text{erf}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c} \\ &\quad +\frac{35d^3e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c}+\frac{21d^3e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c} \\ &\quad +\frac{7d^3e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c}+\frac{d^3e^{-\frac{7a}{b}}\sqrt{7\pi}\,\text{erfi}\left(\frac{\sqrt{7}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c}\end{aligned}$$

output
```
-2*d^3*(c^2*x^2+1)^(7/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-35/64*d^3*exp(a/b)*P
i^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-21/64*d^3*exp(3*a/
b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/
c-7/64*d^3*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2
)/b^(1/2))/b^(3/2)/c-1/64*d^3*exp(7*a/b)*7^(1/2)*Pi^(1/2)*erf(7^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+35/64*d^3*Pi^(1/2)*erfi((a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)+21/64*d^3*3^(1/2)*Pi^(1/2)*erfi
(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(3*a/b)+7/64*d^3*5
^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/e
xp(5*a/b)+1/64*d^3*7^(1/2)*Pi^(1/2)*erfi(7^(1/2)*(a+b*arcsinh(c*x))^(1/2)/
b^(1/2))/b^(3/2)/c/exp(7*a/b)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.28

$$\int \frac{(d+c^2dx^2)^3}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{d^3e^{-7\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}\left(-e^{\frac{7a}{b}}-7e^{\frac{7a}{b}+2\operatorname{arcsinh}(cx)}-21e^{\frac{7a}{b}+4\operatorname{arcsinh}(cx)}-35e^{\frac{7a}{b}+6\operatorname{arcsi}}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^3/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d^3*(-E^((7*a)/b) - 7*E^((7*a)/b + 2*ArcSinh[c*x]) - 21*E^((7*a)/b + 4*Ar
cSinh[c*x]) - 35*E^((7*a)/b + 6*ArcSinh[c*x]) - 35*E^((7*a)/b + 8*ArcSinh[
c*x]) - 21*E^((7*a)/b + 10*ArcSinh[c*x]) - 7*E^((7*a)/b + 12*ArcSinh[c*x])
 - E^((7*a)/b + 14*ArcSinh[c*x]) + 35*E^((8*a)/b + 7*ArcSinh[c*x])*Sqrt[a/
b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[7]*E^(7*ArcSinh[c*
x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a + b*ArcSinh[c*x]))/b
] + 7*Sqrt[5]*E^((2*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]
*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 21*Sqrt[3]*E^((4*a)/b + 7*ArcSi
nh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x
]))/b] + 35*E^((6*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, -((a + b*ArcSinh[c*x])/b)] + 21*Sqrt[3]*E^((10*a)/b + 7*ArcSinh[
c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] + 7*
Sqrt[5]*E^((12*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2,
(5*(a + b*ArcSinh[c*x]))/b] + Sqrt[7]*E^(7*((2*a)/b + ArcSinh[c*x]))*Sqrt[
a/b + ArcSinh[c*x]]*Gamma[1/2, (7*(a + b*ArcSinh[c*x]))/b]))/(64*b*c*E^(7*
(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{14cd^3\int \frac{x(c^2x^2+1)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^3(c^2x^2+1)^{7/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{14d^3\int -\frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d^3(c^2x^2+1)^{7/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{14d^3\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d^3(c^2x^2+1)^{7/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{14d^3\int \left(\frac{\sinh\left(\frac{7a}{b}-\frac{7(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{64\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\sinh\left(\frac{a}{b}-\frac{a+b\text{a}}{}\right.}{64\sqrt{a+b\text{arc}}}\right.}{b^2c}$$

$$\frac{2d^3(c^2x^2+1)^{7/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\downarrow$ 2009

$$\frac{14d^3\left(-\frac{5}{128}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{3}{128}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{128}\sqrt{5\pi}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{}}{}\right.\right.}{}$$

$$\frac{2d^3\left(c^2x^2+1\right)^{7/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)^3/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d^3*(1 + c^2*x^2)^(7/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (14*d^3*((-5
*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/128 - (3*
Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/128 - (Sqrt[b]*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSin
h[c*x]])/Sqrt[b]])/128 - (Sqrt[b]*E^((7*a)/b)*Sqrt[Pi/7]*Erf[(Sqrt[7]*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/128 + (5*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]])/(128*E^(a/b)) + (3*Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3
]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^((3*a)/b)) + (Sqrt[b]*Sqrt[5*
Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128*E^((5*a)/b)) +
(Sqrt[b]*Sqrt[Pi/7]*Erfi[(Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(128
*E^((7*a)/b))))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^3}{(a + b\,\mathrm{arcsinh}\,(xc))^{\frac{3}{2}}} dx$$

input
```
int((c^2*d*x^2+d)^3/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^3/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^3}{(a + b\mathrm{arcsinh}(cx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d^3\left(\int \frac{3c^2x^2}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{3c^4x^4}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{c^6x^6}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)**3/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**3*(Integral(3*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(
c*x))*asinh(c*x)), x) + Integral(3*c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b
*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(c**6*x**6/(a*sqrt(a + b
*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sq
rt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^3}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^3}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^3/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(d\,c^2\,x^2+d)^3}{(a+b\,\operatorname{asinh}(c\,x))^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)^3/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^3}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((c^2*d*x^2+d)^3/(a+b*asinh(c*x))^(3/2),x)
```

output

```
(d**3*(2*asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**8)/(asinh(c*x)**2*b**2*
c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)
*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**9 + 8*asinh(c*x)*int((sqrt(asinh(
c*x)*b + a)*x**6)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*a
sinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**
2*c**7 + 13*asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**4)/(asinh(c*x)**2*b*
*2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c
*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**5 + 7*asinh(c*x)*int((sqrt(asi
nh(c*x)*b + a)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 +
2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*
b**2*c**3 - 6*asinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*a
sinh(c*x)*x**3)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asi
nh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*
c**4 - 6*asinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**3)/
(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2
*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**4 + 2*sqrt(c**
2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*c**2*x**2 - 4*sqrt(c**2*x**2 + 1)*sqrt(
asinh(c*x)*b + a) + 2*int((sqrt(asinh(c*x)*b + a)*x**8)/(asinh(c*x)**2*b**
2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*
x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**9 + 8*int((sqrt(asinh(c*x)*b ...
```

3.97 $\int \frac{(d+c^2dx^2)^2}{(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 346

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d^2(1+c^2x^2)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} - \frac{d^2e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{5d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} + \frac{5d^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c} + \frac{d^2e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$

output
```
-2*d^2*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-5/8*d^2*exp(a/b)*Pi^
(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-5/16*d^2*exp(3*a/b)*
3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-1
/16*d^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b
^(1/2))/b^(3/2)/c+5/8*d^2*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/
b^(3/2)/c/exp(a/b)+5/16*d^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x
))^(1/2)/b^(1/2))/b^(3/2)/c/exp(3*a/b)+1/16*d^2*5^(1/2)*Pi^(1/2)*erfi(5^(1
/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(5*a/b)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.27

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{d^2e^{-5\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{5a}{b}}-5e^{\frac{5a}{b}+2\text{arcsinh}(cx)}-10e^{\frac{5a}{b}+4\text{arcsinh}(cx)}-10e^{\frac{5a}{b}+6\text{arcsi}}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d^2*(-E^((5*a)/b) - 5*E^((5*a)/b + 2*ArcSinh[c*x]) - 10*E^((5*a)/b + 4*Ar
cSinh[c*x]) - 10*E^((5*a)/b + 6*ArcSinh[c*x]) - 5*E^((5*a)/b + 8*ArcSinh[c
*x]) - E^((5*a)/b + 10*ArcSinh[c*x]) + 10*E^((6*a)/b + 5*ArcSinh[c*x])*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSin
h[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]
))/b] + 5*Sqrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])
/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 10*E^((4*a)/b + 5*ArcSinh[c
*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]
 + 5*Sqrt[3]*E^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1
/2, (3*(a + b*ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c*E
^(5*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \; 6205$$

$$\frac{10cd^2\int \frac{x\left(c^2x^2+1\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{10d^2\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{10d^2\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{10d^2\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx}{b^2c}$$

$$\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{10d^2\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b}}{}\right.\right.}{}$$

$$\frac{2d^2\left(c^2x^2+1\right)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (10*d^2*(-1/
16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (Sqr
t[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b
]])/(16*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^2}{(a + b\ \mathrm{arcsinh}\,(xc))^{\frac{3}{2}}} dx$$

input
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d + c^2 dx^2)^2}{(a + b\mathrm{arcsinh}(cx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d + c^2 dx^2)^2}{(a + b\mathrm{arcsinh}(cx))^{3/2}}\, dx = d^2 \left(\int \frac{2c^2 x^2}{a\sqrt{a + b\,\mathrm{asinh}\,(cx)} + b\sqrt{a + b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\, dx + \int \frac{c^4 x^4}{a\sqrt{a + b\,\mathrm{asinh}\,(cx)} + b\sqrt{a + b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\, dx + \int \frac{1}{a\sqrt{a + b\,\mathrm{asinh}\,(cx)} + b\sqrt{a + b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\, dx\right)$$

input
```
integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d**2*(Integral(2*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(
c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b*s
qrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d\,c^2\,x^2+d)^2}{(a+b\,\operatorname{asinh}(c\,x))^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((c^2*d*x^2+d)^2/(a+b*asinh(c*x))^(3/2),x)
```

output

```
(d**2*(2*asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**6)/(asinh(c*x)**2*b**2*
c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)
*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**7 + 7*asinh(c*x)*int((sqrt(asinh(
c*x)*b + a)*x**4)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*a
sinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**
2*c**5 + 5*asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*x)**2*b**
2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*
x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**3 - 6*asinh(c*x)*int((sqrt(c**2
*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**3)/(asinh(c*x)**2*b**2*c**
2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*
b + a**2*c**2*x**2 + a**2),x)*b**2*c**4 - 6*asinh(c*x)*int((sqrt(c**2*x**2
 + 1)*sqrt(asinh(c*x)*b + a)*x**3)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c
*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x*
*2 + a**2),x)*a*b*c**4 + 2*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*c**2
*x**2 - 4*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a) + 2*int((sqrt(asinh(c
*x)*b + a)*x**6)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*as
inh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*
c**7 + 7*int((sqrt(asinh(c*x)*b + a)*x**4)/(asinh(c*x)**2*b**2*c**2*x**2 +
 asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2
*c**2*x**2 + a**2),x)*a*b*c**5 + 5*int((sqrt(asinh(c*x)*b + a)*x**2)/(a...
```

3.98 $\int \frac{d+c^2dx^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{d+c^2dx^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = -\frac{2d(1+c^2x^2)^{3/2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{3de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{3de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{de^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

output
```
-2*d*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-3/4*d*exp(a/b)*Pi^(1/2
)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-1/4*d*exp(3*a/b)*3^(1/2)
*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+3/4*d*Pi
^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)+1/4*d*3^(
1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp
(3*a/b)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.29

$$\int \frac{d + c^2dx^2}{(a + b\text{arcsinh}(cx))^{3/2}} dx = \frac{de^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-e^{\frac{3a}{b}} - 3e^{\frac{3a}{b}+2\text{arcsinh}(cx)} - 3e^{\frac{3a}{b}+4\text{arcsinh}(cx)} - e^{\frac{3a}{b}+6\text{arcsinh}(cx}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*(-E^((3*a)/b) - 3*E^((3*a)/b + 2*ArcSinh[c*x]) - 3*E^((3*a)/b + 4*ArcSi
nh[c*x]) - E^((3*a)/b + 6*ArcSinh[c*x]) + 3*E^((4*a)/b + 3*ArcSinh[c*x])*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcS
inh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*
x]))/b] + 3*E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x]
)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(4*b*c
*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2dx^2 + d}{(a + b\text{arcsinh}(cx))^{3/2}} dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{6cd\int \frac{x\sqrt{c^2x^2+1}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b} - \frac{2d\left(c^2x^2+1\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{6d\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{6d\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 5971$$

$$-\frac{6d\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 2009$$

$$\frac{6d\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}}}{\sqrt{b}}\right.\right.}{b^2c}$$

$$\frac{2d(c^2x^2+1)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (6*d*(-1/8*(Sq
rt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]) - (Sqrt[b]*E
^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 +
 (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*E^(a/b)) + (
Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^
((3*a)/b))))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{c^2 d\,x^2 + d}{\left(a + b\,\operatorname{arcsinh}\left(xc\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = d\left(\int \frac{c^2x^2}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx + \int \frac{1}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx\right)$$

input
```
integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
d*(Integral(c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))
*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x))
```

Maxima [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{c^2dx^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{c^2dx^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{d\,c^2\,x^2+d}{{\left(a+b\,\text{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{d+c^2dx^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((c^2*d*x^2+d)/(a+b*asinh(c*x))^(3/2),x)
```

output

```
(d*(3*asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**4)/(asinh(c*x)**2*b**2*c**
2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*
b + a**2*c**2*x**2 + a**2),x)*b**2*c**5 + 3*asinh(c*x)*int((sqrt(asinh(c*x
)*b + a)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asin
h(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c
**3 - 6*asinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c
*x)*x**3)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x
)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**4 -
 6*asinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**3)/(asinh
(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2
+ 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**4 + 2*sqrt(c**2*x**2
 + 1)*sqrt(asinh(c*x)*b + a)*c**2*x**2 - 4*sqrt(c**2*x**2 + 1)*sqrt(asinh(
c*x)*b + a) + 3*int((sqrt(asinh(c*x)*b + a)*x**4)/(asinh(c*x)**2*b**2*c**2
*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b
 + a**2*c**2*x**2 + a**2),x)*a*b*c**5 + 3*int((sqrt(asinh(c*x)*b + a)*x**2
)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c*
*2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**3 - 6*int((s
qrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**3)/(asinh(c*x)**2*
b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh
(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**4 - 6*int((sqrt(c**2*x**2 ...
```

3.99 $\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

output
```
-2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-exp(a/b)*Pi^(1/2)*erf((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+Pi^(1/2)*erfi((a+b*arcsinh(c*x))
^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(cx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2}\right.\right.}{bc\sqrt{a+b\text{arc}}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output
```
(-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a
 + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a +
 b*ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}} \, dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{2c\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \ 26$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \ 3789$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \ 2611$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c}$$

$$\Big\downarrow \ 2633$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

$$\Big\downarrow \ 2634$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + ((2*I)*((I/2)*Sqrt
[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(xc\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a + b\,\text{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*asinh(c*x))^(3/2),x)
```

output

```
(2*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x) - asinh(c*x)*int(
(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + a
sinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c
**2*x**2 + a**2),x)*b**2*c**3 - asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*asi
nh(c*x))/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)
*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c - 2*a
sinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*x)/(a
sinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x
**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**2 - 2*asinh(c*x)
*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)**2*x)/(asinh(c
*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 +
2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**2 - int((sqrt(asinh(c
*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*
b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a*
*2),x)*a*b*c**3 - int((sqrt(asinh(c*x)*b + a)*asinh(c*x))/(asinh(c*x)**2*b
**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(
c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c - 2*int((sqrt(c**2*x**2 + 1)*sq
rt(asinh(c*x)*b + a)*asinh(c*x)*x)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c
*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x*
*2 + a**2),x)*a**2*c**2 - 2*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b ...
```

3.100 $\int \frac{1}{(d+c^2dx^2)(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{2c\int \frac{x}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd}-\frac{2}{bcd\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{2c\int \frac{x}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd}-\frac{2}{bcd\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[1/((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c^2d\,x^2+d)\,(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(c^2*d*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.28

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\int \frac{1}{ac^2x^2\sqrt{a+b\,\text{asinh}\,(cx)}+a\sqrt{a+b\,\text{asinh}\,(cx)}+bc^2x^2\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)+b\sqrt{a+}}}{d}$$

input
```
integrate(1/(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/(a*c**2*x**2*sqrt(a + b*asinh(c*x)) + a*sqrt(a + b*asinh(c*x))
+ b*c**2*x**2*sqrt(a + b*asinh(c*x))*asinh(c*x) + b*sqrt(a + b*asinh(c*x))
*asinh(c*x)), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(c^2dx^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\text{asinh}\,(c\,x)\right)^{3/2}\,\left(d\,c^2\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.20

$$\int \frac{1}{(d+c^2dx^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{asinh(cx)b+a}}{asinh(cx)^2b^2c^2x^2+asinh(cx)^2b^2+2asinh(cx)ab\,c^2x^2+2asinh(cx)ab+a^2c^2x^2+a^2}dx}{d}$$

input
```
int(1/(c^2*d*x^2+d)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b
**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**
2),x)/d
```

3.101 $\int \frac{1}{(d+c^2dx^2)^2(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{(d+c^2dx^2)^2\,(a+b\,\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(d+c^2dx^2)^2\,(a+b\,\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^2 (a+b\text{arcsinh}(cx))^{3/2}} dx$$
$$\Big\downarrow \text{ 6205}$$
$$-\frac{6c\int \frac{x}{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd^2}-\frac{2}{bcd^2\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}$$
$$\Big\downarrow \text{ 6239}$$
$$-\frac{6c\int \frac{x}{(c^2x^2+1)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd^2}-\frac{2}{bcd^2\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[1/((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c^{2} d\, x^{2}+d\right)^{2} \left(a+b \,\text{arcsinh}\left(x c\right)\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 15.00 (sec) , antiderivative size = 133, normalized size of antiderivative = 5.32

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\int \frac{1}{ac^4x^4\sqrt{a+b\operatorname{asinh}(cx)}+2ac^2x^2\sqrt{a+b\operatorname{asinh}(cx)}+a\sqrt{a+b\operatorname{asinh}(cx)}+bc^4x^4\sqrt{a+b\,a}}}{}$$

input
```
integrate(1/(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/(a*c**4*x**4*sqrt(a + b*asinh(c*x)) + 2*a*c**2*x**2*sqrt(a + b*
asinh(c*x)) + a*sqrt(a + b*asinh(c*x)) + b*c**4*x**4*sqrt(a + b*asinh(c*x)
)*asinh(c*x) + 2*b*c**2*x**2*sqrt(a + b*asinh(c*x))*asinh(c*x) + b*sqrt(a
+ b*asinh(c*x))*asinh(c*x)), x)/d**2
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(c^2dx^2+d)^2(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^2\,(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(a+b\operatorname{asinh}(c\,x))^{3/2}\,(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.88

$$\int \frac{1}{(d+c^2dx^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{asinh(cx)b+a}}{asinh(cx)^2b^2c^4x^4+2asinh(cx)^2b^2c^2x^2+asinh(cx)^2b^2+2asinh(cx)ab\,c^4x^4+4asinh(c}}{d^2}$$

input
```
int(1/(c^2*d*x^2+d)^2/(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**2*b**2*c**4*x**4 + 2*asinh(c*x)**2
*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**4*x**4 + 4*asin
h(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**4*x**4 + 2*a**2*c**2*x**
2 + a**2),x)/d**2
```

3.102 $\int (d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 632

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{5}{16}d^2x\sqrt{d+c^2dx^2}\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\operatorname{arcsinh}(cx)} + \frac{5d^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{24bc\sqrt{1+c^2x^2}} + \frac{3\sqrt{b}d^2e^{\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{512c\sqrt{1+c^2x^2}} + \frac{15\sqrt{b}d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c\sqrt{1+c^2x^2}} + \frac{\sqrt{b}d^2e^{\frac{6a}{b}}\sqrt{\frac{\pi}{6}}\sqrt{d+c^2dx^2}\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{768c\sqrt{1+c^2x^2}} - \frac{3\sqrt{b}d^2e^{-\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{512c\sqrt{1+c^2x^2}} - \frac{15\sqrt{b}d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c\sqrt{1+c^2x^2}} - \frac{\sqrt{b}d^2e^{-\frac{6a}{b}}\sqrt{\frac{\pi}{6}}\sqrt{d+c^2dx^2}\operatorname{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{768c\sqrt{1+c^2x^2}}$$

output
```
5/16*d^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)+5/24*d*x*(c^2*d*x^
2+d)^(3/2)*(a+b*arcsinh(c*x))^(1/2)+1/6*x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh
(c*x))^(1/2)+5/24*d^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(3/2)/b/c/(c^
2*x^2+1)^(1/2)+3/512*b^(1/2)*d^2*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*e
rf(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/(c^2*x^2+1)^(1/2)+15/512*b^(1/2)*
d^2*exp(2*a/b)*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))/c/(c^2*x^2+1)^(1/2)+1/4608*b^(1/2)*d^2*exp(6*a/b)*
6^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/
b^(1/2))/c/(c^2*x^2+1)^(1/2)-3/512*b^(1/2)*d^2*Pi^(1/2)*(c^2*d*x^2+d)^(1/2
)*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(4*a/b)/(c^2*x^2+1)^(1/2)-
15/512*b^(1/2)*d^2*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*
arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(2*a/b)/(c^2*x^2+1)^(1/2)-1/4608*b^(1/2)
*d^2*6^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(6^(1/2)*(a+b*arcsinh(c*x))^
(1/2)/b^(1/2))/c/exp(6*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.02 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.06

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx =$$

$$-\frac{bd^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}\sqrt{-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}}\left(-\sqrt{6}b^2\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\sqrt{-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}}\Gamma\left(\frac{3}{2},-\frac{6(a+b\text{arcsi}}{b}\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/2304*(b*d^2*Sqrt[d + c^2*d*x^2]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)]*(-(
Sqrt[6]*b^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/
b^2)]*Gamma[3/2, (-6*(a + b*ArcSinh[c*x]))/b]) + 9*b*E^((2*a)/b)*(4*a*Sqrt
[a/b + ArcSinh[c*x]] + 4*b*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2,
 (-4*(a + b*ArcSinh[c*x]))/b] + 9*Sqrt[2]*b*E^((4*a)/b)*(16*a*Sqrt[a/b + A
rcSinh[c*x]] + 16*b*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*Sqrt[-((a +
b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, (-2*(a
 + b*ArcSinh[c*x]))/b] + 9*Sqrt[2]*b*E^((8*a)/b)*(-16*a*Sqrt[-((a + b*ArcS
inh[c*x])/b)] - 16*b*ArcSinh[c*x]*Sqrt[-((a + b*ArcSinh[c*x])/b)] + b*Sqrt
[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, (2*(a
 + b*ArcSinh[c*x]))/b] + 9*b*E^((10*a)/b)*(-4*a*Sqrt[-((a + b*ArcSinh[c*x]
)/b)] - 4*b*ArcSinh[c*x]*Sqrt[-((a + b*ArcSinh[c*x])/b)] + b*Sqrt[a/b + Ar
cSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, (4*(a + b*ArcS
inh[c*x]))/b] + E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)]*(480*(a +
b*ArcSinh[c*x])^2 - Sqrt[6]*b^2*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[3/2, (6*(a + b*ArcSinh[c*x]))/b])))/(c*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x])^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 5.62 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.33, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6201, 6201, 6200, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x\left(c^2x^2+1\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+\frac{5}{6}d\int \left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{6201}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\int \sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+\right.$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{ar}}}{\sqrt{c}}}{2\sqrt{c^2x^2+}}\right.\right.$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{6195}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$

$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a}{}\right.}{}}{}\right.\right.$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b}{}\right.}{}}{}\right.\right.$$
$$\frac{1}{6}x(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\downarrow$ 5971

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(}{}\right.}{2\sqrt{a+b}}}{4}\right.\right.$$
$$\frac{1}{6}x(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\downarrow$ 27

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(}{}\right.}{\sqrt{a+b}}}{8}\right.\right.$$
$$\frac{1}{6}x(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\downarrow$ 3042

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int -\frac{i\sin\left(\frac{2ia}{b}-\right.}{\sqrt{a-}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{i\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i}{}\right.}{\sqrt{a+b}}}{8}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}\right.}{8c\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\right.}{8c\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\downarrow$ 2633

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\downarrow$ 2634

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-}\right.}{8c\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\downarrow$ 6198

$$-\frac{bcd^2\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{12\sqrt{c^2x^2+1}}+$$
$$\frac{5}{6}d\left(-\frac{bcd\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{}\right.}{8c\sqrt{c^2x^2+1}}\right.\right.$$
$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\Big\downarrow$ 6234

$$-\frac{d^2\sqrt{c^2dx^2+d}\int -\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{12c\sqrt{c^2x^2+1}}+$$

$$\frac{5}{6}d\left(-\frac{d\sqrt{c^2dx^2+d}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+\cdots}}{\cdots}\right.\right.$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\Big\downarrow$ 25

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{12c\sqrt{c^2x^2+1}}+$$

$$\frac{5}{6}d\left(\frac{d\sqrt{c^2dx^2+d}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}\cdots\right.}{\cdots}\right.\right.$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\Big\downarrow$ 5971

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\cdots}{12c\sqrt{c^2x^2+1}}$$

$$\frac{5}{6}d\left(\frac{d\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{i\sqrt{\cdots}}{\cdots}\right.\right.$$

$$\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$\Big\downarrow$ 2009

$$\frac{1}{6}x\sqrt{a+b\text{arcsinh}(cx)}\left(c^2dx^2+d\right)^{5/2}-$$

$$\frac{d^2\left(-\frac{1}{32}\sqrt{b}e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{5}{64}\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{b}e^{\frac{6a}{b}}\sqrt{\frac{\pi}{6}}\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{ar}}}{\sqrt{}}\right.\right.}{}$$

$$\frac{5}{6}d\left(\frac{1}{4}x\sqrt{a+b\text{arcsinh}(cx)}\left(c^2dx^2+d\right)^{3/2}-\frac{d\left(-\frac{1}{32}\sqrt{b}e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.\right.}{}\right.$$

input
```
Int[(d + c^2*d*x^2)^(5/2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(x*(d + c^2*d*x^2)^(5/2)*Sqrt[a + b*ArcSinh[c*x]])/6 + (5*d*((x*(d + c^2*d
*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]])/4 - (d*Sqrt[d + c^2*d*x^2]*(-1/32*(S
qrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]) - (
Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(3
2*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(8*E^((2*a)/b))))/(8*c*Sqrt[1 + c^2*x^2]) + (3*d*((x*Sqrt[d +
 c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]])/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^(3/2))/(3*b*c*Sqrt[1 + c^2*x^2]) - ((I/8)*Sqrt[d + c^2*d*x^2]*((
I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/E^((2*a)/b)))/(c*Sqrt[1 + c^2*x^2])))/4))/6 - (d^2*Sqrt[d +
c^2*d*x^2]*(-1/32*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]]) - (5*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*E^((6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt
[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (5*Sqrt[b]*Sqrt[Pi/2]*
Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((2*a)/b)) + (Sqrt
[b]*Sqrt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((6
*a)/b))))/(12*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \sqrt{a +b \,\operatorname{arcsinh}\left(x c \right)}d x$$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}\sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\text{asinh}\,(c\,x)}\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int (d + c^2dx^2)^{5/2} \sqrt{a + b\text{arcsinh}(cx)}\, dx = \sqrt{d}\, d^2 \left(\left(\int \sqrt{c^2x^2 + 1}\, \sqrt{asinh\,(cx)\, b + a}\, x^4 dx \right) c^4 + 2 \left(\int \sqrt{c^2x^2 + 1}\, \sqrt{asinh\,(cx)\, b + a}\, x^2 dx \right) c^2 + \int \sqrt{c^2x^2 + 1}\, \sqrt{asinh\,(cx)\, b + a} dx \right)$$

input

```
int((c^2*d*x^2+d)^(5/2)*(a+b*asinh(c*x))^(1/2),x)
```

output

```
sqrt(d)*d**2*(int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**4,x)*c**4
+ 2*int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**2,x)*c**2 + int(sqrt
(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a),x))
```

3.103 $\int (d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 422

$$\int (d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{3}{8}dx\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{4}x(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)} + \frac{d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}{4bc\sqrt{1+c^2x^2}} + \frac{\sqrt{b}de^{\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c\sqrt{1+c^2x^2}} + \frac{\sqrt{b}de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c\sqrt{1+c^2x^2}} - \frac{\sqrt{b}de^{-\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c\sqrt{1+c^2x^2}} - \frac{\sqrt{b}de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c\sqrt{1+c^2x^2}}$$

output
```
3/8*d*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)+1/4*x*(c^2*d*x^2+d)^(
3/2)*(a+b*arcsinh(c*x))^(1/2)+1/4*d*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))
^(3/2)/b/c/(c^2*x^2+1)^(1/2)+1/256*b^(1/2)*d*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^
2+d)^(1/2)*erf(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/(c^2*x^2+1)^(1/2)+1/3
2*b^(1/2)*d*exp(2*a/b)*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a
+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/(c^2*x^2+1)^(1/2)-1/256*b^(1/2)*d*Pi^(1/
2)*(c^2*d*x^2+d)^(1/2)*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(4*a/
b)/(c^2*x^2+1)^(1/2)-1/32*b^(1/2)*d*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*e
rfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(2*a/b)/(c^2*x^2+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.68

$$\int \left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx =$$

$$-\frac{bde^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\sqrt{-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}}\left(32e^{\frac{4a}{b}}(a+b\text{arcsinh}(cx))\sqrt{-\frac{(a+b\text{arcsinh}(cx))^2}{b^2}}+b\sqrt{\frac{a}{b}+\text{arcsinh}(\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/128*(b*d*Sqrt[d + c^2*d*x^2]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)]*(32*E^
((4*a)/b)*(a + b*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)] + b*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[3/2, (-4*(a + b*ArcSinh[c*x]))/b] + 8*Sqrt[2]*
b*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-2*(a + b*ArcSinh[c*x])
)/b] - 8*Sqrt[2]*b*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2,
(2*(a + b*ArcSinh[c*x]))/b] - b*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)
]*Gamma[3/2, (4*(a + b*ArcSinh[c*x]))/b]))/(c*E^((4*a)/b)*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {6201, 6200, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\int \sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{ 6200}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{bc\sqrt{c^2dx^2+d}\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{ 6195}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+\text{bar}}{4c\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcs}}{4c\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 5971$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2d}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2d}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{i\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{}}{}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c\sqrt{c^2x^2+1}}+\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow 2633$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow 2634$$

$$\frac{3}{4}d\left(\frac{\sqrt{c^2dx^2+d}\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{er}\right.}{8c\sqrt{c^2x^2+1}}\right.$$

$$\frac{bcd\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow 6198$$

$$-\frac{bcd\sqrt{c^2dx^2+d}\int\frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2d}}{}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow 6234$$

$$-\frac{d\sqrt{c^2dx^2+d}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2d\cdots}}{\cdots}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2d\cdots}}{\cdots}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2d\cdots}}{\cdots}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\sqrt{c^2dx^2+d}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{er}\ldots}{8c\sqrt{c^2x^2+1}}$$

$$\frac{3}{4}d\left(-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2d\ldots}}{}\right.$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(x*(d + c^2*d*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]])/4 - (d*Sqrt[d + c^2*d*x
^2]*(-1/32*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]]) - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSin
h[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((2*a)/b))))/(8*c*Sqrt[1 + c^2*x^2]) + (3*d
*((x*Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]])/2 + (Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])^(3/2))/(3*b*c*Sqrt[1 + c^2*x^2]) - ((I/8)*Sqrt[d +
c^2*d*x^2]*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c*x]])/Sqrt[b]])/E^((2*a)/b)))/(c*Sqrt[1 + c^2*x^2])))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (c^2 d\,x^2+d)^{\frac{3}{2}}\sqrt{a+b\,\mathrm{arcsinh}\,(xc)}dx$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}\sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int (d + c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \sqrt{d}\,d\left(\left(\int \sqrt{c^2x^2+1}\,\sqrt{asinh\left(cx\right)b+a}\,x^2dx\right)c^2 + \int \sqrt{c^2x^2+1}\,\sqrt{asinh\left(cx\right)b+a}dx\right)$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x))^(1/2),x)
```

output
```
sqrt(d)*d*(int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**2,x)*c**2 + i
nt(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a),x))
```

3.104 $\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 241

$$\begin{aligned}\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx &= \frac{1}{2}x\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}{3bc\sqrt{1+c^2x^2}}\\ &\quad+\frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c\sqrt{1+c^2x^2}}\\ &\quad-\frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
1/2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)+1/3*(c^2*d*x^2+d)^(1/2)
*(a+b*arcsinh(c*x))^(3/2)/b/c/(c^2*x^2+1)^(1/2)+1/32*b^(1/2)*exp(2*a/b)*2^
(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^
(1/2))/c/(c^2*x^2+1)^(1/2)-1/32*b^(1/2)*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/
2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(2*a/b)/(c^2*x^2+1)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.65

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx$$
$$=\frac{\sqrt{d(1+c^2x^2)}\sqrt{a+b\text{arcsinh}(cx)}\left(\frac{16(a+b\text{arcsinh}(cx))}{b}+\frac{3\sqrt{2}e^{-\frac{2a}{b}}\Gamma\left(\frac{3}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}-\frac{3\sqrt{2}e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{2(a+b\text{arcsin}}{}\right.}{\sqrt{\frac{a}{b}+\text{arcsin}}}\right.}{48c\sqrt{1+c^2x^2}}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[d*(1 + c^2*x^2)]*Sqrt[a + b*ArcSinh[c*x]]*((16*(a + b*ArcSinh[c*x]))
/b + (3*Sqrt[2]*Gamma[3/2, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*Sqrt
[-((a + b*ArcSinh[c*x])/b)]) - (3*Sqrt[2]*E^((2*a)/b)*Gamma[3/2, (2*(a + b
*ArcSinh[c*x]))/b])/Sqrt[a/b + ArcSinh[c*x]]))/(48*c*Sqrt[1 + c^2*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6200, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6200}$$

$$-\frac{bc\sqrt{c^2dx^2+d}\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 6195$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 5971$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 26$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{i\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 3789$$

$$-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 2611$$

$$-\frac{i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \; 2633$$

$$-\frac{i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{c^2dx^2+d}\int\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{8c\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}}{3bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(x*Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]])/2 + (Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^(3/2))/(3*b*c*Sqrt[1 + c^2*x^2]) - ((I/8)*Sqrt[d + c^
2*d*x^2]*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*Arc
Sinh[c*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/E^((2*a)/b)))/(c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [F]

$$\int \sqrt{c^2 d\, x^2 + d}\, \sqrt{a + b\,\mathrm{arcsinh}\,(xc)}dx$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{d\,(c^2x^2+1)}\sqrt{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{c^2dx^2+d}\sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\text{asinh}\left(c\,x\right)}\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\,dx = \sqrt{d}\left(\int \sqrt{c^2x^2+1}\,\sqrt{asinh\left(cx\right)b+a}dx\right)$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a),x)
```

3.105 $\int \frac{\sqrt{a+b\,\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 49

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx=\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{3bc\sqrt{d+c^2dx^2}}$$

output
```
2/3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(3/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx=\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{3bc\sqrt{d\,(1+c^2x^2)}}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[d + c^2*d*x^2],x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(3*b*c*Sqrt[d*(1 + c^2*x^
2)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2dx^2+d}}\,dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{3/2}}{3bc\sqrt{c^2dx^2+d}}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[d + c^2*d*x^2],x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(3*b*c*Sqrt[d + c^2*d*x^2
])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2(a+b\,\mathrm{arcsinh}(xc))^{\frac{3}{2}}\sqrt{c^2x^2+1}}{3b\sqrt{d(c^2x^2+1)}\,c}$	43

input
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a+b*arcsinh(x*c))^(3/2)*(c^2*x^2+1)^(1/2)/b/(d*(c^2*x^2+1))^(1/2)/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{\sqrt{a+b\,\mathrm{asinh}\,(cx)}}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{\sqrt{a+b\,\text{asinh}\,(c\,x)}}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{d+c^2dx^2}}\,dx=\frac{2\sqrt{d}\,\sqrt{asinh\,(cx)\,b+a}\,(asinh(cx)\,b+a)}{3bcd}$$

input
```
int((a+b*asinh(c*x))^(1/2)/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt(asinh(c*x)*b + a)*(asinh(c*x)*b + a))/(3*b*c*d)
```

3.106 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 9.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(c^2dx^2+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)\sqrt{a+b\text{arcsinh}(cx)}}dx}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)\sqrt{a+b\text{arcsinh}(cx)}}dx}{2d\sqrt{c^2dx^2+d}}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(xc)}}{(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + c^2dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a + b\,\text{asinh}\,(cx)}}{(d\,(c^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\frac{\sqrt{d}\left(2\sqrt{c^{2}x^{2}+1}\,\sqrt{asinh\left(cx\right)b+a}\,x-\left(\int \frac{\sqrt{asinh(cx)b+a}\,x}{asinh(cx)b\,c^{2}x^{2}+asinh(cx)b+a\,c^{2}x^{2}+a}dx\right)b$$
$$\overline{\qquad 2d^{2}\left(c^{2}x^{2}+1\right)\qquad}$$

input
```
int((a+b*asinh(c*x))^(1/2)/(c^2*d*x^2+d)^(3/2),x)
```

output
```
(sqrt(d)*(2*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x - int((sqrt(asinh
(c*x)*b + a)*x)/(asinh(c*x)*b*c**2*x**2 + asinh(c*x)*b + a*c**2*x**2 + a),
x)*b*c**3*x**2 - int((sqrt(asinh(c*x)*b + a)*x)/(asinh(c*x)*b*c**2*x**2 +
asinh(c*x)*b + a*c**2*x**2 + a),x)*b*c))/(2*d**2*(c**2*x**2 + 1))
```

3.107 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 10.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{6d^2\sqrt{c^2dx^2+d}}+\frac{2\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(c^2dx^2+d)^{3/2}}dx}{3d}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{6d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)\sqrt{a+b\text{arcsinh}(cx)}}dx}{2d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{6d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)\sqrt{a+b\text{arcsinh}(cx)}}dx}{2d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{3d\,(c^2dx^2+d)^{3/2}}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + c^2*d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\,(xc)}}{(c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/(c^2*d*x^2+d)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 22.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+b\,\mathrm{asinh}\,(cx)}}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + c^2*d*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 518, normalized size of antiderivative = 19.19

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d}\left(4\sqrt{c^2x^2+1}\,\sqrt{asinh\left(cx\right)b+a}\,c^2x^3+6\sqrt{c^2x^2+1}\,\sqrt{asinh\left(cx\right)b+a}\,x-\right.}{}$$

input
```
int((a+b*asinh(c*x))^(1/2)/(c^2*d*x^2+d)^(5/2),x)
```

output
```
(sqrt(d)*(4*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*c**2*x**3 + 6*sqrt(
c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x - 2*int((sqrt(asinh(c*x)*b + a)*x*
*3)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asinh(c*x)*b + a*
c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**7*x**4 - 4*int((sqrt(asinh(c*x)*b +
 a)*x**3)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asinh(c*x)*
b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**5*x**2 - 2*int((sqrt(asinh(c*
x)*b + a)*x**3)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asinh
(c*x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**3 - 3*int((sqrt(asinh(c
*x)*b + a)*x)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asinh(c
*x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**5*x**4 - 6*int((sqrt(asin
h(c*x)*b + a)*x)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asin
h(c*x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**3*x**2 - 3*int((sqrt(a
sinh(c*x)*b + a)*x)/(asinh(c*x)*b*c**4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + a
sinh(c*x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c))/(6*d**3*(c**4*x**4
 + 2*c**2*x**2 + 1))
```

3.108 $\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^{3/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 589

$$\int (d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^{3/2} dx = -\frac{27bd\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{256c\sqrt{1+c^2x^2}}$$
$$-\frac{9bcdx^2\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{32\sqrt{1+c^2x^2}} + \frac{3}{8}dx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}$$
$$+\frac{1}{4}x(d+c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))^{3/2} + \frac{3d\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{5/2}}{20bc\sqrt{1+c^2x^2}} - \frac{3bd\sqrt{d+c^2dx^2}\sqrt{a+\text{barcsi}}}{32c}$$

output
```
-27/256*b*d*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c/(c^2*x^2+1)^(1/
2)-9/32*b*c*d*x^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/(c^2*x^2+1)
^(1/2)+3/8*d*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(3/2)+1/4*x*(c^2*d*x
^2+d)^(3/2)*(a+b*arcsinh(c*x))^(3/2)+3/20*d*(c^2*d*x^2+d)^(1/2)*(a+b*arcsi
nh(c*x))^(5/2)/b/c/(c^2*x^2+1)^(1/2)-3/32*b*d*(c^2*d*x^2+d)^(1/2)*(a+b*arc
sinh(c*x))^(1/2)*cosh(a/b-(a+b*arcsinh(c*x))/b)^4/c/(c^2*x^2+1)^(1/2)+3/20
48*b^(3/2)*d*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2*(a+b*arcsinh(c*
x))^(1/2)/b^(1/2))/c/(c^2*x^2+1)^(1/2)+3/128*b^(3/2)*d*exp(2*a/b)*2^(1/2)*
Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
/c/(c^2*x^2+1)^(1/2)+3/2048*b^(3/2)*d*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2*
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(4*a/b)/(c^2*x^2+1)^(1/2)+3/128*b^(
3/2)*d*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))/c/exp(2*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.07

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]]*((7680*a*(a + b*ArcSinh[c*
x]))/b + (2048*(a + b*ArcSinh[c*x])*(-2*a + 3*b*ArcSinh[c*x]))/b - (512*(-
2*a^2 + a*b*ArcSinh[c*x] + 3*b^2*ArcSinh[c*x]^2))/b + (1920*Sqrt[2]*a*Gamm
a[3/2, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*
x])/b)]) - (1920*Sqrt[2]*a*E^((2*a)/b)*Gamma[3/2, (2*(a + b*ArcSinh[c*x]))
/b])/Sqrt[a/b + ArcSinh[c*x]] + (240*a*(Gamma[3/2, (-4*(a + b*ArcSinh[c*x]
))/b]/Sqrt[-((a + b*ArcSinh[c*x])/b)] - (E^((8*a)/b)*Gamma[3/2, (4*(a + b*
ArcSinh[c*x]))/b])/Sqrt[a/b + ArcSinh[c*x]]))/E^((4*a)/b) + (240*Sqrt[b]*(
(4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(C
osh[(2*a)/b] - Sinh[(2*a)/b]) + (-4*a + 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*
Sqrt[a + b*ArcSinh[c*x]]*(-3*Cosh[2*ArcSinh[c*x]] + 4*ArcSinh[c*x]*Sinh[2*
ArcSinh[c*x]])))/Sqrt[a + b*ArcSinh[c*x]] + (15*Sqrt[b]*((8*a + 3*b)*Sqrt[
Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(4*a)/b] - Sinh[(4*a)
/b]) + (-8*a + 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Co
sh[(4*a)/b] + Sinh[(4*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Cosh
[4*ArcSinh[c*x]] + 8*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]])))/Sqrt[a + b*ArcSi
nh[c*x]]))/(30720*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 4.29 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6201, 6200, 6192, 6198, 6213, 6206, 3042, 3793, 2009, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}dx$$

$$\Big\downarrow \text{ 6201}$$

$$-\frac{3bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}dx+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 6200}$$

$$-\frac{3bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\int x\sqrt{a+b\text{arcsinh}(cx)}dx}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arc}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 6192}$$

$$-\frac{3bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}dx}{8\sqrt{c^2x^2+1}}+\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int \frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}}{\sqrt{c^2x^2+}}}{2\sqrt{c^2x^2+1}}\right.$$
$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int \frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}}{5bc\sqrt{c^2x^2+1}}\right.$$
$$\frac{3bcd\sqrt{c^2dx^2+d}\int x\left(c^2x^2+1\right)\sqrt{a+b\text{arcsinh}(cx)}dx}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(}{5bc\sqrt{c^2x^2+1}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{b\int\frac{(c^2x^2+1)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{8c}\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 6206

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(}{5bc\sqrt{c^2x^2+1}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\int\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c^2}\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(}{5bc\sqrt{c^2x^2+1}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{8c^2}\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3793

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(\ldots}{5bc\sqrt{c^2x^2+1}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3}{8\sqrt{a+b\text{arcsi}\ldots}}\right.}{8c^2}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\big\downarrow$ 2009

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int\frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(\ldots}{5bc\sqrt{c^2x^2+1}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\ldots}{\ldots}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\big\downarrow$ 6234

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int\frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+}{5bc\sqrt{c}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int-\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(}{5bc}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 25

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a}{5bc\sqrt{}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 3793

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{\int\left(\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}\right.}{4\sqrt{c^2x^2+1}}\right.$$

$$\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

$\downarrow$ 2009

$$-\frac{3bcd\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}-\frac{\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right.}{}\right.}{8\sqrt{c^2x^2+1}}$$

$$\frac{3}{4}d\left(-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.}{4c^2}\right.}{4\sqrt{c^2x^2+1}}\right.$$

$$\frac{1}{4}x(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^(3/2))/4 - (3*b*c*d*Sqrt[d +
 c^2*d*x^2]*(((1 + c^2*x^2)^2*Sqrt[a + b*ArcSinh[c*x]])/(4*c^2) - ((3*Sqrt
[a + b*ArcSinh[c*x]])/4 + (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/(8*c^2)))/(8*
Sqrt[1 + c^2*x^2]) + (3*d*((x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(3/
2))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(5/2))/(5*b*c*Sqrt[1 + c
^2*x^2]) - (3*b*c*Sqrt[d + c^2*d*x^2]*((x^2*Sqrt[a + b*ArcSinh[c*x]])/2 -
(-Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/(4*c^2)))/(4*Sqrt[1 +
 c^2*x^2])))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}d x$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c\,x))^{3/2}\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\sqrt{d}\,d\left(\left(\int \sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,asinh(cx)\,x^2dx\right)b\,c^2+\left(\int \sqrt{c^2x^2+1}\,\sqrt{as}\right.\right.$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x))^(3/2),x)
```

output
```
sqrt(d)*d*(int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2,
x)*b*c**2 + int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x),x)*b
 + int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**2,x)*a*c**2 + int(sqr
t(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a),x)*a)
```

3.109 $\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 336

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx = -\frac{3b\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{16c\sqrt{1+c^2x^2}} - \frac{3bcx^2\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{8\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2} + \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{1+c^2x^2}} + \frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c\sqrt{1+c^2x^2}} + \frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c\sqrt{1+c^2x^2}}$$

output
```
-3/16*b*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c/(c^2*x^2+1)^(1/2)-3
/8*b*c*x^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/(c^2*x^2+1)^(1/2)+
1/2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(3/2)+1/5*(c^2*d*x^2+d)^(1/2)
*(a+b*arcsinh(c*x))^(5/2)/b/c/(c^2*x^2+1)^(1/2)+3/128*b^(3/2)*exp(2*a/b)*2
^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b
^(1/2))/c/(c^2*x^2+1)^(1/2)+3/128*b^(3/2)*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(
1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(2*a/b)/(c^2*x^2+
1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.06

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx = \frac{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}\left(\frac{128(a+b\text{arcsinh}(cx))(-2a+3b\text{arcsinh}(cx))}{b}+40a\left(\frac{16(a+b\ldots}{}\right.\right.}{}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]]*((128*(a + b*ArcSinh[c*x])*(
-2*a + 3*b*ArcSinh[c*x]))/b + 40*a*((16*(a + b*ArcSinh[c*x]))/b + (3*Sqrt[
2]*Gamma[3/2, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*Sqrt[-((a + b*Arc
Sinh[c*x])/b)]) - (3*Sqrt[2]*E^((2*a)/b)*Gamma[3/2, (2*(a + b*ArcSinh[c*x]
))/b])/Sqrt[a/b + ArcSinh[c*x]]) + (15*Sqrt[b]*((4*a + 3*b)*Sqrt[2*Pi]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/
b]) + (-4*a + 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(
-3*Cosh[2*ArcSinh[c*x]] + 4*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])))/Sqrt[a +
b*ArcSinh[c*x]]))/(1920*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6200, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6200}$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\int x\sqrt{a+b\text{arcsinh}(cx)}dx}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6192}$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int \frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2x^2+1}}dx}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}bc\int \frac{x^2}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{6234}$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int \frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{4c^2}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \ 3793$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{\int \left(\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}\right)}{4\sqrt{c^2x^2+1}}+\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{3bc\sqrt{c^2dx^2+d}\left(\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right.}{4c^2}\right.}{4\sqrt{c^2x^2+1}}$$

$$\frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}$$

input
```
Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(3/2))/2 + (Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])^(5/2))/(5*b*c*Sqrt[1 + c^2*x^2]) - (3*b*c*Sqrt[d +
c^2*d*x^2]*((x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (-Sqrt[a + b*ArcSinh[c*x]]
+ (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/S
qrt[b]])/4 + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/S
qrt[b]])/(4*E^((2*a)/b)))/(4*c^2)))/(4*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{c^2 d\, x^2 + d}\, (a + b\, \mathrm{arcsinh}\,(xc))^{\frac{3}{2}}\, dx$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{d + c^2 dx^2}(a + b\mathrm{arcsinh}(cx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int \sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c\,x))^{3/2}\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}\,dx=\sqrt{d}\left(\left(\left(\int \sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,asinh(cx)\,dx\right)b+\left(\int \sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}dx\right)a\right)\right.$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x))^(3/2),x)
```

output
```
sqrt(d)*(int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x),x)*b +
int(sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a),x)*a)
```

3.110 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 49

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx=\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{d+c^2dx^2}}$$

output
```
2/5*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(5/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx=\frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{d(1+c^2x^2)}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(5/2))/(5*b*c*Sqrt[d*(1 + c^2*x^
2)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^{5/2}}{5bc\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/Sqrt[d + c^2*d*x^2],x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(5/2))/(5*b*c*Sqrt[d + c^2*d*x^2
])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2(a+b\,\operatorname{arcsinh}(xc))^{\frac{5}{2}}\sqrt{c^2x^2+1}}{5b\sqrt{d(c^2x^2+1)}\,c}$	43

input
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a+b*arcsinh(x*c))^(5/2)*(c^2*x^2+1)^(1/2)/b/(d*(c^2*x^2+1))^(1/2)/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/sqrt(c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{d+c^2dx^2}}\,dx = \frac{2\sqrt{d}\,\sqrt{asinh\,(cx)\,b+a}\,\left(asinh(cx)^2\,b^2+2asinh(cx)\,ab+a^2\right)}{5bcd}$$

input
```
int((a+b*asinh(c*x))^(3/2)/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt(asinh(c*x)*b + a)*(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b +
 a**2))/(5*b*c*d)
```

3.111 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(c^2dx^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{d\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{d\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}{(c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + c^2dx^2)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 12.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + c^2dx^2)^{3/2}}\, dx = \int \frac{(a + b\operatorname{asinh}(cx))^{\frac{3}{2}}}{(d(c^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/(d*(c**2*x**2 + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^{3/2}}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.81

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\left(2\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,asinh(cx)\,bx+2\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)}\cdots\right.}{2d^2\,(c^2x^2+\cdots}$$

input
```
int((a+b*asinh(c*x))^(3/2)/(c^2*d*x^2+d)^(3/2),x)
```

output
```
(sqrt(d)*(2*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*b*x + 2*
sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*a*x - 3*int((sqrt(asinh(c*x)*b
+ a)*x)/(c**2*x**2 + 1),x)*b*c**3*x**2 - 3*int((sqrt(asinh(c*x)*b + a)*x)/
(c**2*x**2 + 1),x)*b*c))/(2*d**2*(c**2*x**2 + 1))
```

3.112 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 14.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(c^2dx^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{bc\sqrt{c^2x^2+1}\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{(c^2x^2+1)^2}dx}{2d^2\sqrt{c^2dx^2+d}}+\frac{2\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(c^2dx^2+d)^{3/2}}dx}{3d}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{bc\sqrt{c^2x^2+1}\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{(c^2x^2+1)^2}dx}{2d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{d\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{bc\sqrt{c^2x^2+1}\left(\frac{b\int \frac{1}{\left(c^2x^2+1\right)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4c}-\frac{\sqrt{a+b\text{arcsinh}(cx)}}{2c^2(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{d\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{3d\left(c^2dx^2+d\right)^{3/2}}$$

$$\Big\downarrow \text{6209}$$

$$-\frac{bc\sqrt{c^2x^2+1}\left(\frac{b\int\frac{1}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4c}-\frac{\sqrt{a+b\text{arcsinh}(cx)}}{2c^2(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{d\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{3d\,(c^2dx^2+d)^{3/2}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{bc\sqrt{c^2x^2+1}\left(\frac{b\int\frac{1}{(c^2x^2+1)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx}{4c}-\frac{\sqrt{a+b\text{arcsinh}(cx)}}{2c^2(c^2x^2+1)}\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{2\left(\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{d\sqrt{c^2dx^2+d}}-\frac{3bc\sqrt{c^2x^2+1}\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{c^2x^2+1}dx}{2d\sqrt{c^2dx^2+d}}\right)}{3d}+\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{3d\,(c^2dx^2+d)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + c^2*d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\ \text{arcsinh}\,(xc))^{\frac{3}{2}}}{(c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/(c^2*d*x^2+d)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 78.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}{(d\,(c^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/(d*(c**2*x**2 + 1))**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^{\frac{3}{2}}}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(c^2*d*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{(d + c^2dx^2)^{5/2}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^{3/2}}{(d\,c^2\,x^2 + d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + c^2*d*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 381, normalized size of antiderivative = 14.11

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d}\left(4\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,asinh(cx)\,b\,c^2x^3+6\sqrt{c^2x^2+1}\,\sqrt{asinh\,(\right.}{}$$

input
```
int((a+b*asinh(c*x))^(3/2)/(c^2*d*x^2+d)^(5/2),x)
```

output
```
(sqrt(d)*(4*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*b*c**2*x
**3 + 6*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*b*x + 4*sqrt
(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*a*c**2*x**3 + 6*sqrt(c**2*x**2 + 1)
*sqrt(asinh(c*x)*b + a)*a*x - 6*int((sqrt(asinh(c*x)*b + a)*x**3)/(c**4*x*
*4 + 2*c**2*x**2 + 1),x)*b*c**7*x**4 - 12*int((sqrt(asinh(c*x)*b + a)*x**3
)/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b*c**5*x**2 - 6*int((sqrt(asinh(c*x)*b
+ a)*x**3)/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b*c**3 - 9*int((sqrt(asinh(c*x
)*b + a)*x)/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b*c**5*x**4 - 18*int((sqrt(as
inh(c*x)*b + a)*x)/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b*c**3*x**2 - 9*int((s
qrt(asinh(c*x)*b + a)*x)/(c**4*x**4 + 2*c**2*x**2 + 1),x)*b*c))/(6*d**3*(c
**4*x**4 + 2*c**2*x**2 + 1))
```

3.113 $\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 514

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} + \frac{15}{256}cx(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\operatorname{arcsinh}(ax)} - \frac{45c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{256a\sqrt{1+a^2x^2}} - \frac{15acx^2\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32\sqrt{1+a^2x^2}} - \frac{5c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2} + \frac{1}{4}x(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2} + \frac{3c\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{7/2}}{28a\sqrt{1+a^2x^2}} + \frac{15c\sqrt{\pi}\sqrt{}}{}$$

output
```
225/512*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)+15/256*c*x*(a^2*x^2+1)*
(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)-45/256*c*(a^2*c*x^2+c)^(1/2)*arcsin
h(a*x)^(3/2)/a/(a^2*x^2+1)^(1/2)-15/32*a*c*x^2*(a^2*c*x^2+c)^(1/2)*arcsinh
(a*x)^(3/2)/(a^2*x^2+1)^(1/2)-5/32*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)
*arcsinh(a*x)^(3/2)/a+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(5/2)+1/4*x
*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2)+3/28*c*(a^2*c*x^2+c)^(1/2)*arcsinh
(a*x)^(7/2)/a/(a^2*x^2+1)^(1/2)+15/16384*c*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*er
f(2*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)+15/512*c*2^(1/2)*Pi^(1/2)*(a^2
*c*x^2+c)^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)-15/163
84*c*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)
^(1/2)-15/512*c*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*erfi(2^(1/2)*arcsinh(
a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.39

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(1536\operatorname{arcsinh}(ax)^4 - 4480\operatorname{arcsinh}(ax)^2\cosh(2\operatorname{arcsinh}(ax)) + \cdots$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(1536*ArcSinh[a*x]^4 - 4480*ArcSinh[a*x]^2*Cosh[2*A
rcSinh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[
a*x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]
] - 7*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 7*Sqrt[ArcSinh[a*x
]]*Gamma[7/2, 4*ArcSinh[a*x]] + 3360*ArcSinh[a*x]*Sinh[2*ArcSinh[a*x]] + 3
584*ArcSinh[a*x]^3*Sinh[2*ArcSinh[a*x]]))/(14336*a*Sqrt[1 + a^2*x^2]*Sqrt[
ArcSinh[a*x]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{6201}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\text{arcsinh}(ax)^{5/2}dx+\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^{3/2}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6192}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcs}\right.$$
$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$
$$\frac{5ac\sqrt{a^2cx^2+c}\int x\left(a^2x^2+1\right)\text{arcsinh}(ax)^{3/2}dx}{8\sqrt{a^2x^2+1}}+\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6213}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\int(a^2x^2+1)^{3/2}\sqrt{\text{arcsinh}(ax)}dx}{8a}\right)}{8\sqrt{a^2x^2+1}}+$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6201}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\int\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}dx+\frac{1}{4}x(a^2x^2+1)^{3/2}\sqrt{\text{arcsi}\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{1}{4}a\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{1}{2}x\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{6195}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a}+\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(}{\sqrt{a^2x^2+1}}\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 5971

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsin}}{4a}\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 27

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsin}}{8a}\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcs}}{8a}\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 26

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsi}}{8a}\right.\right.}{8a}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3789

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arc}\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2611

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsi}\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2633

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\right.}{8a}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2634

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx+\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsi}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6198

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right)}{}\right)}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6227}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right)}{}\right)}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)\right)}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 6195}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\ldots}\right.\right.}{\ldots}\right.\right.}{\ldots}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arc}\ldots}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\ldots}\right.\right.}{\ldots}\right.\right.}{\ldots}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\ldots}}{2\ldots}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\ldots}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\ldots}{2\ldots}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\ldots}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+\ldots}}{}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \; 26$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right.}{}}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{a}}}{2a}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 3789

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right.}{}}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a^3}-\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 2611

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\int\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\int\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{3}{4}c\left(-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\right.\right.}{8a^3}\right.\right.}{4\sqrt{a^2x^2+1}}\right.$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}$$

$\downarrow$ 6198

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{1}{8}a\int\frac{x(a^2x^2+1)}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\right.\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}+$$

$$\frac{3}{4}c\left(\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}\right.}{}\right.\right.}{}\right.$$

$\downarrow$ 6234

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{\int \frac{ax\left(a^2x^2+1\right)^{3/2}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a}+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\right.}{}\right.\right.}{}\right.}{8\sqrt{a^2x^2+1}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}+$$

$$\frac{3}{4}c\left(\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5971}$$

$$-\frac{5ac\sqrt{a^2cx^2+c}\left(\frac{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}{4a^2}-\frac{3\left(-\frac{\int \left(\frac{\sinh(2\text{arcsinh}(ax))}{4\sqrt{\text{arcsinh}(ax)}}+\frac{\sinh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{8a}+\frac{3}{4}\left(\frac{1}{2}x\sqrt{a^2x^2+1}\sqrt{}\right.\right.}{}\right.}{8\sqrt{a^2}}$$

$$\frac{1}{4}x\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}+$$

$$\frac{3}{4}c\left(\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}\right.}{}\right.\right.}{}\right.$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(x a \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(x*a)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2} \operatorname{arcsinh}(ax)^{5/2}\,dx = \int \operatorname{asinh}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int (c+a^2cx^2)^{3/2} \operatorname{arcsinh}(ax)^{5/2} \, dx = \sqrt{c}\, c\left(\left(\int \sqrt{a^2x^2+1}\,\sqrt{asinh\,(ax)}\,asinh(ax)^2\,x^2dx\right)a^2+\int \sqrt{a^2x^2+1}\,\sqrt{a$$

input
```
int((a^2*c*x^2+c)^(3/2)*asinh(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*asinh(a*x)**2*x**2,x)*
a**2 + int(sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*asinh(a*x)**2,x))
```

3.114 $\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 298

$$\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx = \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{7/2}}{7a\sqrt{1+a^2x^2}} + \frac{15\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a\sqrt{1+a^2x^2}} - 15$$

output

```
15/32*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)-5/16*(a^2*c*x^2+c)^(1/2)*ar
csinh(a*x)^(3/2)/a/(a^2*x^2+1)^(1/2)-5/8*a*x^2*(a^2*c*x^2+c)^(1/2)*arcsinh
(a*x)^(3/2)/(a^2*x^2+1)^(1/2)+1/2*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(5/2)
+1/7*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(7/2)/a/(a^2*x^2+1)^(1/2)+15/512*2^(
1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a/(a^2*x
^2+1)^(1/2)-15/512*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*erfi(2^(1/2)*arcsi
nh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.45

$$\int \sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}\,dx = \frac{\sqrt{c\,(1+a^2x^2)}\left(105\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)-105\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arc}\right.\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 1
05*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(64*
ArcSinh[a*x]^3 - 140*ArcSinh[a*x]*Cosh[2*ArcSinh[a*x]] + 7*(15 + 16*ArcSin
h[a*x]^2)*Sinh[2*ArcSinh[a*x]])))/(3584*a*Sqrt[1 + a^2*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {6200, 6192, 6198, 6227, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 6200$$

$$-\frac{5a\sqrt{a^2cx^2+c}\int x\text{arcsinh}(ax)^{3/2}dx}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int \frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 6192$$

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\int\frac{\text{arcsinh}(ax)^{5/2}}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 6198

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\int\frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 6227

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}dx}{4a}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{2a^2}\right)\right)}{4\sqrt{a^2x^2+1}}+\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 6195

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{\int\frac{ax\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 5971

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{4a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcs}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 27

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sinh(2\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcs}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 3042

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int-\frac{i\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{ar}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 26

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\int\frac{\sin(2i\text{arcsinh}(ax))}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{8a^3}-\frac{\int\frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{x\sqrt{a^2x^2+1}\sqrt{\text{arcsi}}}{2a^2}\right.\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 3789

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{8a^3}-\frac{\int\sqrt{\text{a}}}{}\right.\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$\downarrow$ 2611

$$-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(i\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}-i\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}-\frac{\int\sqrt{\text{ar}}}{\sqrt{}}\right.\right.}{4\sqrt{a^2x^2+1}}$$
$$\frac{\text{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\text{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\operatorname{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-i\int e^{-2\operatorname{arcsinh}(ax)}d\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3}-\frac{\int\frac{\sqrt{\operatorname{arc}}}{\sqrt{}}}{}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\operatorname{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(-\frac{\int\frac{\sqrt{\operatorname{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}dx}{2a^2}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcs}}\right.\right.}{8a^3}\right.\right.}{4\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\operatorname{arcsinh}(ax)^{7/2}\sqrt{a^2cx^2+c}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5a\sqrt{a^2cx^2+c}\left(\frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2}-\frac{3}{4}a\left(\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)\right)}{8a^3}-\frac{\operatorname{arcsinh}(a}{3a^3}\right.\right.}{4\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSin
h[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*x^2]) - (5*a*Sqrt[c + a^2*c*x^2]*((x^2*Arc
Sinh[a*x]^(3/2))/2 - (3*a*((x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a^2
) - ArcSinh[a*x]^(3/2)/(3*a^3) + ((I/8)*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]]))/a
^3))/4))/(4*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int \sqrt{a^2 c\,x^2+c}\,\operatorname{arcsinh}(xa)^{\frac{5}{2}}\,dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arcsinh(x*a)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx = \int \sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\,\text{arcsinh}(ax)^{5/2}\,dx = \int \text{asinh}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,\sqrt{asinh\,(ax)}\,asinh(ax)^2\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*asinh(a*x)^(5/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*asinh(a*x)**2,x)
```

3.115 $\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

output
```
2/7*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(7/2)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{c\left(1+a^2x^2\right)}}$$

input
```
Integrate[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c*(1 + a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \ 6198$$

$$\frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\operatorname{arcsinh}(xa)^{\frac{7}{2}}\sqrt{a^2x^2+1}}{7\sqrt{c(a^2x^2+1)}\,a}$	36

input
```
int(arcsinh(x*a)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*arcsinh(x*a)^(7/2)/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{\sqrt{c + a^2cx^2}}\, dx = \frac{2\sqrt{c}\,\sqrt{asinh\,(ax)}\,asinh(ax)^3}{7ac}$$

input
```
int(asinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(asinh(a*x))*asinh(a*x)**3)/(7*a*c)
```

3.116 $\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}} - \frac{5a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}} - \frac{5a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\text{arcsinh}\left(xa\right)^{\frac{5}{2}}}{\left(a^{2} c \,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(x*a)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arcsinh(x*a)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{asinh\,(ax)}\,asinh(ax)^2\,x-5\left(\int\frac{\sqrt{asinh(ax)}\,asinh(ax)x}{a^2x^2+1}dx\right)a^3x^2-5\left(\right.}{2c^2\,(a^2x^2+1)}$$

input
```
int(asinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*asinh(a*x)**2*x - 5*int((
sqrt(asinh(a*x))*asinh(a*x)*x)/(a**2*x**2 + 1),x)*a**3*x**2 - 5*int((sqrt(
asinh(a*x))*asinh(a*x)*x)/(a**2*x**2 + 1),x)*a))/(2*c**2*(a**2*x**2 + 1))
```

3.117 $\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\text{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$-\frac{5a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^{3/2}}{(a^2x^2+1)^2}dx}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\int \frac{\text{arcsinh}(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\text{arcsinh}(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{5a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^{3/2}}{(a^2x^2+1)^2}dx}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6213}$$

$$-\frac{5a\sqrt{a^2x^2+1}\left(\frac{3\int \frac{\sqrt{\text{arcsinh}(ax)}}{(a^2x^2+1)^{3/2}}dx}{4a}-\frac{\text{arcsinh}(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\int \frac{x\text{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{5a\sqrt{a^2x^2+1}\left(\frac{3\left(\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}a\int\frac{x}{(a^2x^2+1)\sqrt{\text{arcsinh}(ax)}}dx\right)}{4a}-\frac{\text{arcsinh}(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\int\frac{x\text{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{5a\sqrt{a^2x^2+1}\left(\frac{3\left(\frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}a\int\frac{x}{(a^2x^2+1)\sqrt{\text{arcsinh}(ax)}}dx\right)}{4a}-\frac{\text{arcsinh}(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{6c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{x\text{arcsinh}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\int\frac{x\text{arcsinh}(ax)^{3/2}}{a^2x^2+1}dx}{2c\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\text{arcsinh}(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(xa)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arcsinh(x*a)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arcsinh(x*a)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 318, normalized size of antiderivative = 13.83

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}\,asinh(ax)^2\,a^2x^3+6\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}\,asinh(ax\right.}{}$$

input
```
int(asinh(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*asinh(a*x)**2*a**2*x**3 +
 6*sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*asinh(a*x)**2*x - 10*int((sqrt(asi
nh(a*x))*asinh(a*x)*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a**7*x**4 - 20*
int((sqrt(asinh(a*x))*asinh(a*x)*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a*
*5*x**2 - 10*int((sqrt(asinh(a*x))*asinh(a*x)*x**3)/(a**4*x**4 + 2*a**2*x*
*2 + 1),x)*a**3 - 15*int((sqrt(asinh(a*x))*asinh(a*x)*x)/(a**4*x**4 + 2*a*
*2*x**2 + 1),x)*a**5*x**4 - 30*int((sqrt(asinh(a*x))*asinh(a*x)*x)/(a**4*x
**4 + 2*a**2*x**2 + 1),x)*a**3*x**2 - 15*int((sqrt(asinh(a*x))*asinh(a*x)*
x)/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a))/(6*c**3*(a**4*x**4 + 2*a**2*x**2 +
 1))
```

3.118 $\int (a^2+x^2)^{3/2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 309

$$\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}} + \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\pi}\sqrt{a^2+x^2}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

output
```
3/8*a^2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)+1/4*x*(a^2+x^2)^(3/2)*arcsinh
(x/a)^(1/2)+1/4*a^3*(a^2+x^2)^(1/2)*arcsinh(x/a)^(3/2)/(1+x^2/a^2)^(1/2)+1
/256*a^3*Pi^(1/2)*(a^2+x^2)^(1/2)*erf(2*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1
/2)+1/32*a^3*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)*erf(2^(1/2)*arcsinh(x/a)^(1/
2))/(1+x^2/a^2)^(1/2)-1/256*a^3*Pi^(1/2)*(a^2+x^2)^(1/2)*erfi(2*arcsinh(x/
a)^(1/2))/(1+x^2/a^2)^(1/2)-1/32*a^3*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)*erfi
(2^(1/2)*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.50

$$\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{a^3\sqrt{a^2+x^2}\left(-\sqrt{-\operatorname{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2},-4\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)-8\sqrt{2}\sqrt{-\operatorname{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2}\right.\right.}{12}$$

input
```
Integrate[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

output
```
(a^3*Sqrt[a^2 + x^2]*(-(Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -4*ArcSinh[x/a]]) -
 8*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -2*ArcSinh[x/a]] + Sqrt[ArcSinh[
x/a]]*(32*ArcSinh[x/a]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[x/a]] - Gamm
a[3/2, 4*ArcSinh[x/a]])))/(128*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6201, 27, 6200, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$$

$$\Big\downarrow \; 6201$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx-\frac{a\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{a^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{6200}$$

$$\frac{3}{4}a^2\left(-\frac{\sqrt{a^2+x^2}\int \frac{x}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{6195}$$

$$\frac{3}{4}a^2\left(-\frac{a\sqrt{a^2+x^2}\int \frac{x\sqrt{\frac{x^2}{a^2}+1}}{a\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}(\frac{x}{a})\right)}{2\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x(a^2+x^2)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}(\frac{x}{a})\right)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int -\frac{i\sin\left(2i\text{arcsinh}(\frac{x}{a})\right)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$
$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}(\frac{x}{a})}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\int \frac{\sin\left(2i\text{arcsinh}(\frac{x}{a})\right)}{\sqrt{\text{arcsinh}(\frac{x}{a})}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)$$
$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 3789$$

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\int\frac{e^{2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)-\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}d}{2\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2611}$$

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(i\int e^{2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}d}{2\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}d}{2\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2634}$$

$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{\sqrt{a^2+x^2}\int\frac{x(a^2+x^2)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 6198$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 6234$$

$$-\frac{a^3\sqrt{a^2+x^2}\int \frac{x\left(\frac{x^2}{a^2}+1\right)^{3/2}}{a\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 5971$$

$$-\frac{a^3\sqrt{a^2+x^2}\int \left(\frac{\sinh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}+\frac{\sinh\left(4\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\right)d\operatorname{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{4}a^2\left(\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-$$

$$\frac{a^3\sqrt{a^2+x^2}\left(-\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{er}\right.}{8\sqrt{\frac{x^2}{a^2}+1}}$$

input
```
Int[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

output
```
(x*(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]])/4 - (a^3*Sqrt[a^2 + x^2]*(-1/32*(
Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[x/a]]]) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh
[x/a]]])/8 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[x/a]]])/32 + (Sqrt[Pi/2]*Erfi[S
qrt[2]*Sqrt[ArcSinh[x/a]]])/8))/(8*Sqrt[1 + x^2/a^2]) + (3*a^2*((x*Sqrt[a^
2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(3
*Sqrt[1 + x^2/a^2]) + ((I/8)*a*Sqrt[a^2 + x^2]*((-1/2*I)*Sqrt[Pi/2]*Erf[Sq
rt[2]*Sqrt[ArcSinh[x/a]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a
]]]))/Sqrt[1 + x^2/a^2]))/4
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (a^2 + x^2)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

output
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a**2+x**2)**(3/2)*asinh(x/a)**(1/2),x)
```

output
```
Integral((a**2 + x**2)**(3/2)*sqrt(asinh(x/a)), x)
```

Maxima [F]

$$\int \left(a^2+x^2\right)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Giac [F]

$$\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int (a^2+x^2)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,(a^2+x^2)^{3/2}\,dx$$

input
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2), x)
```

Reduce [F]

$$\int (a^2+x^2)^{3/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx=\int \sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}\,x^2dx+\left(\int \sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}dx\right)a^2$$

input
```
int((a^2+x^2)^(3/2)*asinh(x/a)^(1/2),x)
```

output
```
int(sqrt(a**2 + x**2)*sqrt(asinh(x/a))*x**2,x) + int(sqrt(a**2 + x**2)*sqr
t(asinh(x/a)),x)*a**2
```

3.119 $\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)}{16\sqrt{1+\frac{x^2}{a^2}}}$$

output
```
1/2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)+1/3*a*(a^2+x^2)^(1/2)*arcsinh(x/a
)^(3/2)/(1+x^2/a^2)^(1/2)+1/32*a*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)*erf(2^(1
/2)*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)-1/32*a*2^(1/2)*Pi^(1/2)*(a^2+x^2
)^(1/2)*erfi(2^(1/2)*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$$
$$= \frac{a\sqrt{a^2+x^2}\left(16\text{arcsinh}\left(\frac{x}{a}\right)^2 - 3\sqrt{2}\sqrt{-\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2}, -2\text{arcsinh}\left(\frac{x}{a}\right)\right) - 3\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\Gamma\left(\frac{3}{2}, 2\text{arcsinh}\right.}{48\sqrt{1+\frac{x^2}{a^2}}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}$$

input
```
Integrate[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

output
```
(a*Sqrt[a^2 + x^2]*(16*ArcSinh[x/a]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamm
a[3/2, -2*ArcSinh[x/a]] - 3*Sqrt[2]*Sqrt[ArcSinh[x/a]]*Gamma[3/2, 2*ArcSin
h[x/a]]))/(48*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6200, 6195, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\,dx$$

$$\Big\downarrow \text{ 6200}$$

$$-\frac{\sqrt{a^2+x^2}\int \frac{x}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a\sqrt{\frac{x^2}{a^2}+1}} + \frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 6195}$$

$$-\frac{a\sqrt{a^2+x^2}\int \frac{x\sqrt{\frac{x^2}{a^2}+1}}{a\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{4\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int \frac{\sinh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}-\frac{a\sqrt{a^2+x^2}\int -\frac{i\sin\left(2i\text{arcsinh}\left(\frac{x}{a}\right)\right)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\int \frac{\sin\left(2i\text{arcsinh}\left(\frac{x}{a}\right)\right)}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{3789}$$

$$\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\int \frac{e^{2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)-\frac{1}{2}i\int \frac{e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2611}$$

$$\frac{ia\sqrt{a^2+x^2}\left(i\int e^{2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-i\int e^{-2\text{arcsinh}\left(\frac{x}{a}\right)}d\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{6198}$$

$$\frac{ia\sqrt{a^2+x^2}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}$$

input
```
Int[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

output
```
(x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]
^(3/2))/(3*Sqrt[1 + x^2/a^2]) + ((I/8)*a*Sqrt[a^2 + x^2]*((-1/2*I)*Sqrt[Pi
/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + (I/2)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[A
rcSinh[x/a]]]))/Sqrt[1 + x^2/a^2]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Maple [F]

$$\int \sqrt{a^2+x^2}\,\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

output
```
int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a**2+x**2)**(1/2)*asinh(x/a)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + x**2)*sqrt(asinh(x/a)), x)
```

Maxima [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Giac [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,\sqrt{a^2+x^2}\,dx$$

input
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2+x^2)^(1/2)*asinh(x/a)^(1/2),x)
```

output
```
int(sqrt(a**2 + x**2)*sqrt(asinh(x/a)),x)
```

3.120 $\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}} \, dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

output
```
2/3*a*(1+x^2/a^2)^(1/2)*arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}} \, dx=\frac{2a\sqrt{1+\frac{x^2}{a^2}}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

input
```
Integrate[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}} \, dx$$

$$\Big\downarrow \text{6198}$$

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

input
```
Int[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2 \,\text{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} \sqrt{\frac{a^{2}+x^{2}}{a^{2}}}\, a}{3 \sqrt{a^{2}+x^{2}}}$	34

input
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \frac{2}{3}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{3}{2}}$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*log((x + sqrt(a^2 + x^2))/a)^(3/2)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(1/2),x)
```

output
```
Integral(sqrt(asinh(x/a))/sqrt(a**2 + x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \frac{2\sqrt{asinh\left(\frac{x}{a}\right)}\,asinh\left(\frac{x}{a}\right)}{3}$$

input
```
int(asinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x)
```

output
```
(2*sqrt(asinh(x/a))*asinh(x/a))/3
```

3.121 $\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 6202$$

$$\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}} - \frac{\sqrt{\frac{x^2}{a^2}+1} \int \frac{a^2 x}{(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a^3\sqrt{a^2+x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}} - \frac{\sqrt{\frac{x^2}{a^2}+1} \int \frac{x}{(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2+x^2}}$$

$$\Big\downarrow \ 6239$$

$$\frac{x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}} - \frac{\sqrt{\frac{x^2}{a^2}+1} \int \frac{x}{(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2+x^2}}$$

input
```
Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

output
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(3/2),x)
```

output
```
Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\mathrm{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}}{a^4+2a^2x^2+x^4}dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

output
```
int((sqrt(a**2 + x**2)*sqrt(asinh(x/a)))/(a**4 + 2*a**2*x**2 + x**4),x)
```

3.122 $\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\operatorname{Int}\left(\frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}},x\right)$$

output
```
Defer(Int)(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}dx}{3a^2}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^4x}{(a^2+x^2)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{6a^5\sqrt{a^2+x^2}}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{3/2}}dx}{3a^2}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^2x}{(a^2+x^2)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{2a^3\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)^2\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}(\frac{x}{a})}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \; 6239$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)^2\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{6a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\sqrt{\text{arcsinh}(\frac{x}{a})}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x}{(a^2+x^2)\sqrt{\text{arcsinh}(\frac{x}{a})}}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

input
```
Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}dx$$

input
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

output
```
int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 24.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(5/2),x)
```

output
```
Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2),x)
```

output
```
int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx = \int \frac{\sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}}{a^6+3a^4x^2+3a^2x^4+x^6}dx$$

input
```
int(asinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

output
```
int((sqrt(a**2 + x**2)*sqrt(asinh(x/a)))/(a**6 + 3*a**4*x**2 + 3*a**2*x**4
 + x**6),x)
```

3.123 $\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 433

$$\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3(a^2+x^2)^{5/2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x(a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{3a^3\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a^3\sqrt{\pi}\sqrt{a^2-}}{2}$$

output
```
-27/256*a^3*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-9/32*a*x^
2*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-3/32*(a^2+x^2)^(5/2
)*arcsinh(x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)+3/8*a^2*x*(a^2+x^2)^(1/2)*arcsinh
(x/a)^(3/2)+1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2)+3/20*a^3*(a^2+x^2)^(1
/2)*arcsinh(x/a)^(5/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*Pi^(1/2)*(a^2+x^2)^(1/
2)*erf(2*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)+3/128*a^3*2^(1/2)*Pi^(1/2)*
(a^2+x^2)^(1/2)*erf(2^(1/2)*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)+3/2048*a
^3*Pi^(1/2)*(a^2+x^2)^(1/2)*erfi(2*arcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)+3
/128*a^3*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)*erfi(2^(1/2)*arcsinh(x/a)^(1/2))
/(1+x^2/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.48

$$\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \frac{a^3\sqrt{a^2+x^2}\left(384\operatorname{arcsinh}\left(\frac{x}{a}\right)^3 - 480\operatorname{arcsinh}\left(\frac{x}{a}\right)\cosh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right) + 60\sqrt{2\pi}\sqrt{}\right.}{}$$

input
```
Integrate[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

output
```
(a^3*Sqrt[a^2 + x^2]*(384*ArcSinh[x/a]^3 - 480*ArcSinh[x/a]*Cosh[2*ArcSinh
[x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]]
+ 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 5*Sq
rt[-ArcSinh[x/a]]*Gamma[5/2, -4*ArcSinh[x/a]] - 5*Sqrt[ArcSinh[x/a]]*Gamma
[5/2, 4*ArcSinh[x/a]] + 640*ArcSinh[x/a]^2*Sinh[2*ArcSinh[x/a]]))/(2560*Sq
rt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Rubi [A] (verified)

Time = 2.98 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {6201, 27, 6200, 6192, 6198, 6213, 6206, 3042, 3793, 2009, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2+x^2)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}dx - \frac{3a\sqrt{a^2+x^2}\int \frac{x(a^2+x^2)\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{a^2}dx}{8\sqrt{\frac{x^2}{a^2}+1}} + \frac{1}{4}x(a^2+x^2)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}a^2\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}dx-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6200}$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\int x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\right)-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6192}$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\right.$$

$$-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arc}\right.$$

$$-\frac{3\sqrt{a^2+x^2}\int x\left(a^2+x^2\right)\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}dx}{8a\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 6213

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arc}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^3\int\frac{\left(\frac{x^2}{a^2}+1\right)^{3/2}}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx\right)}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 6206

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int\frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arc}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\int\frac{\left(\frac{x^2}{a^2}+1\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arc}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\int \frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)+\frac{\pi}{2}\right)^4}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8a\sqrt{\frac{x^2}{a^2}+1}}+$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \ 3793$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arc}\right.$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\int\left(\frac{\cosh\left(2\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}+\frac{\cosh\left(4\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{8\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}+\frac{3}{8\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}\right)d\text{arcsi}\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arc}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\Big\downarrow$ 6234

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int\frac{x^2}{a^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\Big\downarrow$ 3042

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int-\frac{\sin\left(i\text{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\Big\downarrow$ 25

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}+\frac{1}{4}a^2\int\frac{\sin\left(i\text{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}d\text{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\Big\downarrow$ 3793

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}a^2\int\left(\frac{1}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}-\frac{\cosh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}\right)d\text{arcsinh}\left(\frac{x}{a}\right)+\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x}}{5}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

$\downarrow$ 2009

$$\frac{3}{4}a^2\left(-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\sqrt{\text{ar}}\right.\right.}{4a\sqrt{\frac{x^2}{a^2}+1}}\right.$$

$$\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}-$$

$$\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}\left(a^2+x^2\right)^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{8}a^4\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{32}\sqrt{\pi}\right.\right.}{8a\sqrt{\frac{x^2}{a^2}+1}}$$

input
```
Int[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

output
```
(x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4 - (3*Sqrt[a^2 + x^2]*(((a^2 + x
^2)^2*Sqrt[ArcSinh[x/a]])/4 - (a^4*((3*Sqrt[ArcSinh[x/a]])/4 + (Sqrt[Pi]*E
rf[2*Sqrt[ArcSinh[x/a]]])/32 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]]
)/4 + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[x/a]]])/32 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*
Sqrt[ArcSinh[x/a]]])/4))/8))/(8*a*Sqrt[1 + x^2/a^2]) + (3*a^2*((x*Sqrt[a^2
 + x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*
Sqrt[1 + x^2/a^2]) - (3*Sqrt[a^2 + x^2]*((x^2*Sqrt[ArcSinh[x/a]])/2 - (a^2
*(-Sqrt[ArcSinh[x/a]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/4 + (
Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/4))/4))/(4*a*Sqrt[1 + x^2/a^2
])))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

output
```
int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a**2+x**2)**(3/2)*asinh(x/a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Giac [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2} \left(a^2+x^2\right)^{3/2} dx$$

input
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a^2+x^2\right)^{3/2} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2} dx = \int \sqrt{a^2+x^2}\, \sqrt{asinh\left(\frac{x}{a}\right)}\, asinh\left(\frac{x}{a}\right) x^2 dx + \left(\int \sqrt{a^2+x^2}\, \sqrt{asinh\left(\frac{x}{a}\right)}\, asinh\left(\frac{x}{a}\right) dx\right) a^2$$

input
```
int((a^2+x^2)^(3/2)*asinh(x/a)^(3/2),x)
```

output
```
int(sqrt(a**2 + x**2)*sqrt(asinh(x/a))*asinh(x/a)*x**2,x) + int(sqrt(a**2
+ x**2)*sqrt(asinh(x/a))*asinh(x/a),x)*a**2
```

3.124 $\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 259

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx = -\frac{3a\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}} + \frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{64\sqrt{1+\frac{x^2}{a^2}}}$$

output
```
-3/16*a*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-3/8*x^2*(a^2+
x^2)^(1/2)*arcsinh(x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)+1/2*x*(a^2+x^2)^(1/2)*ar
csinh(x/a)^(3/2)+1/5*a*(a^2+x^2)^(1/2)*arcsinh(x/a)^(5/2)/(1+x^2/a^2)^(1/2
)+3/128*a*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)*erf(2^(1/2)*arcsinh(x/a)^(1/2))
/(1+x^2/a^2)^(1/2)+3/128*a*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)*erfi(2^(1/2)*a
rcsinh(x/a)^(1/2))/(1+x^2/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx = \frac{a\sqrt{a^2+x^2}\left(15\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right)\right.}{}$$

input
```
Integrate[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

output
```
(a*Sqrt[a^2 + x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 15*Sqr
t[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 8*Sqrt[ArcSinh[x/a]]*(16*ArcSin
h[x/a]^2 - 15*Cosh[2*ArcSinh[x/a]] + 20*ArcSinh[x/a]*Sinh[2*ArcSinh[x/a]])
))/(640*Sqrt[1 + x^2/a^2])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6200, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx$$

$$\Big\downarrow \; 6200$$

$$-\frac{3\sqrt{a^2+x^2}\int x\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}dx}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \; 6192$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{a^2+x^2}\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{\frac{x^2}{a^2}+1}}dx}{2\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{\int \frac{x^2}{\sqrt{\frac{x^2}{a^2}+1}\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{6234}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int \frac{x^2}{a^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int -\frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}+\frac{1}{4}a^2\int \frac{\sin\left(i\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)^2}{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}d\operatorname{arcsinh}\left(\frac{x}{a}\right)\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{4}a^2\int\left(\frac{1}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}-\frac{\cosh\left(2\text{arcsinh}\left(\frac{x}{a}\right)\right)}{2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}\right)d\text{arcsinh}\left(\frac{x}{a}\right)+\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)}{4a\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{a^2+x^2}\left(\frac{1}{2}x^2\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}\right)-\sqrt{\text{arcsinh}}\right.\right.}{4a\sqrt{\frac{x^2}{a^2}+1}}$$

$$\frac{a\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}$$

input
```
Int[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

output
```
(x*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]
^(5/2))/(5*Sqrt[1 + x^2/a^2]) - (3*Sqrt[a^2 + x^2]*((x^2*Sqrt[ArcSinh[x/a]
])/2 - (a^2*(-Sqrt[ArcSinh[x/a]] + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/
a]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/4))/4))/(4*a*Sqrt[
1 + x^2/a^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{a^2+x^2}\, \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
int((a^2+x^2)^(1/2)*arcsinh(x/a)^(3/2),x)
```

output
```
int((a^2+x^2)^(1/2)*arcsinh(x/a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)\,dx$$

input
```
integrate((a**2+x**2)**(1/2)*asinh(x/a)**(3/2),x)
```

output
```
Integral(sqrt(a**2 + x**2)*asinh(x/a)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2+x^2}\,\text{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Giac [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx = \int \sqrt{a^2+x^2}\,\text{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx = \int \text{asinh}\left(\frac{x}{a}\right)^{3/2}\sqrt{a^2+x^2}\,dx$$

input
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a^2+x^2}\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}\,dx = \int \sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}\,asinh\left(\frac{x}{a}\right)dx$$

input
```
int((a^2+x^2)^(1/2)*asinh(x/a)^(3/2),x)
```

output
```
int(sqrt(a**2 + x**2)*sqrt(asinh(x/a))*asinh(x/a),x)
```

3.125 $\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

output
```
2/5*a*(1+x^2/a^2)^(1/2)*arcsinh(x/a)^(5/2)/(a^2+x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\text{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

input
```
Integrate[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$$
$$\Big\downarrow \text{ 6198}$$
$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\operatorname{arcsinh}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

input
```
Int[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

output
```
(2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{5}{2}}\sqrt{\frac{a^2+x^2}{a^2}}\,a}{5\sqrt{a^2+x^2}}$	34

input
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arcsinh(x/a)^(5/2)/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \frac{2}{5}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{5}{2}}$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*log((x + sqrt(a^2 + x^2))/a)^(5/2)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(1/2),x)
```

output
```
Integral(asinh(x/a)**(3/2)/sqrt(a**2 + x**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$$

input
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2),x)
```

output
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.44

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \frac{2\sqrt{asinh\left(\frac{x}{a}\right)}\,asinh\left(\frac{x}{a}\right)^2}{5}$$

input
```
int(asinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x)
```

output
```
(2*sqrt(asinh(x/a))*asinh(x/a)**2)/5
```

3.126 $\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}, x\right)$$

output
```
Defer(Int)(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx = \int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}} \, dx$$

input
```
Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

output
```
Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6202}$$

$$\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^2x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a^3\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 6239}$$

$$\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}$$

input
```
Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

output
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(3/2),x)
```

output
```
Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx$$

input
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2),x)
```

output
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{3/2}}\,dx = \frac{2\sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}\,asinh\left(\frac{x}{a}\right)x - 3\left(\int \frac{\sqrt{asinh\left(\frac{x}{a}\right)}\,x}{a^2+x^2}dx\right)a^2 - 3\left(\int \frac{\sqrt{asinh\left(\frac{x}{a}\right)}\,x}{a^2+x^2}dx\right)x}{2a^2\left(a^2+x^2\right)}$$

input
```
int(asinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

output
```
(2*sqrt(a**2 + x**2)*sqrt(asinh(x/a))*asinh(x/a)*x - 3*int((sqrt(asinh(x/a
))*x)/(a**2 + x**2),x)*a**2 - 3*int((sqrt(asinh(x/a))*x)/(a**2 + x**2),x)*
x**2)/(2*a**2*(a**2 + x**2))
```

3.127 $\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}} \, dx=\text{Int}\left(\frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}},x\right)$$

output
```
Defer(Int)(arcsinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}} \, dx=\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}} \, dx$$

input
```
Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(5/2),x]
```

output
```
Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6203}$$

$$\frac{2\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}dx}{3a^2}-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^4x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^2}dx}{2a^5\sqrt{a^2+x^2}}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^2}dx}{2a\sqrt{a^2+x^2}}+\frac{2\int \frac{\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}dx}{3a^2}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^2}dx}{2a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{a^2x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a^3\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^2}dx}{2a\sqrt{a^2+x^2}}+\frac{2\left(\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int \frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \; 6213$$

$$\frac{2\left(\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}-\frac{\sqrt{\frac{x^2}{a^2}+1}\left(\frac{\int\frac{1}{\left(\frac{x^2}{a^2}+1\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a^3}-\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{2(a^2+x^2)}\right)}{2a\sqrt{a^2+x^2}}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \; 6209$$

$$\frac{2\left(\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}-\frac{\sqrt{\frac{x^2}{a^2}+1}\left(\frac{\int\frac{1}{\left(\frac{x^2}{a^2}+1\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a^3}-\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{2(a^2+x^2)}\right)}{2a\sqrt{a^2+x^2}}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

$$\Big\downarrow \; 6239$$

$$\frac{2\left(\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\int\frac{x\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{a^2+x^2}dx}{2a\sqrt{a^2+x^2}}\right)}{3a^2}-\frac{\sqrt{\frac{x^2}{a^2}+1}\left(\frac{\int\frac{1}{\left(\frac{x^2}{a^2}+1\right)^{3/2}\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}dx}{4a^3}-\frac{\sqrt{\text{arcsinh}\left(\frac{x}{a}\right)}}{2(a^2+x^2)}\right)}{2a\sqrt{a^2+x^2}}+\frac{x\text{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{3a^2\left(a^2+x^2\right)^{3/2}}$$

input

```
Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(5/2),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{5}{2}}}dx$$

input
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x)
```

output
```
int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 54.89 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{5/2}}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(5/2),x)
```

output
```
Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{5/2}}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{5/2}}\,dx=\int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{5/2}}\,dx$$

input
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(5/2),x)
```

output
```
int(asinh(x/a)^(3/2)/(a^2 + x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 13.14

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{3/2}}{(a^2+x^2)^{5/2}}\,dx=\frac{6\sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}\,asinh\left(\frac{x}{a}\right)a^2x+4\sqrt{a^2+x^2}\,\sqrt{asinh\left(\frac{x}{a}\right)}\,asinh\left(\frac{x}{a}\right)x^3-6\left(\int \right.}{}$$

input
```
int(asinh(x/a)^(3/2)/(a^2+x^2)^(5/2),x)
```

output
```
(6*sqrt(a**2 + x**2)*sqrt(asinh(x/a))*asinh(x/a)*a**2*x + 4*sqrt(a**2 + x*
*2)*sqrt(asinh(x/a))*asinh(x/a)*x**3 - 6*int((sqrt(asinh(x/a))*x**3)/(a**4
 + 2*a**2*x**2 + x**4),x)*a**4 - 12*int((sqrt(asinh(x/a))*x**3)/(a**4 + 2*
a**2*x**2 + x**4),x)*a**2*x**2 - 6*int((sqrt(asinh(x/a))*x**3)/(a**4 + 2*a
**2*x**2 + x**4),x)*x**4 - 9*int((sqrt(asinh(x/a))*x)/(a**4 + 2*a**2*x**2
+ x**4),x)*a**6 - 18*int((sqrt(asinh(x/a))*x)/(a**4 + 2*a**2*x**2 + x**4),
x)*a**4*x**2 - 9*int((sqrt(asinh(x/a))*x)/(a**4 + 2*a**2*x**2 + x**4),x)*a
**2*x**4)/(6*a**4*(a**4 + 2*a**2*x**2 + x**4))
```

3.128 $\int \frac{\left(d+c^2dx^2\right)^{5/2}}{\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 535

$$\int \frac{(d+c^2dx^2)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{5d^2\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{8bc\sqrt{1+c^2x^2}} + \frac{3d^2e^{\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{15d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{\pi}{6}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{3d^2e^{-\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{15d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{\pi}{6}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64\sqrt{b}c\sqrt{1+c^2x^2}}$$

output

```
5/8*d^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b/c/(c^2*x^2+1)^(1/2)
+3/64*d^2*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))/b^(1/2)/c/(c^2*x^2+1)^(1/2)+15/128*d^2*exp(2*a/b)*2^(1/2)*
Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
/b^(1/2)/c/(c^2*x^2+1)^(1/2)+1/384*d^2*exp(6*a/b)*6^(1/2)*Pi^(1/2)*(c^2*d*
x^2+d)^(1/2)*erf(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/(c^2*
x^2+1)^(1/2)+3/64*d^2*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2*(a+b*arcsinh(c*x
))^(1/2)/b^(1/2))/b^(1/2)/c/exp(4*a/b)/(c^2*x^2+1)^(1/2)+15/128*d^2*2^(1/2
)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/
2))/b^(1/2)/c/exp(2*a/b)/(c^2*x^2+1)^(1/2)+1/384*d^2*6^(1/2)*Pi^(1/2)*(c^2
*d*x^2+d)^(1/2)*erfi(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/e
xp(6*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.67

$$\int \frac{(d+c^2dx^2)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}\left(240ae^{\frac{6a}{b}}+240be^{\frac{6a}{b}}\text{arcsinh}(cx)+\sqrt{6}b\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(240*a*E^((6*a)/b) + 240*b*E^((6*a)/b)*ArcSinh[c*
x] + Sqrt[6]*b*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSi
nh[c*x]))/b] + 18*b*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-4*(a + b*ArcSinh[c*x]))/b] + 45*Sqrt[2]*b*E^((4*a)/b)*Sqrt[-((a + b*Arc
Sinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] - 45*Sqrt[2]*b*E^((
8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] -
18*b*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*
x]))/b] - Sqrt[6]*b*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a
 + b*ArcSinh[c*x]))/b]))/(384*b*c*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*Sqrt[a + b
*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\cosh^6\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^6}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{\cosh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(cx)}}+\frac{15\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{}{16\sqrt{}}\right.}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\left(\frac{3}{64}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{15}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\text{erf}\left(\right.\right.}{}$$

input
```
Int[(d + c^2*d*x^2)^(5/2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((5*Sqrt[a + b*ArcSinh[c*x]])/8 + (3*Sqrt[b]*E^((
4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 + (15*Sqrt[
b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/64 + (Sqrt[b]*E^((6*a)/b)*Sqrt[Pi/6]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/64 + (3*Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(64*E^((4*a)/b)) + (15*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((2*a)/b)) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqr
t[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*E^((6*a)/b))))/(b*c*Sqrt[1 +
c^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{\sqrt{a +b \,\operatorname{arcsinh}\left(x c \right)}}d x$$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + c^2 d x^2)^{5/2}}{\sqrt{a + b\operatorname{arcsinh}(cx)}}\, dx = \text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d + c^2 d x^2)^{5/2}}{\sqrt{a + b\operatorname{arcsinh}(cx)}}\, dx = \int \frac{(c^2 d x^2 + d)^{\frac{5}{2}}}{\sqrt{b\operatorname{arsinh}(cx) + a}}\, dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(d\,c^2\,x^2+d)^{5/2}}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{5/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \sqrt{d}\,d^2\Bigg(\Bigg(\Bigg(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,x^4}{asinh\,(cx)\,b+a}dx\Bigg)\,c^4 \\ +2\Bigg(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,x^2}{asinh\,(cx)\,b+a}dx\Bigg)\,c^2 \\ +\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)\,b+a}dx\Bigg)\end{aligned}$$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
sqrt(d)*d**2*(int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**4)/(asinh
(c*x)*b + a),x)*c**4 + 2*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x
**2)/(asinh(c*x)*b + a),x)*c**2 + int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)
*b + a))/(asinh(c*x)*b + a),x))
```

3.129 $\int \frac{\left(d+c^2dx^2\right)^{3/2}}{\sqrt{a+b\textbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 359

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{3d\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{4bc\sqrt{1+c^2x^2}} + \frac{de^{\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{de^{-\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c\sqrt{1+c^2x^2}} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c\sqrt{1+c^2x^2}}$$

output
```
3/4*d*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b/c/(c^2*x^2+1)^(1/2)+1
/32*d*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2*(a+b*arcsinh(c*x))^(1/
2)/b^(1/2))/b^(1/2)/c/(c^2*x^2+1)^(1/2)+1/8*d*exp(2*a/b)*2^(1/2)*Pi^(1/2)*
(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/
c/(c^2*x^2+1)^(1/2)+1/32*d*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2*(a+b*arcsin
h(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(4*a/b)/(c^2*x^2+1)^(1/2)+1/8*d*2^(1/2
)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/
2))/b^(1/2)/c/exp(2*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.70

$$\int \frac{(d+c^2dx^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(24e^{\frac{4a}{b}}(a+b\text{arcsinh}(cx))+b\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\ldots}{}\right.\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(24*E^((4*a)/b)*(a + b*ArcSinh[c*x]) + b*Sqrt[-((a
+ b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] + 4*Sqrt[2]*
b*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSin
h[c*x]))/b] - 4*Sqrt[2]*b*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2,
(2*(a + b*ArcSinh[c*x]))/b] - b*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[1/2, (4*(a + b*ArcSinh[c*x]))/b]))/(32*b*c*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*
Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$\Big\downarrow$ 6206

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 3042

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^4}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 3793

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{3}{8\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx}{bc\sqrt{c^2x^2+1}}$$

$\Big\downarrow$ 2009

$$\frac{d\sqrt{c^2dx^2+d}\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\right.\right.}{bc\sqrt{c^2x^2+1}}$$

input

```
Int[(d + c^2*d*x^2)^(3/2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
(d*Sqrt[d + c^2*d*x^2]*((3*Sqrt[a + b*ArcSinh[c*x]])/4 + (Sqrt[b]*E^((4*a)
/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((
2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4 + (S
qrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b
)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(4*E^((2*a)/b))))/(b*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^{\frac{3}{2}}}{\sqrt{a + b\,\mathrm{arcsinh}\,(xc)}}dx$$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2}}{\sqrt{a + b\mathrm{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{\sqrt{a+b\mathrm{arcsinh}(cx)}}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}}{\sqrt{a+b\,\mathrm{asinh}\,(cx)}}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{\sqrt{a+b\mathrm{arcsinh}(cx)}}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{\sqrt{b\,\mathrm{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{(d\,c^2\,x^2+d)^{3/2}}{\sqrt{a+b\,\text{asinh}\,(c\,x)}}\,dx$$

input
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\sqrt{d}\,d\left(\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,x^2}{asinh\,(cx)\,b+a}dx\right)c^2+\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)\,b+a}dx\right)$$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
sqrt(d)*d*(int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*
x)*b + a),x)*c**2 + int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asin
h(c*x)*b + a),x))
```

3.130 $\int \frac{\sqrt{d+c^2dx^2}}{\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 207

$$\int \frac{\sqrt{d+c^2dx^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\frac{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}{bc\sqrt{1+c^2x^2}}+\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c\sqrt{1+c^2x^2}}+\frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c\sqrt{1+c^2x^2}}$$

output
```
(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b/c/(c^2*x^2+1)^(1/2)+1/8*exp
(2*a/b)*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))/b^(1/2)/c/(c^2*x^2+1)^(1/2)+1/8*2^(1/2)*Pi^(1/2)*(c^2*d*x
^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(2
*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{d+c^2dx^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{2a}{b}}\sqrt{d(1+c^2x^2)}\left(8e^{\frac{2a}{b}}(a+b\text{arcsinh}(cx))+\sqrt{2}b\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)-\sqrt{2}be^{\frac{4a}{b}}\right.}{8bc\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[d*(1 + c^2*x^2)]*(8*E^((2*a)/b)*(a + b*ArcSinh[c*x]) + Sqrt[2]*b*Sqr
t[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] - Sqr
t[2]*b*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c
*x]))/b]))/(8*b*c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$\Big\downarrow \text{6206}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)^2}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{1}{2\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{bc\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(c}\right.}{bc\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^((2*a)/b)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*E^((2*a)/b))))/
(b*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{\sqrt{c^2 d\, x^2 + d}}{\sqrt{a + b \operatorname{arcsinh}\left(x c\right)}}dx$$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d + c^2 dx^2}}{\sqrt{a + b\text{arcsinh}(cx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{c^2dx^2+d}}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{c^2dx^2+d}}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d + c^2 d x^2}}{\sqrt{a + b \operatorname{arcsinh}(cx)}}\, dx = \int \frac{\sqrt{d\, c^2\, x^2 + d}}{\sqrt{a + b\, \operatorname{asinh}\left(c\, x\right)}}\, dx$$

input
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d + c^2 d x^2}}{\sqrt{a + b \operatorname{arcsinh}(cx)}}\, dx = \sqrt{d} \left(\int \frac{\sqrt{c^2 x^2 + 1}\, \sqrt{asinh\left(cx\right) b + a}}{asinh\left(cx\right) b + a} dx \right)$$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
sqrt(d)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*x)*b + a
),x)
```

3.131 $\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\textbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{bc\sqrt{d+c^2dx^2}}$$

output
```
2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{bc\sqrt{d(1+c^2x^2)}}$$

input
```
Integrate[1/(Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(b*c*Sqrt[d*(1 + c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}}dx$$
$$\Big\downarrow \text{ 6198}$$
$$\frac{2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{bc\sqrt{c^2dx^2+d}}$$

input
```
Int[1/(Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(b*c*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{2\sqrt{a+b\,\text{arcsinh}(xc)}\,\sqrt{c^2x^2+1}}{b\sqrt{d(c^2x^2+1)}\,c}$	43

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
2*(a+b*arcsinh(x*c))^(1/2)*(c^2*x^2+1)^(1/2)/b/(d*(c^2*x^2+1))^(1/2)/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fri
cas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{d\left(c^2x^2+1\right)}\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(d*(c**2*x**2 + 1))*sqrt(a + b*asinh(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{c^2dx^2+d}\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{c^2dx^2+d}\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{d+c^2dx^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\frac{2\sqrt{d}\,\sqrt{asinh\,(cx)\,b+a}}{bcd}$$

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt(asinh(c*x)*b + a))/(b*c*d)
```

3.132 $\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx = \mathrm{Int}\left(\frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(c^2dx^2+d)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + c^2*d*x^2)^(3/2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c^2d\,x^2+d)^{\frac{3}{2}}\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fri
cas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(d(c^2x^2+1))^{\frac{3}{2}}\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(3/2)*sqrt(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{3}{2}}\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,(d\,c^2\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh(cx)b+a}}{asinh(cx)b\,c^4x^4+2asinh(cx)b\,c^2x^2+asinh(cx)b+a\,c^4x^4+2a\,c^2x^2+a}dx\right)}{d^2}$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*x)*b*c*
*4*x**4 + 2*asinh(c*x)*b*c**2*x**2 + asinh(c*x)*b + a*c**4*x**4 + 2*a*c**2
*x**2 + a),x))/d**2
```

3.133 $\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\,\text{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + c^2*d*x^2)^(5/2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c^2d\,x^2+d)^{\frac{5}{2}}\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fri
cas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 56.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{(d\,(c^2x^2+1))^{\frac{5}{2}}\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(5/2)*sqrt(a + b*asinh(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + c^2*d*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh(cx)b+a}}{asinh(cx)b\,c^6x^6+3asinh(cx)b\,c^4x^4+3asinh(cx)b\,c^2x^2+asinh(cx)b+a\,c^6x^6+3}\right.}{d^3}$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*x)*b*c*
*6*x**6 + 3*asinh(c*x)*b*c**4*x**4 + 3*asinh(c*x)*b*c**2*x**2 + asinh(c*x)
*b + a*c**6*x**6 + 3*a*c**4*x**4 + 3*a*c**2*x**2 + a),x))/d**3
```

3.134 $\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 533

$$\begin{aligned}
\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\
&- \frac{3d^2e^{\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c\sqrt{1+c^2x^2}} \\
&- \frac{15d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c\sqrt{1+c^2x^2}} \\
&- \frac{d^2e^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c\sqrt{1+c^2x^2}} \\
&+ \frac{3d^2e^{-\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c\sqrt{1+c^2x^2}} \\
&+ \frac{15d^2e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c\sqrt{1+c^2x^2}} \\
&+ \frac{d^2e^{-\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{6}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-2*d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-
3/8*d^2*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))/b^(3/2)/c/(c^2*x^2+1)^(1/2)-15/32*d^2*exp(2*a/b)*2^(1/2)*Pi^
(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^
(3/2)/c/(c^2*x^2+1)^(1/2)-1/32*d^2*exp(6*a/b)*6^(1/2)*Pi^(1/2)*(c^2*d*x^2+
d)^(1/2)*erf(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/(c^2*x^2+
1)^(1/2)+3/8*d^2*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2*(a+b*arcsinh(c*x))^(1
/2)/b^(1/2))/b^(3/2)/c/exp(4*a/b)/(c^2*x^2+1)^(1/2)+15/32*d^2*2^(1/2)*Pi^(
1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^
(3/2)/c/exp(2*a/b)/(c^2*x^2+1)^(1/2)+1/32*d^2*6^(1/2)*Pi^(1/2)*(c^2*d*x^2+
d)^(1/2)*erfi(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(6*a/
b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.19

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$

$$-\frac{d^2e^{-6\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\sqrt{d+c^2dx^2}\left(16e^{\frac{8a}{b}+6\text{arcsinh}(cx)}\sqrt{2\pi}\sqrt{a+b\text{arcsinh}(cx)}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-16e^{\frac{4a}{b}}\cdots\right.}{\cdots}$$

input
```
Integrate[(d + c^2*d*x^2)^(5/2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
-1/32*(d^2*Sqrt[d + c^2*d*x^2]*(16*E^((8*a)/b + 6*ArcSinh[c*x])*Sqrt[2*Pi]
*Sqrt[a + b*ArcSinh[c*x]]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]
- 16*E^((4*a)/b + 6*ArcSinh[c*x])*Sqrt[2*Pi]*Sqrt[a + b*ArcSinh[c*x]]*Erfi
[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]] + Sqrt[b]*(E^((6*a)/b) + 52*E
^(6*(a/b + ArcSinh[c*x])) + 6*E^((6*a)/b + 2*ArcSinh[c*x]) - E^((6*a)/b +
4*ArcSinh[c*x]) - E^((6*a)/b + 8*ArcSinh[c*x]) + 6*E^((6*a)/b + 10*ArcSinh
[c*x]) + E^((6*a)/b + 12*ArcSinh[c*x]) + 64*c^2*E^(6*(a/b + ArcSinh[c*x]))
*x^2 - Sqrt[6]*E^(6*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/
2, (-6*(a + b*ArcSinh[c*x]))/b] - 12*E^((2*a)/b + 6*ArcSinh[c*x])*Sqrt[-((
a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] + Sqrt[2]*
E^((4*a)/b + 6*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-
2*(a + b*ArcSinh[c*x]))/b] + Sqrt[2]*E^((8*a)/b + 6*ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] - 12*E^((10*a)/b +
 6*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x
]))/b] - Sqrt[6]*E^(6*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*G
amma[1/2, (6*(a + b*ArcSinh[c*x]))/b])))/(b^(3/2)*c*E^(6*(a/b + ArcSinh[c*
x]))*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{12cd^2\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{12d^2\sqrt{c^2dx^2+d}\int -\frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\big\downarrow$ 25

$$-\frac{12d^2\sqrt{c^2dx^2+d}\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\big\downarrow$ 5971

$$-\frac{12d^2\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{6a}{b}-\frac{6(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{5\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{32\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a}{b^2c\sqrt{c^2x^2+1}}$$
$$\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$\big\downarrow$ 2009

$$\frac{12d^2\sqrt{c^2dx^2+d}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{5}{64}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{64}\sqrt{\frac{\pi}{6}}\sqrt{b}e^{\frac{6a}{b}}\right.}{}$$
$$\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{5/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)^(5/2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2]*(d + c^2*d*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]
) + (12*d^2*Sqrt[d + c^2*d*x^2]*(-1/32*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(
2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]) - (5*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*
Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 - (Sqrt[b]*E^((6*a)/b)
*Sqrt[Pi/6]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/64 + (Sqrt[b]
*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (
5*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64
*E^((2*a)/b)) + (Sqrt[b]*Sqrt[Pi/6]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(64*E^((6*a)/b))))/(b^2*c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^{\frac{5}{2}}}{(a + b\,\operatorname{arcsinh}(xc))^{\frac{3}{2}}} dx$$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d + c^2 dx^2)^{5/2}}{(a + b\text{arcsinh}(cx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + c^2 dx^2)^{5/2}}{(a + b\text{arcsinh}(cx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2+d)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(5/2)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d\,c^2\,x^2+d)^{5/2}}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)^(5/2)/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{5/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \sqrt{d}\,d^2\left(\left(\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,x^4}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)c^4+2\left(\int \frac{\sqrt{c^2x^2+1}}{asinh\,(cx)^2\,b}\right.\right.\right.$$

input
```
int((c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
sqrt(d)*d**2*(int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**4)/(asinh
(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**4 + 2*int((sqrt(c**2*x**2 +
 1)*sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b +
a**2),x)*c**2 + int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*
x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x))
```

3.135 $\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\textbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 352

$$\begin{aligned}\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\ &\quad - \frac{de^{\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c\sqrt{1+c^2x^2}} \\ &\quad - \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c\sqrt{1+c^2x^2}} \\ &\quad + \frac{de^{-\frac{4a}{b}}\sqrt{\pi}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c\sqrt{1+c^2x^2}} \\ &\quad + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-2*d*(c^2*x^2+1)^(3/2)*(c^2*d*x^2+d)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-1/
4*d*exp(4*a/b)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))/b^(3/2)/c/(c^2*x^2+1)^(1/2)-1/2*d*exp(2*a/b)*2^(1/2)*Pi^(1/2)*(c
^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/
(c^2*x^2+1)^(1/2)+1/4*d*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2*(a+b*arcsinh(c
*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(4*a/b)/(c^2*x^2+1)^(1/2)+1/2*d*2^(1/2)*P
i^(1/2)*(c^2*d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
/b^(3/2)/c/exp(2*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.92

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$

$$-\frac{de^{-4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\sqrt{d+c^2dx^2}\left(4e^{\frac{6a}{b}+4\text{arcsinh}(cx)}\sqrt{2\pi}\sqrt{a+b\text{arcsinh}(cx)}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-4e^{\frac{2a}{b}+4\text{ar}}\right.}{}$$

input
```
Integrate[(d + c^2*d*x^2)^(3/2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
-1/8*(d*Sqrt[d + c^2*d*x^2]*(4*E^((6*a)/b + 4*ArcSinh[c*x])*Sqrt[2*Pi]*Sqr
t[a + b*ArcSinh[c*x]]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]] - 4*
E^((2*a)/b + 4*ArcSinh[c*x])*Sqrt[2*Pi]*Sqrt[a + b*ArcSinh[c*x]]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]] + Sqrt[b]*(-2*E^(4*ArcSinh[c*x])*S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] + E
^((4*a)/b)*(1 + E^(8*ArcSinh[c*x]) + 2*E^(4*ArcSinh[c*x])*(7 + 8*c^2*x^2)
- 2*E^(4*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a +
 b*ArcSinh[c*x]))/b]))))/(b^(3/2)*c*E^(4*(a/b + ArcSinh[c*x]))*Sqrt[1 + c^
2*x^2]*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6205, 6234, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2dx^2+d)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{8cd\sqrt{c^2dx^2+d}\int \frac{x(c^2x^2+1)}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6234}$$

$$\frac{8d\sqrt{c^2dx^2+d}\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{8d\sqrt{c^2dx^2+d}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{8d\sqrt{c^2dx^2+d}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(cx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(cx)}}\right)d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}(c^2dx^2+d)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8d\sqrt{c^2dx^2+d}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erf}}{b^2c\sqrt{c^2x^2+1}}$$

$$\frac{2\sqrt{c^2x^2+1}\left(c^2dx^2+d\right)^{3/2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + c^2*d*x^2)^(3/2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2]*(d + c^2*d*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]
) + (8*d*Sqrt[d + c^2*d*x^2]*(-1/32*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*S
qrt[a + b*ArcSinh[c*x]])/Sqrt[b]]) - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]
*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(8*E^((2*a)/b))))/(b^2*
c*Sqrt[1 + c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(c^2 d\,x^2 + d)^{\frac{3}{2}}}{(a + b \operatorname{arcsinh}(xc))^{\frac{3}{2}}}dx$$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d + c^2 dx^2)^{3/2}}{(a + b\operatorname{arcsinh}(cx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}}{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 + d)^(3/2)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{(d\,c^2\,x^2+d)^{3/2}}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)^(3/2)/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{3/2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \sqrt{d}\,d\left(\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\,(cx)\,b+a}\,x^2}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)c^2+\int \frac{\sqrt{c^2x^2+1}\,\sqrt{}}{asinh\,(cx)^2\,b^2+}\right.$$

input
```
int((c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
sqrt(d)*d*(int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*
x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*c**2 + int((sqrt(c**2*x**2 + 1)*s
qrt(asinh(c*x)*b + a))/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x))
```

3.136 $\int \frac{\sqrt{d+c^2dx^2}}{(a+b\textbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c\sqrt{1+c^2x^2}} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\sqrt{d+c^2dx^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c\sqrt{1+c^2x^2}}$$

output

```
-2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-1/2*
exp(2*a/b)*2^(1/2)*Pi^(1/2)*(c^2*d*x^2+d)^(1/2)*erf(2^(1/2)*(a+b*arcsinh(c
*x))^(1/2)/b^(1/2))/b^(3/2)/c/(c^2*x^2+1)^(1/2)+1/2*2^(1/2)*Pi^(1/2)*(c^2*
d*x^2+d)^(1/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c/ex
p(2*a/b)/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{d(1+c^2x^2)}\left(4\sqrt{b}(1+c^2x^2)+\sqrt{2\pi}\sqrt{a+b\text{arcsinh}(cx)}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\left(-\cosh\left(\frac{2a}{b}\right)+\sinh\left(\right.\right.\right.}{2b^{3/2}c\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsin}}}$$

input
```
Integrate[Sqrt[d + c^2*d*x^2]/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
-1/2*(Sqrt[d*(1 + c^2*x^2)]*(4*Sqrt[b]*(1 + c^2*x^2) + Sqrt[2*Pi]*Sqrt[a +
 b*ArcSinh[c*x]]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(-Cosh[(
2*a)/b] + Sinh[(2*a)/b]) + Sqrt[2*Pi]*Sqrt[a + b*ArcSinh[c*x]]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])))/(b^
(3/2)*c*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6205, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6205}$$

$$\frac{4c\sqrt{c^2dx^2+d}\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{b\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6195}$$

$$\frac{4\sqrt{c^2dx^2+d}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{4\sqrt{c^2dx^2+d}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 5971$$

$$-\frac{4\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{2\sqrt{c^2dx^2+d}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2\sqrt{c^2dx^2+d}\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 26$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\sqrt{c^2dx^2+d}\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \ 3789$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\int\frac{e^{-2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int\frac{e^{2\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{2(a+b\text{arcsinh}(cx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\sqrt{c^2dx^2+d}\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(cx))}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\sqrt{c^2dx^2+d}\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + c^2*d*x^2]/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]])
+ ((2*I)*Sqrt[d + c^2*d*x^2]*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/E^((2*a)/b)))/(b^2*c*Sqrt[1 +
 c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

Maple [F]

$$\int \frac{\sqrt{c^{2} d \,x^{2}+d}}{\left(a +b \,\operatorname{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}}d x$$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{d\left(c^2x^2+1\right)}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{c^2dx^2+d}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{c^2dx^2+d}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{\sqrt{d\,c^2\,x^2+d}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + c^2*d*x^2)^(1/2)/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+c^2dx^2}}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\sqrt{d}\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)^2b^2+2asinh\left(cx\right)ab+a^2}dx\right)$$

input
```
int((c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
sqrt(d)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*x)**2*b*
*2 + 2*asinh(c*x)*a*b + a**2),x)
```

3.137 $\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\textbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx=-\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{d+c^2dx^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

output
```
-2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx=-\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{d\left(1+c^2x^2\right)}\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

input
```
Integrate[1/(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[d*(1 + c^2*x^2)]*Sqrt[a + b*ArcSinh[c*x]]
)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6198}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{c^2dx^2+d}\sqrt{a+b\text{arcsinh}(cx)}}$$

input

```
Int[1/(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output

```
(-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[d + c^2*d*x^2]*Sqrt[a + b*ArcSinh[c*x]])
```

Defintions of rubi rules used

rule 6198

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{c^2x^2+1}}{\sqrt{a+b\,\text{arcsinh}(xc)}\,b\sqrt{d(c^2x^2+1)}\,c}$	43

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/(a+b*arcsinh(x*c))^(1/2)*(c^2*x^2+1)^(1/2)/b/(d*(c^2*x^2+1))^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}\sqrt{b\log\left(cx+\sqrt{c^2x^2+1}\right)+a}}{abc^3dx^2+abcd+(b^2c^3dx^2+b^2cd)\log\left(cx+\sqrt{c^2x^2+1}\right)}$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fri
cas")
```

output
```
-2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*sqrt(b*log(c*x + sqrt(c^2*x^2 + 1
)) + a)/(a*b*c^3*d*x^2 + a*b*c*d + (b^2*c^3*d*x^2 + b^2*c*d)*log(c*x + sqr
t(c^2*x^2 + 1)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\left(c^2x^2+1\right)}\left(a+b\,\text{asinh}\,(cx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(1/2)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{d}\,\sqrt{asinh\left(cx\right)b+a}}{bcd\left(asinh\left(cx\right)b+a\right)}$$

input
```
int(1/(c^2*d*x^2+d)^(1/2)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(asinh(c*x)*b + a))/(b*c*d*(asinh(c*x)*b + a))
```

3.138 $\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\textbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}}dx$$
$$\Big\downarrow \text{ 6205}$$
$$-\frac{4c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}}{bc(c^2dx^2+d)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}$$
$$\Big\downarrow \text{ 6239}$$
$$-\frac{4c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^2\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}}{bc(c^2dx^2+d)^{3/2}\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[1/((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c^2d\,x^2+d)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fri
cas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 27.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d\,(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c**2*d*x**2+d)**(3/2)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="gia
c")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}\,\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.96

$$\int \frac{1}{(d+c^2dx^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{c^2x^2+1}\,\sqrt{asinh(cx}}{asinh(cx)^2b^2c^4x^4+2asinh(cx)^2b^2c^2x^2+asinh(cx)^2b^2+2asinh(cx)ab\,c^4x^4-}\right.}{d^2}$$

input
```
int(1/(c^2*d*x^2+d)^(3/2)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*x)**2*b
**2*c**4*x**4 + 2*asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*as
inh(c*x)*a*b*c**4*x**4 + 4*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a
**2*c**4*x**4 + 2*a**2*c**2*x**2 + a**2),x))/d**2
```

3.139 $\int \frac{1}{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2dx^2+d)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{6205}$$

$$-\frac{8c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^3\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}}{bc\,(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{6239}$$

$$-\frac{8c\sqrt{c^2x^2+1}\int \frac{x}{(c^2x^2+1)^3\sqrt{a+b\text{arcsinh}(cx)}}dx}{bd^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}}{bc\,(c^2dx^2+d)^{5/2}\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[1/((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c^2d\,x^2+d)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fri
cas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c**2*d*x**2+d)**(5/2)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c^2*d*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="gia
c")
```

output
```
integrate(1/((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,(d\,c^2\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + c^2*d*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 176, normalized size of antiderivative = 6.52

$$\int \frac{1}{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{}{asinh(cx)^2b^2c^6x^6+3asinh(cx)^2b^2c^4x^4+3asinh(cx)^2b^2c^2x^2+asinh(cx)^2b^2}\right.}{}$$

input
```
int(1/(c^2*d*x^2+d)^(5/2)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a))/(asinh(c*x)**2*b
**2*c**6*x**6 + 3*asinh(c*x)**2*b**2*c**4*x**4 + 3*asinh(c*x)**2*b**2*c**2
*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**6*x**6 + 6*asinh(c*x)*a*b
*c**4*x**4 + 6*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**6*x**
6 + 3*a**2*c**4*x**4 + 3*a**2*c**2*x**2 + a**2),x))/d**3
```

3.140 $\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^{3/2}}{\text{arcsinh}(ax)^{5/2}}\,dx = &-\frac{2c(1+a^2x^2)^{3/2}\sqrt{c+a^2cx^2}}{3a\text{arcsinh}(ax)^{3/2}} \\
&-\frac{16x(c+a^2cx^2)^{3/2}}{3\sqrt{\text{arcsinh}(ax)}}+\frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} \\
&+\frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} \\
&+\frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} \\
&+\frac{2c\sqrt{2\pi}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}
\end{aligned}
$$

output
```
-2/3*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/arcsinh(a*x)^(3/2)-16/3*x*(
a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2)+2/3*c*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*e
rf(2*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)+2/3*c*2^(1/2)*Pi^(1/2)*(a^2*c
*x^2+c)^(1/2)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)+2/3*c*Pi
^(1/2)*(a^2*c*x^2+c)^(1/2)*erfi(2*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)+
2/3*c*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2)
)/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx =$$
$$-\frac{ce^{-4\operatorname{arcsinh}(ax)}\sqrt{c+a^2cx^2}\left(1+14e^{4\operatorname{arcsinh}(ax)}+e^{8\operatorname{arcsinh}(ax)}+16a^2e^{4\operatorname{arcsinh}(ax)}x^2-8\operatorname{arcsinh}(ax)+8e^{8\operatorname{arcsinh}(}\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

output
```
-1/24*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x
]) + 16*a^2*E^(4*ArcSinh[a*x])*x^2 - 8*ArcSinh[a*x] + 8*E^(8*ArcSinh[a*x])
*ArcSinh[a*x] + 64*a*E^(4*ArcSinh[a*x])*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] +
 16*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] +
 16*Sqrt[2]*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh
[a*x]] + 16*Sqrt[2]*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*Arc
Sinh[a*x]] + 16*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh
[a*x]]))/(a*E^(4*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.81, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6205, 6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\text{arcsinh}(ax)^{5/2}}dx$$

$\Big\downarrow$ 6205

$$\frac{8ac\sqrt{a^2cx^2+c}\int \frac{x(a^2x^2+1)}{\text{arcsinh}(ax)^{3/2}}dx}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\Big\downarrow$ 6229

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{2\int \frac{\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx}{a}+8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\Big\downarrow$ 6206

$$\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\int \frac{a^2x^2+1}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\int \frac{\sin\left(i\text{arcsinh}(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}$$

$\Big\downarrow$ 3793

$$\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int \frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\int \left(\frac{\cosh(2\text{arcsinh}(ax))}{2\sqrt{\text{arcsinh}(ax)}}+\frac{1}{2\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^2}-\frac{2x(a^2x^2+1)^{3/2}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(8a\int\frac{x^2\sqrt{a^2x^2+1}}{\sqrt{\text{arcsinh}(ax)}}dx+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\sqrt{\text{arcsinh}(ax)}\right)}{a^2}\cdots\right.}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 6234$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{8\int\frac{a^2x^2\left(a^2x^2+1\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\sqrt{\text{arcsinh}\cdots}\right)}{a^2}\cdots\right.}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 5971$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{8\int\left(\frac{\cosh(4\text{arcsinh}(ax))}{8\sqrt{\text{arcsinh}(ax)}}-\frac{1}{8\sqrt{\text{arcsinh}(ax)}}\right)d\text{arcsinh}(ax)}{a^2}+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{ar}\cdots}\right.\right.}{a^2}\cdots\right.}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{8ac\sqrt{a^2cx^2+c}\left(\frac{8\left(\frac{1}{32}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{32}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)-\frac{1}{4}\sqrt{\text{arcsinh}(ax)}\right)}{a^2}+\frac{2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(\cdots}\right.\right.}{\cdots}\cdots\right.}{3\sqrt{a^2x^2+1}}$$

$$\frac{2\sqrt{a^2x^2+1}\left(a^2cx^2+c\right)^{3/2}}{3a\text{arcsinh}(ax)^{3/2}}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) + (8
*a*c*Sqrt[c + a^2*c*x^2]*((-2*x*(1 + a^2*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]
) + (8*(-1/4*Sqrt[ArcSinh[a*x]] + (Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/32
+ (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/32))/a^2 + (2*(Sqrt[ArcSinh[a*x]]
+ (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/4 + (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/4))/a^2))/(3*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(a^2 c\,x^2 + c)^{\frac{3}{2}}}{\operatorname{arcsinh}(xa)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(x*a)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c + a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(c(a^2x^2 + 1))^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c + a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}\,x^2}{asinh(ax)^3}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}}{asinh(ax)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/asinh(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x))*x**2)/asinh(a*x)**3,x
)*a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/asinh(a*x)**3,x))
```

3.141 $\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}} \, dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1/2)/a/arcsinh(a*x)^(3/2)-8/3*x*(a^2
*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)+2/3*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2
)*erf(2^(1/2)*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)+2/3*2^(1/2)*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)*erfi(2^(1/2)*arcsinh(a*x)^(1/2))/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{c+a^2cx^2}\left(1+a^2x^2+4ax\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)+\sqrt{2}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)+\sqrt{2}\mathrm{a}\right.}{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

output
```
(-2*Sqrt[c + a^2*c*x^2]*(1 + a^2*x^2 + 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x
] + Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*Ar
cSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]))/(3*a*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6205, 6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arcsinh}(ax)^{5/2}}dx$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{4a\sqrt{a^2cx^2+c}\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}dx}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\operatorname{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\int \frac{\cosh(2\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}d\operatorname{arcsinh}(ax)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\operatorname{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{4a\sqrt{a^2cx^2+c}\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\int \frac{\sin\left(2i\text{arcsinh}(ax)+\frac{\pi}{2}\right)}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)}{a^2}\right)}{3\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{4a\sqrt{a^2cx^2+c}\left(-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)-\frac{1}{2}i\int \frac{ie^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}\right)}{3\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{26}$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)+\frac{1}{2}\int \frac{e^{2\text{arcsinh}(ax)}}{\sqrt{\text{arcsinh}(ax)}}d\text{arcsinh}(ax)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\int e^{2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\int e^{-2\text{arcsinh}(ax)}d\sqrt{\text{arcsinh}(ax)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{4a\sqrt{a^2cx^2+c}\left(\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\text{arcsinh}(ax)}}\right)}{3\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{3a\text{arcsinh}(ax)^{3/2}}$$

input

```
Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

output

```
(-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) + (4*a
*Sqrt[c + a^2*c*x^2]*((-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (2
*((Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/2 + (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/2))/a^2))/(3*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

Maple [F]

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\operatorname{arcsinh}\left(xa\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(x*a)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh\left(ax\right)}}{asinh\left(ax\right)^{3}}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/asinh(a*x)^(5/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/asinh(a*x)**3,x)
```

3.142 $\int \frac{1}{\sqrt{c+a^2cx^2}\mathbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{3/2}}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c(1+a^2x^2)}\text{arcsinh}(ax)^{3/2}}$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c*(1 + a^2*x^2)]*ArcSinh[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{6198}$$
$$-\frac{2\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

output
```
(-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\sqrt{a^2x^2+1}}{3\operatorname{arcsinh}(xa)^{\frac{3}{2}}\sqrt{c(a^2x^2+1)}\,a}$	36

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arcsinh(x*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/arcsinh(x*a)^(3/2)/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{3\left(a^3cx^2+ac\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*log(a*x + sq
rt(a^2*x^2 + 1))^(3/2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\,\text{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{\text{asinh}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{c+a^2cx^2}\text{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{asinh\,(ax)}}{3asinh\,(ax)^2\,ac}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/asinh(a*x)^(5/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(asinh(a*x)))/(3*asinh(a*x)**2*a*c)
```

3.143 $\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{5/2} (a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{6205}$$
$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}dx}{3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2} (a^2cx^2+c)^{3/2}}$$
$$\Big\downarrow \text{6239}$$
$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^2\text{arcsinh}(ax)^{3/2}}dx}{3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2} (a^2cx^2+c)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{3}{2}}\operatorname{arcsinh}(xa)^{\frac{5}{2}}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(x*a)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\,\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{\operatorname{asinh}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\,\mathrm{arcsinh}(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}}{asinh(ax)^3a^4x^4+2asinh(ax)^3a^2x^2+asinh(ax)^3}dx\right)}{c^2}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/asinh(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/(asinh(a*x)**3*a**4*x*
*4 + 2*asinh(a*x)**3*a**2*x**2 + asinh(a*x)**3),x))/c**2
```

3.144 $\int \frac{1}{(c+a^2cx^2)^{5/2}\text{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx=\text{Int}\left(\frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6205}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3\text{arcsinh}(ax)^{3/2}}dx}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{ 6239}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^3\text{arcsinh}(ax)^{3/2}}dx}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\text{arcsinh}(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\operatorname{arcsinh}\left(xa\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(x*a)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(x*a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\,\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\,\text{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{\text{asinh}\,(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.22

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{asinh(ax)}}{asinh(ax)^3a^6x^6+3asinh(ax)^3a^4x^4+3asinh(ax)^3a^2x^2+asinh(ax)^3}dx\right)}{c^3}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/asinh(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(asinh(a*x)))/(asinh(a*x)**3*a**6*x*
*6 + 3*asinh(a*x)**3*a**4*x**4 + 3*asinh(a*x)**3*a**2*x**2 + asinh(a*x)**3
),x))/c**3
```

3.145 $\int (d+ex^2)^3 (a+b\,\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 221

$$\begin{aligned}\int (d+ex^2)^3 (a+b\,\text{arcsinh}(cx))\,dx &= -\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\sqrt{1+c^2x^2}}{35c^7} \\ &- \frac{be(35c^4d^2-42c^2de+15e^2)(1+c^2x^2)^{3/2}}{105c^7} \\ &- \frac{3b(7c^2d-5e)e^2(1+c^2x^2)^{5/2}}{175c^7} - \frac{be^3(1+c^2x^2)^{7/2}}{49c^7} \\ &+ d^3x(a+b\,\text{arcsinh}(cx)) + d^2ex^3(a+b\,\text{arcsinh}(cx)) + \frac{3}{5}de^2x^5(a+b\,\text{arcsinh}(cx)) + \frac{1}{7}e^3x^7(a+b\,\text{arcsinh}(cx))\end{aligned}$$

output

```
-1/35*b*(35*c^6*d^3-35*c^4*d^2*e+21*c^2*d*e^2-5*e^3)*(c^2*x^2+1)^(1/2)/c^7
-1/105*b*e*(35*c^4*d^2-42*c^2*d*e+15*e^2)*(c^2*x^2+1)^(3/2)/c^7-3/175*b*(7
*c^2*d-5*e)*e^2*(c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(c^2*x^2+1)^(7/2)/c^7+d^3
*x*(a+b*arcsinh(c*x))+d^2*e*x^3*(a+b*arcsinh(c*x))+3/5*d*e^2*x^5*(a+b*arcs
inh(c*x))+1/7*e^3*x^7*(a+b*arcsinh(c*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))\,dx = a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right) - \frac{b\sqrt{1+c^2x^2}(-240e^3+24c^2e^2(49d+5ex^2)-2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2}{3675c^7} + b\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)\operatorname{arcsinh}(cx)$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 + c^2*x^
2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x
^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3
*x^6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)
*ArcSinh[c*x]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6207, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6207$$

$$-bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{35\sqrt{c^2x^2+1}}dx + d^3x(a+b\operatorname{arcsinh}(cx)) + d^2ex^3(a+b\operatorname{arcsinh}(cx)) + \frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx)) + \frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{35}bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{\sqrt{c^2x^2+1}}dx+d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$\downarrow$ 2331

$$-\frac{1}{70}bc\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{\sqrt{c^2x^2+1}}dx^2+d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$\downarrow$ 2389

$$-\frac{1}{70}bc\int \left(\frac{5(c^2x^2+1)^{5/2}e^3}{c^6}+\frac{3(7c^2d-5e)(c^2x^2+1)^{3/2}e^2}{c^6}+\frac{(35d^2c^4-42dec^2+15e^2)\sqrt{c^2x^2+1}e}{c^6}+\frac{35d^3c^{\cdots}}{\cdots}\right.$$
$$d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))$$

$\downarrow$ 2009

$$d^3x(a+b\text{arcsinh}(cx))+d^2ex^3(a+b\text{arcsinh}(cx))+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))-$$
$$\frac{1}{70}bc\left(\frac{6e^2(c^2x^2+1)^{5/2}(7c^2d-5e)}{5c^8}+\frac{10e^3(c^2x^2+1)^{7/2}}{7c^8}+\frac{2e(c^2x^2+1)^{3/2}(35c^4d^2-42c^2de+15e^2)}{3c^8}+\frac{2\sqrt{c^2x\cdots}}{\cdots}\right.$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/70*(b*c*((2*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 +
 c^2*x^2])/c^8 + (2*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/
2))/(3*c^8) + (6*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(5*c^8) + (10*e^
3*(1 + c^2*x^2)^(7/2))/(7*c^8))) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*
(a + b*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a
+ b*ArcSinh[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.33

method	result
parts	$a\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+x\,d^3\right)+\frac{b\left(\frac{c\,\mathrm{arcsinh}(xc)e^3x^7}{7}+\frac{3c\,\mathrm{arcsinh}(xc)d\,e^2x^5}{5}+c\,\mathrm{arcsinh}(xc)d^2e\,x^3+\right.}{}$
derivativedivides	$\frac{\frac{a\left(x\,c^7d^3+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3x^7c^7\right)}{c^6}+\frac{b\left(\mathrm{arcsinh}(xc)x\,c^7d^3+\mathrm{arcsinh}(xc)d^2c^7e\,x^3+\frac{3\,\mathrm{arcsinh}(xc)d\,c^7e^2x^5}{5}+\frac{\mathrm{arcsinh}(xc)e^3}{7}\right.}{}}{}$
default	$\frac{\frac{a\left(x\,c^7d^3+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3x^7c^7\right)}{c^6}+\frac{b\left(\mathrm{arcsinh}(xc)x\,c^7d^3+\mathrm{arcsinh}(xc)d^2c^7e\,x^3+\frac{3\,\mathrm{arcsinh}(xc)d\,c^7e^2x^5}{5}+\frac{\mathrm{arcsinh}(xc)e^3}{7}\right.}{}}{}$
orering	$\frac{x\left(325c^8e^4x^8+1792c^8d\,e^3x^6+4410c^8d^2e^2x^4-30c^6e^4x^6+9800c^8d^3e\,x^2-294c^6d\,e^3x^4+1225c^8d^4-2450c^6d^2e^2x^2+60c^4\right.}{1225c^8(e\,x^2+d)}$

input
```
int((e*x^2+d)^3*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+x*d^3)+b/c*(1/7*c*arcsinh(x*c)*e^3*
x^7+3/5*c*arcsinh(x*c)*d*e^2*x^5+c*arcsinh(x*c)*d^2*e*x^3+arcsinh(x*c)*x*c
*d^3-1/35/c^6*(5*e^3*(1/7*x^6*c^6*(c^2*x^2+1)^(1/2)-6/35*x^4*c^4*(c^2*x^2+
1)^(1/2)+8/35*x^2*c^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))+35*d^3*c^
6*(c^2*x^2+1)^(1/2)+21*d*c^2*e^2*(1/5*x^4*c^4*(c^2*x^2+1)^(1/2)-4/15*x^2*c
^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+35*d^2*c^4*e*(1/3*x^2*c^2*(c^
2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.09

$$\int \left(d+ex^2\right)^3(a+b\mathrm{arcsinh}(cx))\,dx$$
$$=\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\,(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 +
3675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^
2*e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*e^3*x^6
 + 3675*b*c^6*d^3 - 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^
2 - 10*b*c^4*e^3)*x^4 - 240*b*e^3 + (1225*b*c^6*d^2*e - 588*b*c^4*d*e^2 +
120*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.76

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x \operatorname{asinh}(cx) + bd^2ex^3 \operatorname{asinh}(cx) + \frac{3bde^2x^5 \operatorname{asinh}(cx)}{5} + \frac{be^3x^7 \operatorname{asinh}(cx)}{7} \\ a\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*asinh(c*x) + b*d**2*e*x**3*asinh(c*x) + 3*b*d*e**2*x**5*asinh(c*x
)/5 + b*e**3*x**7*asinh(c*x)/7 - b*d**3*sqrt(c**2*x**2 + 1)/c - b*d**2*e*x
**2*sqrt(c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c)
 - b*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(c**2*x**2 + 1)
/(3*c**3) + 4*b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**4*
sqrt(c**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5)
- 8*b*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(c**2*x**2
+ 1)/(245*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e
**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.30

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{7}\,ae^3x^7 + \frac{3}{5}\,ade^2x^5 + ad^2ex^3 + \frac{1}{3}\left(3\,x^3\,\text{arsinh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2e + \frac{1}{25}\left(15\,x^5\,\text{arsinh}\left(cx\right) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bde^2 + \frac{1}{245}\left(35\,x^7\,\text{arsinh}\left(cx\right) - \left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\sqrt{c^2x^2+1}}{c^8}\right)c\right)be^3 + ad^3x + \frac{\left(cx\,\text{arsinh}\left(cx\right) - \sqrt{c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arcsinh(c*x) -
c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(1
5*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^
2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsinh(c*x) -
 (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x
^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arcs
inh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.48

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))\,dx = \frac{3675asinh(cx)\,b\,c^7d^3x+3675asinh(cx)\,b\,c^7d^2e\,x^3+2205asinh(cx)\,b\,c^7d\,e^2x^5+525asinh(cx)\,b\,c^7e^3x^7-3}{}$$

input
```
int((e*x^2+d)^3*(a+b*asinh(c*x)),x)
```

output
```
(3675*asinh(c*x)*b*c**7*d**3*x + 3675*asinh(c*x)*b*c**7*d**2*e*x**3 + 2205
*asinh(c*x)*b*c**7*d*e**2*x**5 + 525*asinh(c*x)*b*c**7*e**3*x**7 - 3675*sq
rt(c**2*x**2 + 1)*b*c**6*d**3 - 1225*sqrt(c**2*x**2 + 1)*b*c**6*d**2*e*x**
2 - 441*sqrt(c**2*x**2 + 1)*b*c**6*d*e**2*x**4 - 75*sqrt(c**2*x**2 + 1)*b*
c**6*e**3*x**6 + 2450*sqrt(c**2*x**2 + 1)*b*c**4*d**2*e + 588*sqrt(c**2*x*
*2 + 1)*b*c**4*d*e**2*x**2 + 90*sqrt(c**2*x**2 + 1)*b*c**4*e**3*x**4 - 117
6*sqrt(c**2*x**2 + 1)*b*c**2*d*e**2 - 120*sqrt(c**2*x**2 + 1)*b*c**2*e**3*
x**2 + 240*sqrt(c**2*x**2 + 1)*b*e**3 + 3675*a*c**7*d**3*x + 3675*a*c**7*d
**2*e*x**3 + 2205*a*c**7*d*e**2*x**5 + 525*a*c**7*e**3*x**7)/(3675*c**7)
```

3.146 $\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = -\frac{b(15c^4d^2-10c^2de+3e^2)\sqrt{1+c^2x^2}}{15c^5} - \frac{2b(5c^2d-3e)\,e(1+c^2x^2)^{3/2}}{45c^5} - \frac{be^2(1+c^2x^2)^{5/2}}{25c^5} + d^2x(a+b\text{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

output
```
-1/15*b*(15*c^4*d^2-10*c^2*d*e+3*e^2)*(c^2*x^2+1)^(1/2)/c^5-2/45*b*(5*c^2*
d-3*e)*e*(c^2*x^2+1)^(3/2)/c^5-1/25*b*e^2*(c^2*x^2+1)^(5/2)/c^5+d^2*x*(a+b
*arcsinh(c*x))+2/3*d*e*x^3*(a+b*arcsinh(c*x))+1/5*e^2*x^5*(a+b*arcsinh(c*x
))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx = \frac{1}{225}\left(15ax(15d^2+10dex^2+3e^2x^4) - \frac{b\sqrt{1+c^2x^2}(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5} + 15bx(15d^2+10dex^2+3e^2x^4)\,\text{arcsinh}(cx)\right)$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 + c^2*x^2]*(24*e^2 -
 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 +
 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSinh[c*x])/225
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6207, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6207}$$

$$-bc\int \frac{x(3e^2x^4+10dex^2+15d^2)}{15\sqrt{c^2x^2+1}}dx + d^2x(a+b\text{arcsinh}(cx)) + \frac{2}{3}dex^3(a+b\text{arcsinh}(cx)) + \frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{15}bc\int \frac{x(3e^2x^4+10dex^2+15d^2)}{\sqrt{c^2x^2+1}}dx+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}bc\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{c^2x^2+1}}dx^2+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}bc\int \left(\frac{3(c^2x^2+1)^{3/2}e^2}{c^4}+\frac{2(5c^2d-3e)\sqrt{c^2x^2+1}e}{c^4}+\frac{15d^2c^4-10dec^2+3e^2}{c^4\sqrt{c^2x^2+1}}\right)dx^2+d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2x(a+b\text{arcsinh}(cx))+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))-\frac{1}{30}bc\left(\frac{4e(c^2x^2+1)^{3/2}(5c^2d-3e)}{3c^6}+\frac{6e^2(c^2x^2+1)^{5/2}}{5c^6}+\frac{2\sqrt{c^2x^2+1}(15c^4d^2-10c^2de+3e^2)}{c^6}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/30*(b*c*((2*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/c^6 +
(4*(5*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(3*c^6) + (6*e^2*(1 + c^2*x^2)^(
5/2))/(5*c^6))) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c
*x]))/3 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{c\,\operatorname{arcsinh}(xc)e^2x^5}{5}+\frac{2c\,\operatorname{arcsinh}(xc)de\,x^3}{3}+\operatorname{arcsinh}(xc)xc\,d^2-\frac{3e^2\left(\frac{x^4c^4\sqrt{c^2x^2+1}}{5}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(x\,c^5d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2x^5c^5\right)}{c^4}+\frac{b\left(\operatorname{arcsinh}(xc)x\,c^5d^2+\frac{2\,\operatorname{arcsinh}(xc)d\,c^5e\,x^3}{3}+\frac{\operatorname{arcsinh}(xc)e^2x^5c^5}{5}-\frac{e^2\left(\frac{x^4c^4\sqrt{c^2x^2+1}}{5}-\frac{4x^2c^2}{5}\right.}{}\right.}{c^4}}{c}$
default	$\frac{\frac{a\left(x\,c^5d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2x^5c^5\right)}{c^4}+\frac{b\left(\operatorname{arcsinh}(xc)x\,c^5d^2+\frac{2\,\operatorname{arcsinh}(xc)d\,c^5e\,x^3}{3}+\frac{\operatorname{arcsinh}(xc)e^2x^5c^5}{5}-\frac{e^2\left(\frac{x^4c^4\sqrt{c^2x^2+1}}{5}-\frac{4x^2c^2}{5}\right.}{}\right.}{c^4}}{c}$
orering	$\frac{x\left(81e^3x^6c^6+395c^6d\,e^2x^4+1275c^6d^2e\,x^2-12c^4e^3x^4+225c^6d^3-200c^4d\,e^2x^2+900c^4d^2e+48c^2e^3x^2-400c^2d\,e^2+96e^3\right)(}{225c^6\left(e\,x^2+d\right)}$

input
```
int((e*x^2+d)^2*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arcsinh(x*c)*e^2*x^5+2/3*c*ar
csinh(x*c)*d*e*x^3+arcsinh(x*c)*x*c*d^2-1/15/c^4*(3*e^2*(1/5*x^4*c^4*(c^2*
x^2+1)^(1/2)-4/15*x^2*c^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))+15*d^2
*c^4*(c^2*x^2+1)^(1/2)+10*d*c^2*e*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*
x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \left(d+ex^2\right)^2\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-}{225\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^
5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)
) - (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 - 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*
c^4*d*e - 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.63

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{asinh}(cx) + \frac{2bdex^3\operatorname{asinh}(cx)}{3} + \frac{be^2x^5\operatorname{asinh}(cx)}{5} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{2bdex^2\sqrt{c^2x^2+1}}{9c} - \\ a\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asinh(c*x)
 + 2*b*d*e*x**3*asinh(c*x)/3 + b*e**2*x**5*asinh(c*x)/5 - b*d**2*sqrt(c**2
*x**2 + 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(c
**2*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**
2*sqrt(c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 + 1)/(75*c**5),
Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.22

$$\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{1}{5}ae^2x^5 + \frac{2}{3}adex^3 + \frac{2}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde$$
$$+ \frac{1}{75}\left(15x^5\operatorname{arsinh}(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)be^2$$
$$+ ad^2x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arcsinh(c*x)
 - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))
*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int (d + ex^2)^2 (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2)^2 (a + b\text{arcsinh}(cx))\, dx = \int (a + b\,\text{asinh}(c\,x))\, \left(e\,x^2 + d\right)^2\, dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{225 asinh(cx)\,b\,c^5d^2x + 150 asinh(cx)\,b\,c^5de\,x^3 + 45 asinh(cx)\,b\,c^5e^2x^5 - 225\sqrt{c^2x^2+1}\,b\,c^4d^2 - 50\sqrt{c^2x^2}}{}$$

input
```
int((e*x^2+d)^2*(a+b*asinh(c*x)),x)
```

output
```
(225*asinh(c*x)*b*c**5*d**2*x + 150*asinh(c*x)*b*c**5*d*e*x**3 + 45*asinh(
c*x)*b*c**5*e**2*x**5 - 225*sqrt(c**2*x**2 + 1)*b*c**4*d**2 - 50*sqrt(c**2
*x**2 + 1)*b*c**4*d*e*x**2 - 9*sqrt(c**2*x**2 + 1)*b*c**4*e**2*x**4 + 100*
sqrt(c**2*x**2 + 1)*b*c**2*d*e + 12*sqrt(c**2*x**2 + 1)*b*c**2*e**2*x**2 -
 24*sqrt(c**2*x**2 + 1)*b*e**2 + 225*a*c**5*d**2*x + 150*a*c**5*d*e*x**3 +
 45*a*c**5*e**2*x**5)/(225*c**5)
```

3.147 $\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx = -\frac{b(3c^2d-e)\sqrt{1+c^2x^2}}{3c^3}-\frac{be(1+c^2x^2)^{3/2}}{9c^3}+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

output
```
-1/3*b*(3*c^2*d-e)*(c^2*x^2+1)^(1/2)/c^3-1/9*b*e*(c^2*x^2+1)^(3/2)/c^3+d*x
*(a+b*arcsinh(c*x))+1/3*e*x^3*(a+b*arcsinh(c*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx = \frac{1}{9}\left(3ax(3d+ex^2)-\frac{b\sqrt{1+c^2x^2}(-2e+c^2(9d+ex^2))}{c^3}+3bx(3d+ex^2)\,\text{arcsinh}(cx)\right)$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(3*a*x*(3*d + e*x^2) - (b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))/c^
3 + 3*b*x*(3*d + e*x^2)*ArcSinh[c*x])/9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6207, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6207}$$

$$-bc\int \frac{x\left(ex^2+3d\right)}{3\sqrt{c^2x^2+1}}dx+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}bc\int \frac{x\left(ex^2+3d\right)}{\sqrt{c^2x^2+1}}dx+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{1}{6}bc\int \frac{ex^2+3d}{\sqrt{c^2x^2+1}}dx^2+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{6}bc\int \left(\frac{3c^2d-e}{c^2\sqrt{c^2x^2+1}}+\frac{e\sqrt{c^2x^2+1}}{c^2}\right)dx^2+dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$dx(a+b\text{arcsinh}(cx))+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))-\frac{1}{6}bc\left(\frac{2\sqrt{c^2x^2+1}\left(3c^2d-e\right)}{c^4}+\frac{2e\left(c^2x^2+1\right)^{3/2}}{3c^4}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

output
```
-1/6*(b*c*((2*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/c^4 + (2*e*(1 + c^2*x^2)^(3
/2))/(3*c^4))) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\text{arcsinh}(xc)x^3e}{3}+\text{arcsinh}(xc)xcd-\frac{e\left(\frac{x^2c^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)+3d\,c^2\sqrt{c^2x^2+1}}{3c^2}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,x^3c^3\right)}{c^2}+\frac{b\left(\text{arcsinh}(xc)x\,c^3d+\frac{\text{arcsinh}(xc)e\,x^3c^3}{3}-\frac{e\left(\frac{x^2c^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{c^2x^2+1}\right)}{c^2}}{c}$
default	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,x^3c^3\right)}{c^2}+\frac{b\left(\text{arcsinh}(xc)x\,c^3d+\frac{\text{arcsinh}(xc)e\,x^3c^3}{3}-\frac{e\left(\frac{x^2c^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{c^2x^2+1}\right)}{c^2}}{c}$
orering	$\frac{x\left(5e^2x^4c^4+30c^4de\,x^2+9c^4d^2-2c^2e^2x^2+18c^2de-4e^2\right)(a+b\,\text{arcsinh}(xc))}{9c^4(e\,x^2+d)}-\frac{(c^2e\,x^2+9c^2d-2e)\,(c^2x^2+1)\left(2ex(a+b\right.}{9c^4(e\,x^2+d)}$

input
```
int((e*x^2+d)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arcsinh(x*c)*x^3*e+arcsinh(x*c)*x*c*d-1/3/c^2
*(e*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c^2*(c^2*x^2
+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int\left(d+ex^2\right)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{3\,ac^3ex^3+9\,ac^3dx+3\,(bc^3ex^3+3\,bc^3dx)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^2ex^2+9\,bc^2d-2\,be)\sqrt{c^2x^2+1}}{9\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
 sqrt(c^2*x^2 + 1)) - (b*c^2*e*x^2 + 9*b*c^2*d - 2*b*e)*sqrt(c^2*x^2 + 1))
/c^3
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx$$
$$= \begin{cases} adx + \frac{aex^3}{3} + bdx\operatorname{asinh}(cx) + \frac{bex^3\operatorname{asinh}(cx)}{3} - \frac{bd\sqrt{c^2x^2+1}}{c} - \frac{bex^2\sqrt{c^2x^2+1}}{9c} + \frac{2be\sqrt{c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asinh(c*x) + b*e*x**3*asinh(c*x)/3 -
 b*d*sqrt(c**2*x**2 + 1)/c - b*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) + 2*b*e*sq
rt(c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{1}{3}aex^3 + \frac{1}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)be$$
$$+ adx + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*s
qrt(c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1)
)*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d + ex^2)(a + b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2)(a + b\operatorname{arcsinh}(cx))\,dx = \int (a + b\operatorname{asinh}(c\,x))\,\left(e\,x^2 + d\right)\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \left(d+ex^2\right)\left(a+b\operatorname{arcsinh}(cx)\right)dx$$
$$=\frac{9asinh(cx)\,b\,c^3dx+3asinh(cx)\,b\,c^3e\,x^3-9\sqrt{c^2x^2+1}\,b\,c^2d-\sqrt{c^2x^2+1}\,b\,c^2e\,x^2+2\sqrt{c^2x^2+1}\,be+9a\,c^3}{9c^3}$$

input
```
int((e*x^2+d)*(a+b*asinh(c*x)),x)
```

output
```
(9*asinh(c*x)*b*c**3*d*x + 3*asinh(c*x)*b*c**3*e*x**3 - 9*sqrt(c**2*x**2 +
 1)*b*c**2*d - sqrt(c**2*x**2 + 1)*b*c**2*e*x**2 + 2*sqrt(c**2*x**2 + 1)*b
*e + 9*a*c**3*d*x + 3*a*c**3*e*x**3)/(9*c**3)
```

3.148 $\int (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a+b\text{arcsinh}(cx))\,dx = ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\text{arcsinh}(cx)$$

output
```
a*x-b*(c^2*x^2+1)^(1/2)/c+b*x*arcsinh(c*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a+b\text{arcsinh}(cx))\,dx = ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\text{arcsinh}(cx)$$

input
```
Integrate[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}$$

input
```
Int[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
orering	$x(a+b\ \text{arcsinh}\,(xc))-\frac{b\sqrt{c^2x^2+1}}{c}$	29
default	$xa+\frac{b\left(xc\ \text{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	31
parts	$xa+\frac{b\left(xc\ \text{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	31
derivativedivides	$\frac{axc+b\left(xc\ \text{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	33

input
```
int(a+b*arcsinh(x*c),x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arcsinh(x*c))-b*(c^2*x^2+1)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a+b\text{arcsinh}(cx))\,dx=\frac{bcx\log\left(cx+\sqrt{c^2x^2+1}\right)+acx-\sqrt{c^2x^2+1}b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

output
```
(b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (a+b\text{arcsinh}(cx))\,dx=ax+b\left(\begin{cases}x\operatorname{asinh}(cx)-\frac{\sqrt{c^2x^2+1}}{c} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*asinh(c*x),x)
```

output
```
a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a+b\text{arcsinh}(cx))\,dx = ax + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (a+b\text{arcsinh}(cx))\,dx = \left(x\log\left(cx+\sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)b + ax$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

output
```
(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (a+b\text{arcsinh}(cx))\,dx = a\,x - \frac{b\,\sqrt{c^2\,x^2+1}}{c} + b\,x\,\text{asinh}(c\,x)$$

input
```
int(a + b*asinh(c*x),x)
```

output
```
a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\, dx = \frac{asinh(cx)\, bcx - \sqrt{c^2x^2 + 1}\, b + acx}{c}$$

input
```
int(a+b*asinh(c*x),x)
```

output
```
(asinh(c*x)*b*c*x - sqrt(c**2*x**2 + 1)*b + a*c*x)/c
```

3.149 $\int \frac{a+b\,\text{arcsinh}(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 485

$$\begin{aligned}
\int \frac{a+b\,\text{arcsinh}(cx)}{d+ex^2}\,dx &= \frac{(a+b\,\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{(a+b\,\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{(a+b\,\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{(a+b\,\text{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}$$

output
```
1/2*(a+b*arcsinh(c*x))*ln(1-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-
(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+e^(1/2)*
(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2
)+1/2*(a+b*arcsinh(c*x))*ln(1-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2
)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+e^(1/2
)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1
/2)-1/2*b*polylog(2,-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d
+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,e^(1/2)*(c*x+(c^2*x^2+1)^(1
/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-
e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2
)/e^(1/2)+1/2*b*polylog(2,e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-
c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.89

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx = \frac{2a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-b\sqrt{d}\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)+b\sqrt{d}\text{arcsinh}(cx)\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(c}}{-c\sqrt{-d}+\sqrt{-c}}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

output
```
(2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 +
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*A
rcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d
) + e])] + b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^Ar
cSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*PolyLog[2, (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*PolyLo
g[2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - b*Sq
rt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e]))] + b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1
+ (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]
*Sqrt[e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[
2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt
[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 34.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.46

method	result
parts	$\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{bc\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)} \frac{\mathrm{arcsinh}(xc)\ln\left(\frac{\_R1-xc-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\ldots}{\ldots}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)}\ldots\right.}{2}$
derivativedivides	$\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + bc^2\left(\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)} \frac{\mathrm{arcsinh}(xc)\ln\left(\frac{\_R1-xc-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1\ldots}{}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)}}{2}\ldots\right.\right.$
default	$\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + bc^2\left(\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)} \frac{\mathrm{arcsinh}(xc)\ln\left(\frac{\_R1-xc-\sqrt{c^2x^2+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1\ldots}{}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)}}{2}\ldots\right.\right.$

input
```
int((a+b*arcsinh(x*c))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c*sum(1/_R1/(_R1^2*e+2*c^2*d-e
)*(arcsinh(x*c)*ln((_R1-x*c-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-x*c-(c^2*x^
2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/2*b*c*sum(_R1
/(_R1^2*e+2*c^2*d-e)*(arcsinh(x*c)*ln((_R1-x*c-(c^2*x^2+1)^(1/2))/_R1)+dil
og((_R1-x*c-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+
e))
```

Fricas [F]

$$\int \frac{a + b\mathrm{arcsinh}(cx)}{d + ex^2}\, dx = \int \frac{b\,\mathrm{arsinh}\,(cx) + a}{ex^2 + d}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{d + ex^2}\, dx = \int \frac{a + b\operatorname{asinh}(cx)}{d + ex^2}\, dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{d + ex^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex^2}\,dx=\frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a+\left(\int \frac{asinh(cx)}{e\,x^2+d}dx\right)bde}{de}$$

input
```
int((a+b*asinh(c*x))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(asinh(c*x)/(d + e*x
**2),x)*b*d*e)/(d*e)
```

3.150 $\int \frac{a+b\,\mathbf{arcsinh}(cx)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 707

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^2}\,dx = -\frac{a+b\operatorname{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\operatorname{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}$$
$$-\frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\arctan\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$
$$-\frac{(a+b\operatorname{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{(a+b\operatorname{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{\operatorname{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

output
```
-1/4*(a+b*arcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*(a+b*arcsinh(
c*x))/d/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)-1/4*b*c*arctan((e^(1/2)-c^2*(-d)^(1
/2)*x)/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)-1/4*b*
c*arctan((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(
c^2*d-e)^(1/2)/e^(1/2)-1/4*(a+b*arcsinh(c*x))*ln(1-e^(1/2)*(c*x+(c^2*x^2+1
)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcs
inh(c*x))*ln(1+e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1
/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arcsinh(c*x))*ln(1-e^(1/2)*(c*x+(c^2*x^2
+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*ar
csinh(c*x))*ln(1+e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^
(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2)
)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,e^(1
/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^
(1/2)+1/4*b*polylog(2,-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2
*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,e^(1/2)*(c*x+(c^2*x^2+1)^
(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 622, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}\right.$$
$$+\frac{b\left(-2\sqrt{d}\left(-\frac{\text{arcsinh}(cx)}{i\sqrt{d}+\sqrt{e}x}+\frac{c\arctan\left(\frac{\sqrt{e}-ic^2\sqrt{d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right)+2i\sqrt{d}\left(\frac{\text{arcsinh}(cx)}{\sqrt{d}+i\sqrt{e}x}+\frac{c\text{arctanh}\left(\frac{i\sqrt{e}-c^2\sqrt{d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d-e}}\right)+\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*(-2*Sqrt[d]*(-(ArcSinh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*ArcTan[(Sq
rt[e] - I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d
- e]) + (2*I)*Sqrt[d]*(ArcSinh[c*x]/(Sqrt[d] + I*Sqrt[e]*x) + (c*ArcTanh[(
I*Sqrt[e] - c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*
d - e]) + I*(ArcSinh[c*x]*(-ArcSinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c
*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) + e])] + Log[1 + (Sqrt[e]*E^ArcSinh[c*x]
)/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[
c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])] + 2*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]))]) - I*(ArcSinh[c*x]*(-Arc
Sinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c
^2*d) + e])] + Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) + e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sq
rt[-(c^2*d) + e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] +
 Sqrt[-(c^2*d) + e])])))/(4*d^(3/2)*Sqrt[e]))/2
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(-\frac{e(a + b\text{arcsinh}(cx))}{2d(-de - e^2x^2)} - \frac{e(a + b\text{arcsinh}(cx))}{4d\left(\sqrt{-d}\sqrt{e} - ex\right)^2} - \frac{e(a + b\text{arcsinh}(cx))}{4d\left(\sqrt{-d}\sqrt{e} + ex\right)^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\text{arcsinh}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{bc\arctan\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2x^2+1}\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\sqrt{c^2d-e}}-\frac{bc\arctan\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{c^2x^2+1}\sqrt{c^2d-e}}\right)}{4d\sqrt{e}\sqrt{c^2d-e}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcSinh[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcS
inh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^
2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*S
qrt[e]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 +
 c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2
)*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x
])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4
*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLo
g[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-
d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) -
 (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])]
)/(4*(-d)^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 51.42 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.20

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\operatorname{arcsinh}(xc)x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{c^2\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(xc)\ln\left(-\cdots\right.}{\cdots}\cdots\right.}{4d}\cdots\right)}{\cdots}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arcsinh}(xc)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(xc)\ln\left(-\cdots\right.}{\cdots}\cdots}{4c^2d}\cdots\right.}{\cdots}$
default	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arcsinh}(xc)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\operatorname{arcsinh}(xc)\ln\left(-\cdots\right.}{\cdots}\cdots}{4c^2d}\cdots\right.}{\cdots}$

input
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arcsinh(x*c)*x/d/(c^2*e*x^2+c^2*d)+1/4/d*c^2*sum(1/_R1/(_R1^2*e+2*c^2*d
-e)*(arcsinh(x*c)*ln((_R1-x*c-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-x*c-(c^2*
x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/4/d*c^2*sum
(_R1/(_R1^2*e+2*c^2*d-e)*(arcsinh(x*c)*ln((_R1-x*c-(c^2*x^2+1)^(1/2))/_R1)
+dilog((_R1-x*c-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_
Z^2+e))+1/2*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(-2*(c^2*d*(c^
2*d-e))^(1/2)*c^2*d+2*c^4*d^2-2*c^2*d*e+(c^2*d*(c^2*d-e))^(1/2)*e)*c^2*arc
tan(e*(x*c+(c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1
/2))/d/(c^2*d-e)/e^3-1/2*((2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(
2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*arctan(e*(x*c+(c^2*x^2+1)^(1/2))/((2*
c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2))*c^2/d/e^3+1/2*(-(2*c^2*d-2*(c
^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*(c^2*d*(c^2*d-e))^(1/2)*c^2*d+2*c^4*d
^2-2*c^2*d*e-(c^2*d*(c^2*d-e))^(1/2)*e)*c^2*arctanh(e*(x*c+(c^2*x^2+1)^(1/
2))/((-2*c^2*d+2*(c^2*d*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/(c^2*d-e)/e^3-1/2*
(-(2*c^2*d-2*(c^2*d*(c^2*d-e))^(1/2)-e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d-
e))^(1/2)-e)*arctanh(e*(x*c+(c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d-
e))^(1/2)+e)*e)^(1/2))*c^2/d/e^3)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} dx = \int \frac{b \operatorname{arsinh}(cx) + a}{(ex^2 + d)^2} dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx = \int \frac{a + b \operatorname{asinh}{(cx)}}{(d + ex^2)^2} \, dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^2} \, dx = \int \frac{b \operatorname{arsinh}{(cx)} + a}{(ex^2 + d)^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int\frac{asinh(cx)}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^3e+2\left(\int\frac{asinh(cx)}{e^2x^4+2de\,x^2+d^2}dx\right.}{2d^2e\,(e\,x^2+d)}$$

input
```
int((a+b*asinh(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int(asinh(c*x)/(d**2 + 2*d*e*x**2 +
e**2*x**4),x)*b*d**3*e + 2*int(asinh(c*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),
x)*b*d**2*e**2*x**2 + a*d*e*x)/(2*d**2*e*(d + e*x**2))
```

3.151 $\int (d+ex^2)^3 (a+b\,\mathrm{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 559

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))^2\,dx = 2b^2d^3x-\frac{4b^2d^2ex}{3c^2}+\frac{16b^2de^2x}{25c^4}-\frac{32b^2e^3x}{245c^6}+\frac{2}{9}b^2d^2ex^3 -\frac{8b^2de^2x^3}{75c^2}+\frac{16b^2e^3x^3}{735c^4}+\frac{6}{125}b^2de^2x^5-\frac{12b^2e^3x^5}{1225c^2} +\frac{2}{343}b^2e^3x^7-\frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} +\frac{4bd^2e\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} -\frac{16bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^5} +\frac{32be^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^7} -\frac{2bd^2ex^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} +\frac{8bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c^3} -\frac{16be^3x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^5} -\frac{6bde^2x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{25c} +\frac{12be^3x^4\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{245c^3} -\frac{2be^3x^6\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{49c} +d^3x(a+b\operatorname{arcsinh}(cx))^2 +d^2ex^3(a+b\operatorname{arcsinh}(cx))^2 +\frac{3}{5}de^2x^5(a+b\operatorname{arcsinh}(cx))^2 +\frac{1}{7}e^3x^7(a+b\operatorname{arcsinh}(cx))^2$$

output
```
2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^6
+2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2
*d*e^2*x^5-12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7-2*b*d^3*(c^2*x^2+1)^(
1/2)*(a+b*arcsinh(c*x))/c+4/3*b*d^2*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))
/c^3-16/25*b*d*e^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^5+32/245*b*e^3*(
c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^7-2/3*b*d^2*e*x^2*(c^2*x^2+1)^(1/2)*
(a+b*arcsinh(c*x))/c+8/25*b*d*e^2*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))
/c^3-16/245*b*e^3*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^5-6/25*b*d*e^
2*x^4*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+12/245*b*e^3*x^4*(c^2*x^2+1)^
(1/2)*(a+b*arcsinh(c*x))/c^3-2/49*b*e^3*x^6*(c^2*x^2+1)^(1/2)*(a+b*arcsinh
(c*x))/c+d^3*x*(a+b*arcsinh(c*x))^2+d^2*e*x^3*(a+b*arcsinh(c*x))^2+3/5*d*e
^2*x^5*(a+b*arcsinh(c*x))^2+1/7*e^3*x^7*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.79

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{11025a^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-210ab\sqrt{1+c^2x^2}(-240e^3+24c^2e^2(49d+5ex^2)-2c^4}{}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*
a*b*Sqrt[1 + c^2*x^2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1
225*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441
*d*e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(-25200*e^3 + 840*c^2*e^2*(147*d + 5
*e*x^2) - 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3
+ 42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*
(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 + c^2*x^2]*(
-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2 +
 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6
)))*ArcSinh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 +
 5*e^3*x^6)*ArcSinh[c*x]^2)/(385875*c^7)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2 dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(d^3(a+b\text{arcsinh}(cx))^2+3d^2ex^2(a+b\text{arcsinh}(cx))^2+3de^2x^4(a+b\text{arcsinh}(cx))^2+e^3x^6(a+b\text{arcsinh}(cx))^2\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2bd^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{2bd^2ex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}-\frac{6bde^2x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c}-\frac{2be^3x^6\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{49c}+\frac{32be^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^7}-\frac{16bde^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c^5}-\frac{16be^3x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^5}+\frac{4bd^2e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c^3}+\frac{8bde^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c^3}+\frac{12be^3x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{245c^3}+d^3x(a+b\text{arcsinh}(cx))^2+d^2ex^3(a+b\text{arcsinh}(cx))^2+\frac{3}{5}de^2x^5(a+b\text{arcsinh}(cx))^2+\frac{1}{7}e^3x^7(a+b\text{arcsinh}(cx))^2-\frac{32b^2e^3x}{245c^6}+\frac{16b^2de^2x}{25c^4}+\frac{16b^2e^3x^3}{735c^4}-\frac{4b^2d^2ex}{3c^2}-\frac{8b^2de^2x^3}{75c^2}-\frac{12b^2e^3x^5}{1225c^2}+2b^2d^3x+\frac{2}{9}b^2d^2ex^3+\frac{6}{125}b^2de^2x^5+\frac{2}{343}b^2e^3x^7$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^
2*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (1
6*b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*
c^2) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b
*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*
x^2]*(a + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(245*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
(25*c) + (12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) -
 (2*b*e^3*x^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a +
b*ArcSinh[c*x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b
*ArcSinh[c*x])^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{a^2\left(x\,c^7 d^3+d^2 c^7 e\,x^3+\frac{3}{5} d\,c^7 e^2 x^5+\frac{1}{7} e^3 x^7 c^7\right)}{c^6}+\frac{b^2\left(c^6 d^3\left(\operatorname{arcsinh}(xc)^2 xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{c^4 d^2 e\left(9\operatorname{arcsinh}(xc)^2 x^3\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(x\,c^7 d^3+d^2 c^7 e\,x^3+\frac{3}{5} d\,c^7 e^2 x^5+\frac{1}{7} e^3 x^7 c^7\right)}{c^6}+\frac{b^2\left(c^6 d^3\left(\operatorname{arcsinh}(xc)^2 xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{c^4 d^2 e\left(9\operatorname{arcsinh}(xc)^2 x^3\right.}{}\right.}{}}{}$
parts	$a^2\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+x\,d^3\right)+\frac{b^2\left(55125\operatorname{arcsinh}(xc)^2 x^7 c^7 e^3+231525\operatorname{arcsinh}(xc)^2 x^5 c^7 d\,e^2+38\right.}{}$
orering	$\frac{x\left(47625c^8e^5x^{10}+328917c^8d\,e^4x^8+1128666c^8d^2e^3x^6-10080c^6e^5x^8+5951050c^8d^3e^2x^4-146016c^6d\,e^4x^6-385875c^8d\right.}{}$

input
```
int((e*x^2+d)^3*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/c^6*(x*c^7*d^3+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*x^7*c^7)+b
^2/c^6*(c^6*d^3*(arcsinh(x*c)^2*x*c-2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*c
)+1/9*c^4*d^2*e*(9*arcsinh(x*c)^2*x^3*c^3-6*arcsinh(x*c)*(c^2*x^2+1)^(1/2)
*x^2*c^2+2*x^3*c^3+12*arcsinh(x*c)*(c^2*x^2+1)^(1/2)-12*x*c)+1/375*c^2*d*e
^2*(225*arcsinh(x*c)^2*x^5*c^5-90*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^4*c^4+1
8*x^5*c^5+120*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^2*c^2-40*x^3*c^3-240*arcsin
h(x*c)*(c^2*x^2+1)^(1/2)+240*x*c)+1/25725*e^3*(3675*arcsinh(x*c)^2*x^7*c^7
-1050*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^6*c^6+150*x^7*c^7+1260*arcsinh(x*c)
*(c^2*x^2+1)^(1/2)*x^4*c^4-252*x^5*c^5-1680*arcsinh(x*c)*(c^2*x^2+1)^(1/2)
*x^2*c^2+560*x^3*c^3+3360*arcsinh(x*c)*(c^2*x^2+1)^(1/2)-3360*x*c))+2*a*b/
c^6*(arcsinh(x*c)*x*c^7*d^3+arcsinh(x*c)*d^2*c^7*e*x^3+3/5*arcsinh(x*c)*d*
c^7*e^2*x^5+1/7*arcsinh(x*c)*e^3*x^7*c^7-1/7*e^3*(1/7*x^6*c^6*(c^2*x^2+1)^
(1/2)-6/35*x^4*c^4*(c^2*x^2+1)^(1/2)+8/35*x^2*c^2*(c^2*x^2+1)^(1/2)-16/35*
(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^2+1)^(1/2)-3/5*d*c^2*e^2*(1/5*x^4*c^4*(c
^2*x^2+1)^(1/2)-4/15*x^2*c^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-d^2
*c^4*e*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.05

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{1125\,(49\,a^2+2\,b^2)c^7e^3x^7+189\,(49\,(25\,a^2+2\,b^2)c^7de^2-20\,b^2c^5e^3)x^5+35\,(1225\,(9\,a^2+2\,b^2)c^7d^2e-117}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7
*d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e - 1176*b
^2*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^
7*d*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*
x^2 + 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e + 2352*
b^2*c^3*d*e^2 - 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7
*d*e^2*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3
*x^6 + 3675*b^2*c^6*d^3 - 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 - 240*b^
2*e^3 + 9*(49*b^2*c^6*d*e^2 - 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e -
588*b^2*c^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqr
t(c^2*x^2 + 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 - 2450*a*b*c^
4*d^2*e + 1176*a*b*c^2*d*e^2 - 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 - 10*a*b*
c^4*e^3)*x^4 + (1225*a*b*c^6*d^2*e - 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*
x^2)*sqrt(c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.77

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e*
*3*x**7/7 + 2*a*b*d**3*x*asinh(c*x) + 2*a*b*d**2*e*x**3*asinh(c*x) + 6*a*b
*d*e**2*x**5*asinh(c*x)/5 + 2*a*b*e**3*x**7*asinh(c*x)/7 - 2*a*b*d**3*sqrt
(c**2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 + 1)/(3*c) - 6*a*b*d*
e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 + 1)
/(49*c) + 4*a*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c**3) + 8*a*b*d*e**2*x**2*sq
rt(c**2*x**2 + 1)/(25*c**3) + 12*a*b*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c*
*3) - 16*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(
c**2*x**2 + 1)/(245*c**5) + 32*a*b*e**3*sqrt(c**2*x**2 + 1)/(245*c**7) + b
**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*asinh(c*x)**2
+ 2*b**2*d**2*e*x**3/9 + 3*b**2*d*e**2*x**5*asinh(c*x)**2/5 + 6*b**2*d*e**
2*x**5/125 + b**2*e**3*x**7*asinh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**
2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**
2 + 1)*asinh(c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(25*c) - 2*b**2*e**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/(49*c) - 4*b**2
*d**2*e*x/(3*c**2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(122
5*c**2) + 4*b**2*d**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 8*b**2*d
*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c**3) + 12*b**2*e**3*x**4*sq
rt(c**2*x**2 + 1)*asinh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*
b**2*e**3*x**3/(735*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.22

$$\int (d+ex^2)^3 (a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*e^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arcsi
nh(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arcsinh(c*x)^2 + a^2*d^2*e*x
^3 + b^2*d^3*x*arcsinh(c*x)^2 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^
2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c
^2)*b^2*d^2*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 -
 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/3
75*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*
b^2*d*e^2 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*
sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2
+ 1)/c^8)*c)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(
c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/
c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^
6)*b^2*e^3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x
+ 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)^3 (a+b\text{arcsinh}(cx))^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx=\int \left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^3,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^3 (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{3675asinh(cx)^2\,b^2c^7d^3x-7350\sqrt{c^2x^2+1}\,asinh(cx)\,b^2c^6d^3+7350asinh(cx)\,ab\,c^7d^3x+7350asinh(cx)\,ab}{}$$

input
```
int((e*x^2+d)^3*(a+b*asinh(c*x))^2,x)
```

output
```
(3675*asinh(c*x)**2*b**2*c**7*d**3*x - 7350*sqrt(c**2*x**2 + 1)*asinh(c*x)
*b**2*c**6*d**3 + 7350*asinh(c*x)*a*b*c**7*d**3*x + 7350*asinh(c*x)*a*b*c*
*7*d**2*e*x**3 + 4410*asinh(c*x)*a*b*c**7*d*e**2*x**5 + 1050*asinh(c*x)*a*
b*c**7*e**3*x**7 - 7350*sqrt(c**2*x**2 + 1)*a*b*c**6*d**3 - 2450*sqrt(c**2
*x**2 + 1)*a*b*c**6*d**2*e*x**2 - 882*sqrt(c**2*x**2 + 1)*a*b*c**6*d*e**2*
x**4 - 150*sqrt(c**2*x**2 + 1)*a*b*c**6*e**3*x**6 + 4900*sqrt(c**2*x**2 +
1)*a*b*c**4*d**2*e + 1176*sqrt(c**2*x**2 + 1)*a*b*c**4*d*e**2*x**2 + 180*s
qrt(c**2*x**2 + 1)*a*b*c**4*e**3*x**4 - 2352*sqrt(c**2*x**2 + 1)*a*b*c**2*
d*e**2 - 240*sqrt(c**2*x**2 + 1)*a*b*c**2*e**3*x**2 + 480*sqrt(c**2*x**2 +
 1)*a*b*e**3 + 3675*int(asinh(c*x)**2*x**6,x)*b**2*c**7*e**3 + 11025*int(a
sinh(c*x)**2*x**4,x)*b**2*c**7*d*e**2 + 11025*int(asinh(c*x)**2*x**2,x)*b*
*2*c**7*d**2*e + 3675*a**2*c**7*d**3*x + 3675*a**2*c**7*d**2*e*x**3 + 2205
*a**2*c**7*d*e**2*x**5 + 525*a**2*c**7*e**3*x**7 + 7350*b**2*c**7*d**3*x)/
(3675*c**7)
```

3.152 $\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 329

$$\begin{aligned}\int (d+ex^2)^2 (a+b\text{arcsinh}(cx))^2\,dx &= 2b^2d^2x-\frac{8b^2dex}{9c^2}+\frac{16b^2e^2x}{75c^4}+\frac{4}{27}b^2dex^3-\frac{8b^2e^2x^3}{225c^2}\\ &\quad+\frac{2}{125}b^2e^2x^5-\frac{2bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}\\ &\quad+\frac{8bde\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3}\\ &\quad-\frac{16be^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^5}\\ &\quad-\frac{4bdex^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c}\\ &\quad+\frac{8be^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{75c^3}\\ &\quad-\frac{2be^2x^4\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{25c}\\ &\quad+d^2x(a+b\text{arcsinh}(cx))^2\\ &\quad+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2\\ &\quad+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2\end{aligned}$$

output
```
2*b^2*d^2*x-8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3-8/225*b
^2*e^2*x^3/c^2+2/125*b^2*e^2*x^5-2*b*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*
x))/c+8/9*b*d*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^3-16/75*b*e^2*(c^2*
x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^5-4/9*b*d*e*x^2*(c^2*x^2+1)^(1/2)*(a+b*a
rcsinh(c*x))/c+8/75*b*e^2*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^3-2/2
5*b*e^2*x^4*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+d^2*x*(a+b*arcsinh(c*x)
)^2+2/3*d*e*x^3*(a+b*arcsinh(c*x))^2+1/5*e^2*x^5*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4)-30ab\sqrt{1+c^2x^2}(24e^2-4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 + c^2*x^2
]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x
^4)) + 2*b^2*c*x*(360*e^2 - 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*
d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^
4) + b*Sqrt[1 + c^2*x^2]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2
 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcSinh[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e
*x^2 + 3*e^2*x^4)*ArcSinh[c*x]^2)/(3375*c^5)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(d^2(a+b\text{arcsinh}(cx))^2+2dex^2(a+b\text{arcsinh}(cx))^2+e^2x^4(a+b\text{arcsinh}(cx))^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2bd^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c}-\frac{2be^2x^4\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{25c}-\frac{16be^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{75c^5}+\frac{8bde\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^3}+\frac{8be^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{75c^3}+d^2x(a+b\text{arcsinh}(cx))^2+\frac{2}{3}dex^3(a+b\text{arcsinh}(cx))^2+\frac{1}{5}e^2x^5(a+b\text{arcsinh}(cx))^2+\frac{16b^2e^2x}{75c^4}-\frac{8b^2dex}{9c^2}-\frac{8b^2e^2x^3}{225c^2}+2b^2d^2x+\frac{4}{27}b^2dex^3+\frac{2}{125}b^2e^2x^5$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e
*x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(75*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (
8*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^
4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*
x])^2 + (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x
])^2)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{a^2\left(x c^5 d^2+\frac{2}{3} d c^5 e x^3+\frac{1}{5} e^2 x^5 c^5\right)}{c^4}+\frac{b^2\left(c^4 d^2\left(\operatorname{arcsinh}(xc)^2 xc-2 \operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{2c^2 de\left(9 \operatorname{arcsinh}(xc)^2 x^3 c^3-6 \operatorname{arcsi}\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(x c^5 d^2+\frac{2}{3} d c^5 e x^3+\frac{1}{5} e^2 x^5 c^5\right)}{c^4}+\frac{b^2\left(c^4 d^2\left(\operatorname{arcsinh}(xc)^2 xc-2 \operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{2c^2 de\left(9 \operatorname{arcsinh}(xc)^2 x^3 c^3-6 \operatorname{arcsi}\right.}{}\right.}{}}{}$
parts	$a^2\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b^2\left(675 \operatorname{arcsinh}(xc)^2 x^5 c^5 e^2+2250 \operatorname{arcsinh}(xc)^2 x^3 c^5 de+3375 \operatorname{arcsinh}(xc)^2 x c^5\right.}{}$
orering	$\frac{x\left(1647c^6e^4x^8+10924c^6d\,e^3x^6+77050c^6d^2e^2x^4-600c^4e^4x^6-4500c^6d^3e\,x^2-21808c^4d\,e^3x^4+3375c^6d^4+89000c^4d^2e^2\right.}{3375c^6\left(e\,x^2+d\right)^2}$

input
```
int((e*x^2+d)^2*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/c*(a^2/c^4*(x*c^5*d^2+2/3*d*c^5*e*x^3+1/5*e^2*x^5*c^5)+b^2/c^4*(c^4*d^2*
(arcsinh(x*c)^2*x*c-2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*c)+2/27*c^2*d*e*(
9*arcsinh(x*c)^2*x^3*c^3-6*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*x^3*c^
3+12*arcsinh(x*c)*(c^2*x^2+1)^(1/2)-12*x*c)+1/1125*e^2*(225*arcsinh(x*c)^2
*x^5*c^5-90*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^4*c^4+18*x^5*c^5+120*arcsinh(
x*c)*(c^2*x^2+1)^(1/2)*x^2*c^2-40*x^3*c^3-240*arcsinh(x*c)*(c^2*x^2+1)^(1/
2)+240*x*c))+2*a*b/c^4*(arcsinh(x*c)*x*c^5*d^2+2/3*arcsinh(x*c)*d*c^5*e*x^
3+1/5*arcsinh(x*c)*e^2*x^5*c^5-1/5*e^2*(1/5*x^4*c^4*(c^2*x^2+1)^(1/2)-4/15
*x^2*c^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-d^2*c^4*(c^2*x^2+1)^(1/
2)-2/3*d*c^2*e*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.16

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{27\,(25\,a^2+2\,b^2)c^5e^2x^5+10\,(25\,(9\,a^2+2\,b^2)c^5de-12\,b^2c^3e^2)x^3+225\,(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c}{}$$

input

```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output

```
1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e -
 12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^
2*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^
2 - 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c
^5*d*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 - 10
0*b^2*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e - 6*b^2*c^2*e^2)*x^2)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a
*b*c^4*d^2 - 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e - 6*a*b*c^2*
e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.81

$$\int \left(d+ex^2\right)^2 (a+b\operatorname{arcsinh}(cx))^2\, dx$$
$$= \begin{cases} a^2d^2x + \frac{2a^2dex^3}{3} + \frac{a^2e^2x^5}{5} + 2abd^2x\operatorname{asinh}(cx) + \frac{4abdex^3\operatorname{asinh}(cx)}{3} + \frac{2abe^2x^5\operatorname{asinh}(cx)}{5} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{4abdex^2}{9} \\ a^2\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2
*x*asinh(c*x) + 4*a*b*d*e*x**3*asinh(c*x)/3 + 2*a*b*e**2*x**5*asinh(c*x)/5
 - 2*a*b*d**2*sqrt(c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(
9*c) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) + 8*a*b*d*e*sqrt(c**2*x*
*2 + 1)/(9*c**3) + 8*a*b*e**2*x**2*sqrt(c**2*x**2 + 1)/(75*c**3) - 16*a*b*
e**2*sqrt(c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d*
*2*x + 2*b**2*d*e*x**3*asinh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x*
*5*asinh(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 + 1
)*asinh(c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c) - 2*
b**2*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**
2) - 8*b**2*e**2*x**3/(225*c**2) + 8*b**2*d*e*sqrt(c**2*x**2 + 1)*asinh(c*
x)/(9*c**3) + 8*b**2*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**3) +
16*b**2*e**2*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75
*c**5), Ne(c, 0)), (a**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.30

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{1}{5}\,b^2e^2x^5\,\text{arsinh}\,(cx)^2 + \frac{1}{5}\,a^2e^2x^5 + \frac{2}{3}\,b^2dex^3\,\text{arsinh}\,(cx)^2 + \frac{2}{3}\,a^2dex^3 + b^2d^2x\,\text{arsinh}\,(cx)^2 + \frac{4}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abde - \frac{4}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2de + \frac{2}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)abe^2 - \frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\,\text{arsinh}\,(cx) - \frac{9\,c^4x^5-20\,c^2x^3+120\,x}{c^4} + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2d^2x + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*e^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arcsinh
(c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arcsinh(c*x)^2 + 4/9*(3*x^3*arcsinh(
c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e -
4/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*
x) - (c^2*x^3 - 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^
2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6
)*c)*a*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 +
1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2
*x^3 + 120*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)
/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^2 (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{225 asinh(cx)^2\,b^2c^5d^2x - 450\sqrt{c^2x^2+1}\,asinh(cx)\,b^2c^4d^2 + 450 asinh(cx)\,ab\,c^5d^2x + 300 asinh(cx)\,ab\,c^5de}{}$$

input
```
int((e*x^2+d)^2*(a+b*asinh(c*x))^2,x)
```

output
```
(225*asinh(c*x)**2*b**2*c**5*d**2*x - 450*sqrt(c**2*x**2 + 1)*asinh(c*x)*b
**2*c**4*d**2 + 450*asinh(c*x)*a*b*c**5*d**2*x + 300*asinh(c*x)*a*b*c**5*d
*e*x**3 + 90*asinh(c*x)*a*b*c**5*e**2*x**5 - 450*sqrt(c**2*x**2 + 1)*a*b*c
**4*d**2 - 100*sqrt(c**2*x**2 + 1)*a*b*c**4*d*e*x**2 - 18*sqrt(c**2*x**2 +
 1)*a*b*c**4*e**2*x**4 + 200*sqrt(c**2*x**2 + 1)*a*b*c**2*d*e + 24*sqrt(c*
*2*x**2 + 1)*a*b*c**2*e**2*x**2 - 48*sqrt(c**2*x**2 + 1)*a*b*e**2 + 225*in
t(asinh(c*x)**2*x**4,x)*b**2*c**5*e**2 + 450*int(asinh(c*x)**2*x**2,x)*b**
2*c**5*d*e + 225*a**2*c**5*d**2*x + 150*a**2*c**5*d*e*x**3 + 45*a**2*c**5*
e**2*x**5 + 450*b**2*c**5*d**2*x)/(225*c**5)
```

3.153 $\int (d+ex^2)(a+b\,\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int (d+ex^2)(a+b\,\text{arcsinh}(cx))^2\,dx = 2b^2dx-\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))}{c}+\frac{4be\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))}{9c^3}-\frac{2bex^2\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))}{9c}+dx(a+b\,\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\,\text{arcsinh}(cx))^2$$

output
```
2*b^2*d*x-4/9*b^2*e*x/c^2+2/27*b^2*e*x^3-2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcs
inh(c*x))/c+4/9*b*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^3-2/9*b*e*x^2*(
c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+d*x*(a+b*arcsinh(c*x))^2+1/3*e*x^3*(
a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int \left(d+ex^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{9a^2c^3x(3d+ex^2)-6ab\sqrt{1+c^2x^2}(-2e+c^2(9d+ex^2))+2b^2cx(-6e+c^2(27d+ex^2))-6b\big(-3ac^3x(3d}{27c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*
x^2)) + 2*b^2*c*x*(-6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x
^2) + b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))*ArcSinh[c*x] + 9*b^2
*c^3*x*(3*d + e*x^2)*ArcSinh[c*x]^2)/(27*c^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \text{ 6208}$$
$$\int \left(d(a+b\text{arcsinh}(cx))^2+ex^2(a+b\text{arcsinh}(cx))^2\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2bd\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{2bex^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c}+\frac{4be\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{9c^3}+dx(a+b\text{arcsinh}(cx))^2+\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^2-\frac{4b^2ex}{9c^2}+2b^2dx+\frac{2}{27}b^2ex^3$$

input
```
Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
))/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*
x*(a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeqQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

method	result
parts	$a^2\left(\frac{1}{3}x^3e+dx\right)+\frac{b^2\left(\frac{e\left(9\,\mathrm{arcsinh}(xc)^2x^3c^3-6\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^2c^2+2x^3c^3+12\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}-12xc\right)}{27c^2}+d\left(\cdots\right.\right.}{c}$
derivativedivides	$\frac{\frac{a^2\left(c^3dx+\frac{1}{3}e\,x^3c^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\mathrm{arcsinh}(xc)^2xc-2\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{e\left(9\,\mathrm{arcsinh}(xc)^2x^3c^3-6\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^2\cdots\right.}{27}\right.}{c^2}}{\cdots}$
default	$\frac{\frac{a^2\left(c^3dx+\frac{1}{3}e\,x^3c^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\mathrm{arcsinh}(xc)^2xc-2\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{e\left(9\,\mathrm{arcsinh}(xc)^2x^3c^3-6\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^2\cdots\right.}{27}\right.}{c^2}}{\cdots}$
orering	$\frac{x\left(19c^4e^3x^6+209c^4d\,e^2x^4+9c^4d^2e\,x^2-24c^2e^3x^4+27c^4d^3+232c^2d\,e^2x^2-48e^3x^2\right)(a+b\,\mathrm{arcsinh}(xc))^2}{27c^4(e\,x^2+d)^2}-\frac{\left(6c^4e^2x^6+1\cdots\right.}{\cdots}$

input
```
int((e*x^2+d)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*x^3*e+d*x)+b^2/c*(1/27*e*(9*arcsinh(x*c)^2*x^3*c^3-6*arcsinh(x*c)
*(c^2*x^2+1)^(1/2)*x^2*c^2+2*x^3*c^3+12*arcsinh(x*c)*(c^2*x^2+1)^(1/2)-12*
x*c)/c^2+d*(arcsinh(x*c)^2*x*c-2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*c))+2*
a*b/c*(1/3*c*arcsinh(x*c)*x^3*e+arcsinh(x*c)*x*c*d-1/3/c^2*(e*(1/3*x^2*c^2
*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c^2*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.37

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = \frac{(9a^2+2b^2)c^3ex^3+9(b^2c^3ex^3+3b^2c^3dx)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3(9(a^2+2b^2)c^3d-4b^2ce)x+6(3\ldots}{}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d - 4*b^2*c*e)*x + 6*(3*
a*b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d - 2*b^2*e)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c
^2*d - 2*a*b*e)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.82

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = \begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{asinh}(cx)+\frac{2abex^3\operatorname{asinh}(cx)}{3}-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{2abex^2\sqrt{c^2x^2+1}}{9c}+\frac{4abe\sqrt{c^2x^2+1}}{9c^3}+b^2dx\operatorname{asi}\ldots \\ a^2\left(dx+\frac{ex^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*asinh(c*x) + 2*a*b*e*x**3*
asinh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2
 + 1)/(9*c) + 4*a*b*e*sqrt(c**2*x**2 + 1)/(9*c**3) + b**2*d*x*asinh(c*x)**
2 + 2*b**2*d*x + b**2*e*x**3*asinh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d
*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(9*c) - 4*b**2*e*x/(9*c**2) + 4*b**2*e*sqrt(c**2*x**2 + 1)*asinh(c*
x)/(9*c**3), Ne(c, 0)), (a**2*(d*x + e*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.42

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{1}{3}\,b^2ex^3\,\text{arsinh}\,(cx)^2+\frac{1}{3}\,a^2ex^3+b^2dx\,\text{arsinh}\,(cx)^2$$
$$+\frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abe$$
$$-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx)-\frac{c^2x^3-6\,x}{c^2}\right)b^2e$$
$$+2\,b^2d\left(x-\frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right)+a^2dx+\frac{2\left(cx\,\text{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)abd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*e*x^3*arcsinh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arcsinh(c*x)^2 + 2/
9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)
/c^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/
c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2
 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))
*a*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(e\,x^2+d)\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{9asinh(cx)^2\,b^2c^3dx - 18\sqrt{c^2x^2+1}\,asinh(cx)\,b^2c^2d + 18asinh(cx)\,ab\,c^3dx + 6asinh(cx)\,ab\,c^3e\,x^3 - 18\sqrt{c}}{}$$

input
```
int((e*x^2+d)*(a+b*asinh(c*x))^2,x)
```

output
```
(9*asinh(c*x)**2*b**2*c**3*d*x - 18*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2*c*
*2*d + 18*asinh(c*x)*a*b*c**3*d*x + 6*asinh(c*x)*a*b*c**3*e*x**3 - 18*sqrt
(c**2*x**2 + 1)*a*b*c**2*d - 2*sqrt(c**2*x**2 + 1)*a*b*c**2*e*x**2 + 4*sqr
t(c**2*x**2 + 1)*a*b*e + 9*int(asinh(c*x)**2*x**2,x)*b**2*c**3*e + 9*a**2*
c**3*d*x + 3*a**2*c**3*e*x**3 + 18*b**2*c**3*d*x)/(9*c**3)
```

3.154 $\int (a + b\text{arcsinh}(cx))^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a+b\text{arcsinh}(cx))^2 \, dx = 2b^2x - \frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} + x(a+b\text{arcsinh}(cx))^2$$

output
```
2*b^2*x-2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+x*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = (a^2 + 2b^2) x - \frac{2ab\sqrt{1+c^2x^2}}{c} + \frac{2b\left(acx - b\sqrt{1+c^2x^2}\right) \text{arcsinh}(cx)}{c} + b^2x\text{arcsinh}(cx)^2$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2,x]
```

output
```
(a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c
^2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\operatorname{arcsinh}(cx))^2-2bc\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x(a+b\operatorname{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)$$

$$\Big\downarrow \text{24}$$

$$x(a+b\operatorname{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^2,x]
```

output
```
x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+2ab\left(xc\operatorname{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	72
default	$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+2ab\left(xc\operatorname{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	72
parts	$a^2x+\frac{b^2\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)}{c}+\frac{2ab\left(xc\operatorname{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	73
orering	$x(a+b\operatorname{arcsinh}(xc))^2-\frac{2(a+b\operatorname{arcsinh}(xc))b}{c\sqrt{c^2x^2+1}}+\frac{x(c^2x^2+1)\left(\frac{2c^2b^2}{c^2x^2+1}-\frac{2(a+b\operatorname{arcsinh}(xc))bc^3x}{(c^2x^2+1)^{\frac{3}{2}}}\right)}{c^2}$	99

input
```
int((a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arcsinh(x*c)^2*x*c-2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*
c)+2*a*b*(x*c*arcsinh(x*c)-(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(44) = 88.

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2+1}ab+2\left(abcx-\sqrt{c^2x^2+1}b^2\right)\log\left(cx+\sqrt{c^2x^2\right.}}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x
^2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 +
 1)))/c
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = \begin{cases} a^2x + 2abx \operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x \operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**
2*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne
(c, 0)), (a**2*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = b^2 x \operatorname{arsinh}(cx)^2 + 2\, b^2 \left(x - \frac{\sqrt{c^2x^2+1} \operatorname{arsinh}(cx)}{c} \right) + a^2 x + \frac{2 \left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2+1} \right) ab}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*
x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = 2\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)ab + \left(x\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2\,c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2 + a^2x$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))/c^2))*b^2 + a^2*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b\,\text{asinh}(c\,x))^2 \, dx$$

input
```
int((a + b*asinh(c*x))^2,x)
```

output
```
int((a + b*asinh(c*x))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int (a + b\text{arcsinh}(cx))^2 \, dx$$
$$= \frac{asinh(cx)^2 b^2 cx - 2\sqrt{c^2x^2+1}\, asinh(cx)\, b^2 + 2asinh(cx)\, abcx - 2\sqrt{c^2x^2+1}\, ab + a^2cx + 2b^2cx}{c}$$

input
```
int((a+b*asinh(c*x))^2,x)
```

output
```
(asinh(c*x)**2*b**2*c*x - 2*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2 + 2*asinh(
c*x)*a*b*c*x - 2*sqrt(c**2*x**2 + 1)*a*b + a**2*c*x + 2*b**2*c*x)/c
```

3.155 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 739

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx = \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2*ln(1-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2
)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+e^(1
/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
(1/2)+1/2*(a+b*arcsinh(c*x))^2*ln(1-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d
)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(
1+e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2)
)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))
*polylog(2,e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2))
)/(-d)^(1/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-e^(1/2)*(c*x+(c^2*x^2
+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcs
inh(c*x))*polylog(2,e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+
e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylog(3,-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2
))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,e^(1/
2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(
1/2)+b^2*polylog(3,-e^(1/2)*(c*x+(c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+(-c^2*d+
e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,e^(1/2)*(c*x+(c^2*x^2+1)^(1/2)
)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

output
```
(2*a^2*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*L
og[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b^2*S
qrt[d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) + e])] + 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*
x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + 2*a*
b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 - (Sqrt[e]*E^ArcSinh[c
*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1
 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - b^2*Sqrt[
d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^
2*d) + e])] + 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSi
nh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - 2*b*Sqrt[d]*(a + b*ArcSinh[c
*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) +
e])] - 2*a*b*Sqrt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + S
qrt[-(c^2*d) + e]))] - 2*b^2*Sqrt[d]*ArcSinh[c*x]*PolyLog[2, -((Sqrt[e]*E^
ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + 2*a*b*Sqrt[d]*PolyLog[
2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + 2*b^2*Sqr
t[d]*ArcSinh[c*x]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e])] - 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*S...
```

Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}dx$$
$$\Big\downarrow \text{ 6208}$$
$$\int \left(\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\text{arcsinh}(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(1-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{\text{arcsinh}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

output
```
((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*S
qrt[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^
2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*
Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSin
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a +
b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt
[e]) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt
[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e])
 - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) +
e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(xc))^2}{e\,x^2 + d}\,dx$$

input
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex^2} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{ex^2 + d} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex^2} dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex^2} dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex^2}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2+2\left(\int \frac{asinh(cx)}{e\,x^2+d}dx\right)abde+\left(\int \frac{asinh(cx)^2}{e\,x^2+d}dx\right)b^2de}{de}$$

input
```
int((a+b*asinh(c*x))^2/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2 + 2*int(asinh(c*x)/(d
+ e*x**2),x)*a*b*d*e + int(asinh(c*x)**2/(d + e*x**2),x)*b**2*d*e)/(d*e)
```

3.156 $\int \frac{(d+ex^2)^2}{a+b\,\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 388

$$\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} - \frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} + \frac{de\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3} - \frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} + \frac{e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc} + \frac{de\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2bc^3} - \frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8bc^5} - \frac{de\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2bc^3} + \frac{3e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5} - \frac{e^2\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16bc^5}$$

output
```
d^2*cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c-1/2*d*e*cosh(a/b)*Chi((a+b*arc
sinh(c*x))/b)/b/c^3+1/8*e^2*cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c^5+1/2*
d*e*cosh(3*a/b)*Chi(3*(a+b*arcsinh(c*x))/b)/b/c^3-3/16*e^2*cosh(3*a/b)*Chi
(3*(a+b*arcsinh(c*x))/b)/b/c^5+1/16*e^2*cosh(5*a/b)*Chi(5*(a+b*arcsinh(c*x
))/b)/b/c^5-d^2*sinh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c+1/2*d*e*sinh(a/b)*
Shi((a+b*arcsinh(c*x))/b)/b/c^3-1/8*e^2*sinh(a/b)*Shi((a+b*arcsinh(c*x))/b
)/b/c^5-1/2*d*e*sinh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b/c^3+3/16*e^2*sin
h(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b/c^5-1/16*e^2*sinh(5*a/b)*Shi(5*(a+b
*arcsinh(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{2(8c^4d^2-4c^2de+e^2)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+(8c^2d-3e)\,e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
(2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]
] + (8*c^2*d - 3*e)*e*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] +
 e^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 16*c^4*d^2*Sinh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d*e*Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]] - 2*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 8*
c^2*d*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 3*e^2*Sinh[(3
*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - e^2*Sinh[(5*a)/b]*SinhIntegr
al[5*(a/b + ArcSinh[c*x])])/(16*b*c^5)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(\frac{d^2}{a+b\text{arcsinh}(cx)}+\frac{2dex^2}{a+b\text{arcsinh}(cx)}+\frac{e^2x^4}{a+b\text{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8bc^5}+\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16bc^5}-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}+\frac{de\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2bc^3}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (d*e*Cosh[a/b
]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(2*b*c^3) + (e^2*Cosh[a/b]*CoshInt
egral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshIntegral
[(3*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegra
l[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a
 + b*ArcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])
/(8*b*c^5) - (d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(2*b*c^3) + (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])
/(16*b*c^5) - (e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])
/(16*b*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{e^{2} {\mathrm e}^{\frac{5 a}{b}} \operatorname{expIntegral}_{1}\left(5 \,\operatorname{arcsinh}\left(x c \right)+\frac{5 a}{b}\right)}{32 c^{4} b}-\frac{e^{2} {\mathrm e}^{-\frac{5 a}{b}} \operatorname{expIntegral}_{1}\left(-5 \,\operatorname{arcsinh}\left(x c \right)-\frac{5 a}{b}\right)}{32 c^{4} b}-\frac{{\mathrm e}^{\frac{a}{b}} \operatorname{expIntegral}_{1}\left(\operatorname{arcsinh}\left(x c \right)+\frac{a}{b}\right) d^{2}}{2 b}+$
default	$-\frac{e^{2} {\mathrm e}^{\frac{5 a}{b}} \operatorname{expIntegral}_{1}\left(5 \,\operatorname{arcsinh}\left(x c \right)+\frac{5 a}{b}\right)}{32 c^{4} b}-\frac{e^{2} {\mathrm e}^{-\frac{5 a}{b}} \operatorname{expIntegral}_{1}\left(-5 \,\operatorname{arcsinh}\left(x c \right)-\frac{5 a}{b}\right)}{32 c^{4} b}-\frac{{\mathrm e}^{\frac{a}{b}} \operatorname{expIntegral}_{1}\left(\operatorname{arcsinh}\left(x c \right)+\frac{a}{b}\right) d^{2}}{2 b}+$

input
```
int((e*x^2+d)^2/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(x*c)+5*a/b)-1/32/c^4*e^2/b*
exp(-5*a/b)*Ei(1,-5*arcsinh(x*c)-5*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a
/b)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)*d*e-1/16/c^4/b*exp(a/b)*
Ei(1,arcsinh(x*c)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)*d^2+1/4
/c^2/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-ar
csinh(x*c)-a/b)*e^2-1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)*d+3/
32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)
*Ei(1,-3*arcsinh(x*c)-3*a/b)*d+3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
x*c)-3*a/b))
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex^2)^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x**2)**2/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(e\,x^2+d)^2}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)^2/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \left(\int \frac{x^4}{asinh\,(cx)\,b+a}dx\right)e^2 + 2\left(\int \frac{x^2}{asinh\,(cx)\,b+a}dx\right)de + \left(\int \frac{1}{asinh\,(cx)\,b+a}dx\right)d^2$$

input
```
int((e*x^2+d)^2/(a+b*asinh(c*x)),x)
```

output
```
int(x**4/(asinh(c*x)*b + a),x)*e**2 + 2*int(x**2/(asinh(c*x)*b + a),x)*d*e
 + int(1/(asinh(c*x)*b + a),x)*d**2
```

3.157 $\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} + \frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4bc^3} - \frac{e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4bc^3}$$

output
```
d*cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c-1/4*e*cosh(a/b)*Chi((a+b*arcsinh
(c*x))/b)/b/c^3+1/4*e*cosh(3*a/b)*Chi(3*(a+b*arcsinh(c*x))/b)/b/c^3-d*sinh
(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c+1/4*e*sinh(a/b)*Shi((a+b*arcsinh(c*x))
/b)/b/c^3-1/4*e*sinh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{(4c^2d-e)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-4c^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.}{4bc^3}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

output
```
((4*c^2*d - e)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e*Cosh[(3*a)/b
]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c^2*d*Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c*x]] + e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - e*Sinh[
(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \text{6208}$$
$$\int \left(\frac{d}{a+b\text{arcsinh}(cx)}+\frac{ex^2}{a+b\text{arcsinh}(cx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{e\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4bc^3}+\frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc}-\frac{d\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (e*Cosh[a/b]*Co
shIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (e*Cosh[(3*a)/b]*CoshInteg
ral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x
])/b])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/
b])/(4*b*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\operatorname{expIntegral}_1\left(-3\operatorname{arcsinh}(xc)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,\mathrm{e}^{\frac{3a}{b}}\operatorname{expIntegral}_1\left(3\operatorname{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\operatorname{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}}{c}$
default	$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\operatorname{expIntegral}_1\left(-3\operatorname{arcsinh}(xc)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,\mathrm{e}^{\frac{3a}{b}}\operatorname{expIntegral}_1\left(3\operatorname{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\operatorname{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}}{c}$

input
```
int((e*x^2+d)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/8*e/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(x*c)-3*a/b)-1/8*e/c^2/b*exp(
3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)*d+
1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arcsinh(
x*c)-a/b)*d+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)*e)
```

Fricas [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{d+ex^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x**2)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{e\,x^2+d}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{a+b\text{arcsinh}(cx)}\,dx=\left(\int \frac{x^2}{asinh\,(cx)\,b+a}dx\right)e+\left(\int \frac{1}{asinh\,(cx)\,b+a}dx\right)d$$

input
```
int((e*x^2+d)/(a+b*asinh(c*x)),x)
```

output
```
int(x**2/(asinh(c*x)*b + a),x)*e + int(1/(asinh(c*x)*b + a),x)*d
```

3.158 $\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

output
```
cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c-sinh(a/b)*Shi((a+b*arcsinh(c*x))/b
)/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(b*c)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arcsinh}(cx)} \, dx$$

$$\Big\downarrow \text{ 6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3782}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b] - Sinh[a/b]*SinhIntegral[(
a + b*ArcSinh[c*x])/b])/(b*c)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}} \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \text{expIntegral}_1\left(-\text{arcsinh}(xc)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{-\frac{e^{\frac{a}{b}} \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \text{expIntegral}_1\left(-\text{arcsinh}(xc)-\frac{a}{b}\right)}{2b}}{c}$	56

input
```
int(1/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(
x*c)-a/b))
```

Fricas [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a + b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a + b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int(1/(a + b*asinh(c*x)),x)
```

output
```
int(1/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{asinh\,(cx)\,b + a}dx$$

input
```
int(1/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b + a),x)
```

3.159 $\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,(e\,x^2+d)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{asinh\left(cx\right)bd+asinh\left(cx\right)be\,x^2+ad+ae\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b*d + asinh(c*x)*b*e*x**2 + a*d + a*e*x**2),x)
```

3.160 $\int \frac{1}{(d+ex^2)^2(a+b\textbf{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 \left(a+b\,\text{arcsinh}\,(xc)\right)}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b
*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 101.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(ex^2+d)^2 (b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\,(e\,x^2+d)^2} \, dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))} dx$$
$$= \int \frac{1}{asinh(cx)\, b\, d^2 + 2asinh(cx)\, bde\, x^2 + asinh(cx)\, b\, e^2x^4 + a\, d^2 + 2ade\, x^2 + a\, e^2x^4} dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b*d**2 + 2*asinh(c*x)*b*d*e*x**2 + asinh(c*x)*b*e**2*x**
4 + a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4),x)
```

3.161 $\int \frac{(d+ex^2)^2}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 495

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{d^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{de\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2b^2c^3} - \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^5} - \frac{3de\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{2b^2c^3} + \frac{9e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2c^5} - \frac{5e^2\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2c^5} + \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{de\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{2b^2c^3} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{8b^2c^5} + \frac{3de\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{2b^2c^3} - \frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^5} + \frac{5e^2\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(cx))}{b}\right)}{16b^2c^5}$$

output
```
-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-2*d*e*x^2*(c^2*x^2+1)^(1/2)/
b/c/(a+b*arcsinh(c*x))-e^2*x^4*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-d^
2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/2*d*e*Chi((a+b*arcsinh(c*x))
/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^5-
3/2*d*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3+9/16*e^2*Chi(3*(a+
b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*Chi(5*(a+b*arcsinh(c*x))/b
)*sinh(5*a/b)/b^2/c^5+d^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/2*d*
e*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*a
rcsinh(c*x))/b)/b^2/c^5+3/2*d*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^
2/c^3-9/16*e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^5+5/16*e^2*co
sh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b^2/c^5
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx =$$

$$-\frac{\frac{16bc^4d^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{32bc^4dex^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{16bc^4e^2x^4\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+2(8c^4d^2-4c^2de+e^2)\,\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/16*((16*b*c^4*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (32*b*c^4*d
*e*x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (16*b*c^4*e^2*x^4*Sqrt[1
+ c^2*x^2])/(a + b*ArcSinh[c*x]) + 2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*CoshInt
egral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*(8*c^2*d - 3*e)*e*CoshIntegral[3*(
a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] + 5*e^2*CoshIntegral[5*(a/b + ArcSinh[c
*x])]*Sinh[(5*a)/b] - 16*c^4*d^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]
] + 8*c^2*d*e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 2*e^2*Cosh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e*Cosh[(3*a)/b]*SinhIntegral[
3*(a/b + ArcSinh[c*x])] + 9*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSin
h[c*x])] - 5*e^2*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])])/(b^2*
c^5)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d^2}{(a+b\text{arcsinh}(cx))^2}+\frac{2dex^2}{(a+b\text{arcsinh}(cx))^2}+\frac{e^2x^4}{(a+b\text{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(cx))}{b}\right)}{16b^2c^5}+\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}-\frac{de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{2b^2c^3}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{2dex^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{e^2x^4\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1
+ c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*
(a + b*ArcSinh[c*x])) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b
])/(b^2*c) + (d*e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(2*b^2*c
^3) - (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b^2*c^5) - (
3*d*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(2*b^2*c^3)
+ (9*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c
^5) - (5*e^2*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b
^2*c^5) + (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (
d*e*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(2*b^2*c^3) + (e^2*Cos
h[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cosh[
(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*C
osh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(469) = 938$.

Time = 3.77 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.09

method	result	size
derivativedivides	Expression too large to display	1036
default	Expression too large to display	1036

input
```
int((e*x^2+d)^2/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/32*(-16*x^4*c^4*(c^2*x^2+1)^(1/2)+16*x^5*c^5-12*x^2*c^2*(c^2*x^2+1)
^(1/2)+20*x^3*c^3-(c^2*x^2+1)^(1/2)+5*x*c)*e^2/c^4/b/(a+b*arcsinh(x*c))+5/
32*e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(x*c)+5*a/b)-1/32/b*e^2/c^4*(16*x^
5*c^5+20*x^3*c^3+16*x^4*c^4*(c^2*x^2+1)^(1/2)+5*x*c+12*x^2*c^2*(c^2*x^2+1)
^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-5/32/b^2*e^2/c^4*exp(-5*a/b)*
Ei(1,-5*arcsinh(x*c)-5*a/b)+1/2*(x*c-(c^2*x^2+1)^(1/2))*d^2/b/(a+b*arcsinh
(x*c))+1/2*d^2/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/4*(x*c-(c^2*x^2+1)^(1
/2))*d*e/c^2/b/(a+b*arcsinh(x*c))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arcsinh(x*
c)+a/b)+1/16*(x*c-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(x*c))+1/16/c^4
*e^2/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/2/b*d^2*(x*c+(c^2*x^2+1)^(1/2))
/(a+b*arcsinh(x*c))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)+1/4/c^2/
b*d*e*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))+1/4/c^2/b^2*d*e*exp(-a/b)
*Ei(1,-arcsinh(x*c)-a/b)-1/16/c^4/b*e^2*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsi
nh(x*c))-1/16/c^4/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)+1/4*(-4*x^2*c^
2*(c^2*x^2+1)^(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3*x*c)*d*e/c^2/b/(a+b*arcs
inh(x*c))-3/32*(-4*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3
*x*c)*e^2/c^4/b/(a+b*arcsinh(x*c))+3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh
(x*c)+3*a/b)*d-9/32*e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)-1/4/
c^2*e/b*(4*x^3*c^3+3*x*c+4*x^2*c^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a
+b*arcsinh(x*c))*d+3/32/c^4*e^2/b*(4*x^3*c^3+3*x*c+4*x^2*c^2*(c^2*x^2+1...
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c
*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex^2)^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x**2)**2/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e^2*x^7 + (2*c^3*d*e + c*e^2)*x^5 + c*d^2*x + (c^3*d^2 + 2*c*d*e)*x^
3 + (c^2*e^2*x^6 + (2*c^2*d*e + e^2)*x^4 + (c^2*d^2 + 2*d*e)*x^2 + d^2)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1
))) + integrate((5*c^5*e^2*x^8 + 2*(3*c^5*d*e + 5*c^3*e^2)*x^6 + (c^5*d^2
+ 12*c^3*d*e + 5*c*e^2)*x^4 + c*d^2 + 2*(c^3*d^2 + 3*c*d*e)*x^2 + (5*c^3*e
^2*x^6 + 3*(2*c^3*d*e + c*e^2)*x^4 - c*d^2 + (c^3*d^2 + 2*c*d*e)*x^2)*(c^2
*x^2 + 1) + (10*c^4*e^2*x^7 + (12*c^4*d*e + 13*c^2*e^2)*x^5 + 2*(c^4*d^2 +
 7*c^2*d*e + 2*e^2)*x^3 + (c^2*d^2 + 4*d*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5
*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 +
(c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c
^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a
*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(e\,x^2+d)^2}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \left(\int \frac{x^4}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)e^2 + 2\left(\int \frac{x^2}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)de + \left(\int \frac{1}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)d^2$$

input
```
int((e*x^2+d)^2/(a+b*asinh(c*x))^2,x)
```

output
```
int(x**4/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*e**2 + 2*int(x*
*2/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*d*e + int(1/(asinh(c*
x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*d**2
```

3.162 $\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 247

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} \\ &\quad -\frac{d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} \\ &\quad +\frac{e\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3} \\ &\quad -\frac{3e\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3} \\ &\quad +\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} \\ &\quad -\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} \\ &\quad +\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}\end{aligned}$$

output
```
-d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a
+b*arcsinh(c*x))-d*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e*Chi((a+
b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^3-3/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*sin
h(3*a/b)/b^2/c^3+d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/4*e*cosh(a/
b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^3+3/4*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(
c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx =$$
$$-\frac{\frac{4bc^2d\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{4bc^2ex^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+(4c^2d-e)\,\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right)+3e\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/4*((4*b*c^2*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e*x^2*
Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*c^2*d - e)*CoshIntegral[a/b +
 ArcSinh[c*x]]*Sinh[a/b] + 3*e*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(
3*a)/b] - 4*c^2*d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e*Cosh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - 3*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b
+ ArcSinh[c*x])])/(b^2*c^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d}{(a+b\text{arcsinh}(cx))^2}+\frac{ex^2}{(a+b\text{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*
x^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]
*Sinh[a/b])/(b^2*c) + (e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(
4*b^2*c^3) - (3*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/
(4*b^2*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) -
 (e*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) + (3*e*Cos
h[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\left(-4x^2c^2\sqrt{c^2x^2+1}+4x^3c^3-\sqrt{c^2x^2+1}+3xc\right)e}{8c^2b(a+b\,\mathrm{arcsinh}(xc))}+\frac{3e\,e^{\frac{3a}{b}}\,\mathrm{expIntegral}_1\left(3\,\mathrm{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4x^3c^3+3xc+4x^2c^2\sqrt{c^2x^2+1}+\sqrt{c^2x\right.}{8c^2b(a+b\,\mathrm{arcsinh}(xc))}$
default	$\frac{\left(-4x^2c^2\sqrt{c^2x^2+1}+4x^3c^3-\sqrt{c^2x^2+1}+3xc\right)e}{8c^2b(a+b\,\mathrm{arcsinh}(xc))}+\frac{3e\,e^{\frac{3a}{b}}\,\mathrm{expIntegral}_1\left(3\,\mathrm{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4x^3c^3+3xc+4x^2c^2\sqrt{c^2x^2+1}+\sqrt{c^2x\right.}{8c^2b(a+b\,\mathrm{arcsinh}(xc))}$

input
```
int((e*x^2+d)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/8*(-4*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3*x*c)*
e/c^2/b/(a+b*arcsinh(x*c))+3/8*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*
a/b)-1/8*e/c^2/b*(4*x^3*c^3+3*x*c+4*x^2*c^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^
(1/2))/(a+b*arcsinh(x*c))-3/8*e/c^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(x*c)-3
*a/b)+1/2*(x*c-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(x*c))-1/8*(x*c-(c^2*x^2
+1)^(1/2))*e/c^2/b/(a+b*arcsinh(x*c))+1/2/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a
/b)*d-1/8/c^2/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)*e-1/2/b*(x*c+(c^2*x^2+1)
^(1/2))/(a+b*arcsinh(x*c))*d+1/8/c^2/b*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsin
h(x*c))*e-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)*d+1/8/c^2/b^2*exp(-a/b
)*Ei(1,-arcsinh(x*c)-a/b)*e)
```

Fricas [F]

$$\int \frac{d+ex^2}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{ex^2+d}{\left(b\,\mathrm{arsinh}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{d+ex^2}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e*x^5 + (c^3*d + c*e)*x^3 + c*d*x + (c^2*e*x^4 + (c^2*d + e)*x^2 + d
)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
 + 1))) + integrate((3*c^5*e*x^6 + (c^5*d + 6*c^3*e)*x^4 + (2*c^3*d + 3*c*
e)*x^2 + (3*c^3*e*x^4 + (c^3*d + c*e)*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (6*
c^4*e*x^5 + (2*c^4*d + 7*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c^2*x^2 + 1))/
(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^
5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3
 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{e\,x^2+d}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^2}\,dx=\left(\int \frac{x^2}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)e+\left(\int \frac{1}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)d$$

input
```
int((e*x^2+d)/(a+b*asinh(c*x))^2,x)
```

output
```
int(x**2/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*e + int(1/(asin
h(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*d
```

3.163 $\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}$$

output
```
-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-Chi((a+b*arcsinh(c*x))/b)*sinh(a
/b)/b^2/c+cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
(-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSin
h[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6188, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{c\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx}{b}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$\downarrow$ 3784

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 3042

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 3779

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
-(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(a +
b*ArcSinh[c*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x
])/b]))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{\frac{xc-\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}_1\left(\mathrm{arcsinh}(xc)+\frac{a}{b}\right)}{2b^2}-\frac{xc+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}_1\left(-\mathrm{arcsinh}(xc)-\frac{a}{b}\right)}{2b^2}}{c}$	118
default	$\frac{\frac{xc-\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}_1\left(\mathrm{arcsinh}(xc)+\frac{a}{b}\right)}{2b^2}-\frac{xc+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}_1\left(-\mathrm{arcsinh}(xc)-\frac{a}{b}\right)}{2b^2}}{c}$	118

input
```
int(1/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(x*c-(c^2*x^2+1)^(1/2))/b/(a+b*arcsinh(x*c))+1/2/b^2*exp(a/b)*Ei(
1,arcsinh(x*c)+a/b)-1/2/b*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-1/2/b
^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b))
```

Fricas [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(
c*x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)
*(c^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (
c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 +
 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c
^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^2,x)
```

output
```
int(1/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx$$

input
```
int(1/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)
```

3.164 $\int \frac{1}{(d+ex^2)(a+b\,\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6209}$$

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\text{arcsinh}\,(xc))^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*
b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 45.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 766, normalized size of antiderivative = 38.30

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^4 + (c^3*d + c*e)*a*b*
x^2 + a*b*c*d + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*c^
2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (
a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^6 - (
c^5*d - 2*c^3*e)*x^4 - (2*c^3*d - c*e)*x^2 + (c^3*e*x^4 - (c^3*d - 3*c*e)*
x^2 + c*d)*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^5 - (2*c^4*d - 5*c^2*e)*x^3 -
(c^2*d - 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^
2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^
2 + c*d*e)*a*b*x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^
2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6 + (c^5
*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*b^2*x^
2 + (b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1)
+ 2*(b^2*c^4*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^
4*d^2 + 2*c^2*d*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (
c^4*d^2 + 2*c^2*d*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^2}\,dx$$
$$= \int \frac{1}{asinh\left(cx\right)^2 b^2 d + asinh\left(cx\right)^2 b^2 e\,x^2 + 2asinh\left(cx\right) abd + 2asinh\left(cx\right) abe\,x^2 + a^2 d + a^2 e\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2*d + asinh(c*x)**2*b**2*e*x**2 + 2*asinh(c*x)*a*b
*d + 2*asinh(c*x)*a*b*e*x**2 + a**2*d + a**2*e*x**2),x)
```

3.165 $\int \frac{1}{(d+ex^2)^2(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2(a+b\,\mathrm{arcsinh}(cx))^2}\,dx=\mathrm{Int}\left(\frac{1}{(d+ex^2)^2(a+b\,\mathrm{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 22.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\,\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{1}{(d+ex^2)^2(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\,\text{arcsinh}\,(xc))^2} dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2 (b \operatorname{arsinh}(cx)+a)^2} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d
*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^
2)*arcsinh(c*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 1027, normalized size of antiderivative = 51.35

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2 (b \operatorname{arsinh}(cx)+a)^2} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^6 + (2*c^3*d*e + c*e
^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2 + (b^2*c^3*e^2*x^6 +
 (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e)*b^2*x^2 + (
b^2*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^
2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*e*x^6 - (c^5*d - 6*c^3*e)*x
^4 - (2*c^3*d - 3*c*e)*x^2 + (3*c^3*e*x^4 - (c^3*d - 5*c*e)*x^2 + c*d)*(c^
2*x^2 + 1) - c*d + (6*c^4*e*x^5 - (2*c^4*d - 11*c^2*e)*x^3 - (c^2*d - 4*e)
*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x
^8 + (3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e
+ 3*c*d*e^2)*a*b*x^4 + a*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2 + (a*b*
c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)
*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (
3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*
d*e^2)*b^2*x^4 + b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2 + (b^2*c^3*e^
3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c^2*
x^2 + 1) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*
d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^
3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^9 +
(3*c^4*d*e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*...
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(ex^2+d)^2 (b \operatorname{arsinh}(cx)+a)^2} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(e\,x^2+d\right)}^2} \,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.75

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^2} \, dx$$
$$= \int \frac{1}{asinh\left(cx\right)^{2} b^{2} d^{2}+2 asinh\left(cx\right)^{2} b^{2} d e\,x^{2}+asinh\left(cx\right)^{2} b^{2} e^{2} x^{4}+2 asinh\left(cx\right) a b\,d^{2}+4 asinh\left(cx\right) a b d e\,x^{2}}$$

input
```
int(1/(e*x^2+d)^2/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2*d**2 + 2*asinh(c*x)**2*b**2*d*e*x**2 + asinh(c*x
)**2*b**2*e**2*x**4 + 2*asinh(c*x)*a*b*d**2 + 4*asinh(c*x)*a*b*d*e*x**2 +
2*asinh(c*x)*a*b*e**2*x**4 + a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4)
,x)
```

3.166 $\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\text{arcsinh}(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6209}$$

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \sqrt{e\,x^2+d}\,(a+b\,\text{arcsinh}\,(xc))\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsinh(x*c)),x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((a + b*asinh(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int\sqrt{e\,x^2+d}\,asinh(cx)\,dx\right)be}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int(sqrt(d + e*x**2)*asinh(c*x),x)*b*e)/(2*e)
```

3.167 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\text{arcsinh}\,(xc)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+ex^2}}\,dx=\frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int\frac{asinh(cx)}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input
```
int((a+b*asinh(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a + int(asinh(c*x)/sq
rt(d + e*x**2),x)*b*e)/e
```

3.168 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

output
```
x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(1/2)-b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)
/c/(e*x^2+d)^(1/2))/d/e^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x\left(-bcx\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{ex^2}{d}\right)+2(a+b\text{arcsinh}(cx))\right)}{2d\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(-(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*
x^2)/d)]) + 2*(a + b*ArcSinh[c*x])))/(2*d*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6207, 27, 353, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{6207}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-bc\int \frac{x}{d\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx^2}{2d}$$

$$\Big\downarrow \text{66}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2+1}}{\sqrt{ex^2+d}}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{x(a+b\text{arcsinh}(cx))}{d\sqrt{d+ex^2}}-\frac{b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Maple [F]

$$\int \frac{a + b \operatorname{arcsinh}(xc)}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(60) = 120$.

Time = 0.14 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.66

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{3/2}}\, dx = \left[\frac{4\sqrt{ex^2 + d}bex\log\left(cx + \sqrt{c^2x^2 + 1}\right) + 4\sqrt{ex^2 + d}aex + (bex^2 + bd)\sqrt{e}\log\left(8c\right.}{}\right.$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(e*x^2
+ d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d
*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^
2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*
x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b
*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 +
 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)
))/(d*e^2*x^2 + d^2*e)]
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{a + b\operatorname{asinh}(cx)}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\,ad+\sqrt{e}\,ae\,x^2+\left(\int \frac{asinh(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)b\,d^2e+\left(\int \frac{}{\sqrt{e\,x^2}}\right.}{de\,(e\,x^2+d)}$$

input
```
int((a+b*asinh(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*a*d + sqrt(e)*a*e*x**2 + int(asinh(c*x)/
(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d**2*e + int(asinh(c*x
)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2*x**2)/(d*e*(d
 + e*x**2))
```

3.169 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{3d(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{2b\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

output
```
-1/3*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(1/2)+1/3*x*(a+b*arcsinh(
c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/(e*x^2+d)^(1/2)-2/3*b
*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{bcd\sqrt{1+c^2x^2}(d+ex^2)}{c^2d-e} + ax(3d+2ex^2) - bcx^2(d+ex^2)\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},\right.}{3d^2(d+ex^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(-((b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^2))/(c^2*d - e)) + a*x*(3*d + 2*e*x^2
) - b*c*x^2*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2
*x^2), -((e*x^2)/d)] + b*x*(3*d + 2*e*x^2)*ArcSinh[c*x])/(3*d^2*(d + e*x^2
)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6207, 27, 435, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{6207}$$

$$-bc\int \frac{x(2ex^2+3d)}{3d^2\sqrt{c^2x^2+1}(ex^2+d)^{3/2}}dx+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x(2ex^2+3d)}{\sqrt{c^2x^2+1}(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bc\int \frac{2ex^2+3d}{\sqrt{c^2x^2+1}(ex^2+d)^{3/2}}dx^2}{6d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{bc\left(2\int \frac{1}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx^2+\frac{2d\sqrt{c^2x^2+1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$-\frac{bc\left(4\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2+1}}{\sqrt{ex^2+d}}+\frac{2d\sqrt{c^2x^2+1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2x(a+b\text{arcsinh}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{arcsinh}(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(\frac{4\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}+\frac{2d\sqrt{c^2x^2+1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x
]))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*((2*d*Sqrt[1 + c^2*x^2])/((c^2*d - e)*S
qrt[d + e*x^2]) + (4*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2
])])/(c*Sqrt[e])))/(6*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsinh}\left(xc\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(122) = 244$.

Time = 0.20 (sec) , antiderivative size = 738, normalized size of antiderivative = 5.05

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(bc^2d^3+(bc^2de^2-be^3)x^4-bd^2e+2\,(bc^2d^2e-bde^2)x^2)\sqrt{e}\log\left(8\,c^4e^2x^4+c^4d^2\right.}{}\right.$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e -
b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e
 + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3*(b*c^2*d^2*e -
 b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(2*(a*c^2*d*
e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*
e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3
 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2
*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*ar
ctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(
-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (2*(b*c^2*d*e^2 - b*e
^3)*x^3 + 3*(b*c^2*d^2*e - b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*
x^2 + 1)) + (2*(a*c^2*d*e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (
b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e
- d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\operatorname{asinh}(c\,x)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{5/2}}\,dx=\frac{3\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3-2\sqrt{e}\,a\,d^2-4\sqrt{e}\,ade\,x^2-2\sqrt{e}\,a\,e^2x^4+3}{}$$

input
```
int((a+b*asinh(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - 2*sqrt(e)*a
*d**2 - 4*sqrt(e)*a*d*e*x**2 - 2*sqrt(e)*a*e**2*x**4 + 3*int(asinh(c*x)/(s
qrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2
*x**4),x)*b*d**4*e + 6*int(asinh(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d +
e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2*x**2 + 3*int
(asinh(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d
+ e*x**2)*e**2*x**4),x)*b*d**2*e**3*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 + e
**2*x**4))
```

3.170 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{15d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{2bc(3c^2d-2e)\sqrt{1+c^2x^2}}{15d^2\,(c^2d-e)^2\sqrt{d+ex^2}} + \frac{x(a+b\text{arcsinh}(cx))}{5d\,(d+ex^2)^{5/2}} + \frac{4x(a+b\text{arcsinh}(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}} - \frac{8b\text{arctan}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

output
```
-1/15*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(3/2)-2/15*b*c*(3*c^2*d-
2*e)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d-e)^2/(e*x^2+d)^(1/2)+1/5*x*(a+b*arcsinh(
c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arcsinh(c*x))/d^2/(e*x^2+d)^(3/2)+8/15
*x*(a+b*arcsinh(c*x))/d^3/(e*x^2+d)^(1/2)-8/15*b*arctan(e^(1/2)*(c^2*x^2+
1)^(1/2)/c/(e*x^2+d)^(1/2))/d^3/e^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{ax(15d^2+20dex^2+8e^2x^4)-\frac{bcd\sqrt{1+c^2x^2}(d+ex^2)(-e(5d+4ex^2)+c^2d(7d+6ex^2))}{(-c^2d+e)^2}-4bcx^2(a}{1}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x
^2)*(-(e*(5*d + 4*e*x^2)) + c^2*d*(7*d + 6*e*x^2)))/(-(c^2*d) + e)^2 - 4*b
*c*x^2*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^
2), -((e*x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcSinh[c*x])/(1
5*d^3*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6207, 27, 7266, 1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{6207}$$

$$-bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{15d^3\sqrt{c^2x^2+1}(ex^2+d)^{5/2}}dx+\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x\left(8e^2x^4+20dex^2+15d^2\right)}{\sqrt{c^2x^2+1}\left(ex^2+d\right)^{5/2}}dx}{15d^3}+\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{bc\int \frac{8e^2x^4+20dex^2+15d^2}{\sqrt{c^2x^2+1}\left(ex^2+d\right)^{5/2}}dx^2}{30d^3}+\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{1193}$$

$$-\frac{bc\left(\frac{2\int \frac{3\left(4\left(c^2d-e\right)ex^2+d\left(7c^2d-6e\right)\right)}{\sqrt{c^2x^2+1}\left(ex^2+d\right)^{3/2}}dx^2}{3(c^2d-e)}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\left(\frac{2\int \frac{4\left(c^2d-e\right)ex^2+d\left(7c^2d-6e\right)}{\sqrt{c^2x^2+1}\left(ex^2+d\right)^{3/2}}dx^2}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{bc\left(\frac{2\left(4\left(c^2d-e\right)\int \frac{1}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}}dx^2+\frac{2d\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\operatorname{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\operatorname{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\operatorname{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{66}$$

$$-\frac{bc\left(\frac{2\left(8(c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2+1}}{\sqrt{ex^2+d}}+\frac{2d\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}+$$
$$\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{8x(a+b\text{arcsinh}(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\text{arcsinh}(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\text{arcsinh}(cx))}{5d\left(d+ex^2\right)^{5/2}}-$$
$$\frac{bc\left(\frac{2\left(\frac{8\left(c^2d-e\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}+\frac{2d\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{c^2d-e}+\frac{2d^2\sqrt{c^2x^2+1}}{(c^2d-e)(d+ex^2)^{3/2}}\right)}{30d^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(x*(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x
]))/(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d
 + e*x^2]) - (b*c*((2*d^2*Sqrt[1 + c^2*x^2])/((c^2*d - e)*(d + e*x^2)^(3/2
)) + (2*((2*d*(3*c^2*d - 2*e)*Sqrt[1 + c^2*x^2])/((c^2*d - e)*Sqrt[d + e*x
^2]) + (8*(c^2*d - e)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^
2])])/(c*Sqrt[e])))/(c^2*d - e)))/(30*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 6207
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSinh[c*x])   u,
 x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int \frac{a + b \,\mathrm{arcsinh}\left(xc\right)}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(7/2),x)
```

output
```
int((a+b*arcsinh(x*c))/(e*x^2+d)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(193) = 386$.

Time = 0.22 (sec) , antiderivative size = 1354, normalized size of antiderivative = 5.96

$$\int \frac{a + b\mathrm{arcsinh}(cx)}{(d + ex^2)^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(2*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e
^5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 +
3*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x
^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^
3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^
2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 +
 b*d*e^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x
^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 +
 a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c
^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2
 + 2*(3*b*c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e
^3)*x^2)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^4*d^8*e - 2*c^2*d^7*e^2 +
d^6*e^3 + (c^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2
*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 - 2*c^2*d^6*e^3 + d^5*e^4)*x^
2), 1/15*(4*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 +
b*e^5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4
 + 3*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*
e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 - 2*b*c^2*d*
e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(ex^2 + d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*
x^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^
(7/2), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(ex^2 + d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex^2)^{7/2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(e\,x^2 + d)^{7/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x^2)^(7/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{15\sqrt{e\,x^2+d}\,a\,d^2ex+20\sqrt{e\,x^2+d}\,ad\,e^2x^3+8\sqrt{e\,x^2+d}\,a\,e^3x^5-8\sqrt{e}\,a\,d^3-24\sqrt{}}{}$$

input
```
int((a+b*asinh(c*x))/(e*x^2+d)^(7/2),x)
```

output
```
(15*sqrt(d + e*x**2)*a*d**2*e*x + 20*sqrt(d + e*x**2)*a*d*e**2*x**3 + 8*sq
rt(d + e*x**2)*a*e**3*x**5 - 8*sqrt(e)*a*d**3 - 24*sqrt(e)*a*d**2*e*x**2 -
 24*sqrt(e)*a*d*e**2*x**4 - 8*sqrt(e)*a*e**3*x**6 + 15*int(asinh(c*x)/(sqr
t(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d + e*x**2)*d
*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**6*e + 45*int(asinh(c*x)/(
sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d + e*x**2
)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**5*e**2*x**2 + 45*int(a
sinh(c*x)/(sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt
(d + e*x**2)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**4*e**3*x**4
 + 15*int(asinh(c*x)/(sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x*
*2 + 3*sqrt(d + e*x**2)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**
3*e**4*x**6)/(15*d**3*e*(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6)
)
```

3.171 $\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2,x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 11.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6209}$$

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{e\,x^2+d}\,(a+b\,\text{arcsinh}\,(xc))^2\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^2,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d),
x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}\,(cx))^2\,\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**2*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int \sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,a^2ex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a^2d+4\left(\int\sqrt{e\,x^2+d}\,asinh(cx)\,dx\right)abe+2\left(\int\sqrt{e\,x^2+d}\,asinh(cx\right.}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(d + e*x**2)*a**2*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a**2*d + 4*int(sqrt(d + e*x**2)*asinh(c*x),x)*a*b*e + 2*int(sqrt(d
+ e*x**2)*asinh(c*x)**2,x)*b**2*e)/(2*e)
```

3.172 $\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 7.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \; 6209$$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^2}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(e*x^2 + d),
x)
```

Sympy [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \int \frac{(a + b\operatorname{asinh}(cx))^2}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a^2+2\left(\int\frac{asinh(cx)}{\sqrt{e\,x^2+d}}dx\right)abe+\left(\int\frac{asinh(cx)^2}{\sqrt{e\,x^2+d}}dx\right)b^2e}{e}$$

input
```
int((a+b*asinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2 + 2*int(asinh(c*
x)/sqrt(d + e*x**2),x)*a*b*e + int(asinh(c*x)**2/sqrt(d + e*x**2),x)*b**2*
e)/e
```

3.173 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^2}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(
e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + ex^2)^{3/2}}\,dx = \int \frac{(a + b\,\text{asinh}\,(cx))^2}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 10.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a^2ex+\sqrt{e}\,a^2d+\sqrt{e}\,a^2e\,x^2+2\left(\int \frac{asinh(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)ab\,d^2e+2}{}$$

input
```
int((a+b*asinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a**2*e*x + sqrt(e)*a**2*d + sqrt(e)*a**2*e*x**2 + 2*int(
asinh(c*x)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a*b*d**2*e +
2*int(asinh(c*x)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a*b*d*e
**2*x**2 + int(asinh(c*x)**2/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2
),x)*b**2*d**2*e + int(asinh(c*x)**2/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2
)*e*x**2),x)*b**2*d*e**2*x**2)/(d*e*(d + e*x**2))
```

3.174 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}} \, dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}} \, dx = \int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}} \, dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^2}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(
e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 61.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.50

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(
b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 488, normalized size of antiderivative = 22.18

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,a^2dex+2\sqrt{e\,x^2+d}\,a^2e^2x^3-2\sqrt{e}\,a^2d^2-4\sqrt{e}\,a^2de\,x^2-2\sqrt{e}\,a^2e^2x}{}$$

input
```
int((a+b*asinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a**2*d*e*x + 2*sqrt(d + e*x**2)*a**2*e**2*x**3 - 2*sqr
t(e)*a**2*d**2 - 4*sqrt(e)*a**2*d*e*x**2 - 2*sqrt(e)*a**2*e**2*x**4 + 6*in
t(asinh(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d
 + e*x**2)*e**2*x**4),x)*a*b*d**4*e + 12*int(asinh(c*x)/(sqrt(d + e*x**2)*
d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*a*b*d*
*3*e**2*x**2 + 6*int(asinh(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2
)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*a*b*d**2*e**3*x**4 + 3*int(asi
nh(c*x)**2/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d +
 e*x**2)*e**2*x**4),x)*b**2*d**4*e + 6*int(asinh(c*x)**2/(sqrt(d + e*x**2)
*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b**2*
d**3*e**2*x**2 + 3*int(asinh(c*x)**2/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e
*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b**2*d**2*e**3*x**4)/(3*d
**2*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.175 $\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)} dx = \text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}, x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)} dx = \int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)} dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\,\operatorname{arcsinh}\left(xc\right)}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\,\text{asinh}\left(cx\right)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{asinh\,(cx)\,b+a}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*asinh(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*asinh(c*x)),x)
```

3.176 $\int \frac{1}{\sqrt{d+ex^2}(a+b\,\mathbf{arcsinh}(cx))} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))} \, dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))} \, dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx$$
$$\Big\downarrow \; 6209$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{e\,x^2+d}\;(a+b\;\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x
)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))\,\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,asinh\,(cx)\,b+\sqrt{e\,x^2+d}\,a}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*asinh(c*x)*b + sqrt(d + e*x**2)*a),x)
```

3.177 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2
*b*d*e*x^2 + b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,asinh\,(cx)\,bd+\sqrt{e\,x^2+d}\,asinh\,(cx)\,be\,x^2+\sqrt{e\,x^2+}$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*asinh(c*x)*b*d + sqrt(d + e*x**2)*asinh(c*x)*b*e*x
**2 + sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2),x)
```

3.178 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))} dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}, x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))} dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.95

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^
3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,asinh\,(cx)\,b\,d^2+2\sqrt{e\,x^2+d}\,asinh\,(cx)\,bde\,x^2+\sqrt{e\,x^2+d}\,asinh\,(cx)\,b\,e^2x^4+\sqrt{e\,x^2+d}\,a\,d^2+2\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4}\,dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*asinh(c*x)*b*d**2 + 2*sqrt(d + e*x**2)*asinh(c*x)*
b*d*e*x**2 + sqrt(d + e*x**2)*asinh(c*x)*b*e**2*x**4 + sqrt(d + e*x**2)*a*
d**2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d + e*x**2)*a*e**2*x**4),x)
```

3.179 $\int \frac{\sqrt{d+ex^2}}{(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2} \, dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}}{\left(a+b\ \text{arcsinh}\left(xc\right)\right)^2}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{d+ex^2}}{\left(a+b\,\text{asinh}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 575, normalized size of antiderivative = 26.14

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 + 1)^(3/2)*sqrt(e*x^2 + d) + (c^3*x^3 + c*x)*sqrt(e*x^2 + d))/(
a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^
2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((
2*c^3*e*x^4 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (4*c^4*e*x^
5 + 2*(c^4*d + 2*c^2*e)*x^3 + (c^2*d + e)*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2
+ d) + (2*c^5*e*x^6 + (c^5*d + 4*c^3*e)*x^4 + 2*(c^3*d + c*e)*x^2 + c*d)*s
qrt(e*x^2 + d))/(a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*
e)*a*b*x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^
2*c^5*e*x^6 + (c^5*d + 2*c^3*e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*
d + (b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^5 + b^2
*c^2*d*x + (c^4*d + c^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt
(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{e\,x^2+d}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{\sqrt{e\,x^2+d}}{\left(asinh\left(cx\right)b+a\right)^2}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*asinh(c*x))^2,x)
```

3.180 $\int \frac{1}{\sqrt{d+ex^2}(a+b\,\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 6.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \; 6209$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{e\,x^2+d}\;\left(a+b\,\text{arcsinh}\left(xc\right)\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(
c*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 591, normalized size of antiderivative = 26.86

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*
a*b*c^2*x + (sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*b^2*c^2*x + (b^2*c^3*x^2 +
b^2*c)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*
c)*sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 + 2*c^3*d*x^2 + (c^2*x^2 + 1)*(
(c^3*d - 2*c*e)*x^2 - c*d) + c*d + sqrt(c^2*x^2 + 1)*(2*(c^4*d - c^2*e)*x^
3 + (c^2*d - e)*x))/((a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt(e*
x^2 + d) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(
c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 +
 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x + (c^4*d + c^2*e)*b^2
*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e*x^6 + (c^5*d + 2*c^3*
e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sqrt(e*x^2 + d))*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*
d + c*e)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,\left(asinh\left(cx\right)b+a\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*asinh(c*x))^2,x)
```

3.181 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2}, x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\text{arcsinh}(cx))^2} dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}} (a+b\,\text{arcsinh}\,(xc))^2} dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2
*x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*
e*x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 864, normalized size of antiderivative = 39.27

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt
(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^
2 + 1)*sqrt(e*x^2 + d) + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d)
*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e*x^4 + (c^3*d +
 c*e)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)) - integrate((2*c^5*e*x^6 - (c^5*
d - 4*c^3*e)*x^4 - 2*(c^3*d - c*e)*x^2 + (2*c^3*e*x^4 - (c^3*d - 4*c*e)*x^
2 + c*d)*(c^2*x^2 + 1) - c*d + (4*c^4*e*x^5 - 2*(c^4*d - 4*c^2*e)*x^3 - (c
^2*d - 3*e)*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 +
a*b*c^3*d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^2*x^7 + (2*c
^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*a*b*x^3)
*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4
 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^2*x^7 + (
2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*b^2*x
^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^
3*e^2)*b^2*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^
3*d^2 + c*d*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a
*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*
e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^2 + c*d*e)*a*b*x^2)*sqrt(e*x^2 + d)),
x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(asinh\left(cx\right)b+a\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*asinh(c*x))^2,x)
```

3.182 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\textbf{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$
$$\Big\downarrow \; 6209$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(xc))^2}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 6.77

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2
+ a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*ar
csinh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^
3)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 28.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 1123, normalized size of antiderivative = 51.05

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input

```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output

```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x
^3 + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e^2*x^5
+ 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (
b^2*c^3*e^2*x^6 + (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c
*d*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e^
2*x^6 + (2*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*
x^2)*sqrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d - 8*c^3*e)*x^4 - 2
*(c^3*d - 2*c*e)*x^2 + (4*c^3*e*x^4 - (c^3*d - 6*c*e)*x^2 + c*d)*(c^2*x^2
+ 1) - c*d + (8*c^4*e*x^5 - 2*(c^4*d - 7*c^2*e)*x^3 - (c^2*d - 5*e)*x)*sqr
t(c^2*x^2 + 1))/((a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*
x^4 + a*b*c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^3*x^9
+ (3*c^4*d*e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e
^2)*a*b*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^
2 + d) + ((b^2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b
^2*c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^3*x^9 + (3*c^
4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2
*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)
+ (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e + 6
*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4
+ b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x...
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 187, normalized size of antiderivative = 8.50

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,asinh\,(cx)^2\,b^2d^2+2\sqrt{e\,x^2+d}\,asinh\,(cx)^2\,b^2de\,x^2+}$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*asinh(c*x)**2*b**2*d**2 + 2*sqrt(d + e*x**2)*asinh
(c*x)**2*b**2*d*e*x**2 + sqrt(d + e*x**2)*asinh(c*x)**2*b**2*e**2*x**4 + 2
*sqrt(d + e*x**2)*asinh(c*x)*a*b*d**2 + 4*sqrt(d + e*x**2)*asinh(c*x)*a*b*
d*e*x**2 + 2*sqrt(d + e*x**2)*asinh(c*x)*a*b*e**2*x**4 + sqrt(d + e*x**2)*
a**2*d**2 + 2*sqrt(d + e*x**2)*a**2*d*e*x**2 + sqrt(d + e*x**2)*a**2*e**2*
x**4),x)
```

3.183 $\int (d+ex^2)^2 \sqrt{a+b\,\mathrm{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 672

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = d^2x\sqrt{a+b\text{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\text{arcsinh}(cx)} + \frac{\sqrt{b}d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}dee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} + \frac{\sqrt{b}dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} + \frac{\sqrt{b}e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5} - \frac{\sqrt{b}d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} + \frac{\sqrt{b}dee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3} - \frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5} - \frac{\sqrt{b}dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} + \frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5} - \frac{\sqrt{b}e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}$$

output
```
d^2*x*(a+b*arcsinh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arcsinh(c*x))^(1/2)+1/5*e^
2*x^5*(a+b*arcsinh(c*x))^(1/2)+1/4*b^(1/2)*d^2*exp(a/b)*Pi^(1/2)*erf((a+b*
arcsinh(c*x))^(1/2)/b^(1/2))/c-1/8*b^(1/2)*d*e*exp(a/b)*Pi^(1/2)*erf((a+b*
arcsinh(c*x))^(1/2)/b^(1/2))/c^3+1/32*b^(1/2)*e^2*exp(a/b)*Pi^(1/2)*erf((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^5+1/72*b^(1/2)*d*e*exp(3*a/b)*3^(1/2)*Pi
^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3-1/192*b^(1/2)*e^2
*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
/c^5+1/1600*b^(1/2)*e^2*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))/c^5-1/4*b^(1/2)*d^2*Pi^(1/2)*erfi((a+b*arcsinh(c*x
))^(1/2)/b^(1/2))/c/exp(a/b)+1/8*b^(1/2)*d*e*Pi^(1/2)*erfi((a+b*arcsinh(c*
x))^(1/2)/b^(1/2))/c^3/exp(a/b)-1/32*b^(1/2)*e^2*Pi^(1/2)*erfi((a+b*arcsin
h(c*x))^(1/2)/b^(1/2))/c^5/exp(a/b)-1/72*b^(1/2)*d*e*3^(1/2)*Pi^(1/2)*erfi
(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3/exp(3*a/b)+1/192*b^(1/2)*e^
2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^5/exp(
3*a/b)-1/1600*b^(1/2)*e^2*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))/c^5/exp(5*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 4.23 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\, dx =$$

$$-\frac{be^{-\frac{5a}{b}}\left(450e^{\frac{6a}{b}}\left(8ac^4d^2\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+8bc^4d^2\text{arcsinh}(cx)\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}+b\left(4c^2d-e\right)e\sqrt{-\frac{a}{\cdots}}\right.\right.}{\cdots}$$

input
```
Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
-1/7200*(b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcSinh[c*x]] + 8*b*c^
4*d^2*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*(4*c^2*d - e)*e*Sqrt[-((a
+ b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b
+ ArcSinh[c*x]] + 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*A
rcSinh[c*x])^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] + 25*Sqrt[3]*
b*(8*c^2*d - 3*e)*e*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*Arc
Sinh[c*x])^2/b^2)]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] + 450*E^((4*a)/
b)*(8*a*c^4*d^2*Sqrt[-((a + b*ArcSinh[c*x])/b)] + 8*b*c^4*d^2*ArcSinh[c*x]
*Sqrt[-((a + b*ArcSinh[c*x])/b)] + b*e*(-4*c^2*d + e)*Sqrt[a/b + ArcSinh[c
*x]]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcSinh[c*x]
)/b)] - b*e*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcS
inh[c*x])^2/b^2)]*(25*Sqrt[3]*(8*c^2*d - 3*e)*Gamma[3/2, (3*(a + b*ArcSinh
[c*x]))/b] + 9*Sqrt[5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b
])))/(c^5*E^((5*a)/b)*(a + b*ArcSinh[c*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(d^2\sqrt{a+b\text{arcsinh}(cx)}+2dex^2\sqrt{a+b\text{arcsinh}(cx)}+e^2x^4\sqrt{a+b\text{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^5}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^5}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{-\frac{5a}{b}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}d^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}d^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+d^2x\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\operatorname{arcsinh}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\operatorname{arcsinh}(cx)}$$

input

```
Int[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

output

```
d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 +
(e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Er
f[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)
/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) -
 (Sqrt[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sq
rt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*
e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^(
(3*a)/b)) + (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(64*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*S
qrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{2} \sqrt{a+b\,\mathrm{arcsinh}\left(x c\right)}dx$$

input
```
int((e*x^2+d)^2*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((e*x^2+d)^2*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\operatorname{asinh}{(cx)}}\left(d+ex^2\right)^2\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(ex^2+d\right)^2\sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(ex^2+d\right)^2\sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\, dx = \int \sqrt{a+b\,\text{asinh}\,(c\,x)}\,(e\,x^2+d)^2\, dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int (d+ex^2)^2 \sqrt{a+b\text{arcsinh}(cx)}\, dx = \left(\int \sqrt{asinh\,(cx)\,b+a}dx\right) d^2 + \left(\int \sqrt{asinh\,(cx)\,b+a}\,x^4dx\right) e^2 + 2\left(\int \sqrt{asinh\,(cx)\,b+a}\,x^2dx\right) de$$

input
```
int((e*x^2+d)^2*(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)*d**2 + int(sqrt(asinh(c*x)*b + a)*x**4,x)*e*
*2 + 2*int(sqrt(asinh(c*x)*b + a)*x**2,x)*d*e
```

3.184 $\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 322

$$\begin{aligned}\int (d+ex^2) \sqrt{a+b\text{arcsinh}(cx)}\,dx &= dx\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)} \\ &+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\ &-\frac{\sqrt{b}ee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\ &+\frac{\sqrt{b}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3} \\ &-\frac{\sqrt{b}de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} \\ &+\frac{\sqrt{b}ee^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\ &-\frac{\sqrt{b}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}\end{aligned}$$

output
```
d*x*(a+b*arcsinh(c*x))^(1/2)+1/3*e*x^3*(a+b*arcsinh(c*x))^(1/2)+1/4*b^(1/2
)*d*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c-1/16*b^(1/2)
*e*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3+1/144*b^(1/
2)*e*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))/c^3-1/4*b^(1/2)*d*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/e
xp(a/b)+1/16*b^(1/2)*e*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3
/exp(a/b)-1/144*b^(1/2)*e*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.99

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx$$
$$=\frac{de^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}$$
$$+\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{3}{2},-\right.\right.}{72c}$$

input
```
Integrate[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]]
)/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcSinh[c*x]]*(9
*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]
] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/
b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*
x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (
3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x
])^2/b^2)])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(d\sqrt{a+b\text{arcsinh}(cx)}+ex^2\sqrt{a+b\text{arcsinh}(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+dx\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\text{arcsinh}(cx)}$$

input
```
Int[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[
b]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt
[b]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (
Sqrt[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[
b]])/(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqr
t[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)\sqrt{a+b\,\mathrm{arcsinh}\left(xc\right)}dx$$

input
```
int((e*x^2+d)*(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((e*x^2+d)*(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)\,\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(d+ex^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\operatorname{asinh}{(cx)}}\left(d+ex^2\right)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(ex^2+d\right)\sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \left(d+ex^2\right)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \left(ex^2+d\right)\sqrt{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}\,dx=\left(\int \sqrt{asinh\left(cx\right)b+a}dx\right)d+\left(\int \sqrt{asinh\left(cx\right)b+a}\,x^2dx\right)e$$

input
```
int((e*x^2+d)*(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)*d + int(sqrt(asinh(c*x)*b + a)*x**2,x)*e
```

3.185 $\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

output
```
x*(a+b*arcsinh(c*x))^(1/2)+1/4*b^(1/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(
c*x))^(1/2)/b^(1/2))/c-1/4*b^(1/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/
Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6187}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}bc\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6234}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}+x\sqrt{a+b\text{arcsinh}(cx)}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \; 26$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c}$$

$$\Big\downarrow \; 3789$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{2c}$$

$$\Big\downarrow \; 2611$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{2c}$$

$$\Big\downarrow \; 2633$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

$$\Big\downarrow \; 2634$$

$$x\sqrt{a+b\text{arcsinh}(cx)}-\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{2c}$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
x*Sqrt[a + b*ArcSinh[c*x]] - ((I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{a + b\,\mathrm{arcsinh}\,(xc)}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\mathrm{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{a + b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{b\,\text{arsinh}\,(cx) + a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{b\,\text{arsinh}\,(cx) + a}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{a+b\,\text{asinh}\left(c\,x\right)}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\int \sqrt{asinh\left(cx\right)b+a}dx$$

input
```
int((a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)
```

3.186 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2} dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}, x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2} dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2} dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}dx$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(xc)}}{e\,x^2+d}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/(e*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/(e*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\,\text{asinh}\,(cx)}}{d+ex^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(cx)+a}}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\,\text{asinh}\,(c\,x)}}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 13.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{d+ex^2}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{e\,x^2+d}dx$$

input
```
int((a+b*asinh(c*x))^(1/2)/(e*x^2+d),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(d + e*x**2),x)
```

3.187 $\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2} dx = \text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}, x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2} dx = \int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2} dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2} \, dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{(d+ex^2)^2}dx$$

input
```
Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\,\text{arcsinh}\,(xc)}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(x*c))^(1/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(1/2)/(e*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 15.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\,\mathrm{asinh}\,(cx)}}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

output
```
Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{b\,\mathrm{arsinh}\,(cx)+a}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + ex^2)^2} \, dx = \int \frac{\sqrt{b \, \text{arsinh}\,(cx) + a}}{(ex^2 + d)^2} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\text{arcsinh}(cx)}}{(d + ex^2)^2} \, dx = \int \frac{\sqrt{a + b \, \text{asinh}\,(c \, x)}}{(e \, x^2 + d)^2} \, dx$$

input
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{\left(e\,x^2+d\right)^2}dx$$

input
```
int((a+b*asinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*asinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

3.188 $\int (d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 427

$$\begin{aligned}\int (d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}\,dx =\\ & -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3}\\ & -\frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c}+dx(a+b\text{arcsinh}(cx))^{3/2}\\ & +\frac{1}{3}ex^3(a+b\text{arcsinh}(cx))^{3/2}+\frac{3b^{3/2}de^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}\\ & -\frac{3b^{3/2}ee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{b^{3/2}ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\\ & +\frac{3b^{3/2}de^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b^{3/2}ee^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}\\ & +\frac{b^{3/2}ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\end{aligned}$$

output
```
-3/2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+1/3*b*e*(c^2*x^2+1)^
(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*e*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcs
inh(c*x))^(1/2)/c+d*x*(a+b*arcsinh(c*x))^(3/2)+1/3*e*x^3*(a+b*arcsinh(c*x)
)^(3/2)+3/8*b^(3/2)*d*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/
2))/c-3/32*b^(3/2)*e*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))/c^3+1/288*b^(3/2)*e*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsin
h(c*x))^(1/2)/b^(1/2))/c^3+3/8*b^(3/2)*d*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))/c/exp(a/b)-3/32*b^(3/2)*e*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))/c^3/exp(a/b)+1/288*b^(3/2)*e*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c^3/exp(3*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.73 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.80

$$\int (d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x
]])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt
[-((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcSinh[c*x]
]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[
c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x
]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSin
h[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/
2, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh
[c*x])^2/b^2)]) + (Sqrt[b]*d*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[
1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt
[b]*e*(-9*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*
x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[
b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh
[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (
-2*a + b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh
[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*
ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c^3)
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) (a+b\text{arcsinh}(cx))^{3/2} \, dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(d(a+b\text{arcsinh}(cx))^{3/2}+ex^2(a+b\text{arcsinh}(cx))^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$
-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}de^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}de^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3bd\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}{2c}-\frac{bex^2\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c}+\frac{be\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3}+dx(a+b\operatorname{arcsinh}(cx))^{3/2}+\frac{1}{3}ex^3(a+b\operatorname{arcsinh}(cx))^{3/2}
$$

input

```
Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

output

```
(-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 +
c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sq
rt[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a
 + b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*
e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(
3/2)*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^
3*E^((3*a)/b))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \left(e\,x^{2}+d\right) \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right)^{\frac{3}{2}}d x$$

input
```
int((e*x^2+d)*(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((e*x^2+d)*(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}\,dx = \int \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{\frac{3}{2}} \left(d + e x^{2}\right)\, dx$$

input
```
integrate((e*x**2+d)*(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)*(d + e*x**2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)\left(a+b\text{arcsinh}(cx)\right)^{3/2}\,dx=\int \left(ex^2+d\right)\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)\left(a+b\text{arcsinh}(cx)\right)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a+b\text{arcsinh}(cx)\right)^{3/2}\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)*(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}\,dx = \left(\int \sqrt{asinh\,(cx)\,b+a}dx\right)ad + \left(\int \sqrt{asinh\,(cx)\,b+a}\,asinh(cx)\,x^2dx\right)be + \left(\int \sqrt{asinh\,(cx)\,b+a}\,asinh(cx)\,dx\right)bd + \left(\int \sqrt{asinh\,(cx)\,b+a}\,x^2dx\right)ae$$

input
```
int((e*x^2+d)*(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)*a*d + int(sqrt(asinh(c*x)*b + a)*asinh(c*x)*
x**2,x)*b*e + int(sqrt(asinh(c*x)*b + a)*asinh(c*x),x)*b*d + int(sqrt(asin
h(c*x)*b + a)*x**2,x)*a*e
```

3.189 $\int (a + b\text{arcsinh}(cx))^{3/2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 135

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}$$

output
```
-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c+x*(a+b*arcsinh(c*x))^(
3/2)+3/8*b^(3/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c
+3/8*b^(3/2)*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)(\cos$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]]
)/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b
*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (
-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Si
nh[a/b])))/(8*c)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\int \frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{c^2x^2+1}}dx$$

$\Big\downarrow$ 6213

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{b\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx}{2c}\right)$$

$\Big\downarrow$ 6189

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$\Big\downarrow$ 3042

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{2c^2}\right)$$

$\Big\downarrow$ 3788

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b}{2c^2}\right.$$

$\Big\downarrow$ 26

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+\text{barcs}}{2c^2}\right.$$

$\Big\downarrow$ 2611

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsin}}}{2c^2}\right.$$

$\downarrow$ 2633

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right.}{2c^2}\right.$$

$\downarrow$ 2634

$$x(a+b\text{arcsinh}(cx))^{3/2}-\frac{3}{2}bc\left(\frac{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{c^2}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2c^2}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
x*(a + b*ArcSinh[c*x])^(3/2) - (3*b*c*((Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSi
nh[c*x]])/c^2 - ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E
^(a/b)))/(2*c^2)))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [F]

$$\int (a + b \operatorname{arcsinh}(xc))^{\frac{3}{2}} dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a + b\text{arcsinh}(cx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a + b\text{arcsinh}(cx))^{3/2} dx = \int (a + b \operatorname{asinh}(cx))^{\frac{3}{2}} dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \left(\int \sqrt{asinh\left(cx\right)b+a}dx\right)a + \left(\int \sqrt{asinh\left(cx\right)b+a}\, asinh(cx)\,dx\right)b$$

input
```
int((a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a),x)*a + int(sqrt(asinh(c*x)*b + a)*asinh(c*x),x)
*b
```

3.190 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}, x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx = \int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2} \, dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b \,\text{arcsinh}\,(xc))^{\frac{3}{2}}}{e\,x^2+d}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/(e*x^2+d),x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/(e*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{d + ex^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 10.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{d + ex^2} dx = \int \frac{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}}{d + ex^2} dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d),x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^{3/2}}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{ex^2+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2),x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{d+ex^2}\,dx=\int \frac{(asinh(cx)\,b+a)^{\frac{3}{2}}}{e\,x^2+d}dx$$

input
```
int((a+b*asinh(c*x))^(3/2)/(e*x^2+d),x)
```

output
```
int((a+b*asinh(c*x))^(3/2)/(e*x^2+d),x)
```

3.191 $\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}, x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx = \int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} dx$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

output
```
Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2} \, dx$$
$$\Big\downarrow \text{6209}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}dx$$

input
```
Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*arcsinh(x*c))^(3/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*arcsinh(x*c))^(3/2)/(e*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 106.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^{3/2}}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2,x)
```

output
```
int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(asinh(cx)\,b+a)^{\frac{3}{2}}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*asinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*asinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

3.192 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\textbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{dee^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5} + \frac{dee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3} - \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5} + \frac{e^2e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}$$

output
```
1/2*d^2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c-
1/4*d*e*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^
3+1/16*e^2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)
/c^5+1/12*d*e*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))/b^(1/2)/c^3-1/32*e^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)
*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5+1/160*e^2*exp(5*a/b)*5^(1/2
)*Pi^(1/2)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5+1/2*d
^2*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(a/b)-1/4*
d*e*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^3/exp(a/b)+1
/16*e^2*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5/exp(a/
b)+1/12*d*e*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)/b^(1/2)/c^3/exp(3*a/b)-1/32*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5/exp(3*a/b)+1/160*e^2*5^(1/2)*Pi^(1/2)*
erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5/exp(5*a/b)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{5a}{b}}\left(-30(8c^4d^2-4c^2de+e^2)\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+3\sqrt{5}e^2\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-30*(8*c^4*d^2 - 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Ga
mma[1/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcSinh[c*x])/b
)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b
)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b]
- 15*Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-
3*(a + b*ArcSinh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh
[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - 120*c^2*d*e*E^((4*a)/b)
*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 3
0*e^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c*x])/b)] - 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Ga
mma[1/2, (3*(a + b*ArcSinh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*e^2*E^
((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]
)/(480*c^5*E^((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(\frac{d^2}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2dex^2}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\text{arcsinh}(cx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{-\frac{5a}{b}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}-\frac{\sqrt{\pi}dee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\pi}d^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}d^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
 - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]
*c^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*S
qrt[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3
]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*
Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^
5) + (d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^
(a/b)) - (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*
c^3*E^(a/b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*S
qrt[b]*c^5*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sq
rt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5
*E^((5*a)/b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \frac{(e\,x^2+d)^2}{\sqrt{a+b\,\mathrm{arcsinh}\,(xc)}}dx$$

input
```
int((e*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((e*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\mathrm{arcsinh}(cx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral((d + e*x**2)**2/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\left(\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b+a}dx\right)d^2+\left(\int \frac{\sqrt{asinh\left(cx\right)b+a}\,x^4}{asinh\left(cx\right)b+a}dx\right)e^2+2\left(\int \frac{\sqrt{asinh\left(cx\right)b+a}\,x^2}{asinh\left(cx\right)b+a}dx\right)de$$

input
```
int((e*x^2+d)^2/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x)*d**2 + int((sqrt(asinh(c*
x)*b + a)*x**4)/(asinh(c*x)*b + a),x)*e**2 + 2*int((sqrt(asinh(c*x)*b + a)
*x**2)/(asinh(c*x)*b + a),x)*d*e
```

3.193 $\int \frac{d+ex^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 287

$$\int \frac{d+ex^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

output
```
1/2*d*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c-1/
8*e*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^3+1/
24*e*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))/b^(1/2)/c^3+1/2*d*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(
1/2)/c/exp(a/b)-1/8*e*Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1
/2)/c^3/exp(a/b)+1/24*e*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))/b^(1/2)/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{3a}{b}}\left(-3(4c^2d-e)\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}e\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\text{arc}\right.\right.}{24\ldots}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-3*(4*c^2*d - e)*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Ar
cSinh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c*x]))/b] + 3*(4*c^2*d - e)*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[
c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*e*E^((6*a)/b)*Sq
rt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^(
(3*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6208}$$

$$\int \left(\frac{d}{\sqrt{a+b\text{arcsinh}(cx)}}+\frac{ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{\pi}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\pi}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

input
```
Int[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
 (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3
) + (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(2*Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
])/(8*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c*x]])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \frac{e\,x^{2}+d}{\sqrt{a +b \,\mathrm{arcsinh}\left(x c \right)}}d x$$

input
```
int((e*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int((e*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{d+ex^2}{\sqrt{a+b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{ex^2+d}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{ex^2+d}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{e\,x^2+d}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \left(\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b+a}dx\right)d+\left(\int \frac{\sqrt{asinh\left(cx\right)b+a}\,x^2}{asinh\left(cx\right)b+a}dx\right)e$$

input
```
int((e*x^2+d)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x)*d + int((sqrt(asinh(c*x)*
b + a)*x**2)/(asinh(c*x)*b + a),x)*e
```

3.194 $\int \frac{1}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

output
```
1/2*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c+1/2*
Pi^(1/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(1/2)/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx =\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{2c\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
(-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) +
Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*
c*E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{ie^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))+\frac{1}{2}\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}}{bc}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{bc}$$

input
```
Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

output
```
((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/2 + (Sqr
t[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*E^(a/b)))/(b*c)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b \operatorname{arcsinh}(xc)}} dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\operatorname{arcsinh}(cx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a + b\,\text{asinh}\,(cx)}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\,(cx) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\,(cx) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^(1/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b+a}dx$$

input
```
int(1/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b + a),x)
```

3.195 $\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\mathrm{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\mathrm{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\mathrm{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)\sqrt{b\,\mathrm{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{(ex^2+d)\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)\,bd+asinh\,(cx)\,be\,x^2+ad+ae\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b*d + asinh(c*x)*b*e*x**2 + a*d + a
*e*x**2),x)
```

3.196 $\int \frac{1}{(d+ex^2)^2\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex^2)^2\sqrt{a+b\operatorname{arcsinh}(cx)}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = \int \frac{1}{(d+ex^2)^2\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}dx$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^2\sqrt{a+b\,\text{arcsinh}\,(xc)}}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 45.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(cx)}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c\,x)}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$
$$=\int \frac{\sqrt{asinh\left(cx\right)b+a}}{asinh\left(cx\right)b\,d^2+2asinh\left(cx\right)bde\,x^2+asinh\left(cx\right)b\,e^2x^4+a\,d^2+2ade\,x^2+a\,e^2x^4}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*asinh(c*x))^(1/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)*b*d**2 + 2*asinh(c*x)*b*d*e*x**2 +
asinh(c*x)*b*e**2*x**4 + a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4),x)
```

3.197 $\int \frac{d+ex^2}{(a+b\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 349

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}-\frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}}\\
&-\frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{ee^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\\
&-\frac{ee^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{de^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}\\
&-\frac{ee^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{ee^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\end{aligned}$$

output
```
-2*d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-2*e*x^2*(c^2*x^2+1)^(1
/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-d*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))/b^(3/2)/c+1/4*e*exp(a/b)*Pi^(1/2)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))/b^(3/2)/c^3-1/4*e*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(
a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+d*Pi^(1/2)*erfi((a+b*arcsinh(
c*x))^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)-1/4*e*Pi^(1/2)*erfi((a+b*arcsinh(c
*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(a/b)+1/4*e*3^(1/2)*Pi^(1/2)*erfi(3^(1/
2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.87

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(\left(4c^2d-e\right)e^{\frac{4a}{b}+3\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsin}\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
((4*c^2*d - e)*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*e*E^(3*ArcSinh[c*x])*Sqrt[-((a + b*Arc
Sinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + (4*c^2*d - e)*E^(
(2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a
+ b*ArcSinh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcSinh[c*x]))*(4*c^2*d*E
^(2*ArcSinh[c*x]) + e*(-1 + E^(2*ArcSinh[c*x]))^2)) + Sqrt[3]*e*E^(3*(a/b
+ ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x
]))/b]))/(4*b*c^3*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6208}$$

$$\int \left(\frac{d}{(a+b\text{arcsinh}(cx))^{3/2}}+\frac{ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}ee^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2d\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2ex^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

input
```
Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

output
```
(-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1
+ c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
 + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*S
qrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3
*E^((3*a)/b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Maple [F]

$$\int \frac{e\,x^{2}+d}{\left(a+b\,\operatorname{arcsinh}\left(x c\right)\right)^{\frac{3}{2}}}d x$$

input
```
int((e*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int((e*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{d+ex^{2}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((d + e*x**2)/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{ex^2+d}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{e\,x^2+d}{(a+b\,\text{asinh}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int((d + e*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{d + ex^2}{(a + b\text{arcsinh}(cx))^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((e*x^2+d)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
(asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**4)/(asinh(c*x)**2*b**2*c**2*x**
2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a
**2*c**2*x**2 + a**2),x)*b**2*c**5*d + 2*asinh(c*x)*int((sqrt(asinh(c*x)*b
 + a)*x**4)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c
*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**3
*e + asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*x**2)/(asinh(c*x)**2*b**2*c**2
*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b
 + a**2*c**2*x**2 + a**2),x)*b**2*c**3*d + 2*asinh(c*x)*int((sqrt(asinh(c*
x)*b + a)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asi
nh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*
c*e - 6*asinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c
*x)*x**3)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x
)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**4*d
 - 6*asinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*x**3)/(asi
nh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**
2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**4*d + 2*sqrt(c**2*
x**2 + 1)*sqrt(asinh(c*x)*b + a)*c**2*d*x**2 - 4*sqrt(c**2*x**2 + 1)*sqrt(
asinh(c*x)*b + a)*d + int((sqrt(asinh(c*x)*b + a)*x**4)/(asinh(c*x)**2*b**
2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*
x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**5*d + 2*int((sqrt(asinh(c*x)*...
```

3.198 $\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

output
```
-2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-exp(a/b)*Pi^(1/2)*erf((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+Pi^(1/2)*erfi((a+b*arcsinh(c*x))
^(1/2)/b^(1/2))/b^(3/2)/c/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(cx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2}\right.\right.}{bc\sqrt{a+b\text{arc}}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output
```
(-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a
 + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a +
 b*ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}dx$$

$$\Big\downarrow \text{ 6188}$$

$$\frac{2c\int \frac{x}{\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}dx}{b}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \ 26$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \ 3789$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{-\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))-\frac{1}{2}i\int \frac{e^{\text{arcsinh}(cx)}}{\sqrt{a+b\text{arcsinh}(cx)}}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \ 2611$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-i\int e^{\frac{a+b\text{arcsinh}(cx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c}$$

$$\Big\downarrow \ 2633$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}}d\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

$$\Big\downarrow \ 2634$$

$$-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

output
```
(-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + ((2*I)*((I/2)*Sqrt
[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]] - ((I/2)*Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/E^(a/b)))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

input
```
int(1/(a + b*asinh(c*x))^(3/2),x)
```

output
```
int(1/(a + b*asinh(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*asinh(c*x))^(3/2),x)
```

output

```
(2*sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x) - asinh(c*x)*int(
(sqrt(asinh(c*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + a
sinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c
**2*x**2 + a**2),x)*b**2*c**3 - asinh(c*x)*int((sqrt(asinh(c*x)*b + a)*asi
nh(c*x))/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)
*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c - 2*a
sinh(c*x)*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)*x)/(a
sinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x
**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c**2 - 2*asinh(c*x)
*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b + a)*asinh(c*x)**2*x)/(asinh(c
*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 +
2*asinh(c*x)*a*b + a**2*c**2*x**2 + a**2),x)*b**2*c**2 - int((sqrt(asinh(c
*x)*b + a)*asinh(c*x)*x**2)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c*x)**2*
b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x**2 + a*
*2),x)*a*b*c**3 - int((sqrt(asinh(c*x)*b + a)*asinh(c*x))/(asinh(c*x)**2*b
**2*c**2*x**2 + asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(
c*x)*a*b + a**2*c**2*x**2 + a**2),x)*a*b*c - 2*int((sqrt(c**2*x**2 + 1)*sq
rt(asinh(c*x)*b + a)*asinh(c*x)*x)/(asinh(c*x)**2*b**2*c**2*x**2 + asinh(c
*x)**2*b**2 + 2*asinh(c*x)*a*b*c**2*x**2 + 2*asinh(c*x)*a*b + a**2*c**2*x*
*2 + a**2),x)*a**2*c**2 - 2*int((sqrt(c**2*x**2 + 1)*sqrt(asinh(c*x)*b ...
```

3.199 $\int \frac{1}{(d+ex^2)(a+b\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}dx$$
$$\Big\downarrow \text{ 6209}$$
$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\text{arcsinh}\,(xc))^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.36

$$\int \frac{1}{(d+ex^2)\,(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{\sqrt{asinh\,(cx)\,b+a}}{asinh\,(cx)^2\,b^2d+asinh\,(cx)^2\,b^2e\,x^2+2asinh\,(cx)\,abd+2asinh}$$

input
```
int(1/(e*x^2+d)/(a+b*asinh(c*x))^(3/2),x)
```

output
```
int(sqrt(asinh(c*x)*b + a)/(asinh(c*x)**2*b**2*d + asinh(c*x)**2*b**2*e*x*
*2 + 2*asinh(c*x)*a*b*d + 2*asinh(c*x)*a*b*e*x**2 + a**2*d + a**2*e*x**2),
x)
```

3.200 $\int \frac{1}{(d+ex^2)^2(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6209}$$

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \left(a+b \,\operatorname{arcsinh}\left(x c\right)\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c))^(3/2),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arcsinh(x*c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}} dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}} dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}} dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{3/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(e\,x^2+d)^2\,(asinh\,(cx)\,b+a)^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*asinh(c*x))^(3/2),x)
```

output

```
int(1/(e*x^2+d)^2/(a+b*asinh(c*x))^(3/2),x)
```

3.201 $\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 416

$$\begin{aligned}\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = &-\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}\\ &-\frac{2ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}\\ &+\frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\\ &-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\\ &+\frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c}\\ &+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
-2/3*I*b*d^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*
b*c*d^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*
c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c
^3*d^2*x^4*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+3/8*d^2*x
*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))-1/4*c^2*d^2*x^3*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+2/3*I*d^2*(d+I*c*d*x)
^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+5/16*d^2*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.87

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{48ad^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}(16i+9cx+16ic^2x^2-6c^3x^3)+720ad^{5/2}\sqrt{f}\sqrt{1}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(48*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*(16*I + 9*
c*x + (16*I)*c^2*x^2 - 6*c^3*x^3) + 720*a*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2
]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 144
*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^2 - Cosh[2*ArcS
inh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]) - (64*I)*b*d^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + C
osh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]) + 9*b*d^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]
*Sinh[4*ArcSinh[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d^2(icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int \left(-c^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x^2+2ic\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x+\sqrt{c^2x^2+1}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{2i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}-\frac{1}{4}c^2x^3\sqrt{c^2x^2+1}(a+b\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(((-2*I)/3)*b*x - (3*b*c*x^2)/16
- ((2*I)/9)*b*c^2*x^3 + (b*c^3*x^4)/16 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/8 - (c^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 + (((2*
I)/3)*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (5*(a + b*ArcSinh[c*x]
)^2)/(16*b*c)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(339) = 678$.

Time = 5.95 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.53

method	result
default	$\frac{ia(icdx+d)^{\frac{5}{2}}(-icfx+f)^{\frac{3}{2}}}{4cf}+\frac{5iad(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}{12cf}+\frac{5ia\,d^2\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{8cf}-\frac{5ia\,d^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{8c}+$
parts	$\frac{ia(icdx+d)^{\frac{5}{2}}(-icfx+f)^{\frac{3}{2}}}{4cf}+\frac{5iad(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}{12cf}+\frac{5ia\,d^2\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{8cf}-\frac{5ia\,d^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{8c}+$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/4*I*a/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)+5/12*I*a*d/c/f*(d+I*c*d*x)
^(3/2)*(f-I*c*f*x)^(3/2)+5/8*I*a*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/
2)-5/8*I*a*d^2/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+5/8*a*d^3*f*((f-I*c*f
*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c
^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(5/16*(I*(x*c-I)*
d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*d^2-1/256
*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)
^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1
+4*arcsinh(x*c))*d^2/c/(c^2*x^2+1)+1/36*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*
f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1
/2)*x*c+1)*(-1+3*arcsinh(x*c))*d^2/c/(c^2*x^2+1)+1/16*(I*(x*c-I)*d)^(1/2)*
(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2
+1)^(1/2))*(-1+2*arcsinh(x*c))*d^2/c/(c^2*x^2+1)+1/4*I*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)-1)*d
^2/c/(c^2*x^2+1)+1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(
c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)+1)*d^2/c/(c^2*x^2+1)+1/16*(I*(x*c-I)
*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*
c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(x*c))*d^2/c/(c^2*x^2+1)+1/36*I*(I*(x*c-I
)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c
^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*(1+3*arcsinh(x*c))*d^2/c/(c^2*x^2+1)-...
```

Fricas [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral(-(b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x
 - a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^{5/2}\sqrt{f - icfx}(a + b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{5/2}\sqrt{f - icfx}(a + b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,d^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-6\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^3x^3+16\sqrt{cix+1}\,\sqrt{-cix}\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - 6*sqrt(c*
i*x + 1)*sqrt( - c*i*x + 1)*a*c**3*x**3 + 16*sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)*a*c**2*i*x**2 + 9*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c*x + 16*sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i - 24*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)*x**2,x)*b*c**3 + 48*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)
*asinh(c*x)*x,x)*b*c**2*i + 24*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin
h(c*x),x)*b*c))/(24*c)
```

3.202 $\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 304

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = -\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} -\frac{ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) +\frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} +\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
-1/3*I*b*d*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c
*d*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*I*b*c^2*d
*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*d*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+1/3*I*d*(d+I*c*d*x)^(1/2)*
(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+1/4*d*(d+I*c*d*x)^(1/2)
*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{12ad\sqrt{d+icdx}\sqrt{f-icfx}(2i+3cx+2ic^2x^2)+36ad^{3/2}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(12*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*I + 3*c*x + (2*I)*c^2*x^2)
+ 36*a*d^(3/2)*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]] + (9*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*
x]^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1
 + c^2*x^2] - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*Ar
cSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c
*x]]))/Sqrt[1 + c^2*x^2])/(72*c)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d(icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 6253

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int \left(icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)dx}{\sqrt{c^2x^2+1}}$$

$\big\downarrow$ 2009

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{9}ibc\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1/3*I)*b*x - (b*c*x^2)/4 - (I/9)
*b*c^2*x^3 + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (a + b*ArcSinh[c*x])^2/(4*b*c)))/Sq
rt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(245) = 490$.

Time = 4.88 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.50

method	result
default	$\frac{ia(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}{3cf}+\frac{iad\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{2cf}-\frac{iad\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c}+\frac{a\,d^{2}f\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}\right.}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{}}$
parts	$\frac{ia(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}{3cf}+\frac{iad\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{2cf}-\frac{iad\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c}+\frac{a\,d^{2}f\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}\right.}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{}}$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/3*I*a/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+1/2*I*a*d/c/f*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(3/2)-1/2*I*a*d/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+1/
2*a*d^2*f*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1
/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+
b*(1/4*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsin
h(x*c)^2*d+1/72*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c
^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(-1+3*arcsinh
(x*c))*d/(c^2*x^2+1)/c+1/16*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(2*x^
3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x
*c))*d/(c^2*x^2+1)/c+1/8*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x
^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)-1)*d/(c^2*x^2+1)/c+1/8*I*(I*(x*c
-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(arcsi
nh(x*c)+1)*d/(c^2*x^2+1)/c+1/16*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(
2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsin
h(x*c))*d/(c^2*x^2+1)/c+1/72*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4
*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*
(1+3*arcsinh(x*c))*d/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((I*b*c*d*x + b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (I*a*c*d*x + a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
), x)
```

Sympy [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\int \left(id(cx-i)\right)^{\frac{3}{2}}\sqrt{-if\left(cx+i\right)}(a+b\operatorname{asinh}\left(cx\right))\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a + b\operatorname{asinh}(c\,x))\;(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,d\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+2\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^2 i\,x^2+3\sqrt{cix+1}\,\sqrt{-cix+$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + 2*sqrt(c*i*x
+ 1)*sqrt( - c*i*x + 1)*a*c**2*i*x**2 + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x +
1)*a*c*x + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i + 6*int(sqrt(c*i*x + 1
)*sqrt( - c*i*x + 1)*asinh(c*x)*x,x)*b*c**2*i + 6*int(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*asinh(c*x),x)*b*c))/(6*c)
```

3.203 $\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 147

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
-1/4*b*c*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*x*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+1/4*(d+I*c*d*x)^(1/2
)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.59

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{2}ax\sqrt{id(-i+cx)}\sqrt{-if(i+cx)} + \frac{a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(-i+cx)}\sqrt{-if(i+cx)}\right)}{2c} - \frac{b\sqrt{i(-id+cdx)}\sqrt{-i(if+cfx)}\sqrt{-df(1+c^2x^2)}(\cosh(2\text{arcsinh}(cx))-2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\text{s}}{8c\sqrt{-((-id+cdx)(if+cfx))}\sqrt{1+c^2x^2}}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(a*x*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)])/2 + (a*Sqrt[d]*Sqrt[f]*L
og[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])
/(2*c) - (b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(
1 + c^2*x^2))]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh
[2*ArcSinh[c*x]])))/(8*c*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 +
c^2*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6200}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-1/4*(b*c*x^2) + (x*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*c)))/Sqrt[1 + c^2*
x^2]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(119) = 238.

Time = 4.24 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.56

method	result
default	$\frac{ia\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{2cf}-\frac{ia\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c}+\frac{adf\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\sqrt{c^{2}df\,x^{2}+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^{2}df}}+b\Bigg(\frac{\sqrt{i(xc-}$
parts	$\frac{ia\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{2cf}-\frac{ia\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c}+\frac{adf\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\sqrt{c^{2}df\,x^{2}+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^{2}df}}+b\Bigg(\frac{\sqrt{i(xc-}$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/2*I*a/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)-1/2*I*a/c*(f-I*c*f*x)^(1/2
)*(d+I*c*d*x)^(1/2)+1/2*a*d*f*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^
(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/
2))/(c^2*d*f)^(1/2)+b*(1/4*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x
^2+1)^(1/2)/c*arcsinh(x*c)^2+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)
*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arc
sinh(x*c))/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*
x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(
x*c))/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1
)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a, x)
```

Sympy [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$
$$=\int \sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}{(cx)})\;dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$
$$= \int (a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{\sqrt{f}\,\sqrt{d}\,\left(2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+\sqrt{cix+1}\,\sqrt{-cix+1}\,acx+2\left(\int\sqrt{cix+1}\,\sqrt{-cix+1}\,asinh(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*a*c*x + 2*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh
(c*x),x)*b*c))/(2*c)
```

3.204 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 158

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
I*b*f*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*f*(c^2*x^2
+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*f*(c^2*x^
2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)-2ib\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b\sqrt{d+i}}{2cd\sqrt{1+c^2x}}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
((2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) -
 (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqr
t[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]])/(2*c*d*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{c^2x^2+1}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f\sqrt{c^2x^2+1}\int \left(\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\sqrt{c^2x^2+1}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc}+ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
(f*Sqrt[1 + c^2*x^2]*(I*b*x - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
 + (a + b*ArcSinh[c*x])^2/(2*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(132) = 264$.

Time = 4.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.98

method	result
default	$-\frac{ia\sqrt{-icfx+f}\,\sqrt{icdx+d}}{cd}+\frac{af\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}}+b\left(\frac{\operatorname{arcsinh}(xc)^2\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{2\sqrt{c^2x^2+1}\,cd}\right.$
parts	$-\frac{ia\sqrt{-icfx+f}\,\sqrt{icdx+d}}{cd}+\frac{af\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}}+b\left(\frac{\operatorname{arcsinh}(xc)^2\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{2\sqrt{c^2x^2+1}\,cd}\right.$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-I*a/c/d*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+a*f*((f-I*c*f*x)*(d+I*c*d*x))
^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c
^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(1/2*arcsinh(x*c)^2*(I*(x*c-I)*d)
^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c/d-1/2*I*(arcsinh(x*c)-1)*(
c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/
c/(c^2*x^2+1)/d-1/2*I*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(
I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/(c^2*x^2+1)/d)
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*d*x - I*d), x)
```

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
a*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + b*i
ntegrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x + d)
, x)
```

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{f}\left(2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-\sqrt{cix+1}\,\sqrt{-cix+1}\,ai+\left(\int \frac{\sqrt{-cix+1}\,asinh(cx)}{\sqrt{cix+1}}dx\right)bc\right)}{\sqrt{d}\,c}$$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

output
```
(sqrt(f)*(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)*a*i + int((sqrt( - c*i*x + 1)*asinh(c*x))/sqrt(c*i*x + 1),x)*b
*c))/(sqrt(d)*c)
```

3.205 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}} dx = \frac{2if^2(1-icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bf^2\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
2*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-1/2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f^2*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}} dx = -\frac{-\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{-i+cx} + 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + \frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(\text{arcsinh}(cx)\left(-\right.\right.}{}}{}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
-1/2*((-4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) + 2*a*Sqrt[d]*
Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
 + (b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSi
nh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2
] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[
1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-\frac{2i(cx+i)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2(c^2x^2+1)^{3/2}\left(\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{2b\log(-cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^2*(1 + c^2*x^2)^(3/2)*(((2*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt
[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/(2*b*c) - (2*b*Log[I - c*x])/c))/(
(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\operatorname{arcsinh}(xc)^{2} b \sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{2\sqrt{c^{2}x^{2}+1}\,c\,d^{2}}-\frac{a\operatorname{arcsinh}(xc)\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^{2}x^{2}+1}\,c\,d^{2}}+\frac{4\operatorname{arcsinh}(xc) b\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^{2}x^{2}+1}\,c\,d^{2}}$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*arcsinh(x*c)^2*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)
^(1/2)/c/d^2-a*arcsinh(x*c)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*
x^2+1)^(1/2)/c/d^2+4*arcsinh(x*c)*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/
2)/(c^2*x^2+1)^(1/2)/c/d^2+2*(a+b*arcsinh(x*c))*(x*c+I-(c^2*x^2+1)^(1/2))*
(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/c/(c^2*x^2+1)/d^2-4*ln(x*c+(c^2*x
^2+1)^(1/2)-I)*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2
)/c/d^2
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\,(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*x^2 - 2*I*c*d^2*x
- d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{(d + icdx)^{3/2}}\,dx = \int \frac{\sqrt{-if\,(cx + i)}(a + b\,\text{asinh}\,(cx))}{(id\,(cx - i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2), x
)
```

Maxima [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{(d + icdx)^{3/2}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)}{(i\,cdx + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + b*integrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1
))/(I*c*d*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))}{(d + icdx)^{3/2}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)}{(i\,cdx + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{(d+c\,d\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\frac{\sqrt{f}\left(-2\sqrt{cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+2\sqrt{-cix+1}\,ai+\sqrt{cix+1}\,\left(\right.\right.}{\sqrt{d}\,\sqrt{cix+1}\,cd}$$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

output
```
(sqrt(f)*( - 2*sqrt(c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + 2*sq
rt( - c*i*x + 1)*a*i + sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))
/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b*c))/(sqrt(d)*sqrt(c*i*x +
1)*c*d)
```

3.206 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 192

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{2ibf\sqrt{1+c^2x^2}}{3cd^2(i-cx)\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{if(1-icx)^3(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{bf\sqrt{1+c^2x^2}\log(i-cx)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
2/3*I*b*f*(c^2*x^2+1)^(1/2)/c/d^2/(I-c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+1/3*I*f*(1-I*c*x)^3*(a+b*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*
f*x)^(1/2)/(c^2*x^2+1)+1/3*b*f*(c^2*x^2+1)^(1/2)*ln(I-c*x)/c/d^2/(d+I*c*d*
x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-\left((i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)\right)-b(i\right.}{3cd^3(-i+cx)^2\sqrt{\ }}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-((I + c*x)*((-I)*b + b*c*x + a*Sqrt
[1 + c^2*x^2])) - b*(I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(-I + c*x
)^2*Log[d + I*c*d*x]))/(3*c*d^3*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6211, 27, 6252, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\int \frac{i(1-icx)^3}{3c(c^2x^2+1)^2}dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\int \frac{(1-icx)^3}{(c^2x^2+1)^2}dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{456}$$

$$\frac{f^3(c^2x^2+1)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\int \frac{1-icx}{(icx+1)^2}dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\int\left(\frac{i}{cx-i}-\frac{2}{(cx-i)^2}\right)dx\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(\frac{i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{1}{3}ib\left(\frac{i\log(-cx+i)}{c}-\frac{2}{c(-cx+i)}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(5/2)*(((I/3)*(1 - I*c*x)^3*(a + b*ArcSinh[c*x]))/(c*(1
 + c^2*x^2)^(3/2)) - (I/3)*b*(-2/(c*(I - c*x)) + (I*Log[I - c*x])/c)))/((d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(159) = 318$.

Time = 4.98 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.27

method	result
default	$a\left(\frac{i\sqrt{-icfx+f}}{cd(icdx+d)^{\frac{3}{2}}}-f\left(\frac{i\sqrt{-icfx+f}}{3fcd(icdx+d)^{\frac{3}{2}}}+\frac{i\sqrt{-icfx+f}}{3cf\,d^{2}\sqrt{icdx+d}}\right)\right)+\frac{b\left(i\ln\left(xc+\sqrt{c^{2}x^{2}+1}-i\right)x^{3}c^{3}-i\ln\left(xc+\sqrt{c^{2}x^{2}+1}-i\right)\sqrt{c^{2}\right.}{}$
parts	$a\left(\frac{i\sqrt{-icfx+f}}{cd(icdx+d)^{\frac{3}{2}}}-f\left(\frac{i\sqrt{-icfx+f}}{3fcd(icdx+d)^{\frac{3}{2}}}+\frac{i\sqrt{-icfx+f}}{3cf\,d^{2}\sqrt{icdx+d}}\right)\right)+\frac{b\left(i\ln\left(xc+\sqrt{c^{2}x^{2}+1}-i\right)x^{3}c^{3}-i\ln\left(xc+\sqrt{c^{2}x^{2}+1}-i\right)\sqrt{c^{2}\right.}{}$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(I/c/d*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2)-f*(1/3*I/f/c/d/(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(1/2)+1/3*I/c/f/d^2/(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)))+
1/3*b*(I*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^3*c^3-I*ln(x*c+(c^2*x^2+1)^(1/2)-I)
*(c^2*x^2+1)^(1/2)*x^2*c^2+3*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^2*c^2-3*arcsinh
(x*c)*c^2*x^2-3*I*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x*c+c^2*x^2-I*ln(x*c+(c^2*x^
2+1)^(1/2)-I)*(c^2*x^2+1)^(1/2)-2*I*x*c-(c^2*x^2+1)^(1/2)*x*c-I*(c^2*x^2+1
)^(1/2)-ln(x*c+(c^2*x^2+1)^(1/2)-I)+arcsinh(x*c)-1)*(x^3*c^3+3*I*c^2*x^2+x
^2*c^2*(c^2*x^2+1)^(1/2)-3*x*c-I+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(-
I*(I+x*c)*f)^(1/2)/d^3/(3*c^2*x^2-1)/(c^2*x^2+1)^2/c
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(147) = 294.

Time = 0.20 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx =$$

$$-\frac{4\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx+2\left(bc^2x^2+2i\,bcx-b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(
b*c^2*x^2 + 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) - (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x - I*c*d^3)*s
qrt(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*x^4 + 2*c^8
*d^3*x^3 + I*c^7*d^3*x^2 + 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(b*c^3*x^3
- I*b*c^2*x^2 + b*c*x - I*b)) + (c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d^3*x -
 I*c*d^3)*sqrt(b^2*f/(c^2*d^5))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b
*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*d^3
*x^4 - 2*c^8*d^3*x^3 - I*c^7*d^3*x^2 - 2*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))
/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*(a*c^2*x^2 + 2*I*a*c*x - a)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*d^3*x^3 - I*c^3*d^3*x^2 + c^2*d
^3*x - I*c*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{\sqrt{-if\left(cx+i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(id\left(cx-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2), x
)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{1}{3}\,bc\left(\frac{6\,\sqrt{f}}{3i\,c^3d^{\frac{5}{2}}x+3\,c^2d^{\frac{5}{2}}}+\frac{\sqrt{f}\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}}\right)\\ &\quad-\frac{1}{3}\,b\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\operatorname{arsinh}\left(cx\right)\\ &\quad-\frac{1}{3}\,a\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\end{aligned}$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="maxima")
```

output
```
1/3*b*c*(6*sqrt(f)/(3*I*c^3*d^(5/2)*x + 3*c^2*d^(5/2)) + sqrt(f)*log(c*x -
 I)/(c^2*d^(5/2))) - 1/3*b*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I
*c^2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3
))*arcsinh(c*x) - 1/3*a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c^
2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))
```

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\text{i}}}{(d+c\,d\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{f}\left(-3\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{\sqrt{-cix+1}\,asinh(cx)}{\sqrt{cix+1}\,c^2x^2-2\sqrt{cix+1}\,cix-\sqrt{cix+1}}dx\right)b\,c}{3\sqrt{d}\,\sqrt{c}}$$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

output
```
(sqrt(f)*( - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*
asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*
i*x + 1)),x)*b*c**2*i*x - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(
- c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*
i*x - sqrt(c*i*x + 1)),x)*b*c - a*c**2*i*x**2 + 2*a*c*x + a*i))/(3*sqrt(d)
*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d**2*(c*i*x + 1))
```

3.207 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 483

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{ibd^2fx\sqrt{d+icdx}\sqrt{f-icfx}}{5\sqrt{1+c^2x^2}} - \frac{3bcd^2fx^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}}{15\sqrt{1+c^2x^2}} - \frac{ibc^4d^2fx^5\sqrt{d+icdx}\sqrt{f-icfx}}{25\sqrt{1+c^2x^2}} - \frac{bd^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}}{16c} + \frac{3}{8}d^2fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{1}{4}d^2fx\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))$$

output
```
-1/5*I*b*d^2*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/1
6*b*c*d^2*f*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/15
*I*b*c^2*d^2*f*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1
/25*I*b*c^4*d^2*f*x^5*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2
)-1/16*b*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^(3/2)/c+3/8
*d^2*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+1/4*d^2*f*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/5*I
*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x)
)/c+3/16*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/
c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.81 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.41

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-1200ibcd^2fx\sqrt{d+icdx}\sqrt{f-icfx}+1920iad^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((-1200*I)*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (1920*I)*a*d^
2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d^2*f
*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (3840*I)*a*c^2*
d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c
^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1920
*I)*a*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 1800*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b
*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(5/
2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]] - (200*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*Sinh[3*ArcSinh[c*x]] + 60*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x]*((10*I)*Cosh[3*ArcSinh[c*x]] + (2*I)*Cosh[5*ArcSinh[c*x]] + 5*(
(4*I)*Sqrt[1 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]]))
- (24*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])
/(9600*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int d(icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int (icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 6253$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(icx(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}+(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}\right)dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{i(c^2x^2+1)^5}{\cdots}\right.}{\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((-1/5*I)*b*x - (5*b*c*x^2)/16
- ((2*I)/15)*b*c^2*x^3 - (b*c^3*x^4)/16 - (I/25)*b*c^4*x^5 + (3*x*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/4 + ((I/5)*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/c + (3*(a + b*
ArcSinh[c*x])^2)/(16*b*c)))/(1 + c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(395) = 790$.

Time = 5.14 (sec) , antiderivative size = 1359, normalized size of antiderivative = 2.81

method	result	size
default	Expression too large to display	1359
parts	Expression too large to display	1359

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/5*I*a/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)+1/4*I*a*d/c/f*(d+I*c*d*x)^
(3/2)*(f-I*c*f*x)^(5/2)+1/4*I*a*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2
)-1/8*I*a*d^2/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/8*I*a*d^2*f/c*(f-I*c
*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/8*a*d^3*f^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2
)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*
f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(3/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*
f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*f*d^2+1/800*I*(I*(x*c-I)*d)^(1
/2)*(-I*(I+x*c)*f)^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x
^4+20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*(-1+
5*arcsinh(x*c))*f*d^2/(c^2*x^2+1)/c+1/256*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*
f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*
x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*c))*f*d^2/(c^2*x^2+1
)/c+1/96*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+
1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(-1+3*arcsinh(x*c))*
f*d^2/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c
^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c)
)*f*d^2/(c^2*x^2+1)/c+1/16*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2
*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)-1)*f*d^2/(c^2*x^2+1)/c+1/16*I*
(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)
*(arcsinh(x*c)+1)*f*d^2/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x...
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((I*b*c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 + I*b*c*d^2*f*x + b*d^2*f)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^
3*d^2*f*x^3 + a*c^2*d^2*f*x^2 + I*a*c*d^2*f*x + a*d^2*f)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\mathrm{arcsinh}(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,d^2f\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+8\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^4i\,x^4+10\sqrt{c}$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*f*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + 8*sqrt(
c*i*x + 1)*sqrt( - c*i*x + 1)*a*c**4*i*x**4 + 10*sqrt(c*i*x + 1)*sqrt( - c
*i*x + 1)*a*c**3*x**3 + 16*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c**2*i*x**
2 + 25*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c*x + 8*sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*a*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x*
*3,x)*b*c**4*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**2
,x)*b*c**3 + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x,x)*b*c
**2*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*b*c))/(40*
c)
```

3.208 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 252

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))\,dx =$$
$$-\frac{3bcdfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}-\frac{bdf\sqrt{d+icdx}\sqrt{f-icfx}\left(1+c^2x^2\right)^{3/2}}{16c}$$
$$+\frac{3}{8}dfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))+\frac{1}{4}dfx\sqrt{d+icdx}\sqrt{f-icfx}\left(1+c^2x^2\right)(a+b\,\mathrm{arcsinh}(cx))+$$

output
```
-3/16*b*c*d*f*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/
16*b*d*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^(3/2)/c+3/8*d*f*x
*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+1/4*d*f*x*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))+3/16*d*f*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.40

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{80acdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+32ac^3dfx^3\sqrt{d+icdx}\sqrt{f-ic}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(80*a*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32*a
*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 24*b*
d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 16*b*d*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - b*d*f*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 48*a*d^(3/2)*f^(3/2)*Sqrt[1 + c^
2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
+ 4*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(8*Sinh[2*ArcSi
nh[c*x]] + Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6211, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{6201}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 244

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int\left(c^2x^3+x\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc\left(\frac{c^2}{4}\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 6200

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x\left(c^2x^2\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 15

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(c^2x^2+1\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 6198

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{(a+b\text{arcs}}{4}\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(-1/4*(b*c*(x^2/2 + (c^2*x^4)/4))
 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (
x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x])^2/(4*b*
c)))/4))/(1 + c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(206) = 412$.

Time = 4.31 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.73

method	result
default	$\frac{ia(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{5}{2}}}{4cf}+\frac{iad\sqrt{icdx+d}\,(-icfx+f)^{\frac{5}{2}}}{4cf}-\frac{iad(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{8c}-\frac{3iadf\sqrt{-icfx+f}\,\sqrt{icdx+d}}{8c}+\frac{3a\,d^{2}f}{}$
parts	$\frac{ia(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{5}{2}}}{4cf}+\frac{iad\sqrt{icdx+d}\,(-icfx+f)^{\frac{5}{2}}}{4cf}-\frac{iad(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{8c}-\frac{3iadf\sqrt{-icfx+f}\,\sqrt{icdx+d}}{8c}+\frac{3a\,d^{2}f}{}$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/4*I*a/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)+1/4*I*a*d/c/f*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(5/2)-1/8*I*a*d/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/
8*I*a*d*f/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/8*a*d^2*f^2*((f-I*c*f*x)
*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*
d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(3/16*(I*(x*c-I)*d)^
(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*d*f+1/256*(I
*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1
/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*
arcsinh(x*c))*d*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1
/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*
arcsinh(x*c))*d*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1
/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*a
rcsinh(x*c))*d*f/(c^2*x^2+1)/c+1/256*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1
/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3-8*x^2*c^2*(c^2*x^2+1
)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(x*c))*d*f/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((b*c^2*d*f*x^2 + b*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d*f*x^2 + a*d*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,df\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+2\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^3x^3+5\sqrt{cix+\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d*f*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + 2*sqrt(c*i*
x + 1)*sqrt( - c*i*x + 1)*a*c**3*x**3 + 5*sqrt(c*i*x + 1)*sqrt( - c*i*x +
1)*a*c*x + 8*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**2,x)*b*c
**3 + 8*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*b*c))/(8*c)
```

3.209 $\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 304

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{ibfx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c} + \frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output

```
1/3*I*b*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*
f*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/9*I*b*c^2*f*
x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*f*x*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))-1/3*I*f*(d+I*c*d*x)^(1/2)*(
f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+1/4*f*(d+I*c*d*x)^(1/2)*
(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{12af\sqrt{d+icdx}\sqrt{f-icfx}(-2i+3cx-2ic^2x^2)+36a\sqrt{d}f^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+\cdots}\right.}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(12*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*I + 3*c*x - (2*I)*c^2*x^2)
 + 36*a*Sqrt[d]*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]] + (9*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c
*x]^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[
1 + c^2*x^2] + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*A
rcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[
c*x]]))/Sqrt[1 + c^2*x^2])/(72*c)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f(1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int(1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int\left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}+\frac{(a+b\text{arcsinh}(cx))^2}{4bc}+\frac{1}{9}ibc\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((I/3)*b*x - (b*c*x^2)/4 + (I/9)*b*
c^2*x^3 + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (a + b*ArcSinh[c*x])^2/(4*b*c)))/Sqrt[
1 + c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(245) = 490$.

Time = 4.96 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.49

method	result
default	$\frac{ia\sqrt{icdx+d}\,(-icfx+f)^{\frac{5}{2}}}{3cf}-\frac{ia(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{6c}-\frac{iaf\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c}+\frac{ad\,f^{2}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\right.}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^{2}d}}$
parts	$\frac{ia\sqrt{icdx+d}\,(-icfx+f)^{\frac{5}{2}}}{3cf}-\frac{ia(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{6c}-\frac{iaf\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c}+\frac{ad\,f^{2}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\right.}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^{2}d}}$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/3*I*a/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)-1/6*I*a/c*(f-I*c*f*x)^(3/2
)*(d+I*c*d*x)^(1/2)-1/2*I*a*f/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+1/2*a*
d*f^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*
ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(1
/4*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*
c)^2*f-1/72*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x
^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(-1+3*arcsinh(x*c
))*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^
3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c))
*f/(c^2*x^2+1)/c-1/8*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+(
c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)-1)*f/(c^2*x^2+1)/c-1/8*I*(I*(x*c-I)*
d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x
*c)+1)*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^
3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(x*
c))*f/(c^2*x^2+1)/c-1/72*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4
*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*(1+3
*arcsinh(x*c))*f/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((-I*b*c*f*x + b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a*c*f*x + a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f), x)
```

Sympy [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int \sqrt{id\,(cx-i)}(-if\,(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}\,(cx))\;dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (a + b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,f\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-2\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^2i\,x^2+3\sqrt{cix+1}\,\sqrt{-cix+ }\right.}{}$$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*f*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - 2*sqrt(c*i*x
+ 1)*sqrt( - c*i*x + 1)*a*c**2*i*x**2 + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x +
1)*a*c*x - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i - 6*int(sqrt(c*i*x + 1
)*sqrt( - c*i*x + 1)*asinh(c*x)*x,x)*b*c**2*i + 6*int(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*asinh(c*x),x)*b*c))/(6*c)
```

3.210 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 266

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output

```
2*I*b*f^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b*c*
f^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*f^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*f^2*x
*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*f^
2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{16ibcfx\sqrt{d+icdx}\sqrt{f-icfx}-16iaf\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+}}{}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
((16*I)*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a*f*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*f*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*f*(4*I + c*x)*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*f*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Cosh[2*ArcSinh[c*x]] + 12*a*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f
*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*d*Sqrt[1 +
 c^2*x^2])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f^2\sqrt{c^2x^2+1}\int \left(-\frac{c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}-\frac{2ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\sqrt{c^2x^2+1}\left(-\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{2i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{3(a+b\text{arcsinh}(cx))^2}{4bc}+\frac{1}{4}bcx^2+2ibx\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
(f^2*Sqrt[1 + c^2*x^2]*((2*I)*b*x + (b*c*x^2)/4 - ((2*I)*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/c - (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 +
(3*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(222) = 444.

Time = 4.36 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.04

method	result
default	$-\frac{ia(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{2cd}-\frac{3iaf\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2cd}+\frac{3a\,f^{2}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\sqrt{c^{2}df\,x^{2}+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^{2}df}}+b\Big(3f$
parts	$-\frac{ia(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{2cd}-\frac{3iaf\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2cd}+\frac{3a\,f^{2}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\sqrt{c^{2}df\,x^{2}+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^{2}df}}+b\Big(3f$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*I*a/c/d*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/2*I*a*f/c/d*(f-I*c*f*x)
^(1/2)*(d+I*c*d*x)^(1/2)+3/2*a*f^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*
f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f
)^(1/2))/(c^2*d*f)^(1/2)+b*(3/4*f*arcsinh(x*c)^2*(-I*(I+x*c)*f)^(1/2)*(I*(
x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/d/c-1/16*f*(-1+2*arcsinh(x*c))*(2*x^3*c^
3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-I*f*(arcsinh(x*c)-1)*(c^2*x^2+(c^2*x
^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1
)-I*f*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/
2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-1/16*f*(1+2*arcsinh(x*c))*(2*x^3*c
^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f))/(c*d*x - I*d), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\frac{\sqrt{f}\,f\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-\sqrt{cix+1}\,\sqrt{-cix+1}\,acx-4\sqrt{cix+}\right.}{}$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

output
```
(sqrt(f)*f*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - sqrt(c*i*x + 1)*sqrt(
 - c*i*x + 1)*a*c*x - 4*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i - 2*int((sq
rt( - c*i*x + 1)*asinh(c*x)*x)/sqrt(c*i*x + 1),x)*b*c**2*i + 2*int((sqrt(
- c*i*x + 1)*asinh(c*x))/sqrt(c*i*x + 1),x)*b*c))/(2*sqrt(d)*c)
```

3.211 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 288

$$\begin{aligned}\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx &= -\frac{ibf^2x\sqrt{1+c^2x^2}}{d\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{4if^2(1-icx)(a+b\text{arcsinh}(cx))}{cd\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{if^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{3f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bcd\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{4bf^2\sqrt{1+c^2x^2}\log(i-cx)}{cd\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output

```
-I*b*f^2*x*(c^2*x^2+1)^(1/2)/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*f^2
*(1-I*c*x)*(a+b*arcsinh(c*x))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*f^
2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3
/2*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/d/(d+I*c*d*x)^(1/2)/(f-I
*c*f*x)^(1/2)-4*b*f^2*(c^2*x^2+1)^(1/2)*ln(I-c*x)/c/d/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.97 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.78

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{\frac{2af(5+icx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(-i+cx)} - \frac{6af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}} - \frac{bf\sqrt{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x
]
```

output
```
((2*a*f*(5 + I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^2*(-I + c*x))
- (6*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]])/d^(3/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*
((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSi
nh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x
]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])) + (2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh
[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh
[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[
c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] -
(-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$$

$$\Big\downarrow \ 6211$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\int \left(\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{4i(cx+i)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3(c^2x^2+1)^{3/2}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{4i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{4b\log(-cx+i)}{c}-ibx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(3/2)*((-I)*b*x + ((4*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x
]))/(c*Sqrt[1 + c^2*x^2]) + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c -
 (3*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (4*b*Log[I - c*x])/c))/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.05

method	result
default	$-\frac{f\left(-2i\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2-2i\sqrt{c^2x^2+1}\,a\,c^2x^2+3\operatorname{arcsinh}(xc)^2b\,c^2x^2-10i\sqrt{c^2x^2+1}\,a+6\operatorname{arcsinh}(xc)a\,c^2x^2-8\operatorname{arcsinh}(x\right.}{}$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*f*(-2*I*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*b*c^2*x^2-2*I*(c^2*x^2+1)^(1/2
)*a*c^2*x^2+3*arcsinh(x*c)^2*b*c^2*x^2-10*I*(c^2*x^2+1)^(1/2)*a+6*arcsinh(
x*c)*a*c^2*x^2-8*arcsinh(x*c)*b*c^2*x^2+16*ln(x*c+(c^2*x^2+1)^(1/2)-I)*b*c
^2*x^2-8*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c*x+8*a*c^2*x^2+2*I*b*c^3*x^3+2*
I*b*x*c-8*(c^2*x^2+1)^(1/2)*a*c*x+3*b*arcsinh(x*c)^2-10*I*(c^2*x^2+1)^(1/2
)*arcsinh(x*c)*b+6*arcsinh(x*c)*a-8*b*arcsinh(x*c)+16*b*ln(x*c+(c^2*x^2+1)
^(1/2)-I)+8*a)*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(3/2)/
d^2/c
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((I*b*c*f*x - b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (I*a*c*f*x - a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2)
, x)
```

Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*I
*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3/2)*log(c*x + sqrt(c^2*x^2 +
 1))/(I*c*d*x + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{971846465379212459847603531931675639761825792OO,[3,10,0,10
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \frac{\sqrt{f}\,f\left(-6\sqrt{cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-\sqrt{-cix+1}\,acx+5\sqrt{-ci\right.}{}$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

output
```
(sqrt(f)*f*( - 6*sqrt(c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - sq
rt( - c*i*x + 1)*a*c*x + 5*sqrt( - c*i*x + 1)*a*i - sqrt(c*i*x + 1)*int((s
qrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),
x)*b*c**2*i + sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))/(sqrt(c*
i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b*c))/(sqrt(d)*sqrt(c*i*x + 1)*c*d)
```

3.212 $\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 373

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{4ibf^2\sqrt{1+c^2x^2}}{3cd^2(i-cx)\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{bf^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2cd^2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2(1-icx)(a+b\text{arcsinh}(cx))}{cd^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2if^2(1-icx)^3(a+b\text{arcsinh}(cx))}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{f^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{cd^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{8bf^2\sqrt{1+c^2x^2}\log(i-cx)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
4/3*I*b*f^2*(c^2*x^2+1)^(1/2)/c/d^2/(I-c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-1/2*b*f^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2/c/d^2/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)-2*I*f^2*(1-I*c*x)*(a+b*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1
/2)/(f-I*c*f*x)^(1/2)+2/3*I*f^2*(1-I*c*x)^3*(a+b*arcsinh(c*x))/c/d^2/(d+I*
c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+f^2*(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*(a+b*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*b*f^2
*(c^2*x^2+1)^(1/2)*ln(I-c*x)/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.28 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.89

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x
]
```

output
```
((-16*a*f*(-I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^3*(-I + c*x
)^2) + (12*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/d^(5/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 +
(3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (7*I)
*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]]
 - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*Log[1 + c^2*x^2])) +
2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[
c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I +
3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]
))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Ar
cSinh[c*x]/2])^4) + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[A
rcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(A
rcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1 + c^2*x^2]) + C
osh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*
x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]
 + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*(4 + (2
+ Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c
*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$$

$\Big\downarrow$ 6211

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{f^4(c^2x^2+1)^{5/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6252

$$\frac{f^4(c^2x^2+1)^{5/2}\left(-bc\int \left(\frac{2i(1-icx)^3}{3c(c^2x^2+1)^2}-\frac{2i(1-icx)}{c(c^2x^2+1)}+\frac{\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}\right)dx+\frac{2i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{2i(1-icx)(a+b\text{arcsi}}{c\sqrt{c^2x^2+1}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{f^4(c^2x^2+1)^{5/2}\left(\frac{2i(1-icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}+\frac{\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c}-bc\left(\frac{\text{arcsinh}(}{2c^2}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*(1 - I*c*x)^3*(a + b*ArcSinh[c*x]))/(
c*(1 + c^2*x^2)^(3/2)) - ((2*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt[
1 + c^2*x^2]) + (ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*(((-4*I)/3)/(c
^2*(I - c*x)) + ArcSinh[c*x]^2/(2*c^2) - (8*Log[I - c*x])/(3*c^2))))/((d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 7.53 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.14

method	result
default	$\frac{f\left(16a+8b+64\ln\left(xc+\sqrt{c^2x^2+1}-i\right)b\,c^2x^2-8ib\,c^3x^3-8ibxc-8i\sqrt{c^2x^2+1}\ \operatorname{arcsinh}(xc)b-24i\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2-16a\,c^3x^3}{}$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*f*(16*a+8*b-8*I*b*c^3*x^3-16*a*c^3*x^3*(c^2*x^2+1)^(1/2)-24*I*arcsinh(
x*c)*(c^2*x^2+1)^(1/2)*b*c^2*x^2+6*arcsinh(x*c)^2*b*c^2*x^2+12*arcsinh(x*c
)*a*c^2*x^2-32*arcsinh(x*c)*b*c^2*x^2+64*ln(x*c+(c^2*x^2+1)^(1/2)-I)*b*c^2
*x^2-24*I*(c^2*x^2+1)^(1/2)*a*c^2*x^2+8*b*c^2*x^2-16*b*c^3*x^3*arcsinh(x*c
)*(c^2*x^2+1)^(1/2)-8*I*(c^2*x^2+1)^(1/2)*a+3*arcsinh(x*c)^2*b*c^4*x^4+6*a
rcsinh(x*c)*a*c^4*x^4-16*arcsinh(x*c)*b*c^4*x^4+32*ln(x*c+(c^2*x^2+1)^(1/2
)-I)*b*c^4*x^4-8*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b-16*b*arcsinh(x*c)-8*I*
b*x*c+32*a*c^2*x^2+16*a*c^4*x^4+3*b*arcsinh(x*c)^2+32*b*ln(x*c+(c^2*x^2+1)
^(1/2)-I)+6*arcsinh(x*c)*a)*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(c^2*
x^2+1)^(1/2)/c/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/d^3
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/(I*d*(c*x - I))**(5/2)
, x)
```

Maxima [F]

$$\int \frac{(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))}{(d + icdx)^{5/2}} \, dx = \int \frac{(-i\, cfx + f)^{\frac{3}{2}}(b\, \text{arsinh}\,(cx) + a)}{(i\, cdx + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*a*(-3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(-3*I*c^4*d^4*x^3 - 9*c^3*d^4*x^2 +
 9*I*c^2*d^4*x + 3*c*d^4) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*f/(c^3*d^3*x^2 - 2
*I*c^2*d^3*x - c*d^3) + 21*I*sqrt(c^2*d*f*x^2 + d*f)*f/(3*I*c^2*d^3*x + 3*
c*d^3) - 3*f^2*arcsinh(c*x)/(c*d^3*sqrt(f/d))) + b*integrate((-I*c*f*x + f
)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))}{(d + icdx)^{5/2}} \, dx = \int \frac{(-i\, cfx + f)^{\frac{3}{2}}(b\, \text{arsinh}\,(cx) + a)}{(i\, cdx + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\int \frac{(a+b\,\operatorname{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx=\frac{\sqrt{f}\,f\left(-6\sqrt{cix+1}\,\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)acx+6\sqrt{cix+1}\,\sqrt{}\right.}{}$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

output
```
(sqrt(f)*f*( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1
)/sqrt(2))*a*c*x + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x
 + 1)/sqrt(2))*a*i - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i
*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x
 - sqrt(c*i*x + 1)),x)*b*c**3*x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int
((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i
*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b*c**2*i - 3*sqrt(c*i*x + 1)*sqrt( - c
*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 -
 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b*c**2*i*x - 3*sqrt(c*i*x +
 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1
)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b*c - 8*a*c**2
*i*x**2 + 4*a*c*x - 4*a*i))/(3*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*
c*d**2*(c*i*x + 1))
```

3.213 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{5bcd^2f^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{32\sqrt{1+c^2x^2}} - \frac{5bd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}}{96c} - \frac{bd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{5/2}}{36c} + \frac{5}{16}d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{5}{24}d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}($$

output
```
-5/32*b*c*d^2*f^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2
)-5/96*b*d^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^(3/2)/c-1
/36*b*d^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^(5/2)/c+5/16
*d^2*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+5/24*d^2
*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))+
1/6*d^2*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2*(a+b*arcsi
nh(c*x))+5/32*d^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x
))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.26

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1584acd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+1248ac^3d^2f^2x^3\sqrt{d+icdx}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(1584*a*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 1248*a*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] + 384*a*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2] + 360*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^
2 - 270*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]]
 - 27*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] -
 2*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSinh[c*x]] + 72
0*a*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]] + 12*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + S
inh[6*ArcSinh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.53, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {6211, 6201, 241, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\int \left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{(c^2x^2+1)^{5/2}}$$

$\downarrow$ 6201

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx-\frac{1}{6}bc\int x\left(c^2x^2+1\right)^2dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(\right.}{(c^2x^2+1)^{5/2}}$$

$\downarrow$ 241

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx+\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))-\frac{b(c^2}{}\right.}{(c^2x^2+1)^{5/2}}$$

$\downarrow$ 6201

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int x\left(c^2x^2+1\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a\right.\right.}{(c^2x^2+1)^{5/2}}$$

$\downarrow$ 244

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx-\frac{1}{4}bc\int \left(c^2x^3+x\right)dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\right.\right.}{(c^2x^2+1)^{5/2}}$$

$\downarrow$ 2009

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-\frac{1}{4}bc(\right.\right.}{(c^2x^2+1)^{5/2}}$$

$\downarrow$ 6200

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)+\frac{1}{4}x(c^2\right.\right.}{(c^2x^2+1)^{5}}$$

$\downarrow$ 15

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x(c^2x^2\right.\right.}{(c^2x^2+1)^{5/2}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{1}{6}x(c^2x^2+1)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{6}\left(\frac{1}{4}x(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{4}\left(\frac{1}{2}\right.\right.\right.}{(c^2x^2+1)^{5/2}}$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(-1/36*(b*(1 + c^2*x^2)^3)/c + (x
*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*(-1/4*(b*c*(x^2/2 + (c^2
*x^4)/4)) + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3*(-1/4*(b*c
*x^2) + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (a + b*ArcSinh[c*x]
)^2/(4*b*c)))/4))/6))/(1 + c^2*x^2)^(5/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(317) = 634$.

Time = 4.62 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.84

method	result	size
default	Expression too large to display	1083
parts	Expression too large to display	1083

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*I*a/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(7/2)+1/6*I*a*d/c/f*(d+I*c*d*x)^
(3/2)*(f-I*c*f*x)^(7/2)+1/8*I*a*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(7/2
)-1/24*I*a*d^2/c*(f-I*c*f*x)^(5/2)*(d+I*c*d*x)^(1/2)-5/48*I*a*d^2*f/c*(f-I
*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-5/16*I*a*d^2*f^2/c*(f-I*c*f*x)^(1/2)*(d+I*
c*d*x)^(1/2)+5/16*a*d^3*f^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1
/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2)
)/(c^2*d*f)^(1/2)+b*(5/32*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^
2+1)^(1/2)/c*arcsinh(x*c)^2*f^2*d^2+1/2304*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)
*f)^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x^5*c^5+48*x^4*c^4*(
c^2*x^2+1)^(1/2)+38*x^3*c^3+18*x^2*c^2*(c^2*x^2+1)^(1/2)+6*x*c+(c^2*x^2+1)
^(1/2))*(-1+6*arcsinh(x*c))*f^2*d^2/(c^2*x^2+1)/c+3/512*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8
*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*c))*f^
2*d^2/(c^2*x^2+1)/c+15/256*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3
*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*
c))*f^2*d^2/(c^2*x^2+1)/c+15/256*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*
(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsi
nh(x*c))*f^2*d^2/(c^2*x^2+1)/c+3/512*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1
/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3-8*x^2*c^2*(c^2*x^2+1
)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*(1+4*arcsinh(x*c))*f^2*d^2/(c^2*x^2+1)...
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((b*c^4*d^2*f^2*x^4 + 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^4*d^2*f^2*x^
4 + 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^{5/2} (f - icfx)^{5/2} (a + b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d + icdx)^{5/2} (f - icfx)^{5/2} (a + b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\operatorname{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,d^2f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+8\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^5x^5+26\sqrt{c}\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + 8*sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c**5*x**5 + 26*sqrt(c*i*x + 1)*sqrt( -
c*i*x + 1)*a*c**3*x**3 + 33*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c*x + 48*
int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**4,x)*b*c**5 + 96*int(
sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**2,x)*b*c**3 + 48*int(sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*b*c))/(48*c)
```

3.214 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 483

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{ibdf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{5\sqrt{1+c^2x^2}} - \frac{3bcdf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{2ibc^2df^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{15\sqrt{1+c^2x^2}} + \frac{ibc^4df^2x^5\sqrt{d+icdx}\sqrt{f-icfx}}{25\sqrt{1+c^2x^2}} - \frac{bdf^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}}{16c} + \frac{3}{8}df^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx)) + \frac{1}{4}df^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx)) -$$

output

```
1/5*I*b*d*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16
*b*c*d*f^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/15*
I*b*c^2*d*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/
25*I*b*c^4*d*f^2*x^5*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)
-1/16*b*d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^(3/2)/c+3/8*
d*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))+1/4*d*f^2*x
*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))-1/5*I*
d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))
/c+3/16*d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c
/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.41

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{1200ibcdf^2x\sqrt{d+icdx}\sqrt{f-icfx}-1920iadf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1-}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((1200*I)*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (1920*I)*a*d*f
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d*f^2*
x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (3840*I)*a*c^2*d
*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^
3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1920*
I)*a*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
 1800*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*
d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(3/2
)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]] + (200*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Sinh[3*ArcSinh[c*x]] + 60*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Arc
Sinh[c*x]*((-10*I)*Cosh[3*ArcSinh[c*x]] - (2*I)*Cosh[5*ArcSinh[c*x]] + 5*(
(-4*I)*Sqrt[1 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]]))
 + (24*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]]
)/(9600*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$\Big\downarrow$ 6211

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int f(1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int (1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 6253

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))-icx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))\right)dx}{\left(c^2x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{i(c^2x^2+1)^5}{}\right.}{\left(c^2x^2+1\right)^{3/2}}$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((I/5)*b*x - (5*b*c*x^2)/16 + (
(2*I)/15)*b*c^2*x^3 - (b*c^3*x^4)/16 + (I/25)*b*c^4*x^5 + (3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
]))/4 - ((I/5)*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/c + (3*(a + b*Arc
Sinh[c*x])^2)/(16*b*c)))/(1 + c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(395) = 790$.

Time = 5.11 (sec) , antiderivative size = 1353, normalized size of antiderivative = 2.80

method	result	size
default	Expression too large to display	1353
parts	Expression too large to display	1353

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/5*I*a/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(7/2)+3/20*I*a*d/c/f*(d+I*c*d*x)
^(1/2)*(f-I*c*f*x)^(7/2)-1/20*I*a*d/c*(f-I*c*f*x)^(5/2)*(d+I*c*d*x)^(1/2)-
1/8*I*a*d*f/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/8*I*a*d*f^2/c*(f-I*c*f
*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/8*a*d^2*f^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/
(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*
x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(3/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)
^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*f^2*d-1/800*I*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4
+20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*(-1+5*
arcsinh(x*c))*f^2*d/(c^2*x^2+1)/c+1/256*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)
^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^
2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*c))*f^2*d/(c^2*x^2+1)/
c-1/96*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)
^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(-1+3*arcsinh(x*c))*f^
2*d/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3
+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c))*
f^2*d/(c^2*x^2+1)/c-1/16*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x
^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)-1)*f^2*d/(c^2*x^2+1)/c-1/16*I*(I
*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(
arcsinh(x*c)+1)*f^2*d/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c...
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral((-I*b*c^3*d*f^2*x^3 + b*c^2*d*f^2*x^2 - I*b*c*d*f^2*x + b*d*f^2)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*
c^3*d*f^2*x^3 + a*c^2*d*f^2*x^2 - I*a*c*d*f^2*x + a*d*f^2)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d\,f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-8\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^4i\,x^4+10\sqrt{}\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - 8*sqrt(
c*i*x + 1)*sqrt( - c*i*x + 1)*a*c**4*i*x**4 + 10*sqrt(c*i*x + 1)*sqrt( - c
*i*x + 1)*a*c**3*x**3 - 16*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c**2*i*x**
2 + 25*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c*x - 8*sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*a*i - 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x*
*3,x)*b*c**4*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**2
,x)*b*c**3 - 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x,x)*b*c
**2*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*b*c))/(40*
c)
```

3.215 $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 416

$$\begin{aligned}\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx &= \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}\\
&-\frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}+\frac{2ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{9\sqrt{1+c^2x^2}}\\
&+\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}+\frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\\
&-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))\\
&-\frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c}\\
&+\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}\end{aligned}$$

output

```
2/3*I*b*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b
*c*f^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c
^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^
3*f^2*x^4*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+3/8*f^2*x*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))-1/4*c^2*f^2*x^3*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))-2/3*I*f^2*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+5/16*f^2*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.36

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \frac{576ibcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-768iaf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+432acf^2}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
((576*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (768*I)*a*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 432*a*c*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (768*I)*a*c^2*f^2*x^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 288*a*c^3*f^2*x^3*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 360*b*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 144*b*f^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 9*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*Cosh[4*ArcSinh[c*x]] + 720*a*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[
c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (64*I)*b*
f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 12*b*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-48*I)*Sqrt[1 + c^2*x^2]
- (16*I)*Cosh[3*ArcSinh[c*x]] + 24*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh
[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f^2(1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int(1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int\Big(-c^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x^2-2ic\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))x+\sqrt{c^2x^2+1}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{2i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c}-\frac{1}{4}c^2x^3\sqrt{c^2x^2+1}(a+\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(((2*I)/3)*b*x - (3*b*c*x^2)/16 +
 ((2*I)/9)*b*c^2*x^3 + (b*c^3*x^4)/16 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x]))/8 - (c^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 - (((2*I
)/3)*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c + (5*(a + b*ArcSinh[c*x])
^2)/(16*b*c)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(339) = 678$.

Time = 5.23 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.51

method	result
default	$\frac{ia\sqrt{icdx+d}\,(-icfx+f)^{\frac{7}{2}}}{4cf} - \frac{ia(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}{12c} - \frac{5iaf(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{24c} - \frac{5ia\,f^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{8c} + \frac{5ad\,f^3}{}$
parts	$\frac{ia\sqrt{icdx+d}\,(-icfx+f)^{\frac{7}{2}}}{4cf} - \frac{ia(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}{12c} - \frac{5iaf(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{24c} - \frac{5ia\,f^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{8c} + \frac{5ad\,f^3}{}$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/4*I*a/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(7/2)-1/12*I*a/c*(f-I*c*f*x)^(5/
2)*(d+I*c*d*x)^(1/2)-5/24*I*a*f/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-5/8*
I*a*f^2/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+5/8*a*d*f^3*((f-I*c*f*x)*(d+
I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)
^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(5/16*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2*f^2-1/256*(I*(x*
c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+
12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(-1+4*arcs
inh(x*c))*f^2/(c^2*x^2+1)/c-1/36*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2
)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c
+1)*(-1+3*arcsinh(x*c))*f^2/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+
x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/
2))*(-1+2*arcsinh(x*c))*f^2/(c^2*x^2+1)/c-1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I
+x*c)*f)^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)-1)*f^2/(c^2
*x^2+1)/c-1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2
+1)^(1/2)*x*c+1)*(arcsinh(x*c)+1)*f^2/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/
2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*
x^2+1)^(1/2))*(1+2*arcsinh(x*c))*f^2/(c^2*x^2+1)/c-1/36*I*(I*(x*c-I)*d)^(1
/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-
3*(c^2*x^2+1)^(1/2)*x*c+1)*(1+3*arcsinh(x*c))*f^2/(c^2*x^2+1)/c-1/256*(...
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="fricas")
```

output
```
integral(-(b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x
 - a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\operatorname{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-6\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^3x^3-16\sqrt{cix+1}\,\sqrt{-cix+1}\right.}{}$$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - 6*sqrt(c*
i*x + 1)*sqrt( - c*i*x + 1)*a*c**3*x**3 - 16*sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)*a*c**2*i*x**2 + 9*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*c*x - 16*sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i - 24*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)*x**2,x)*b*c**3 - 48*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)
*asinh(c*x)*x,x)*b*c**2*i + 24*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin
h(c*x),x)*b*c))/(24*c)
```

3.216 $\int \frac{(f-icfx)^{5/2}(a+b\mathbf{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx &= \frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{11if^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{icf^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
11/3*I*b*f^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b
*c*f^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/9*I*b*c
^2*f^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-11/3*I*f^
3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2
*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+
1/3*I*c*f^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+5/4*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(1
/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.22

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \frac{264ibcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-8ibc^3f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
((264*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (8*I)*b*c^3*f^2*x
^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (264*I)*a*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (24*I)*a*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^2 + 27*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[
2*ArcSinh[c*x]] - 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]
*(9*(5*I + 2*c*x)*Sqrt[1 + c^2*x^2] - I*Cosh[3*ArcSinh[c*x]]) + 180*a*Sqrt
[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]])/(72*c*d*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 6253

$$\frac{f^3\sqrt{c^2x^2+1}\int \left(\frac{ic^3(a+b\text{arcsinh}(cx))x^3}{\sqrt{c^2x^2+1}}-\frac{3c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}-\frac{3ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 2009

$$\frac{f^3\sqrt{c^2x^2+1}\left(\frac{1}{3}icx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{3}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{11i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

output
```
(f^3*Sqrt[1 + c^2*x^2]*(((11*I)/3)*b*x + (3*b*c*x^2)/4 - (I/9)*b*c^2*x^3 -
 (((11*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c - (3*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/2 + (I/3)*c*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]) + (5*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(313) = 626$.

Time = 4.62 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.10

method	result
default	$-\frac{ia(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}{3cd}-\frac{5iaf(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{6cd}-\frac{5ia\,f^{2}\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2cd}+\frac{5a\,f^{3}\sqrt{(-icfx+f)(icdx+d)}\,\ln\Big(}{2\sqrt{-icfx+f}\,\sqrt{icdx}}$
parts	$-\frac{ia(-icfx+f)^{\frac{5}{2}}\sqrt{icdx+d}}{3cd}-\frac{5iaf(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{6cd}-\frac{5ia\,f^{2}\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2cd}+\frac{5a\,f^{3}\sqrt{(-icfx+f)(icdx+d)}\,\ln\Big(}{2\sqrt{-icfx+f}\,\sqrt{icdx}}$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/3*I*a/c/d*(f-I*c*f*x)^(5/2)*(d+I*c*d*x)^(1/2)-5/6*I*a*f/c/d*(f-I*c*f*x)
^(3/2)*(d+I*c*d*x)^(1/2)-5/2*I*a*f^2/c/d*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/
2)+5/2*a*f^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)
^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/
2)+b*(5/4*f^2*arcsinh(x*c)^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2
*x^2+1)^(1/2)/d/c+1/72*I*f^2*(-1+3*arcsinh(x*c))*(4*c^4*x^4+4*(c^2*x^2+1)^
(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I
*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-3/16*f^2*(-1+2*arcsinh(x*c))*(2*x^3*c^3+
2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(
-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-15/8*I*f^2*(arcsinh(x*c)-1)*(c^2*x^2+(
c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*
x^2+1)-15/8*I*f^2*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x
*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-3/16*f^2*(1+2*arcsinh(
x*c))*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(I*(
x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)+1/72*I*f^2*(1+3*arcsi
nh(x*c))*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1
/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(((I*b*c^2*f^2*x^2 - 2*b*c*f^2*x - I*b*f^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*f^2*x^2 - 2*a*c*f^2
*x - I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}}\,dx=\frac{\sqrt{f}\,f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+2\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^2i\,x^2-9\sqrt{}\right.}{}$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

output
```
(sqrt(f)*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + 2*sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*a*c**2*i*x**2 - 9*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a
*c*x - 22*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i - 6*int((sqrt( - c*i*x +
1)*asinh(c*x)*x**2)/sqrt(c*i*x + 1),x)*b*c**3 - 12*int((sqrt( - c*i*x + 1)
*asinh(c*x)*x)/sqrt(c*i*x + 1),x)*b*c**2*i + 6*int((sqrt( - c*i*x + 1)*asi
nh(c*x))/sqrt(c*i*x + 1),x)*b*c))/(6*sqrt(d)*c)
```

3.217 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 454

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = -\frac{4ibf^3x\sqrt{1+c^2x^2}}{d\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{bcf^3x^2\sqrt{1+c^2x^2}}{4d\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{15bf^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{4cd\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{8if^3(1-icx)(a+b\text{arcsinh}(cx))}{cd\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{4if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{cd\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2d\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{15f^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{2cd\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{8bf^3\sqrt{1+c^2x^2}\log(i-cx)}{cd\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-4*I*b*f^3*x*(c^2*x^2+1)^(1/2)/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b
*c*f^3*x^2*(c^2*x^2+1)^(1/2)/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+15/4*b*
f^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)+8*I*f^3*(1-I*c*x)*(a+b*arcsinh(c*x))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)+4*I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*
f*x)^(1/2)+1/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/d/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)-15/2*f^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*(a+b*arcsinh(c*x))
/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b*f^3*(c^2*x^2+1)^(1/2)*ln(I-c*
x)/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.72

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx = \text{Too large to display}$$

input

```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x
]
```

output

```
((4*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 + (7*I)*c*x + c^2*x^2))/
(d^2*(-I + c*x)) - (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]])/d^(3/2) - (4*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2
]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((
4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2
] + I*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2])) + (16*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*
x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSin
h[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2]
)*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/
(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) +
(b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSi
nh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - (Cosh[2*ArcSinh[c*x]] + 8*((2*I)*c*
x + (4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(
8 - 8*Sqrt[1 + c^2*x^2] + I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*(
(8*I)*(1 + Sqrt[1 + c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(d^2*Sqrt[1 + c^2
*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\left(-bc\int \left(\frac{x}{2}-\frac{15\text{arcsinh}(cx)}{2c\sqrt{c^2x^2+1}}+\frac{4i}{c}+\frac{8i(1-icx)}{c(c^2x^2+1)}\right)dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{4i\sqrt{c^2x^2+1}(a\ldots}{\ldots}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{8i(1-icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{15\text{arcs}\ldots}{\ldots}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(3/2)*(((8*I)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt
[1 + c^2*x^2]) + ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (x*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (15*ArcSinh[c*x]*(a + b*ArcSinh[c
*x]))/(2*c) - b*c*(((4*I)*x)/c + x^2/4 - (15*ArcSinh[c*x]^2)/(4*c^2) + (8*
Log[I - c*x])/c^2)))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 7.46 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.85

method	result
default	$-\frac{f^2\left(-4b\,c^3x^3\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}+2b\,c^4x^4-32i\,\mathrm{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2-32i\sqrt{c^2x^2+1}\,a\,c^2x^2-4a\,c^3x^3\sqrt{c^2x^2+1}+30\,\mathrm{arcsin}\right.}{}$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/8*f^2*(-4*b*c^3*x^3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*b*c^4*x^4-32*I*arc
sinh(x*c)*(c^2*x^2+1)^(1/2)*b*c^2*x^2-32*I*(c^2*x^2+1)^(1/2)*a*c^2*x^2-4*a
*c^3*x^3*(c^2*x^2+1)^(1/2)+30*arcsinh(x*c)^2*b*c^2*x^2-96*I*(c^2*x^2+1)^(1
/2)*a+60*arcsinh(x*c)*a*c^2*x^2-64*arcsinh(x*c)*b*c^2*x^2+128*ln(x*c+(c^2*
x^2+1)^(1/2)-I)*b*c^2*x^2-68*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c*x+64*a*c^2
*x^2+3*b*c^2*x^2+32*I*b*c^3*x^3+32*I*b*x*c-68*(c^2*x^2+1)^(1/2)*a*c*x+30*b
*arcsinh(x*c)^2-96*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b+60*arcsinh(x*c)*a-64
*b*arcsinh(x*c)+128*b*ln(x*c+(c^2*x^2+1)^(1/2)-I)+64*a+b)*(-I*(I+x*c)*f)^(
1/2)*(I*(x*c-I)*d)^(1/2)*(c^2*x^2+1)^(1/2)/c/(c^4*x^4+2*c^2*x^2+1)/d^2
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x
 - a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x
 - d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x
)/(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a + b*integrat
e((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}}\,dx=\frac{\sqrt{f}\,f^2\left(-30\sqrt{cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+\sqrt{-cix+1}\,a\,c^2i\,x^2-7\right.}{}$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

output
```
(sqrt(f)*f**2*( - 30*sqrt(c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i
+ sqrt( - c*i*x + 1)*a*c**2*i*x**2 - 7*sqrt( - c*i*x + 1)*a*c*x + 24*sqrt(
 - c*i*x + 1)*a*i - 2*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x
**2)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b*c**3 - 4*sqrt(c*i*x +
1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i
*x + 1)),x)*b*c**2*i + 2*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x
))/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b*c))/(2*sqrt(d)*sqrt(c*i*
x + 1)*c*d)
```

3.218 $\int \frac{(f-icfx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 485

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx &= \frac{ibf^3x\sqrt{1+c^2x^2}}{d^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8ibf^3\sqrt{1+c^2x^2}}{3cd^2(i-cx)\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{5bf^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^2}{2cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{20if^3(1-icx)(a+b\operatorname{arcsinh}(cx))}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2if^3(1-icx)^4(a+b\operatorname{arcsinh}(cx))}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} \\
&- \frac{5if^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{5f^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{cd^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{28bf^3\sqrt{1+c^2x^2}\log(i-cx)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
I*b*f^3*x*(c^2*x^2+1)^(1/2)/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*I*
b*f^3*(c^2*x^2+1)^(1/2)/c/d^2/(I-c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-
5/2*b*f^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*
f*x)^(1/2)-20/3*I*f^3*(1-I*c*x)*(a+b*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+2/3*I*f^3*(1-I*c*x)^4*(a+b*arcsinh(c*x))/c/d^2/(d+I*c*d
*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)-5/3*I*f^3*(c^2*x^2+1)*(a+b*arcsinh
(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5*f^3*(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*(a+b*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2
8/3*b*f^3*(c^2*x^2+1)^(1/2)*ln(I-c*x)/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1005 vs. $2(485) = 970$.

Time = 10.84 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.07

$$\int \frac{(f - icfx)^{5/2}(a + b\text{arcsinh}(cx))}{(d + icdx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x
]
```

output
```
(((-4*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 - (34*I)*c*x + 3*c
^2*x^2))/(d^3*(-I + c*x)^2) + (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/d^(5/2) - (2*b*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(
3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Ta
nh[ArcSinh[c*x]/2]] + (7*I)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*A
rcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2
 + 21*Log[1 + c^2*x^2])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 +
(56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x
^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((2*I)*b*f^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I
)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
- (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] -
 (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sq
rt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcS
inh[c*x]/2]] + (I/2)*(4 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[
ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*
x]/2])^4) + (b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(I*Cosh[ArcSinh[...
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^5(1-icx)^5(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^5(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6252}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\left(-bc\int \left(\frac{2i(1-icx)^4}{3c(c^2x^2+1)^2}-\frac{20i(1-icx)}{3c(c^2x^2+1)}+\frac{5\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}-\frac{5i}{3c}\right)dx+\frac{2i(1-icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{20i(1-icx)}{3c}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\left(\frac{2i(1-icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-\frac{20i(1-icx)(a+b\text{arcsinh}(cx))}{3c\sqrt{c^2x^2+1}}-\frac{5i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}+\frac{5\text{arcsinh}(cx)(}{}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^5*(1 + c^2*x^2)^(5/2)*(((((2*I)/3)*(1 - I*c*x)^4*(a + b*ArcSinh[c*x]))/(
c*(1 + c^2*x^2)^(3/2)) - (((20*I)/3)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*
Sqrt[1 + c^2*x^2]) - (((5*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
+ (5*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*(((-I)*x)/c - ((8*I)/3)/(c
^2*(I - c*x)) + (5*ArcSinh[c*x]^2)/(2*c^2) - (28*Log[I - c*x])/(3*c^2))))/
((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 7.61 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.06

method	result
default	$\frac{f^2\left(56a+16b+224\ln\left(xc+\sqrt{c^2x^2+1}-i\right)b\,c^2x^2-4ib\,c^3x^3-10ibxc-46i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(xc)b-84i\,\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2-56a\right.}{}$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))/(d+I*c*d*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*f^2*(56*a+16*b-46*I*(c^2*x^2+1)^(1/2)*a-84*I*(c^2*x^2+1)^(1/2)*arcsinh
(x*c)*b*c^2*x^2-6*I*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*b*c^4*x^4-56*a*c^3*x^3*
(c^2*x^2+1)^(1/2)-24*(c^2*x^2+1)^(1/2)*a*c*x+30*arcsinh(x*c)^2*b*c^2*x^2+6
0*arcsinh(x*c)*a*c^2*x^2-112*arcsinh(x*c)*b*c^2*x^2+224*ln(x*c+(c^2*x^2+1)
^(1/2)-I)*b*c^2*x^2-24*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c*x-10*I*b*x*c-4*I
*b*c^3*x^3+6*I*b*c^5*x^5+16*b*c^2*x^2-46*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*
b-56*b*c^3*x^3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+15*arcsinh(x*c)^2*b*c^4*x^4+
30*arcsinh(x*c)*a*c^4*x^4-56*arcsinh(x*c)*b*c^4*x^4+112*ln(x*c+(c^2*x^2+1)
^(1/2)-I)*b*c^4*x^4-56*b*arcsinh(x*c)-84*I*(c^2*x^2+1)^(1/2)*a*c^2*x^2-6*I
*(c^2*x^2+1)^(1/2)*a*c^4*x^4+112*a*c^2*x^2+56*a*c^4*x^4+15*b*arcsinh(x*c)^
2+112*b*ln(x*c+(c^2*x^2+1)^(1/2)-I)+30*arcsinh(x*c)*a)*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/d^3/(c^4*x^4+2*c^2*x^2+1)/c
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((-I*b*c^2*f^2*x^2 + 2*b*c*f^2*x + I*b*f^2)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*f^2*x^2 + 2*a*c*f
^2*x + I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c
^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*(3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*d^5*x^4 - 4*I*c^4*d^5*x^3 - 6*c^3
*d^5*x^2 + 4*I*c^2*d^5*x + c*d^5) - 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*f/(-3*I
*c^4*d^4*x^3 - 9*c^3*d^4*x^2 + 9*I*c^2*d^4*x + 3*c*d^4) + 10*I*sqrt(c^2*d*
f*x^2 + d*f)*f^2/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 105*I*sqrt(c^2*d*
f*x^2 + d*f)*f^2/(3*I*c^2*d^3*x + 3*c*d^3) - 15*f^3*arcsinh(c*x)/(c*d^3*sq
rt(f/d)))*a + b*integrate((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(I*c*d*x + d)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))}{(d+icdx)^{5/2}}\,dx = \frac{\sqrt{f}\,f^2\left(-30\sqrt{cix+1}\,\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)acx+30\sqrt{cix+}\right.}{}$$

input

```
int((f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

output

```
(sqrt(f)*f**2*( - 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x
 + 1)/sqrt(2))*a*c*x + 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( -
c*i*x + 1)/sqrt(2))*a*i + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(
- c*i*x + 1)*asinh(c*x)*x**2)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x +
1)*c*i*x - sqrt(c*i*x + 1)),x)*b*c**4*i*x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*
x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x**2)/(sqrt(c*i*x + 1)*c**2*x**2
 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b*c**3 - 6*sqrt(c*i*x + 1
)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1
)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b*c**3*x + 6*s
qrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(s
qrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b
*c**2*i - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asi
nh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x
 + 1)),x)*b*c**2*i*x - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c
*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x
 - sqrt(c*i*x + 1)),x)*b*c + 3*a*c**3*x**3 - 31*a*c**2*i*x**2 + 11*a*c*x -
 23*a*i))/(3*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d**2*(c*i*x + 1)
)
```

3.219 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{11id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{icd^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output

```
-11/3*I*b*d^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*
b*c*d^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/9*I*b*
c^2*d^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+11/3*I*d
^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/
2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
-1/3*I*c*d^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+5/4*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(
1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 6.29 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.22

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-264ibcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+8ibc^3d^2x^3\sqrt{d+icdx}\sqrt{f-icf}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
((-264*I)*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (8*I)*b*c^3*d^2*
x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (264*I)*a*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (24*I)*a*c^2*d^2*x^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*ArcSinh[c*x]^2 + 27*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[2*ArcSinh[c*x]] - 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]*(9*(-5*I + 2*c*x)*Sqrt[1 + c^2*x^2] + I*Cosh[3*ArcSinh[c*x]]) + 180*a*d^
(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]])/(72*c*f*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\sqrt{c^2x^2+1}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^3\sqrt{c^2x^2+1}\int \left(-\frac{ic^3(a+b\text{arcsinh}(cx))x^3}{\sqrt{c^2x^2+1}}-\frac{3c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}+\frac{3ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\sqrt{c^2x^2+1}\left(-\frac{1}{3}icx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{3}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{11i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx}{3c}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
(d^3*Sqrt[1 + c^2*x^2]*(((-11*I)/3)*b*x + (3*b*c*x^2)/4 + (I/9)*b*c^2*x^3
+ (((11*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c - (3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/2 - (I/3)*c*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x]) + (5*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(313) = 626$.

Time = 6.93 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.10

method	result
default	$\frac{ia(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}{3cf}+\frac{5iad(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}{6cf}+\frac{5ia\,d^{2}\sqrt{icdx+d}\,\sqrt{-icfx+f}}{2cf}+\frac{5a\,d^{3}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}d}{\sqrt{c^{2}}}\right.}{2\sqrt{icdx+d}\,\sqrt{-icfx+f}}$
parts	$\frac{ia(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}{3cf}+\frac{5iad(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}{6cf}+\frac{5ia\,d^{2}\sqrt{icdx+d}\,\sqrt{-icfx+f}}{2cf}+\frac{5a\,d^{3}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}d}{\sqrt{c^{2}}}\right.}{2\sqrt{icdx+d}\,\sqrt{-icfx+f}}$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/3*I*a/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)+5/6*I*a*d/c/f*(d+I*c*d*x)^
(3/2)*(f-I*c*f*x)^(1/2)+5/2*I*a*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)+5/2*a*d^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2
)+b*(5/4*d^2*arcsinh(x*c)^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*
x^2+1)^(1/2)/c/f-1/72*I*d^2*(-1+3*arcsinh(x*c))*(4*c^4*x^4+4*(c^2*x^2+1)^(
1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*
(I+x*c)*f)^(1/2)/c/(c^2*x^2+1)/f-3/16*d^2*(-1+2*arcsinh(x*c))*(2*x^3*c^3+2
*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/2)*(
I*(x*c-I)*d)^(1/2)/c/(c^2*x^2+1)/f+15/8*I*d^2*(arcsinh(x*c)-1)*(c^2*x^2+(c
^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/(c^2*x^2
+1)/f+15/8*I*d^2*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*
c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/(c^2*x^2+1)/f-3/16*d^2*(1+2*arcsinh(x
*c))*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(-I*(
I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/c/(c^2*x^2+1)/f-1/72*I*d^2*(1+3*arcsin
h(x*c))*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/
2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/(c^2*x^2+1)/f)
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(((-I*b*c^2*d^2*x^2 - 2*b*c*d^2*x + I*b*d^2)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*d^2*x^2 - 2*a*c*d
^2*x + I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\frac{\sqrt{d}\,d^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-2\sqrt{cix+1}\,\sqrt{-cix+1}\,a\,c^2i\,x^2-9\sqrt{}\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

output
```
(sqrt(d)*d**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - 2*sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*a*c**2*i*x**2 - 9*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a
*c*x + 22*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i - 6*int((sqrt(c*i*x + 1)*
asinh(c*x)*x**2)/sqrt( - c*i*x + 1),x)*b*c**3 + 12*int((sqrt(c*i*x + 1)*as
inh(c*x)*x)/sqrt( - c*i*x + 1),x)*b*c**2*i + 6*int((sqrt(c*i*x + 1)*asinh(
c*x))/sqrt( - c*i*x + 1),x)*b*c))/(6*sqrt(f)*c)
```

3.220 $\int \frac{(d+icdx)^{3/2}(a+b\,\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 266

$$\int \frac{(d+icdx)^{3/2}(a+b\,\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = -\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)\,(a+b\,\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x(1+c^2x^2)\,(a+b\,\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3d^2\sqrt{1+c^2x^2}(a+b\,\text{arcsinh}(cx))^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-2*I*b*d^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b*c
*d^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*d^2*(c^
2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*d^2*
x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*d
^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Mathematica [A] (verified)

Time = 4.77 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{-16ibcdx\sqrt{d+icdx}\sqrt{f-icfx}+16iad\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1}}{}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
((-16*I)*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a*d*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*d*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*d*(-4*I + c*x)*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*d*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*Cosh[2*ArcSinh[c*x]] + 12*a*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*f*Sqrt[1
 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{c^2x^2+1}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{c^2x^2+1}\int \left(-\frac{c^2(a+b\text{arcsinh}(cx))x^2}{\sqrt{c^2x^2+1}}+\frac{2ic(a+b\text{arcsinh}(cx))x}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{c^2x^2+1}\left(-\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{2i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{3(a+b\text{arcsinh}(cx))^2}{4bc}+\frac{1}{4}bcx^2-2ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
(d^2*Sqrt[1 + c^2*x^2]*((-2*I)*b*x + (b*c*x^2)/4 + ((2*I)*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x]))/c - (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 +
 (3*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(222) = 444$.

Time = 6.36 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.04

method	result
default	$\frac{ia(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}{2cf}+\frac{3iad\sqrt{icdx+d}\,\sqrt{-icfx+f}}{2cf}+\frac{3a\,d^{2}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\sqrt{c^{2}df\,x^{2}+df}\right)}{2\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^{2}df}}+b\Bigg(\frac{3d\,\mathrm{ar}}{}$
parts	$\frac{ia(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}{2cf}+\frac{3iad\sqrt{icdx+d}\,\sqrt{-icfx+f}}{2cf}+\frac{3a\,d^{2}\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^{2}dfx}{\sqrt{c^{2}df}}+\sqrt{c^{2}df\,x^{2}+df}\right)}{2\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^{2}df}}+b\Bigg(\frac{3d\,\mathrm{ar}}{}$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2*I*a/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)+3/2*I*a*d/c/f*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)+3/2*a*d^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(d+I*c*d
*x)^(1/2)/(f-I*c*f*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)
^(1/2))/(c^2*d*f)^(1/2)+b*(3/4*d*arcsinh(x*c)^2*(-I*(I+x*c)*f)^(1/2)*(I*(x
*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/f/c-1/16*d*(-1+2*arcsinh(x*c))*(2*x^3*c^3
+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*
(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)+I*d*(arcsinh(x*c)-1)*(c^2*x^2+(c^2*x^
2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)
+I*d*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)-1/16*d*(1+2*arcsinh(x*c))*(2*x^3*c^
3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f))/(c*f*x + I*f), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{3/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\frac{\sqrt{d}\,d\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-\sqrt{cix+1}\,\sqrt{-cix+1}\,acx+4\sqrt{cix+1}\cdots\right.}{\cdots}$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

output

```
(sqrt(d)*d*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - sqrt(c*i*x + 1)*sqrt(
 - c*i*x + 1)*a*c*x + 4*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*i + 2*int((sq
rt(c*i*x + 1)*asinh(c*x)*x)/sqrt( - c*i*x + 1),x)*b*c**2*i + 2*int((sqrt(c
*i*x + 1)*asinh(c*x))/sqrt( - c*i*x + 1),x)*b*c))/(2*sqrt(f)*c)
```

3.221 $\int \frac{\sqrt{d+icdx}(a+b\,\mathbf{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 158

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-I*b*d*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*d*(c^2*x^
2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*d*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx$$
$$= \frac{-2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{1+c^2x^2}\right)+2ib\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b\sqrt{d+}}{2cf\sqrt{1+c^2}}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
((-2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2])
+ (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sq
rt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]])/(2*c*f*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{c^2x^2+1}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{c^2x^2+1}\int \left(\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}+\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2x^2+1}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}+\frac{(a+b\text{arcsinh}(cx))^2}{2bc}-ibx\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*((-I)*b*x + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
)/c + (a + b*ArcSinh[c*x])^2/(2*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(132) = 264.

Time = 6.51 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.98

method	result
default	$\frac{ia\sqrt{icdx+d}\,\sqrt{-icfx+f}}{cf}+\frac{ad\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+b\left(\frac{\operatorname{arcsinh}(xc)^2\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}}{2\sqrt{c^2x^2+1}\,fc}\right.$
parts	$\frac{ia\sqrt{icdx+d}\,\sqrt{-icfx+f}}{cf}+\frac{ad\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+b\left(\frac{\operatorname{arcsinh}(xc)^2\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}}{2\sqrt{c^2x^2+1}\,fc}\right.$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
I*a/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+a*d*((f-I*c*f*x)*(d+I*c*d*x))^
(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^
2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b*(1/2*arcsinh(x*c)^2*(-I*(I+x*c)*f)
^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/f/c+1/2*I*(arcsinh(x*c)-1)*(c
^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f
/c/(c^2*x^2+1)+1/2*I*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I
*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
+ 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*f*x + I*f), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\operatorname{asinh}\,(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
a*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + b*i
ntegrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x + f)
, x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}\,(cx)+a)}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{d}\left(2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+\sqrt{cix+1}\,\sqrt{-cix+1}\,ai+\left(\int \frac{\sqrt{cix+1}\,asinh(cx)}{\sqrt{-cix+1}}dx\right)bc\right)}{\sqrt{f}\,c}$$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

output
```
(sqrt(d)*(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)*a*i + int((sqrt(c*i*x + 1)*asinh(c*x))/sqrt( - c*i*x + 1),x)*b
*c))/(sqrt(f)*c)
```

3.222 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
1/2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{a\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (a*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]])/(c*Sqrt[d]*Sqrt[f])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {6211, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(49) = 98.

Time = 4.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.31

method	result	size
default	$\frac{a\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+\frac{b\,\mathrm{arcsinh}(xc)^2\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{2\sqrt{c^2x^2+1}\,fcd}$	136
parts	$\frac{a\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+\frac{b\,\mathrm{arcsinh}(xc)^2\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{2\sqrt{c^2x^2+1}\,fcd}$	136

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)*ln(c
^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+1/2*b*ar
csinh(x*c)^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/f/
c/d
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f*x^2 + d*f), x)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{b\,\text{arsinh}\,(cx)^2}{2\,\sqrt{df}c} + \frac{a\,\text{arsinh}\,(cx)}{\sqrt{df}c}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a*arcsinh(c*x)/(sqrt(d*f)*c)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)bc}{\sqrt{f}\,\sqrt{d}\,c}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i + int(asinh(c*x)/(sqrt(c*i*x + 1)*
sqrt( - c*i*x + 1)),x)*b*c)/(sqrt(f)*sqrt(d)*c)
```

3.223 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 111

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{f(i+cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
f*(I+c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-b*f*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a\sqrt{1+c^2x^2}+b\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b(i-cx)\,\text{l}\right.}{cd^2f(-i+cx)\sqrt{1+c^2x^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x] + b*(I - c*x)*Log[d + I*c*d*x]))/(c*d^2*f*(-I + c*x)*Sq
rt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {6211, 27, 6252, 27, 451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(c^2x^2+1)^{3/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-bc\int \frac{cx+i}{c(c^2x^2+1)}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-b\int \frac{cx+i}{c^2x^2+1}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{451}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(b\int \frac{1}{i-cx}dx+\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(\frac{(cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{b\log(-cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f*(1 + c^2*x^2)^(3/2)*(((I + c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt[1 + c^2*x
^2]) - (b*Log[I - c*x])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(95) = 190.

Time = 6.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.98

method	result
default	$\frac{ia\sqrt{-icfx+f}}{cdf\sqrt{icdx+d}}+b\left(\frac{2\,\mathrm{arcsinh}(xc)\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^2x^2+1}\,c\,d^2f}+\frac{\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}\left(xc+i-\sqrt{c^2x^2+1}\right)\mathrm{arcsinh}(xc)}{(c^2x^2+1)c\,d^2f}-\frac{2\ln}{}\right.$
parts	$\frac{ia\sqrt{-icfx+f}}{cdf\sqrt{icdx+d}}+b\left(\frac{2\,\mathrm{arcsinh}(xc)\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^2x^2+1}\,c\,d^2f}+\frac{\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}\left(xc+i-\sqrt{c^2x^2+1}\right)\mathrm{arcsinh}(xc)}{(c^2x^2+1)c\,d^2f}-\frac{2\ln}{}\right.$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
I*a/c/d/f/(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+b*(2*arcsinh(x*c)*(I*(x*c-I)
*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2/f+(I*(x*c-I)*d)^(1/
2)*(-I*(I+x*c)*f)^(1/2)*(x*c+I-(c^2*x^2+1)^(1/2))*arcsinh(x*c)/(c^2*x^2+1)
/c/d^2/f-2*ln(x*c+(c^2*x^2+1)^(1/2)-I)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^
(1/2)/(c^2*x^2+1)^(1/2)/c/d^2/f)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(89) = 178.

Time = 0.17 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.99

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\frac{2\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(c^2d^2fx-i\,cd^2f\right)\sqrt{c^2}}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
 + (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2
+ 2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f) - (I*c^9*d^2*f*x^4 + 2*c^8*d^2*f*x^3 + I*c^7*d^2*f*x^2 + 2*c^6*d^2*f
*x)*sqrt(b^2/(c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^2
*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c
^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) -
 (-I*c^9*d^2*f*x^4 - 2*c^8*d^2*f*x^3 - I*c^7*d^2*f*x^2 - 2*c^6*d^2*f*x)*sq
rt(b^2/(c^2*d^3*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 2*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*f*x - I*c*d^2*f)
```

Sympy [F]

$$\int \frac{a + b\,\mathrm{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \int \frac{a + b\,\mathrm{asinh}\,(cx)}{(id\,(cx - i))^{\frac{3}{2}}\,\sqrt{-if\,(cx + i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{a + b\,\mathrm{arcsinh}(cx)}{(d + icdx)^{3/2}\sqrt{f - icfx}}\,dx = \frac{i\,\sqrt{c^2dfx^2 + df}\,b\,\mathrm{arsinh}\,(cx)}{i\,c^2d^2fx + cd^2f} + \frac{i\,\sqrt{c^2dfx^2 + df}\,a}{i\,c^2d^2fx + cd^2f} - \frac{b\log\,(i\,cx + 1)}{cd^{\frac{3}{2}}\sqrt{f}}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(I*c^2*d^2*f*x + c*d^2*f) + I*sqr
t(c^2*d*f*x^2 + d*f)*a/(I*c^2*d^2*f*x + c*d^2*f) - b*log(I*c*x + 1)/(c*d^(
3/2)*sqrt(f))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(d+c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,cix+\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)bc+acx+a}{\sqrt{f}\,\sqrt{d}\,\sqrt{cix+1}\,\sqrt{-cix+1}\,cd}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c + a*c*x +
a*i)/(sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d)
```

3.224 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{ibf^2(1+c^2x^2)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\,(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x(1+c^2x^2)^2\,(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibf^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
1/3*I*b*f^2*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)+2/3*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)-1/3*I*b*f^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*f^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d
+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.48

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-2i+cx)\left(-ib+bcx+a\sqrt{1+c^2x^2}\right)+b(-2i+c\right.}{3cd^3f(-i+cx)^2\sqrt{1+}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-2*I + c*x)*((-I)*b + b*c*x + a*Sqr
t[1 + c^2*x^2]) + b*(-2*I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*(-I +
c*x)^2*Log[d + I*c*d*x]))/(3*c*d^3*f*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\left(-bc\int\left(\frac{x}{3(c^2x^2+1)}+\frac{2i(1-icx)}{3c(c^2x^2+1)^2}\right)dx+\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{2i(1-icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(\frac{i\arctan(cx)}{3c^2}+\frac{i(cx+i)}{3c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{6c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f^2*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*(1 - I*c*x)*(a + b*ArcSinh[c*x]))/(c*
(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b*
c*(((I/3)*(I + c*x))/(c^2*(1 + c^2*x^2)) + ((I/3)*ArcTan[c*x])/c^2 + Log[1
 + c^2*x^2]/(6*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.01

method	result
default	$a\left(\frac{i\sqrt{-icfx+f}}{3fcd(icdx+d)^{\frac{3}{2}}}+\frac{i\sqrt{-icfx+f}}{3cf\,d^2\sqrt{icdx+d}}\right)+\frac{b\left(\operatorname{arcsinh}(xc)c^4x^4-2\ln\left(xc+\sqrt{c^2x^2+1}-i\right)x^4c^4+\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^3c^3-ix^3c^3\right.}{}$
parts	$a\left(\frac{i\sqrt{-icfx+f}}{3fcd(icdx+d)^{\frac{3}{2}}}+\frac{i\sqrt{-icfx+f}}{3cf\,d^2\sqrt{icdx+d}}\right)+\frac{b\left(\operatorname{arcsinh}(xc)c^4x^4-2\ln\left(xc+\sqrt{c^2x^2+1}-i\right)x^4c^4+\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,x^3c^3-ix^3c^3\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(1/3*I/f/c/d/(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)+1/3*I/c/f/d^2/(d+I*c*d*
x)^(1/2)*(f-I*c*f*x)^(1/2))+1/3*b*(arcsinh(x*c)*c^4*x^4-2*ln(x*c+(c^2*x^2+
1)^(1/2)-I)*x^4*c^4+arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-I*x^3*c^3+2*arc
sinh(x*c)*c^2*x^2-4*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^2*c^2+3*arcsinh(x*c)*(c^
2*x^2+1)^(1/2)*x*c+c^2*x^2+2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)-I*x*c+arcsin
h(x*c)-2*ln(x*c+(c^2*x^2+1)^(1/2)-I)+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)
^(1/2)/(c^2*x^2+1)^(5/2)/d^3/c/f
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(228) = 456$.

Time = 0.21 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.95

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d + icdx)^{5/2}\sqrt{f - icfx}}\, dx =$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx - 2\,(bc^2x^2 - i\,bcx + 2\,b)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algori
thm="fricas")
```

output
```
-1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(
b*c^2*x^2 - I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (c^4*d^3*f*x^3 - I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*
d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6*x^2 + 2*b*c^5*x - 2*I*b*c^
4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d^3*f*x
^4 + 2*c^8*d^3*f*x^3 + I*c^7*d^3*f*x^2 + 2*c^6*d^3*f*x)*sqrt(b^2/(c^2*d^5*
f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - (c^4*d^3*f*x^3 - I*c^3*d^3
*f*x^2 + c^2*d^3*f*x - I*c*d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(-1/8*((I*b*c^6
*x^2 + 2*b*c^5*x - 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f) + (-I*c^9*d^3*f*x^4 - 2*c^8*d^3*f*x^3 - I*c^7*d^3*f*x^2 - 2*c^6
*d^3*f*x)*sqrt(b^2/(c^2*d^5*f)))/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b))
- 2*(a*c^2*x^2 - I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4
*d^3*f*x^3 - I*c^3*d^3*f*x^2 + c^2*d^3*f*x - I*c*d^3*f)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}\sqrt{f - icfx}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(id\,(cx - i))^{\frac{5}{2}}\,\sqrt{-if\,(cx + i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.79

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}\sqrt{f - icfx}}\,dx = \frac{1}{3}\,bc\left(\frac{3}{3i\,c^3d^{\frac{5}{2}}\sqrt{f}x + 3\,c^2d^{\frac{5}{2}}\sqrt{f}} - \frac{\log\,(cx - i)}{c^2d^{\frac{5}{2}}\sqrt{f}}\right) - \frac{1}{3}\,b\left(\frac{i\,\sqrt{c^2dfx^2 + df}}{c^3d^3fx^2 - 2i\,c^2d^3fx - cd^3f} - \frac{3i\,\sqrt{c^2dfx^2 + df}}{3i\,c^2d^3fx + 3\,cd^3f}\right)\text{arsinh}\,(cx) - \frac{1}{3}\,a\left(\frac{i\,\sqrt{c^2dfx^2 + df}}{c^3d^3fx^2 - 2i\,c^2d^3fx - cd^3f} - \frac{3i\,\sqrt{c^2dfx^2 + df}}{3i\,c^2d^3fx + 3\,cd^3f}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algori
thm="maxima")
```

output
```
1/3*b*c*(3/(3*I*c^3*d^(5/2)*sqrt(f)*x + 3*c^2*d^(5/2)*sqrt(f)) - log(c*x -
 I)/(c^2*d^(5/2)*sqrt(f))) - 1/3*b*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*f*x
^2 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3
*f*x + 3*c*d^3*f))*arcsinh(c*x) - 1/3*a*(I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^
3*f*x^2 - 2*I*c^2*d^3*f*x - c*d^3*f) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^
2*d^3*f*x + 3*c*d^3*f))
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{a+b\operatorname{asinh}(c\,x)}{(d+c\,d\,x\,1\mathrm{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\frac{-3\sqrt{cix+1}\,\sqrt{-cix+1}\,\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2-2\sqrt{cix+1}\,\sqrt{-cix+1}\,cix-\sqrt{cix+1}\,\sqrt{-}\right.}{}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
( - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x -
 sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c**3*x**2 - 3*sqrt(c*i*x + 1)*sq
rt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x
**2 - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)),x)*b*c + a*c**3*x**3 + 3*a*c*x + 2*a*i)/(3*sqrt(f)*sqrt(d)*sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d**2*(c**2*x**2 + 1))
```

3.225 $\int \frac{(d+icdx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 453

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{4ibd^3x\sqrt{1+c^2x^2}}{f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{bcd^3x^2\sqrt{1+c^2x^2}}{4f\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{15bd^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^2}{4cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8id^3(1+icx)(a+b\operatorname{arcsinh}(cx))}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4id^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{cf\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^3x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))}{2f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{15d^3\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{2cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8bd^3\sqrt{1+c^2x^2}\log(i+cx)}{cf\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
4*I*b*d^3*x*(c^2*x^2+1)^(1/2)/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b*
c*d^3*x^2*(c^2*x^2+1)^(1/2)/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+15/4*b*d
^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)-8*I*d^3*(1+I*c*x)*(a+b*arcsinh(c*x))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-4*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+1/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/f/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)-15/2*d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*(a+b*arcsinh(c*x))/
c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b*d^3*(c^2*x^2+1)^(1/2)*ln(I+c*x
)/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.74 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.72

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x
]
```

output
```
((4*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 - (7*I)*c*x + c^2*x^2))/
(f^2*(I + c*x)) - (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]])/f^(3/2) + (4*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))
+ 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4
*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2
] + Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])) + (16*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x
- 4*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x
]/2] + Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*
Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f
^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (b
*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (16*c*x + 32*ArcTan[Tanh[ArcSinh[c*x]
/2]] + I*Cosh[2*ArcSinh[c*x]] + (8*I)*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*
x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 -
8*Sqrt[1 + c^2*x^2] - I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((-8*
I)*(1 + Sqrt[1 + c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(f^2*Sqrt[1 + c^2*x^
2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^4(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 6252$$

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\left(-bc\int \left(\frac{x}{2}-\frac{15\text{arcsinh}(cx)}{2c\sqrt{c^2x^2+1}}-\frac{4i}{c}-\frac{8i(icx+1)}{c(c^2x^2+1)}\right)dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{4i\sqrt{c^2x^2+1}(a}{}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))-\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{8i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{15\text{arcs}}{}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(3/2)*(((-8*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqr
t[1 + c^2*x^2]) - ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (x*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (15*ArcSinh[c*x]*(a + b*ArcSinh[
c*x]))/(2*c) - b*c*(((-4*I)*x)/c + x^2/4 - (15*ArcSinh[c*x]^2)/(4*c^2) + (
8*Log[I + c*x])/c^2)))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 10.69 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.85

method	result
default	$-\frac{d^2\left(-4b\,c^3x^3\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2b\,c^4x^4+32i\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2+32i\sqrt{c^2x^2+1}\,a\,c^2x^2-4a\,c^3x^3\sqrt{c^2x^2+1}+30\operatorname{arcsin}\right.}{}$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/8*d^2*(-4*b*c^3*x^3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*b*c^4*x^4+32*I*(c^
2*x^2+1)^(1/2)*arcsinh(x*c)*b*c^2*x^2+32*I*(c^2*x^2+1)^(1/2)*a*c^2*x^2-4*a
*c^3*x^3*(c^2*x^2+1)^(1/2)+30*arcsinh(x*c)^2*b*c^2*x^2+96*I*(c^2*x^2+1)^(1
/2)*a+60*arcsinh(x*c)*a*c^2*x^2-64*arcsinh(x*c)*b*c^2*x^2+128*ln(x*c+(c^2*
x^2+1)^(1/2)+I)*b*c^2*x^2-68*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c*x+64*a*c^2
*x^2+3*b*c^2*x^2-32*I*b*c^3*x^3-32*I*b*x*c-68*(c^2*x^2+1)^(1/2)*a*c*x+30*b
*arcsinh(x*c)^2+96*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b+60*arcsinh(x*c)*a-64
*b*arcsinh(x*c)+128*b*ln(x*c+(c^2*x^2+1)^(1/2)+I)+64*a+b)*(I*(x*c-I)*d)^(1
/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)/c/(c^4*x^4+2*c^2*x^2+1)/f^2
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x
 - a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x
 - f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x
)/(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a + b*integrat
e((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\frac{\sqrt{d}\,d^2\left(-30\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai+2\sqrt{-cix+1}\left(\int \frac{\sqrt{cix}}{\sqrt{-cix}}\right.\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a
*i + 2*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x**2)/(sqrt( - c
*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**3 - 4*sqrt( - c*i*x + 1)*int
((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x
+ 1)),x)*b*c**2*i - 2*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/
(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c - sqrt(c*i*x + 1)*a
*c**2*i*x**2 - 7*sqrt(c*i*x + 1)*a*c*x - 24*sqrt(c*i*x + 1)*a*i))/(2*sqrt(
f)*sqrt( - c*i*x + 1)*c*f)
```

3.226 $\int \frac{(d+icdx)^{3/2}(a+b\,\mathbf{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{ibd^2x\sqrt{1+c^2x^2}}{f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4id^2(1+icx)(a+b\operatorname{arcsinh}(cx))}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{id^2(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4bd^2\sqrt{1+c^2x^2}\log(i+cx)}{cf\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
I*b*d^2*x*(c^2*x^2+1)^(1/2)/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4*I*d^2*
(1+I*c*x)*(a+b*arcsinh(c*x))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*d^2
*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/
2*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/f/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)-4*b*d^2*(c^2*x^2+1)^(1/2)*ln(I+c*x)/c/f/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.79

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{2ad(5-icx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(i+cx)} - \frac{6ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{bd\sqrt{d}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x
]
```

output
```
((2*a*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x)) -
 (6*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]])/f^(3/2) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]
^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)
*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c
*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/
2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2
])) + (2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[A
rcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c
*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/
2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-
2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cos
h[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{icx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{4i(i-cx)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c}-\frac{4i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{4b\log(cx+i)}{c}+ibx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(3/2)*(I*b*x - ((4*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))
/(c*Sqrt[1 + c^2*x^2]) - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c - (3
*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (4*b*Log[I + c*x])/c))/((d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 10.61 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.05

method	result
default	$-\frac{d\left(2i \operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\, b\, c^2x^2+2i\sqrt{c^2x^2+1}\, a\, c^2x^2+3 \operatorname{arcsinh}(xc)^2 b\, c^2x^2+10i\sqrt{c^2x^2+1}\, a+6 \operatorname{arcsinh}(xc) a\, c^2x^2-8 \operatorname{arcsinh}(xc\right.}{}$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*d*(2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c^2*x^2+2*I*(c^2*x^2+1)^(1/2)
*a*c^2*x^2+3*arcsinh(x*c)^2*b*c^2*x^2+10*I*(c^2*x^2+1)^(1/2)*a+6*arcsinh(x
*c)*a*c^2*x^2-8*arcsinh(x*c)*b*c^2*x^2+16*ln(x*c+(c^2*x^2+1)^(1/2)+I)*b*c^
2*x^2-8*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c*x+8*a*c^2*x^2-2*I*b*c^3*x^3-2*I
*b*x*c-8*(c^2*x^2+1)^(1/2)*a*c*x+3*b*arcsinh(x*c)^2+10*I*(c^2*x^2+1)^(1/2)
*arcsinh(x*c)*b+6*arcsinh(x*c)*a-8*b*arcsinh(x*c)+16*b*ln(x*c+(c^2*x^2+1)^
(1/2)+I)+8*a)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(3/2)/f
^2/c
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(((-I*b*c*d*x - b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (-I*a*c*d*x - a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
+ f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2)
, x)
```

Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6*
I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2)*log(c*x + sqrt(c^2*x^2
+ 1))/(-I*c*f*x + f)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionUnable to transpose Error: Bad Argument Valuesym2poly/r2sy
m(const g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d+c\,d\,x\,1\text{i})^{3/2}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d}\,d\Big(-6\sqrt{-cix+1}\,asin\Big(\frac{\sqrt{-cix+1}}{\sqrt{2}}\Big)\,ai - \sqrt{-cix+1}\,\Big(\int \frac{\sqrt{cix+1}\,a}{\sqrt{-cix+1}\,ci}$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(d)*d*( - 6*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i -
 sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)
*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**2*i - sqrt( - c*i*x + 1)*int((sqrt(c*
i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*
c - sqrt(c*i*x + 1)*a*c*x - 5*sqrt(c*i*x + 1)*a*i))/(sqrt(f)*sqrt( - c*i*x
 + 1)*c*f)
```

3.227 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 180

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = -\frac{2id^2(1+icx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}(a+b\text{arcsinh}(cx))^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2bd^2\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
-2*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-1/2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d^2*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{i+cx} - 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icf\ldots}\right.}{\ldots}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
((4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 2*a*Sqrt[d]*Sqrt[f]
*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] +
Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*
x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]
*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c*f^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2x^2+1}}-\frac{2i(i-cx)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(-\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{2b\log(cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^2*(1 + c^2*x^2)^(3/2)*(((-2*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqr
t[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/(2*b*c) - (2*b*Log[I + c*x])/c))/
((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 10.96 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\operatorname{arcsinh}(xc)^{2} b \sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{2\sqrt{c^{2}x^{2}+1}\,c\,f^{2}}-\frac{a\,\operatorname{arcsinh}(xc)\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^{2}x^{2}+1}\,c\,f^{2}}+\frac{4\,\operatorname{arcsinh}(xc) b\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^{2}x^{2}+1}\,c\,f^{2}}-$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*arcsinh(x*c)^2*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)
^(1/2)/c/f^2-a*arcsinh(x*c)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*
x^2+1)^(1/2)/c/f^2+4*arcsinh(x*c)*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/
2)/(c^2*x^2+1)^(1/2)/c/f^2+2*(a+b*arcsinh(x*c))*(x*c-(c^2*x^2+1)^(1/2)-I)*
(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)/c/f^2-4*ln(x*c+(c^2*x
^2+1)^(1/2)+I)*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2
)/c/f^2
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*f^2*x^2 + 2*I*c*f^2*x
- f^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\,\text{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2), x
)
```

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + b*integrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 +
1))/(-I*c*f*x + f)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d}\left(-2\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)ai-\sqrt{-cix+1}\left(\int \frac{\sqrt{cix+1}\,asinh}{\sqrt{-cix+1}\,cix-\sqrt{}}\right.\right.}{\sqrt{f}\,\sqrt{-cix+1}\,cf}$$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a*i - s
qrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c*i
*x - sqrt( - c*i*x + 1)),x)*b*c - 2*sqrt(c*i*x + 1)*a*i))/(sqrt(f)*sqrt( -
 c*i*x + 1)*c*f)
```

3.228 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 112

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = -\frac{d(i-cx)\left(1+c^2x^2\right)(a+b\text{arcsinh}(cx))}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bd\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
-d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-b*d*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{f-icfx}\left(a+iacx+(b+ibcx)\text{arcsinh}(cx)-ib\sqrt{1+c^2x^2}\log(d(-1+\right.}{cf^2(i+cx)\sqrt{d+icdx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

output
```
(Sqrt[f - I*c*f*x]*(a + I*a*c*x + (b + I*b*c*x)*ArcSinh[c*x] - I*b*Sqrt[1
+ c^2*x^2]*Log[d*(-1 + I*c*x)]))/(c*f^2*(I + c*x)*Sqrt[d + I*c*d*x])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6211, 27, 6252, 25, 27, 451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\left(-bc\int -\frac{i-cx}{c(c^2x^2+1)}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\left(bc\int \frac{i-cx}{c(c^2x^2+1)}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(c^2x^2+1\right)^{3/2}\left(b\int \frac{i-cx}{c^2x^2+1}dx-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{451}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(-b\int \frac{1}{cx+i}dx - \frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \ 16$$

$$\frac{d(c^2x^2+1)^{3/2}\left(-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}} - \frac{b\log(cx+i)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(3/2)*(-(((I - c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt[1 + c^2
*x^2])) - (b*Log[I + c*x])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(96) = 192$.

Time = 6.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.96

method	result
default	$-\frac{ia\sqrt{icdx+d}}{dcf\sqrt{-icfx+f}}+b\left(\frac{2\,\mathrm{arcsinh}(xc)\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^2x^2+1}\,f^2cd}+\frac{\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}\,\left(xc-\sqrt{c^2x^2+1}-i\right)\mathrm{arcsinh}(xc)}{(c^2x^2+1)f^2cd}-\right.$
parts	$-\frac{ia\sqrt{icdx+d}}{dcf\sqrt{-icfx+f}}+b\left(\frac{2\,\mathrm{arcsinh}(xc)\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{\sqrt{c^2x^2+1}\,f^2cd}+\frac{\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}\,\left(xc-\sqrt{c^2x^2+1}-i\right)\mathrm{arcsinh}(xc)}{(c^2x^2+1)f^2cd}-\right.$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-I*a/d/c/f/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+b*(2*arcsinh(x*c)*(I*(x*c-I
)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/f^2/c/d+(I*(x*c-I)*d)^(1
/2)*(-I*(I+x*c)*f)^(1/2)*(x*c-(c^2*x^2+1)^(1/2)-I)*arcsinh(x*c)/(c^2*x^2+1
)/f^2/c/d-2*ln(x*c+(c^2*x^2+1)^(1/2)+I)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)
^(1/2)/(c^2*x^2+1)^(1/2)/f^2/c/d)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(88) = 176$.

Time = 0.14 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.96

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{2\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)-(c^2df^2x+i\,cdf^2)\sqrt{\frac{b^2}{c^2d\cdots}}\cdots}{\cdots}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
 - (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2
 + 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f) - (I*c^9*d*f^2*x^4 - 2*c^8*d*f^2*x^3 + I*c^7*d*f^2*x^2 - 2*c^6*d*f^
2*x)*sqrt(b^2/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^
2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(-1/8*((-I*b*c^6*x^2 + 2*b
*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
 - (-I*c^9*d*f^2*x^4 + 2*c^8*d*f^2*x^3 - I*c^7*d*f^2*x^2 + 2*c^6*d*f^2*x)*
sqrt(b^2/(c^2*d*f^3)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f^2*x + I*c*d*f^2)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = -\frac{i\sqrt{c^2dfx^2+df}\,b\,\text{arsinh}\,(cx)}{-i\,c^2df^2x+cdf^2} - \frac{i\sqrt{c^2dfx^2+df}\,a}{-i\,c^2df^2x+cdf^2} - \frac{b\log\,(i\,cx-1)}{c\sqrt{d}f^{\frac{3}{2}}}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
-I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(-I*c^2*d*f^2*x + c*d*f^2) - I*s
qrt(c^2*d*f*x^2 + d*f)*a/(-I*c^2*d*f^2*x + c*d*f^2) - b*log(I*c*x - 1)/(c*
sqrt(d)*f^(3/2))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{-\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,cix-\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)bc+acx-}{\sqrt{f}\,\sqrt{d}\,\sqrt{cix+1}\,\sqrt{-cix+1}\,cf}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
( - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqr
t( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c + a*c*x
 - a*i)/(sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*f)
```

3.229 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 103

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b(1+c^2x^2)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b
*(c^2*x^2+1)^(3/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{i\sqrt{f-icfx}\left(2acx+2bcx\text{arcsinh}(cx)-b\sqrt{1+c^2x^2}\log(d(-1+icx))-\right.}{2cdf^2(i+cx)\sqrt{d+icdx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x
]
```

output
```
((I/2)*Sqrt[f - I*c*f*x]*(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*
x^2]*Log[d*(-1 + I*c*x)] - b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(c*d*f^2
*(I + c*x)*Sqrt[d + I*c*d*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {6211, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{a+b\text{arcsinh}(cx)}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int \frac{x}{c^2x^2+1}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
((1 + c^2*x^2)^(3/2)*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*Log[
1 + c^2*x^2])/(2*c)))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(87) = 174$.

Time = 6.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.13

method	result
default	$a\left(\frac{i}{cdf\sqrt{icdx+d}\,\sqrt{-icfx+f}}-\frac{i\sqrt{icdx+d}}{cf\,d^2\sqrt{-icfx+f}}\right)+\frac{b\left(-\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)x^2c^2+\sqrt{c^2x^2+1}\,\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)}{c}$
parts	$a\left(\frac{i}{cdf\sqrt{icdx+d}\,\sqrt{-icfx+f}}-\frac{i\sqrt{icdx+d}}{cf\,d^2\sqrt{-icfx+f}}\right)+\frac{b\left(-\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)x^2c^2+\sqrt{c^2x^2+1}\,\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)}{c}$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(I/c/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I/c/f/d^2/(f-I*c*f*x)^(1/2)
*(d+I*c*d*x)^(1/2))+b*(-ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^2*c^2+(c^2*x^2+1
)^(1/2)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x*c+arcsinh(x*c)-ln(1+(x*c+(c^2*x^
2+1)^(1/2))^2))*(x*c+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)
^(1/2)/c/f^2/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
1/4*(4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*x*log(c*x + sqrt(c^2*x^2 + 1
)) + 4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*x + (c^2*d^2*f^2*x^2 + d^2*f
^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^
4*x^4 + 2*b*c^2*x^2 + b)) - (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*
f^3))*log((b*c^2*x^4 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 +
b)) - 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3
 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2
/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sq
rt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)
) + 4*(c^2*d^2*f^2*x^2 + d^2*f^2)*integral(-sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*b*c*x/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*
f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f^2)
```

Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\operatorname{asinh}(cx)}{(id\,(cx-i))^{\frac{3}{2}}\,(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2
)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\frac{bx\,\text{arsinh}\,(cx)}{\sqrt{c^2dfx^2+df}\,df}+\frac{ax}{\sqrt{c^2dfx^2+df}\,df}-\frac{b\sqrt{\frac{1}{df}}\log\left(x^2+\frac{1}{c^2}\right)}{2\,cdf}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(c^2*d*f*x^2 + d
*f)*d*f) - 1/2*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*f)
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\frac{\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)b+ax}{\sqrt{f}\,\sqrt{d}\,\sqrt{cix+1}\,\sqrt{-cix+1}\,df}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b + a*x)/(
sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*d*f)
```

3.230 $\int \frac{a+b\,\mathrm{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 298

$$\int \frac{a+b\,\mathrm{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{ib\sqrt{1+c^2x^2}}{6cd^2f(i-cx)\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2x(a+b\,\mathrm{arcsinh}(cx))}{3d^2f\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{(i+cx)(a+b\,\mathrm{arcsinh}(cx))}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} - \frac{ib\sqrt{1+c^2x^2}\arctan(cx)}{6cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
1/6*I*b*(c^2*x^2+1)^(1/2)/c/d^2/f/(I-c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3
*(I+c*x)*(a+b*arcsinh(c*x))/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c
^2*x^2+1)-1/6*I*b*(c^2*x^2+1)^(1/2)*arctan(c*x)/c/d^2/f/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)-1/3*b*(c^2*x^2+1)^(1/2)*ln(c^2*x^2+1)/c/d^2/f/(d+I*c*d*x
)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.67

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{f-icfx}\left(4ia+8acx+8iac^2x^2+2b\sqrt{1+c^2x^2}+4b(i+2cx+2ic^2x\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x
]
```

output
```
(Sqrt[f - I*c*f*x]*((4*I)*a + 8*a*c*x + (8*I)*a*c^2*x^2 + 2*b*Sqrt[1 + c^2
*x^2] + 4*b*(I + 2*c*x + (2*I)*c^2*x^2)*ArcSinh[c*x] + 3*b*(-1 - I*c*x)*Sq
rt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] - 5*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*
x] - (5*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*d^2*f^2*Sqrt[d +
 I*c*d*x]*(c + c^3*x^2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\left(-bc\int\left(\frac{2x}{3(c^2x^2+1)}+\frac{cx+i}{3c(c^2x^2+1)^2}\right)dx+\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{(cx+i)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\left(\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}+\frac{(cx+i)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(\frac{i\arctan(cx)}{6c^2}-\frac{1-icx}{6c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{3c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
(f*(1 + c^2*x^2)^(5/2)*(((I + c*x)*(a + b*ArcSinh[c*x]))/(3*c*(1 + c^2*x^2
)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b*c*(-1/6*(1
 - I*c*x)/(c^2*(1 + c^2*x^2)) + ((I/6)*ArcTan[c*x])/c^2 + Log[1 + c^2*x^2]
/(3*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 6.72 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.47

method	result
default	$a\left(\frac{i}{3cdf(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}+\frac{\frac{2i}{3cdf\sqrt{icdx+d}\,\sqrt{-icfx+f}}-\frac{2i\sqrt{icdx+d}}{3cf\,d^2\sqrt{-icfx+f}}}{d}\right)+\frac{b\left(4\,\mathrm{arcsinh}(xc)c^4x^4-5\ln\left(xc+\sqrt{c^2x^2+1}-i\right)\right.}{}$
parts	$a\left(\frac{i}{3cdf(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}+\frac{\frac{2i}{3cdf\sqrt{icdx+d}\,\sqrt{-icfx+f}}-\frac{2i\sqrt{icdx+d}}{3cf\,d^2\sqrt{-icfx+f}}}{d}\right)+\frac{b\left(4\,\mathrm{arcsinh}(xc)c^4x^4-5\ln\left(xc+\sqrt{c^2x^2+1}-i\right)\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(1/3*I/c/d/f/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2)+2/3/d*(I/c/d/f/(d+I*c*d
*x)^(1/2)/(f-I*c*f*x)^(1/2)-I/c/f/d^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2))
)+1/6*b*(4*arcsinh(x*c)*c^4*x^4-5*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^4*c^4-3*ln
(x*c+(c^2*x^2+1)^(1/2)+I)*x^4*c^4+4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3
-I*x*c+8*arcsinh(x*c)*c^2*x^2-10*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^2*c^2-6*ln(
x*c+(c^2*x^2+1)^(1/2)+I)*x^2*c^2+6*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c+c^2*
x^2+2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)-I*x^3*c^3+4*arcsinh(x*c)-5*ln(x*c+(
c^2*x^2+1)^(1/2)-I)-3*ln(x*c+(c^2*x^2+1)^(1/2)+I)+1)*(I*(x*c-I)*d)^(1/2)*(
-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)/f^2/d^3/c/(c^6*x^6+3*c^4*x^4+3*c^2*x
^2+1)
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algori
thm="fricas")
```

output
```
-1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*
(2*b*c^2*x^2 - 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) - 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f
^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c
^2*x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 3*(c^4*d^3*f^2*
x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f
^3))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*
f*x*sqrt(b^2/(c^2*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^
2 + b*c*x + I*b)) + 5*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x
 - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + I*b*c^2*
x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 3*(c^4*d^3*f^2*x^3
 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3)
)*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*
x*sqrt(b^2/(c^2*d^5*f^3)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2
+ b*c*x + I*b)) + 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x -
 I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + b*c^2*x^3 + b*
x)/(b*c^2*x^2 + b)) - 8*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.80

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{1}{12}\,bc\left(-\frac{2i\,\sqrt{d}\sqrt{f}}{c^3d^3f^2x-i\,c^2d^3f^2}-\frac{3\,\log\left(cx+i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}-\frac{5\,\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}\right)-\frac{1}{3}\,b\left(-\frac{3i}{3i\,\sqrt{c^2dfx^2+df}c^2d^2fx+3\,\sqrt{c^2dfx^2+df}cd^2f}-\frac{2\,x}{\sqrt{c^2dfx^2+df}d^2f}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(-\frac{3i}{3i\,\sqrt{c^2dfx^2+df}c^2d^2fx+3\,\sqrt{c^2dfx^2+df}cd^2f}-\frac{2\,x}{\sqrt{c^2dfx^2+df}d^2f}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algori
thm="maxima")
```

output
```
1/12*b*c*(-2*I*sqrt(d)*sqrt(f)/(c^3*d^3*f^2*x - I*c^2*d^3*f^2) - 3*log(c*x
 + I)/(c^2*d^(5/2)*f^(3/2)) - 5*log(c*x - I)/(c^2*d^(5/2)*f^(3/2))) - 1/3*
b*(-3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*
f)*c*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))*arcsinh(c*x) - 1/3*a*(-
3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c
*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+c\,d\,x\,1\mathrm{i}\right)}^{5/2}\,{\left(f-c\,f\,x\,1\mathrm{i}\right)}^{3/2}} \,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\frac{3\sqrt{cix+1}\,\sqrt{-cix+1}\,\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^3i\,x^3+\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+1}\,\sqrt{}}\right.}{}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
(3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**3*i*x**3 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2
+ sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)),x)*b*c**3*x**2 + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x
)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**3*i*x**3 + sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x + sqrt(c
*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c + 2*a*c**3*x**3 + 3*a*c*x + a*i)/(3*s
qrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d**2*f*(c**2*x**2 + 1)
)
```

3.231 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 483

$$\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx &= \\
&-\frac{ibd^3x\sqrt{1+c^2x^2}}{f^2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{8ibd^3\sqrt{1+c^2x^2}}{3cf^2(i+cx)\sqrt{d+icdx}\sqrt{f-icfx}} \\
&-\frac{5bd^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2cf^2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{20id^3(1+icx)(a+b\text{arcsinh}(cx))}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&-\frac{2id^3(1+icx)^4(a+b\text{arcsinh}(cx))}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)}+\frac{5id^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+\frac{5d^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{cf^2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{28bd^3\sqrt{1+c^2x^2}\log(i+cx)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
-I*b*d^3*x*(c^2*x^2+1)^(1/2)/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*I
*b*d^3*(c^2*x^2+1)^(1/2)/c/f^2/(I+c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
-5/2*b*d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c
*f*x)^(1/2)+20/3*I*d^3*(1+I*c*x)*(a+b*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)-2/3*I*d^3*(1+I*c*x)^4*(a+b*arcsinh(c*x))/c/f^2/(d+I*c*
d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+5/3*I*d^3*(c^2*x^2+1)*(a+b*arcsin
h(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5*d^3*(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*(a+b*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+
28/3*b*d^3*(c^2*x^2+1)^(1/2)*ln(I+c*x)/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1083 vs. $2(483) = 966$.

Time = 12.48 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.24

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x
]
```

output
```
(((4*I)*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 + (34*I)*c*x + 3*c^
2*x^2))/(f^3*(I + c*x)^2) + (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/f^(5/2) - ((2*I)*b*d^2*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Co
sh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I
/2)*Log[1 + c^2*x^2])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*Ar
cTan[Coth[ArcSinh[c*x]/2]] + ((3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*Ar
cSinh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt
[1 + c^2*x^2]*((2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + L
og[1 + c^2*x^2]))/2)*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (2*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSin
h[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] - 7*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 21*Log[
1 + c^2*x^2]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(Ar
cSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] +
 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSin
h[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - (I*b*d^2*Sqrt[d + I*c*d*x]*Sqr...
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^5(icx+1)^5(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^5(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6252

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\left(-bc\int\left(-\frac{2i(icx+1)^4}{3c(c^2x^2+1)^2}+\frac{20i(icx+1)}{3c(c^2x^2+1)}+\frac{5\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}+\frac{5i}{3c}\right)dx-\frac{2i(1+icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{20i(1+ic\cdots}{}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\left(-\frac{2i(1+icx)^4(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{20i(1+icx)(a+b\text{arcsinh}(cx))}{3c\sqrt{c^2x^2+1}}+\frac{5i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{3c}+\frac{5\text{arcsinh}(c\cdots}{}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^5*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*(1 + I*c*x)^4*(a + b*ArcSinh[c*x]))/
(c*(1 + c^2*x^2)^(3/2)) + (((20*I)/3)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c
*Sqrt[1 + c^2*x^2]) + (((5*I)/3)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c
 + (5*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*((I*x)/c - ((8*I)/3)/(c^2
*(I + c*x)) + (5*ArcSinh[c*x]^2)/(2*c^2) - (28*Log[I + c*x])/(3*c^2))))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 10.80 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.06

method	result
default	$\frac{d^2\left(56a+16b+4ib\,c^3x^3+10ibxc+46i\sqrt{c^2x^2+1}\,\operatorname{arcsinh}(xc)b+84i\,\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2-56a\,c^3x^3\sqrt{c^2x^2+1}-24\sqrt{c^2x^2+1}\,acx\right.}{}$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*d^2*(56*a+16*b-56*a*c^3*x^3*(c^2*x^2+1)^(1/2)-24*(c^2*x^2+1)^(1/2)*a*c
*x+30*arcsinh(x*c)^2*b*c^2*x^2+60*arcsinh(x*c)*a*c^2*x^2-112*arcsinh(x*c)*
b*c^2*x^2-24*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c*x+16*b*c^2*x^2-56*b*c^3*x^
3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+224*ln(x*c+(c^2*x^2+1)^(1/2)+I)*b*c^2*x^2
+112*ln(x*c+(c^2*x^2+1)^(1/2)+I)*b*c^4*x^4+46*I*(c^2*x^2+1)^(1/2)*arcsinh(
x*c)*b+15*arcsinh(x*c)^2*b*c^4*x^4+30*arcsinh(x*c)*a*c^4*x^4-56*arcsinh(x*
c)*b*c^4*x^4-56*b*arcsinh(x*c)+112*a*c^2*x^2+4*I*b*c^3*x^3+10*I*b*x*c-6*I*
b*c^5*x^5+56*a*c^4*x^4+84*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c^2*x^2+6*I*(
c^2*x^2+1)^(1/2)*arcsinh(x*c)*b*c^4*x^4+15*b*arcsinh(x*c)^2+30*arcsinh(x*c
)*a+84*I*(c^2*x^2+1)^(1/2)*a*c^2*x^2+6*I*(c^2*x^2+1)^(1/2)*a*c^4*x^4+46*I*
(c^2*x^2+1)^(1/2)*a+112*b*ln(x*c+(c^2*x^2+1)^(1/2)+I))*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/f^3/(c^4*x^4+2*c^2*x^2+1)/c
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((I*b*c^2*d^2*x^2 + 2*b*c*d^2*x - I*b*d^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*d^2*x^2 + 2*a*c*d^2
*x - I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2
*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*(-3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*f^5*x^4 + 4*I*c^4*f^5*x^3 - 6*c^
3*f^5*x^2 - 4*I*c^2*f^5*x + c*f^5) + 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*d/(3*I
*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*I*c^2*f^4*x + 3*c*f^4) - 10*I*sqrt(c^2*d*
f*x^2 + d*f)*d^2/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 105*I*sqrt(c^2*d*
f*x^2 + d*f)*d^2/(-3*I*c^2*f^3*x + 3*c*f^3) - 15*d^3*arcsinh(c*x)/(c*f^3*s
qrt(d/f)))*a + b*integrate((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(-I*c*f*x + f)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d+c\,d\,x\,1\text{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d}\,d^2\left(-30\sqrt{cix+1}\,\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)acx-30\sqrt{cix+1}\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x
 + 1)/sqrt(2))*a*c*x - 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( -
c*i*x + 1)/sqrt(2))*a*i + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c
*i*x + 1)*asinh(c*x)*x**2)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x
 + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**4*i*x - 3*sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x**2)/(sqrt( - c*i*x + 1)*c**
2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**3 + 6*sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt(
 - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1))
,x)*b*c**3*x + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*a
sinh(c*x)*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x -
sqrt( - c*i*x + 1)),x)*b*c**2*i - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int
((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*
i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**2*i*x + 3*sqrt(c*i*x + 1)*sqr
t( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c**2*
x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c - 3*a*c**3*
x**3 - 31*a*c**2*i*x**2 - 11*a*c*x - 23*a*i))/(3*sqrt(f)*sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c*f**2*(c*i*x - 1))
```

3.232 $\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 371

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{4ibd^2\sqrt{1+c^2x^2}}{3cf^2(i+cx)\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{bd^2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^2}{2cf^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+icx)(a+b\operatorname{arcsinh}(cx))}{cf^2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2id^2(1+icx)^3(a+b\operatorname{arcsinh}(cx))}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{d^2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)(a+b\operatorname{arcsinh}(cx))}{cf^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{8bd^2\sqrt{1+c^2x^2}\log(i+cx)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
4/3*I*b*d^2*(c^2*x^2+1)^(1/2)/c/f^2/(I+c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-1/2*b*d^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2/c/f^2/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)+2*I*d^2*(1+I*c*x)*(a+b*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1
/2)/(f-I*c*f*x)^(1/2)-2/3*I*d^2*(1+I*c*x)^3*(a+b*arcsinh(c*x))/c/f^2/(d+I*
c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+d^2*(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*(a+b*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*b*d^2
*(c^2*x^2+1)^(1/2)*ln(I+c*x)/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.75 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.90

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x
]
```

output
```
((-16*a*d*(I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^3*(I + c*x)^
2) + (12*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/f^(5/2) - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cos
h[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(A
rcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*Log[1 + c^2*x^2])) +
Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]
] + ((3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c*x] + (4*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2*x^2]*((2*I)*ArcS
inh[c*x] + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))/2)*Sinh
[ArcSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^4) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]
/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh
[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 7*Log[1 + c^2*
x^2]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 -
(84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) - (2*I)*(4 + (4
*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
+ 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[
c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[Ar
cSinh[c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*
x]/2])^4))/(12*c)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4(c^2x^2+1)^{5/2}\int \frac{(icx+1)^4(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d^4(c^2x^2+1)^{5/2}\left(-bc\int \left(-\frac{2i(icx+1)^3}{3c(c^2x^2+1)^2}+\frac{2i(icx+1)}{c(c^2x^2+1)}+\frac{\text{arcsinh}(cx)}{c\sqrt{c^2x^2+1}}\right)dx-\frac{2i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{2i(1+icx)(a+b\text{arc}}{c\sqrt{c^2x^2+}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^4(c^2x^2+1)^{5/2}\left(-\frac{2i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}+\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{c\sqrt{c^2x^2+1}}+\frac{\text{arcsinh}(cx)(a+b\text{arcsinh}(cx))}{c}-bc\left(\frac{\text{arcsin}}{2c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*(1 + I*c*x)^3*(a + b*ArcSinh[c*x]))/
(c*(1 + c^2*x^2)^(3/2)) + ((2*I)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c*Sqrt
[1 + c^2*x^2]) + (ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/c - b*c*(((-4*I)/3)/(
c^2*(I + c*x)) + ArcSinh[c*x]^2/(2*c^2) - (8*Log[I + c*x])/(3*c^2))))/((d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 10.98 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.15

method	result
default	$\frac{d\left(16a+8b+8ib\,c^3x^3+8ibxc+8i\sqrt{c^2x^2+1}\ \text{arcsinh}(xc)b+24i\ \text{arcsinh}(xc)\sqrt{c^2x^2+1}\,b\,c^2x^2-16a\,c^3x^3\sqrt{c^2x^2+1}+6\ \text{arcsinh}(xc)^2b\,c^2x^2\right.}{}$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*d*(16*a+8*b-16*a*c^3*x^3*(c^2*x^2+1)^(1/2)+6*arcsinh(x*c)^2*b*c^2*x^2+
12*arcsinh(x*c)*a*c^2*x^2-32*arcsinh(x*c)*b*c^2*x^2+8*b*c^2*x^2-16*b*c^3*x
^3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+64*ln(x*c+(c^2*x^2+1)^(1/2)+I)*b*c^2*x^2
+32*ln(x*c+(c^2*x^2+1)^(1/2)+I)*b*c^4*x^4+3*arcsinh(x*c)^2*b*c^4*x^4+6*arc
sinh(x*c)*a*c^4*x^4-16*arcsinh(x*c)*b*c^4*x^4-16*b*arcsinh(x*c)+24*I*arcsi
nh(x*c)*(c^2*x^2+1)^(1/2)*b*c^2*x^2+32*a*c^2*x^2+8*I*(c^2*x^2+1)^(1/2)*arc
sinh(x*c)*b+16*a*c^4*x^4+8*I*b*c^3*x^3+8*I*b*x*c+24*I*(c^2*x^2+1)^(1/2)*a*
c^2*x^2+3*b*arcsinh(x*c)^2+6*arcsinh(x*c)*a+8*I*(c^2*x^2+1)^(1/2)*a+32*b*l
n(x*c+(c^2*x^2+1)^(1/2)+I))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*
x^2+1)^(1/2)/c/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/f^3
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\mathrm{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2)
, x)
```

Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*a*(3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(3*I*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9
*I*c^2*f^4*x + 3*c*f^4) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*d/(c^3*f^3*x^2 + 2*I
*c^2*f^3*x - c*f^3) - 21*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-3*I*c^2*f^3*x + 3*c
*f^3) - 3*d^2*arcsinh(c*x)/(c*f^3*sqrt(d/f))) + b*integrate((I*c*d*x + d)^
(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d}\,d\left(-6\sqrt{cix+1}\,\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)acx-6\sqrt{cix+1}\,\sqrt{}\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

output
```
(sqrt(d)*d*( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1
)/sqrt(2))*a*c*x - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x
 + 1)/sqrt(2))*a*i + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x
+ 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*
i*x - sqrt( - c*i*x + 1)),x)*b*c**3*x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x +
1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sq
rt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**2*i - 3*sqrt(c*i*x +
1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)
*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c**2*i*
x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/
(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x
 + 1)),x)*b*c - 8*a*c**2*i*x**2 - 4*a*c*x - 4*a*i))/(3*sqrt(f)*sqrt(c*i*x
+ 1)*sqrt( - c*i*x + 1)*c*f**2*(c*i*x - 1))
```

3.233 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 190

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{2ibd\sqrt{1+c^2x^2}}{3cf^2(i+cx)\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{id(1+icx)^3(a+b\text{arcsinh}(cx))}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{bd\sqrt{1+c^2x^2}\log(i+cx)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
2/3*I*b*d*(c^2*x^2+1)^(1/2)/c/f^2/(I+c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)-1/3*I*d*(1+I*c*x)^3*(a+b*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*
f*x)^(1/2)/(c^2*x^2+1)+1/3*b*d*(c^2*x^2+1)^(1/2)*ln(I+c*x)/c/f^2/(d+I*c*d*
x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = -\frac{id\sqrt{f-icfx}\left((-i+cx)\left(-ia+acx+b\sqrt{1+c^2x^2}\right)+b(-i+cx)^2\text{arcsinh}(cx)-b(i+cx)\sqrt{1+c^2x^2}\log(\right.}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
((-1/3*I)*d*Sqrt[f - I*c*f*x]*((-I + c*x)*((-I)*a + a*c*x + b*Sqrt[1 + c^2
*x^2]) + b*(-I + c*x)^2*ArcSinh[c*x] - b*(I + c*x)*Sqrt[1 + c^2*x^2]*Log[d
*(-1 + I*c*x)]))/(c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6211, 27, 6252, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(-bc\int -\frac{i(icx+1)^3}{3c\left(c^2x^2+1\right)^2}dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c\left(c^2x^2+1\right)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(\frac{1}{3}ib\int \frac{(icx+1)^3}{\left(c^2x^2+1\right)^2}dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c\left(c^2x^2+1\right)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{456}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(\frac{1}{3}ib\int\frac{icx+1}{(1-icx)^2}dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(\frac{1}{3}ib\int\left(-\frac{i}{cx+i}-\frac{2}{(cx+i)^2}\right)dx-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(\frac{1}{3}ib\left(\frac{2}{c(cx+i)}-\frac{i\log(cx+i)}{c}\right)-\frac{i(1+icx)^3(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(5/2)*(((-1/3*I)*(1 + I*c*x)^3*(a + b*ArcSinh[c*x]))/(c
*(1 + c^2*x^2)^(3/2)) + (I/3)*b*(2/(c*(I + c*x)) - (I*Log[I + c*x])/c)))/(
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(157) = 314$.

Time = 7.76 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.29

method	result
default	$a\left(-\frac{i\sqrt{icdx+d}}{cf(-icfx+f)^{\frac{3}{2}}}-d\left(-\frac{i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}\right)\right)-\frac{b\left(i\ln\left(xc+\sqrt{c^{2}x^{2}+1}+i\right)x^{3}c^{3}-i\sqrt{c^{2}x^{2}+1}\,\ln\left(\right.\right.}{}$
parts	$a\left(-\frac{i\sqrt{icdx+d}}{cf(-icfx+f)^{\frac{3}{2}}}-d\left(-\frac{i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}\right)\right)-\frac{b\left(i\ln\left(xc+\sqrt{c^{2}x^{2}+1}+i\right)x^{3}c^{3}-i\sqrt{c^{2}x^{2}+1}\,\ln\left(\right.\right.}{}$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*arcsinh(x*c))/(f-I*c*f*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(-I/c/f*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2)-d*(-1/3*I/d/c/f/(f-I*c*f*x)^
(3/2)*(d+I*c*d*x)^(1/2)-1/3*I/c/d/f^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2))
)-1/3*b*(I*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^3*c^3-I*(c^2*x^2+1)^(1/2)*ln(x*c+
(c^2*x^2+1)^(1/2)+I)*x^2*c^2+3*arcsinh(x*c)*c^2*x^2-3*ln(x*c+(c^2*x^2+1)^(
1/2)+I)*x^2*c^2-3*I*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x*c-c^2*x^2-I*(c^2*x^2+1)^
(1/2)*ln(x*c+(c^2*x^2+1)^(1/2)+I)-2*I*x*c+(c^2*x^2+1)^(1/2)*x*c-I*(c^2*x^2
+1)^(1/2)-arcsinh(x*c)+ln(x*c+(c^2*x^2+1)^(1/2)+I)+1)*(x^3*c^3-3*I*c^2*x^2
+x^2*c^2*(c^2*x^2+1)^(1/2)-3*x*c+I+(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/2)
*(I*(x*c-I)*d)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)^2/c
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(146) = 292$.

Time = 0.16 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.88

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx =$$

$$-\frac{4\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx+2\left(bc^2x^2-2i\,bcx-b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 2*(
b*c^2*x^2 - 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x + I*c*f^3)*s
qrt(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*sqrt
(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*f^3*x^4 - 2*c^
8*f^3*x^3 + I*c^7*f^3*x^2 - 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(b*c^3*x^3
 + I*b*c^2*x^2 + b*c*x + I*b)) - (c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2*f^3*x
+ I*c*f^3)*sqrt(b^2*d/(c^2*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I
*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (-I*c^9*f
^3*x^4 + 2*c^8*f^3*x^3 - I*c^7*f^3*x^2 + 2*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)
))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + 2*(a*c^2*x^2 - 2*I*a*c*x - a
)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^4*f^3*x^3 + I*c^3*f^3*x^2 + c^2
*f^3*x + I*c*f^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{id\,(cx-i)}(a+b\operatorname{asinh}(cx))}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(5/2), x
)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.16

$$\begin{aligned}&\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx=\\&-\frac{1}{3}\,bc\left(\frac{6\,\sqrt{d}}{3i\,c^3f^{\frac{5}{2}}x-3\,c^2f^{\frac{5}{2}}}-\frac{\sqrt{d}\log\left(cx+i\right)}{c^2f^{\frac{5}{2}}}\right)\\&-\frac{1}{3}\,b\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)\operatorname{arsinh}\left(cx\right)\\&-\frac{1}{3}\,a\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{3i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)\end{aligned}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*b*c*(6*sqrt(d)/(3*I*c^3*f^(5/2)*x - 3*c^2*f^(5/2)) - sqrt(d)*log(c*x
+ I)/(c^2*f^(5/2))) - 1/3*b*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2
*I*c^2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*
f^3))*arcsinh(c*x) - 1/3*a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 + 2*
I*c^2*f^3*x - c*f^3) - 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f
^3))
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d}\left(-3\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{\sqrt{cix+1}\,asinh(cx)}{\sqrt{-cix+1}\,c^2x^2+2\sqrt{-cix+1}\,cix-\sqrt{-cix+1}}dx\right)\right.}{3\sqrt{f}}$$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*asinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

output
```
(sqrt(d)*( - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asi
nh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt
( - c*i*x + 1)),x)*b*c**2*i*x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((
sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*
x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b*c - a*c**2*i*x**2 - 2*a*c*x + a*i)
)/(3*sqrt(f)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*f**2*(c*i*x - 1))
```

3.234 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 294

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}} dx = \frac{ibd^2(1+c^2x^2)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)(1+c^2x^2)(a+b\text{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)^2(a+b\text{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibd^2(1+c^2x^2)^{5/2}\arctan(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2(1+c^2x^2)^{5/2}\log(1+c^2x^2)}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
1/3*I*b*d^2*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)-2/3*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+1/3*I*b*d^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*d^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d
+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\frac{\sqrt{f-icfx}\left((2i+cx)\left(a+iacx+ib\sqrt{1+c^2x^2}\right)+ib(2+icx+c^2x^2)\,\text{arcs}\right.}{3cf^3(i+cx)^2\sqrt{d+icdx}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

output
```
(Sqrt[f - I*c*f*x]*((2*I + c*x)*(a + I*a*c*x + I*b*Sqrt[1 + c^2*x^2]) + I*
b*(2 + I*c*x + c^2*x^2)*ArcSinh[c*x] + b*(1 - I*c*x)*Sqrt[1 + c^2*x^2]*Log
[d*(-1 + I*c*x)]))/(3*c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 6252$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\left(-bc\int\left(\frac{x}{3(c^2x^2+1)}-\frac{2i(icx+1)}{3c(c^2x^2+1)^2}\right)dx+\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{2i(1+icx)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(-\frac{i\arctan(cx)}{3c^2}+\frac{i(-cx+i)}{3c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{6c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d^2*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*(1 + I*c*x)*(a + b*ArcSinh[c*x]))/(c
*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b
*c*(((I/3)*(I - c*x))/(c^2*(1 + c^2*x^2)) - ((I/3)*ArcTan[c*x])/c^2 + Log[
1 + c^2*x^2]/(6*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.01

method	result
default	$a\left(-\frac{i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}\right)+\frac{b\left(\operatorname{arcsinh}(xc)c^{4}x^{4}-2\ln\left(xc+\sqrt{c^{2}x^{2}+1}+i\right)x^{4}c^{4}+\operatorname{arcsinh}(xc)\sqrt{c^{2}x^{2}+1}\,x^{3}c^{3}\right.}{}$
parts	$a\left(-\frac{i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}\right)+\frac{b\left(\operatorname{arcsinh}(xc)c^{4}x^{4}-2\ln\left(xc+\sqrt{c^{2}x^{2}+1}+i\right)x^{4}c^{4}+\operatorname{arcsinh}(xc)\sqrt{c^{2}x^{2}+1}\,x^{3}c^{3}\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(-1/3*I/d/c/f/(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-1/3*I/c/d/f^2/(f-I*c*f
*x)^(1/2)*(d+I*c*d*x)^(1/2))+1/3*b*(arcsinh(x*c)*c^4*x^4-2*ln(x*c+(c^2*x^2
+1)^(1/2)+I)*x^4*c^4+arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3+I*x^3*c^3+2*ar
csinh(x*c)*c^2*x^2-4*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^2*c^2+3*arcsinh(x*c)*(c
^2*x^2+1)^(1/2)*x*c+c^2*x^2-2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)+I*x*c+arcsi
nh(x*c)-2*ln(x*c+(c^2*x^2+1)^(1/2)+I)+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f
)^(1/2)/(c^2*x^2+1)^(5/2)/f^3/c/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(228) = 456$.

Time = 0.20 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.96

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx - 2\left(bc^2x^2+i\,bcx+2\,b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right) \cdots}{\cdots}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 2*(
b*c^2*x^2 + I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) - (c^4*d*f^3*x^3 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*
d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c^6*x^2 + 2*b*c^5*x + 2*I*b*c
^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + (I*c^9*d*f^3*
x^4 - 2*c^8*d*f^3*x^3 + I*c^7*d*f^3*x^2 - 2*c^6*d*f^3*x)*sqrt(b^2/(c^2*d*f
^5)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)) + (c^4*d*f^3*x^3 + I*c^3*d*
f^3*x^2 + c^2*d*f^3*x + I*c*d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(-1/8*((-I*b*c
^6*x^2 + 2*b*c^5*x + 2*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f) + (-I*c^9*d*f^3*x^4 + 2*c^8*d*f^3*x^3 - I*c^7*d*f^3*x^2 + 2*c
^6*d*f^3*x)*sqrt(b^2/(c^2*d*f^5)))/(b*c^3*x^3 + I*b*c^2*x^2 + b*c*x + I*b)
) - 2*(a*c^2*x^2 + I*a*c*x + 2*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c
^4*d*f^3*x^3 + I*c^3*d*f^3*x^2 + c^2*d*f^3*x + I*c*d*f^3)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{\sqrt{id\,(cx - i)}\,(-if\,(cx + i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + icdx}(f - icfx)^{5/2}}\,dx &= -\frac{1}{3}\,bc\left(\frac{3}{3i\,c^3\sqrt{d}f^{\frac{5}{2}}x - 3\,c^2\sqrt{d}f^{\frac{5}{2}}} + \frac{\log\,(cx + i)}{c^2\sqrt{d}f^{\frac{5}{2}}}\right)\\ &\quad - \frac{1}{3}\,b\left(-\frac{i\,\sqrt{c^2dfx^2 + df}}{c^3df^3x^2 + 2i\,c^2df^3x - cdf^3} + \frac{3i\,\sqrt{c^2dfx^2 + df}}{-3i\,c^2df^3x + 3\,cdf^3}\right)\text{arsinh}\,(cx)\\ &\quad - \frac{1}{3}\,a\left(-\frac{i\,\sqrt{c^2dfx^2 + df}}{c^3df^3x^2 + 2i\,c^2df^3x - cdf^3} + \frac{3i\,\sqrt{c^2dfx^2 + df}}{-3i\,c^2df^3x + 3\,cdf^3}\right)\end{aligned}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*b*c*(3/(3*I*c^3*sqrt(d)*f^(5/2)*x - 3*c^2*sqrt(d)*f^(5/2)) + log(c*x
+ I)/(c^2*sqrt(d)*f^(5/2))) - 1/3*b*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d*f^3
*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*
d*f^3*x + 3*c*d*f^3))*arcsinh(c*x) - 1/3*a*(-I*sqrt(c^2*d*f*x^2 + d*f)/(c^
3*d*f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(-3
*I*c^2*d*f^3*x + 3*c*d*f^3))
```

Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\frac{-3\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+2\sqrt{cix+1}\,\sqrt{-cix+1}\,cix-\sqrt{cix+1}\,\sqrt{-}\right.}{}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

output

```
( - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x -
 sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c**3*x**2 - 3*sqrt(c*i*x + 1)*sq
rt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x
**2 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)),x)*b*c + a*c**3*x**3 + 3*a*c*x - 2*a*i)/(3*sqrt(f)*sqrt(d)*sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*f**2*(c**2*x**2 + 1))
```

3.235 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 298

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{ib\sqrt{1+c^2x^2}}{6cdf^2(i+cx)\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2x(a+b\operatorname{arcsinh}(cx))}{3df^2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{(i-cx)(a+b\operatorname{arcsinh}(cx))}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{ib\sqrt{1+c^2x^2}\arctan(cx)}{6cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}}$$

output

```
1/6*I*b*(c^2*x^2+1)^(1/2)/c/d/f^2/(I+c*x)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+2/3*x*(a+b*arcsinh(c*x))/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3
*(I-c*x)*(a+b*arcsinh(c*x))/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c
^2*x^2+1)+1/6*I*b*(c^2*x^2+1)^(1/2)*arctan(c*x)/c/d/f^2/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)-1/3*b*(c^2*x^2+1)^(1/2)*ln(c^2*x^2+1)/c/d/f^2/(d+I*c*d*x
)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.68

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{f-icfx}\left(4ia-8acx+8iac^2x^2-2b\sqrt{1+c^2x^2}+4ib(1+2icx+2c^2\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x
]
```

output
```
(Sqrt[f - I*c*f*x]*((4*I)*a - 8*a*c*x + (8*I)*a*c^2*x^2 - 2*b*Sqrt[1 + c^2
*x^2] + (4*I)*b*(1 + (2*I)*c*x + 2*c^2*x^2)*ArcSinh[c*x] + 5*b*(1 - I*c*x)
*Sqrt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] + 3*b*Sqrt[1 + c^2*x^2]*Log[d + I*c
*d*x] - (3*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*c*d*f^3*(I +
c*x)^2*Sqrt[d + I*c*d*x])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6211, 27, 6252, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6252}$$

$$\frac{d(c^2x^2+1)^{5/2}\left(-bc\int\left(\frac{2x}{3(c^2x^2+1)}-\frac{i-cx}{3c(c^2x^2+1)^2}\right)dx+\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d(c^2x^2+1)^{5/2}\left(\frac{2x(a+b\text{arcsinh}(cx))}{3\sqrt{c^2x^2+1}}-\frac{(-cx+i)(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)^{3/2}}-bc\left(-\frac{i\arctan(cx)}{6c^2}-\frac{1+icx}{6c^2(c^2x^2+1)}+\frac{\log(c^2x^2+1)}{3c^2}\right)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(5/2)*(-1/3*((I - c*x)*(a + b*ArcSinh[c*x]))/(c*(1 + c^2*
x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - b*c*(-1/6
*(1 + I*c*x)/(c^2*(1 + c^2*x^2)) - ((I/6)*ArcTan[c*x])/c^2 + Log[1 + c^2*x
^2]/(3*c^2))))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6252
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Simp[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[1/Sqrt[1 + c^2*x^
2]   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IGtQ
[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Maple [A] (verified)

Time = 6.99 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.47

method	result
default	$a\left(\frac{i}{cdf\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}+\frac{-\frac{2i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{2i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}}{d}\right)+\frac{b\left(4\,\text{arcsinh}(xc)c^{4}x^{4}-5\ln\left(xc+\sqrt{c^{2}x^{2}+1}+i\right)x^{4}c^{4}-\right.}{}$
parts	$a\left(\frac{i}{cdf\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}+\frac{-\frac{2i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{2i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}}{d}\right)+\frac{b\left(4\,\text{arcsinh}(xc)c^{4}x^{4}-5\ln\left(xc+\sqrt{c^{2}x^{2}+1}+i\right)x^{4}c^{4}-\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(I/c/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2)+2/d*(-1/3*I/d/c/f/(f-I*c*f*
x)^(3/2)*(d+I*c*d*x)^(1/2)-1/3*I/c/d/f^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/
2)))+1/6*b*(4*arcsinh(x*c)*c^4*x^4-5*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^4*c^4-3
*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^4*c^4+4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*
c^3+I*x*c+8*arcsinh(x*c)*c^2*x^2-10*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^2*c^2-6*
ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^2*c^2+6*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c+c
^2*x^2-2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)+I*x^3*c^3+4*arcsinh(x*c)-5*ln(x*
c+(c^2*x^2+1)^(1/2)+I)-3*ln(x*c+(c^2*x^2+1)^(1/2)-I)+1)*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)/f^3/d^2/c/(c^6*x^6+3*c^4*x^4+3*c^
2*x^2+1)
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/24*(4*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 8*
(2*b*c^2*x^2 + 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) - 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f
^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c
^2*x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) + 5*(c^4*d^2*f^3*
x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f
^5))*log(-(I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^
2*x*sqrt(b^2/(c^2*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^
2 + b*c*x + I*b)) + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x
 + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + I*b*c^2*
x^3 + I*b*x)/(b*c^3*x^3 - I*b*c^2*x^2 + b*c*x - I*b)) - 5*(c^4*d^2*f^3*x^3
 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5)
)*log(-(-I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*
x*sqrt(b^2/(c^2*d^3*f^5)) - I*b*c^2*x^3 - I*b*x)/(b*c^3*x^3 + I*b*c^2*x^2
+ b*c*x + I*b)) + 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x +
 I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + b*c^2*x^3 + b*
x)/(b*c^2*x^2 + b)) - 8*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.80

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{1}{12}\,bc\left(\frac{2i\,\sqrt{d}\sqrt{f}}{c^3d^2f^3x+i\,c^2d^2f^3}-\frac{5\,\log\left(cx+i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}-\frac{3\,\log\left(cx-i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}\right) - \frac{1}{3}\,b\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,df^2}\right)\operatorname{arsinh}\left(cx\right) - \frac{1}{3}\,a\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{c^2dfx^2+df}\,df^2}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
1/12*b*c*(2*I*sqrt(d)*sqrt(f)/(c^3*d^2*f^3*x + I*c^2*d^2*f^3) - 5*log(c*x
+ I)/(c^2*d^(3/2)*f^(5/2)) - 3*log(c*x - I)/(c^2*d^(3/2)*f^(5/2))) - 1/3*b
*(3*I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f
)*c*d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))*arcsinh(c*x) - 1/3*a*(3*
I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*
d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{(d+c\,d\,x\,1\mathrm{i})^{3/2}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{-3\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^3i\,x^3-\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+}}\right.}{}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
( - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**3*i*x**3 - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x*
*2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)),x)*b*c**3*x**2 - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(
c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**3*i*x**3 - sqrt(c*i*x + 1)*sqr
t( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqr
t(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c + 2*a*c**3*x**3 + 3*a*c*x - a*i)/(
3*sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d*f**2*(c**2*x**2 +
 1))
```

3.236 $\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 218

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{b}{6cd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}} + \frac{2x(a+b\text{arcsinh}(cx))}{3d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{x(a+b\text{arcsinh}(cx))}{3d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} - \frac{b\sqrt{1+c^2x^2}\log(1+c^2x^2)}{3cd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
1/6*b/c/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*
x*(a+b*arcsinh(c*x))/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*x*(a+
b*arcsinh(c*x))/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)-1/
3*b*(c^2*x^2+1)^(1/2)*ln(c^2*x^2+1)/c/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{i\sqrt{f-icfx}\left(6acx+4ac^3x^3+b\sqrt{1+c^2x^2}+2bcx(3+2c^2x^2)\text{arcsinh}(c\right.}{6c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x
]
```

output
```
((I/6)*Sqrt[f - I*c*f*x]*(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*
b*c*x*(3 + 2*c^2*x^2)*ArcSinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[d*(-1 + I
*c*x)] - 2*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x] - 2*b*c^2*x^2*Sqrt[1 + c^2
*x^2]*Log[d + I*c*d*x]))/(c*d^2*f^3*(-I + c*x)*(I + c*x)^2*Sqrt[d + I*c*d*
x])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6211, 6203, 241, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6203}$$

$$\frac{(c^2x^2+1)^{5/2}\left(\frac{2}{3}\int \frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx-\frac{1}{3}bc\int \frac{x}{(c^2x^2+1)^2}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 241$$

$$\frac{(c^2x^2+1)^{5/2}\left(\frac{2}{3}\int\frac{a+b\text{arcsinh}(cx)}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 6202$$

$$\frac{(c^2x^2+1)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-bc\int\frac{x}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{b}{6c(c^2x^2+1)}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 240$$

$$\frac{(c^2x^2+1)^{5/2}\left(\frac{x(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}-\frac{b\log(c^2x^2+1)}{2c}\right)+\frac{b}{6c(c^2x^2+1)}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
((1 + c^2*x^2)^(5/2)*(b/(6*c*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*
(1 + c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b
*Log[1 + c^2*x^2])/(2*c)))/3))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(182) = 364$.

Time = 6.45 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.46

method	result
default	$a\left(\frac{i}{3cdf(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}+\frac{\frac{i}{cdf\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}+\frac{-\frac{2i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{2i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}}{d}}{d}\right)+\frac{b\left(-8\ln\left(1+\left(xc+\sqrt{c^{\cdots}}\right.\right.\right.}{}$
parts	$a\left(\frac{i}{3cdf(icdx+d)^{\frac{3}{2}}(-icfx+f)^{\frac{3}{2}}}+\frac{\frac{i}{cdf\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}+\frac{-\frac{2i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{2i\sqrt{icdx+d}}{3cd\,f^{2}\sqrt{-icfx+f}}}{d}}{d}\right)+\frac{b\left(-8\ln\left(1+\left(xc+\sqrt{c^{\cdots}}\right.\right.\right.}{}$

input
```
int((a+b*arcsinh(x*c))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
a*(1/3*I/c/d/f/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/d*(I/c/d/f/(d+I*c*d*x
)^(1/2)/(f-I*c*f*x)^(3/2)+2/d*(-1/3*I/d/c/f/(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^
(1/2)-1/3*I/c/d/f^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2))))+1/6*b*(-8*ln(1+
(x*c+(c^2*x^2+1)^(1/2))^2)*x^6*c^6+8*(c^2*x^2+1)^(1/2)*ln(1+(x*c+(c^2*x^2+
1)^(1/2))^2)*x^5*c^5-24*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^4*c^4+20*(c^2*x^
2+1)^(1/2)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^3*c^3+2*c^4*x^4-2*(c^2*x^2+1)
^(1/2)*c^3*x^3+6*arcsinh(x*c)*c^2*x^2-24*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x
^2*c^2+12*(c^2*x^2+1)^(1/2)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x*c+4*c^2*x^2-
3*(c^2*x^2+1)^(1/2)*x*c+8*arcsinh(x*c)-8*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)+2
)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+3*x*c+2*(c^2*x^2+1)^(1/2))*(I*(x*
c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d^3/f^3/c/(c^2*x^2+1)^2/(3*c^2*x^2+4)
```

Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{b\,\operatorname{arsinh}\left(cx\right)+a}{\left(i\,cdx+d\right)^{\frac{5}{2}}\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algori
thm="fricas")
```

output
```
-1/6*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x^2 - 2*(
2*b*c^2*x^3 + 3*b*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c
^2*x^2 + 1)) - (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c
^2*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c
*d^2*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^4 + b*x^2)/(b*c^4*x^4 + 2*b
*c^2*x^2 + b)) + (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/
(c^2*d^5*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*c*d^2*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^4 - b*x^2)/(b*c^4*x^4 +
2*b*c^2*x^2 + b)) + 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt
(b^2/(c^2*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f)*c*d^2*f^2*x*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 +
b)) - 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d^5*
f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f
^2*x*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 2*(2*a*
c^2*x^3 + 3*a*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 6*(c^4*d^3*f^3*x^4
 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*integral(-2/3*sqrt(c^2*x^2 + 1)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f)*b*c*x/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^
3*f^3), x))/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.73

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\,dx = \frac{1}{6}\,bc\left(\frac{1}{c^4 d^{\frac{5}{2}} f^{\frac{5}{2}} x^2 + c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}} - \frac{2\,\log\left(c^2x^2+1\right)}{c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}} df} + \frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)\text{arsinh}\left(cx\right) + \frac{1}{3}\,a\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}} df} + \frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algori
thm="maxima")
```

output
```
1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^2
 + 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f) +
2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a*(x/((c^2*d*f*x
^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + icdx)^{5/2}(f - icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{(d+c\,d\,x\,1{}\mathrm{i})}^{5/2}\,{(f-c\,f\,x\,1{}\mathrm{i})}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\frac{3\sqrt{cix+1}\,\sqrt{-cix+1}\,\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^4x^4+2\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+1}\,\sqrt{}}\right.}{}$$

input
```
int((a+b*asinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
(3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**4*x**4 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2
+ sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*c**2*x**2 + 3*sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**4*
x**4 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sq
rt( - c*i*x + 1)),x)*b + 2*a*c**2*x**3 + 3*a*x)/(3*sqrt(f)*sqrt(d)*sqrt(c*
i*x + 1)*sqrt( - c*i*x + 1)*d**2*f**2*(c**2*x**2 + 1))
```

3.237 $\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\,\mathbf{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 680

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} + \frac{4ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} - \frac{15b^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} - \frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{4ibc^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} + \frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 + \frac{2id^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{3c} + \frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}$$

output
```
8/9*I*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d^2*x*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)+4/27*I*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1
)/c-15/64*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsinh(c*x)/c/(c^2*
x^2+1)^(1/2)-4/3*I*b*d^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsin
h(c*x))/(c^2*x^2+1)^(1/2)-3/8*b*c*d^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1
/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-4/9*I*b*c^2*d^2*x^3*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+1/8*b*c^3*d^2*
x^4*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/
2)+3/8*d^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2-1/4*
c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+2/3*I
*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/
c+5/24*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c
^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.31

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-6912iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+4608ia^2d^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+6}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((-6912*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (4608*I)*a^2*
d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (6912*I)*b^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*d^2*
x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (4608*I)*a^2*c^2
*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*
c^3*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b
^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*d^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (256*I)*b^2*d^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*d^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*S
qrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]] + 864*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2
*ArcSinh[c*x]] - (768*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[
3*ArcSinh[c*x]] - 27*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*Ar
cSinh[c*x]] + 12*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((
-576*I)*b*c*x + (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] +
 (192*I)*a*Cosh[3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*
ArcSinh[c*x]] - (64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]])
 + 72*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (48
*I)*b*Sqrt[1 + c^2*x^2] + (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*A...
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d^2(icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\int \left(-c^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+2icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\sqrt{c^2x^2+\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{8}bc^3x^4(a+b\text{arcsinh}(cx))-\frac{4}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx\right.}{}$$

input
```
Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((8*I)/9)*b^2*Sqrt[1 + c^2*x^2]
)/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 - (b^2*c^2*x^3*Sqrt[1 + c^2*x^2])/32
 + (((4*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c - (15*b^2*ArcSinh[c*x])/(64*c) -
 ((4*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8 -
 ((4*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (b*c^3*x^4*(a + b*ArcSinh[c*x]
))/8 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*(1 + c^2*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(24*b*c)))/Sqrt[1 + c^2
*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1971 vs. $2(559) = 1118$.

Time = 8.14 (sec) , antiderivative size = 1972, normalized size of antiderivative = 2.90

method	result	size
default	Expression too large to display	1972
parts	Expression too large to display	1972

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/4*I*a^2/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)+5/12*I*a^2*d/c/f*(d+I*c*
d*x)^(3/2)*(f-I*c*f*x)^(3/2)+5/8*I*a^2*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*
x)^(3/2)-5/8*I*a^2*d^2/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+5/8*a^2*d^3*f
*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^
2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(5/24
*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)
^3*d^2-1/512*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(8*x^5*c^5+8*x^4*c^4
*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1
)^(1/2))*(8*arcsinh(x*c)^2-4*arcsinh(x*c)+1)*d^2/(c^2*x^2+1)/c+1/108*I*(I*
(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x
^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(9*arcsinh(x*c)^2-6*arcsinh(x*c)+2
)*d^2/(c^2*x^2+1)/c+1/16*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(2*x^3*c
^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-
2*arcsinh(x*c)+1)*d^2/(c^2*x^2+1)/c+1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*
f)^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2-2*arcsinh(x*c)+
2)*d^2/(c^2*x^2+1)/c+1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x
^2-(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2+2*arcsinh(x*c)+2)*d^2/(c^2*x^2
+1)/c+1/16*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(
c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2+2*arcsinh(x*c)
+1)*d^2/(c^2*x^2+1)/c+1/108*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*...
```

Fricas [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\int (i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral(-(b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*d^2*x^2 - 2
*I*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) - (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-6\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^3x^3+16\sqrt{cix+1}\,\sqrt{-ci\cdots}\right.}{\cdots}$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - 6*sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c**3*x**3 + 16*sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)*a**2*c**2*i*x**2 + 9*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c
*x + 16*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i - 48*int(sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*asinh(c*x)*x**2,x)*a*b*c**3 + 96*int(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*asinh(c*x)*x,x)*a*b*c**2*i + 48*int(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*asinh(c*x),x)*a*b*c - 24*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)**2*x**2,x)*b**2*c**3 + 48*int(sqrt(c*i*x + 1)*sqrt( - c*i*
x + 1)*asinh(c*x)**2*x,x)*b**2*c**2*i + 24*int(sqrt(c*i*x + 1)*sqrt( - c*i
*x + 1)*asinh(c*x)**2,x)*b**2*c))/(24*c)
```

3.238 $\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 508

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx}+\frac{2ib^2d\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c}-\frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}-\frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2+\frac{id\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c}+\frac{d\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}$$

output
```
4/9*I*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*d*x*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)+2/27*I*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(
c^2*x^2+1)/c-1/4*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsinh(c*x)/c/
(c^2*x^2+1)^(1/2)-2/3*I*b*d*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arc
sinh(c*x))/(c^2*x^2+1)^(1/2)-1/2*b*c*d*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(
1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d*x^3*(d+I*c*d*x)^(1
/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+1/2*d*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+1/3*I*d*(d+I*c*d*x)^(1/2
)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/6*d*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.39

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \frac{-108iabcdx\sqrt{d+icdx}\sqrt{f-icfx}+72ia^2d\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+108ib^2\cdots}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((-108*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (72*I)*a^2*d*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (108*I)*b^2*d*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*d*x*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*d*x^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*d*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*Cosh[3*ArcSinh[c*x]] + 108*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c
*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*d*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a + (3*I)*b*Sqrt[1 + c^2*x^2]
 + I*b*Cosh[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*d*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*d*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((-9*I
)*b*c*x + (9*I)*a*Sqrt[1 + c^2*x^2] + (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*S
inh[2*ArcSinh[c*x]] - I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2]
)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int d(icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int (icx+1)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\int \left(icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(-\frac{2}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsin}}{3c}\right.}{}$$

input
```
Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((4*I)/9)*b^2*Sqrt[1 + c^2*x^2])/
c + (b^2*x*Sqrt[1 + c^2*x^2])/4 + (((2*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c -
 (b^2*ArcSinh[c*x])/(4*c) - ((2*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (b*c*x^2*
(a + b*ArcSinh[c*x]))/2 - ((2*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (x*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 + ((I/3)*(1 + c^2*x^2)^(3/2)*(a
+ b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(6*b*c)))/Sqrt[1 + c^2*x^2
]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(413) = 826$.

Time = 7.92 (sec) , antiderivative size = 1403, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	1403
parts	Expression too large to display	1403

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/3*I*a^2/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+1/2*I*a^2*d/c/f*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(3/2)-1/2*I*a^2*d/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1
/2)+1/2*a^2*d^2*f*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c
*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f
)^(1/2)+b^2*(1/6*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2
)/c*arcsinh(x*c)^3*d+1/216*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c
^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(9
*arcsinh(x*c)^2-6*arcsinh(x*c)+2)*d/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^
2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)*d/(c^2*x^2+1)/c+1/8*I*(I*(
x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(ar
csinh(x*c)^2-2*arcsinh(x*c)+2)*d/(c^2*x^2+1)/c+1/8*I*(I*(x*c-I)*d)^(1/2)*(
-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2+2*ar
csinh(x*c)+2)*d/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2
)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsi
nh(x*c)^2+2*arcsinh(x*c)+1)*d/(c^2*x^2+1)/c+1/216*I*(I*(x*c-I)*d)^(1/2)*(-
I*(I+x*c)*f)^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2
*x^2+1)^(1/2)*x*c+1)*(9*arcsinh(x*c)^2+6*arcsinh(x*c)+2)*d/(c^2*x^2+1)/c)+
2*a*b*(1/4*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*ar
csinh(x*c)^2*d+1/72*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x...
```

Fricas [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\int (i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((I*b^2*c*d*x + b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*d*x - a*b*d)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*d*x + a^2*d)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \int (id(cx-i))^{\frac{3}{2}}\sqrt{-if(cx+i)}(a+b\,\text{asinh}\,(cx))^2\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+2\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^2i\,x^2+3\sqrt{cix+1}\,\sqrt{-cix+}\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(f)*sqrt(d)*d*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 2*sqrt(c*i
*x + 1)*sqrt( - c*i*x + 1)*a**2*c**2*i*x**2 + 3*sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)*a**2*c*x + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i + 12*int(s
qrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x,x)*a*b*c**2*i + 12*int(sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*a*b*c + 6*int(sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*asinh(c*x)**2*x,x)*b**2*c**2*i + 6*int(sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*asinh(c*x)**2,x)*b**2*c))/(6*c)
```

3.239 $\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 244

$$\begin{aligned}\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx}-\frac{b^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\&\quad-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\&\quad+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\&\quad+\frac{\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
1/4*b^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b^2*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+1/2*x*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+1/6*(d+I*c*d*x)^(1/2)*(f-
I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.44

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{12a^2cx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+4b^2\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)^3-6ab\sqrt{d+icdx}\sqrt{f-ic}}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(12*a^2*c*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 6*a*b*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1
+ c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]] + 3*b^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - 6*b
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(b*Cosh[2*ArcSinh[c*x]]
- 2*a*Sinh[2*ArcSinh[c*x]]) + 6*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcS
inh[c*x]^2*(2*a + b*Sinh[2*ArcSinh[c*x]]))/(24*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6200}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-bc\int x(a+b\text{arcsinh}(cx))dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{c^2x^2+1}}dx\right)+\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}\right)\right)+\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}\int\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\frac{x\sqrt{c^2x^2+1}}{2c^2}-\frac{\text{arcsinh}(}{2c^3}\right.\right.\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])^2)/2 + (a + b*ArcSinh[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSinh[c*x
]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)))/
Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(198) = 396$.

Time = 6.52 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.60

method	result
default	$\frac{ia^2\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{2cf} - \frac{ia^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c} + \frac{a^2df\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}} + b^2\Bigg(\frac{\sqrt{i(}}{}$
parts	$\frac{ia^2\sqrt{icdx+d}\,(-icfx+f)^{\frac{3}{2}}}{2cf} - \frac{ia^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2c} + \frac{a^2df\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}} + b^2\Bigg(\frac{\sqrt{i(}}{}$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/2*I*a^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)-1/2*I*a^2/c*(f-I*c*f*x)^
(1/2)*(d+I*c*d*x)^(1/2)+1/2*a^2*d*f*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c
*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*
f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/6*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2
)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*
f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(
2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*
(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2
+1)^(1/2))*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)/(c^2*x^2+1)/c)+2*a*b*(1/4*(
I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^2
+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x
^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(-1+2*arcsinh(x*c))/(c^2*x^2+1)/c+1/1
6*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1
)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(1+2*arcsinh(x*c))/(c^2*x^2+1)/c)
```

Fricas [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\int \sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2, x)
```

Sympy [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\int \sqrt{id\,(cx-i)}\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}(cx))^2\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{\sqrt{f}\,\sqrt{d}\left(2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2cx+4\left(\int \sqrt{cix+1}\,\sqrt{-cix+1}\,asinh(cx)\,dx\right)abc\right.}{2c}$$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(f)*sqrt(d)*(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + sqrt(c*i*x +
 1)*sqrt( - c*i*x + 1)*a**2*c*x + 4*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)
*asinh(c*x),x)*a*b*c + 2*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)
**2,x)*b**2*c))/(2*c)
```

3.240 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 214

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = -\frac{2ib^2f(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibfx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output

```
-2*I*b^2*f*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b*f*x*(c^
2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*f*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3
*f*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$
$$= \frac{-3i\sqrt{d+icdx}\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)+6ib\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\right.}{}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
((-3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2
*x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) + (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*(a - I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 +
 c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]])/(3*c*d*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$
$$\Big\downarrow \; 6211$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{f(1-icx)(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$\Big\downarrow \; 27$$

$$\frac{f\sqrt{c^2x^2+1}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{f\sqrt{c^2x^2+1}\int \left(\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f\sqrt{c^2x^2+1}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{(a+b\text{arcsinh}(cx))^3}{3bc}+2iabx+2ib^2x\text{arcsinh}(cx)-\frac{2ib^2\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
(f*Sqrt[1 + c^2*x^2]*((2*I)*a*b*x - ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c + (2*I
)*b^2*x*ArcSinh[c*x] - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (a
 + b*ArcSinh[c*x])^3/(3*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(181) = 362$.

Time = 5.74 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.50

method	result
default	$-\frac{ia^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{cd}+\frac{a^2 f\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}}+b^2\left(\frac{\operatorname{arcsinh}(xc)^3\sqrt{i(xc-i)d}\,\sqrt{-i(xc-}}{3\sqrt{c^2x^2+1}\,dc}\right.$
parts	$-\frac{ia^2\sqrt{-icfx+f}\,\sqrt{icdx+d}}{cd}+\frac{a^2 f\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}}+b^2\left(\frac{\operatorname{arcsinh}(xc)^3\sqrt{i(xc-i)d}\,\sqrt{-i(xc-}}{3\sqrt{c^2x^2+1}\,dc}\right.$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-I*a^2/c/d*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+a^2*f*((f-I*c*f*x)*(d+I*c*d
*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2
)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/3*arcsinh(x*c)^3*(I*(x*c
-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/d/c-1/2*I*(arcsinh(x*c
)^2-2*arcsinh(x*c)+2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-1/2*I*(arcsinh(x*c)^2+2*arcsinh(x*c
)+2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^
(1/2)/d/c/(c^2*x^2+1))+2*a*b*(1/2*arcsinh(x*c)^2*(I*(x*c-I)*d)^(1/2)*(-I*(
I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c/d-1/2*I*(arcsinh(x*c)-1)*(c^2*x^2+(c^2
*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/(c^2*x^2+1
)/d-1/2*I*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)
^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/(c^2*x^2+1)/d)
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2 - 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c
^2*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*d*x - I*d),
x)
```

Sympy [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-if\,(cx+i)}(a+b\operatorname{asinh}(cx))^2}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
a^2*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + i
ntegrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(I*c*d*
x + d) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*
x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\,(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{f-c\,f\,x\,1\text{i}}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$
$$=\frac{\sqrt{f}\left(2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2i+2\left(\int\frac{\sqrt{-cix+1}\,asinh(cx)}{\sqrt{cix+1}}dx\right)abc+\left(\int\frac{\sqrt{-cix+1}\,asinh(cx)^2}{\sqrt{cix+1}}\right.\right.}{\sqrt{d}\,c}$$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

output
```
(sqrt(f)*(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*a**2*i + 2*int((sqrt( - c*i*x + 1)*asinh(c*x))/sqrt(c*i*x +
 1),x)*a*b*c + int((sqrt( - c*i*x + 1)*asinh(c*x)**2)/sqrt(c*i*x + 1),x)*b
**2*c))/(sqrt(d)*c)
```

3.241 $\int \frac{\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 544

$$\int \frac{\sqrt{f-icfx}(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \frac{2if^2(1+c^2x^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x(1+c^2x^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ibf^2(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))\arctan\left(e^{\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4bf^2(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))\log\left(1+e^{2\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4b^2f^2(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4b^2f^2(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b^2f^2(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

output
```
2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)+2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)+2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(
f-I*c*f*x)^(3/2)-1/3*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c
*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x
))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b
*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/
c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(
2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*
f^2*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(
3/2)/(f-I*c*f*x)^(3/2)-2*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x
]
```

output
```
((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) - 3*a^2*Sqrt[d]*Sq
rt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] -
 (3*a*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[Arc
Sinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]
/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Lo
g[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1
+ c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*((-6 + 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]
/2] - Sinh[ArcSinh[c*x]/2]) - ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2]) + (12*I)*Pi*(Log[1 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^Ar
cSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*
x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 24*PolyLog[2, I
/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSi
nh[c*x]*(Pi - (4*I)*Log[1 - I/E^ArcSinh[c*x]])*((-I)*Cosh[ArcSinh[c*x]/2]
+ Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])))/(3*c*d^2)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\int\left(-\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2i(cx+i)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(-\frac{8ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{2x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}}{3bc}\right.}{}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^2*(1 + c^2*x^2)^(3/2)*((2*(a + b*ArcSinh[c*x])^2)/c + ((2*I)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (2*x*(a + b*ArcSinh[c*x])^2)/Sqrt[1
+ c^2*x^2] - (a + b*ArcSinh[c*x])^3/(3*b*c) - ((8*I)*b*(a + b*ArcSinh[c*x]
)*ArcTan[E^ArcSinh[c*x]])/c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSi
nh[c*x])])/c - (4*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c + (4*b^2*PolyLog[
2, I*E^ArcSinh[c*x]])/c - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d
+ I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 6.77 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.53

method	result
default	$-\frac{(a+b\,\mathrm{arcsinh}(xc))^{3}\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}}{3\sqrt{c^{2}x^{2}+1}\,b\,d^{2}c}+\frac{2\left(\mathrm{arcsinh}(xc)^{2}b^{2}+2\,\mathrm{arcsinh}(xc)ab+a^{2}\right)\left(xc+i-\sqrt{c^{2}x^{2}+1}\right)\sqrt{i(xc-i)d}\,\sqrt{-i(x}}{d^{2}c(c^{2}x^{2}+1)}$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3*(a+b*arcsinh(x*c))^3*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^
2+1)^(1/2)/b/d^2/c+2*(arcsinh(x*c)^2*b^2+2*arcsinh(x*c)*a*b+a^2)*(x*c+I-(c
^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d^2/c/(c^2*x^2+1
)+4*(b*arcsinh(x*c)^2-2*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*b-2*a
*ln(x*c+(c^2*x^2+1)^(1/2)-I)+2*a*ln(x*c+(c^2*x^2+1)^(1/2))-2*polylog(2,-I*
(x*c+(c^2*x^2+1)^(1/2)))*b)*b*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^
2*x^2+1)^(1/2)/d^2/c
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\,(b\,\mathrm{arsinh}\,(cx)+a)^{2}}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x
^2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d^2*x^2 - 2*I*c*
d^2*x - d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \int \frac{\sqrt{-if\,(cx + i)}\,(a + b\,\text{asinh}\,(cx))^2}{(id\,(cx - i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\,dx = \int \frac{\sqrt{-i\,cfx + f}\,(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c
*d^2*sqrt(f/d))) + integrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2/(I*c*d*x + d)^(3/2) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2
*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{f - icfx}(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\, dx = \int \frac{\sqrt{-i\, cfx + f}(b\operatorname{arsinh}(cx) + a)^2}{(i\, cdx + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\, dx = \int \frac{(a + b\operatorname{asinh}(c\, x))^2\, \sqrt{f - c\, f\, x\, 1\mathrm{i}}}{(d + c\, d\, x\, 1\mathrm{i})^{3/2}}\, dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f - icfx}(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}}\, dx = \frac{\sqrt{f}\left(-2\sqrt{cix + 1}\, asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right) a^2 i + 2\sqrt{-cix + 1}\, a^2 i + 2\sqrt{cix +}\right.}{\sqrt{d}\,\sqrt{}}$$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

output
```
(sqrt(f)*( - 2*sqrt(c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 2
*sqrt( - c*i*x + 1)*a**2*i + 2*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asi
nh(c*x))/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*a*b*c + sqrt(c*i*x +
 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c
*i*x + 1)),x)*b**2*c))/(sqrt(d)*sqrt(c*i*x + 1)*c*d)
```

3.242 $\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 525

$$\begin{aligned}
\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= -\frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4ib^2f\sqrt{1+c^2x^2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{if\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{2bf\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{if\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4bf\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4b^2f\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
-1/3*f*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^(1/2)/(f-I
*c*f*x)^(1/2)-4/3*I*b^2*f*(c^2*x^2+1)^(1/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))
/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*f*(c^2*x^2+1)^(1/2)*(a+b*
arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)+2/3*b*f*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2
*I*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*f*(c^2*
x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*P
i+1/2*I*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4/3*b*f*
(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2
/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4/3*b^2*f*(c^2*x^2+1)^(1/2)*polylog(2
,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.31 (sec) , antiderivative size = 783, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x
]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-2*I)/3)*a^2)/(d^3*(-I +
c*x)^2) - a^2/(3*d^3*(-I + c*x))))/c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]
*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] -
2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[
c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*L
og[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Cot
h[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*A
rcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]
/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSi
nh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*(I + c*x)*Sqrt[I*((-I
)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I
)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*
I)*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x
] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(P
i + (2*I)*ArcSinh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh
[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2
] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x)
)]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
```

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$$

$$\Big\downarrow \ 6211$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{f^3\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6259}$$

$$\frac{f^3\left(c^2x^2+1\right)^{5/2}\int \left(\frac{i(a+b\text{arcsinh}(cx))^2}{(cx-i)\sqrt{c^2x^2+1}}-\frac{2(a+b\text{arcsinh}(cx))^2}{(cx-i)^2\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(-\frac{(a+b\text{arcsinh}(cx))^2}{3c}+\frac{4b\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{i\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx}{3c}\right.}{}$$

input
```
Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(5/2)*(-1/3*(a + b*ArcSinh[c*x])^2/c - (((4*I)/3)*b^2*C
ot[Pi/4 + (I/2)*ArcSinh[c*x]])/c - ((I/3)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4
+ (I/2)*ArcSinh[c*x]])/c + (2*b*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcS
inh[c*x]]^2)/(3*c) + ((I/3)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSin
h[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/c + (4*b*(a + b*ArcSinh[c*x])*Lo
g[1 + I*E^ArcSinh[c*x]])/(3*c) + (4*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(
3*c)))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2756 vs. $2(443) = 886$.

Time = 7.90 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.25

method	result	size
default	Expression too large to display	2757
parts	Expression too large to display	2757

input
```
int((f-I*c*f*x)^(1/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
-2/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/(c^2*x
^2+1)^2*arcsinh(x*c)*x+4*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/
(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*c^3*x^4-4/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x
*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/(c^2*x^2+1)*c^2*x^3-1/3*b^2*(-I*(I+x*c)*f
)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/c*arcsinh(
x*c)^2+2/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/
(c^2*x^2+1)*arcsinh(x*c)*x-8/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2
)/d^3/(3*c^2*x^2-1)/(c^2*x^2+1)^2*c^4*x^5+4/3*b^2/(c^2*x^2+1)^(1/2)*(I*(x*
c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/d^3*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2
+1)^(1/2)))+4/3*I*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*
x^2-1)/(c^2*x^2+1)/c-b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c
^2*x^2-1)/(c^2*x^2+1)^2*arcsinh(x*c)^2*x+8/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(
x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*c*x^2-4/3*b^2*(-I*(I+x
*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/c*arc
sinh(x*c)-4/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-
1)/(c^2*x^2+1)*x+8/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c
^2*x^2-1)/(c^2*x^2+1)^2*x-2/3*b^2/(c^2*x^2+1)^(1/2)*(I*(x*c-I)*d)^(1/2)*(-
I*(I+x*c)*f)^(1/2)/c/d^3*arcsinh(x*c)^2+4/3*b^2/(c^2*x^2+1)^(1/2)*(I*(x*c-
I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/c/d^3*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)
))-4/3*b^2*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d^3/(3*c^2*x^2-1)/(...
```

Fricas [F]

$$\int \frac{\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{\sqrt{-i\,cfx+f}\,(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input

```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="fricas")
```

output

```
-1/3*((b^2*c*x + I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 - 3*(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)*integral(1/3*(
3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*
c*d^3*x + I*d^3), x))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}}\,dx = \int \frac{\sqrt{-if\,(cx + i)}(a + b\,\text{asinh}\,(cx))^2}{(id\,(cx - i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(1/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(5/2)
, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}}\,dx = \int \frac{\sqrt{-i\,cfx + f}(b\,\text{arsinh}\,(cx) + a)^2}{(i\,cdx + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}} \, dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2\,\sqrt{f - c\,f\,x\,1\text{i}}}{(d + c\,d\,x\,1\text{i})^{5/2}} \, dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f - icfx}(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}} \, dx = \frac{\sqrt{f}\left(-6\sqrt{cix + 1}\,\sqrt{-cix + 1}\left(\int \frac{\sqrt{-cix+1}\,asinh(cx)}{\sqrt{cix+1}\,c^2x^2 - 2\sqrt{cix+1}\,cix - \sqrt{cix+1}}dx\right) a$$

input
```
int((f-I*c*f*x)^(1/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

output
```
(sqrt(f)*( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*
asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*
i*x + 1)),x)*a*b*c**2*i*x - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt
( - c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*
c*i*x - sqrt(c*i*x + 1)),x)*a*b*c - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*i
nt((sqrt( - c*i*x + 1)*asinh(c*x)**2)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(
c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b**2*c**2*i*x - 3*sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2)/(sqrt(c*i*x + 1)*
c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b**2*c - a**2*c*
*2*i*x**2 + 2*a**2*c*x + a**2*i))/(3*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)*c*d**2*(c*i*x + 1))
```

3.243 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 817

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{16ib^2d^2f\sqrt{d+icdx}\sqrt{f-icfx}}{75c}$$
$$+\frac{15}{64}b^2d^2fx\sqrt{d+icdx}\sqrt{f-icfx}+\frac{8ib^2d^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{225c}$$
$$+\frac{1}{32}b^2d^2fx\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)+\frac{2ib^2d^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^2}{125c}$$
$$-\frac{9b^2d^2f\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}}$$
$$-\frac{2ibd^2fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{5\sqrt{1+c^2x^2}}$$
$$-\frac{3bcd^2fx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{8\sqrt{1+c^2x^2}}$$
$$-\frac{4ibc^2d^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{15\sqrt{1+c^2x^2}}$$
$$-\frac{2ibc^4d^2fx^5\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{25\sqrt{1+c^2x^2}}$$
$$-\frac{bd^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))}{8c}$$
$$+\frac{3}{8}d^2fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2+\frac{1}{4}d^2fx\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx)$$

output
```
16/75*I*b^2*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d^2*f*x*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+8/225*I*b^2*d^2*f*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)*(c^2*x^2+1)/c+1/32*b^2*d^2*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*
x)^(1/2)*(c^2*x^2+1)+2/125*I*b^2*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
*(c^2*x^2+1)^2/c-9/64*b^2*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsin
h(c*x)/c/(c^2*x^2+1)^(1/2)-2/5*I*b*d^2*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(
1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-3/8*b*c*d^2*f*x^2*(d+I*c*d*x)^(1
/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-4/15*I*b*c^2*d^
2*f*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)
^(1/2)-2/25*I*b*c^4*d^2*f*x^5*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arc
sinh(c*x))/(c^2*x^2+1)^(1/2)-1/8*b*d^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/
2)*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c+3/8*d^2*f*x*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+1/4*d^2*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*
f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/5*I*d^2*f*(d+I*c*d*x)^(1/2)*
(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c+1/8*d^2*f*(d+I*c*d*
x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.33

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
((-72000*I)*a*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (57600*I)*
a^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72000*I
)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000
*a^2*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (11
5200*I)*a^2*c^2*d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + 72000*a^2*c^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1
 + c^2*x^2] + (57600*I)*a^2*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*Sqrt[1 + c^2*x^2] + 36000*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^3 - 72000*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[2*ArcSinh[c*x]] + (4000*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[3*ArcSinh[c*x]] - 4500*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[4*ArcSinh[c*x]] + (288*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*Cosh[5*ArcSinh[c*x]] + 108000*a^2*d^(5/2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b
^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - (12000
*I)*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1
125*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1
800*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (20
*I)*b*Sqrt[1 + c^2*x^2] + (10*I)*b*Cosh[3*ArcSinh[c*x]] + (2*I)*b*Cosh[5*A
rcSinh[c*x]] + 40*b*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) - ...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int d(icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int (icx+1)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(icx(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))^2+\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2\right)dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{2}{25}ibc^4x^5(a+b\text{arcsinh}(cx))-\frac{4}{15}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a-\right.$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(((((16*I)/75)*b^2*Sqrt[1 + c^2*
x^2])/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 + (((8*I)/225)*b^2*(1 + c^2*x^2)
^(3/2))/c + (b^2*x*(1 + c^2*x^2)^(3/2))/32 + (((2*I)/125)*b^2*(1 + c^2*x^2
)^(5/2))/c - (9*b^2*ArcSinh[c*x])/(64*c) - ((2*I)/5)*b*x*(a + b*ArcSinh[c*
x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8 - ((4*I)/15)*b*c^2*x^3*(a + b*Arc
Sinh[c*x]) - ((2*I)/25)*b*c^4*x^5*(a + b*ArcSinh[c*x]) - (b*(1 + c^2*x^2)^
2*(a + b*ArcSinh[c*x]))/(8*c) + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)^2)/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + ((I/5)*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(8*b*c)))/
(1 + c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(676) = 1352$.

Time = 8.14 (sec) , antiderivative size = 2569, normalized size of antiderivative = 3.14

method	result	size
default	Expression too large to display	2569
parts	Expression too large to display	2569

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/5*I*a^2/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)+1/4*I*a^2*d/c/f*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(5/2)+1/4*I*a^2*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x
)^(5/2)-1/8*I*a^2*d^2/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/8*I*a^2*d^2*
f/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/8*a^2*d^3*f^2*((f-I*c*f*x)*(d+I*
c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(
1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/8*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3*f*d^2+1/4000*I*(I
*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^
5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/
2)*x*c+1)*(25*arcsinh(x*c)^2-10*arcsinh(x*c)+2)*f*d^2/(c^2*x^2+1)/c+1/512*
(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^
(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(8*a
rcsinh(x*c)^2-4*arcsinh(x*c)+1)*f*d^2/(c^2*x^2+1)/c+1/288*I*(I*(x*c-I)*d)^
(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^
2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(9*arcsinh(x*c)^2-6*arcsinh(x*c)+2)*f*d^2/(c^
2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*
c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh
(x*c)+1)*f*d^2/(c^2*x^2+1)/c+1/16*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/
2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2-2*arcsinh(x*c)+2)*f*d
^2/(c^2*x^2+1)/c+1/16*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x...
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((I*b^2*c^3*d^2*f*x^3 + b^2*c^2*d^2*f*x^2 + I*b^2*c*d^2*f*x + b^2*
d^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2
 - 2*(-I*a*b*c^3*d^2*f*x^3 - a*b*c^2*d^2*f*x^2 - I*a*b*c*d^2*f*x - a*b*d^2
*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I
*a^2*c^3*d^2*f*x^3 + a^2*c^2*d^2*f*x^2 + I*a^2*c*d^2*f*x + a^2*d^2*f)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d^2f\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+8\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^4i\,x^4+10\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))^2,x)
```

output

```
(sqrt(f)*sqrt(d)*d**2*f*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 8*sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c**4*i*x**4 + 10*sqrt(c*i*x + 1)*sqr
t( - c*i*x + 1)*a**2*c**3*x**3 + 16*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**
2*c**2*i*x**2 + 25*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c*x + 8*sqrt(c*
i*x + 1)*sqrt( - c*i*x + 1)*a**2*i + 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)*x**3,x)*a*b*c**4*i + 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)*x**2,x)*a*b*c**3 + 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x +
1)*asinh(c*x)*x,x)*a*b*c**2*i + 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*
asinh(c*x),x)*a*b*c + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)
**2*x**3,x)*b**2*c**4*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(
c*x)**2*x**2,x)*b**2*c**3 + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin
h(c*x)**2*x,x)*b**2*c**2*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asi
nh(c*x)**2,x)*b**2*c))/(40*c)
```

3.244 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 408

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{15}{64}b^2dfx\sqrt{d+icdx}\sqrt{f-icfx} + \frac{1}{32}b^2dfx\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2) - \frac{9b^2df\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} - \frac{3bcdfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} - \frac{bdf\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{8c} + \frac{3}{8}dfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 + \frac{1}{4}dfx\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2 -$$

output
```
15/64*b^2*d*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/32*b^2*d*f*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)-9/64*b^2*d*f*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(1/2)-3/8*b*c*d*f*x^2*(d+I*c*d*
x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-1/8*b*d*f*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c
+3/8*d*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+1/4*d*
f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1
/8*d*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x
^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.28

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{160a^2cdfx\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+64a^2c^3dfx^3\sqrt{d+icdx}\sqrt{f-}}{}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
(160*a^2*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6
4*a^2*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 64*a*b*d*f
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 4*a*b*d*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 96*a^2*d^(3/2)*f^(3
/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]] + 32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*Arc
Sinh[c*x]] + b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*
x]] + 8*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(12*a + 8
*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) - 4*b*d*f*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(16*b*Cosh[2*ArcSinh[c*x]] + b*Cosh[4*A
rcSinh[c*x]] - 4*a*(8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(256*
c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.67, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {6211, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 6201$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\int \sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx+\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 6200$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx-bc\int x(a+\right.\right.}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 6191$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\int\right.\right.\right.}{(c^2x^2}$$

$$\Big\downarrow \; 262$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(-bc\left(\frac{1}{2}x^2(a+b\text{arcsinh}(cx))-\frac{1}{2}bc\left(\right.\right.\right.\right.}{}$$

$$\Big\downarrow \; 222$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{3}{4}\left(\frac{1}{2}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+\right.\right.}{}$$

$$\Big\downarrow \; 6198$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(cx))dx+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2+\right.}{(c^2}$$

$$\Big\downarrow \; 6213$$

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\int(c^2x^2+1)^{3/2}dx}{4c}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}\left(a+b\text{arcsinh}\right.\right.$$

$$\Big\downarrow \textbf{211}$$

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)+\frac{1}{4}x\left(c^2x^2+1\right)^{3}\right.$$

$$\Big\downarrow \textbf{211}$$

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)}{4c}\right)\right.$$

$$\Big\downarrow \textbf{222}$$

$$(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2-\frac{1}{2}bc\left(\frac{(c^2x^2+1)^2(a+b\text{arcsinh}(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\text{arcsin}}{2c}\right.\right.}{}\right.\right.$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((x*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x])^2)/4 + (3*((x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (a
 + b*ArcSinh[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSinh[c*x]))/2 - (b*c*((
x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)))/2)))/4 - (b*c*(((1 +
 c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*(1 + c^2*x^2)^(3/2))/4
+ (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/(4*c)))/2))/(1 +
c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(336) = 672$.

Time = 6.77 (sec) , antiderivative size = 1216, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1216
parts	Expression too large to display	1216

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/4*I*a^2/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)+1/4*I*a^2*d/c/f*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(5/2)-1/8*I*a^2*d/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1
/2)-3/8*I*a^2*d*f/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/8*a^2*d^2*f^2*((
f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d
*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/8*(I*
(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3*d
*f+1/512*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^
2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1
/2))*(8*arcsinh(x*c)^2-4*arcsinh(x*c)+1)*d*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)
*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*
c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)*d*f/(c^2*x^2+1)/c
+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x
^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)*d
*f/(c^2*x^2+1)/c+1/512*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5
-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3-8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c-
(c^2*x^2+1)^(1/2))*(8*arcsinh(x*c)^2+4*arcsinh(x*c)+1)*d*f/(c^2*x^2+1)/c)+
2*a*b*(3/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*a
rcsinh(x*c)^2*d*f+1/256*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^
5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c
+(c^2*x^2+1)^(1/2))*(-1+4*arcsinh(x*c))*d*f/(c^2*x^2+1)/c+1/16*(I*(x*c-...
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((b^2*c^2*d*f*x^2 + b^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d*f*x^2 + a*b*d*f)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^2*d*f*x^2
+ a^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{\sqrt{f}\,\sqrt{d}\,df\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+2\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^3x^3+5\sqrt{cix}\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(f)*sqrt(d)*d*f*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 2*sqrt(c
*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c**3*x**3 + 5*sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)*a**2*c*x + 16*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x
**2,x)*a*b*c**3 + 16*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*
a*b*c + 8*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)**2*x**2,x)*b**
2*c**3 + 8*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)**2,x)*b**2*c)
)/(8*c)
```

3.245 $\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 508

$$
\begin{aligned}
\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = &-\frac{4ib^2f\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx}\\
&-\frac{2ib^2f\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c}\\
&-\frac{b^2f\sqrt{d+icdx}\sqrt{f-icfx}\text{arcsinh}(cx)}{4c\sqrt{1+c^2x^2}}\\
&+\frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{3\sqrt{1+c^2x^2}}\\
&-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{2\sqrt{1+c^2x^2}}\\
&+\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))}{9\sqrt{1+c^2x^2}}\\
&+\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^2\\
&-\frac{if\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c}\\
&+\frac{f\sqrt{d+icdx}\sqrt{f-icfx}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
-4/9*I*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*f*x*(d+I*c*d*x)
^(1/2)*(f-I*c*f*x)^(1/2)-2/27*I*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*
(c^2*x^2+1)/c-1/4*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsinh(c*x)/c
/(c^2*x^2+1)^(1/2)+2/3*I*b*f*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*ar
csinh(c*x))/(c^2*x^2+1)^(1/2)-1/2*b*c*f*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^
(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f*x^3*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+1/2*f*x*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2-1/3*I*f*(d+I*c*d*x)^(1/
2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/6*f*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.39

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{108iabcfx\sqrt{d+icdx}\sqrt{f-icfx}-72ia^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-108ib^2f}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((108*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (72*I)*a^2*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (108*I)*b^2*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*f*x*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*f*x^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*f*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Cosh[3*ArcSinh[c*x]] + 108*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*
d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a - (3*I)*b*Sqrt[1 + c^2*x^2]
- I*b*Cosh[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*f*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((9*I)*
b*c*x - (9*I)*a*Sqrt[1 + c^2*x^2] - (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sin
h[2*ArcSinh[c*x]] + I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f(1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int (1-icx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\int\left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$f\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{2}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-\frac{i(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(}{3c}\right.$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((-4*I)/9)*b^2*Sqrt[1 + c^2*x^2])
/c + (b^2*x*Sqrt[1 + c^2*x^2])/4 - (((2*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c
- (b^2*ArcSinh[c*x])/(4*c) + ((2*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (b*c*x^2
*(a + b*ArcSinh[c*x]))/2 + ((2*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (x*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((I/3)*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(6*b*c)))/Sqrt[1 + c^2*x^
2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1398 vs. $2(413) = 826$.

Time = 6.88 (sec) , antiderivative size = 1399, normalized size of antiderivative = 2.75

method	result	size
default	Expression too large to display	1399
parts	Expression too large to display	1399

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/3*I*a^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)-1/6*I*a^2/c*(f-I*c*f*x)^
(3/2)*(d+I*c*d*x)^(1/2)-1/2*I*a^2*f/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+
1/2*a^2*d*f^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x
)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1
/2)+b^2*(1/6*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*
arcsinh(x*c)^3*f-1/216*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x
^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(9*arc
sinh(x*c)^2-6*arcsinh(x*c)+2)*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I
*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)
^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)*f/(c^2*x^2+1)/c-1/8*I*(I*(x*c-
I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsin
h(x*c)^2-2*arcsinh(x*c)+2)*f/(c^2*x^2+1)/c-1/8*I*(I*(x*c-I)*d)^(1/2)*(-I*(
I+x*c)*f)^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2+2*arcsin
h(x*c)+2)*f/(c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2
*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x
*c)^2+2*arcsinh(x*c)+1)*f/(c^2*x^2+1)/c-1/216*I*(I*(x*c-I)*d)^(1/2)*(-I*(I
+x*c)*f)^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2
+1)^(1/2)*x*c+1)*(9*arcsinh(x*c)^2+6*arcsinh(x*c)+2)*f/(c^2*x^2+1)/c)+2*a*
b*(1/4*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsin
h(x*c)^2*f-1/72*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4...
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((-I*b^2*c*f*x + b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*f*x - a*b*f)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*f*x + a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F]

$$\int \sqrt{d + icdx}(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2\, dx = \int \sqrt{id\,(cx - i)}(-if(cx + i))^{\frac{3}{2}}\,(a + b\,\text{asinh}\,(cx))^2\, dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d + icdx}(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d + icdx}(f - icfx)^{3/2}(a + b\text{arcsinh}(cx))^2\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int (a+b\,\mathrm{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{3/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,f\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-2\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^2i\,x^2+3\sqrt{cix+1}\,\sqrt{-cix}\right.}{}$$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(f)*sqrt(d)*f*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - 2*sqrt(c*i
*x + 1)*sqrt( - c*i*x + 1)*a**2*c**2*i*x**2 + 3*sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)*a**2*c*x - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i - 12*int(s
qrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x,x)*a*b*c**2*i + 12*int(sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*a*b*c - 6*int(sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*asinh(c*x)**2*x,x)*b**2*c**2*i + 6*int(sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*asinh(c*x)**2,x)*b**2*c))/(6*c)
```

3.246 $\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 436

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = -\frac{4ib^2f^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{b^2f^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2f^2\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{4ibf^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
-4*I*b^2*f^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*f^2
*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*f^2*(c^2*x^2+1)
^(1/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*b*f^2*x*(c^2
*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*b
*c*f^2*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)-2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)-1/2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)+1/2*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(d+
I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 7.58 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.22

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{32iabcfx\sqrt{d+icdx}\sqrt{f-icfx}-16ia^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{}}{}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x
]
```

output
```
((32*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (32*I)*b^2*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*f*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]^3 + 2*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]
*((4*I)*(4*b*c*x + a*(-4 + I*c*x)*Sqrt[1 + c^2*x^2]) + b*Cosh[2*ArcSinh[c*
x]]) + 12*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt
[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^2*(6*a - (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]
))/(8*c*d*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 27

$$\frac{f^2\sqrt{c^2x^2+1}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 6258

$$\frac{f^2\sqrt{c^2x^2+1}\int \left(c-ic^2x\right)^2(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3042

$$\frac{f^2\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(c-c\sin(i\text{arcsinh}(cx)))^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3798

$$\frac{f^2\sqrt{c^2x^2+1}\int \left(-x^2(a+b\text{arcsinh}(cx))^2c^4-2ix(a+b\text{arcsinh}(cx))^2c^3+(a+b\text{arcsinh}(cx))^2c^2\right)d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 2009

$$\frac{f^2\sqrt{c^2x^2+1}\left(\frac{1}{2}bc^4x^2(a+b\text{arcsinh}(cx))+4ibc^3x(a+b\text{arcsinh}(cx))-2ic^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{c^2(a+\cdots}{}\right.}{c^3\sqrt{d+icd\cdots}}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
(f^2*Sqrt[1 + c^2*x^2]*((-4*I)*b^2*c^2*Sqrt[1 + c^2*x^2] - (b^2*c^3*x*Sqrt
[1 + c^2*x^2])/4 + (b^2*c^2*ArcSinh[c*x])/4 + (4*I)*b*c^3*x*(a + b*ArcSinh
[c*x]) + (b*c^4*x^2*(a + b*ArcSinh[c*x]))/2 - (2*I)*c^2*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x])^2 - (c^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/
2 + (c^2*(a + b*ArcSinh[c*x])^3)/(2*b)))/(c^3*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(367) = 734$.

Time = 4.20 (sec) , antiderivative size = 978, normalized size of antiderivative = 2.24

method	result
default	$-\frac{ia^2(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{2cd}-\frac{3ia^2f\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2cd}+\frac{3a^2f^2\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}}+b^2$
parts	$-\frac{ia^2(-icfx+f)^{\frac{3}{2}}\sqrt{icdx+d}}{2cd}-\frac{3ia^2f\sqrt{-icfx+f}\,\sqrt{icdx+d}}{2cd}+\frac{3a^2f^2\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{2\sqrt{-icfx+f}\,\sqrt{icdx+d}\,\sqrt{c^2df}}+b^2$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*I*a^2/c/d*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/2*I*a^2*f/c/d*(f-I*c*
f*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/2*a^2*f^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(
f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x
^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/2*f*arcsinh(x*c)^3*(I*(x*c-I)*d)^(1/
2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/d/c-1/16*f*(2*arcsinh(x*c)^2-2*a
rcsinh(x*c)+1)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1
/2))*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d/c/(c^2*x^2+1)-I*f*(arcsinh
(x*c)^2-2*arcsinh(x*c)+2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^
(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-I*f*(arcsinh(x*c)^2+2*arcsinh(x
*c)+2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f
)^(1/2)/d/c/(c^2*x^2+1)-1/16*f*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)*(2*x^3*
c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1
/2)*(I*(x*c-I)*d)^(1/2)/d/c/(c^2*x^2+1))+2*a*b*(3/4*f*arcsinh(x*c)^2*(-I*(
I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/d/c-1/16*f*(-1+2*arc
sinh(x*c))*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))
*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-I*f*(arcsinh(x*c
)-1)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^
(1/2)/d/c/(c^2*x^2+1)-I*f*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+
1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-1/16*f*(1+2*ar
csinh(x*c))*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1...
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{\sqrt{id\,(cx-i)}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,(f-c\,f\,x\,1\text{i})^{3/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{f}\,f\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2cx-4\sqrt{cix}\right.}{}$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

output
```
(sqrt(f)*f*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - sqrt(c*i*x + 1)*sq
rt( - c*i*x + 1)*a**2*c*x - 4*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i -
4*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/sqrt(c*i*x + 1),x)*a*b*c**2*i + 4*
int((sqrt( - c*i*x + 1)*asinh(c*x))/sqrt(c*i*x + 1),x)*a*b*c - 2*int((sqrt
( - c*i*x + 1)*asinh(c*x)**2*x)/sqrt(c*i*x + 1),x)*b**2*c**2*i + 2*int((sq
rt( - c*i*x + 1)*asinh(c*x)**2)/sqrt(c*i*x + 1),x)*b**2*c))/(2*sqrt(d)*c)
```

3.247 $\int \frac{(f-icfx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 719

$$\begin{aligned}
\int \frac{(f-icfx)^{3/2}(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx &= \frac{2ib^2f^2(1+c^2x^2)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{2ibf^2x\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))}{d\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4if^2(a+b\,\mathrm{arcsinh}(cx))^2}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4f^2x(a+b\,\mathrm{arcsinh}(cx))^2}{d\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4f^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))^2}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{if^2(1+c^2x^2)(a+b\,\mathrm{arcsinh}(cx))^2}{cd\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))^3}{bcd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{16ibf^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))\arctan\left(e^{\mathrm{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{8bf^2\sqrt{1+c^2x^2}(a+b\,\mathrm{arcsinh}(cx))\log\left(1+e^{2\mathrm{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{8b^2f^2\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8b^2f^2\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4b^2f^2\sqrt{1+c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
2*I*b^2*f^2*(c^2*x^2+1)/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b*f^2*
x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)+4*I*f^2*(a+b*arcsinh(c*x))^2/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*
f^2*x*(a+b*arcsinh(c*x))^2/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*f^2*(c^
2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)+I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)-f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/d/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)-16*I*b*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*arctan(c
*x+(c^2*x^2+1)^(1/2))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b*f^2*(c^2
*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/d/(d+I*
c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b^2*f^2*(c^2*x^2+1)^(1/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8*b^2*f^2*(
c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)-4*b^2*f^2*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1
)^(1/2))^2)/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 12.86 (sec) , antiderivative size = 1174, normalized size of antiderivative = 1.63

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2)
,x]
```

output
```
((I/3)*f*(-3*a^2*(-5*I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
 c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 9*a^2*Sqrt[d]*
Sqrt[f]*(-I + c*x)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*
x]/2]) + 6*a*b*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh
[c*x]/2]*(-(c*x) + (2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2
 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2]) + I*(-(c*x) + (-2
+ Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + (3*I)*a*b*(I - c
*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x
]*(-4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*Log[1 + c
^2*x^2]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSin
h[c*x]/2]] + 2*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + I*b^2*(I - c*x)*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((6 - 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSinh[c*x]*(I*Pi + 4*Log[1 - I/E^ArcSinh[c*x
]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 24*PolyLog[2, I/E^Ar
cSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 12*Pi*(Log[1
 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^ArcSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]
/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*((-I)*Cosh[ArcSinh[c*x]/2...
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3\left(c^2x^2+1\right)^{3/2}\int\frac{(1-icx)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^3\left(c^2x^2+1\right)^{3/2}\int\left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4i(cx+i)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3\left(c^2x^2+1\right)^{3/2}\left(-\frac{16ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{4x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{4i(a-}{}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^3*(1 + c^2*x^2)^(3/2)*((-2*I)*a*b*x + ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c -
 (2*I)*b^2*x*ArcSinh[c*x] + (4*(a + b*ArcSinh[c*x])^2)/c + ((4*I)*(a + b*A
rcSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (4*x*(a + b*ArcSinh[c*x])^2)/Sqrt[
1 + c^2*x^2] + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c - (a + b*Arc
Sinh[c*x])^3/(b*c) - ((16*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]]
)/c - (8*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c - (8*b^2*Po
lyLog[2, (-I)*E^ArcSinh[c*x]])/c + (8*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c
- (4*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.16

method	result
default	$\frac{if\left(-4i\sqrt{c^2x^2+1}\,a^2bcx+3i\,\operatorname{arcsinh}(xc)^2 a\,b^2c^2x^2+16i\,\operatorname{arcsinh}(xc)\ln\left(1+i\left(xc+\sqrt{c^2x^2+1}\right)\right)b^3c^2x^2+3i\,\operatorname{arcsinh}(xc)a^2b\,c^2x^2+8i\,\operatorname{arcs}\right.}{}$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
I*f*(-2*a*b^2*c*x+(c^2*x^2+1)^(1/2)*a^2*b*c^2*x^2-16*arctan(x*c+(c^2*x^2+1
)^(1/2))*a*b^2*c^2*x^2+arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*b^3*c^2*x^2+I*arcs
inh(x*c)^3*b^3*c^2*x^2-4*I*arcsinh(x*c)^2*b^3*c^2*x^2+16*I*polylog(2,-I*(x
*c+(c^2*x^2+1)^(1/2)))*b^3*c^2*x^2+3*I*a*b^2*arcsinh(x*c)^2+16*I*ln(1+I*(x
*c+(c^2*x^2+1)^(1/2)))*b^3*arcsinh(x*c)+3*I*a^2*b*arcsinh(x*c)-16*I*ln(x*c
+(c^2*x^2+1)^(1/2))*a*b^2+8*I*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*a*b^2+8*I*a*
b^2*arcsinh(x*c)+10*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*a*b^2-2*arcsinh(x*c)*b^
3*c*x-2*arcsinh(x*c)*b^3*c^3*x^3+2*(c^2*x^2+1)^(1/2)*b^3*c^2*x^2+I*a^3*c^2
*x^2-4*I*(c^2*x^2+1)^(1/2)*a^2*b*c*x-2*a*b^2*c^3*x^3+5*(c^2*x^2+1)^(1/2)*a
^2*b+I*a^3+2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*a*b^2*c^2*x^2+3*I*arcsinh(x*c)
^2*a*b^2*c^2*x^2+16*I*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*b^3*c^2
*x^2+3*I*arcsinh(x*c)*a^2*b*c^2*x^2+8*I*arcsinh(x*c)*a*b^2*c^2*x^2-16*I*ln
(x*c+(c^2*x^2+1)^(1/2))*a*b^2*c^2*x^2+8*I*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*
a*b^2*c^2*x^2-4*I*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*b^3*c*x+4*I*a^2*b*c^2*x
^2-16*arctan(x*c+(c^2*x^2+1)^(1/2))*a*b^2-8*I*arcsinh(x*c)*(c^2*x^2+1)^(1/
2)*a*b^2*c*x+2*(c^2*x^2+1)^(1/2)*b^3+4*I*a^2*b+5*(c^2*x^2+1)^(1/2)*arcsinh
(x*c)^2*b^3+I*b^3*arcsinh(x*c)^3-4*I*b^3*arcsinh(x*c)^2+16*I*polylog(2,-I*
(x*c+(c^2*x^2+1)^(1/2)))*b^3)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^
2*x^2+1)^(1/2)/b/d^2/(c^4*x^4+2*c^2*x^2+1)/c
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((I*b^2*c*f*x - b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*f*x + a*b*f)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*f*x - a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}{(cx)})^2}{(id\,(cx-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3
/2), x)
```

Maxima [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}{(cx)}+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6
*I*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c
*d^2*sqrt(f/d))) + integrate((-I*c*f*x + f)^(3/2)*b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2/(I*c*d*x + d)^(3/2) + 2*(-I*c*f*x + f)^(3/2)*a*b*log(c*x + sqrt
(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-21414341606035747530220161080998646913688299520000,[3,1
0,0,10]%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}{(d+c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx=\frac{\sqrt{f}\,f\left(-6\sqrt{cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-\sqrt{-cix+1}\,a^2cx+5\sqrt{-}\right.}{}$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

output
```
(sqrt(f)*f*( - 6*sqrt(c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i -
 sqrt( - c*i*x + 1)*a**2*c*x + 5*sqrt( - c*i*x + 1)*a**2*i - 2*sqrt(c*i*x
+ 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c
*i*x + 1)),x)*a*b*c**2*i + 2*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh
(c*x))/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*a*b*c - sqrt(c*i*x + 1
)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2*x)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c
*i*x + 1)),x)*b**2*c**2*i + sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(
c*x)**2)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b**2*c))/(sqrt(d)*sq
rt(c*i*x + 1)*c*d)
```

3.248 $\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 604

$$\begin{aligned}\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= -\frac{8f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{3bcd^2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8ib^2f^2\sqrt{1+c^2x^2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{8if^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4bf^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{2if^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{32bf^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{\operatorname{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{32b^2f^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
-8/3*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)+1/3*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/d^2/(d+
I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8/3*I*b^2*f^2*(c^2*x^2+1)^(1/2)*cot(1/4*P
i+1/2*I*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8/3*I*f^2*
(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/d^
2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4/3*b*f^2*(c^2*x^2+1)^(1/2)*(a+b*arc
sinh(c*x))*csc(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c
*f*x)^(1/2)+2/3*I*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/
2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+32/3*b*f^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+I*
(c*x+(c^2*x^2+1)^(1/2)))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+32/3*b^
2*f^2*(c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2/(d+I*c
*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1609 vs. $2(604) = 1208$.

Time = 15.78 (sec) , antiderivative size = 1609, normalized size of antiderivative = 2.66

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-4*I)/3)*a^2*f)/(d^3*(-I
+ c*x)^2) - (8*a^2*f)/(3*d^3*(-I + c*x))))/c + (a^2*f^(3/2)*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2))
+ ((I/3)*a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*
f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Co
sh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*
Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (
6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1
+ c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
 c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log
[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)
*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]
)^4) - (a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f
*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*
ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh
[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*
(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c
*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSin
h[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]]
 + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*Ar...
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\int \left(\frac{4i(a+b\text{arcsinh}(cx))^2}{(cx-i)\sqrt{c^2x^2+1}}-\frac{4(a+b\text{arcsinh}(cx))^2}{(cx-i)^2\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(\frac{(a+b\text{arcsinh}(cx))^3}{3bc}-\frac{8(a+b\text{arcsinh}(cx))^2}{3c}+\frac{32b\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{8i\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsi}\right.}{}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(5/2)*((-8*(a + b*ArcSinh[c*x])^2)/(3*c) + (a + b*ArcSi
nh[c*x])^3/(3*b*c) - (((8*I)/3)*b^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c - ((
(8*I)/3)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c + (4*b*(
a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((2*I)/3)*(
a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcS
inh[c*x]]^2)/c + (32*b*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*
c) + (32*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c)))/((d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.39

method	result
default	$\frac{f(a+b\,\operatorname{arcsinh}(xc))^3\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}}{3\sqrt{c^2x^2+1}\,bc\,d^3} - \frac{4f\left(20ab+105\operatorname{arcsinh}(xc)^2b^2c^2x^2+25a^2+25\operatorname{arcsinh}(xc)^2b^2+20b^2\operatorname{arcsinh}(xc)+\right.}{}$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
1/3*f*(a+b*arcsinh(x*c))^3*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x
^2+1)^(1/2)/b/c/d^3-4/3*f*(20*a*b+105*arcsinh(x*c)^2*b^2*c^2*x^2-6*I*(c^2*
x^2+1)^(1/2)*b^2*c^2*x^2-20*I*a*b*c*x-36*I*a*b*c^3*x^3+25*a^2+25*arcsinh(x
*c)^2*b^2+20*b^2*arcsinh(x*c)+30*b^2+48*a*b*c^4*x^4+52*a*b*c^2*x^2-24*(c^2
*x^2+1)^(1/2)*b^2*c^3*x^3-20*(c^2*x^2+1)^(1/2)*b^2*c*x+48*(c^2*x^2+1)^(1/2
)*arcsinh(x*c)*b^2*c^3*x^3+10*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2*c*x+48*ar
csinh(x*c)*b^2*c^4*x^4+52*arcsinh(x*c)*b^2*c^2*x^2+36*I*(c^2*x^2+1)^(1/2)*
a*b*c^2*x^2+48*(c^2*x^2+1)^(1/2)*a*b*c^3*x^3+10*(c^2*x^2+1)^(1/2)*a*b*c*x+
36*I*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*b^2*c^2*x^2+210*arcsinh(x*c)*a*b*c^2*x
^2+50*arcsinh(x*c)*a*b-20*I*arcsinh(x*c)*b^2*c*x-42*I*b^2*c^3*x^3-10*I*b^2
*c*x+10*I*(c^2*x^2+1)^(1/2)*a*b-10*I*(c^2*x^2+1)^(1/2)*b^2+120*b^2*c^4*x^4
+118*b^2*c^2*x^2-36*I*arcsinh(x*c)*b^2*c^3*x^3+288*arcsinh(x*c)*a*b*c^4*x^
4+144*arcsinh(x*c)^2*b^2*c^4*x^4+10*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2+1
05*a^2*c^2*x^2+144*a^2*c^4*x^4)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+3*I
*c^2*x^2-2*(c^2*x^2+1)^(1/2)+I)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d
^3/(144*c^4*x^4+105*c^2*x^2+25)/(c^2*x^2+1)^2/c-16/3*f*(b*arcsinh(x*c)^2-2
*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*b-2*a*ln(x*c+(c^2*x^2+1)^(1/
2)-I)+2*a*ln(x*c+(c^2*x^2+1)^(1/2))-2*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2))
)*b)*b*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3
```

Fricas [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input

```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="fricas")
```

output

```
integral(((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

Sympy [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{(-if(cx+i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

output
```
Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(5
/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{3/2}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((f-I*c*f*x)^(3/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

output
```
(sqrt(f)*f*( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1
)/sqrt(2))*a**2*c*x + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*
i*x + 1)/sqrt(2))*a**2*i - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(
 - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)
*c*i*x - sqrt(c*i*x + 1)),x)*a*b*c**3*x + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*
sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*b*c**2*i - 6*sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c
**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*b*c**2*i*x - 6*
sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))/(sq
rt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*
b*c - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c
*x)**2*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*
x + 1)),x)*b**2*c**3*x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( -
 c*i*x + 1)*asinh(c*x)**2*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1
)*c*i*x - sqrt(c*i*x + 1)),x)*b**2*c**2*i - 3*sqrt(c*i*x + 1)*sqrt( - c*i*
x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2)/(sqrt(c*i*x + 1)*c**2*x**2 -
 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b**2*c**2*i*x - 3*sqrt(c*i*
x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2)/(sqrt(c*i
*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b**2*...
```

3.249 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 610

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{245b^2d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}}{1152}$$
$$+\frac{65b^2d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{1728}$$
$$+\frac{1}{108}b^2d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^2$$
$$-\frac{115b^2d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{1152c\sqrt{1+c^2x^2}}$$
$$-\frac{5bcd^2f^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{16\sqrt{1+c^2x^2}}$$
$$-\frac{5bd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{48c}$$
$$-\frac{bd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{18c}$$
$$+\frac{5}{16}d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2+\frac{5}{24}d^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}$$

output
```
245/1152*b^2*d^2*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+65/1728*b^2*d^2
*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)+1/108*b^2*d^2*f^2*x
*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2-115/1152*b^2*d^2*f^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(1/2)-5/16*b*
c*d^2*f^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*
x^2+1)^(1/2)-5/48*b*d^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1
)^(3/2)*(a+b*arcsinh(c*x))/c-1/18*b*d^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^
(1/2)*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c+5/16*d^2*f^2*x*(d+I*c*d*x)^(1
/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+5/24*d^2*f^2*x*(d+I*c*d*x)^(1/2
)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/6*d^2*f^2*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+5/48*d^2*f
^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1
)^(1/2)
```

Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.20

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{9504a^2cd^2f^2x\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}+7488a^2c^3d^2f^2x^3\sqrt{d+i}}{}$$

input
```
Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
(9504*a^2*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2
] + 7488*a^2*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2] + 2304*a^2*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sq
rt[1 + c^2*x^2] + 1440*b^2*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Arc
Sinh[c*x]^3 - 3240*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*
ArcSinh[c*x]] - 324*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4
*ArcSinh[c*x]] - 24*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6
*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 1620*b^2*d^2*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 81*b^2*d^2*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d^2*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[6*ArcSinh[c*x]] - 12*b*d^2*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] +
27*b*Cosh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSin
h[c*x]] - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d
^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + 45*b*Sin
h[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(1
3824*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {6211, 6201, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\int \left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2dx}{\left(c^2x^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{6201}$$

$$\frac{(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\int \left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2d\right.}{\left(c^2x^2+1\right)^{5/2}}$$

$\downarrow$ 6201

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 6200

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 6191

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 262

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 222

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 6198

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\int x\left(c^2x^2+1\right)^2(a+b\text{arcsinh}(cx))dx+\frac{5}{6}\left(-\frac{1}{2}bc\int x\left(c^2x^2+1\right)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 6213

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{\left(c^2x^2+1\right)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\int\left(c^2x^2+1\right)^{5/2}dx}{6c}\right)+\frac{5}{6}\left(-\frac{1}{2}bc\left(\frac{\left(c^2x^2+1\right)^2(a+b\text{arcsinh}}{4c^2}\right.\right.\right.$$

$\downarrow$ 211

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\int(c^2x^2+1)^{3/2}dx+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)+\frac{5}{6}\left(-\frac{1}{2}bc\left(\right.\right.\right.$$

$\downarrow$ 211

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{c^2x^2+1}dx+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)+\frac{1}{6}x(c^2x^2+1)^{5/2}\right)}{6c}\right)\right.$$

$\downarrow$ 211

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c^2x^2+1}}dx+\frac{1}{2}x\sqrt{c^2x^2+1}\right)+\frac{1}{4}x(c^2x^2+1)^{3/2}\right)\right.}{6c}\right.\right.$$

$\downarrow$ 222

$$(d+icdx)^{5/2}(f-icfx)^{5/2}\left(\frac{1}{6}x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))^2-\frac{1}{3}bc\left(\frac{(c^2x^2+1)^3(a+b\text{arcsinh}(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arc}}{}\right.\right.\right.}{}\right.\right.$$

input
```
Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*((x*(1 + c^2*x^2)^(5/2)*(a + b*Ar
cSinh[c*x])^2)/6 - (b*c*(((1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2) -
(b*((x*(1 + c^2*x^2)^(5/2))/6 + (5*((x*(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqr
t[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c)))/4))/6))/(6*c)))/3 + (5*((x*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*((x*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/2 + (a + b*ArcSinh[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*Arc
Sinh[c*x]))/2 - (b*c*((x*Sqrt[1 + c^2*x^2])/(2*c^2) - ArcSinh[c*x]/(2*c^3)
))/2)))/4 - (b*c*(((1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4*c^2) - (b*((x*
(1 + c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 + c^2*x^2])/2 + ArcSinh[c*x]/(2*c))
)/4))/(4*c)))/2))/6))/(1 + c^2*x^2)^(5/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1953 vs. $2(512) = 1024$.

Time = 4.58 (sec) , antiderivative size = 1954, normalized size of antiderivative = 3.20

method	result	size
default	Expression too large to display	1954
parts	Expression too large to display	1954

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/6*I*a^2/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(7/2)+1/6*I*a^2*d/c/f*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(7/2)+1/8*I*a^2*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x
)^(7/2)-1/24*I*a^2*d^2/c*(f-I*c*f*x)^(5/2)*(d+I*c*d*x)^(1/2)-5/48*I*a^2*d^
2*f/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-5/16*I*a^2*d^2*f^2/c*(f-I*c*f*x)
^(1/2)*(d+I*c*d*x)^(1/2)+5/16*a^2*d^3*f^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/
(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*
x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(5/48*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*
f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3*f^2*d^2+1/6912*(-I*(I+x*c)*f)^
(1/2)*(I*(x*c-I)*d)^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x^5*
c^5+48*x^4*c^4*(c^2*x^2+1)^(1/2)+38*x^3*c^3+18*x^2*c^2*(c^2*x^2+1)^(1/2)+6
*x*c+(c^2*x^2+1)^(1/2))*(18*arcsinh(x*c)^2-6*arcsinh(x*c)+1)*f^2*d^2/(c^2*
x^2+1)/c+3/1024*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(8*x^5*c^5+8*x^4*
c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^
2+1)^(1/2))*(8*arcsinh(x*c)^2-4*arcsinh(x*c)+1)*f^2*d^2/(c^2*x^2+1)/c+15/2
56*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+
1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x*c)+1)*f^2*
d^2/(c^2*x^2+1)/c+15/256*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)*(2*x^3*c
^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2+
2*arcsinh(x*c)+1)*f^2*d^2/(c^2*x^2+1)/c+3/1024*(-I*(I+x*c)*f)^(1/2)*(I*(x*
c-I)*d)^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3-8*x^2*c...
```

Fricas [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((b^2*c^4*d^2*f^2*x^4 + 2*b^2*c^2*d^2*f^2*x^2 + b^2*d^2*f^2)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^
4*d^2*f^2*x^4 + 2*a*b*c^2*d^2*f^2*x^2 + a*b*d^2*f^2)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^4*d^2*f^2*x^4 + 2*a^
2*c^2*d^2*f^2*x^2 + a^2*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}\,(f-c\,f\,x\,1\text{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d^2f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+8\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^5x^5+26\cdots\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))^2,x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 8
*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c**5*x**5 + 26*sqrt(c*i*x + 1)*sq
rt( - c*i*x + 1)*a**2*c**3*x**3 + 33*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a*
*2*c*x + 96*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**4,x)*a*b*
c**5 + 192*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)*x**2,x)*a*b*c
**3 + 96*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x),x)*a*b*c + 48*i
nt(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)**2*x**4,x)*b**2*c**5 + 96
*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)**2*x**2,x)*b**2*c**3 +
48*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)**2,x)*b**2*c))/(48*c)
```

3.250 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 817

$$\begin{aligned}
&\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \\
&-\frac{16ib^2df^2\sqrt{d+icdx}\sqrt{f-icfx}}{75c}+\frac{15}{64}b^2df^2x\sqrt{d+icdx}\sqrt{f-icfx} \\
&-\frac{8ib^2df^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{225c}+\frac{1}{32}b^2df^2x\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2) \\
&-\frac{2ib^2df^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^2}{125c}-\frac{9b^2df^2\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} \\
&+\frac{2ibdf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{5\sqrt{1+c^2x^2}} \\
&-\frac{3bcdf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} \\
&+\frac{4ibc^2df^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{15\sqrt{1+c^2x^2}} \\
&+\frac{2ibc^4df^2x^5\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{25\sqrt{1+c^2x^2}} \\
&-\frac{bdf^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{8c} \\
&+\frac{3}{8}df^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2+\frac{1}{4}df^2x\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))
\end{aligned}$$

output
```
-16/75*I*b^2*d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d*f^2*x
*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-8/225*I*b^2*d*f^2*(d+I*c*d*x)^(1/2)*(
f-I*c*f*x)^(1/2)*(c^2*x^2+1)/c+1/32*b^2*d*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f
*x)^(1/2)*(c^2*x^2+1)-2/125*I*b^2*d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)*(c^2*x^2+1)^2/c-9/64*b^2*d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsi
nh(c*x)/c/(c^2*x^2+1)^(1/2)+2/5*I*b*d*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^
(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)-3/8*b*c*d*f^2*x^2*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+4/15*I*b*c^2*d
*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1
)^(1/2)+2/25*I*b*c^4*d*f^2*x^5*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*ar
csinh(c*x))/(c^2*x^2+1)^(1/2)-1/8*b*d*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1
/2)*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c+3/8*d*f^2*x*(d+I*c*d*x)^(1/2)*(
f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2+1/4*d*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c
*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-1/5*I*d*f^2*(d+I*c*d*x)^(1/2)
*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c+1/8*d*f^2*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.33

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x
]
```

output
```
((72000*I)*a*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (57600*I)*a
^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72000*I)
*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*
a^2*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (115
200*I)*a^2*c^2*d*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] + 72000*a^2*c^3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1
+ c^2*x^2] - (57600*I)*a^2*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Sqrt[1 + c^2*x^2] + 36000*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*ArcSinh[c*x]^3 - 72000*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[2*ArcSinh[c*x]] - (4000*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[3*ArcSinh[c*x]] - 4500*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[4*ArcSinh[c*x]] - (288*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Cosh[5*ArcSinh[c*x]] + 108000*a^2*d^(3/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log
[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^
2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (12000*
I)*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 11
25*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 18
00*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (20*
I)*b*Sqrt[1 + c^2*x^2] - (10*I)*b*Cosh[3*ArcSinh[c*x]] - (2*I)*b*Cosh[5*Ar
cSinh[c*x]] + 40*b*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) + (...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\int f(1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{\left(c^2x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int(1-icx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\int\left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2-icx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))^2\right)dx}{(c^2x^2+1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(\frac{2}{25}ibc^4x^5(a+b\text{arcsinh}(cx))+\frac{4}{15}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{1}{4}x\left(c^2x^2+1\right)^{3/2}(a+\right.}{}$$

input
```
Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*((((-16*I)/75)*b^2*Sqrt[1 + c^2
*x^2])/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 - (((8*I)/225)*b^2*(1 + c^2*x^2
)^(3/2))/c + (b^2*x*(1 + c^2*x^2)^(3/2))/32 - (((2*I)/125)*b^2*(1 + c^2*x^
2)^(5/2))/c - (9*b^2*ArcSinh[c*x])/(64*c) + ((2*I)/5)*b*x*(a + b*ArcSinh[c
*x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8 + ((4*I)/15)*b*c^2*x^3*(a + b*Ar
cSinh[c*x]) + ((2*I)/25)*b*c^4*x^5*(a + b*ArcSinh[c*x]) - (b*(1 + c^2*x^2)
^2*(a + b*ArcSinh[c*x]))/(8*c) + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/8 + (x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((I/5)*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/c + (a + b*ArcSinh[c*x])^3/(8*b*c)))
/(1 + c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(676) = 1352$.

Time = 4.81 (sec) , antiderivative size = 2563, normalized size of antiderivative = 3.14

method	result	size
default	Expression too large to display	2563
parts	Expression too large to display	2563

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/5*I*a^2/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(7/2)+3/20*I*a^2*d/c/f*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(7/2)-1/20*I*a^2*d/c*(f-I*c*f*x)^(5/2)*(d+I*c*d*x)^
(1/2)-1/8*I*a^2*d*f/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-3/8*I*a^2*d*f^2/
c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+3/8*a^2*d^2*f^3*((f-I*c*f*x)*(d+I*c*
d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/
2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/8*(I*(x*c-I)*d)^(1/2)*(
-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3*f^2*d-1/4000*I*(I*(
x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*
c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)
*x*c+1)*(25*arcsinh(x*c)^2-10*arcsinh(x*c)+2)*f^2*d/(c^2*x^2+1)/c+1/512*(I
*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1
/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*(8*arc
sinh(x*c)^2-4*arcsinh(x*c)+1)*f^2*d/(c^2*x^2+1)/c-1/288*I*(I*(x*c-I)*d)^(1
/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+
3*(c^2*x^2+1)^(1/2)*x*c+1)*(9*arcsinh(x*c)^2-6*arcsinh(x*c)+2)*f^2*d/(c^2*
x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^2*c^
2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsinh(x
*c)+1)*f^2*d/(c^2*x^2+1)/c-1/16*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)
*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2-2*arcsinh(x*c)+2)*f^2*d
/(c^2*x^2+1)/c-1/16*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2...
```

Fricas [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((-I*b^2*c^3*d*f^2*x^3 + b^2*c^2*d*f^2*x^2 - I*b^2*c*d*f^2*x + b^2
*d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^
2 - 2*(I*a*b*c^3*d*f^2*x^3 - a*b*c^2*d*f^2*x^2 + I*a*b*c*d*f^2*x - a*b*d*f
^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-
I*a^2*c^3*d*f^2*x^3 + a^2*c^2*d*f^2*x^2 - I*a^2*c*d*f^2*x + a^2*d*f^2)*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d\,f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-8\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^4i\,x^4+10\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))^2,x)
```

output

```
(sqrt(f)*sqrt(d)*d*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - 8*sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c**4*i*x**4 + 10*sqrt(c*i*x + 1)*sqr
t( - c*i*x + 1)*a**2*c**3*x**3 - 16*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**
2*c**2*i*x**2 + 25*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c*x - 8*sqrt(c*
i*x + 1)*sqrt( - c*i*x + 1)*a**2*i - 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)*x**3,x)*a*b*c**4*i + 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)*x**2,x)*a*b*c**3 - 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x +
1)*asinh(c*x)*x,x)*a*b*c**2*i + 80*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*
asinh(c*x),x)*a*b*c - 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(c*x)
**2*x**3,x)*b**2*c**4*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asinh(
c*x)**2*x**2,x)*b**2*c**3 - 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin
h(c*x)**2*x,x)*b**2*c**2*i + 40*int(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asi
nh(c*x)**2,x)*b**2*c))/(40*c)
```

3.251 $\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 680

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{8ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} -\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} -\frac{4ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)}{27c} -\frac{15b^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\operatorname{arcsinh}(cx)}{64c\sqrt{1+c^2x^2}} +\frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{3\sqrt{1+c^2x^2}} -\frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} +\frac{4ibc^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{9\sqrt{1+c^2x^2}} +\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))}{8\sqrt{1+c^2x^2}} +\frac{3}{8}f^2x\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 -\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^2 -\frac{2if^2\sqrt{d+icdx}\sqrt{f-icfx}(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c} +\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}(a+b\operatorname{arcsinh}(cx))^3}{24bc\sqrt{1+c^2x^2}}$$

output
```
-8/9*I*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*f^2*x*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)-4/27*I*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+
1)/c-15/64*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*arcsinh(c*x)/c/(c^2
*x^2+1)^(1/2)+4/3*I*b*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsi
nh(c*x))/(c^2*x^2+1)^(1/2)-3/8*b*c*f^2*x^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(
1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+4/9*I*b*c^2*f^2*x^3*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2)+1/8*b*c^3*f^2
*x^4*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1
/2)+3/8*f^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2-1/4
*c^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^2-2/3*
I*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2
/c+5/24*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(
c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.31

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx = \frac{6912iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-4608ia^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}-69}{}$$

input
```
Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((6912*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (4608*I)*a^2*f
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (6912*I)*b^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*f^2*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (4608*I)*a^2*c^2*
f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c
^3*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^
2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (256*I)*b^2*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*f^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*Sqrt[d]*f^
(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]] + 864*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*
ArcSinh[c*x]] + (768*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3
*ArcSinh[c*x]] - 27*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*Arc
Sinh[c*x]] + 12*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((5
76*I)*b*c*x - (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] - (
192*I)*a*Cosh[3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*Ar
cSinh[c*x]] + (64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) +
 72*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (48*I
)*b*Sqrt[1 + c^2*x^2] - (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*Arc...
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\int f^2(1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int(1-icx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\int\left(-c^2x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-2icx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\sqrt{c^2x^2+}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{1}{8}bc^3x^4(a+b\text{arcsinh}(cx))+\frac{4}{9}ibc^2x^3(a+b\text{arcsinh}(cx))+\frac{3}{8}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx\right.}{}$$

input
```
Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((((-8*I)/9)*b^2*Sqrt[1 + c^2*x^2
])/c + (15*b^2*x*Sqrt[1 + c^2*x^2])/64 - (b^2*c^2*x^3*Sqrt[1 + c^2*x^2])/3
2 - (((4*I)/27)*b^2*(1 + c^2*x^2)^(3/2))/c - (15*b^2*ArcSinh[c*x])/(64*c)
+ ((4*I)/3)*b*x*(a + b*ArcSinh[c*x]) - (3*b*c*x^2*(a + b*ArcSinh[c*x]))/8
+ ((4*I)/9)*b*c^2*x^3*(a + b*ArcSinh[c*x]) + (b*c^3*x^4*(a + b*ArcSinh[c*x
]))/8 + (3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*x^3*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*(1 + c^2*x^2)^(3/2)*(a
+ b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(24*b*c)))/Sqrt[1 + c^
2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1962 vs. $2(559) = 1118$.

Time = 4.98 (sec), antiderivative size = 1963, normalized size of antiderivative = 2.89

method	result	size
default	Expression too large to display	1963
parts	Expression too large to display	1963

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/4*I*a^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(7/2)-1/12*I*a^2/c*(f-I*c*f*x)
^(5/2)*(d+I*c*d*x)^(1/2)-5/24*I*a^2*f/c*(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2
)-5/8*I*a^2*f^2/c*(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+5/8*a^2*d*f^3*((f-I*
c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2)*ln(c^2*d*f*x
/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(5/24*(I*(x*
c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(x*c)^3*f^2-
1/512*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x
^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2)
)*(8*arcsinh(x*c)^2-4*arcsinh(x*c)+1)*f^2/(c^2*x^2+1)/c-1/108*I*(I*(x*c-I)
*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^
2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*(9*arcsinh(x*c)^2-6*arcsinh(x*c)+2)*f^2/(
c^2*x^2+1)/c+1/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3+2*x^
2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2-2*arcsi
nh(x*c)+1)*f^2/(c^2*x^2+1)/c-1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2
)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2-2*arcsinh(x*c)+2)*f^2/
(c^2*x^2+1)/c-1/4*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2-(c^2
*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)^2+2*arcsinh(x*c)+2)*f^2/(c^2*x^2+1)/c+1
/16*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2
+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)*f^2
/(c^2*x^2+1)/c-1/108*I*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(4*c^4*...
```

Fricas [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int \sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral(-(b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*f^2*x^2 + 2
*I*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) - (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\int (a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}\,(f-c\,f\,x\,1\text{i})^{5/2}\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2\,dx=\frac{\sqrt{f}\,\sqrt{d}\,f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-6\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^3x^3-16\sqrt{cix+1}\,\sqrt{-ci}\right.}{}$$

input
```
int((d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))^2,x)
```

output

```
(sqrt(f)*sqrt(d)*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - 6*sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c**3*x**3 - 16*sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)*a**2*c**2*i*x**2 + 9*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*c
*x - 16*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i - 48*int(sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*asinh(c*x)*x**2,x)*a*b*c**3 - 96*int(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*asinh(c*x)*x,x)*a*b*c**2*i + 48*int(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*asinh(c*x),x)*a*b*c - 24*int(sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*asinh(c*x)**2*x**2,x)*b**2*c**3 - 48*int(sqrt(c*i*x + 1)*sqrt( - c*i*
x + 1)*asinh(c*x)**2*x,x)*b**2*c**2*i + 24*int(sqrt(c*i*x + 1)*sqrt( - c*i
*x + 1)*asinh(c*x)**2,x)*b**2*c))/(24*c)
```

3.252 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 615

$$\begin{aligned}\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx &= -\frac{68ib^2f^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{3b^2f^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3b^2f^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{22ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{3bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{11if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{icf^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
-68/9*I*b^2*f^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*
f^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2/27*I*b^2*f^3*(c^2*
x^2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*f^3*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+22/3*I*b*f^3*x*(c^2*
x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/2*b*
c*f^3*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)-2/9*I*b*c^2*f^3*x^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d
*x)^(1/2)/(f-I*c*f*x)^(1/2)-11/3*I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/
(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x
))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*c*f^3*x^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*f^3*(c^2*x^2+1)^(1
/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 13.36 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.18

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \frac{1620iabcf^2x\sqrt{d+icdx}\sqrt{f-icfx}-792ia^2f^2\sqrt{d+icdx}\sqrt{f-ic}}{}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x
]
```

output
```
((1620*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (792*I)*a^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1620*I)*b^2*f^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*f^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*f^2*
x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(-4*b*c*x
*(-33 + c^2*x^2) + 27*a*(-5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] + 3*a*Cosh[3*Ar
cSinh[c*x]])) + 540*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*f^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(30*a - (45*I)*b*Sqrt[1 + c^2*x^2] + I*
b*Cosh[3*ArcSinh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*d*Sqrt[1 + c^2*
x^2])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{f^3(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3\sqrt{c^2x^2+1}\int \frac{(1-icx)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6258}$$

$$\frac{f^3\sqrt{c^2x^2+1}\int\left(c-ic^2x\right)^3(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3042

$$\frac{f^3\sqrt{c^2x^2+1}\int(a+b\text{arcsinh}(cx))^2(c-c\sin(i\text{arcsinh}(cx)))^3d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3798

$$\frac{f^3\sqrt{c^2x^2+1}\int\left(ix^3(a+b\text{arcsinh}(cx))^2c^6-3x^2(a+b\text{arcsinh}(cx))^2c^5-3ix(a+b\text{arcsinh}(cx))^2c^4+(a+b\text{arcsin}\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 2009

$$\frac{f^3\sqrt{c^2x^2+1}\left(-\frac{2}{9}ibc^6x^3(a+b\text{arcsinh}(cx))+\frac{3}{2}bc^5x^2(a+b\text{arcsinh}(cx))+\frac{22}{3}ibc^4x(a+b\text{arcsinh}(cx))+\frac{5c^3(a+b\text{arc}}{6}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

output
```
(f^3*Sqrt[1 + c^2*x^2]*(((-68*I)/9)*b^2*c^3*Sqrt[1 + c^2*x^2] - (3*b^2*c^4
*x*Sqrt[1 + c^2*x^2])/4 + ((2*I)/27)*b^2*c^3*(1 + c^2*x^2)^(3/2) + (3*b^2*
c^3*ArcSinh[c*x])/4 + ((22*I)/3)*b*c^4*x*(a + b*ArcSinh[c*x]) + (3*b*c^5*x
^2*(a + b*ArcSinh[c*x]))/2 - ((2*I)/9)*b*c^6*x^3*(a + b*ArcSinh[c*x]) - ((
11*I)/3)*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2 - (3*c^4*x*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (I/3)*c^5*x^2*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])^2 + (5*c^3*(a + b*ArcSinh[c*x])^3)/(6*b)))/(c^4*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1476 vs. $2(511) = 1022$.

Time = 4.11 (sec) , antiderivative size = 1477, normalized size of antiderivative = 2.40

method	result	size
default	Expression too large to display	1477
parts	Expression too large to display	1477

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/3*I*a^2/c/d*(f-I*c*f*x)^(5/2)*(d+I*c*d*x)^(1/2)-5/6*I*a^2*f/c/d*(f-I*c*
f*x)^(3/2)*(d+I*c*d*x)^(1/2)-5/2*I*a^2*f^2/c/d*(f-I*c*f*x)^(1/2)*(d+I*c*d*
x)^(1/2)+5/2*a^2*f^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(f-I*c*f*x)^(1/2)/(d+
I*c*d*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*
d*f)^(1/2)+b^2*(5/6*f^2*arcsinh(x*c)^3*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^
(1/2)/(c^2*x^2+1)^(1/2)/d/c+1/216*I*f^2*(9*arcsinh(x*c)^2-6*arcsinh(x*c)+2
)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c
+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-3/16*f^2*(2*a
rcsinh(x*c)^2-2*arcsinh(x*c)+1)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x
*c+(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d/c/(c^2*x^
2+1)-15/8*I*f^2*(arcsinh(x*c)^2-2*arcsinh(x*c)+2)*(c^2*x^2+(c^2*x^2+1)^(1/
2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-15/8*I*
f^2*(arcsinh(x*c)^2+2*arcsinh(x*c)+2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I
*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1)-3/16*f^2*(2*arcsinh
(x*c)^2+2*arcsinh(x*c)+1)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^
2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/d/c/(c^2*x^2+1)+1
/216*I*f^2*(9*arcsinh(x*c)^2+6*arcsinh(x*c)+2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1
/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(
I+x*c)*f)^(1/2)/d/c/(c^2*x^2+1))+2*a*b*(5/4*f^2*arcsinh(x*c)^2*(I*(x*c-I)*
d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/d/c+1/72*I*f^2*(-1+3*ar...
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}}\,dx$$

input

```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="fricas")
```

output

```
integral(((I*b^2*c^2*f^2*x^2 - 2*b^2*c*f^2*x - I*b^2*f^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*f^2*x^
2 + 2*a*b*c*f^2*x + I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*f^2*x^2 - 2*a^2*c*f^2*x - I*a^2*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\int \frac{\left(a+b\,\text{asinh}(c\,x)\right)^2\,\left(f-c\,f\,x\,1\text{i}\right)^{5/2}}{\sqrt{d+c\,d\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}}\,dx=\frac{\sqrt{f}\,f^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+2\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^2i\,x^2-\right.}{}$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

output
```
(sqrt(f)*f**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 2*sqrt(c*i*x +
 1)*sqrt( - c*i*x + 1)*a**2*c**2*i*x**2 - 9*sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*a**2*c*x - 22*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i - 12*int((sqr
t( - c*i*x + 1)*asinh(c*x)*x**2)/sqrt(c*i*x + 1),x)*a*b*c**3 - 24*int((sqr
t( - c*i*x + 1)*asinh(c*x)*x)/sqrt(c*i*x + 1),x)*a*b*c**2*i + 12*int((sqrt
( - c*i*x + 1)*asinh(c*x))/sqrt(c*i*x + 1),x)*a*b*c - 6*int((sqrt( - c*i*x
 + 1)*asinh(c*x)**2*x**2)/sqrt(c*i*x + 1),x)*b**2*c**3 - 12*int((sqrt( - c
*i*x + 1)*asinh(c*x)**2*x)/sqrt(c*i*x + 1),x)*b**2*c**2*i + 6*int((sqrt( -
 c*i*x + 1)*asinh(c*x)**2)/sqrt(c*i*x + 1),x)*b**2*c))/(6*sqrt(d)*c)
```

3.253 $\int \frac{(f-icfx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 947

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \frac{8ib^2f^3(1+c^2x^2)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2f^3x(1+c^2x^2)}{4d\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2f^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4cd\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{d\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{bcf^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2d\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{8if^3(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{8f^3x(a+b\text{arcsinh}(cx))^2}{d\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{8f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4if^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{cd\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^3x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2d\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bcd\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{32ibf^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{16bf^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{16b^2f^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{16b^2f^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8b^2f^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cd\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
8*I*f^3*(a+b*arcsinh(c*x))^2/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b
^2*f^3*x*(c^2*x^2+1)/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*f^3*(c^
2*x^2+1)^(1/2)*arcsinh(c*x)/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*f^
3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
-1/2*b*c*f^3*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/d/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)-8*I*b*f^3*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/d/(d+I*
c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8*f^3*x*(a+b*arcsinh(c*x))^2/d/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)+8*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d/(
d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8*I*b^2*f^3*(c^2*x^2+1)/c/d/(d+I*c*d*x)
^(1/2)/(f-I*c*f*x)^(1/2)+1/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/d/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-5/2*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x
))^3/b/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-32*I*b*f^3*(c^2*x^2+1)^(1/2
)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)-16*b*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-16*b^2*f^3*(c
^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)+16*b^2*f^3*(c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b^2*f^3*(c^2*x^2+1)^
(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/d/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2492 vs. $2(947) = 1894$.

Time = 22.39 (sec) , antiderivative size = 2492, normalized size of antiderivative = 2.63

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((4*I)*a^2*f^2)/d^2 + (a^2*c
*f^2*x)/(2*d^2) + (8*a^2*f^2)/(d^2*(-I + c*x))))/c - (15*a^2*f^(5/2)*Log[c
*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*
c*d^(3/2)) + ((4*I)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*
x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c
*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[Ar
cSinh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]]) + I*(-(c*x) - 2*ArcSinh[c*x
] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcS
inh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c
*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(Arc
Sinh[c*x]*(-4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*L
og[Sqrt[1 + c^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcT
an[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2])
)/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[A
rcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (b^2*f^2*Sqrt[I*((-I)*d + c*d*
x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]
/2]*((6*I)*Pi*ArcSinh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + ArcSinh[c*x]^3 + 1
2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24*I)*Pi*Log[...
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{f^4(1-icx)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\int \frac{(1-icx)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\downarrow$ 6259

$$\frac{f^4\left(c^2x^2+1\right)^{3/2}\int \left(\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{4icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{7(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{8i(cx+i)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right) d}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\downarrow$ 2009

$$f^4\left(c^2x^2+1\right)^{3/2}\left(-\frac{32ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsi}}{c}\right.$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

output
```
(f^4*(1 + c^2*x^2)^(3/2)*((-8*I)*a*b*x + ((8*I)*b^2*Sqrt[1 + c^2*x^2])/c +
 (b^2*x*Sqrt[1 + c^2*x^2])/4 - (b^2*ArcSinh[c*x])/(4*c) - (8*I)*b^2*x*ArcS
inh[c*x] - (b*c*x^2*(a + b*ArcSinh[c*x]))/2 + (8*(a + b*ArcSinh[c*x])^2)/c
 + ((8*I)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (8*x*(a + b*ArcS
inh[c*x])^2)/Sqrt[1 + c^2*x^2] + ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - (5*(a + b*Arc
Sinh[c*x])^3)/(2*b*c) - ((32*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*
x]])/c - (16*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c - (16*b
^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c + (16*b^2*PolyLog[2, I*E^ArcSinh[c*x
]])/c - (8*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.01

method	result
default	$-\frac{f^2\left(32a^2b+a\,b^2+10a^3-128\ln\left(xc+\sqrt{c^2x^2+1}\right)a\,b^2+32a^2b\,c^2x^2+3a\,b^2c^2x^2-\sqrt{c^2x^2+1}\,b^3cx+30\ \operatorname{arcsinh}(xc)a^2b\,c^2x^2+128\,\mathrm{polylo}}{}$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/4*f^2*(32*a^2*b+a*b^2+128*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*b^3+10*
a^3+128*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*b^3*arcsinh(x*c)-128*ln(x*c+(c^2*x
^2+1)^(1/2))*a*b^2+128*ln(x*c+(c^2*x^2+1)^(1/2)-I)*a*b^2+32*a^2*b*c^2*x^2+
3*a*b^2*c^2*x^2-(c^2*x^2+1)^(1/2)*b^3*c*x-32*I*(c^2*x^2+1)^(1/2)*b^3*c^2*x
^2+32*I*a*b^2*c^3*x^3+32*I*a*b^2*c*x+30*arcsinh(x*c)*a^2*b*c^2*x^2-32*I*(c
^2*x^2+1)^(1/2)*b^3-16*I*(c^2*x^2+1)^(1/2)*a^2*b*c^2*x^2-128*ln(x*c+(c^2*x
^2+1)^(1/2))*a*b^2*c^2*x^2+128*ln(x*c+(c^2*x^2+1)^(1/2)-I)*a*b^2*c^2*x^2+3
2*I*arcsinh(x*c)*b^3*c^3*x^3-96*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*a*b^2+32*
I*arcsinh(x*c)*b^3*c*x-34*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*b^3*c*x+64*arcs
inh(x*c)*a*b^2*c^2*x^2-2*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*b^3*c^3*x^3+30*a
rcsinh(x*c)^2*a*b^2*c^2*x^2+128*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)
))*b^3*c^2*x^2+30*a*b^2*arcsinh(x*c)^2-68*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*a
*b^2*c*x-16*I*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*b^3*c^2*x^2-4*arcsinh(x*c)*
(c^2*x^2+1)^(1/2)*a*b^2*c^3*x^3+2*a*b^2*c^4*x^4-48*I*(c^2*x^2+1)^(1/2)*a^2
*b-34*(c^2*x^2+1)^(1/2)*a^2*b*c*x-32*I*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*a*b^
2*c^2*x^2+10*b^3*arcsinh(x*c)^3-32*b^3*arcsinh(x*c)^2+b^3*arcsinh(x*c)+30*
a^2*b*arcsinh(x*c)+64*a*b^2*arcsinh(x*c)-(c^2*x^2+1)^(1/2)*b^3*c^3*x^3-32*
arcsinh(x*c)^2*b^3*c^2*x^2+3*arcsinh(x*c)*b^3*c^2*x^2+2*arcsinh(x*c)*b^3*c
^4*x^4+10*arcsinh(x*c)^3*b^3*c^2*x^2-48*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2
*b^3+128*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*b^3*c^2*x^2-2*(c^2*x^2+1...
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input

```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="fricas")
```

output

```
integral(((b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*f^2*x^2 + 2
*I*a*b*c*f^2*x - a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x
)/(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a^2 + integrat
e((-I*c*f*x + f)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(I*c*d*x + d)^(3
/2) + 2*(-I*c*f*x + f)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d
)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}{(d+c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}}\,dx = \frac{\sqrt{f}\,f^2\left(-30\sqrt{cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+\sqrt{-cix+1}\,a^2c^2i\,x^2-\right.}{}$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

output

```
(sqrt(f)*f**2*( - 30*sqrt(c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2
*i + sqrt( - c*i*x + 1)*a**2*c**2*i*x**2 - 7*sqrt( - c*i*x + 1)*a**2*c*x +
 24*sqrt( - c*i*x + 1)*a**2*i - 4*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*
asinh(c*x)*x**2)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*a*b*c**3 - 8
*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqrt(c*i*x + 1)*c*
i*x + sqrt(c*i*x + 1)),x)*a*b*c**2*i + 4*sqrt(c*i*x + 1)*int((sqrt( - c*i*
x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*a*b*c - 2*
sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2*x**2)/(sqrt(c*i*x +
1)*c*i*x + sqrt(c*i*x + 1)),x)*b**2*c**3 - 4*sqrt(c*i*x + 1)*int((sqrt( -
c*i*x + 1)*asinh(c*x)**2*x)/(sqrt(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b
**2*c**2*i + 2*sqrt(c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2)/(sqr
t(c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)),x)*b**2*c))/(2*sqrt(d)*sqrt(c*i*x +
1)*c*d)
```

3.254 $\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 772

$$\begin{aligned}
\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx &= -\frac{2ib^2f^3(1+c^2x^2)}{cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{2ibf^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{d^2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{28f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{if^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{cd^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bcd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{16ib^2f^3\sqrt{1+c^2x^2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{28if^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8bf^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4if^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{112bf^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{112b^2f^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3cd^2\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
-2*I*b^2*f^3*(c^2*x^2+1)/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b*f
^3*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)-28/3*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d^2/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)-I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/d^2/(d+I*
c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/3*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x)
)^3/b/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-16/3*I*b^2*f^3*(c^2*x^2+1)
^(1/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-28/3*I*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*a
rcsinh(c*x))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*b*f^3*(c^2*x^2+
1)^(1/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/d^2/(d+I*c*
d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4/3*I*f^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x)
)^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/d^2/
(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+112/3*b*f^3*(c^2*x^2+1)^(1/2)*(a+b*arc
sinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*
f*x)^(1/2)+112/3*b^2*f^3*(c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(
1/2)))/c/d^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2622 vs. $2(772) = 1544$.

Time = 23.50 (sec) , antiderivative size = 2622, normalized size of antiderivative = 3.40

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*f^2)/d^3 - (((8*I)
/3)*a^2*f^2)/(d^3*(-I + c*x)^2) - (28*a^2*f^2)/(3*d^3*(-I + c*x))))/c + (5
*a^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*
f*(I + c*x)]])/(c*d^(5/2)) + ((I/3)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[
(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*S
inh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcT
an[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2
]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqr
t[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcS
inh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[
Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/
(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x
]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*
Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x]
)*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2
*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (
6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4
- (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/
2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*...
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-icfx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^5(1-icx)^5(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^5(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6259

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\int \left(-\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{12i(a+b\text{arcsinh}(cx))^2}{(cx-i)\sqrt{c^2x^2+1}}-\frac{8(a+b\text{arcsinh}(cx))^2}{(cx-i)^2\sqrt{c^2x^2+1}}+\frac{5(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{f^5\left(c^2x^2+1\right)^{5/2}\left(-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{5(a+b\text{arcsinh}(cx))^3}{3bc}-\frac{28(a+b\text{arcsinh}(cx))^2}{3c}+\frac{112b\log\left(1+ie^{\text{arcsinh}(cx)}\right)(a}{3c}\right.}{}$$

input
```
Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

output
```
(f^5*(1 + c^2*x^2)^(5/2)*((2*I)*a*b*x - ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c +
(2*I)*b^2*x*ArcSinh[c*x] - (28*(a + b*ArcSinh[c*x])^2)/(3*c) - (I*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(3*b*c) -
 (((16*I)/3)*b^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c - (((28*I)/3)*(a + b*Ar
cSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/c + (8*b*(a + b*ArcSinh[c*x]
)*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((4*I)/3)*(a + b*ArcSinh[c*x]
)^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/c + (
112*b*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c) + (112*b^2*Pol
yLog[2, (-I)*E^ArcSinh[c*x]])/(3*c)))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 1132, normalized size of antiderivative = 1.47

method	result	size
default	Expression too large to display	1132

input
```
int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
5/3*f^2*(a+b*arcsinh(x*c))^3*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2
*x^2+1)^(1/2)/b/d^3/c-1/2*I*f^2*(arcsinh(x*c)^2*b^2+2*arcsinh(x*c)*a*b-2*b
^2*arcsinh(x*c)+a^2-2*a*b+2*b^2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c
-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d^3/c/(c^2*x^2+1)-1/2*I*f^2*(arcsinh(x*c
)^2*b^2+2*arcsinh(x*c)*a*b+2*b^2*arcsinh(x*c)+a^2+2*a*b+2*b^2)*(c^2*x^2-(c
^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/d^3/c/(c^2
*x^2+1)-4/3*f^2*(518*a*b+4554*arcsinh(x*c)^2*b^2*c^2*x^2+1369*a^2+1369*arc
sinh(x*c)^2*b^2+518*b^2*arcsinh(x*c)+888*b^2+882*a*b*c^4*x^4+1288*a*b*c^2*
x^2-252*(c^2*x^2+1)^(1/2)*b^2*c^3*x^3-236*(c^2*x^2+1)^(1/2)*b^2*c*x+882*(c
^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2*c^3*x^3+442*(c^2*x^2+1)^(1/2)*arcsinh(x*c
)*b^2*c*x+882*arcsinh(x*c)*b^2*c^4*x^4+1288*arcsinh(x*c)*b^2*c^2*x^2+882*(
c^2*x^2+1)^(1/2)*a*b*c^3*x^3+442*(c^2*x^2+1)^(1/2)*a*b*c*x+9108*arcsinh(x*
c)*a*b*c^2*x^2+370*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2+714*I*arcsinh(x*c)
*(c^2*x^2+1)^(1/2)*b^2*c^2*x^2+2738*arcsinh(x*c)*a*b-232*I*b^2*c*x-456*I*b
^2*c^3*x^3+370*I*(c^2*x^2+1)^(1/2)*a*b-148*I*(c^2*x^2+1)^(1/2)*b^2+714*I*(
c^2*x^2+1)^(1/2)*a*b*c^2*x^2-420*I*arcsinh(x*c)*b^2*c^3*x^3-308*I*arcsinh(
x*c)*b^2*c*x+2016*b^2*c^4*x^4+2680*b^2*c^2*x^2+7938*arcsinh(x*c)*a*b*c^4*x
^4+3969*arcsinh(x*c)^2*b^2*c^4*x^4-420*I*a*b*c^3*x^3-308*I*a*b*c*x-132*I*(
c^2*x^2+1)^(1/2)*b^2*c^2*x^2+4554*a^2*c^2*x^2+3969*a^2*c^4*x^4)*(7*x^3*c^3
+9*I*c^2*x^2-7*x^2*c^2*(c^2*x^2+1)^(1/2)+3*x*c+5*I-7*(c^2*x^2+1)^(1/2))...
```

Fricas [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{(-i\,cfx+f)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((-I*b^2*c^2*f^2*x^2 + 2*b^2*c*f^2*x + I*b^2*f^2)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*f^2*x^
2 - 2*a*b*c*f^2*x - I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*f^2*x^2 + 2*a^2*c*f^2*x + I*a^2*f^2)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c
*d^3*x + I*d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f - icfx)^{5/2}(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)^2\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(f-icfx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((f-I*c*f*x)^(5/2)*(a+b*asinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

output

```
(sqrt(f)*f**2*( - 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x
 + 1)/sqrt(2))*a**2*c*x + 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt(
 - c*i*x + 1)/sqrt(2))*a**2*i + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((
sqrt( - c*i*x + 1)*asinh(c*x)*x**2)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*
i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*b*c**4*i*x + 6*sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x**2)/(sqrt(c*i*x + 1)*c
**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*b*c**3 - 12*sqr
t(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)*x)/(sqr
t(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*b
*c**3*x + 12*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*as
inh(c*x)*x)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*
i*x + 1)),x)*a*b*c**2*i - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(
- c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*
i*x - sqrt(c*i*x + 1)),x)*a*b*c**2*i*x - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x +
 1)*int((sqrt( - c*i*x + 1)*asinh(c*x))/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqr
t(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*a*b*c + 3*sqrt(c*i*x + 1)*sqrt( -
 c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2*x**2)/(sqrt(c*i*x + 1)*c
**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)),x)*b**2*c**4*i*x + 3
*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt( - c*i*x + 1)*asinh(c*x)**2*
x**2)/(sqrt(c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*c*i*x - sqrt(c*i*x...
```

3.255 $\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 615

$$\begin{aligned}\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{68ib^2d^3(1+c^2x^2)}{9c\sqrt{d+icdx}\sqrt{f-icfx}}\\
&- \frac{3b^2d^3x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3(1+c^2x^2)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}}\\
&+ \frac{3b^2d^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{22ibd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3\sqrt{d+icdx}\sqrt{f-icfx}}\\
&+ \frac{3bcd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}}\\
&+ \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9\sqrt{d+icdx}\sqrt{f-icfx}}\\
&+ \frac{11id^3(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3d^3x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}\\
&- \frac{icd^3x^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
68/9*I*b^2*d^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*d
^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/27*I*b^2*d^3*(c^2*x
^2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*d^3*(c^2*x^2+1)^(1/2
)*arcsinh(c*x)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-22/3*I*b*d^3*x*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/2*b*c
*d^3*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)+2/9*I*b*c^2*d^3*x^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*
x)^(1/2)/(f-I*c*f*x)^(1/2)+11/3*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(
d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x)
)^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*c*d^3*x^2*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*d^3*(c^2*x^2+1)^(1/
2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 13.27 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.18

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{-1620iabcd^2x\sqrt{d+icdx}\sqrt{f-icfx}+792ia^2d^2\sqrt{d+icdx}\sqrt{f-}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x
]
```

output
```
((-1620*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (792*I)*a^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1620*I)*b^2*d^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*d^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*d^2
*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*d^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*d^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*d^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(4*b*c*x
*(-33 + c^2*x^2) + 27*a*(5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] - 3*a*Cosh[3*Arc
Sinh[c*x]])) + 540*a^2*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqr
t[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*d^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(30*a + (45*I)*b*Sqrt[1 + c^2*x^2] - I*b
*Cosh[3*ArcSinh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*d^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*f*Sqrt[1 + c^2*x
^2])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\sqrt{c^2x^2+1}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6258}$$

$$\frac{d^3\sqrt{c^2x^2+1}\int \left(ixc^2+c\right)^3(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\big\downarrow$ 3042

$$\frac{d^3\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(\sin(i\text{arcsinh}(cx))c+c)^3d\text{arcsinh}(cx)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\big\downarrow$ 3798

$$\frac{d^3\sqrt{c^2x^2+1}\int \left(-ix^3(a+b\text{arcsinh}(cx))^2c^6-3x^2(a+b\text{arcsinh}(cx))^2c^5+3ix(a+b\text{arcsinh}(cx))^2c^4+(a+b\text{arcsi}\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\big\downarrow$ 2009

$$d^3\sqrt{c^2x^2+1}\left(\frac{2}{9}ibc^6x^3(a+b\text{arcsinh}(cx))+\frac{3}{2}bc^5x^2(a+b\text{arcsinh}(cx))-\frac{22}{3}ibc^4x(a+b\text{arcsinh}(cx))+\frac{5c^3(a+b\text{arcsi}}{6b}\right.$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
(d^3*Sqrt[1 + c^2*x^2]*(((68*I)/9)*b^2*c^3*Sqrt[1 + c^2*x^2] - (3*b^2*c^4*
x*Sqrt[1 + c^2*x^2])/4 - ((2*I)/27)*b^2*c^3*(1 + c^2*x^2)^(3/2) + (3*b^2*c
^3*ArcSinh[c*x])/4 - ((22*I)/3)*b*c^4*x*(a + b*ArcSinh[c*x]) + (3*b*c^5*x^
2*(a + b*ArcSinh[c*x]))/2 + ((2*I)/9)*b*c^6*x^3*(a + b*ArcSinh[c*x]) + ((1
1*I)/3)*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2 - (3*c^4*x*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - (I/3)*c^5*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])^2 + (5*c^3*(a + b*ArcSinh[c*x])^3)/(6*b)))/(c^4*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1476 vs. $2(511) = 1022$.

Time = 6.34 (sec) , antiderivative size = 1477, normalized size of antiderivative = 2.40

method	result	size
default	Expression too large to display	1477
parts	Expression too large to display	1477

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3*I*a^2/c/f*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(1/2)+5/6*I*a^2*d/c/f*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(1/2)+5/2*I*a^2*d^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x
)^(1/2)+5/2*a^2*d^3*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(d+I*c*d*x)^(1/2)/(f-I
*c*f*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d
*f)^(1/2)+b^2*(5/6*d^2*arcsinh(x*c)^3*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(
1/2)/(c^2*x^2+1)^(1/2)/f/c-1/216*I*d^2*(9*arcsinh(x*c)^2-6*arcsinh(x*c)+2)
*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+
1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)-3/16*d^2*(2*ar
csinh(x*c)^2-2*arcsinh(x*c)+1)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*
c+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2
+1)+15/8*I*d^2*(arcsinh(x*c)^2-2*arcsinh(x*c)+2)*(c^2*x^2+(c^2*x^2+1)^(1/2
)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)+15/8*I*d
^2*(arcsinh(x*c)^2+2*arcsinh(x*c)+2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*
(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)-3/16*d^2*(2*arcsinh(
x*c)^2+2*arcsinh(x*c)+1)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2
*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)-1/
216*I*d^2*(9*arcsinh(x*c)^2+6*arcsinh(x*c)+2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/
2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I
+x*c)*f)^(1/2)/f/c/(c^2*x^2+1))+2*a*b*(5/4*d^2*arcsinh(x*c)^2*(-I*(I+x*c)*
f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/c/f-1/72*I*d^2*(-1+3*arc...
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(((-I*b^2*c^2*d^2*x^2 - 2*b^2*c*d^2*x + I*b^2*d^2)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c^2*d^2*x^
2 + 2*a*b*c*d^2*x - I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*d^2*x^2 - 2*a^2*c*d^2*x + I*a^2*d^2)*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{d}\,d^2\left(30asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-2\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2c^2i\,x^2-9$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

output
```
(sqrt(d)*d**2*(30*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - 2*sqrt(c*i*x +
 1)*sqrt( - c*i*x + 1)*a**2*c**2*i*x**2 - 9*sqrt(c*i*x + 1)*sqrt( - c*i*x
+ 1)*a**2*c*x + 22*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i - 12*int((sqr
t(c*i*x + 1)*asinh(c*x)*x**2)/sqrt( - c*i*x + 1),x)*a*b*c**3 + 24*int((sqr
t(c*i*x + 1)*asinh(c*x)*x)/sqrt( - c*i*x + 1),x)*a*b*c**2*i + 12*int((sqrt
(c*i*x + 1)*asinh(c*x))/sqrt( - c*i*x + 1),x)*a*b*c - 6*int((sqrt(c*i*x +
1)*asinh(c*x)**2*x**2)/sqrt( - c*i*x + 1),x)*b**2*c**3 + 12*int((sqrt(c*i*
x + 1)*asinh(c*x)**2*x)/sqrt( - c*i*x + 1),x)*b**2*c**2*i + 6*int((sqrt(c*
i*x + 1)*asinh(c*x)**2)/sqrt( - c*i*x + 1),x)*b**2*c))/(6*sqrt(f)*c)
```

3.256 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 436

$$\begin{aligned}
\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx &= \frac{4ib^2d^2(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{b^2d^2x(1+c^2x^2)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4ibd^2x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{bcd^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output

```
4*I*b^2*d^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*d^2*
x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*d^2*(c^2*x^2+1)^
(1/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4*I*b*d^2*x*(c^2*
x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*b*
c*d^2*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I
*c*f*x)^(1/2)-1/2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+1/2*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (verified)

Time = 7.61 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{-32iabcdx\sqrt{d+icdx}\sqrt{f-icfx}+16ia^2d\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{}}{}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x
]
```

output
```
((-32*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a^2*d*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (32*I)*b^2*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*d*x*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*ArcSinh[c*x]^3 + 2*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Co
sh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]*((-16*I)*b*c*x - 4*a*(-4*I + c*x)*Sqrt[1 + c^2*x^2] + b*Cosh[2*ArcSinh[c
*x]]) + 12*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqr
t[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*ArcSinh[c*x]^2*(6*a + (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]
]))/(8*c*f*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {6211, 27, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 27

$$\frac{d^2\sqrt{c^2x^2+1}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 6258

$$\frac{d^2\sqrt{c^2x^2+1}\int \left(ixc^2+c\right)^2(a+b\text{arcsinh}(cx))^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3042

$$\frac{d^2\sqrt{c^2x^2+1}\int (a+b\text{arcsinh}(cx))^2(\sin(i\text{arcsinh}(cx))c+c)^2d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 3798

$$\frac{d^2\sqrt{c^2x^2+1}\int \left(-x^2(a+b\text{arcsinh}(cx))^2c^4+2ix(a+b\text{arcsinh}(cx))^2c^3+(a+b\text{arcsinh}(cx))^2c^2\right)d\text{arcsinh}(cx)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 2009

$$\frac{d^2\sqrt{c^2x^2+1}\left(\frac{1}{2}bc^4x^2(a+b\text{arcsinh}(cx))-4ibc^3x(a+b\text{arcsinh}(cx))+2ic^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2+\frac{c^2(a+\cdots}{}\right.}{c^3\sqrt{d+icdx}\cdots}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
(d^2*Sqrt[1 + c^2*x^2]*((4*I)*b^2*c^2*Sqrt[1 + c^2*x^2] - (b^2*c^3*x*Sqrt[
1 + c^2*x^2])/4 + (b^2*c^2*ArcSinh[c*x])/4 - (4*I)*b*c^3*x*(a + b*ArcSinh[
c*x]) + (b*c^4*x^2*(a + b*ArcSinh[c*x]))/2 + (2*I)*c^2*Sqrt[1 + c^2*x^2]*(
a + b*ArcSinh[c*x])^2 - (c^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2
 + (c^2*(a + b*ArcSinh[c*x])^3)/(2*b)))/(c^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(367) = 734$.

Time = 6.21 (sec) , antiderivative size = 978, normalized size of antiderivative = 2.24

method	result
default	$\frac{ia^2(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}{2cf}+\frac{3ia^2d\sqrt{icdx+d}\,\sqrt{-icfx+f}}{2cf}+\frac{3a^2d^2\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{2\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+b^2\Big($
parts	$\frac{ia^2(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}{2cf}+\frac{3ia^2d\sqrt{icdx+d}\,\sqrt{-icfx+f}}{2cf}+\frac{3a^2d^2\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{2\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+b^2\Big($

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2*I*a^2/c/f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)+3/2*I*a^2*d/c/f*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)+3/2*a^2*d^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(d
+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^
2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/2*d*arcsinh(x*c)^3*(I*(x*c-I)*d)^(1/2
)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/f/c-1/16*d*(2*arcsinh(x*c)^2-2*ar
csinh(x*c)+1)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/
2))*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/f/c/(c^2*x^2+1)+I*d*(arcsinh(
x*c)^2-2*arcsinh(x*c)+2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(
1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)+I*d*(arcsinh(x*c)^2+2*arcsinh(x*
c)+2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)
^(1/2)/f/c/(c^2*x^2+1)-1/16*d*(2*arcsinh(x*c)^2+2*arcsinh(x*c)+1)*(2*x^3*c
^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/
2)*(I*(x*c-I)*d)^(1/2)/f/c/(c^2*x^2+1))+2*a*b*(3/4*d*arcsinh(x*c)^2*(-I*(I
+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/f/c-1/16*d*(-1+2*arcs
inh(x*c))*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*
(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)+I*d*(arcsinh(x*c)
-1)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(
1/2)/f/c/(c^2*x^2+1)+I*d*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1
)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)-1/16*d*(1+2*arc
sinh(x*c))*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/...
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{3/2}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{d}\,d\left(6asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2cx+4\sqrt{cix}\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

output
```
(sqrt(d)*d*(6*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i - sqrt(c*i*x + 1)*sq
rt( - c*i*x + 1)*a**2*c*x + 4*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*a**2*i +
4*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/sqrt( - c*i*x + 1),x)*a*b*c**2*i + 4*
int((sqrt(c*i*x + 1)*asinh(c*x))/sqrt( - c*i*x + 1),x)*a*b*c + 2*int((sqrt
(c*i*x + 1)*asinh(c*x)**2*x)/sqrt( - c*i*x + 1),x)*b**2*c**2*i + 2*int((sq
rt(c*i*x + 1)*asinh(c*x)**2)/sqrt( - c*i*x + 1),x)*b**2*c))/(2*sqrt(f)*c)
```

3.257 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 214

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \frac{2ib^2d(1+c^2x^2)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibdx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
2*I*b^2*d*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b*d*x*(c^2
*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*d*(
c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*
d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$$
$$= \frac{3i\sqrt{d+icdx}\sqrt{f-icfx}\left(-2abcx+a^2\sqrt{1+c^2x^2}+2b^2\sqrt{1+c^2x^2}\right)-6ib\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{\right.}}{}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
((3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*
x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) - (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*(a + I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 +
c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
])/(3*c*f*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$$

$$\Big\downarrow \; 6211$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\sqrt{c^2x^2+1}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 6253

$$\frac{d\sqrt{c^2x^2+1}\int \left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$\Big\downarrow$ 2009

$$\frac{d\sqrt{c^2x^2+1}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{(a+b\text{arcsinh}(cx))^3}{3bc}-2iabx-2ib^2x\text{arcsinh}(cx)+\frac{2ib^2\sqrt{c^2x^2+1}}{c}\right)}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

output
```
(d*Sqrt[1 + c^2*x^2]*((-2*I)*a*b*x + ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c - (2*
I)*b^2*x*ArcSinh[c*x] + (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (
a + b*ArcSinh[c*x])^3/(3*b*c)))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(181) = 362$.

Time = 6.21 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.50

method	result
default	$\frac{ia^2\sqrt{icdx+d}\,\sqrt{-icfx+f}}{cf}+\frac{a^2d\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+b^2\left(\frac{\operatorname{arcsinh}(xc)^3\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)}}{3\sqrt{c^2x^2+1}\,fc}\right.$
parts	$\frac{ia^2\sqrt{icdx+d}\,\sqrt{-icfx+f}}{cf}+\frac{a^2d\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+b^2\left(\frac{\operatorname{arcsinh}(xc)^3\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)}}{3\sqrt{c^2x^2+1}\,fc}\right.$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
I*a^2/c/f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+a^2*d*((f-I*c*f*x)*(d+I*c*d*
x))^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)*ln(c^2*d*f*x/(c^2*d*f)^(1/2)
+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+b^2*(1/3*arcsinh(x*c)^3*(-I*(I+x
*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/f/c+1/2*I*(arcsinh(x*c)
^2-2*arcsinh(x*c)+2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1)+1/2*I*(arcsinh(x*c)^2+2*arcsinh(x*c)
+2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(
1/2)/f/c/(c^2*x^2+1))+2*a*b*(1/2*arcsinh(x*c)^2*(-I*(I+x*c)*f)^(1/2)*(I*(x
*c-I)*d)^(1/2)/(c^2*x^2+1)^(1/2)/f/c+1/2*I*(arcsinh(x*c)-1)*(c^2*x^2+(c^2*
x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+
1)+1/2*I*(arcsinh(x*c)+1)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^
(1/2)*(-I*(I+x*c)*f)^(1/2)/f/c/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{\sqrt{i\,cdx+d}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^
2*x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*f*x + I*f), x
)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{\sqrt{id\left(cx-i\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{\sqrt{-if\left(cx+i\right)}}\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx = \int \frac{\sqrt{i\,cdx+d}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
a^2*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + i
ntegrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(-I*c*f*
x + f) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*
x + f), x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}}{\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{\sqrt{f-icfx}}\,dx$$
$$=\frac{\sqrt{d}\left(2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+\sqrt{cix+1}\,\sqrt{-cix+1}\,a^2i+2\left(\int\frac{\sqrt{cix+1}\,asinh(cx)}{\sqrt{-cix+1}}dx\right)abc+\left(\int\frac{\sqrt{cix+1}\,asinh(cx)^2}{\sqrt{-cix+1}}dx\right.\right.}{\sqrt{f}\,c}$$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

output
```
(sqrt(d)*(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*a**2*i + 2*int((sqrt(c*i*x + 1)*asinh(c*x))/sqrt( - c*i*x +
 1),x)*a*b*c + int((sqrt(c*i*x + 1)*asinh(c*x)**2)/sqrt( - c*i*x + 1),x)*b
**2*c))/(sqrt(f)*c)
```

3.258 $\int \frac{(a+b\textbf{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 59

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
1/3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 168 vs. 2(59) = 118.

Time = 1.99 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.85

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx=\frac{ab\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^3}{3c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{a^2\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^3)/(3*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (a^2*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/(c*Sqrt[d]*Sqrt[f])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {6211, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(49) = 98$.

Time = 4.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.27

method	result
default	$\frac{a^2\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+\frac{b^2\operatorname{arcsinh}(xc)^3\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{3\sqrt{c^2x^2+1}\,cdf}+\frac{ab\operatorname{arcsinh}(xc)^2\sqrt{i(xc-i)d}}{\sqrt{c^2x^2+1}\,cdf}$
parts	$\frac{a^2\sqrt{(-icfx+f)(icdx+d)}\,\ln\left(\frac{c^2dfx}{\sqrt{c^2df}}+\sqrt{c^2df\,x^2+df}\right)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\sqrt{c^2df}}+\frac{b^2\operatorname{arcsinh}(xc)^3\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{3\sqrt{c^2x^2+1}\,cdf}+\frac{ab\operatorname{arcsinh}(xc)^2\sqrt{i(xc-i)d}}{\sqrt{c^2x^2+1}\,cdf}$

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*((f-I*c*f*x)*(d+I*c*d*x))^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)*ln
(c^2*d*f*x/(c^2*d*f)^(1/2)+(c^2*d*f*x^2+d*f)^(1/2))/(c^2*d*f)^(1/2)+1/3*b^
2*arcsinh(x*c)^3*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2
)/c/d/f+a*b*arcsinh(x*c)^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x
^2+1)^(1/2)/c/d/f
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^
2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d*f*x^2 + d*f), x
)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{(a + b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx - i)}\sqrt{-if\,(cx + i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \frac{b^2\,\text{arsinh}\,(cx)^3}{3\sqrt{df}c} + \frac{ab\,\text{arsinh}\,(cx)^2}{\sqrt{df}c} + \frac{a^2\,\text{arsinh}\,(cx)}{\sqrt{df}c}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
1/3*b^2*arcsinh(c*x)^3/(sqrt(d*f)*c) + a*b*arcsinh(c*x)^2/(sqrt(d*f)*c) +
a^2*arcsinh(c*x)/(sqrt(d*f)*c)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}\sqrt{f - icfx}}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{\sqrt{d + c\,d\,x\,1\text{i}}\,\sqrt{f - c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$$
$$=\frac{2asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+2\left(\int\frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)abc+\left(\int\frac{asinh(cx)^2}{\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)b^2c}{\sqrt{f}\,\sqrt{d}\,c}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
(2*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i + 2*int(asinh(c*x)/(sqrt(c*i*x
+ 1)*sqrt( - c*i*x + 1)),x)*a*b*c + int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqr
t( - c*i*x + 1)),x)*b**2*c)/(sqrt(f)*sqrt(d)*c)
```

3.259 $\int \frac{(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 464

$$
\begin{aligned}
\int \frac{(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx &= \frac{if(1+c^2x^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{fx(1+c^2x^2)\,(a+b\,\mathrm{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{4ibf(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))\arctan\left(e^{\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2bf(1+c^2x^2)^{3/2}\,(a+b\,\mathrm{arcsinh}(cx))\log\left(1+e^{2\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2b^2f(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2b^2f(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{b^2f(1+c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

output
```
I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+
f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-4*I*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(
1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f*(c^2*x^2+1)^(3/2)*(a+b*a
rcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)-2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,I
*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*f*(c^2
*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-1+i)b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\left(\cosh\left(\tfrac{1}{2}\text{arcsinh}(\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 + I)*b^2*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + (I*a^2 + a^2*c*x
 - (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Pi
*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x
^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - (4*
I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi*Sqrt[
1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2]
+ I*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh
[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*(I*Cosh[ArcSinh[c*x]/2]*(2*a - b*Pi + (4*I)*b*Log[1 - I/E^
ArcSinh[c*x]]) + (2*a + b*Pi - (4*I)*b*Log[1 - I/E^ArcSinh[c*x]])*Sinh[Arc
Sinh[c*x]/2])))/(c*d^2*f*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2]))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(c^2x^2+1)^{3/2}\int \frac{(1-icx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f(c^2x^2+1)^{3/2}\int \left(\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}-\frac{icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f(c^2x^2+1)^{3/2}\left(-\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx}{c}\right.}{(d+icd}$$

input

```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f*(1 + c^2*x^2)^(3/2)*((a + b*ArcSinh[c*x])^2/c + (I*(a + b*ArcSinh[c*x])
^2)/(c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] -
 ((4*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/c - (2*b*(a + b*Arc
Sinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c - (2*b^2*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/c + (2*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c - (b^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.87

method	result
default	$\frac{i a^2 \sqrt{-icfx+f}}{cdf\sqrt{icdx+d}}+b^2\left(\frac{\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}\,\left(xc+i-\sqrt{c^2x^2+1}\right)\operatorname{arcsinh}(xc)^2}{f\,d^2c(c^2x^2+1)}-\frac{2\left(2\operatorname{arcsinh}(xc)\ln\left(1+i\left(xc+\sqrt{c^2x^2+1}\right)\right)-\right.}{}\right.$
parts	$\frac{i a^2 \sqrt{-icfx+f}}{cdf\sqrt{icdx+d}}+b^2\left(\frac{\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}\,\left(xc+i-\sqrt{c^2x^2+1}\right)\operatorname{arcsinh}(xc)^2}{f\,d^2c(c^2x^2+1)}-\frac{2\left(2\operatorname{arcsinh}(xc)\ln\left(1+i\left(xc+\sqrt{c^2x^2+1}\right)\right)-\right.}{}\right.$

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
I*a^2/c/d/f/(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+b^2*((-I*(I+x*c)*f)^(1/2)*
(I*(x*c-I)*d)^(1/2)*(x*c+I-(c^2*x^2+1)^(1/2))*arcsinh(x*c)^2/f/d^2/c/(c^2*
x^2+1)-2*(2*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))-arcsinh(x*c)^2+2*
polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2))))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^
(1/2)/(c^2*x^2+1)^(1/2)/f/d^2/c)+2*a*b*(2*arcsinh(x*c)*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2/f+(I*(x*c-I)*d)^(1/2)*(-I*(I
+x*c)*f)^(1/2)*(x*c+I-(c^2*x^2+1)^(1/2))*arcsinh(x*c)/(c^2*x^2+1)/c/d^2/f-
2*ln(x*c+(c^2*x^2+1)^(1/2)-I)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^
2*x^2+1)^(1/2)/c/d^2/f)
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
 (c^2*d^2*f*x - I*c*d^2*f)*integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + I
*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(
c^3*d^2*f*x^3 - I*c^2*d^2*f*x^2 + c*d^2*f*x - I*d^2*f), x))/(c^2*d^2*f*x -
 I*c*d^2*f)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(id(cx-i))^{\frac{3}{2}}\sqrt{-if(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I)
)), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx=\int \frac{(b\,\text{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*sqrt(-I*
c*f*x + f)), x) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(I*c^2*d^2*
f*x + c*d^2*f) + I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(I*c^2*d^2*f*x + c*d^2*f) -
 2*a*b*log(I*c*x + 1)/(c*d^(3/2)*sqrt(f))
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{2\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,cix+\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)abc+\sqrt{cix}}{\sqrt{f}\,\sqrt{d}\,\sqrt{cix+1}}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
(2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c + sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqrt( -
c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c + a**2*c*
x + a**2*i)/(sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d)
```

3.260 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 942

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx &= -\frac{2ib^2f^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{bf^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2ibf^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{bcf^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2if^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{f^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{c^2f^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2f^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{f^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{4ibf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2bf^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{2b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&+ \frac{2b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \\
&- \frac{b^2f^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}
\end{aligned}
$$

output
```
2/3*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*b^2*f^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3
*b^2*f^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+1/3*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)-4/3*I*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x
+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*f^2*x^2*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2
/3*I*b^2*f^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*f^2*x
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*
c^2*f^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I
*c*f*x)^(5/2)+1/3*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*b*f^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x)
)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*
b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.09 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.56

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(-2i+cx)}{(-i+cx)^2}-\frac{ab\left(-i\cosh\left(\frac{3}{2}\text{arcsinh}(cx)\right)\left(\text{arcsinh}(cx)-2\,\text{ar}\right.\right.}{}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(-2*I + c*x))/(-I + c*x)^2 - (a
*b*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*
x]/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 - (3*I)*ArcSin
h[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*
((-1 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[
Coth[ArcSinh[c*x]/2]] + (I/2)*(-2 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^
2]))*Sinh[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])^3) - (b^2*((1 - I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(-2
*I + ArcSinh[c*x]))/(-I + c*x) + 2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^
ArcSinh[c*x]] + I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cos
h[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 4*PolyLog[
2, I/E^ArcSinh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSi
nh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[Arc
Sinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c
^2*x^2]))/(3*c*d^3*f)
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f^2(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\int \frac{(1-icx)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f^2\left(c^2x^2+1\right)^{5/2}\int\left(-\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}-\frac{2icx(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$f^2\left(c^2x^2+1\right)^{5/2}\left(-\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{bcx^2(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}}+\frac{x(a+b\text{arcsi}}{3(c^2x^2+1}\right.$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

output
```
(f^2*(1 + c^2*x^2)^(5/2)*((((-2*I)/3)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (2*b^2*
x)/(3*Sqrt[1 + c^2*x^2]) + (b^2*ArcSinh[c*x])/(3*c) + (b*(a + b*ArcSinh[c*
x]))/(3*c*(1 + c^2*x^2)) - (((2*I)/3)*b*x*(a + b*ArcSinh[c*x]))/(1 + c^2*x
^2) - (b*c*x^2*(a + b*ArcSinh[c*x]))/(3*(1 + c^2*x^2)) + (a + b*ArcSinh[c*
x])^2/(3*c) + (((2*I)/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) +
 (x*(a + b*ArcSinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) - (c^2*x^3*(a + b*ArcS
inh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sqr
t[1 + c^2*x^2]) - (((4*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]]
)/c - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) - (2*b^
2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) + (2*b^2*PolyLog[2, I*E^ArcSinh[c
*x]])/(3*c) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 775, normalized size of antiderivative = 0.82

method	result
default	$a^2\left(\frac{i\sqrt{-icfx+f}}{3fcd(icdx+d)^{\frac{3}{2}}}+\frac{i\sqrt{-icfx+f}}{3cf\,d^2\sqrt{icdx+d}}\right)-\frac{b^2\left(4\ln\left(1+i\left(xc+\sqrt{c^2x^2+1}\right)\right)\operatorname{arcsinh}(xc)x^4c^4-\operatorname{arcsinh}(xc)^2x^4c^4-\operatorname{arcsinh}(xc)^2\cdots\right.}{\cdots}$
parts	$a^2\left(\frac{i\sqrt{-icfx+f}}{3fcd(icdx+d)^{\frac{3}{2}}}+\frac{i\sqrt{-icfx+f}}{3cf\,d^2\sqrt{icdx+d}}\right)-\frac{b^2\left(4\ln\left(1+i\left(xc+\sqrt{c^2x^2+1}\right)\right)\operatorname{arcsinh}(xc)x^4c^4-\operatorname{arcsinh}(xc)^2x^4c^4-\operatorname{arcsinh}(xc)^2\cdots\right.}{\cdots}$

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(1/3*I/f/c/d/(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(1/2)+1/3*I/c/f/d^2/(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2))-1/3*b^2*(4*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*a
rcsinh(x*c)*x^4*c^4-arcsinh(x*c)^2*x^4*c^4-arcsinh(x*c)^2*(c^2*x^2+1)^(1/2
)*x^3*c^3+4*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4-2*I*(c^2*x^2+1)^
(1/2)*arcsinh(x*c)^2-2*c^4*x^4+2*(c^2*x^2+1)^(1/2)*c^3*x^3+2*I*(c^2*x^2+1)
^(1/2)*x^2*c^2+8*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*arcsinh(x*c)*x^2*c^2-2*ar
csinh(x*c)^2*x^2*c^2-3*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*x*c+8*polylog(2,-I
*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-2*arcsinh(x*c)*c^2*x^2+2*I*arcsinh(x*c)*
x*c+2*I*arcsinh(x*c)*x^3*c^3-4*c^2*x^2+2*(c^2*x^2+1)^(1/2)*x*c+2*I*(c^2*x^
2+1)^(1/2)+4*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))-arcsinh(x*c)^2+4
*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))-2*arcsinh(x*c)-2)*(I*(x*c-I)*d)^(1/
2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/
d^3/c/f+2/3*a*b*(arcsinh(x*c)*c^4*x^4-2*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^4*c^
4+arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-I*x^3*c^3+2*arcsinh(x*c)*c^2*x^2-
4*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^2*c^2+3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c
+c^2*x^2+2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)-I*x*c+arcsinh(x*c)-2*ln(x*c+(c
^2*x^2+1)^(1/2)-I)+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)
^(5/2)/d^3/c/f
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*((b^2*c*x - 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 3*(c^3*d^3*f*x^2 - 2*I*c^2*d^3*f*x - c*d^3*f)*integra
l(-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*a*b + (b^2*c*x - 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^3*f*x^4
- 2*I*c^3*d^3*f*x^3 - 2*I*c*d^3*f*x - d^3*f), x))/(c^3*d^3*f*x^2 - 2*I*c^2
*d^3*f*x - c*d^3*f)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(id(cx-i))^{\frac{5}{2}}\sqrt{-if(cx+i)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I)
)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}\sqrt{-i\,cfx+f}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+c\,d\,x\,1\text{i})^{5/2}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx=\frac{-6\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2-2\sqrt{cix+1}\,\sqrt{-cix+1}\,cix-\sqrt{cix+1}\,\sqrt{-}}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

output
```
( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x -
 sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c**3*x**2 - 6*sqrt(c*i*x + 1)*
sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2
*x**2 - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt(
 - c*i*x + 1)),x)*a*b*c - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c
*x)**2/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c**3
*x**2 - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x
 + 1)*sqrt( - c*i*x + 1)*c**2*x**2 - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*
c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c + a**2*c**3*x**3 + 3
*a**2*c*x + 2*a**2*i)/(3*sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1
)*c*d**2*(c**2*x**2 + 1))
```

3.261 $\int \frac{(d+icdx)^{5/2}(a+b\,\mathbf{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 947

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = -\frac{8ib^2d^3(1+c^2x^2)}{cf\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2d^3x(1+c^2x^2)}{4f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2d^3\sqrt{1+c^2x^2}\text{arcsinh}(cx)}{4cf\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{8ibd^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{bcd^3x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8id^3(a+b\text{arcsinh}(cx))^2}{cf\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{8d^3x(a+b\text{arcsinh}(cx))^2}{f\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{8d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cf\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{4id^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{cf\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^3x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))^2}{2f\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{5d^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{2bcf\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{32ibd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{16bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{16b^2d^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{16b^2d^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{8b^2d^3\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
8*I*b*d^3*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/f/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)+1/4*b^2*d^3*x*(c^2*x^2+1)/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-1/4*b^2*d^3*(c^2*x^2+1)^(1/2)*arcsinh(c*x)/c/f/(d+I*c*d*x)^(1/2)/(f-I*c
*f*x)^(1/2)-8*I*d^3*(a+b*arcsinh(c*x))^2/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)-1/2*b*c*d^3*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/f/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)-8*I*b^2*d^3*(c^2*x^2+1)/c/f/(d+I*c*d*x)^(1/2)/(f-I
*c*f*x)^(1/2)+8*d^3*x*(a+b*arcsinh(c*x))^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)+8*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/f/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)-4*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/f/(d+I*c*d*x)
^(1/2)/(f-I*c*f*x)^(1/2)+1/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/f/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-5/2*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x
))^3/b/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+32*I*b*d^3*(c^2*x^2+1)^(1/2
)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/f/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)-16*b*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+16*b^2*d^3*(c
^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/f/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)-16*b^2*d^3*(c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b^2*d^3*(c^2*x^2+1)^
(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2143 vs. $2(947) = 1894$.

Time = 24.23 (sec) , antiderivative size = 2143, normalized size of antiderivative = 2.26

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-4*I)*a^2*d^2)/f^2 + (a^2*
c*d^2*x)/(2*f^2) + (8*a^2*d^2)/(f^2*(I + c*x))))/c - (15*a^2*d^(5/2)*Log[c
*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*
c*f^(3/2)) - ((4*I)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*
x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c
*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[Ar
cSinh[c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh
[c*x] + I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[C
oth[ArcSinh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c
*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) - (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*
Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*
(8*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4
*Log[Sqrt[1 + c^2*x^2]])) + (ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*Ar
cTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2
]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Co
sh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d^2*(-I + c*x)*Sqrt[I*(
(-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*
Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (
2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[...
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\int\frac{(icx+1)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 6259}$$

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\int\left(\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{7(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{8i(i-cx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}\right)d}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^4\left(c^2x^2+1\right)^{3/2}\left(\frac{32ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{1}{2}x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2-\frac{4i\sqrt{c^2x^2+1}(a+b\text{arcsin}}{c}\right.}{}$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(3/2)*((8*I)*a*b*x - ((8*I)*b^2*Sqrt[1 + c^2*x^2])/c +
(b^2*x*Sqrt[1 + c^2*x^2])/4 - (b^2*ArcSinh[c*x])/(4*c) + (8*I)*b^2*x*ArcSi
nh[c*x] - (b*c*x^2*(a + b*ArcSinh[c*x]))/2 + (8*(a + b*ArcSinh[c*x])^2)/c
- ((8*I)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (8*x*(a + b*ArcSi
nh[c*x])^2)/Sqrt[1 + c^2*x^2] - ((4*I)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x])^2)/c + (x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/2 - (5*(a + b*ArcS
inh[c*x])^3)/(2*b*c) + ((32*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x
]])/c - (16*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c + (16*b^
2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c - (16*b^2*PolyLog[2, I*E^ArcSinh[c*x]
])/c - (8*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 5.91 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.01

method	result
default	$-\frac{d^2\left(32a^2b+a\,b^2+10a^3-128\ln\left(xc+\sqrt{c^2x^2+1}\right)a\,b^2+32a^2b\,c^2x^2+3a\,b^2c^2x^2-\sqrt{c^2x^2+1}\,b^3cx+30\ \mathrm{arcsinh}(xc)a^2b\,c^2x^2-128\ln\left(xc\right.\right.}{}$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4*d^2*(32*a^2*b+a*b^2+10*a^3-128*ln(x*c+(c^2*x^2+1)^(1/2))*a*b^2+32*a^2
*b*c^2*x^2+3*a*b^2*c^2*x^2-(c^2*x^2+1)^(1/2)*b^3*c*x+128*ln(1-I*(x*c+(c^2*
x^2+1)^(1/2)))*b^3*arcsinh(x*c)+128*ln(x*c+(c^2*x^2+1)^(1/2)+I)*a*b^2+30*a
rcsinh(x*c)*a^2*b*c^2*x^2+128*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*b^3*c^2
*x^2+48*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*b^3+32*I*(c^2*x^2+1)^(1/2)*b^3*
c^2*x^2-32*I*a*b^2*c^3*x^3-32*I*a*b^2*c*x-128*ln(x*c+(c^2*x^2+1)^(1/2))*a*
b^2*c^2*x^2-34*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*b^3*c*x+64*arcsinh(x*c)*a*
b^2*c^2*x^2-2*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*b^3*c^3*x^3+30*arcsinh(x*c)
^2*a*b^2*c^2*x^2+30*a*b^2*arcsinh(x*c)^2-68*(c^2*x^2+1)^(1/2)*arcsinh(x*c)
*a*b^2*c*x-4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*a*b^2*c^3*x^3+2*a*b^2*c^4*x^4+
128*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*b^3-34*(c^2*x^2+1)^(1/2)*a^2*b*c*
x+16*I*(c^2*x^2+1)^(1/2)*a^2*b*c^2*x^2+10*b^3*arcsinh(x*c)^3-32*b^3*arcsin
h(x*c)^2+b^3*arcsinh(x*c)+30*a^2*b*arcsinh(x*c)+64*a*b^2*arcsinh(x*c)+32*I
*(c^2*x^2+1)^(1/2)*b^3-(c^2*x^2+1)^(1/2)*b^3*c^3*x^3-32*arcsinh(x*c)^2*b^3
*c^2*x^2+3*arcsinh(x*c)*b^3*c^2*x^2+2*arcsinh(x*c)*b^3*c^4*x^4+10*arcsinh(
x*c)^3*b^3*c^2*x^2-2*(c^2*x^2+1)^(1/2)*a^2*b*c^3*x^3+48*I*(c^2*x^2+1)^(1/2
)*a^2*b+16*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*b^3*c^2*x^2+10*a^3*c^2*x^2+9
6*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*a*b^2-32*I*arcsinh(x*c)*b^3*c^3*x^3-32*
I*arcsinh(x*c)*b^3*c*x+128*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*b^
3*c^2*x^2+128*ln(x*c+(c^2*x^2+1)^(1/2)+I)*a*b^2*c^2*x^2+32*I*(c^2*x^2+1...
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d^2*x^2 - 2
*I*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f
*x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x
)/(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a^2 + integrat
e((I*c*d*x + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(-I*c*f*x + f)^(3
/2) + 2*(I*c*d*x + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f
)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d}\,d^2\left(-30\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i+4\sqrt{-cix+1}\left(\int\frac{\sqrt{c}}{\sqrt{-c}}\right.\right.}{}$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a
**2*i + 4*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x**2)/(sqrt(
- c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**3 - 8*sqrt( - c*i*x + 1
)*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c
*i*x + 1)),x)*a*b*c**2*i - 4*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh
(c*x))/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c + 2*sqrt(
- c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2*x**2)/(sqrt( - c*i*x + 1)*
c*i*x - sqrt( - c*i*x + 1)),x)*b**2*c**3 - 4*sqrt( - c*i*x + 1)*int((sqrt(
c*i*x + 1)*asinh(c*x)**2*x)/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)
),x)*b**2*c**2*i - 2*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2
)/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b**2*c - sqrt(c*i*x +
 1)*a**2*c**2*i*x**2 - 7*sqrt(c*i*x + 1)*a**2*c*x - 24*sqrt(c*i*x + 1)*a**
2*i))/(2*sqrt(f)*sqrt( - c*i*x + 1)*c*f)
```

3.262 $\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 719

$$
\begin{aligned}
\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx &= -\frac{2ib^2d^2(1+c^2x^2)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{2ibd^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{f\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4id^2(a+b\operatorname{arcsinh}(cx))^2}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4d^2x(a+b\operatorname{arcsinh}(cx))^2}{f\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4d^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{id^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{cf\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{bcf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{16ibd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{8bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8b^2d^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{8b^2d^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4b^2d^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{cf\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

output
```
-2*I*b^2*d^2*(c^2*x^2+1)/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b*d^2
*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)-4*I*d^2*(a+b*arcsinh(c*x))^2/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4
*d^2*x*(a+b*arcsinh(c*x))^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*d^2*(c
^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)-d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/f/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+16*I*b*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*arctan(
c*x+(c^2*x^2+1)^(1/2))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b*d^2*(c^
2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/f/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8*b^2*d^2*(c^2*x^2+1)^(1/2)*polylog(2,-I*(
c*x+(c^2*x^2+1)^(1/2)))/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-8*b^2*d^2*
(c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/f/(d+I*c*d*x)^(1/
2)/(f-I*c*f*x)^(1/2)-4*b^2*d^2*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+
1)^(1/2))^2)/c/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 16.39 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.51

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2)
,x]
```

output
```
((3*a^2*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x))
 - (9*a^2*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]])/f^(3/2) - (b^2*d*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*(-18*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*
(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcS
inh[c*x]] + 12*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[
ArcSinh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSin
h[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]
/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^2) - (I*b^2*d*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(((-6*I)*
c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[c*x]^2)/Sqrt[1 +
c^2*x^2] + (2*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2 + ArcSinh[c*x]^
2) + ((6*I)*(2*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3
*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh
[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSin
h[c*x]]))/Sqrt[1 + c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sq
rt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(f^2*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*d*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[Arc
Sinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSi...
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 6259}$$

$$\frac{d^3\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{3(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4i(i-cx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$d^3\left(c^2x^2+1\right)^{3/2}\left(\frac{16ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}-\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{4x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{4i(a+b\text{a}}{c}\right.$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(3/2)*((2*I)*a*b*x - ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c +
(2*I)*b^2*x*ArcSinh[c*x] + (4*(a + b*ArcSinh[c*x])^2)/c - ((4*I)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (4*x*(a + b*ArcSinh[c*x])^2)/Sqrt[1
 + c^2*x^2] - (I*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c - (a + b*ArcS
inh[c*x])^3/(b*c) + ((16*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/c - (8*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c + (8*b^2*Pol
yLog[2, (-I)*E^ArcSinh[c*x]])/c - (8*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c -
 (4*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 7.32 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.16

method	result
default	$-\frac{id\left(4i\sqrt{c^2x^2+1}\,a^2bcx-8i\,\mathrm{arcsinh}(xc)a\,b^2c^2x^2-8i\ln\left(1+\left(xc+\sqrt{c^2x^2+1}\right)^2\right)a\,b^2c^2x^2+4i\,\mathrm{arcsinh}(xc)^2\sqrt{c^2x^2+1}\,b^3cx-2a\,b^2cx+\right.}{}$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-I*d*(8*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*a*b^2*c*x-16*I*ln(1-I*(x*c+(c^2*x
^2+1)^(1/2)))*b^3*arcsinh(x*c)-3*I*a*b^2*arcsinh(x*c)^2+16*I*ln(x*c+(c^2*x
^2+1)^(1/2))*a*b^2-8*I*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*a*b^2-3*I*a^2*b*arc
sinh(x*c)-8*I*a*b^2*arcsinh(x*c)-16*I*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*arcs
inh(x*c)*b^3*c^2*x^2-3*I*arcsinh(x*c)^2*a*b^2*c^2*x^2+16*I*ln(x*c+(c^2*x^2
+1)^(1/2))*a*b^2*c^2*x^2-8*I*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*a*b^2*c^2*x^2
+4*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*b^3*c*x-3*I*arcsinh(x*c)*a^2*b*c^2*x
^2-8*I*arcsinh(x*c)*a*b^2*c^2*x^2-2*a*b^2*c*x+(c^2*x^2+1)^(1/2)*a^2*b*c^2*
x^2-16*arctan(x*c+(c^2*x^2+1)^(1/2))*a*b^2*c^2*x^2+arcsinh(x*c)^2*(c^2*x^2
+1)^(1/2)*b^3*c^2*x^2-4*I*a^2*b*c^2*x^2+10*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*
a*b^2-2*arcsinh(x*c)*b^3*c*x-2*arcsinh(x*c)*b^3*c^3*x^3+2*(c^2*x^2+1)^(1/2
)*b^3*c^2*x^2-I*b^3*arcsinh(x*c)^3+4*I*b^3*arcsinh(x*c)^2-16*I*polylog(2,I
*(x*c+(c^2*x^2+1)^(1/2)))*b^3-2*a*b^2*c^3*x^3-I*a^3*c^2*x^2+4*I*(c^2*x^2+1
)^(1/2)*a^2*b*c*x+5*(c^2*x^2+1)^(1/2)*a^2*b+2*arcsinh(x*c)*(c^2*x^2+1)^(1/
2)*a*b^2*c^2*x^2-16*arctan(x*c+(c^2*x^2+1)^(1/2))*a*b^2+2*(c^2*x^2+1)^(1/2
)*b^3-4*I*a^2*b-I*arcsinh(x*c)^3*b^3*c^2*x^2+4*I*arcsinh(x*c)^2*b^3*c^2*x^
2-16*I*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*b^3*c^2*x^2-I*a^3+5*(c^2*x^2+1
)^(1/2)*arcsinh(x*c)^2*b^3)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*
x^2+1)^(1/2)/b/f^2/(c^4*x^4+2*c^2*x^2+1)/c
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(((-I*b^2*c*d*x - b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*d*x + a*b*d)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*d*x - a^2*d)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3
/2), x)
```

Maxima [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) -
6*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/
(c*f^2*sqrt(d/f))) + integrate((I*c*d*x + d)^(3/2)*b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*(I*c*d*x + d)^(3/2)*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionUnable to transpose Error: Bad Argument ValueDegree mismat
ch inside
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}}{{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\frac{\sqrt{d}\,d\left(-6\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-2\sqrt{-cix+1}\left(\int \frac{\sqrt{cix}}{\sqrt{-cix+}}\right.\right.}{}$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(d)*d*( - 6*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*
i - 2*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x
 + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**2*i - 2*sqrt( - c*i*x + 1)*int
((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x +
1)),x)*a*b*c - sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2*x)/(s
qrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b**2*c**2*i - sqrt( - c*i
*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2)/(sqrt( - c*i*x + 1)*c*i*x - sq
rt( - c*i*x + 1)),x)*b**2*c - sqrt(c*i*x + 1)*a**2*c*x - 5*sqrt(c*i*x + 1)
*a**2*i))/(sqrt(f)*sqrt( - c*i*x + 1)*c*f)
```

3.263 $\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 544

$$\begin{aligned}
\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx =
&-\frac{2id^2(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2x(1+c^2x^2)(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{2d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{8ibd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{4bd^2(1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&+\frac{4b^2d^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{4b^2d^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&-\frac{2b^2d^2(1+c^2x^2)^{3/2}\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

output
```
-2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)+2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/
(f-I*c*f*x)^(3/2)-1/3*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*
x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*
b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2
*d^2*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.80 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{i+cx} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-i\right.}{}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x
]
```

output
```
((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 3*a^2*Sqrt[d]*Sqr
t[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] -
(b^2*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-18*Pi*ArcSinh[c*x] -
 (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x]
)*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-C
os[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I
)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*
x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2
]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[A
rcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSin
h[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Co
sh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcS
inh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6259}$$

$$\frac{d^2\left(c^2x^2+1\right)^{3/2}\int \left(-\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2i(i-cx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\downarrow \text{2009}$$

$$d^2(c^2x^2+1)^{3/2}\left(\frac{8ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{2x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}-\frac{(a+b\text{arcsinh}(cx}{3bc}\right.$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

output
```
(d^2*(1 + c^2*x^2)^(3/2)*((2*(a + b*ArcSinh[c*x])^2)/c - ((2*I)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[1 + c^2*x^2]) + (2*x*(a + b*ArcSinh[c*x])^2)/Sqrt[1
+ c^2*x^2] - (a + b*ArcSinh[c*x])^3/(3*b*c) + ((8*I)*b*(a + b*ArcSinh[c*x]
)*ArcTan[E^ArcSinh[c*x]])/c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSi
nh[c*x])])/c + (4*b^2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c - (4*b^2*PolyLog[
2, I*E^ArcSinh[c*x]])/c - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/c))/((d
+ I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.53

method	result
default	$-\frac{(a+b\,\operatorname{arcsinh}(xc))^3\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}}{3\sqrt{c^2x^2+1}\,b\,f^2c}+\frac{2\left(\operatorname{arcsinh}(xc)^2b^2+2\,\operatorname{arcsinh}(xc)ab+a^2\right)\left(xc-\sqrt{c^2x^2+1}-i\right)\sqrt{i(xc-i)d}\,\sqrt{-i(x}}{f^2c(c^2x^2+1)}$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3*(a+b*arcsinh(x*c))^3*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2*x^
2+1)^(1/2)/b/f^2/c+2*(arcsinh(x*c)^2*b^2+2*arcsinh(x*c)*a*b+a^2)*(x*c-(c^2
*x^2+1)^(1/2)-I)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^2/c/(c^2*x^2+1
)+4*(b*arcsinh(x*c)^2-2*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*b-2*a
*ln(x*c+(c^2*x^2+1)^(1/2)+I)+2*a*ln(x*c+(c^2*x^2+1)^(1/2))-2*polylog(2,I*(
x*c+(c^2*x^2+1)^(1/2)))*b)*b*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/(c^2
*x^2+1)^(1/2)/f^2/c
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x
^2 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*f^2*x^2 + 2*I*c*
f^2*x - f^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d + icdx}(a + b\text{arcsinh}(cx))^2}{(f - icfx)^{3/2}}\,dx = \int \frac{\sqrt{id\,(cx - i)}(a + b\,\text{asinh}\,(cx))^2}{(-if\,(cx + i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{d + icdx}(a + b\text{arcsinh}(cx))^2}{(f - icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx + d}(b\,\text{arsinh}\,(cx) + a)^2}{(-i\,cfx + f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
a^2*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/
(c*f^2*sqrt(d/f))) + integrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^
2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,\sqrt{d+c\,d\,x\,1\text{i}}}{(f-c\,f\,x\,1\text{i})^{3/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d}\left(-2\sqrt{-cix+1}\,asin\left(\frac{\sqrt{-cix+1}}{\sqrt{2}}\right)a^2i-2\sqrt{-cix+1}\left(\int \frac{\sqrt{cix+1}\,a}{\sqrt{-cix+1}\,cix}\right.\right.}{\sqrt{f}}$$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1)/sqrt(2))*a**2*i
- 2*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1
)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c - sqrt( - c*i*x + 1)*int((sqrt(c*i*
x + 1)*asinh(c*x)**2)/(sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b
**2*c - 2*sqrt(c*i*x + 1)*a**2*i))/(sqrt(f)*sqrt( - c*i*x + 1)*c*f)
```

3.264 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 464

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx &= -\frac{id(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{dx(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{4ibd(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2bd(1+c^2x^2)^{3/2}\,(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&+ \frac{2b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{2b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} \\
&- \frac{b^2d(1+c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

output
```
-I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)+d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+4*I*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^
(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f
*x)^(3/2)+2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/
c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,
I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*d*(c^
2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(
f-I*c*f*x)^(3/2)
```

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((-1-i)b^2\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2\left(\cosh\left(\frac{1}{2}\text{arcsinh}(\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 - I)*b^2*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ((-I)*a^2 + a^2*
c*x + (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - (2*I)*b^2
*Pi*Sqrt[1 + c^2*x^2]*Log[1 + I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^
2*x^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] +
(2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4
*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]])*(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^
ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 +
 c^2*x^2]*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2]*(2*a + 3*b*Pi - (4*I)*b*
Log[1 + I/E^ArcSinh[c*x]]) + (2*a - 3*b*Pi + (4*I)*b*Log[1 + I/E^ArcSinh[c
*x]])*Sinh[ArcSinh[c*x]/2])))/(c*d*f^2*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[
c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d(c^2x^2+1)^{3/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d(c^2x^2+1)^{3/2}\int \left(\frac{icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d(c^2x^2+1)^{3/2}\left(\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{c}+\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{i(a+b\text{arcsinh}(cx))^2}{c\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{c}\right.}{(d+icd\ldots}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(3/2)*((a + b*ArcSinh[c*x])^2/c - (I*(a + b*ArcSinh[c*x])
^2)/(c*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] +
 ((4*I)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/c - (2*b*(a + b*Arc
Sinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/c + (2*b^2*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/c - (2*b^2*PolyLog[2, I*E^ArcSinh[c*x]])/c - (b^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/c))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.86

method	result
default	$-\frac{ia^2\sqrt{icdx+d}}{dcf\sqrt{-icfx+f}}+b^2\left(\frac{\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}\,\left(xc-\sqrt{c^2x^2+1}-i\right)\operatorname{arcsinh}(xc)^2}{d\,f^2c(c^2x^2+1)}+\frac{2\left(\operatorname{arcsinh}(xc)^2-2\operatorname{arcsinh}(xc)\ln\left(1-i\left(\right.\right.\right.}{}\right.$
parts	$-\frac{ia^2\sqrt{icdx+d}}{dcf\sqrt{-icfx+f}}+b^2\left(\frac{\sqrt{-i(xc+i)f}\,\sqrt{i(xc-i)d}\,\left(xc-\sqrt{c^2x^2+1}-i\right)\operatorname{arcsinh}(xc)^2}{d\,f^2c(c^2x^2+1)}+\frac{2\left(\operatorname{arcsinh}(xc)^2-2\operatorname{arcsinh}(xc)\ln\left(1-i\left(\right.\right.\right.}{}\right.$

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-I*a^2/d/c/f/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2)+b^2*((-I*(I+x*c)*f)^(1/2)
*(I*(x*c-I)*d)^(1/2)*(x*c-(c^2*x^2+1)^(1/2)-I)*arcsinh(x*c)^2/d/f^2/c/(c^2
*x^2+1)+2*(arcsinh(x*c)^2-2*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))-2
*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2))))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^
(1/2)/(c^2*x^2+1)^(1/2)/d/f^2/c)+2*a*b*(2*arcsinh(x*c)*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/f^2/c/d+(I*(x*c-I)*d)^(1/2)*(-I*(I
+x*c)*f)^(1/2)*(x*c-(c^2*x^2+1)^(1/2)-I)*arcsinh(x*c)/(c^2*x^2+1)/f^2/c/d-
2*ln(x*c+(c^2*x^2+1)^(1/2)+I)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^
2*x^2+1)^(1/2)/f^2/c/d)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
 (c^2*d*f^2*x + I*c*d*f^2)*integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - I*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c
^3*d*f^2*x^3 + I*c^2*d*f^2*x^2 + c*d*f^2*x + I*d*f^2), x))/(c^2*d*f^2*x +
I*c*d*f^2)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}(f - icfx)^{3/2}}\, dx = \int \frac{(a + b\,\text{asinh}\,(cx))^2}{\sqrt{id\,(cx - i)}\,(-if\,(cx + i))^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2
)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{\sqrt{d + icdx}(f - icfx)^{3/2}}\, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{i\,cdx + d}(-i\,cfx + f)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(I*c*d*x + d)*(-I*c*f*x
+ f)^(3/2)), x) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(-I*c^2*d*f
^2*x + c*d*f^2) - I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(-I*c^2*d*f^2*x + c*d*f^2)
 - 2*a*b*log(I*c*x - 1)/(c*sqrt(d)*f^(3/2))
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx=\frac{-2\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,cix-\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)abc-\sqrt{c}}{\sqrt{f}\,\sqrt{d}\,\sqrt{cix+}}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

output

```
( - 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c - s
qrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqrt(
 - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c + a**2
*c*x - a**2*i)/(sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*f)
```

3.265 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 239

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\frac{x(a+b\text{arcsinh}(cx))^2}{df\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{cdf\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{cdf\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{cdf\sqrt{d+icdx}\sqrt{f-icfx}}$$

output
```
x*(a+b*arcsinh(c*x))^2/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+(c^2*x^2+1)
^(1/2)*(a+b*arcsinh(c*x))^2/c/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*b*
(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/d/f
/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-b^2*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))^2)/c/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 488 vs. 2(239) = 478.

Time = 2.90 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{a^2cx+2abcx\text{arcsinh}(cx)-2ib^2\pi\sqrt{1+c^2x^2}\text{arcsinh}(cx)+b^2cx\text{arcsinh}(}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
,x]
```

output
```
(a^2*c*x + 2*a*b*c*x*ArcSinh[c*x] - (2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + b^2*c*x*ArcSinh[c*x]^2 - b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + I*
b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log
[1 + I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^
ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 + E^ArcSinh[c*x]] - a
*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Co
s[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[
ArcSinh[c*x]/2]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[
c*x])/4]] + 2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + 2*b^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c*d*f*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {6211, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{3/2}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6202}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{6212}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(cx)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{4201}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(c^2x^2+1)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{2}b\int \log\left(1+e^{2\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)\right)\right.}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\frac{1}{4}b\int e^{-2\text{arcsinh}(cx)}\log\left(1+e^{2\text{arcsinh}(cx)}\right)d\right.}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$\downarrow$ 2838

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+b\text{arc}}{}\right.}{c}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
((1 + c^2*x^2)^(3/2)*((x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + ((2*I
)*b*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/c)
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(227) = 454.

Time = 5.68 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.77

method	result
default	$a^{2}\left(\frac{i}{cdf\sqrt{icdx+d}\,\sqrt{-icfx+f}}-\frac{i\sqrt{icdx+d}}{cf\,d^{2}\sqrt{-icfx+f}}\right)+\frac{b^{2}\left(2\sqrt{c^{2}x^{2}+1}\,\operatorname{arcsinh}(xc)\ln\left(1+i\left(xc+\sqrt{c^{2}x^{2}+1}\right)\right)xc+2\sqrt{c^{2}x^{2}+1}\,\operatorname{arcs}\ldots}{}$
parts	$a^{2}\left(\frac{i}{cdf\sqrt{icdx+d}\,\sqrt{-icfx+f}}-\frac{i\sqrt{icdx+d}}{cf\,d^{2}\sqrt{-icfx+f}}\right)+\frac{b^{2}\left(2\sqrt{c^{2}x^{2}+1}\,\operatorname{arcsinh}(xc)\ln\left(1+i\left(xc+\sqrt{c^{2}x^{2}+1}\right)\right)xc+2\sqrt{c^{2}x^{2}+1}\,\operatorname{arcs}\ldots}{}$

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(I/c/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I/c/f/d^2/(f-I*c*f*x)^(1/
2)*(d+I*c*d*x)^(1/2))+b^2*(2*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*ln(1+I*(x*c+(c
^2*x^2+1)^(1/2)))*x*c+2*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^
2+1)^(1/2)))*x*c+arcsinh(x*c)^2-2*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*arcsinh(
x*c)*x^2*c^2-2*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2+2*(c^2
*x^2+1)^(1/2)*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x*c+2*(c^2*x^2+1)^(1/2
)*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x*c-2*polylog(2,-I*(x*c+(c^2*x^2+1)
^(1/2)))*x^2*c^2-2*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-2*polylog(
2,-I*(x*c+(c^2*x^2+1)^(1/2)))-2*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))-2*arc
sinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))-2*arcsinh(x*c)*ln(1-I*(x*c+(c^2*
x^2+1)^(1/2))))*(x*c+(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)
^(1/2)/c/f^2/d^2/(c^2*x^2+1)+2*a*b*(-ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x^2*c
^2+(c^2*x^2+1)^(1/2)*ln(1+(x*c+(c^2*x^2+1)^(1/2))^2)*x*c+arcsinh(x*c)-ln(1
+(x*c+(c^2*x^2+1)^(1/2))^2))*(x*c+(c^2*x^2+1)^(1/2))*(I*(x*c-I)*d)^(1/2)*(
-I*(I+x*c)*f)^(1/2)/c/f^2/d^2/(c^2*x^2+1)
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2
 + (c^2*d^2*f^2*x^2 + d^2*f^2)*integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*c*
x - sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1))
)/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 +
d^2*f^2)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}\,(-if\,(cx+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(
3/2)), x)
```

Maxima [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*(-I*c*f*
x + f)^(3/2)), x) + 2*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a
^2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f) - a*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c
*d*f)
```

Giac [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx) + a)^2}{(i\,cdx + d)^{\frac{3}{2}}(-i\,cfx + f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{3/2}}\,dx = \int \frac{(a + b\operatorname{asinh}(c\,x))^2}{(d + c\,d\,x\,1\mathrm{i})^{3/2}\,(f - c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{2\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+1}\,\sqrt{-cix+1}}dx\right)ab+\sqrt{c}}{\sqrt{f}\,\sqrt{d}\,\sqrt{cix+1}}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

output
```
(2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b + sq
rt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2 + a**
2*x)/(sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*d*f)
```

3.266 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 779

$$\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx &= -\frac{ib^2}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{b^2x}{3d^2f\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b(a+b\text{arcsinh}(cx))}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}} \\
&- \frac{ibx(a+b\text{arcsinh}(cx))}{3d^2f\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}} + \frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{i(a+b\text{arcsinh}(cx))^2}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} \\
&+ \frac{x(a+b\text{arcsinh}(cx))^2}{3d^2f\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{2ib\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cd^2f\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
-1/3*I*b^2/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*b^2*x/d^2/f/(d+
I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*b*(a+b*arcsinh(c*x))/c/d^2/f/(d+I*c*d
*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/3*I*b*x*(a+b*arcsinh(c*x))
/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*x*(a+b*ar
csinh(c*x))^2/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*(a+b*arcsinh
(c*x))^2/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+1/3*x*(a+
b*arcsinh(c*x))^2/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+2/
3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*
f*x)^(1/2)-2/3*I*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^
2+1)^(1/2))/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4/3*b*(c^2*x^2+1)^
(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/d^2/f/(d+I*c*d*
x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*b^2*(c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*b^2*(c^2*
x^2+1)^(1/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2/f/(d+I*c*d*x)^(1/2
)/(f-I*c*f*x)^(1/2)-2/3*b^2*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))^2)/c/d^2/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.59 (sec) , antiderivative size = 754, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2))
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-1/6*I)*a^2)/(d^3*f^2*(-I
+ c*x)^2) + (5*a^2)/(12*d^3*f^2*(-I + c*x)) + a^2/(4*d^3*f^2*(I + c*x))))/
c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*Ar
cSinh[c*x] + (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(
1 - (2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2
]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d^2*f*
(-I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2
))]) + ((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1
 + c^2*x^2]*(7*Pi*ArcSinh[c*x] + ((2 + I*ArcSinh[c*x])*ArcSinh[c*x])/(-I +
 c*x) - (1 + 4*I)*ArcSinh[c*x]^2 - 5*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E
^ArcSinh[c*x]] + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 1
6*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]]
 + 16*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 5*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x]
)/4]] + (6*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (10*I)*PolyLog[2, I/E^ArcS
inh[c*x]] + ((3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2]) + ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]
)/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + ((-4 + 5*ArcSinh[c*x
]^2)*Sinh[ArcSinh[c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/
2])))/(c*d^2*f*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2
*x^2))])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{f(1-icx)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\int\frac{(1-icx)(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\int\left(\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}-\frac{icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\left(c^2x^2+1\right)^{5/2}\left(-\frac{2ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{ibx(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{b(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(c\right.}{3\sqrt{c^2x^2+1}}}{}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

output
```
(f*(1 + c^2*x^2)^(5/2)*(((-1/3*I)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (b^2*x)/(3*
Sqrt[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*(1 + c^2*x^2)) - ((I/3)
*b*x*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2) + (2*(a + b*ArcSinh[c*x])^2)/(3*c
) + ((I/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) + (x*(a + b*Arc
Sinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - (((2*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]
])/c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) - (b^2
*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) + (b^2*PolyLog[2, I*E^ArcSinh[c*x]
])/(3*c) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.39

method	result	size
default	Expression too large to display	1084
parts	Expression too large to display	1084

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(1/3*I/c/d/f/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2)+2/3/d*(I/c/d/f/(d+I*c
*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I/c/f/d^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2
)))+1/3*b^2*(1-3*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))-5*polylog(2,-I*(x*c+
(c^2*x^2+1)^(1/2)))+2*arcsinh(x*c)^2+arcsinh(x*c)-3*arcsinh(x*c)*ln(1-I*(x
*c+(c^2*x^2+1)^(1/2)))+3*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*x*c-I*(c^2*x^2+1
)^(1/2)-5*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*arcsinh(x*c)*x^4*c^4+2*arcsinh(x
*c)^2*(c^2*x^2+1)^(1/2)*x^3*c^3-10*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*arcsinh
(x*c)*x^2*c^2+c^4*x^4-3*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c
^4-I*arcsinh(x*c)*x^3*c^3-I*arcsinh(x*c)*x*c-I*(c^2*x^2+1)^(1/2)*x^2*c^2-6
*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-(c^2*x^2+1)^(1/2)*x*
c-(c^2*x^2+1)^(1/2)*c^3*x^3+2*c^2*x^2+arcsinh(x*c)*c^2*x^2+2*arcsinh(x*c)^
2*x^4*c^4-5*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4+4*arcsinh(x*c)^2
*x^2*c^2-10*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-6*polylog(2,I*(x
*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-3*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*
c^4+I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2-5*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2
+1)^(1/2))))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)/f^
2/d^3/c/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)-1/3*a*b*(5*ln(x*c+(c^2*x^2+1)^(1/2
)-I)*x^4*c^4+3*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^4*c^4-4*arcsinh(x*c)*c^4*x^4-
4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3+I*x*c+10*ln(x*c+(c^2*x^2+1)^(1/2)
-I)*x^2*c^2+6*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^2*c^2-8*arcsinh(x*c)*c^2*x^...
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^
2 + c^2*d^3*f^2*x - I*c*d^3*f^2)*integral(1/3*(-3*I*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*a^2 - 2*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*
b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^3*f^2*x^5 - I*c^4*d^3*
f^2*x^4 + 2*c^3*d^3*f^2*x^3 - 2*I*c^2*d^3*f^2*x^2 + c*d^3*f^2*x - I*d^3*f^
2), x))/(c^4*d^3*f^2*x^3 - I*c^3*d^3*f^2*x^2 + c^2*d^3*f^2*x - I*c*d^3*f^2
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}(f - icfx)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}(f - icfx)^{3/2}}\, dx = \int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{5/2}(f - icfx)^{3/2}}\, dx = \frac{6\sqrt{cix+1}\,\sqrt{-cix+1}\,\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^3 i\,x^3+\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+1}\,\sqrt{}

output
```
(6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**3*i*x**3 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2
+ sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)),x)*a*b*c**3*x**2 + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c
*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**3*i*x**3 + sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x + sqrt
(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**3*i*x**3 +
sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i
*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c**3*x**2 + 3*
sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**3*i*x**3 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2
+ sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x + sqrt(c*i*x + 1)*sqrt( - c*i*x
 + 1)),x)*b**2*c + 2*a**2*c**3*x**3 + 3*a**2*c*x + a**2*i)/(3*sqrt(f)*sqrt
(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d**2*f*(c**2*x**2 + 1))
```

3.267 $\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 776

$$\begin{aligned}
\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{2ib^2d^3(1+c^2x^2)}{cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{2ibd^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{f^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{28d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{id^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))^2}{cf^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{5d^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^3}{3bcf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{112bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{-\operatorname{arcsinh}(cx)}\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{112b^2d^3\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arcsinh}(cx)}\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{16ib^2d^3\sqrt{1+c^2x^2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{28id^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4id^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
2*I*b^2*d^3*(c^2*x^2+1)/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b*d^
3*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)+28/3*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/f^2/(d+I*c*d*x)^(
1/2)/(f-I*c*f*x)^(1/2)+I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/f^2/(d+I*c
*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/3*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))
^3/b/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+112/3*b*d^3*(c^2*x^2+1)^(1/
2)*(a+b*arcsinh(c*x))*ln(1+I/(c*x+(c^2*x^2+1)^(1/2)))/c/f^2/(d+I*c*d*x)^(1
/2)/(f-I*c*f*x)^(1/2)-112/3*b^2*d^3*(c^2*x^2+1)^(1/2)*polylog(2,-I/(c*x+(c
^2*x^2+1)^(1/2)))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*b*d^3*(c^2
*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/f^2/(d
+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+16/3*I*b^2*d^3*(c^2*x^2+1)^(1/2)*tan(1/4
*Pi+1/2*I*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+28/3*I*d
^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c
/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4/3*I*d^3*(c^2*x^2+1)^(1/2)*(a+b*
arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh(
c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2552 vs. 2(776) = 1552.

Time = 23.76 (sec) , antiderivative size = 2552, normalized size of antiderivative = 3.29

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*d^2)/f^3 + (((8*I)/3)
*a^2*d^2)/(f^3*(I + c*x)^2) - (28*a^2*d^2)/(3*f^3*(I + c*x))))/c + (5*a^2*
d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I
+ c*x)]])/(c*f^(5/2)) - ((I/3)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*
(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[A
rcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[
ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I
+ 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2
*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh
[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(
c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqr
t[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*
ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x
^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 -
(84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(
4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x
]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14...
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^5(icx+1)^5(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^5(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6259}$$

$$\frac{d^5\left(c^2x^2+1\right)^{5/2}\int \left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{12i(a+b\text{arcsinh}(cx))^2}{(cx+i)\sqrt{c^2x^2+1}}-\frac{8(a+b\text{arcsinh}(cx))^2}{(cx+i)^2\sqrt{c^2x^2+1}}+\frac{5(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$d^5\left(c^2x^2+1\right)^{5/2}\left(\frac{i\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{c}+\frac{5(a+b\text{arcsinh}(cx))^3}{3bc}+\frac{28(a+b\text{arcsinh}(cx))^2}{3c}+\frac{112b\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a-}{3c}\right.$$

input
```
Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^5*(1 + c^2*x^2)^(5/2)*((-2*I)*a*b*x + ((2*I)*b^2*Sqrt[1 + c^2*x^2])/c -
 (2*I)*b^2*x*ArcSinh[c*x] + (28*(a + b*ArcSinh[c*x])^2)/(3*c) + (I*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x])^2)/c + (5*(a + b*ArcSinh[c*x])^3)/(3*b*c)
+ (112*b*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c) - (112*b^2*
PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c) + (8*b*(a + b*ArcSinh[c*x])*Sec[Pi/
4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((16*I)/3)*b^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/c + (((28*I)/3)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c
*x]])/c - (((4*I)/3)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]
^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 7.73 (sec) , antiderivative size = 1134, normalized size of antiderivative = 1.46

method	result	size
default	Expression too large to display	1134

input
```
int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
5/3*d^2*(a+b*arcsinh(x*c))^3*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2
*x^2+1)^(1/2)/b/c/f^3+1/2*I*d^2*(arcsinh(x*c)^2*b^2+2*arcsinh(x*c)*a*b-2*b
^2*arcsinh(x*c)+a^2-2*a*b+2*b^2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c
-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)/c/f^3+1/2*I*d^2*(arcsinh(x*c
)^2*b^2+2*arcsinh(x*c)*a*b+2*b^2*arcsinh(x*c)+a^2+2*a*b+2*b^2)*(c^2*x^2-(c
^2*x^2+1)^(1/2)*x*c+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1
)/c/f^3-4/3*d^2*(518*a*b+4554*arcsinh(x*c)^2*b^2*c^2*x^2+1369*a^2+1369*arc
sinh(x*c)^2*b^2+518*b^2*arcsinh(x*c)+888*b^2+882*a*b*c^4*x^4+1288*a*b*c^2*
x^2-252*(c^2*x^2+1)^(1/2)*b^2*c^3*x^3-236*(c^2*x^2+1)^(1/2)*b^2*c*x+882*(c
^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2*c^3*x^3+442*(c^2*x^2+1)^(1/2)*arcsinh(x*c
)*b^2*c*x+882*arcsinh(x*c)*b^2*c^4*x^4+1288*arcsinh(x*c)*b^2*c^2*x^2+132*I
*(c^2*x^2+1)^(1/2)*b^2*c^2*x^2+308*I*a*b*c*x+420*I*a*b*c^3*x^3+882*(c^2*x^
2+1)^(1/2)*a*b*c^3*x^3+442*(c^2*x^2+1)^(1/2)*a*b*c*x+9108*arcsinh(x*c)*a*b
*c^2*x^2+2738*arcsinh(x*c)*a*b-714*I*(c^2*x^2+1)^(1/2)*a*b*c^2*x^2+308*I*a
rcsinh(x*c)*b^2*c*x+420*I*arcsinh(x*c)*b^2*c^3*x^3-370*I*(c^2*x^2+1)^(1/2)
*arcsinh(x*c)*b^2+2016*b^2*c^4*x^4+2680*b^2*c^2*x^2+7938*arcsinh(x*c)*a*b*
c^4*x^4+3969*arcsinh(x*c)^2*b^2*c^4*x^4+4554*a^2*c^2*x^2-714*I*arcsinh(x*c
)*(c^2*x^2+1)^(1/2)*b^2*c^2*x^2+456*I*b^2*c^3*x^3+232*I*b^2*c*x-370*I*(c^2
*x^2+1)^(1/2)*a*b+148*I*(c^2*x^2+1)^(1/2)*b^2+3969*a^2*c^4*x^4)*(7*x^3*c^3
-9*I*c^2*x^2-7*x^2*c^2*(c^2*x^2+1)^(1/2)+3*x*c-5*I-7*(c^2*x^2+1)^(1/2))...
```

Fricas [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^{2}}{\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((I*b^2*c^2*d^2*x^2 + 2*b^2*c*d^2*x - I*b^2*d^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c^2*d^2*x^
2 - 2*a*b*c*d^2*x + I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x - I*a^2*d^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*
f^3*x - I*f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2\,(d+c\,d\,x\,1\text{i})^{5/2}}{(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{5/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d+I*c*d*x)^(5/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

output

```
(sqrt(d)*d**2*( - 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x
 + 1)/sqrt(2))*a**2*c*x - 30*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt(
 - c*i*x + 1)/sqrt(2))*a**2*i + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((
sqrt(c*i*x + 1)*asinh(c*x)*x**2)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( -
 c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**4*i*x - 6*sqrt(c*i*x + 1
)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x**2)/(sqrt( - c*i*x
+ 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c
**3 + 12*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x
)*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( -
c*i*x + 1)),x)*a*b*c**3*x + 12*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqr
t(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x
 + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**2*i - 6*sqrt(c*i*x + 1)*sqrt(
- c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**
2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**2*i*x + 6*s
qrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt(
- c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),
x)*a*b*c + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh
(c*x)**2*x**2)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x
- sqrt( - c*i*x + 1)),x)*b**2*c**4*i*x - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x +
 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2*x**2)/(sqrt( - c*i*x + 1)*c**2*x...
```

3.268 $\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 608

$$\begin{aligned}
\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{8d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{d^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^3}{3bcf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{32bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+ie^{-\text{arcsinh}(cx)}\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{32b^2d^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{-\text{arcsinh}(cx)}\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{4bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8ib^2d^2\sqrt{1+c^2x^2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{8id^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{2id^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}$$

output
```
8/3*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/f^2/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)+1/3*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^3/b/c/f^2/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+32/3*b*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(
c*x))*ln(1+I/(c*x+(c^2*x^2+1)^(1/2)))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-32/3*b^2*d^2*(c^2*x^2+1)^(1/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))
/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4/3*b*d^2*(c^2*x^2+1)^(1/2)*(a+
b*arcsinh(c*x))*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2/c/f^2/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)+8/3*I*b^2*d^2*(c^2*x^2+1)^(1/2)*tan(1/4*Pi+1/2*I*arcsinh(
c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+8/3*I*d^2*(c^2*x^2+1)^(1/2
)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1
/2)/(f-I*c*f*x)^(1/2)-2/3*I*d^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*sec
(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/f^2/(d+I*c*
d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1617 vs. $2(608) = 1216$.

Time = 16.46 (sec) , antiderivative size = 1617, normalized size of antiderivative = 2.66

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2)
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((4*I)/3)*a^2*d)/(f^3*(I +
c*x)^2) - (8*a^2*d)/(3*f^3*(I + c*x))))/c + (a^2*d^(3/2)*Log[c*d*f*x + Sqr
t[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(5/2)) - (
(I/3)*a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(
1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*
ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sq
rt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan
[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x
^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^
2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Lo
g[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((
-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]
/2])^4) + (a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(
d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[
(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[
Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*
(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*Ar
cSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x
^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)...
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^4(icx+1)^4(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)^4(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6259}$$

$$\frac{d^4\left(c^2x^2+1\right)^{5/2}\int \left(-\frac{4i(a+b\text{arcsinh}(cx))^2}{(cx+i)\sqrt{c^2x^2+1}}-\frac{4(a+b\text{arcsinh}(cx))^2}{(cx+i)^2\sqrt{c^2x^2+1}}+\frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^4\left(c^2x^2+1\right)^{5/2}\left(\frac{(a+b\text{arcsinh}(cx))^3}{3bc}+\frac{8(a+b\text{arcsinh}(cx))^2}{3c}+\frac{32b\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}+\frac{8i\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcs}\right.}{}\right.}{}$$

input
```
Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^4*(1 + c^2*x^2)^(5/2)*((8*(a + b*ArcSinh[c*x])^2)/(3*c) + (a + b*ArcSin
h[c*x])^3/(3*b*c) + (32*b*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/
(3*c) - (32*b^2*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c) + (4*b*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c) + (((8*I)/3)*b^2*Tan[Pi/4
 + (I/2)*ArcSinh[c*x]])/c + (((8*I)/3)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/c - (((2*I)/3)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)
*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c))/((d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 6.26 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.38

method	result
default	$\frac{d(a+b\,\mathrm{arcsinh}(xc))^3\sqrt{i(xc-i)d}\,\sqrt{-i(xc+i)f}}{3\sqrt{c^2x^2+1}\,b\,f^3c}-\frac{4d\left(20ab+105\,\mathrm{arcsinh}(xc)^2b^2c^2x^2+25a^2+25\,\mathrm{arcsinh}(xc)^2b^2+20b^2\,\mathrm{arcsinh}(xc)+\right.}{}$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3*d*(a+b*arcsinh(x*c))^3*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x
^2+1)^(1/2)/b/f^3/c-4/3*d*(20*a*b+105*arcsinh(x*c)^2*b^2*c^2*x^2+25*a^2+25
*arcsinh(x*c)^2*b^2+20*b^2*arcsinh(x*c)+30*b^2+48*a*b*c^4*x^4+52*a*b*c^2*x
^2-24*(c^2*x^2+1)^(1/2)*b^2*c^3*x^3-20*(c^2*x^2+1)^(1/2)*b^2*c*x-10*I*(c^2
*x^2+1)^(1/2)*arcsinh(x*c)*b^2+48*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2*c^3*x
^3+10*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*b^2*c*x+48*arcsinh(x*c)*b^2*c^4*x^4+5
2*arcsinh(x*c)*b^2*c^2*x^2-10*I*(c^2*x^2+1)^(1/2)*a*b+6*I*(c^2*x^2+1)^(1/2
)*b^2*c^2*x^2+10*I*(c^2*x^2+1)^(1/2)*b^2+42*I*b^2*c^3*x^3+10*I*b^2*c*x-36*
I*(c^2*x^2+1)^(1/2)*a*b*c^2*x^2+48*(c^2*x^2+1)^(1/2)*a*b*c^3*x^3+10*(c^2*x
^2+1)^(1/2)*a*b*c*x+210*arcsinh(x*c)*a*b*c^2*x^2+20*I*a*b*c*x+36*I*a*b*c^3
*x^3+50*arcsinh(x*c)*a*b+120*b^2*c^4*x^4+118*b^2*c^2*x^2+288*arcsinh(x*c)*
a*b*c^4*x^4+144*arcsinh(x*c)^2*b^2*c^4*x^4+105*a^2*c^2*x^2-36*I*arcsinh(x*
c)*(c^2*x^2+1)^(1/2)*b^2*c^2*x^2+20*I*arcsinh(x*c)*b^2*c*x+36*I*arcsinh(x*
c)*b^2*c^3*x^3+144*a^2*c^4*x^4)*(2*x^3*c^3-3*I*c^2*x^2-2*x^2*c^2*(c^2*x^2+
1)^(1/2)-I-2*(c^2*x^2+1)^(1/2))*(-I*(I+x*c)*f)^(1/2)*(I*(x*c-I)*d)^(1/2)/f
^3/(144*c^4*x^4+105*c^2*x^2+25)/(c^2*x^2+1)^2/c-16/3*d*(b*arcsinh(x*c)^2-2
*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*b-2*a*ln(x*c+(c^2*x^2+1)^(1/
2)+I)+2*a*ln(x*c+(c^2*x^2+1)^(1/2))-2*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))
*b)*b*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/f^3/c
```

Fricas [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x
 - I*f^3), x)
```

Sympy [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{(id(cx-i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(5
/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+icdx)^{3/2}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+icdx)^{3/2}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d+I*c*d*x)^(3/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

output

```
(sqrt(d)*d*( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*i*x + 1
)/sqrt(2))*a**2*c*x - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*asin(sqrt( - c*
i*x + 1)/sqrt(2))*a**2*i + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(
c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x +
 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**3*x + 6*sqrt(c*i*x + 1)*sqrt( -
c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)*x)/(sqrt( - c*i*x + 1)*c**2*x**
2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c**2*i - 6*sqr
t(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( -
c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)
*a*b*c**2*i*x + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*
asinh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - s
qrt( - c*i*x + 1)),x)*a*b*c + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sq
rt(c*i*x + 1)*asinh(c*x)**2*x)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c
*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b**2*c**3*x + 3*sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2*x)/(sqrt( - c*i*x + 1
)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b**2*c**
2*i - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)
**2)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( -
c*i*x + 1)),x)*b**2*c**2*i*x + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((s
qrt(c*i*x + 1)*asinh(c*x)**2)/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( -...
```

3.269 $\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 529

$$\begin{aligned}\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx &= \frac{d\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{4bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+ie^{-\operatorname{arcsinh}(cx)}\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{4b^2d\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arcsinh}(cx)}\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{2bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{4ib^2d\sqrt{1+c^2x^2}\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{id\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{id\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}i\operatorname{arcsinh}(cx)\right)}{3cf^2\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output
```
1/3*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)+4/3*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*x+(c^2
*x^2+1)^(1/2)))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4/3*b^2*d*(c^2*x
^2+1)^(1/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/f^2/(d+I*c*d*x)^(1/2)/
(f-I*c*f*x)^(1/2)+2/3*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi+
1/2*I*arcsinh(c*x))^2/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4/3*I*b^2*
d*(c^2*x^2+1)^(1/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)+1/3*I*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*tan(1/4*
Pi+1/2*I*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*d*(
c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan
(1/4*Pi+1/2*I*arcsinh(c*x))/c/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.65 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x
]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((2*I)/3)*a^2)/(f^3*(I + c*
x)^2) - a^2/(3*f^3*(I + c*x))))/c - ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcT
an[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/
2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt
[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth
[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I
)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x
]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*(-I + c*x)*Sqrt[I*
((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1
 - I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (
2*I)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh
[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4
]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4
*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSi
nh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSin
h[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f
+ c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x...
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d^3(icx+1)^3(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(c^2x^2+1)^{5/2}\int \frac{(icx+1)^3(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6259

$$\frac{d^3(c^2x^2+1)^{5/2}\int \left(-\frac{i(a+b\text{arcsinh}(cx))^2}{(cx+i)\sqrt{c^2x^2+1}}-\frac{2(a+b\text{arcsinh}(cx))^2}{(cx+i)^2\sqrt{c^2x^2+1}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{d^3(c^2x^2+1)^{5/2}\left(\frac{(a+b\text{arcsinh}(cx))^2}{3c}+\frac{4b\log\left(1+ie^{-\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}+\frac{i\tan\left(\frac{\pi}{4}+\frac{1}{2}i\text{arcsinh}(cx)\right)(a+b\text{arcsinh}(cx)}{3c}\right.}{}$$

input
```
Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

output
```
(d^3*(1 + c^2*x^2)^(5/2)*((a + b*ArcSinh[c*x])^2/(3*c) + (4*b*(a + b*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c) - (4*b^2*PolyLog[2, (-I)/E^ArcSi
nh[c*x]])/(3*c) + (2*b*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]
^2)/(3*c) + (((4*I)/3)*b^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c + ((I/3)*(a +
 b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/c - ((I/3)*(a + b*ArcSi
nh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]]
)/c))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6259
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2756 vs. $2(447) = 894$.

Time = 7.65 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.21

method	result	size
default	Expression too large to display	2757
parts	Expression too large to display	2757

input
```
int((d+I*c*d*x)^(1/2)*(a+b*arcsinh(x*c))^2/(f-I*c*f*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1
)^2*arcsinh(x*c)^2*x-2/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/
(3*c^2*x^2-1)/(c^2*x^2+1)^2*arcsinh(x*c)*x-8/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I
*(I+x*c)*f)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)^2*c^4*x^5+4/3*b^2*(I*(x*c-
I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(1/2)/c/f^3*arcsinh(x*c)*ln(1
-I*(x*c+(c^2*x^2+1)^(1/2)))-4/3*I*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(
1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)/c+8/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*
c)*f)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*c*x^2+2/3*b^2*(I*(x*c-I)*d
)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)*arcsinh(x*c)*x+
4*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+
1)^(3/2)*c^3*x^4-4/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/(3*c
^2*x^2-1)/(c^2*x^2+1)*c^2*x^3-1/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(
1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/c*arcsinh(x*c)^2-4/3*b^2*(I*(x*c-
I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/c*arc
sinh(x*c)+a^2*(-I/c/f*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2)-d*(-1/3*I/d/c/f/
(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-1/3*I/c/d/f^2/(f-I*c*f*x)^(1/2)*(d+I*c
*d*x)^(1/2)))-4/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/(3*c^2*
x^2-1)/(c^2*x^2+1)*x+8/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/f^3/
(3*c^2*x^2-1)/(c^2*x^2+1)^2*x-4/3*b^2*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(
1/2)/f^3/(3*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/c-2/3*b^2*(I*(x*c-I)*d)^(1/2)*...
```

Fricas [F]

$$\int \frac{\sqrt{d+icdx}(a+b\text{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx=\int \frac{\sqrt{i\,cdx+d}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
-1/3*((b^2*c*x - I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqr
t(c^2*x^2 + 1))^2 - 3*(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)*integral(1/3*(
-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
+ f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3
*c*f^3*x - I*f^3), x))/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{id\,(cx-i)}(a+b\operatorname{asinh}\,(cx))^2}{(-if\,(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)**(1/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(5/2)
, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\sqrt{i\,cdx+d}(b\operatorname{arsinh}\,(cx)+a)^2}{(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+I*c*d*x)^(1/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{{\left(f-c\,f\,x\,1\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

output
```
int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+icdx}(a+b\operatorname{arcsinh}(cx))^2}{(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d}\left(-6\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{\sqrt{cix+1}\,asinh(cx)}{\sqrt{-cix+1}\,c^2x^2+2\sqrt{-cix+1}\,cix-\sqrt{-cix+1}}dx\right.\right.}{}$$

input
```
int((d+I*c*d*x)^(1/2)*(a+b*asinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

output
```
(sqrt(d)*( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asi
nh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt
( - c*i*x + 1)),x)*a*b*c**2*i*x + 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int
((sqrt(c*i*x + 1)*asinh(c*x))/(sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*
i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*a*b*c - 3*sqrt(c*i*x + 1)*sqrt( -
c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2)/(sqrt( - c*i*x + 1)*c**2*x*
*2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x + 1)),x)*b**2*c**2*i*x + 3
*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int((sqrt(c*i*x + 1)*asinh(c*x)**2)/(s
qrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt( - c*i*x + 1)*c*i*x - sqrt( - c*i*x +
 1)),x)*b**2*c - a**2*c**2*i*x**2 - 2*a**2*c*x + a**2*i))/(3*sqrt(f)*sqrt(
c*i*x + 1)*sqrt( - c*i*x + 1)*c*f**2*(c*i*x - 1))
```

3.270 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 942

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{2ib^2d^2(1+c^2x^2)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x(1+c^2x^2)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{arcsinh}(cx)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcd^2x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2d^2x^3(1+c^2x^2)(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2d^2x(1+c^2x^2)^2(a+b\operatorname{arcsinh}(cx))^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ibd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\arctan\left(e^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2bd^2(1+c^2x^2)^{5/2}(a+b\operatorname{arcsinh}(cx))\log\left(1+e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2d^2(1+c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

output
```
2/3*I*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)-2/3*b^2*d^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+1/3*b^2*d^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/
2)/(f-I*c*f*x)^(5/2)+4/3*I*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*d^
2*x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*I*b^2*d^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3
*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)-1/3*c^2*d^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+2/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2
)/(f-I*c*f*x)^(5/2)+1/3*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*
c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))
/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*
b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.04 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.56

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(2i+cx)}{(i+cx)^2} - \frac{ab\left(i\cosh\left(\frac{3}{2}\text{arcsinh}(cx)\right)\left(\text{arcsinh}(cx)-2\,\text{arcta}\right.\right.}{\ldots}\ldots\right.}{\ldots}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x
]
```

output
```
(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(2*I + c*x))/(I + c*x)^2 - (a*b
*(I*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]
] + (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 + (3*I)*ArcSinh[c*x
] + (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*(I +
(-1 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[C
oth[ArcSinh[c*x]/2]] - (I/2)*(2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2])*Sin
h[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcS
inh[c*x]/2])^3) - (b^2*((1 + I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(2*I + ArcS
inh[c*x]))/(I + c*x) + 2*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]]
 + I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*
I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) - 4*PolyLog[2, (-I)/E^
ArcSinh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*
x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]
))/(3*c*d*f^3)
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}dx$$

$$\Big\downarrow \text{ 6211}$$

$$\frac{(c^2x^2+1)^{5/2}\int \frac{d^2(icx+1)^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2(c^2x^2+1)^{5/2}\int \frac{(icx+1)^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\int\left(-\frac{c^2x^2(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}+\frac{2icx(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(c^2x^2+1\right)^{5/2}\left(\frac{4ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}-\frac{bcx^2(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx))^2}{3\sqrt{c^2x^2+1}}+\frac{x(a+b\text{arcsinh}}{3(c^2x^2+1)^3}\right.}{}$$

input

```
Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

output

```
(d^2*(1 + c^2*x^2)^(5/2)*((((2*I)/3)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (2*b^2*x
)/(3*Sqrt[1 + c^2*x^2]) + (b^2*ArcSinh[c*x])/(3*c) + (b*(a + b*ArcSinh[c*x
]))/(3*c*(1 + c^2*x^2)) + (((2*I)/3)*b*x*(a + b*ArcSinh[c*x]))/(1 + c^2*x^
2) - (b*c*x^2*(a + b*ArcSinh[c*x]))/(3*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x
])^2/(3*c) - (((2*I)/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) +
(x*(a + b*ArcSinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) - (c^2*x^3*(a + b*ArcSi
nh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sqrt
[1 + c^2*x^2]) + (((4*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/c - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) + (2*b^2
*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) - (2*b^2*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 5.69 (sec) , antiderivative size = 771, normalized size of antiderivative = 0.82

method	result
default	$a^2\left(-\frac{i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{i\sqrt{icdx+d}}{3cd\,f^2\sqrt{-icfx+f}}\right)+\frac{b^2\left(\operatorname{arcsinh}(xc)^2x^4c^4-4\operatorname{arcsinh}(xc)\ln\left(1-i\left(xc+\sqrt{c^2x^2+1}\right)\right)x^4c^4+\operatorname{arcsin}\right.}{}$
parts	$a^2\left(-\frac{i\sqrt{icdx+d}}{3dcf(-icfx+f)^{\frac{3}{2}}}-\frac{i\sqrt{icdx+d}}{3cd\,f^2\sqrt{-icfx+f}}\right)+\frac{b^2\left(\operatorname{arcsinh}(xc)^2x^4c^4-4\operatorname{arcsinh}(xc)\ln\left(1-i\left(xc+\sqrt{c^2x^2+1}\right)\right)x^4c^4+\operatorname{arcsin}\right.}{}$

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(-1/3*I/d/c/f/(f-I*c*f*x)^(3/2)*(d+I*c*d*x)^(1/2)-1/3*I/c/d/f^2/(f-I*c
*f*x)^(1/2)*(d+I*c*d*x)^(1/2))+1/3*b^2*(arcsinh(x*c)^2*x^4*c^4-4*arcsinh(x
*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4+arcsinh(x*c)^2*(c^2*x^2+1)^(1/
2)*x^3*c^3-4*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4+2*I*(c^2*x^2+1)^
(1/2)*x^2*c^2+2*c^4*x^4-2*(c^2*x^2+1)^(1/2)*c^3*x^3+2*arcsinh(x*c)^2*x^2*c
^2-8*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2+2*I*arcsinh(x*c)
*x*c+3*(c^2*x^2+1)^(1/2)*arcsinh(x*c)^2*x*c+2*arcsinh(x*c)*c^2*x^2-8*polyl
og(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)
^2+2*I*arcsinh(x*c)*x^3*c^3+4*c^2*x^2-2*(c^2*x^2+1)^(1/2)*x*c+arcsinh(x*c)
^2-4*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))+2*I*(c^2*x^2+1)^(1/2)+2*
arcsinh(x*c)-4*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))+2)*(I*(x*c-I)*d)^(1/2)
*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)/f^
3/c/d+2/3*a*b*(arcsinh(x*c)*c^4*x^4-2*ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^4*c^4+
arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3+I*x^3*c^3+2*arcsinh(x*c)*c^2*x^2-4*
ln(x*c+(c^2*x^2+1)^(1/2)+I)*x^2*c^2+3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c+c
^2*x^2-2*I*(c^2*x^2+1)^(1/2)*arcsinh(x*c)+I*x*c+arcsinh(x*c)-2*ln(x*c+(c^2
*x^2+1)^(1/2)+I)+1)*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)/(c^2*x^2+1)^(
5/2)/f^3/c/d
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((b^2*c*x + 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 3*(c^3*d*f^3*x^2 + 2*I*c^2*d*f^3*x - c*d*f^3)*integra
l(-1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*a*b + (b^2*c*x + 2*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^4*d*f^3*x^4
+ 2*I*c^3*d*f^3*x^3 + 2*I*c*d*f^3*x - d*f^3), x))/(c^3*d*f^3*x^2 + 2*I*c^2
*d*f^3*x - c*d*f^3)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))^2}{\sqrt{id(cx-i)}\,(-if(cx+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2
)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{i\,cdx+d}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))^2}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,(f-c\,f\,x\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{-6\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+2\sqrt{cix+1}\,\sqrt{-cix+1}\,cix-\sqrt{cix+1}\,\sqrt{-}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x -
 sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c**3*x**2 - 6*sqrt(c*i*x + 1)*
sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2
*x**2 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt(
 - c*i*x + 1)),x)*a*b*c - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c
*x)**2/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c**3
*x**2 - 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x
 + 1)*sqrt( - c*i*x + 1)*c**2*x**2 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*
c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c + a**2*c**3*x**3 + 3
*a**2*c*x - 2*a**2*i)/(3*sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1
)*c*f**2*(c**2*x**2 + 1))
```

3.271 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 779

$$
\begin{aligned}
\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx &= \frac{ib^2}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{b^2x}{3df^2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b(a+b\text{arcsinh}(cx))}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}} \\
&+ \frac{ibx(a+b\text{arcsinh}(cx))}{3df^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}} + \frac{2x(a+b\text{arcsinh}(cx))^2}{3df^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{i(a+b\text{arcsinh}(cx))^2}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} \\
&+ \frac{x(a+b\text{arcsinh}(cx))^2}{3df^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{2ib\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\arctan\left(e^{\text{arcsinh}(cx)}\right)}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&+ \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-ie^{\text{arcsinh}(cx)}\right)}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,ie^{\text{arcsinh}(cx)}\right)}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}} \\
&- \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cdf^2\sqrt{d+icdx}\sqrt{f-icfx}}
\end{aligned}
$$

output
```
1/3*I*b^2/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*b^2*x/d/f^2/(d+I
*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*b*(a+b*arcsinh(c*x))/c/d/f^2/(d+I*c*d*
x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*I*b*x*(a+b*arcsinh(c*x))/
d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*x*(a+b*arc
sinh(c*x))^2/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*(a+b*arcsinh(
c*x))^2/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+1/3*x*(a+b
*arcsinh(c*x))^2/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)+2/3
*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+2/3*I*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2
+1)^(1/2))/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4/3*b*(c^2*x^2+1)^(
1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/d/f^2/(d+I*c*d*x
)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*b^2*(c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+(c^2
*x^2+1)^(1/2)))/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*b^2*(c^2*x
^2+1)^(1/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d/f^2/(d+I*c*d*x)^(1/2)
/(f-I*c*f*x)^(1/2)-2/3*b^2*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(
1/2))^2)/c/d/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.56 (sec) , antiderivative size = 757, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2))
,x]
```

output
```
(Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(a^2/(4*d^2*f^3*(-I + c*x)) +
 ((I/6)*a^2)/(d^2*f^3*(I + c*x)^2) + (5*a^2)/(12*d^2*f^3*(I + c*x))))/c -
((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSin
h[c*x] - (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 +
(2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] +
 (2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d*f^2*(I +
 c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
- ((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^
2*x^2]*(-9*Pi*ArcSinh[c*x] + ((2 - I*ArcSinh[c*x])*ArcSinh[c*x])/(I + c*x)
 - (1 - 4*I)*ArcSinh[c*x]^2 + 3*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcS
inh[c*x]] - 5*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 16*Pi*
Log[1 + E^ArcSinh[c*x]] + 5*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 16
*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]
 - (10*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[2, I/E^ArcSinh[c
*x]] - ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])^3 + (I*(4 - 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2
])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) - ((3*I)*ArcSinh[c*x]^2
*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(
c*d*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]
)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6211, 27, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6211}$$

$$\frac{\left(c^2x^2+1\right)^{5/2}\int \frac{d(icx+1)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(c^2x^2+1\right)^{5/2}\int \frac{(icx+1)(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6253

$$\frac{d\left(c^2x^2+1\right)^{5/2}\int \left(\frac{icx(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}+\frac{(a+b\text{arcsinh}(cx))^2}{\left(c^2x^2+1\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$d\left(c^2x^2+1\right)^{5/2}\left(\frac{2ib\arctan\left(e^{\text{arcsinh}(cx)}\right)(a+b\text{arcsinh}(cx))}{3c}+\frac{ibx(a+b\text{arcsinh}(cx))}{3(c^2x^2+1)}+\frac{b(a+b\text{arcsinh}(cx))}{3c(c^2x^2+1)}+\frac{2x(a+b\text{arcsinh}(cx)}{3\sqrt{c^2x^2+1}}\right.$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
(d*(1 + c^2*x^2)^(5/2)*(((I/3)*b^2)/(c*Sqrt[1 + c^2*x^2]) - (b^2*x)/(3*Sqr
t[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*(1 + c^2*x^2)) + ((I/3)*b*
x*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2) + (2*(a + b*ArcSinh[c*x])^2)/(3*c) -
 ((I/3)*(a + b*ArcSinh[c*x])^2)/(c*(1 + c^2*x^2)^(3/2)) + (x*(a + b*ArcSin
h[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[
1 + c^2*x^2]) + (((2*I)/3)*b*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/
c - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c) + (b^2*Po
lyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c) - (b^2*PolyLog[2, I*E^ArcSinh[c*x]])/
(3*c) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c)))/((d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 6.77 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.39

method	result	size
default	Expression too large to display	1084
parts	Expression too large to display	1084

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(I/c/d/f/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2)+2/d*(-1/3*I/d/c/f/(f-I*c*
f*x)^(3/2)*(d+I*c*d*x)^(1/2)-1/3*I/c/d/f^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(
1/2)))+1/3*b^2*(1-5*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))-3*polylog(2,-I*(x
*c+(c^2*x^2+1)^(1/2)))+I*arcsinh(x*c)*x^3*c^3+I*arcsinh(x*c)*x*c+2*arcsinh
(x*c)^2+arcsinh(x*c)-5*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))+3*(c^2
*x^2+1)^(1/2)*arcsinh(x*c)^2*x*c+I*(c^2*x^2+1)^(1/2)-3*ln(1+I*(x*c+(c^2*x^
2+1)^(1/2)))*arcsinh(x*c)*x^4*c^4+2*arcsinh(x*c)^2*(c^2*x^2+1)^(1/2)*x^3*c
^3-6*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*arcsinh(x*c)*x^2*c^2+c^4*x^4-5*arcsin
h(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4+I*(c^2*x^2+1)^(1/2)*x^2*c^2
-10*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-(c^2*x^2+1)^(1/2)
*x*c-(c^2*x^2+1)^(1/2)*c^3*x^3+2*c^2*x^2+arcsinh(x*c)*c^2*x^2+2*arcsinh(x*
c)^2*x^4*c^4-3*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4+4*arcsinh(x*c
)^2*x^2*c^2-6*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-I*(c^2*x^2+1)^
(1/2)*arcsinh(x*c)^2-10*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2-5*pol
ylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4-3*arcsinh(x*c)*ln(1+I*(x*c+(c^2*
x^2+1)^(1/2))))*(I*(x*c-I)*d)^(1/2)*(-I*(I+x*c)*f)^(1/2)*(c^2*x^2+1)^(1/2)
/f^3/d^2/c/(c^6*x^6+3*c^4*x^4+3*c^2*x^2+1)-1/3*a*b*(5*ln(x*c+(c^2*x^2+1)^(
1/2)+I)*x^4*c^4+3*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^4*c^4-4*arcsinh(x*c)*c^4*x
^4-4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-I*x*c+10*ln(x*c+(c^2*x^2+1)^(1
/2)+I)*x^2*c^2+6*ln(x*c+(c^2*x^2+1)^(1/2)-I)*x^2*c^2-8*arcsinh(x*c)*c^2...
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^
2 + c^2*d^2*f^3*x + I*c*d^2*f^3)*integral(1/3*(3*I*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*a^2 - 2*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*
b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^5*d^2*f^3*x^5 + I*c^4*d^2*
f^3*x^4 + 2*c^3*d^2*f^3*x^3 + 2*I*c^2*d^2*f^3*x^2 + c*d^2*f^3*x + I*d^2*f^
3), x))/(c^4*d^2*f^3*x^3 + I*c^3*d^2*f^3*x^2 + c^2*d^2*f^3*x + I*c*d^2*f^3
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + icdx)^{3/2}(f - icfx)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+c\,d\,x\,1\text{i})^{3/2}\,(f-c\,f\,x\,1\text{i})^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{-6\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^3i\,x^3-\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+}}\right.}{}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
( - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**3*i*x**3 - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x*
*2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)),x)*a*b*c**3*x**2 - 6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asin
h(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**3*i*x**3 - sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - s
qrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c - 3*sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**3*i*x**3
 - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( -
c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2*c**3*x**2 -
 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqrt(c*i*x + 1)*s
qrt( - c*i*x + 1)*c**3*i*x**3 - sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x*
*2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*i*x - sqrt(c*i*x + 1)*sqrt( - c*
i*x + 1)),x)*b**2*c + 2*a**2*c**3*x**3 + 3*a**2*c*x - a**2*i)/(3*sqrt(f)*s
qrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c*d*f**2*(c**2*x**2 + 1))
```

3.272 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 408

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx &= -\frac{b^2x}{3d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{b(a+b\text{arcsinh}(cx))}{3cd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\sqrt{1+c^2x^2}} + \frac{2x(a+b\text{arcsinh}(cx))^2}{3d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &+ \frac{x(a+b\text{arcsinh}(cx))^2}{3d^2f^2\sqrt{d+icdx}\sqrt{f-icfx}\,(1+c^2x^2)} + \frac{2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{3cd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{4b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+e^{2\text{arcsinh}(cx)}\right)}{3cd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}} \\ &- \frac{2b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)}{3cd^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

output

```
-1/3*b^2*x/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*b*(a+b*arcsinh(
c*x))/c/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*
x*(a+b*arcsinh(c*x))^2/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*x*(
a+b*arcsinh(c*x))^2/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)/(c^2*x^2+1
)+2/3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/c/d^2/f^2/(d+I*c*d*x)^(1/2)/(
f-I*c*f*x)^(1/2)-4/3*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2
*x^2+1)^(1/2))^2)/c/d^2/f^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/3*b^2*(c
^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/d^2/f^2/(d+I*c*d*x
)^(1/2)/(f-I*c*f*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 7.79 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.57

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
,x]
```

output
```
(4*a^2*c*x*(3 + 2*c^2*x^2) - b^2*(c*x - 6*c*x*ArcSinh[c*x]^2 + (4*I)*Pi*Ar
cSinh[c*x]*Cosh[3*ArcSinh[c*x]] + 2*ArcSinh[c*x]^2*Cosh[3*ArcSinh[c*x]] -
(2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*C
osh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + (2*I)*Pi*Cosh[3*ArcSinh[c*
x]]*Log[1 + I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1
+ I/E^ArcSinh[c*x]] - (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + E^ArcSinh[c*x]
] - (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] +
 (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[Cosh[ArcSinh[c*x]/2]] + (2*I)*Pi*Cosh[3
*ArcSinh[c*x]]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + 2*Sqrt[1 + c^2*x^2]
*(((-3*I)*Pi + 6*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + I*((2*I)*ArcSin
h[c*x] + 6*Pi*ArcSinh[c*x] - (3*I)*ArcSinh[c*x]^2 + 3*(Pi - (2*I)*ArcSinh[
c*x])*Log[1 + I/E^ArcSinh[c*x]] - 12*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log
[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 12*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 3*
Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])) - 16*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, (-I)/E^ArcSinh[c*x]] - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[
c*x]] + Sinh[3*ArcSinh[c*x]] - 2*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]]) + 2*
a*b*(Sqrt[1 + c^2*x^2]*(2 - 3*Log[1 + c^2*x^2]) - Cosh[3*ArcSinh[c*x]]*Log
[1 + c^2*x^2] + 2*ArcSinh[c*x]*(3*c*x + Sinh[3*ArcSinh[c*x]])))/(12*d^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(c + c^3*x^2))
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.56, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {6211, 6203, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838, 6213, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$$

$\Big\downarrow$ 6211

$$\frac{(c^2x^2+1)^{5/2}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6203

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\int \frac{(a+b\text{arcsinh}(cx))^2}{(c^2x^2+1)^{3/2}}dx+\frac{x(a+b\text{arcsinh}(cx))^2}{3(c^2x^2+1)^{3/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6202

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{c^2x^2+1}dx\right)+\frac{x(a+b\text{arcsinh}(c}{3(c^2x^2+1)^{3/2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 6212

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int \frac{cx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}d\text{arcsinh}(cx)}{c}\right)+\frac{x(a+b}{3(}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}-\frac{2b\int -i(a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsin}}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 26

$$\frac{(c^2x^2+1)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\int (a+b\text{arcsinh}(cx))\tan(i\text{arcsinh}(cx))d\text{arcsinh}(}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\Big\downarrow$ 4201

$$\frac{(c^2x^2+1)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\int \frac{e^{2\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{1+e^{2\text{arcsinh}(cx)}}d\text{arcsinh}(cx)-\frac{i(a+b\text{arcsinh}(cx))^2}{2b}\right)}{c}\right)-\frac{2}{3}bc\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 2620

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 2715

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))-\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5}}$$

$\downarrow$ 2838

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\int \frac{x(a+b\text{arcsinh}(cx))}{(c^2x^2+1)^2}dx+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 6213

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(-\frac{2}{3}bc\left(\frac{b\int \frac{1}{(c^2x^2+1)^{3/2}}dx}{2c}-\frac{a+b\text{arcsinh}(cx)}{2c^2(c^2x^2+1)}\right)+\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a\right.\right.}{}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$\downarrow$ 208

$$\frac{\left(c^2x^2+1\right)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\text{arcsinh}(cx))^2}{\sqrt{c^2x^2+1}}+\frac{2ib\left(2i\left(\frac{1}{2}\log\left(e^{2\text{arcsinh}(cx)}+1\right)(a+b\text{arcsinh}(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(cx)}\right)\right)-\frac{i(a+}{}\right)}{c}\right.\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

output
```
((1 + c^2*x^2)^(5/2)*((x*(a + b*ArcSinh[c*x])^2)/(3*(1 + c^2*x^2)^(3/2)) -
 (2*b*c*((b*x)/(2*c*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*(1 +
c^2*x^2))))/3 + (2*((x*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + ((2*I)*
b*(((-1/2*I)*(a + b*ArcSinh[c*x])^2)/b + (2*I)*(((a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/2 + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/4)))/c))/
3))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6203
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*d*(p +
 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*
ArcSinh[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 +
c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 6211
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 + c^2*x
^2)^q)   Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^
2 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1764 vs. $2(362) = 724$.

Time = 6.17 (sec) , antiderivative size = 1765, normalized size of antiderivative = 4.33

method	result	size
default	Expression too large to display	1765
parts	Expression too large to display	1765

input
```
int((a+b*arcsinh(x*c))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(1/3*I/c/d/f/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/d*(I/c/d/f/(d+I*c*d
*x)^(1/2)/(f-I*c*f*x)^(3/2)+2/d*(-1/3*I/d/c/f/(f-I*c*f*x)^(3/2)*(d+I*c*d*x
)^(1/2)-1/3*I/c/d/f^2/(f-I*c*f*x)^(1/2)*(d+I*c*d*x)^(1/2))))+1/3*b^2*(2-8*
polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))-8*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)
))+8*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^5*c^
5+12*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*x*c-2*
arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3-3*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*
c+4*arcsinh(x*c)^2+2*arcsinh(x*c)-8*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(
1/2)))-24*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*arcsinh(x*c)*x^4*c^4-24*ln(1+I*(
x*c+(c^2*x^2+1)^(1/2)))*arcsinh(x*c)*x^2*c^2+20*(c^2*x^2+1)^(1/2)*arcsinh(
x*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*x^3*c^3+20*(c^2*x^2+1)^(1/2)*arcsinh(
x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^3*c^3+8*(c^2*x^2+1)^(1/2)*arcsinh(x
*c)*ln(1+I*(x*c+(c^2*x^2+1)^(1/2)))*x^5*c^5-4*(c^2*x^2+1)^(1/2)*x^5*c^5+11
*c^4*x^4-24*arcsinh(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^4*c^4-24*arcsin
h(x*c)*ln(1-I*(x*c+(c^2*x^2+1)^(1/2)))*x^2*c^2+12*(c^2*x^2+1)^(1/2)*polylo
g(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x*c+12*(c^2*x^2+1)^(1/2)*polylog(2,I*(x*c+
(c^2*x^2+1)^(1/2)))*x*c+8*(c^2*x^2+1)^(1/2)*polylog(2,I*(x*c+(c^2*x^2+1)^(
1/2)))*x^5*c^5+8*(c^2*x^2+1)^(1/2)*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x
^5*c^5+20*(c^2*x^2+1)^(1/2)*polylog(2,I*(x*c+(c^2*x^2+1)^(1/2)))*x^3*c^3+2
0*(c^2*x^2+1)^(1/2)*polylog(2,-I*(x*c+(c^2*x^2+1)^(1/2)))*x^3*c^3-8*arc...
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*b^2*c^2*x^3 + 3*b^2*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^
3)*integral(1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f)*a*b - (2*b^2*c^3*x^3 + 3*b^2*c*x)*sqrt(c^2*x
^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1))
)/(c^6*d^3*f^3*x^6 + 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 + d^3*f^3), x))
/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}(-i\,cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="maxima")
```

output
```
1/3*a*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x
^2 + 1)/(c^2*d^(5/2)*f^(5/2))) + 2/3*a*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f
) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a^2*(x/((c^2
*d*f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2)) + b^2*
integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(5/2)*(-I*c*f*x +
f)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}(c\,x)\right)^2}{\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

output
```
int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{6\sqrt{cix+1}\,\sqrt{-cix+1}\left(\int \frac{asinh(cx)}{\sqrt{cix+1}\,\sqrt{-cix+1}\,c^4x^4+2\sqrt{cix+1}\,\sqrt{-cix+1}\,c^2x^2+\sqrt{cix+1}\,\cdots}\right.}{\cdots}$$

input
```
int((a+b*asinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

output
```
(6*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt
( - c*i*x + 1)*c**4*x**4 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2
+ sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*a*b*c**2*x**2 + 6*sqrt(c*i*x + 1)
*sqrt( - c*i*x + 1)*int(asinh(c*x)/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**
4*x**4 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*
sqrt( - c*i*x + 1)),x)*a*b + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asin
h(c*x)**2/(sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*c**4*x**4 + 2*sqrt(c*i*x + 1
)*sqrt( - c*i*x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b*
*2*c**2*x**2 + 3*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*int(asinh(c*x)**2/(sqr
t(c*i*x + 1)*sqrt( - c*i*x + 1)*c**4*x**4 + 2*sqrt(c*i*x + 1)*sqrt( - c*i*
x + 1)*c**2*x**2 + sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)),x)*b**2 + 2*a**2*c*
*2*x**3 + 3*a**2*x)/(3*sqrt(f)*sqrt(d)*sqrt(c*i*x + 1)*sqrt( - c*i*x + 1)*
d**2*f**2*(c**2*x**2 + 1))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file