Computer Algebra Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/329-7.1.5

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**57**]. This is test number [329].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (57)	0.00 (0)
Mathematica	98.25 (56)	1.75 (1)
Maple	87.72 (50)	12.28 (7)
Maxima	36.84 (21)	63.16 (36)
Fricas	35.09 (20)	64.91 (37)
Reduce	29.82 (17)	70.18 (40)
Sympy	28.07 (16)	71.93 (41)
Giac	22.81 (13)	77.19 (44)
Mupad	19.30 (11)	80.70 (46)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

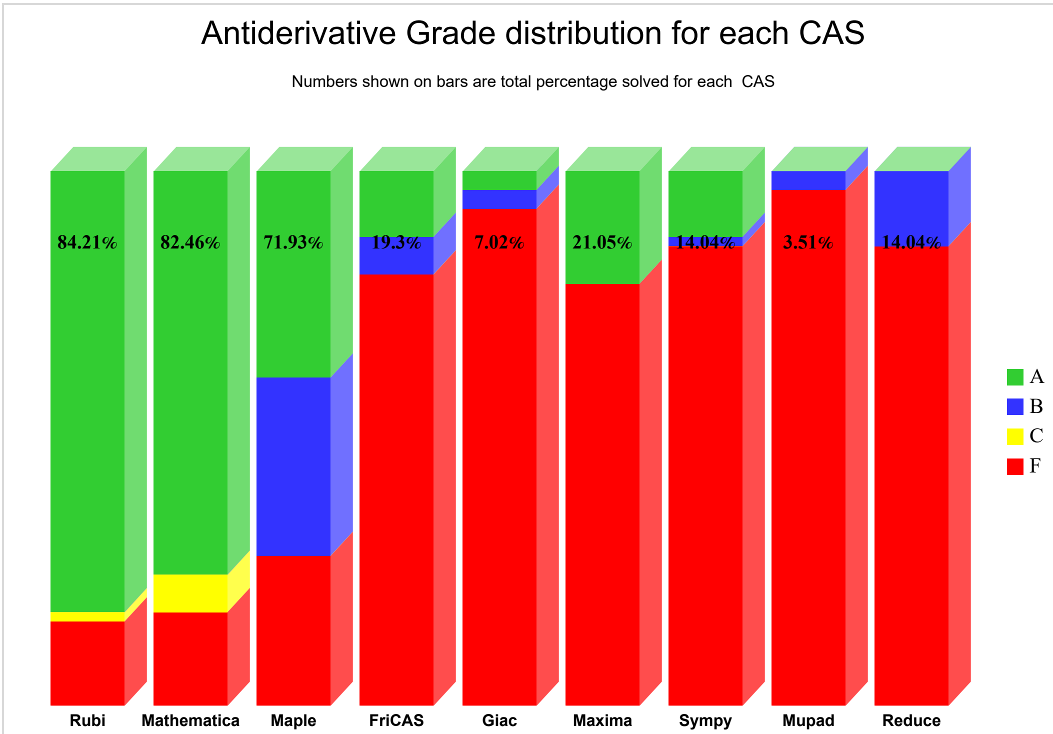
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

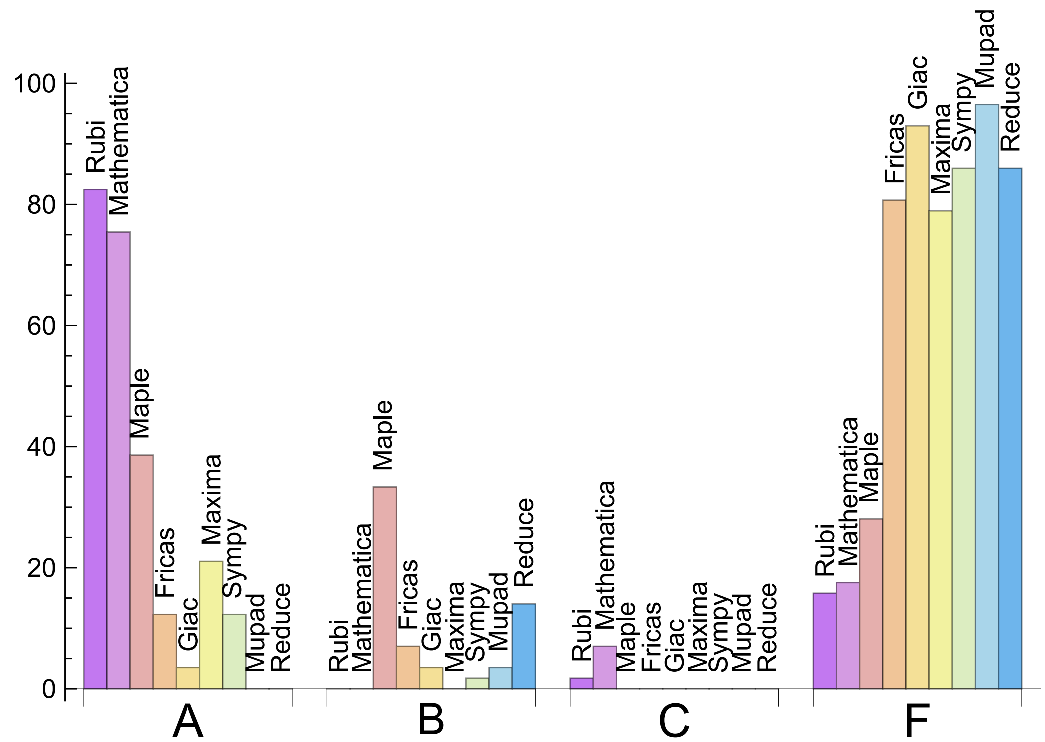
System	% A grade	% B grade	% C grade	% F grade
Rubi	82.456	0.000	1.754	15.789
Mathematica	75.439	0.000	7.018	17.544
Maple	38.596	33.333	0.000	28.070
Maxima	21.053	0.000	0.000	78.947
Fricas	12.281	7.018	0.000	80.702
Sympy	12.281	1.754	0.000	85.965
Giac	3.509	3.509	0.000	92.982
Mupad	0.000	3.509	0.000	96.491
Reduce	0.000	14.035	0.000	85.965

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	7	100.00	0.00	0.00
Maxima	36	63.89	0.00	36.11
Fricas	37	100.00	0.00	0.00
Reduce	40	100.00	0.00	0.00
Sympy	41	92.68	7.32	0.00
Giac	44	54.55	0.00	45.45
Mupad	46	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.12
Maxima	0.26
Rubi	0.98
Mathematica	1.47
Maple	2.22
Sympy	2.55
Mupad	2.73
Giac	4.02
Reduce	11.95

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	28.55	1.05	20.00	1.11
Giac	55.08	1.36	20.00	1.11
Reduce	89.65	2.09	58.00	1.50
Sympy	143.19	1.24	28.50	1.00
Fricas	187.40	2.14	89.50	1.47
Rubi	230.65	0.89	196.00	0.99
Maxima	234.71	8.02	103.00	1.23
Mathematica	420.27	1.08	205.50	0.94
Maple	646.12	1.74	276.00	1.58

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

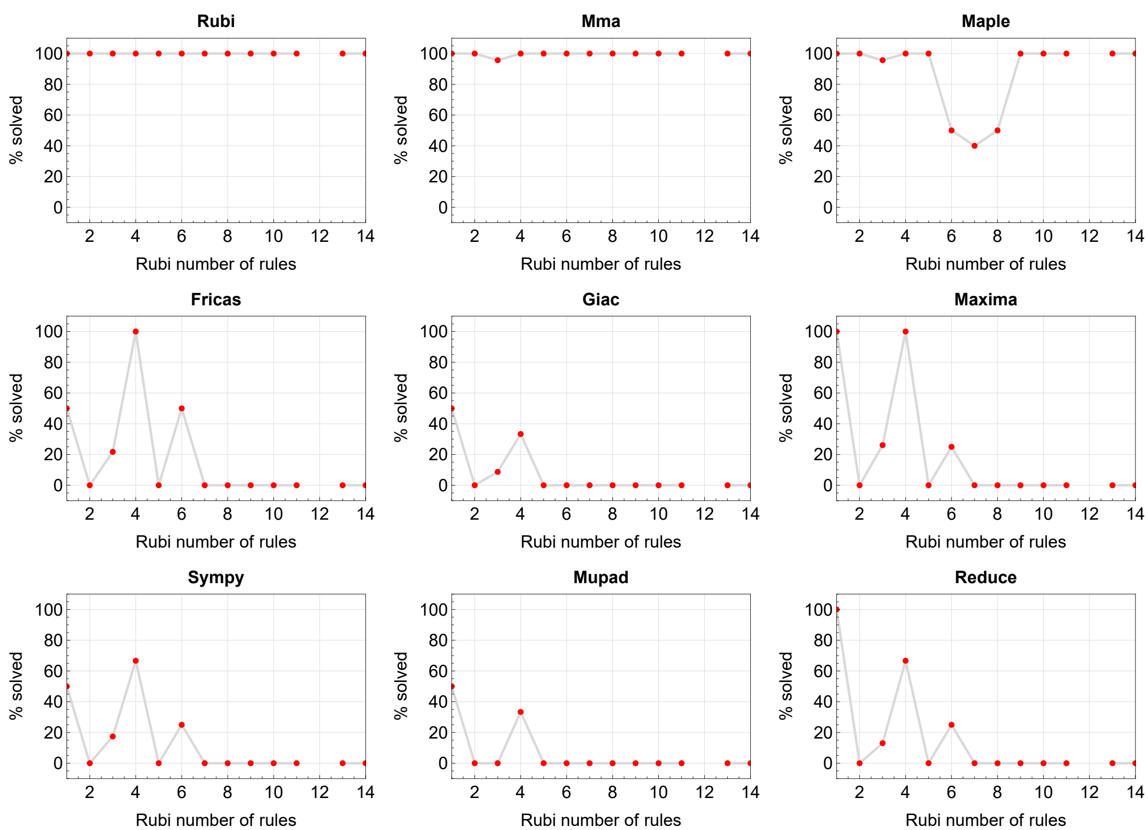


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

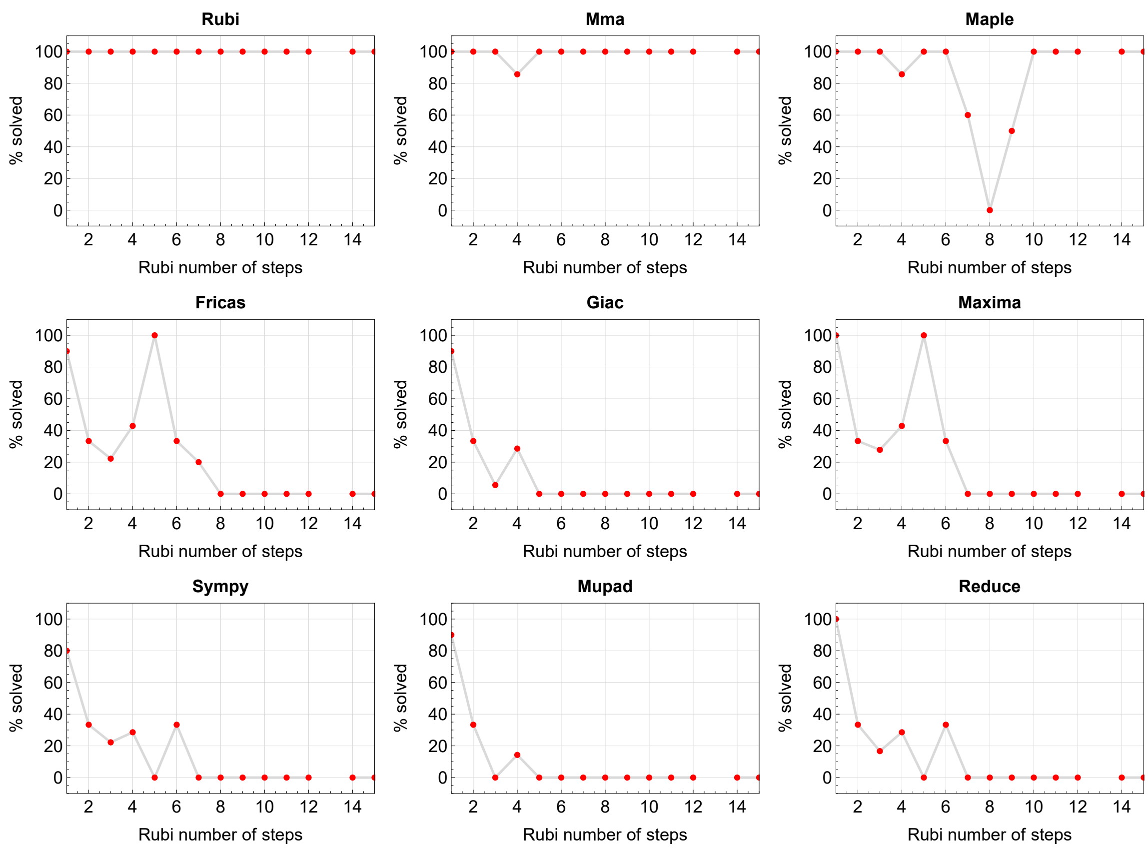


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

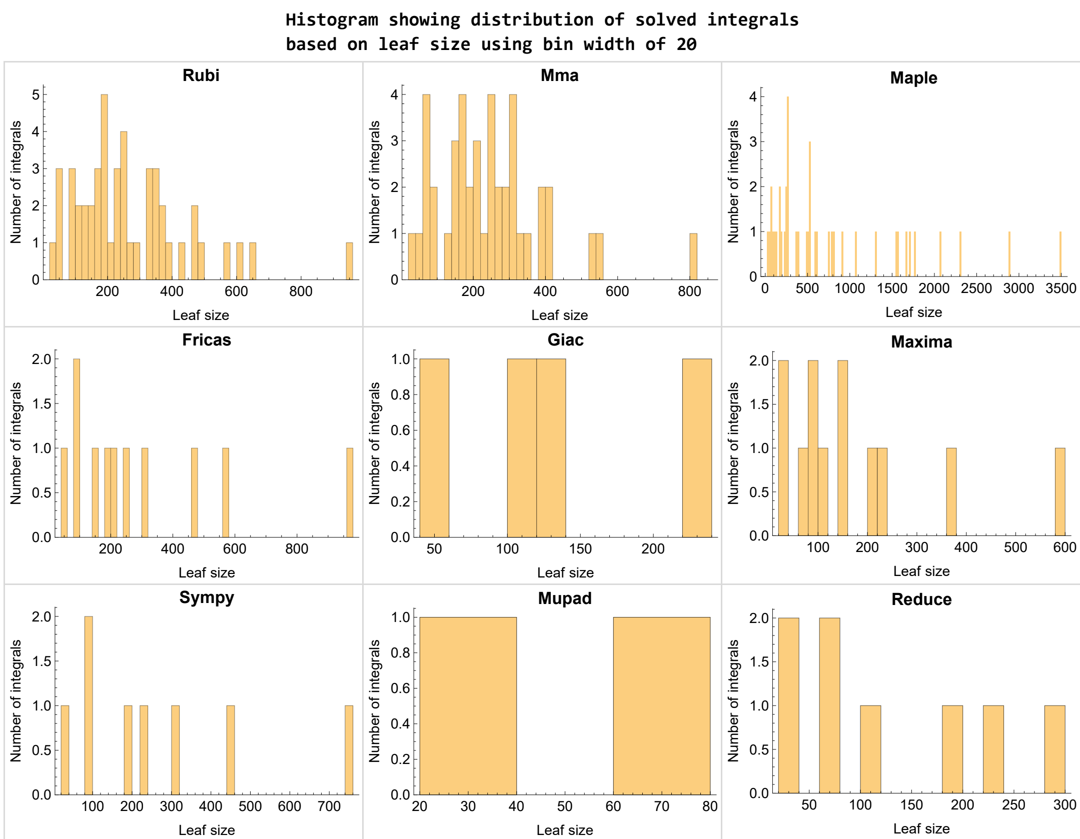


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

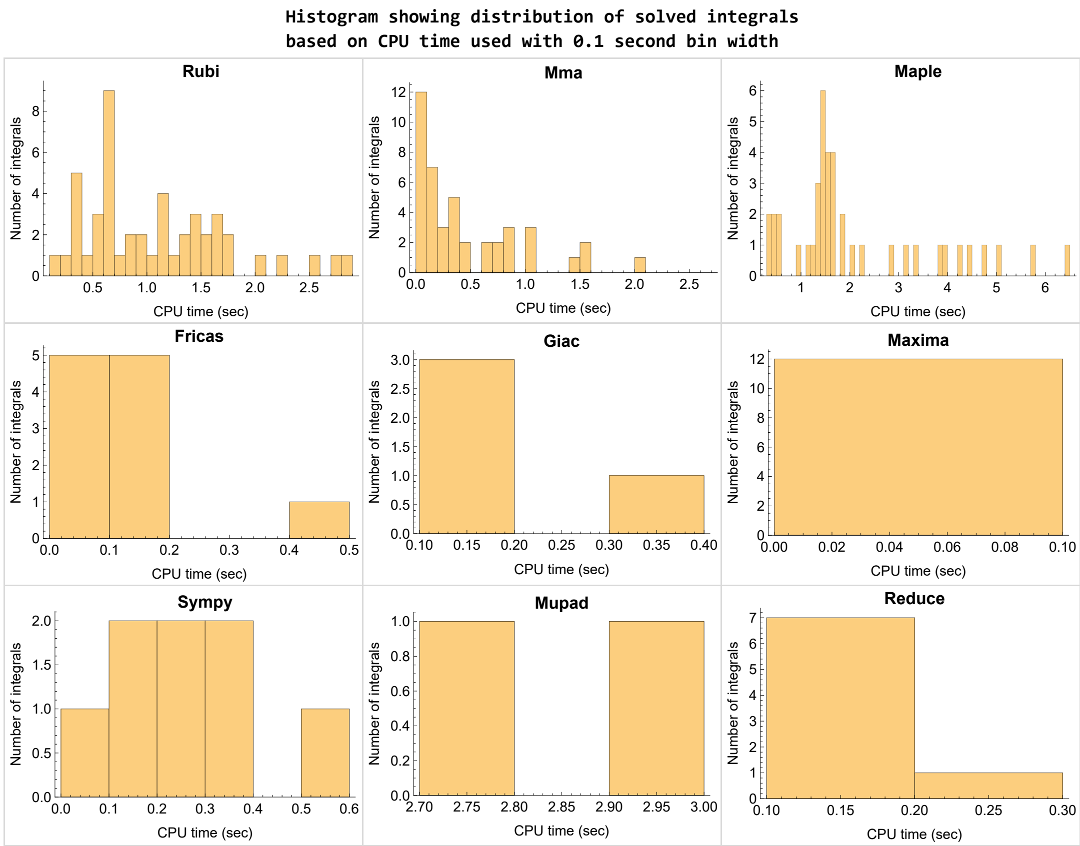


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

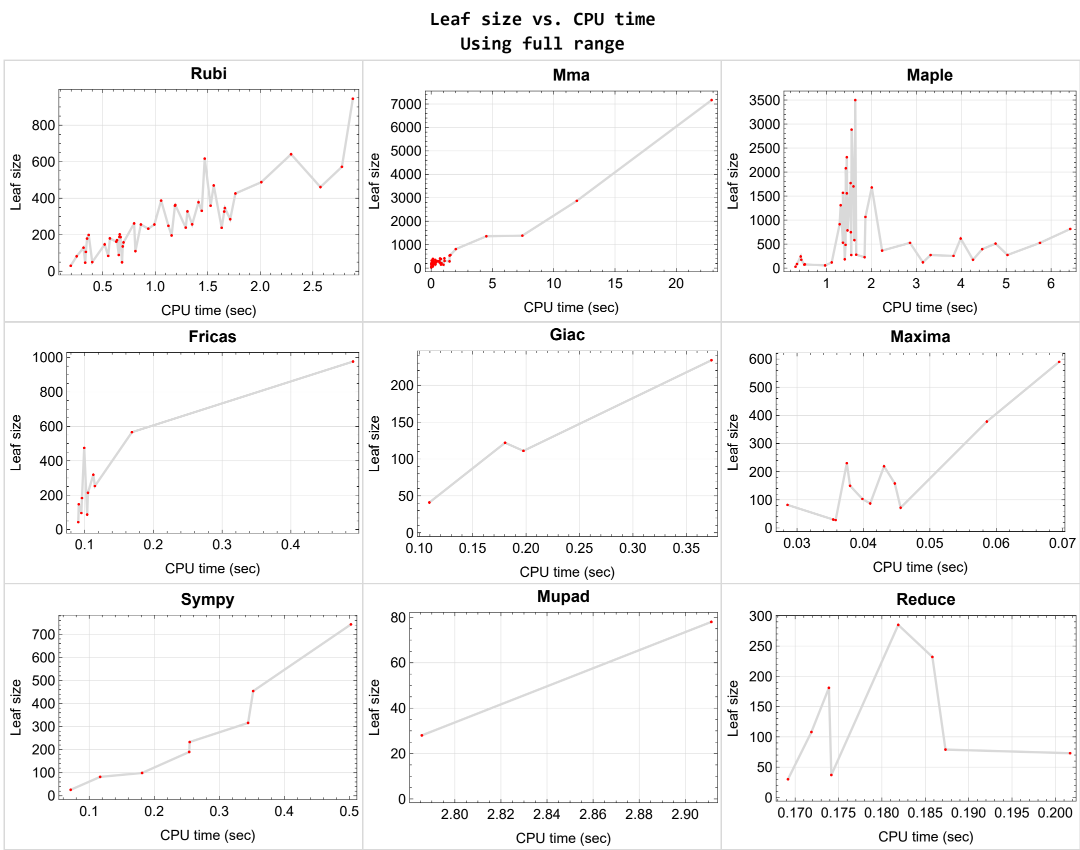


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{23, 24, 28, 29, 30, 32, 33, 53, 57\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{37, 38, 42, 46\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

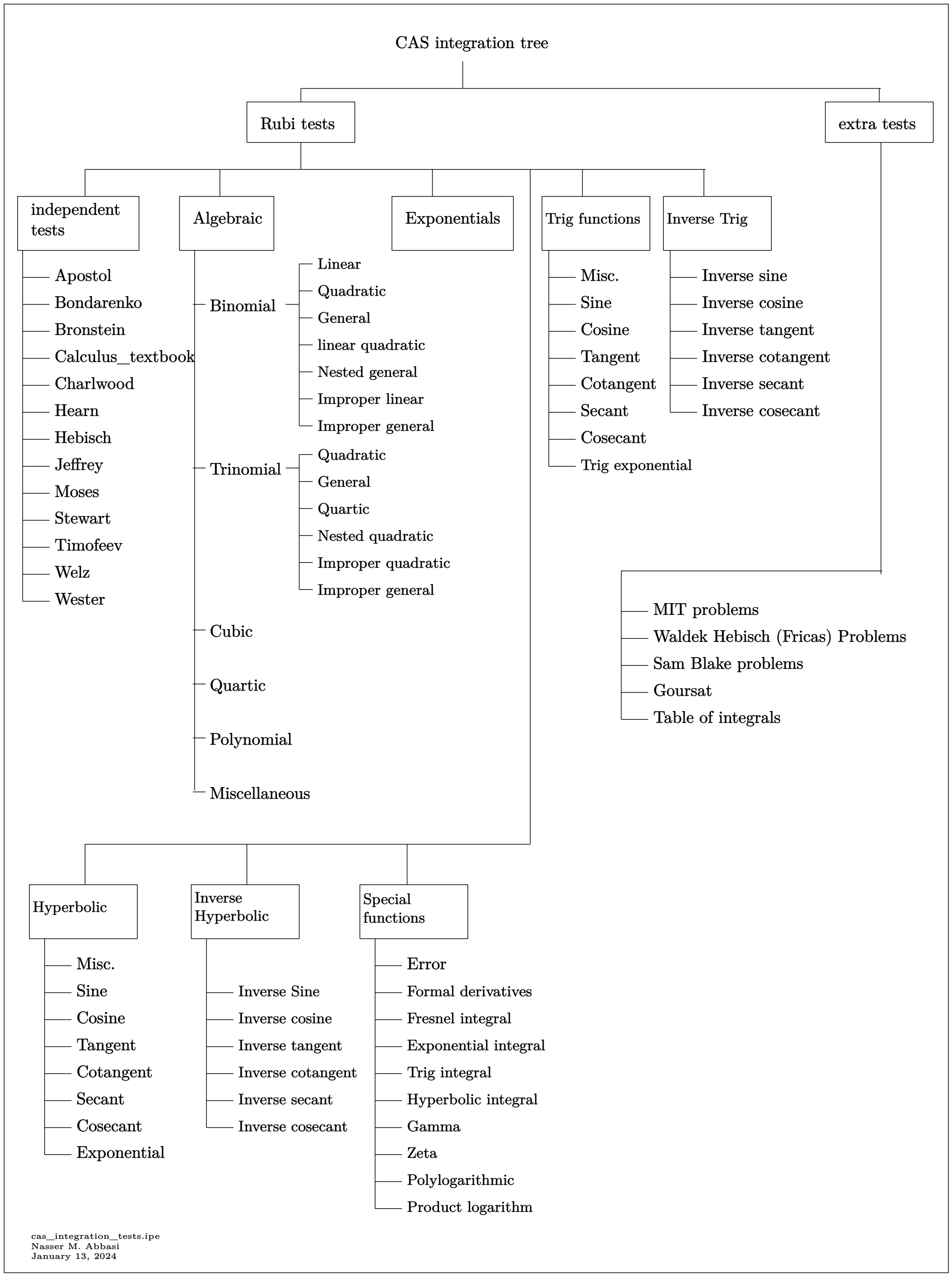
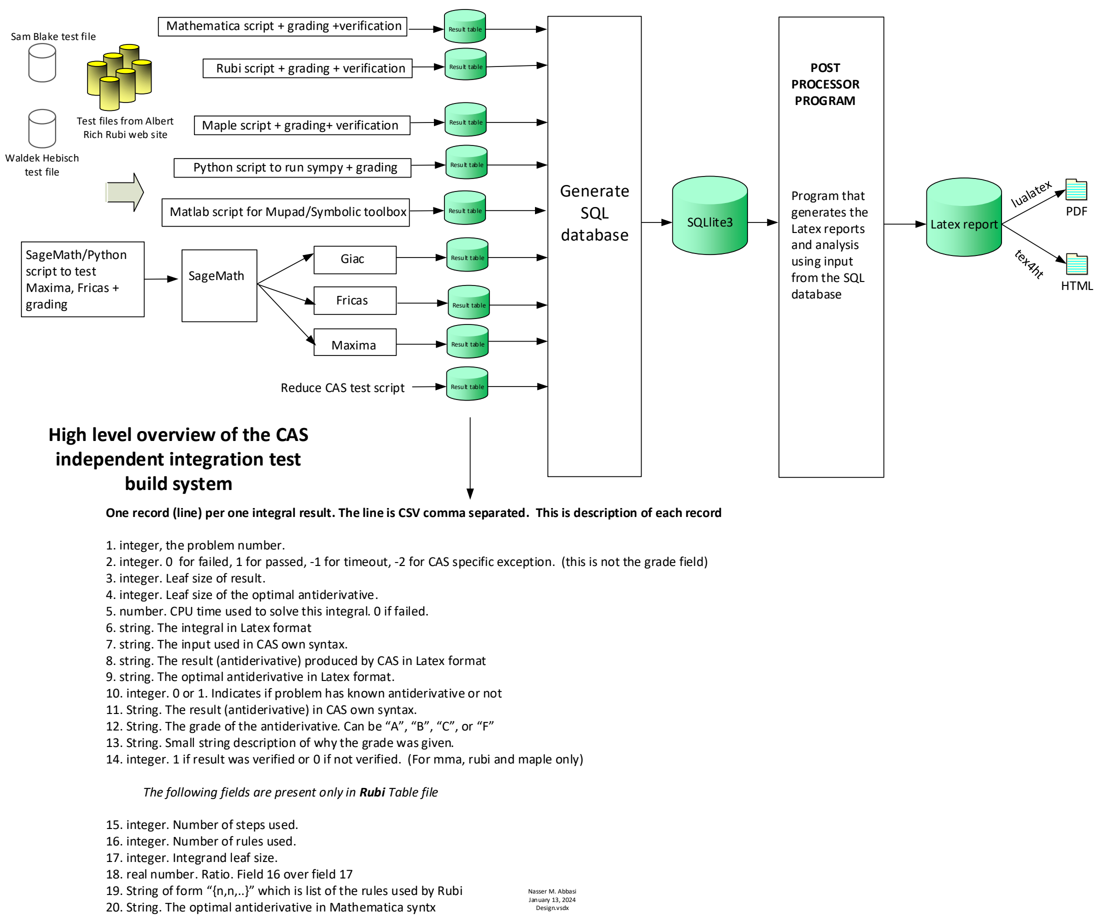


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56 }

B grade { }

C grade { 27 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 34, 35, 36, 39, 40, 41, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 56 }

B grade { }

C grade { 37, 38, 42, 46 }

F normal fail { 31 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { $1, 4, 5, 6, 7, 8, 12, 13, 14, 15, 17, 19, 20, 21, 22, 25, 26, 27, 37, 42, 50, 51$ }

B grade { $9, 10, 11, 18, 34, 35, 36, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 52$ }

C grade { }

F normal fail { $2, 3, 16, 31, 54, 55, 56$ }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { $4, 5, 6, 7, 12, 13, 14$ }

B grade { $9, 10, 11, 15$ }

C grade { }

F normal fail { $1, 2, 3, 8, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56$ }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { $4, 5, 6, 7, 9, 10, 12, 13, 14, 15, 49, 50$ }

B grade { }

C grade { }

F normal fail { $1, 2, 3, 8, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 37, 38, 51, 52, 54, 55, 56$ }

F(-1) timedout fail { }

F(-2) exception fail { $34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48$ }

Giac

A grade { 6, 7 }

B grade { 9, 15 }

C grade { }

F normal fail { 1, 2, 3, 8, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 47, 48, 49, 50, 51, 52, 56 }

F(-1) timedout fail { }

F(-2) exception fail { 4, 5, 12, 13, 14, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 54, 55 }

Mupad

A grade { }

B grade { 6, 7 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 7, 13, 14, 15 }

B grade { 12 }

C grade { }

F normal fail { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56 }

F(-1) timedout fail { 43, 45, 53 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 4, 5, 6, 7, 14, 15, 49, 50 }

C grade { }

F normal fail { 1, 2, 3, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 54, 55, 56 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	168	272	0	0	0	0	14	0
N.S.	1	1.00	0.99	1.60	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.636	0.010	1.553	0.000	0.000	0.000	0.000	0.171	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	256	240	0	0	0	0	0	16	0
N.S.	1	0.98	0.92	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.992	0.097	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	348	346	322	0	0	0	0	0	16	0
N.S.	1	0.99	0.93	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.663	0.039	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	202	166	242	230	214	316	0	285	0
N.S.	1	1.10	0.90	1.32	1.25	1.16	1.72	0.00	1.55	0.00
time (sec)	N/A	0.662	0.110	0.428	0.037	0.105	0.344	0.000	0.182	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	147	121	174	150	147	190	0	181	0
N.S.	1	1.11	0.91	1.31	1.13	1.11	1.43	0.00	1.36	0.00
time (sec)	N/A	0.518	0.073	0.441	0.038	0.091	0.254	0.000	0.174	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	106	91	84	82	87	99	122	108	78
N.S.	1	1.15	0.99	0.91	0.89	0.95	1.08	1.33	1.17	0.85
time (sec)	N/A	0.340	0.027	0.345	0.029	0.104	0.181	0.180	0.172	2.911

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	30	43	26	41	30	28
N.S.	1	1.00	1.00	0.97	1.00	1.43	0.87	1.37	1.00	0.93
time (sec)	N/A	0.196	0.006	0.312	0.035	0.091	0.072	0.109	0.169	2.786

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	175	279	0	0	0	0	30	0
N.S.	1	1.00	0.94	1.49	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.670	0.047	1.663	0.000	0.000	0.000	0.000	0.179	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	174	103	253	0	234	75	0
N.S.	1	1.00	0.96	2.12	1.26	3.09	0.00	2.85	0.91	0.00
time (sec)	N/A	0.252	0.073	4.262	0.040	0.115	0.000	0.374	0.195	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	129	166	273	158	566	0	0	159	0
N.S.	1	1.01	1.30	2.13	1.23	4.42	0.00	0.00	1.24	0.00
time (sec)	N/A	0.317	0.239	5.029	0.045	0.169	0.000	0.000	0.222	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	199	205	510	0	977	0	0	262	0
N.S.	1	1.09	1.12	2.79	0.00	5.34	0.00	0.00	1.43	0.00
time (sec)	N/A	0.367	0.280	4.764	0.000	0.490	0.000	0.000	5.556	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	387	354	526	590	475	743	0	488	0
N.S.	1	1.05	0.96	1.43	1.60	1.29	2.02	0.00	1.33	0.00
time (sec)	N/A	1.056	0.329	2.860	0.069	0.099	0.502	0.000	0.217	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	262	248	364	378	319	454	0	344	0
N.S.	1	1.10	1.04	1.52	1.58	1.33	1.90	0.00	1.44	0.00
time (sec)	N/A	0.799	0.222	2.239	0.059	0.113	0.352	0.000	0.183	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	162	142	183	219	183	233	0	232	0
N.S.	1	1.16	1.01	1.31	1.56	1.31	1.66	0.00	1.66	0.00
time (sec)	N/A	0.629	1.004	1.408	0.043	0.096	0.254	0.000	0.186	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	74	72	72	96	82	111	73	0
N.S.	1	1.09	1.61	1.57	1.57	2.09	1.78	2.41	1.59	0.00
time (sec)	N/A	0.400	0.033	0.508	0.046	0.095	0.117	0.198	0.202	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	285	273	0	0	0	0	0	55	0
N.S.	1	0.98	0.94	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.714	0.176	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	239	191	525	0	0	0	0	149	0
N.S.	1	0.91	0.73	2.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	1.290	0.136	5.752	0.000	0.000	0.000	0.000	10.286	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	331	270	815	0	0	0	0	308	0
N.S.	1	0.95	0.77	2.34	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	1.443	0.399	6.426	0.000	0.000	0.000	0.000	0.340	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	378	305	394	0	0	0	0	79	0
N.S.	1	0.96	0.77	1.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.412	0.460	4.467	0.000	0.000	0.000	0.000	0.195	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	233	188	254	0	0	0	0	55	0
N.S.	1	0.95	0.77	1.04	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.934	0.301	3.828	0.000	0.000	0.000	0.000	0.196	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	110	98	120	0	0	0	0	31	0
N.S.	1	0.95	0.84	1.03	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.811	0.133	3.147	0.000	0.000	0.000	0.000	0.180	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	45	56	0	0	0	0	12	0
N.S.	1	0.91	0.83	1.04	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.684	0.041	0.971	0.000	0.000	0.000	0.000	0.184	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	25	15	20	27	20
N.S.	1	1.00	1.11	1.00	1.11	1.39	0.83	1.11	1.50	1.11
time (sec)	N/A	0.344	0.155	1.362	0.097	0.090	0.944	0.132	0.204	2.624

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	49	17	20	55	20
N.S.	1	1.00	1.11	1.00	1.11	2.72	0.94	1.11	3.06	1.11
time (sec)	N/A	0.343	0.272	0.908	0.107	0.095	1.897	0.770	0.207	2.663

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	288	616	0	0	0	0	97	0
N.S.	1	1.00	0.80	1.72	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.188	1.499	3.988	0.000	0.000	0.000	0.000	0.188	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	150	272	0	0	0	0	59	0
N.S.	1	1.00	0.83	1.51	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.568	0.800	3.318	0.000	0.000	0.000	0.000	0.207	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	71	118	0	0	0	0	26	0
N.S.	1	1.05	0.84	1.39	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.653	0.139	1.118	0.000	0.000	0.000	0.000	0.178	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	746	51	17	20	58	20
N.S.	1	1.00	1.11	1.00	41.44	2.83	0.94	1.11	3.22	1.11
time (sec)	N/A	0.234	5.589	0.863	0.642	0.087	1.788	0.144	0.196	2.639

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1050	92	19	20	109	20
N.S.	1	1.00	1.11	1.00	58.33	5.11	1.06	1.11	6.06	1.11
time (sec)	N/A	0.227	4.021	0.919	1.084	0.094	7.032	1.215	0.225	2.764

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	271	32	17	20	117	20
N.S.	1	1.00	1.11	1.00	15.06	1.78	0.94	1.11	6.50	1.11
time (sec)	N/A	0.577	4.654	1.961	0.782	0.099	6.686	0.214	0.206	2.866

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	66	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.351	0.000	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11	1.11
time (sec)	N/A	0.239	0.268	2.231	0.088	0.085	0.986	0.130	200.026	2.627

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	607	34	17	20	34	20
N.S.	1	1.00	1.11	1.00	33.72	1.89	0.94	1.11	1.89	1.11
time (sec)	N/A	0.229	0.578	2.095	0.775	0.092	10.613	0.133	0.217	2.655

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	363	410	1309	0	0	0	0	335	0
N.S.	1	0.59	0.66	2.11	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.191	0.724	1.317	0.000	0.000	0.000	0.000	0.201	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	257	301	913	0	0	0	0	231	0
N.S.	1	0.60	0.71	2.15	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.351	0.684	1.294	0.000	0.000	0.000	0.000	0.197	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	136	208	582	0	0	0	0	129	0
N.S.	1	0.62	0.94	2.63	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.691	0.847	1.622	0.000	0.000	0.000	0.000	0.193	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	461	1358	747	0	0	0	0	124	0
N.S.	1	0.69	2.05	1.12	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.573	4.502	1.544	0.000	0.000	0.000	0.000	0.188	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	572	1384	1701	0	0	0	0	461	0
N.S.	1	0.73	1.77	2.18	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	2.777	7.441	1.602	0.000	0.000	0.000	0.000	0.193	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	868	488	536	2079	0	0	0	0	544	0
N.S.	1	0.56	0.62	2.40	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	2.010	1.504	1.432	0.000	0.000	0.000	0.000	0.229	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	359	416	1568	0	0	0	0	383	0
N.S.	1	0.58	0.67	2.53	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.527	1.082	1.368	0.000	0.000	0.000	0.000	0.210	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	188	289	1065	0	0	0	0	224	0
N.S.	1	0.57	0.87	3.22	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.659	0.600	1.868	0.000	0.000	0.000	0.000	0.204	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	974	641	2869	1557	0	0	0	0	320	0
N.S.	1	0.66	2.95	1.60	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	2.293	11.884	1.454	0.000	0.000	0.000	0.000	0.194	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1142	617	810	2884	0	0	0	0	754	0
N.S.	1	0.54	0.71	2.53	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	1.471	2.010	1.560	0.000	0.000	0.000	0.000	0.255	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	837	470	555	2309	0	0	0	0	536	0
N.S.	1	0.56	0.66	2.76	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	1.557	1.571	1.454	0.000	0.000	0.000	0.000	0.230	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	249	390	1679	0	0	0	0	320	0
N.S.	1	0.55	0.86	3.69	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	1.125	0.801	2.009	0.000	0.000	0.000	0.000	0.214	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1500	945	7168	3499	0	0	0	0	564	0
N.S.	1	0.63	4.78	2.33	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	2.881	22.883	1.642	0.000	0.000	0.000	0.000	5.622	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	256	304	785	0	0	0	0	249	0
N.S.	1	0.63	0.75	1.93	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.863	1.097	1.469	0.000	0.000	0.000	0.000	0.183	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	158	233	484	0	0	0	0	162	0
N.S.	1	0.64	0.95	1.97	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.698	0.702	1.426	0.000	0.000	0.000	0.000	0.180	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	84	158	227	87	0	0	0	79	0
N.S.	1	0.74	1.39	1.99	0.76	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.552	0.352	1.849	0.041	0.000	0.000	0.000	0.187	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	76	77	28	0	0	0	37	0
N.S.	1	1.00	1.62	1.64	0.60	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.333	0.036	0.520	0.036	0.000	0.000	0.000	0.174	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	238	240	529	0	0	0	0	157	0
N.S.	1	0.73	0.74	1.63	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	1.632	0.340	1.368	0.000	0.000	0.000	0.000	0.182	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	327	251	1770	0	0	0	0	607	0
N.S.	1	0.75	0.57	4.04	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	1.657	0.418	1.538	0.000	0.000	0.000	0.000	0.188	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	34	34	0	34	35	34
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.00	1.00	1.03	1.00
time (sec)	N/A	0.391	0.107	8.298	0.737	0.126	0.000	48.234	0.232	2.781

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	438	426	397	0	0	0	0	0	98	0
N.S.	1	0.97	0.91	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.763	0.143	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	328	304	0	0	0	0	0	59	0
N.S.	1	0.99	0.92	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.307	0.150	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	196	206	0	0	0	0	0	25	0
N.S.	1	0.99	1.05	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.156	0.016	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	34	51	31	34	44	34
N.S.	1	1.00	1.06	0.94	1.00	1.50	0.91	1.00	1.29	1.00
time (sec)	N/A	0.550	0.286	5.250	0.704	0.082	8.790	0.461	0.212	2.699

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [27] had the largest ratio of [1.10000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	12	0.417
2	A	7	6	0.98	14	0.429
3	A	8	7	0.99	14	0.500
4	A	6	6	1.10	16	0.375
5	A	4	4	1.11	16	0.250
6	A	4	4	1.15	14	0.286
7	A	1	1	1.00	8	0.125
8	A	6	5	1.00	16	0.312
9	A	4	3	1.00	16	0.188
10	A	5	4	1.01	16	0.250
11	A	7	6	1.09	16	0.375
12	A	3	3	1.05	18	0.167
13	A	3	3	1.10	18	0.167
14	A	3	3	1.16	16	0.188
15	A	3	3	1.09	10	0.300
16	A	7	6	0.98	18	0.333
17	A	11	10	0.91	18	0.556
18	A	15	14	0.95	18	0.778
19	A	4	3	0.96	18	0.167
20	A	4	3	0.95	18	0.167
21	A	4	3	0.95	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	9	8	0.91	10	0.800
23	N/A	1	0	1.00	18	0.000
24	N/A	1	0	1.00	18	0.000
25	A	2	2	1.00	18	0.111
26	A	2	2	1.00	16	0.125
27	C	12	11	1.05	10	1.100
28	N/A	1	0	1.00	18	0.000
29	N/A	1	0	1.00	18	0.000
30	N/A	2	0	1.00	18	0.000
31	A	4	3	1.00	16	0.188
32	N/A	1	0	1.00	18	0.000
33	N/A	1	0	1.00	18	0.000
34	A	3	3	0.59	30	0.100
35	A	3	3	0.60	30	0.100
36	A	3	3	0.62	28	0.107
37	A	7	7	0.69	30	0.233
38	A	7	7	0.73	30	0.233
39	A	3	3	0.56	30	0.100
40	A	3	3	0.58	30	0.100
41	A	3	3	0.57	28	0.107
42	A	3	3	0.66	30	0.100
43	A	3	3	0.54	30	0.100
44	A	3	3	0.56	30	0.100
45	A	3	3	0.55	28	0.107
46	A	3	3	0.63	30	0.100
47	A	3	3	0.63	30	0.100
48	A	3	3	0.64	30	0.100
49	A	3	3	0.74	28	0.107
50	A	1	1	1.00	23	0.043
51	A	10	9	0.73	30	0.300
52	A	14	13	0.75	30	0.433

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	N/A	1	0	1.00	34	0.000
54	A	9	8	0.97	34	0.235
55	A	8	7	0.99	32	0.219
56	A	8	7	0.99	24	0.292
57	N/A	1	0	1.00	34	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 170

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

output
```
-1/2*arcsinh(c*x)^2/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^
2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2
*d^2+e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2
)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))
)/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,\frac{ee^{\operatorname{arcsinh}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

input
```
Integrate[ArcSinh[c*x]/(d + e*x),x]
```

output
```
-1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sq
rt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])])/e + PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2
*d^2 + e^2])]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^
2]))]/e
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}dx$$

$$\Big\downarrow \text{ 6242}$$

$$\int \frac{\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)}{cd+cex}d\operatorname{arcsinh}(cx)$$

$$\Big\downarrow \text{ 6095}$$

$$\int \frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)+\int \frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)-\frac{\text{arcsinh}(cx)^2}{2e}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^2}{2e}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^2}{2e}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^2}{2e}$$

input
```
Int[ArcSinh[c*x]/(d + e*x),x]
```

output
```
-1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sq
rt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*
d^2 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^
2]))]/e
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{c\,\operatorname{arcsinh}(xc)^2}{2e}+\frac{c\,\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right.}{e}}{c}$
default	$\frac{-\frac{c\,\operatorname{arcsinh}(xc)^2}{2e}+\frac{c\,\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right.}{e}}{c}$

input
```
int(arcsinh(x*c)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*c*arcsinh(x*c)^2/e+c/e*arcsinh(x*c)*ln((-c*d-e*(x*c+(c^2*x^2+1)^
(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+c/e*arcsinh(x*c)*l
n((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(
1/2)))+c/e*dilog((-c*d-e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*
d+(c^2*d^2+e^2)^(1/2)))+c/e*dilog((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+
e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arcsinh(c*x)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{asinh}(cx)}{d+ex}\,dx$$

input
```
integrate(asinh(c*x)/(e*x+d),x)
```

output
```
Integral(asinh(c*x)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)}{d+e\,x}\,dx$$

input
```
int(asinh(c*x)/(d + e*x),x)
```

output
```
int(asinh(c*x)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx=\int \frac{asinh(cx)}{ex+d}dx$$

input
```
int(asinh(c*x)/(e*x+d),x)
```

output
```
int(asinh(c*x)/(d + e*x),x)
```

3.2 $\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 260

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = & -\frac{\operatorname{arcsinh}(cx)^3}{3e}+\frac{\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\ & +\frac{\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\ & +\frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\ & +\frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\ & -\frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

output
```
-1/3*arcsinh(c*x)^3/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(
c^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2)
)/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2
))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*
d+(c^2*d^2+e^2)^(1/2)))/e
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.92

$$\int \frac{\text{arcsinh}(cx)^2}{d+ex}\,dx =$$
$$-\frac{\text{arcsinh}(cx)^3 - 3\text{arcsinh}(cx)^2\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) - 3\text{arcsinh}(cx)^2\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) - 6\text{arcsinh}$$

input
```
Integrate[ArcSinh[c*x]^2/(d + e*x),x]
```

output
```
-1/3*(ArcSinh[c*x]^3 - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d -
Sqrt[c^2*d^2 + e^2])] - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c
*d) + Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 6*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] + 6*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c
^2*d^2 + e^2]))])/e
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(cx)^2}{d+ex}\,dx$$
$$\Big\downarrow \text{ 6242}$$
$$\int \frac{\sqrt{c^2x^2+1}\text{arcsinh}(cx)^2}{cd+cex}d\text{arcsinh}(cx)$$
$$\Big\downarrow \text{ 6095}$$

$$\int \frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)^2}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)+\int \frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)^2}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2\int \text{arcsinh}(cx)\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{2\int \text{arcsinh}(cx)\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2\left(\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2\left(\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2\left(\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2\left(\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2\left(\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2\left(\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

input
```
Int[ArcSinh[c*x]^2/(d + e*x),x]
```

output
```
-1/3*ArcSinh[c*x]^3/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d -
Sqrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d
 + Sqrt[c^2*d^2 + e^2])])/e - (2*(-(ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh
[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]) + PolyLog[3, -((e*E^ArcSinh[c*x])/(c
*d - Sqrt[c^2*d^2 + e^2]))]))/e - (2*(-(ArcSinh[c*x]*PolyLog[2, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]) + PolyLog[3, -((e*E^ArcSinh[c*x]
)/(c*d + Sqrt[c^2*d^2 + e^2]))]))/e
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\operatorname{arcsinh}(xc)^2}{ex+d}dx$$

input
```
int(arcsinh(x*c)^2/(e*x+d),x)
```

output
```
int(arcsinh(x*c)^2/(e*x+d),x)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx=\int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arcsinh(c*x)^2/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{asinh}^2(cx)}{d+ex}\,dx$$

input
```
integrate(asinh(c*x)**2/(e*x+d),x)
```

output
```
Integral(asinh(c*x)**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)^2/(e*x + d), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\text{asinh}(c\,x)^2}{d+e\,x}\,dx$$

input
```
int(asinh(c*x)^2/(d + e*x),x)
```

output
```
int(asinh(c*x)^2/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\text{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{asinh(cx)^2}{ex+d}dx$$

input
```
int(asinh(c*x)^2/(e*x+d),x)
```

output
```
int(asinh(c*x)**2/(d + e*x),x)
```

3.3 $\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 348

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = &-\frac{\operatorname{arcsinh}(cx)^4}{4e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\
&+\frac{\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\
&+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\
&+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\
&-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\
&-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\
&+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

output
```
-1/4*arcsinh(c*x)^4/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(
c^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/
2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*x
+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,
-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*
x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx$$
$$= \frac{-\operatorname{arcsinh}(cx)^4 + 4\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) + 4\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) + 12\operatorname{arcsinh}}{}$$

input
```
Integrate[ArcSinh[c*x]^3/(d + e*x),x]
```

output
```
(-ArcSinh[c*x]^4 + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2])] + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*
d) + Sqrt[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] - 24*ArcSinh[c*x]*PolyLog[3, (e*E^ArcSi
nh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 24*ArcSinh[c*x]*PolyLog[3, -((e
*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 24*PolyLog[4, (e*E^ArcSin
h[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 24*PolyLog[4, -((e*E^ArcSinh[c*x
])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(4*e)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6242, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx$$

$$\Big\downarrow \text{ 6242}$$

$$\int \frac{\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)^3}{cd+cex}d\operatorname{arcsinh}(cx)$$

$$\Big\downarrow \text{ 6095}$$

$$\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)^3}{cd+ee^{\operatorname{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)+\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)^3}{cd+ee^{\operatorname{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)-\frac{\operatorname{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{3\int \operatorname{arcsinh}(cx)^2\log\left(\frac{e^{\operatorname{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\operatorname{arcsinh}(cx)}{e}-\frac{3\int \operatorname{arcsinh}(cx)^2\log\left(\frac{e^{\operatorname{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\operatorname{arcsinh}(cx)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\operatorname{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3\left(2\int \operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\operatorname{arcsinh}(cx)-\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{3\left(2\int \operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\operatorname{arcsinh}(cx)-\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\operatorname{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \text{ 7163}$$

$$-\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\int \text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)^2\,\text{Po}\right.}{e}$$

$$\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\int \text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)^2\,\text{Poly}\right.}{e}$$

$$\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \; 2720$$

$$-\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}\right)-\text{arcsi}\right.}{e}$$

$$\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}\right)-\text{arcsinh}\right.}{e}$$

$$\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \; 7143$$

$$-\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\text{PolyLog}\left(4,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)-\text{arcsinh}(cx)^2\,\text{PolyLog}\left(2,-\frac{ee^{\text{a}}}{cd-}\right.\right.}{e}$$

$$\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\text{PolyLog}\left(4,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)-\text{arcsinh}(cx)^2\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcs}}}{cd+\sqrt{}}\right.\right.}{e}$$

$$\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^4}{4e}$$

input
```
Int[ArcSinh[c*x]^3/(d + e*x),x]
```

output
```
-1/4*ArcSinh[c*x]^4/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d -
Sqrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d
 + Sqrt[c^2*d^2 + e^2])])/e - (3*(-(ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]) + 2*(ArcSinh[c*x]*PolyLog[3, -((e*
E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))] - PolyLog[4, -((e*E^ArcSinh[
c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])))/e - (3*(-(ArcSinh[c*x]^2*PolyLog[2,
 -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]) + 2*(ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] - PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])))/e
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\operatorname{arcsinh}(xc)^3}{ex+d}dx$$

input
```
int(arcsinh(x*c)^3/(e*x+d),x)
```

output
```
int(arcsinh(x*c)^3/(e*x+d),x)
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arcsinh(c*x)^3/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{asinh}^3(cx)}{d+ex}\,dx$$

input
```
integrate(asinh(c*x)**3/(e*x+d),x)
```

output
```
Integral(asinh(c*x)**3/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^3}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)^3/(e*x + d), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^3}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)^3/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)^3}{d+e\,x}\,dx$$

input
```
int(asinh(c*x)^3/(d + e*x),x)
```

output
```
int(asinh(c*x)^3/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\text{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{asinh(cx)^3}{ex+d}dx$$

input
```
int(asinh(c*x)^3/(e*x+d),x)
```

output
```
int(asinh(c*x)**3/(d + e*x),x)
```

3.4 $\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 184

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx = -\frac{bd(cd-e)(cd+e)\sqrt{1+c^2x^2}}{c^3} - \frac{3be(8c^2d^2-e^2)\,x\sqrt{1+c^2x^2}}{32c^3} - \frac{be^3x^3\sqrt{1+c^2x^2}}{16c} - \frac{bde^2(1+c^2x^2)^{3/2}}{3c^3} - \frac{b(8c^4d^4-24c^2d^2e^2+3e^4)\,\text{arcsinh}(cx)}{32c^4e} + \frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}$$

output
```
-b*d*(c*d-e)*(c*d+e)*(c^2*x^2+1)^(1/2)/c^3-3/32*b*e*(8*c^2*d^2-e^2)*x*(c^2
*x^2+1)^(1/2)/c^3-1/16*b*e^3*x^3*(c^2*x^2+1)^(1/2)/c-1/3*b*d*e^2*(c^2*x^2+
1)^(3/2)/c^3-1/32*b*(8*c^4*d^4-24*c^2*d^2*e^2+3*e^4)*arcsinh(c*x)/c^4/e+1/
4*(e*x+d)^4*(a+b*arcsinh(c*x))/e
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.90

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx = \frac{24ac^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-bc\sqrt{1+c^2x^2}(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+}{96c^4}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[1 + c^2
*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6
*e^3*x^3)) + 3*b*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*
e^2*x^2 + e^3*x^3))*ArcSinh[c*x])/(96*c^4)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6243, 497, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e}-\frac{bc\int \frac{(d+ex)^4}{\sqrt{c^2x^2+1}}dx}{4e}$$

$$\Big\downarrow \text{497}$$

$$\frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\int \frac{(d+ex)^2(4d^2c^2+7dexc^2-3e^2)}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^3}{4c^2}\right)}{4e}$$

$$\Big\downarrow \text{687}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{\int \frac{c^2(d+ex)\left(d\left(12c^2d^2-23e^2\right)+e\left(26c^2d^2-9e^2\right)x\right)}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^3}{4c^2}\right)}{4e}$$

$$\Big\downarrow \; 27$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{1}{3}\int \frac{(d+ex)\left(d\left(12c^2d^2-23e^2\right)+e\left(26c^2d^2-9e^2\right)x\right)}{\sqrt{c^2x^2+1}}dx+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^3}{4c^2}\right)}{4e}$$

$$\Big\downarrow \; 676$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{1}{3}\left(\frac{3\left(8c^4d^4-24c^2d^2e^2+3e^4\right)\int \frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{1}{2}e^2x\sqrt{c^2x^2+1}\left(26d^2-\frac{9e^2}{c^2}\right)+2de\sqrt{c^2x^2+1}\left(19d^2-\frac{16e^2}{c^2}\right)\right)+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2}}{}\right.}{4e}$$

$$\Big\downarrow \; 222$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{1}{3}\left(\frac{3\text{arcsinh}(cx)\left(8c^4d^4-24c^2d^2e^2+3e^4\right)}{2c^3}+\frac{1}{2}e^2x\sqrt{c^2x^2+1}\left(26d^2-\frac{9e^2}{c^2}\right)+2de\sqrt{c^2x^2+1}\left(19d^2-\frac{16e^2}{c^2}\right)\right)+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+}}{4}\right.}{4e}$$

input
```
Int[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e) - (b*c*((e*(d + e*x)^3*Sqrt[1 + c
^2*x^2])/(4*c^2) + ((7*d*e*(d + e*x)^2*Sqrt[1 + c^2*x^2])/3 + (2*d*e*(19*d
^2 - (16*e^2)/c^2)*Sqrt[1 + c^2*x^2] + (e^2*(26*d^2 - (9*e^2)/c^2)*x*Sqrt[
1 + c^2*x^2])/2 + (3*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSinh[c*x])/(2
*c^3))/3)/(4*c^2)))/(4*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\left(\frac{c\,e^3\operatorname{arcsinh}(xc)x^4}{4}+c\,e^2\operatorname{arcsinh}(xc)x^3d+\frac{3ce\operatorname{arcsinh}(xc)x^2d^2}{2}+\operatorname{arcsinh}(xc)xc\,d^3+\frac{c\operatorname{arcsinh}(xc)d^4}{4e}-\frac{c^4d^4}{}\right.}{}$
orering	$\frac{(14c^4e^4x^5+72c^4d\,e^3x^4+152c^4d^2e^2x^3+176c^4d^3e\,x^2+32c^4d^4x-3c^2e^4x^3-32c^2d\,e^3x^2+96c^2d^2e^2x+120c^2d^3e-12e^4x-}{32c^4(ex+d)}$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)c^4d^4}{4e}+\operatorname{arcsinh}(xc)c^4d^3x+\frac{3e\operatorname{arcsinh}(xc)c^4d^2x^2}{2}+e^2\operatorname{arcsinh}(xc)c^4d\,x^3+\frac{e^3\operatorname{arcsinh}(xc)x^4c^4}{4}-\frac{c^4d^4\operatorname{ar}}{}\right.}{}}{}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)c^4d^4}{4e}+\operatorname{arcsinh}(xc)c^4d^3x+\frac{3e\operatorname{arcsinh}(xc)c^4d^2x^2}{2}+e^2\operatorname{arcsinh}(xc)c^4d\,x^3+\frac{e^3\operatorname{arcsinh}(xc)x^4c^4}{4}-\frac{c^4d^4\operatorname{ar}}{}\right.}{}}{}$

input
```
int((e*x+d)^3*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arcsinh(x*c)*x^4+c*e^2*arcsinh(x*c)*x^3*d
+3/2*c*e*arcsinh(x*c)*x^2*d^2+arcsinh(x*c)*x*c*d^3+1/4*c/e*arcsinh(x*c)*d^
4-1/4/c^3/e*(c^4*d^4*arcsinh(x*c)+e^4*(1/4*(c^2*x^2+1)^(1/2)*c^3*x^3-3/8*(
c^2*x^2+1)^(1/2)*x*c+3/8*arcsinh(x*c))+4*d*c*e^3*(1/3*x^2*c^2*(c^2*x^2+1)^
(1/2)-2/3*(c^2*x^2+1)^(1/2))+6*d^2*c^2*e^2*(1/2*(c^2*x^2+1)^(1/2)*x*c-1/2*
arcsinh(x*c))+4*d^3*c^3*e*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.16

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{24\,ac^4e^3x^4+96\,ac^4de^2x^3+144\,ac^4d^2ex^2+96\,ac^4d^3x+3\,(8\,bc^4e^3x^4+32\,bc^4de^2x^3+48\,bc^4d^2ex^2+32\,bc^4}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c
^4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 +
32*b*c^4*d^3*x + 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 + 1)) -
(6*b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 - 64*b*c*d*e^2 + 9*(8
*b*c^3*d^2*e - b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.72

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}+bd^3x\,\text{asinh}\,(cx)+\frac{3bd^2ex^2\,\text{asinh}\,(cx)}{2}+bde^2x^3\,\text{asinh}\,(cx)+\frac{be^3x^4\,\text{asinh}\,(cx)}{4} \\ a\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right) \end{cases}$$

input
```
integrate((e*x+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 +
b*d**3*x*asinh(c*x) + 3*b*d**2*e*x**2*asinh(c*x)/2 + b*d*e**2*x**3*asinh(c
*x) + b*e**3*x**4*asinh(c*x)/4 - b*d**3*sqrt(c**2*x**2 + 1)/c - 3*b*d**2*e
*x*sqrt(c**2*x**2 + 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(3*c) - b
*e**3*x**3*sqrt(c**2*x**2 + 1)/(16*c) + 3*b*d**2*e*asinh(c*x)/(4*c**2) + 2
*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*b*e**3*x*sqrt(c**2*x**2 + 1)/(3
2*c**3) - 3*b*e**3*asinh(c*x)/(32*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*
x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.25

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2$$
$$+ \frac{3}{4}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd^2e$$
$$+ \frac{1}{3}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde^2$$
$$+ \frac{1}{32}\left(8x^4\operatorname{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)be^3$$
$$+ ad^3x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arcsinh(c*x) -
c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2*e + 1/3*(3*x^3*arcsi
nh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e^2
 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^
2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*x)
- sqrt(c^2*x^2 + 1))*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx=\int (a+b\,\text{asinh}(c\,x))\,\left(d+e\,x\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.55

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{96asinh(cx)\,b\,c^4d^3x+144asinh(cx)\,b\,c^4d^2e\,x^2+96asinh(cx)\,b\,c^4d\,e^2x^3+24asinh(cx)\,b\,c^4e^3x^4-96\sqrt{c^2x}}{}$$

input
```
int((e*x+d)^3*(a+b*asinh(c*x)),x)
```

output
```
(96*asinh(c*x)*b*c**4*d**3*x + 144*asinh(c*x)*b*c**4*d**2*e*x**2 + 96*asin
h(c*x)*b*c**4*d*e**2*x**3 + 24*asinh(c*x)*b*c**4*e**3*x**4 - 96*sqrt(c**2*
x**2 + 1)*b*c**3*d**3 - 72*sqrt(c**2*x**2 + 1)*b*c**3*d**2*e*x - 32*sqrt(c
**2*x**2 + 1)*b*c**3*d*e**2*x**2 - 6*sqrt(c**2*x**2 + 1)*b*c**3*e**3*x**3
+ 64*sqrt(c**2*x**2 + 1)*b*c*d*e**2 + 9*sqrt(c**2*x**2 + 1)*b*c*e**3*x + 7
2*log(sqrt(c**2*x**2 + 1) + c*x)*b*c**2*d**2*e - 9*log(sqrt(c**2*x**2 + 1)
 + c*x)*b*e**3 + 96*a*c**4*d**3*x + 144*a*c**4*d**2*e*x**2 + 96*a*c**4*d*e
**2*x**3 + 24*a*c**4*e**3*x**4)/(96*c**4)
```

3.5 $\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx = -\frac{b(3c^2d^2-e^2)\sqrt{1+c^2x^2}}{3c^3} - \frac{bdex\sqrt{1+c^2x^2}}{2c} - \frac{be^2(1+c^2x^2)^{3/2}}{9c^3} - \frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\text{arcsinh}(cx)}{6e} + \frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}$$

output
```
-1/3*b*(3*c^2*d^2-e^2)*(c^2*x^2+1)^(1/2)/c^3-1/2*b*d*e*x*(c^2*x^2+1)^(1/2)
/c-1/9*b*e^2*(c^2*x^2+1)^(3/2)/c^3-1/6*b*d*(2*d^2-3*e^2/c^2)*arcsinh(c*x)/
e+1/3*(e*x+d)^3*(a+b*arcsinh(c*x))/e
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx = \frac{6ac^3x(3d^2+3dex+e^2x^2) - b\sqrt{1+c^2x^2}(-4e^2+c^2(18d^2+9dex+2e^2x^2)) + 3bc(6c^2d^2x+2c^2e^2x^3+3c}{18c^3}$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2
*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*b*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*
d*(e + 2*c^2*e*x^2))*ArcSinh[c*x])/(18*c^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6243, 497, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\int \frac{(d+ex)^3}{\sqrt{c^2x^2+1}}dx}{3e}$$

$$\Big\downarrow \text{ 497}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\left(\frac{\int \frac{(d+ex)\left(3d^2c^2+5dexc^2-2e^2\right)}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^2}{3c^2}\right)}{3e}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\left(\frac{\frac{3}{2}d(2c^2d^2-3e^2)\int \frac{1}{\sqrt{c^2x^2+1}}dx+2e\sqrt{c^2x^2+1}\left(4d^2-\frac{e^2}{c^2}\right)+\frac{5}{2}de^2x\sqrt{c^2x^2+1}}{3c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^2}{3c^2}\right)}{3e}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\left(\frac{\frac{3d\text{arcsinh}(cx)\left(2c^2d^2-3e^2\right)}{2c}+2e\sqrt{c^2x^2+1}\left(4d^2-\frac{e^2}{c^2}\right)+\frac{5}{2}de^2x\sqrt{c^2x^2+1}}{3c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^2}{3c^2}\right)}{3e}$$

input
```
Int[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e) - (b*c*((e*(d + e*x)^2*Sqrt[1 + c
^2*x^2])/(3*c^2) + (2*e*(4*d^2 - e^2/c^2)*Sqrt[1 + c^2*x^2] + (5*d*e^2*x*S
qrt[1 + c^2*x^2])/2 + (3*d*(2*c^2*d^2 - 3*e^2)*ArcSinh[c*x])/(2*c))/(3*c^2
)))/(3*e)
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\left(\frac{c\,e^2 \operatorname{arcsinh}(xc)x^3}{3}+ce \operatorname{arcsinh}(xc)x^2 d+\operatorname{arcsinh}(xc)xc\,d^2+\frac{c \operatorname{arcsinh}(xc)d^3}{3e}-\frac{c^3 d^3 \operatorname{arcsinh}(xc)+e^3\left(\frac{x^2c^2\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{c}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)c^3d^3}{3e}+\operatorname{arcsinh}(xc)c^3d^2x+e \operatorname{arcsinh}(xc)c^3d\,x^2+\frac{e^2 \operatorname{arcsinh}(xc)x^3c^3}{3}-\frac{c^3d^3 \operatorname{arcsinh}(xc)+e^3\left(\frac{x^2c^2\sqrt{c^2\cdots}}{3}\right.}{\cdots}\right.}{c^2}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arcsinh}(xc)c^3d^3}{3e}+\operatorname{arcsinh}(xc)c^3d^2x+e \operatorname{arcsinh}(xc)c^3d\,x^2+\frac{e^2 \operatorname{arcsinh}(xc)x^3c^3}{3}-\frac{c^3d^3 \operatorname{arcsinh}(xc)+e^3\left(\frac{x^2c^2\sqrt{c^2\cdots}}{3}\right.}{\cdots}\right.}{c^2}}{c}$
orering	$\frac{(10c^4e^3x^4+42c^4d\,e^2x^3+72c^4d^2e\,x^2+18c^4d^3x-4c^2e^3x^2+27c^2d\,e^2x+45c^2d^2e-8e^3)(a+b \operatorname{arcsinh}(xc))}{18c^4(ex+d)}-\frac{(2c^2e^2x^2+\cdots}{\cdots}$

input
```
int((e*x+d)^2*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arcsinh(x*c)*x^3+c*e*arcsinh(x*c)*x^2*d+a
rcsinh(x*c)*x*c*d^2+1/3*c/e*arcsinh(x*c)*d^3-1/3/c^2/e*(c^3*d^3*arcsinh(x*
c)+e^3*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d*c*e^2*(1/
2*(c^2*x^2+1)^(1/2)*x*c-1/2*arcsinh(x*c))+3*d^2*c^2*e*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{6\,ac^3e^2x^3+18\,ac^3dex^2+18\,ac^3d^2x+3\,(2\,bc^3e^2x^3+6\,bc^3dex^2+6\,bc^3d^2x+3\,bcde)\log\left(cx+\sqrt{c^2x^2+1}\right)}{18\,c^3}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2
*x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x + 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2
 + 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 - 4*b*e^2)*sqrt(c
^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.43

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^2x+adex^2+\frac{ae^2x^3}{3}+bd^2x\operatorname{asinh}(cx)+bdex^2\operatorname{asinh}(cx)+\frac{be^2x^3\operatorname{asinh}(cx)}{3}-\frac{bd^2\sqrt{c^2x^2+1}}{c}-\frac{bdex\sqrt{c^2x^2+1}}{2c}- \\ a\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asinh(c*x) + b
*d*e*x**2*asinh(c*x) + b*e**2*x**3*asinh(c*x)/3 - b*d**2*sqrt(c**2*x**2 +
1)/c - b*d*e*x*sqrt(c**2*x**2 + 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 + 1)
/(9*c) + b*d*e*asinh(c*x)/(2*c**2) + 2*b*e**2*sqrt(c**2*x**2 + 1)/(9*c**3)
, Ne(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{1}{3}ae^2x^3 + adex^2 + \frac{1}{2}\left(2x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)bde$$
$$+ \frac{1}{9}\left(3x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)be^2$$
$$+ ad^2x + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)
*x/c^2 - arcsinh(c*x)/c^3))*b*d*e + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*
x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*e^2 + a*d^2*x + (c*x*arcsin
h(c*x) - sqrt(c^2*x^2 + 1))*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\ (d+e\,x)^2\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.36

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx = \frac{18asinh(cx)\,b\,c^3d^2x+18asinh(cx)\,b\,c^3de\,x^2+6asinh(cx)\,b\,c^3e^2x^3-18\sqrt{c^2x^2+1}\,b\,c^2d^2-9\sqrt{c^2x^2+1}\,b\,c}{}$$

input
```
int((e*x+d)^2*(a+b*asinh(c*x)),x)
```

output
```
(18*asinh(c*x)*b*c**3*d**2*x + 18*asinh(c*x)*b*c**3*d*e*x**2 + 6*asinh(c*x
)*b*c**3*e**2*x**3 - 18*sqrt(c**2*x**2 + 1)*b*c**2*d**2 - 9*sqrt(c**2*x**2
 + 1)*b*c**2*d*e*x - 2*sqrt(c**2*x**2 + 1)*b*c**2*e**2*x**2 + 4*sqrt(c**2*
x**2 + 1)*b*e**2 + 9*log(sqrt(c**2*x**2 + 1) + c*x)*b*c*d*e + 18*a*c**3*d*
*2*x + 18*a*c**3*d*e*x**2 + 6*a*c**3*e**2*x**3)/(18*c**3)
```

3.6 $\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = -\frac{bd\sqrt{1+c^2x^2}}{c} - \frac{bex\sqrt{1+c^2x^2}}{4c} - \frac{b\left(2d^2-\frac{e^2}{c^2}\right)\text{arcsinh}(cx)}{4e} + \frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}$$

output
```
-b*d*(c^2*x^2+1)^(1/2)/c-1/4*b*e*x*(c^2*x^2+1)^(1/2)/c-1/4*b*(2*d^2-e^2/c^
2)*arcsinh(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = adx + \frac{1}{2}aex^2 - \frac{bd\sqrt{1+c^2x^2}}{c} - \frac{bex\sqrt{1+c^2x^2}}{4c} + \frac{be\text{arcsinh}(cx)}{4c^2} + bdx\text{arcsinh}(cx) + \frac{1}{2}bex^2\text{arcsinh}(cx)$$

input
```
Integrate[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[1 + c^2*x^2])/c - (b*e*x*Sqrt[1 + c^2*x^2]
)/(4*c) + (b*e*ArcSinh[c*x])/(4*c^2) + b*d*x*ArcSinh[c*x] + (b*e*x^2*ArcSi
nh[c*x])/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6243, 497, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\int \frac{(d+ex)^2}{\sqrt{c^2x^2+1}}dx}{2e}$$

$$\Big\downarrow \text{497}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\left(\frac{\int \frac{2d^2c^2+3dexc^2-e^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)}{2c^2}\right)}{2e}$$

$$\Big\downarrow \text{455}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\left(\frac{(2c^2d^2-e^2)\int \frac{1}{\sqrt{c^2x^2+1}}dx+3de\sqrt{c^2x^2+1}}{2c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)}{2c^2}\right)}{2e}$$

$$\Big\downarrow \text{222}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\left(\frac{\frac{\text{arcsinh}(cx)(2c^2d^2-e^2)}{c}+3de\sqrt{c^2x^2+1}}{2c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)}{2c^2}\right)}{2e}$$

input
```
Int[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + e*x)^2*(a + b*ArcSinh[c*x]))/(2*e) - (b*c*((e*(d + e*x)*Sqrt[1 + c^2
*x^2])/(2*c^2) + (3*d*e*Sqrt[1 + c^2*x^2] + ((2*c^2*d^2 - e^2)*ArcSinh[c*x
])/c)/(2*c^2)))/(2*e)
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

method	result
parts	$a\left(\frac{1}{2}e\,x^{2}+dx\right)+\frac{b\left(\frac{c\,\mathrm{arcsinh}(xc)e\,x^{2}}{2}+\mathrm{arcsinh}(xc)xcd-\frac{e\left(\frac{\sqrt{c^{2}x^{2}+1}\,xc}{2}-\frac{\mathrm{arcsinh}(xc)}{2}\right)+2dc\sqrt{c^{2}x^{2}+1}}{2c}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(c^{2}dx+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\mathrm{arcsinh}(xc)x\,c^{2}d+\frac{\mathrm{arcsinh}(xc)e\,x^{2}c^{2}}{2}-\frac{e\left(\frac{\sqrt{c^{2}x^{2}+1}\,xc}{2}-\frac{\mathrm{arcsinh}(xc)}{2}\right)}{2}-dc\sqrt{c^{2}x^{2}+1}\right)}{c}}{c}$
default	$\frac{\frac{a\left(c^{2}dx+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\mathrm{arcsinh}(xc)x\,c^{2}d+\frac{\mathrm{arcsinh}(xc)e\,x^{2}c^{2}}{2}-\frac{e\left(\frac{\sqrt{c^{2}x^{2}+1}\,xc}{2}-\frac{\mathrm{arcsinh}(xc)}{2}\right)}{2}-dc\sqrt{c^{2}x^{2}+1}\right)}{c}}{c}$
orering	$\frac{(3c^{2}e^{2}x^{3}+10c^{2}de\,x^{2}+4c^{2}d^{2}x+2e^{2}x+5de)(a+b\,\mathrm{arcsinh}(xc))}{4c^{2}(ex+d)}-\frac{(ex+4d)(c^{2}x^{2}+1)\left(e(a+b\,\mathrm{arcsinh}(xc))+\frac{(ex+d)bc}{\sqrt{c^{2}x^{2}+1}}\right)}{4c^{2}(ex+d)}$

input
```
int((e*x+d)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsinh(x*c)*e*x^2+arcsinh(x*c)*x*c*d-1/2/c*(
e*(1/2*(c^2*x^2+1)^(1/2)*x*c-1/2*arcsinh(x*c))+2*d*c*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int (d+ex)(a+b\mathrm{arcsinh}(cx))\,dx$$
$$=\frac{2\,ac^{2}ex^{2}+4\,ac^{2}dx+(2\,bc^{2}ex^{2}+4\,bc^{2}dx+be)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-(bcex+4\,bcd)\sqrt{c^{2}x^{2}+1}}{4\,c^{2}}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x + b*e)*log
(c*x + sqrt(c^2*x^2 + 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} adx+\frac{aex^2}{2}+bdx\operatorname{asinh}(cx)+\frac{bex^2\operatorname{asinh}(cx)}{2}-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex\sqrt{c^2x^2+1}}{4c}+\frac{be\operatorname{asinh}(cx)}{4c^2} & \text{for } c\neq 0\\ a\left(dx+\frac{ex^2}{2}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**2/2 + b*d*x*asinh(c*x) + b*e*x**2*asinh(c*x)/2 -
 b*d*sqrt(c**2*x**2 + 1)/c - b*e*x*sqrt(c**2*x**2 + 1)/(4*c) + b*e*asinh(c
*x)/(4*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{2}aex^2+\frac{1}{4}\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)be$$
$$+adx+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsi
nh(c*x)/c^3))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.33

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{1}{2}aex^2+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)bd$$
$$+\frac{1}{4}\left(2\,x^2\log\left(cx+\sqrt{c^2x^2+1}\right)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}+\frac{\log\left(-x|c|+\sqrt{c^2x^2+1}\right)}{c^2|c|}\right)\right)be$$
$$+adx$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
1/2*a*e*x^2 + (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d +
 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + lo
g(-x*abs(c) + sqrt(c^2*x^2 + 1))/(c^2*abs(c))))*b*e + a*d*x
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx=\frac{a\,x\,(2\,d+e\,x)}{2}-\frac{b\,d\left(\sqrt{c^2\,x^2+1}-c\,x\,\text{asinh}(c\,x)\right)}{c}$$
$$-\frac{b\,e\,x\,\sqrt{c^2\,x^2+1}}{4\,c}+b\,e\,x\,\text{asinh}(c\,x)\,\left(\frac{x}{2}+\frac{1}{4\,c^2\,x}\right)$$

input
```
int((a + b*asinh(c*x))*(d + e*x),x)
```

output
```
(a*x*(2*d + e*x))/2 - (b*d*((c^2*x^2 + 1)^(1/2) - c*x*asinh(c*x)))/c - (b*
e*x*(c^2*x^2 + 1)^(1/2))/(4*c) + b*e*x*asinh(c*x)*(x/2 + 1/(4*c^2*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{4asinh(cx)\,b\,c^2dx + 2asinh(cx)\,b\,c^2e\,x^2 + 2asinh(cx)\,be - 4\sqrt{c^2x^2+1}\,bcd - \sqrt{c^2x^2+1}\,bcex - \log\left(\sqrt{c^2x\right.}{4c^2}$$

input
```
int((e*x+d)*(a+b*asinh(c*x)),x)
```

output
```
(4*asinh(c*x)*b*c**2*d*x + 2*asinh(c*x)*b*c**2*e*x**2 + 2*asinh(c*x)*b*e -
 4*sqrt(c**2*x**2 + 1)*b*c*d - sqrt(c**2*x**2 + 1)*b*c*e*x - log(sqrt(c**2
*x**2 + 1) + c*x)*b*e + 4*a*c**2*d*x + 2*a*c**2*e*x**2)/(4*c**2)
```

3.7 $\int (a + b\text{arcsinh}(cx))\, dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a + b\text{arcsinh}(cx))\, dx = ax - \frac{b\sqrt{1 + c^2x^2}}{c} + bx\text{arcsinh}(cx)$$

output
```
a*x-b*(c^2*x^2+1)^(1/2)/c+b*x*arcsinh(c*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\, dx = ax - \frac{b\sqrt{1 + c^2x^2}}{c} + bx\text{arcsinh}(cx)$$

input
```
Integrate[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}$$

input
```
Int[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
orering	$x(a+b\,\text{arcsinh}\,(xc))-\frac{b\sqrt{c^2x^2+1}}{c}$	29
default	$xa+\frac{b\left(xc\,\text{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	31
parts	$xa+\frac{b\left(xc\,\text{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	31
derivativedivides	$\frac{axc+b\left(xc\,\text{arcsinh}(xc)-\sqrt{c^2x^2+1}\right)}{c}$	33

input
```
int(a+b*arcsinh(x*c),x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arcsinh(x*c))-b*(c^2*x^2+1)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a + b\text{arcsinh}(cx))\,dx = \frac{bcx\log\left(cx + \sqrt{c^2x^2 + 1}\right) + acx - \sqrt{c^2x^2 + 1}b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

output
```
(b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (a + b\text{arcsinh}(cx))\,dx = ax + b\left(\begin{cases} x\operatorname{asinh}(cx) - \frac{\sqrt{c^2x^2+1}}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*asinh(c*x),x)
```

output
```
a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\,dx = ax + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (a + b\text{arcsinh}(cx))\,dx = \left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)b + ax$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

output
```
(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (a + b\text{arcsinh}(cx))\,dx = a\,x - \frac{b\,\sqrt{c^2\,x^2+1}}{c} + b\,x\,\text{asinh}(c\,x)$$

input
```
int(a + b*asinh(c*x),x)
```

output
```
a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\, dx = \frac{asinh(cx)\, bcx - \sqrt{c^2x^2 + 1}\, b + acx}{c}$$

input
```
int(a+b*asinh(c*x),x)
```

output
```
(asinh(c*x)*b*c*x - sqrt(c**2*x**2 + 1)*b + a*c*x)/c
```

3.8 $\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 187

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx = -\frac{(a+b\text{arcsinh}(cx))^2}{2be} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2/b/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(
1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2
+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(
c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx$$
$$= \frac{-\left((a+b\operatorname{arcsinh}(cx))\left(a+b\operatorname{arcsinh}(cx)-2b\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-2b\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)\right)+2b^2}{2be}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

output
```
(-((a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 2*b*Log[1 + (e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2])] - 2*b*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sq
rt[c^2*d^2 + e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt
[c^2*d^2 + e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*
d^2 + e^2]))])/(2*b*e)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx$$
$$\Big\downarrow \text{ 6242}$$
$$\int \frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{cd+cex}d\operatorname{arcsinh}(cx)$$
$$\Big\downarrow \text{ 6095}$$
$$\int \frac{e^{\operatorname{arcsinh}(cx)}(a+b\operatorname{arcsinh}(cx))}{cd+ee^{\operatorname{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)+\int \frac{e^{\operatorname{arcsinh}(cx)}(a+b\operatorname{arcsinh}(cx))}{cd+ee^{\operatorname{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)-\frac{(a+b\operatorname{arcsinh}(cx))^2}{2be}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2be}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2be}$$

$$\Big\downarrow \; 2838$$

$$\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2be}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^Arc
Sinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 +
 (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E
^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

method	result
parts	$\frac{a\ln(ex+d)}{e}-\frac{b\operatorname{arcsinh}(xc)^2}{2e}+\frac{b\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc\right.}{}\right.}{e}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{\operatorname{arcsinh}(xc)^2}{2e}+\frac{\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc+\sqrt{c^2x^2+\right.}{cd+\sqrt{c^2d}}\right.}{e}\right.}{c}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{\operatorname{arcsinh}(xc)^2}{2e}+\frac{\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc+\sqrt{c^2x^2+\right.}{cd+\sqrt{c^2d}}\right.}{e}\right.}{c}$

input
```
int((a+b*arcsinh(x*c))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e-1/2*b*arcsinh(x*c)^2/e+b/e*arcsinh(x*c)*ln((-c*d-e*(x*c+(c^2
*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*arcsin
h(x*c)*ln((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^
2+e^2)^(1/2)))+b/e*dilog((-c*d-e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/
2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*dilog((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(
c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{d + ex}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{ex + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x) + a*log(e*x + d)/e
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{d + ex}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{ex + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{a+b\,\mathrm{asinh}(c\,x)}{d+e\,x}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx = \frac{\left(\int \frac{asinh(cx)}{ex+d}dx\right)be+\log(ex+d)\,a}{e}$$

input
```
int((a+b*asinh(c*x))/(e*x+d),x)
```

output
```
(int(asinh(c*x)/(d + e*x),x)*b*e + log(d + e*x)*a)/e
```

3.9 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\text{arcsinh}(cx)}{e(d+ex)} - \frac{bc\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

output
```
-(a+b*arcsinh(c*x))/e/(e*x+d)-b*c*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)
/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2}\,dx = -\frac{\frac{a+b\text{arcsinh}(cx)}{d+ex} + \frac{bc\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d^2+e^2}}}{e}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

output
```
-(((a + b*ArcSinh[c*x])/(d + e*x) + (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d
^2 + e^2]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d^2 + e^2])/e)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6243, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{bc\int \frac{1}{(d+ex)\sqrt{c^2x^2+1}}dx}{e}-\frac{a+b\text{arcsinh}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{bc\int \frac{1}{c^2d^2+e^2-\frac{\left(e-c^2dx\right)^2}{c^2x^2+1}}d\frac{e-c^2dx}{\sqrt{c^2x^2+1}}}{e}-\frac{a+b\text{arcsinh}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a+b\text{arcsinh}(cx)}{e(d+ex)}-\frac{b\text{carctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c
^2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(76) = 152$.

Time = 4.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.12

method	result
parts	$-\frac{a}{(ex+d)e}-\frac{bc\,\mathrm{arcsinh}(xc)}{(cex+cd)e}-\frac{bc\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{xc+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arcsinh}(xc)}{(cex+cd)e}-\frac{\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{xc+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arcsinh}(xc)}{(cex+cd)e}-\frac{\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{xc+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$

input
```
int((a+b*arcsinh(x*c))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e-b*c/(c*e*x+c*d)/e*arcsinh(x*c)-b*c/e^2/((c^2*d^2+e^2)/e^2)^(1
/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(x*c+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2
)*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(x*c+d*c/e)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(78) = 156$.

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.09

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+ex)^2}\,dx =$$

$$-\frac{ac^2d^3+ade^2-(bc^2d^2e+be^3)x\log\left(cx+\sqrt{c^2x^2+1}\right)-(bcdex+bcd^2)\sqrt{c^2d^2+e^2}\log\left(-\frac{c^3d^2x-cde+\sqrt{c^2}}{}\right.}{c^2d^4e+d^2e^3+(}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-(a*c^2*d^3 + a*d*e^2 - (b*c^2*d^2*e + b*e^3)*x*log(c*x + sqrt(c^2*x^2 + 1
)) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + s
qrt(c^2*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^
2)*sqrt(c^2*x^2 + 1))/(e*x + d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b
*e^3)*x)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^
2 + d*e^4)*x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex)^2}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(d + ex)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex)^2}\,dx = -b\left(\frac{\text{arsinh}\,(cx)}{e^2x + de} - \frac{c\,\text{arsinh}\left(\frac{cd\sqrt{e^4}x}{e|e^2x+de|} - \frac{\sqrt{e^4}}{c|e^2x+de|}\right)}{\sqrt{\frac{c^2d^2}{e^2} + 1}e^2}\right) - \frac{a}{e^2x + de}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsinh(c*d*sqrt(e^4)*x/(e*abs(e^2*x +
d*e)) - sqrt(e^4)/(c*abs(e^2*x + d*e)))/(sqrt(c^2*d^2/e^2 + 1)*e^2)) - a/(
e^2*x + d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(78) = 156$.

Time = 0.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.85

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex)^2}\,dx$$
$$= \left(\frac{c\log\left(\left|c^2de - \sqrt{c^2d^2 + e^2}|c||e|\right|\right)\text{sgn}\left(\frac{1}{ex+d}\right)\text{sgn}(e)}{\sqrt{c^2d^2 + e^2}|e|} - \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{(ex + d)e} - \frac{c\log\left(\left|c^2de - \sqrt{c^2d^2 + e\right.\right.}{\sqrt{}}\right.$$
$$- \frac{a}{(ex + d)e}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
(c*log(abs(c^2*d*e - sqrt(c^2*d^2 + e^2)*abs(c)*abs(e)))*sgn(1/(e*x + d))*
sgn(e)/(sqrt(c^2*d^2 + e^2)*abs(e)) - log(c*x + sqrt(c^2*x^2 + 1))/((e*x +
 d)*e) - c*log(abs(c^2*d*e - sqrt(c^2*d^2 + e^2)*(sqrt(c^2 - 2*c^2*d/(e*x
+ d) + c^2*d^2/(e*x + d)^2 + e^2/(e*x + d)^2) + sqrt(c^2*d^2*e^2 + e^4)/((
e*x + d)*e))*abs(e)))/(sqrt(c^2*d^2 + e^2)*abs(e)*sgn(1/(e*x + d))*sgn(e))
)*b - a/((e*x + d)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex)^2}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(d + e\,x)^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^2}\, dx = \frac{\left(\int \frac{asinh(cx)}{e^2x^2+2dex+d^2}dx\right) b\, d^2 + \left(\int \frac{asinh(cx)}{e^2x^2+2dex+d^2}dx\right) bdex + ax}{d\,(ex+d)}$$

input
```
int((a+b*asinh(c*x))/(e*x+d)^2,x)
```

output
```
(int(asinh(c*x)/(d**2 + 2*d*e*x + e**2*x**2),x)*b*d**2 + int(asinh(c*x)/(d
**2 + 2*d*e*x + e**2*x**2),x)*b*d*e*x + a*x)/(d*(d + e*x))
```

3.10 $\int \frac{a+b\,\text{arcsinh}(cx)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(d+ex)^3}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\,\text{arcsinh}(cx)}{2e(d+ex)^2} - \frac{bc^3d\,\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

output
```
-1/2*b*c*(c^2*x^2+1)^(1/2)/(c^2*d^2+e^2)/(e*x+d)-1/2*(a+b*arcsinh(c*x))/e/
(e*x+d)^2-1/2*b*c^3*d*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)/(c^2*x^2+1)
^(1/2))/e/(c^2*d^2+e^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.30

$$\int \frac{a+b\,\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bc\sqrt{1+c^2x^2}}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{b\,\text{arcsinh}(cx)}{e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{e\left(c^2d^2+e^2\right)^{3/2}} - \frac{bc^3d\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}\right)$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x)
) - (b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*(c^2*d^2
+ e^2)^(3/2)) - (b*c^3*d*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^
2*x^2]])/(e*(c^2*d^2 + e^2)^(3/2)))/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6243, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{bc\int \frac{1}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\left(\frac{c^2d\int \frac{1}{(d+ex)\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{bc\left(-\frac{c^2d\int \frac{1}{c^2d^2+e^2-\frac{\left(e-c^2dx\right)^2}{c^2x^2+1}}d\frac{e-c^2dx}{\sqrt{c^2x^2+1}}}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{bc\left(-\frac{c^2 d\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{(c^2d^2+e^2)^{3/2}}-\frac{e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2e}-\frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c*(-((e*Sqrt[1 + c^2*x^2])/
((c^2*d^2 + e^2)*(d + e*x))) - (c^2*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2
+ e^2]*Sqrt[1 + c^2*x^2])])/(c^2*d^2 + e^2)^(3/2)))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(114) = 228.

Time = 5.03 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{a}{2(ex+d)^2 e}-\frac{b\,c^2 \operatorname{arcsinh}(xc)}{2(cex+cd)^2 e}-\frac{b\,c^2\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e(c^2d^2+e^2)\left(xc+\frac{dc}{e}\right)}-\frac{b\,c^3 d\ln\left(\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(xc+\frac{dc}{e}\right.}{e}\right.}{2e}$
derivativedivides	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\operatorname{arcsinh}(xc)}{2(cex+cd)^2 e}+\frac{-\frac{e^2\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{\left(c^2d^2+e^2\right)\left(xc+\frac{dc}{e}\right)}-\frac{dce\ln\left(\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+2\sqrt{c}\right.}{\left(c^2d^2+\right.}}{2e^3}\right.}{c}$
default	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\operatorname{arcsinh}(xc)}{2(cex+cd)^2 e}+\frac{-\frac{e^2\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{\left(c^2d^2+e^2\right)\left(xc+\frac{dc}{e}\right)}-\frac{dce\ln\left(\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+2\sqrt{c}\right.}{\left(c^2d^2+\right.}}{2e^3}\right.}{c}$

input
```
int((a+b*arcsinh(x*c))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(e*x+d)^2/e-1/2*b*c^2/(c*e*x+c*d)^2/e*arcsinh(x*c)-1/2*b*c^2/e/(c^2
*d^2+e^2)/(x*c+d*c/e)*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2
)^(1/2)-1/2*b*c^3/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2
*d^2+e^2)/e^2-2*d*c/e*(x*c+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((x*c+d*c/e)
^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(x*c+d*c/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(116) = 232$.

Time = 0.17 (sec) , antiderivative size = 566, normalized size of antiderivative = 4.42

$$\int \frac{a + b\text{arcsinh}(cx)}{(d+ex)^3} dx =$$

$$-\frac{(a+b)c^4d^6 + (2a+b)c^2d^4e^2 + ad^2e^4 + (bc^4d^4e^2 + bc^2d^2e^4)x^2 - (bc^3d^3e^2x^2 + 2bc^3d^4ex + bc^3d^5)\sqrt{c^2d}}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*((a + b)*c^4*d^6 + (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
 + b*c^2*d^2*e^4)*x^2 - (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*
sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqrt(c^2*d^2 + e^2)*(c^2*d*x
 - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x^2 + 1))/(e*x
+ d)) + 2*(b*c^4*d^5*e + b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 + 2*b*c^2*d^2*
e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x
+ sqrt(c^2*x^2 + 1)) - (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d
^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 +
 b*d*e^5)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (b*c^3*d^5*e + b*c*d^3*e^3 +
(b*c^3*d^4*e^2 + b*c*d^2*e^4)*x)*sqrt(c^2*x^2 + 1))/(c^4*d^8*e + 2*c^2*d^6
*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*
e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d+ex)^3} dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(d+ex)^3} dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d)**3,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.23

$$\int \frac{a + b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = -\frac{1}{2}\left(c\left(\frac{\sqrt{c^2x^2+1}}{c^2d^2ex + c^2d^3 + e^3x + de^2} - \frac{c^2d\,\text{arsinh}\left(\frac{cdx}{e\left|x+\frac{d}{e}\right|} - \frac{1}{c\left|x+\frac{d}{e}\right|}\right)}{\left(\frac{c^2d^2}{e^2}+1\right)^{\frac{3}{2}}e^4}\right) + \frac{\text{arsinh}\,(cx)}{e^3x^2 + 2\,de^2x + d^2e}\right)b - \frac{a}{2\,(e^3x^2 + 2\,de^2x + d^2e)}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d
*arcsinh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(
3/2)*e^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^
2 + 2*d*e^2*x + d^2*e)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \frac{2\left(\int \frac{asinh(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)b\,d^2e+4\left(\int \frac{asinh(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)bd\,e^2x+2\left(\int \frac{asinh(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)b\,e}{2e\,(e^2x^2+2dex+d^2)}$$

input
```
int((a+b*asinh(c*x))/(e*x+d)^3,x)
```

output
```
(2*int(asinh(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)*b*d**
2*e + 4*int(asinh(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)*
b*d*e**2*x + 2*int(asinh(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x*
*3),x)*b*e**3*x**2 - a)/(2*e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.11 $\int \frac{a+b\,\mathrm{arcsinh}(cx)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 183

$$\int \frac{a+b\,\mathrm{arcsinh}(cx)}{(d+ex)^4}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\,\mathrm{arcsinh}(cx)}{3e(d+ex)^3} - \frac{bc^3(2c^2d^2-e^2)\,\mathrm{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

output
```
-1/6*b*c*(c^2*x^2+1)^(1/2)/(c^2*d^2+e^2)/(e*x+d)^2-1/2*b*c^3*d*(c^2*x^2+1)
^(1/2)/(c^2*d^2+e^2)^2/(e*x+d)-1/3*(a+b*arcsinh(c*x))/e/(e*x+d)^3-1/6*b*c^
3*(2*c^2*d^2-e^2)*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)/(c^2*x^2+1)^(1/
2))/e/(c^2*d^2+e^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = \frac{1}{6}\left(-\frac{2a}{e(d+ex)^3} - \frac{bc\sqrt{1+c^2x^2}(e^2+c^2d(4d+3ex))}{(c^2d^2+e^2)^2(d+ex)^2} - \frac{2b\operatorname{arcsinh}(cx)}{e(d+ex)^3} - \frac{bc^3(-2c^2d^2+e^2)\log(d+ex)}{e(c^2d^2+e^2)^{5/2}} + \frac{bc^3(-2c^2d^2+e^2)\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e(c^2d^2+e^2)^{5/2}}\right)$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

output
```
((-2*a)/(e*(d + e*x)^3) - (b*c*Sqrt[1 + c^2*x^2]*(e^2 + c^2*d*(4*d + 3*e*x
)))/((c^2*d^2 + e^2)^2*(d + e*x)^2) - (2*b*ArcSinh[c*x])/(e*(d + e*x)^3) -
 (b*c^3*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(e*(c^2*d^2 + e^2)^(5/2)) + (b*c^
3*(-2*c^2*d^2 + e^2)*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x^
2]])/(e*(c^2*d^2 + e^2)^(5/2)))/6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6243, 498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{bc\int \frac{1}{(d+ex)^3\sqrt{c^2x^2+1}}dx}{3e} - \frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{498}$$

$$\frac{bc\left(-\frac{c^2\int -\frac{2d-ex}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{bc\left(\frac{c^2\int \frac{2d-ex}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \; 679$$

$$\frac{bc\left(\frac{c^2\left(\frac{\left(2c^2d^2-e^2\right)\int \frac{1}{(d+ex)\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{3de\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}\right)}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \; 488$$

$$\frac{bc\left(\frac{c^2\left(-\frac{\left(2c^2d^2-e^2\right)\int \frac{1}{c^2d^2+e^2-\frac{\left(e-c^2dx\right)^2}{c^2x^2+1}}d\frac{e-c^2dx}{\sqrt{c^2x^2+1}}}{c^2d^2+e^2}-\frac{3de\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}\right)}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{bc\left(\frac{c^2\left(-\frac{\left(2c^2d^2-e^2\right)\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{\left(c^2d^2+e^2\right)^{3/2}}-\frac{3de\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}\right)}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(e*(d + e*x)^3) + (b*c*(-1/2*(e*Sqrt[1 + c^2*x^2
])/((c^2*d^2 + e^2)*(d + e*x)^2) + (c^2*((-3*d*e*Sqrt[1 + c^2*x^2])/((c^2*
d^2 + e^2)*(d + e*x)) - ((2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2
*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(c^2*d^2 + e^2)^(3/2)))/(2*(c^2*d^2 + e^2
))))/(3*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(165) = 330$.

Time = 4.76 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.79

method	result
parts	$-\frac{a}{3(ex+d)^3 e}-\frac{b\,c^3 \operatorname{arcsinh}(xc)}{3(cex+cd)^3 e}-\frac{b\,c^3\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2(c^2d^2+e^2)\left(xc+\frac{dc}{e}\right)^2}-\frac{b\,c^4 d\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}}}{2e(c^2d^2+e^2)^2\left(xc+\frac{dc}{e}\right)}$
derivativedivides	$-\frac{a\,c^4}{3(cex+cd)^3 e}-\frac{b\,c^4 \operatorname{arcsinh}(xc)}{3(cex+cd)^3 e}-\frac{b\,c^4\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(xc+\frac{dc}{e}\right)^2}-\frac{b\,c^5 d\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(xc+\frac{dc}{e}\right)}-$
default	$-\frac{a\,c^4}{3(cex+cd)^3 e}-\frac{b\,c^4 \operatorname{arcsinh}(xc)}{3(cex+cd)^3 e}-\frac{b\,c^4\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(xc+\frac{dc}{e}\right)^2}-\frac{b\,c^5 d\sqrt{\left(xc+\frac{dc}{e}\right)^2-\frac{2dc\left(xc+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(xc+\frac{dc}{e}\right)}-$

input
```
int((a+b*arcsinh(x*c))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/(e*x+d)^3/e-1/3*b*c^3/(c*e*x+c*d)^3/e*arcsinh(x*c)-1/6*b*c^3/e^2/(c
^2*d^2+e^2)/(x*c+d*c/e)^2*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)
/e^2)^(1/2)-1/2*b*c^4/e*d/(c^2*d^2+e^2)^2/(x*c+d*c/e)*((x*c+d*c/e)^2-2*d*c
/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2)-1/2*b*c^5/e^2*d^2/(c^2*d^2+e^2)^2/
((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(x*c+d*c/e)+2*((
c^2*d^2+e^2)/e^2)^(1/2)*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e
^2)^(1/2))/(x*c+d*c/e))+1/6*b*c^3/e^2/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1
/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(x*c+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2
)*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(x*c+d*c/e)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(167) = 334$.

Time = 0.49 (sec) , antiderivative size = 977, normalized size of antiderivative = 5.34

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex)^4}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*((2*a + 3*b)*c^6*d^9 + 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 + 2*
a*d^3*e^6 + 3*(b*c^6*d^6*e^3 + b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 + b*c
^4*d^5*e^4)*x^2 + (2*b*c^5*d^8 - b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 - b*c^3*
d^3*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 - b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e
- b*c^3*d^5*e^3)*x)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e - sqrt(c^2
*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 - sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt
(c^2*x^2 + 1))/(e*x + d)) + 9*(b*c^6*d^8*e + b*c^4*d^6*e^3)*x - 2*((b*c^6*
d^6*e^3 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^
2 + 3*b*c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*
b*c^4*d^6*e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 + 1
)) - 2*(b*c^6*d^9 + 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 + b*d^3*e^6 + (b*c^6
*d^6*e^3 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e
^2 + 3*b*c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3
*b*c^4*d^6*e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 +
 1)) + (4*b*c^5*d^8*e + 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 +
 b*c^3*d^4*e^5)*x^2 + (7*b*c^5*d^7*e^2 + 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)
*sqrt(c^2*x^2 + 1))/(c^6*d^12*e + 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 + d^6*e^7
 + (c^6*d^9*e^4 + 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 + d^3*e^10)*x^3 + 3*(c^6*d
^10*e^3 + 3*c^4*d^8*e^5 + 3*c^2*d^6*e^7 + d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 +
 3*c^4*d^9*e^4 + 3*c^2*d^7*e^6 + d^5*e^8)*x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d + ex)^4}\, dx = \int \frac{a + b\operatorname{asinh}(cx)}{(d + ex)^4}\, dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d)**4,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx=\int \frac{b\,\operatorname{arsinh}(cx)+a}{(ex+d)^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 + 3*c*d^2*e^2*x^2 +
c*d^3*e*x + (3*c^3*d^2*e^2 + c*e^4)*x^4 + (c^3*d^3*e + 3*c*d*e^3)*x^3 + (c
^2*e^4*x^5 + 3*c^2*d*e^3*x^4 + 3*d^2*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)
*x^3 + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(c^6*d^3 - 3*
c^4*d*e^2)*log(e*x + d)/(c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7)
+ (3*c^6*d^6 + 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e^2 - c^2*e^6)*x^2
 + (5*c^6*d^5*e + 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x + (c^6*d^6 - 3*c^4*d^4*e^
2 + (c^6*d^3*e^3 - 3*c^4*d*e^5)*x^3 + 3*(c^6*d^4*e^2 - 3*c^4*d^2*e^4)*x^2
+ 3*(c^6*d^5*e - 3*c^4*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*(c^6*d^6 + 3*c^4*d
^4*e^2 + 3*c^2*d^2*e^4 + e^6)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^6*d^9*e + 3
*c^4*d^7*e^3 + 3*c^2*d^5*e^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*
c^2*d^2*e^8 + e^10)*x^3 + 3*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 +
 d*e^9)*x^2 + 3*(c^6*d^8*e^2 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x)
 - I*(3*c^6*d^2 - c^4*e^2)*(log(I*c*x + 1) - log(-I*c*x + 1))/((c^6*d^6 +
3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 + e^6)*c))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2
+ 3*d^2*e^2*x + d^3*e)
```

Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx=\int \frac{b\,\operatorname{arsinh}(cx)+a}{(ex+d)^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{(d+e\,x)^4}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x)^4,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx$$
$$= \frac{3\left(\int \frac{asinh(cx)}{e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4}dx\right)b\,d^3e+9\left(\int \frac{asinh(cx)}{e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4}dx\right)b\,d^2e^2x+9\left(\int \frac{}{e^4x^4+4d\,e^3x^3}\right.}{3e\,(e^3x^3+3d\,e^2x^2+3d^2ex+}$$

input
```
int((a+b*asinh(c*x))/(e*x+d)^4,x)
```

output
```
(3*int(asinh(c*x)/(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 +
e**4*x**4),x)*b*d**3*e + 9*int(asinh(c*x)/(d**4 + 4*d**3*e*x + 6*d**2*e**2
*x**2 + 4*d*e**3*x**3 + e**4*x**4),x)*b*d**2*e**2*x + 9*int(asinh(c*x)/(d*
*4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4),x)*b*d*e**
3*x**2 + 3*int(asinh(c*x)/(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3
*x**3 + e**4*x**4),x)*b*e**4*x**3 - a)/(3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*
x**2 + e**3*x**3))
```

3.12 $\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 368

$$\begin{aligned}\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx &= 2b^2d^3x-\frac{4b^2de^2x}{3c^2}+\frac{3}{4}b^2d^2ex^2-\frac{3b^2e^3x^2}{32c^2}+\frac{2}{9}b^2de^2x^3\\&+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}\\&+\frac{4bde^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^3}\\&-\frac{3bd^2ex\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c}\\&+\frac{3be^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c^3}\\&-\frac{2bde^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c}\\&-\frac{be^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c}\\&-\frac{d^4(a+b\text{arcsinh}(cx))^2}{4e}+\frac{3d^2e(a+b\text{arcsinh}(cx))^2}{4c^2}\\&-\frac{3e^3(a+b\text{arcsinh}(cx))^2}{32c^4}\\&+\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}\end{aligned}$$

output
```
2*b^2*d^3*x-4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2-3/32*b^2*e^3*x^2/c^2+2/9
*b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-2*b*d^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x
))/c+4/3*b*d*e^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c^3-3/2*b*d^2*e*x*(c
^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+3/16*b*e^3*x*(c^2*x^2+1)^(1/2)*(a+b*a
rcsinh(c*x))/c^3-2/3*b*d*e^2*x^2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-1/
8*b*e^3*x^3*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-1/4*d^4*(a+b*arcsinh(c*
x))^2/e+3/4*d^2*e*(a+b*arcsinh(c*x))^2/c^2-3/32*e^3*(a+b*arcsinh(c*x))^2/c
^4+1/4*(e*x+d)^4*(a+b*arcsinh(c*x))^2/e
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.96

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{c\left(72a^2c^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-6ab\sqrt{1+c^2x^2}(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x\right.}{}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[
1 + c^2*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*
x^2 + 6*e^3*x^3)) + b^2*c*x*(-3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d
^2*e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*(-3*a*(24*c^2*d^2*e - 3*e^3 + 8
*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + b*c*Sqrt[1 + c^2*x^2
]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3
*x^3)))*ArcSinh[c*x] + 9*b^2*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^
2*e*x + 4*d*e^2*x^2 + e^3*x^3))*ArcSinh[c*x]^2)/(288*c^4)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}-\frac{bc\int \frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{2e}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}-$$

$$\frac{bc\int \left(\frac{(a+b\text{arcsinh}(cx))d^4}{\sqrt{c^2x^2+1}}+\frac{4ex(a+b\text{arcsinh}(cx))d^3}{\sqrt{c^2x^2+1}}+\frac{6e^2x^2(a+b\text{arcsinh}(cx))d^2}{\sqrt{c^2x^2+1}}+\frac{4e^3x^3(a+b\text{arcsinh}(cx))d}{\sqrt{c^2x^2+1}}+\frac{e^4x^4(a+b\text{arcsin}}{\sqrt{c^2x^2+1}}\right.}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}-$$

$$bc\left(\frac{3e^4(a+b\text{arcsinh}(cx))^2}{16bc^5}-\frac{3d^2e^2(a+b\text{arcsinh}(cx))^2}{2bc^3}+\frac{4d^3e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{3d^2e^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{4}{}\right.$$

input
```
Int[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e) - (b*c*((-4*b*d^3*e*x)/c + (8*b
*d*e^3*x)/(3*c^3) - (3*b*d^2*e^2*x^2)/(2*c) + (3*b*e^4*x^2)/(16*c^3) - (4*
b*d*e^3*x^3)/(9*c) - (b*e^4*x^4)/(16*c) + (4*d^3*e*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/c^2 - (8*d*e^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3
*c^4) + (3*d^2*e^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (3*e^4*
x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c^4) + (4*d*e^3*x^2*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (e^4*x^3*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(4*c^2) + (d^4*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (3*d^2*e^
2*(a + b*ArcSinh[c*x])^2)/(2*b*c^3) + (3*e^4*(a + b*ArcSinh[c*x])^2)/(16*b
*c^5)))/(2*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{a^2(cex+cd)^4}{4c^3e}+\frac{b^2\left(d^3c^3\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{3d^2c^2e\left(2\operatorname{arcsinh}(xc)^2x^2c^2-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\right.}{4}}{}}{}$
default	$\frac{\frac{a^2(cex+cd)^4}{4c^3e}+\frac{b^2\left(d^3c^3\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{3d^2c^2e\left(2\operatorname{arcsinh}(xc)^2x^2c^2-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\right.}{4}}{}}{}$
orering	$\frac{(111e^5c^4x^6+699e^4c^4x^5d+1928e^3c^4x^4d^2+3480e^2c^4x^3d^3+672c^4d^4e\,x^2-63e^5c^2x^4+192c^4d^5x-1079e^4c^2x^3d+1632e^3c}{192c^4(ex+d)^2}$
parts	$\frac{a^2(ex+d)^4}{4e}+\frac{b^2\left(72\operatorname{arcsinh}(xc)^2x^4c^4e^3+288\operatorname{arcsinh}(xc)^2x^3c^4d\,e^2+432\operatorname{arcsinh}(xc)^2x^2c^4d^2e+288\operatorname{arcsinh}(xc)^2x\,c^4\right.}{}$

input
```
int((e*x+d)^3*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/4*a^2/c^3*(c*e*x+c*d)^4/e+b^2/c^3*(d^3*c^3*(arcsinh(x*c)^2*x*c-2*ar
csinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*c)+3/4*d^2*c^2*e*(2*arcsinh(x*c)^2*x^2*c^
2-2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c+arcsinh(x*c)^2+c^2*x^2+1)+1/9*d*c*e
^2*(9*arcsinh(x*c)^2*x^3*c^3-6*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*x^2*c^2+2*x^
3*c^3+12*arcsinh(x*c)*(c^2*x^2+1)^(1/2)-12*x*c)+1/32*e^3*(8*arcsinh(x*c)^2
*x^4*c^4-4*arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x^3*c^3+c^4*x^4+6*arcsinh(x*c)*(
c^2*x^2+1)^(1/2)*x*c-3*arcsinh(x*c)^2-3*c^2*x^2-3))+2*a*b/c^3*(1/4/e*arcsi
nh(x*c)*c^4*d^4+arcsinh(x*c)*c^4*d^3*x+3/2*e*arcsinh(x*c)*c^4*d^2*x^2+e^2*
arcsinh(x*c)*c^4*d*x^3+1/4*e^3*arcsinh(x*c)*x^4*c^4-1/4/e*(c^4*d^4*arcsinh
(x*c)+e^4*(1/4*(c^2*x^2+1)^(1/2)*c^3*x^3-3/8*(c^2*x^2+1)^(1/2)*x*c+3/8*arc
sinh(x*c))+4*d*c*e^3*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))
+6*d^2*c^2*e^2*(1/2*(c^2*x^2+1)^(1/2)*x*c-1/2*arcsinh(x*c))+4*d^3*c^3*e*(c
^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.29

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{9\,(8\,a^2+b^2)c^4e^3x^4+32\,(9\,a^2+2\,b^2)c^4de^2x^3+27\,(8\,(2\,a^2+b^2)c^4d^2e-b^2c^2e^3)x^2+9\,(8\,b^2c^4e^3x^4+32\,b^2c}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27
*(8*(2*a^2 + b^2)*c^4*d^2*e - b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32
*b^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x + 24*b^2*c^2*
d^2*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 96*(3*(a^2 + 2*b^2)*c^
4*d^3 - 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3
+ 144*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x + 72*a*b*c^2*d^2*e - 9*a*b*e^3
- (6*b^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 - 64*b^2*c*d*
e^2 + 9*(8*b^2*c^3*d^2*e - b^2*c*e^3)*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt
(c^2*x^2 + 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*
d^3 - 64*a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e - a*b*c*e^3)*x)*sqrt(c^2*x^2 + 1
))/c^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(364) = 728.

Time = 0.50 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.02

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e*
*3*x**4/4 + 2*a*b*d**3*x*asinh(c*x) + 3*a*b*d**2*e*x**2*asinh(c*x) + 2*a*b
*d*e**2*x**3*asinh(c*x) + a*b*e**3*x**4*asinh(c*x)/2 - 2*a*b*d**3*sqrt(c**
2*x**2 + 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 + 1)/(2*c) - 2*a*b*d*e**2*x*
*2*sqrt(c**2*x**2 + 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 + 1)/(8*c) + 3
*a*b*d**2*e*asinh(c*x)/(2*c**2) + 4*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3
) + 3*a*b*e**3*x*sqrt(c**2*x**2 + 1)/(16*c**3) - 3*a*b*e**3*asinh(c*x)/(16
*c**4) + b**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*as
inh(c*x)**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*asinh(c*x)**2 + 2*
b**2*d*e**2*x**3/9 + b**2*e**3*x**4*asinh(c*x)**2/4 + b**2*e**3*x**4/32 -
2*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x
**2 + 1)*asinh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(3*c) - b**2*e**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(8*c) + 3*b**2*d
**2*e*asinh(c*x)**2/(4*c**2) - 4*b**2*d*e**2*x/(3*c**2) - 3*b**2*e**3*x**2
/(32*c**2) + 4*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 3*b**
2*e**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c**3) - 3*b**2*e**3*asinh(c*x)
**2/(32*c**4), Ne(c, 0)), (a**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 +
e**3*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.60

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*e^3*x^4*arcsinh(c*x)^2 + b^2*d*e^2*x^3*arcsinh(c*x)^2 + 1/4*a^2*e^
3*x^4 + 3/2*b^2*d^2*e*x^2*arcsinh(c*x)^2 + a^2*d*e^2*x^3 + b^2*d^3*x*arcsi
nh(c*x)^2 + 3/2*a^2*d^2*e*x^2 + 3/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2*e + 3/4*(c^2*(x^2/c^2 - log(c*x +
sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^
3)*arcsinh(c*x))*b^2*d^2*e + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e^2 - 2/9*(3*c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)
*b^2*d*e^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*s
qrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*e^3 + 1/32*((x^4/c^2 -
 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2
 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(
c*x))*b^2*e^3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3
*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)^3\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x)^3, x)
```

Reduce [F]

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{144\left(\int asinh(cx)^2\,x^2dx\right)b^2c^4d\,e^2+144asinh(cx)\,ab\,c^4d^2e\,x^2+96asinh(cx)\,ab\,c^4d\,e^2x^3-72\sqrt{c^2x^2+1}\,ab}{}$$

input
```
int((e*x+d)^3*(a+b*asinh(c*x))^2,x)
```

output
```
(48*asinh(c*x)**2*b**2*c**4*d**3*x + 72*asinh(c*x)**2*b**2*c**4*d**2*e*x**
2 + 36*asinh(c*x)**2*b**2*c**2*d**2*e - 96*sqrt(c**2*x**2 + 1)*asinh(c*x)*
b**2*c**3*d**3 - 72*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2*c**3*d**2*e*x + 96
*asinh(c*x)*a*b*c**4*d**3*x + 144*asinh(c*x)*a*b*c**4*d**2*e*x**2 + 96*asi
nh(c*x)*a*b*c**4*d*e**2*x**3 + 24*asinh(c*x)*a*b*c**4*e**3*x**4 - 96*sqrt(
c**2*x**2 + 1)*a*b*c**3*d**3 - 72*sqrt(c**2*x**2 + 1)*a*b*c**3*d**2*e*x -
32*sqrt(c**2*x**2 + 1)*a*b*c**3*d*e**2*x**2 - 6*sqrt(c**2*x**2 + 1)*a*b*c*
*3*e**3*x**3 + 64*sqrt(c**2*x**2 + 1)*a*b*c*d*e**2 + 9*sqrt(c**2*x**2 + 1)
*a*b*c*e**3*x + 48*int(asinh(c*x)**2*x**3,x)*b**2*c**4*e**3 + 144*int(asin
h(c*x)**2*x**2,x)*b**2*c**4*d*e**2 + 72*log(sqrt(c**2*x**2 + 1) + c*x)*a*b
*c**2*d**2*e - 9*log(sqrt(c**2*x**2 + 1) + c*x)*a*b*e**3 + 48*a**2*c**4*d*
*3*x + 72*a**2*c**4*d**2*e*x**2 + 48*a**2*c**4*d*e**2*x**3 + 12*a**2*c**4*
e**3*x**4 + 96*b**2*c**4*d**3*x + 36*b**2*c**4*d**2*e*x**2)/(48*c**4)
```

3.13 $\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx = 2b^2d^2x-\frac{4b^2e^2x}{9c^2}+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3-\frac{2bd^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+\frac{4be^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c^3}-\frac{bdex\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}-\frac{2be^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{9c}-\frac{d^3(a+b\text{arcsinh}(cx))^2}{3e}+\frac{de(a+b\text{arcsinh}(cx))^2}{2c^2}+\frac{(d+ex)^3(a+b\text{arcsinh}(cx))^2}{3e}$$

output
```
2*b^2*d^2*x-4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-2*b*d^2*(c^
2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+4/9*b*e^2*(c^2*x^2+1)^(1/2)*(a+b*arcsi
nh(c*x))/c^3-b*d*e*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-2/9*b*e^2*x^2*
(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-1/3*d^3*(a+b*arcsinh(c*x))^2/e+1/2*
d*e*(a+b*arcsinh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*arcsinh(c*x))^2/e
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{18a^2c^3x(3d^2+3dex+e^2x^2)-6ab\sqrt{1+c^2x^2}(-4e^2+c^2(18d^2+9dex+2e^2x^2))+b^2cx(-24e^2+c^2(108d$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(18*a^2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-4*e^
2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + b^2*c*x*(-24*e^2 + c^2*(108*d^2
+ 27*d*e*x + 4*e^2*x^2)) - 6*b*(-3*a*(3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x +
 e^2*x^2)) + b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x
^2)))*ArcSinh[c*x] + 9*b^2*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*(e + 2*c^2
*e*x^2))*ArcSinh[c*x]^2)/(54*c^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \text{ 6243}$$
$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))^2}{3e}-\frac{2bc\int \frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3e}$$
$$\Big\downarrow \text{ 6253}$$

$$\frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e}-\frac{2bc\int\left(\frac{(a+b\operatorname{arcsinh}(cx))d^3}{\sqrt{c^2x^2+1}}+\frac{3ex(a+b\operatorname{arcsinh}(cx))d^2}{\sqrt{c^2x^2+1}}+\frac{3e^2x^2(a+b\operatorname{arcsinh}(cx))d}{\sqrt{c^2x^2+1}}+\frac{e^3x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{3e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e}-\frac{2bc\left(-\frac{3de^2(a+b\operatorname{arcsinh}(cx))^2}{4bc^3}+\frac{3d^2e\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c^2}+\frac{3de^2x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{2c^2}+\frac{e^3x^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}}{3c^2}\right.}{3e}$$

input
```
Int[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e) - (2*b*c*((-3*b*d^2*e*x)/c + (2
*b*e^3*x)/(3*c^3) - (3*b*d*e^2*x^2)/(4*c) - (b*e^3*x^3)/(9*c) + (3*d^2*e*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (2*e^3*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(3*c^4) + (3*d*e^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/(2*c^2) + (e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) +
(d^3*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (3*d*e^2*(a + b*ArcSinh[c*x])^2)/(4
*b*c^3)))/(3*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{\frac{a^2(cex+cd)^3}{3c^2e}+\frac{b^2\left(d^2c^2\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{dce\left(2\operatorname{arcsinh}(xc)^2x^2c^2-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\operatorname{arc}\right.}{2}\right.}{}}{}$
default	$\frac{\frac{a^2(cex+cd)^3}{3c^2e}+\frac{b^2\left(d^2c^2\left(\operatorname{arcsinh}(xc)^2xc-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{dce\left(2\operatorname{arcsinh}(xc)^2x^2c^2-2\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\operatorname{arc}\right.}{2}\right.}{}}{}$
parts	$\frac{a^2(ex+d)^3}{3e}+\frac{b^2\left(18\operatorname{arcsinh}(xc)^2x^3c^3e^2+54\operatorname{arcsinh}(xc)^2x^2c^3de+54\operatorname{arcsinh}(xc)^2x\,c^3d^2-12\operatorname{arcsinh}(xc)\sqrt{c^2x^2+1}\,x\right.}{}$
orering	$\frac{\left(38e^4c^4x^5+206c^4d\,e^3x^4+531c^4d^2e^2x^3+162c^4d^3e\,x^2+54c^4d^4x-48c^2e^4x^3+174c^2d\,e^3x^2+540c^2d^2e^2x+135c^2d^3e-96e\right.}{54c^4(ex+d)^2}$

input
```
int((e*x+d)^2*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*a^2/c^2*(c*e*x+c*d)^3/e+b^2/c^2*(d^2*c^2*(arcsinh(x*c)^2*x*c-2*ar
csinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*c)+1/2*d*c*e*(2*arcsinh(x*c)^2*x^2*c^2-2*
arcsinh(x*c)*(c^2*x^2+1)^(1/2)*x*c+arcsinh(x*c)^2+c^2*x^2+1)+1/27*e^2*(9*a
rcsinh(x*c)^2*x^3*c^3-6*(c^2*x^2+1)^(1/2)*arcsinh(x*c)*x^2*c^2+2*x^3*c^3+1
2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)-12*x*c))+2*a*b/c^2*(1/3/e*arcsinh(x*c)*c^
3*d^3+arcsinh(x*c)*c^3*d^2*x+e*arcsinh(x*c)*c^3*d*x^2+1/3*e^2*arcsinh(x*c)
*x^3*c^3-1/3/e*(c^3*d^3*arcsinh(x*c)+e^3*(1/3*x^2*c^2*(c^2*x^2+1)^(1/2)-2/
3*(c^2*x^2+1)^(1/2))+3*d*c*e^2*(1/2*(c^2*x^2+1)^(1/2)*x*c-1/2*arcsinh(x*c)
)+3*d^2*c^2*e*(c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.33

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{2\,(9\,a^2+2\,b^2)c^3e^2x^3+27\,(2\,a^2+b^2)c^3dex^2+9\,(2\,b^2c^3e^2x^3+6\,b^2c^3dex^2+6\,b^2c^3d^2x+3\,b^2cde)\log\left(cx+\right.}{}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*
b^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x + 3*b^2*c*d*e)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 - 4*b^2*c*e^2)*x +
6*(6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x + 9*a*b*c*d*e
 - (2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 - 4*b^2*e^2)*sqrt
(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b
*c^2*d*e*x + 18*a*b*c^2*d^2 - 4*a*b*e^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.90

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} a^2d^2x+a^2dex^2+\frac{a^2e^2x^3}{3}+2abd^2x\,\text{asinh}\,(cx)+2abdex^2\,\text{asinh}\,(cx)+\frac{2abe^2x^3\,\text{asinh}\,(cx)}{3}-\frac{2abd^2\sqrt{c^2x^2+1}}{c}-\frac{ab}{} \\ a^2\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*a
sinh(c*x) + 2*a*b*d*e*x**2*asinh(c*x) + 2*a*b*e**2*x**3*asinh(c*x)/3 - 2*a
*b*d**2*sqrt(c**2*x**2 + 1)/c - a*b*d*e*x*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*
*2*x**2*sqrt(c**2*x**2 + 1)/(9*c) + a*b*d*e*asinh(c*x)/c**2 + 4*a*b*e**2*s
qrt(c**2*x**2 + 1)/(9*c**3) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x +
b**2*d*e*x**2*asinh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*asinh(c*x)*
*2/3 + 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c
- b**2*d*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(9*c) + b**2*d*e*asinh(c*x)**2/(2*c**2) - 4*b**2*e**
2*x/(9*c**2) + 4*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3), Ne(c,
0)), (a**2*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.58

$$\begin{aligned}
&\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx \\
&= \frac{1}{3}\,b^2e^2x^3\,\text{arsinh}\,(cx)^2 + b^2dex^2\,\text{arsinh}\,(cx)^2 + \frac{1}{3}\,a^2e^2x^3 + b^2d^2x\,\text{arsinh}\,(cx)^2 \\
&\quad + a^2dex^2 + \left(2\,x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)abde \\
&\quad + \frac{1}{2}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\text{arsinh}\,(cx)\right)b^2de \\
&\quad + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abe^2 \\
&\quad - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2e^2 \\
&\quad + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2d^2x + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}
\end{aligned}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*e^2*x^3*arcsinh(c*x)^2 + b^2*d*e*x^2*arcsinh(c*x)^2 + 1/3*a^2*e^2*
x^3 + b^2*d^2*x*arcsinh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arcsinh(c*x) - c*(sq
rt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d*e + 1/2*(c^2*(x^2/c^2 - l
og(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsin
h(c*x)/c^3)*arcsinh(c*x))*b^2*d*e + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*
x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*e^2 - 2/27*(3*c*(sqrt(c^2
*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x
)/c^2)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^
2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+e\,x)^2\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x)^2, x)
```

Reduce [F]

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{18asinh(cx)^2\,b^2c^3d^2x+18asinh(cx)^2\,b^2c^3de\,x^2+9asinh(cx)^2\,b^2cde-36\sqrt{c^2x^2+1}\,asinh(cx)\,b^2c^2d^2-18}{}$$

input
```
int((e*x+d)^2*(a+b*asinh(c*x))^2,x)
```

output
```
(18*asinh(c*x)**2*b**2*c**3*d**2*x + 18*asinh(c*x)**2*b**2*c**3*d*e*x**2 +
 9*asinh(c*x)**2*b**2*c*d*e - 36*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2*c**2*
d**2 - 18*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2*c**2*d*e*x + 36*asinh(c*x)*a
*b*c**3*d**2*x + 36*asinh(c*x)*a*b*c**3*d*e*x**2 + 12*asinh(c*x)*a*b*c**3*
e**2*x**3 - 36*sqrt(c**2*x**2 + 1)*a*b*c**2*d**2 - 18*sqrt(c**2*x**2 + 1)*
a*b*c**2*d*e*x - 4*sqrt(c**2*x**2 + 1)*a*b*c**2*e**2*x**2 + 8*sqrt(c**2*x*
*2 + 1)*a*b*e**2 + 18*int(asinh(c*x)**2*x**2,x)*b**2*c**3*e**2 + 18*log(sq
rt(c**2*x**2 + 1) + c*x)*a*b*c*d*e + 18*a**2*c**3*d**2*x + 18*a**2*c**3*d*
e*x**2 + 6*a**2*c**3*e**2*x**3 + 36*b**2*c**3*d**2*x + 9*b**2*c**3*d*e*x**
2)/(18*c**3)
```

3.14 $\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 140

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = 2b^2dx+\frac{1}{4}b^2ex^2-\frac{2bd\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} - \frac{bex\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c} - \frac{d^2(a+b\text{arcsinh}(cx))^2}{2e}+\frac{e(a+b\text{arcsinh}(cx))^2}{4c^2} + \frac{(d+ex)^2(a+b\text{arcsinh}(cx))^2}{2e}$$

output
```
2*b^2*d*x+1/4*b^2*e*x^2-2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-1/2*b
*e*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c-1/2*d^2*(a+b*arcsinh(c*x))^2/e
+1/4*e*(a+b*arcsinh(c*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))^2/e
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{c\left(2a^2cx(2d+ex)+b^2cx(8d+ex)-2ab(4d+ex)\sqrt{1+c^2x^2}\right)+2b\left(-bc(4d+ex)\sqrt{1+c^2x^2}+a(e+4c^2\right.}{4c^2}$$

input
```
Integrate[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(c*(2*a^2*c*x*(2*d + e*x) + b^2*c*x*(8*d + e*x) - 2*a*b*(4*d + e*x)*Sqrt[1
 + c^2*x^2]) + 2*b*(-(b*c*(4*d + e*x)*Sqrt[1 + c^2*x^2]) + a*(e + 4*c^2*d*
x + 2*c^2*e*x^2))*ArcSinh[c*x] + b^2*(e + 4*c^2*d*x + 2*c^2*e*x^2)*ArcSinh
[c*x]^2)/(4*c^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \ 6243$$

$$\frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e}-\frac{bc\int \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{e}$$

$$\Big\downarrow \ 6253$$

$$\frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e}-\frac{bc\int \left(\frac{(a+b\operatorname{arcsinh}(cx))d^2}{\sqrt{c^2x^2+1}}+\frac{2ex(a+b\operatorname{arcsinh}(cx))d}{\sqrt{c^2x^2+1}}+\frac{e^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))^2}{2e} - \frac{bc\left(-\frac{e^2(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{2de\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{e^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}+\frac{d^2(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{2bdex}{c}-\right.}{e}$$

input
```
Int[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + e*x)^2*(a + b*ArcSinh[c*x])^2)/(2*e) - (b*c*((-2*b*d*e*x)/c - (b*e^2
*x^2)/(4*c) + (2*d*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 + (e^2*x*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) + (d^2*(a + b*ArcSinh[c*x]
)^2)/(2*b*c) - (e^2*(a + b*ArcSinh[c*x])^2)/(4*b*c^3)))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

method	result
parts	$a^2\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b^2\left(\frac{e\left(2\,\text{arcsinh}(xc)^2x^2c^2-2\,\text{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\text{arcsinh}(xc)^2+c^2x^2+1\right)}{4c}+d\left(\text{arcsinh}(xc)^2xc-\right.}{c}$
derivativedivides	$\frac{\frac{a^2\left(c^2dx+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(dc\left(\text{arcsinh}(xc)^2xc-2\,\text{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{e\left(2\,\text{arcsinh}(xc)^2x^2c^2-2\,\text{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\text{a}\right.}{4}}{c}}{c}$
default	$\frac{\frac{a^2\left(c^2dx+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(dc\left(\text{arcsinh}(xc)^2xc-2\,\text{arcsinh}(xc)\sqrt{c^2x^2+1}+2xc\right)+\frac{e\left(2\,\text{arcsinh}(xc)^2x^2c^2-2\,\text{arcsinh}(xc)\sqrt{c^2x^2+1}\,xc+\text{a}\right.}{4}}{c}}{c}$
orering	$\frac{(7c^2e^3x^4+33c^2d\,e^2x^3+20c^2d^2e\,x^2+8c^2d^3x+6e^3x^2+30d\,e^2x+10e\,d^2)(a+b\,\text{arcsinh}(xc))^2}{8c^2(ex+d)^2}-\frac{(3c^2e^2x^4+17c^2de\,x^3+4}{}$

input
```
int((e*x+d)*(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*e*x^2+d*x)+b^2/c*(1/4*e*(2*arcsinh(x*c)^2*x^2*c^2-2*arcsinh(x*c)*
(c^2*x^2+1)^(1/2)*x*c+arcsinh(x*c)^2+c^2*x^2+1)/c+d*(arcsinh(x*c)^2*x*c-2*
arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*c))+2*a*b/c*(1/2*c*arcsinh(x*c)*e*x^2+a
rcsinh(x*c)*x*c*d-1/2/c*(e*(1/2*(c^2*x^2+1)^(1/2)*x*c-1/2*arcsinh(x*c))+2*
d*c*(c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{(2\,a^2+b^2)c^2ex^2+4\,(a^2+2\,b^2)c^2dx+(2\,b^2c^2ex^2+4\,b^2c^2dx+b^2e)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(2\,abc^2ex^2\right.}{4\,c^2}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2
+ 4*b^2*c^2*d*x + b^2*e)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(2*a*b*c^2*e*x
^2 + 4*a*b*c^2*d*x + a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 + 1))/
c^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.66

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} a^2dx + \frac{a^2ex^2}{2} + 2abdx\,\text{asinh}\,(cx) + abex^2\,\text{asinh}\,(cx) - \frac{2abd\sqrt{c^2x^2+1}}{c} - \frac{abex\sqrt{c^2x^2+1}}{2c} + \frac{abe\,\text{asinh}\,(cx)}{2c^2} + b^2dx\,\text{as} \\ a^2\left(dx + \frac{ex^2}{2}\right) \end{cases}$$

input
```
integrate((e*x+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*asinh(c*x) + a*b*e*x**2*as
inh(c*x) - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - a*b*e*x*sqrt(c**2*x**2 + 1)/(2*
c) + a*b*e*asinh(c*x)/(2*c**2) + b**2*d*x*asinh(c*x)**2 + 2*b**2*d*x + b**
2*e*x**2*asinh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 + 1)*as
inh(c*x)/c - b**2*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c) + b**2*e*asinh(
c*x)**2/(4*c**2), Ne(c, 0)), (a**2*(d*x + e*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{2}\,b^2ex^2\,\text{arsinh}\,(cx)^2 + b^2dx\,\text{arsinh}\,(cx)^2 + \frac{1}{2}\,a^2ex^2 + \frac{1}{2}\left(2\,x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)abe + \frac{1}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\text{arsinh}\,(cx)\right)b^2e + 2\,b^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2dx + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd}{c}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*e*x^2*arcsinh(c*x)^2 + b^2*d*x*arcsinh(c*x)^2 + 1/2*a^2*e*x^2 + 1/
2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*
b*e + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(
c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*e + 2*b^2*d*(x -
sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c
^2*x^2 + 1))*a*b*d/c
```

Giac [F(-2)]

Exception generated.

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.66

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{4asinh(cx)^2\,b^2c^2dx + 2asinh(cx)^2\,b^2c^2e\,x^2 + asinh(cx)^2\,b^2e - 8\sqrt{c^2x^2+1}\,asinh(cx)\,b^2cd - 2\sqrt{c^2x^2+1}\,c}{}$$

input
```
int((e*x+d)*(a+b*asinh(c*x))^2,x)
```

output
```
(4*asinh(c*x)**2*b**2*c**2*d*x + 2*asinh(c*x)**2*b**2*c**2*e*x**2 + asinh(
c*x)**2*b**2*e - 8*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2*c*d - 2*sqrt(c**2*x
**2 + 1)*asinh(c*x)*b**2*c*e*x + 8*asinh(c*x)*a*b*c**2*d*x + 4*asinh(c*x)*
a*b*c**2*e*x**2 + 4*asinh(c*x)*a*b*e - 8*sqrt(c**2*x**2 + 1)*a*b*c*d - 2*s
qrt(c**2*x**2 + 1)*a*b*c*e*x - 2*log(sqrt(c**2*x**2 + 1) + c*x)*a*b*e + 4*
a**2*c**2*d*x + 2*a**2*c**2*e*x**2 + 8*b**2*c**2*d*x + b**2*c**2*e*x**2 +
b**2*e)/(4*c**2)
```

3.15 $\int (a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a+b\text{arcsinh}(cx))^2\,dx=2b^2x-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+x(a+b\text{arcsinh}(cx))^2$$

output
```
2*b^2*x-2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/c+x*(a+b*arcsinh(c*x))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int (a+b\text{arcsinh}(cx))^2\,dx=(a^2+2b^2)\,x-\frac{2ab\sqrt{1+c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)}{c}+b^2x\text{arcsinh}(cx)^2$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2,x]
```

output
```
(a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c
^2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6187}$$

$$x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6213}$$

$$x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)$$

$$\Big\downarrow \text{24}$$

$$x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^2,x]
```

output
```
x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{a^{2} c x+b^{2} \left(\operatorname{arcsinh}\left(x c \right)^{2} x c -2 \,\operatorname{arcsinh}\left(x c \right) \sqrt{c^{2} x^{2}+1}+2 x c \right)+2 a b \left(x c \,\operatorname{arcsinh}\left(x c \right)-\sqrt{c^{2} x^{2}+1}\right)}{c}$	72
default	$\frac{a^{2} c x+b^{2} \left(\operatorname{arcsinh}\left(x c \right)^{2} x c -2 \,\operatorname{arcsinh}\left(x c \right) \sqrt{c^{2} x^{2}+1}+2 x c \right)+2 a b \left(x c \,\operatorname{arcsinh}\left(x c \right)-\sqrt{c^{2} x^{2}+1}\right)}{c}$	72
parts	$a^{2} x +\frac{b^{2} \left(\operatorname{arcsinh}\left(x c \right)^{2} x c -2 \,\operatorname{arcsinh}\left(x c \right) \sqrt{c^{2} x^{2}+1}+2 x c \right)}{c}+\frac{2 a b \left(x c \,\operatorname{arcsinh}\left(x c \right)-\sqrt{c^{2} x^{2}+1}\right)}{c}$	73
orering	$x \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right)^{2}-\frac{2 \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right) b}{c \sqrt{c^{2} x^{2}+1}}+\frac{x \left(c^{2} x^{2}+1\right) \left(\frac{2 c^{2} b^{2}}{c^{2} x^{2}+1}-\frac{2 \left(a +b \,\operatorname{arcsinh}\left(x c \right)\right) b \,c^{3} x}{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}\right)}{c^{2}}$	99

input
```
int((a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arcsinh(x*c)^2*x*c-2*arcsinh(x*c)*(c^2*x^2+1)^(1/2)+2*x*
c)+2*a*b*(x*c*arcsinh(x*c)-(c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int (a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2+1}ab+2\,\left(abcx-\sqrt{c^2x^2+1}b^2\right)\log\left(cx+\sqrt{c^2x^2}\right.}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x
^2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 +
 1)))/c
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int (a+b\text{arcsinh}(cx))^2\,dx = \begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**
2*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne
(c, 0)), (a**2*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int (a+b\text{arcsinh}(cx))^2\,dx = b^2x\operatorname{arsinh}(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)ab}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*
x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int (a + b\text{arcsinh}(cx))^2\,dx$$
$$= 2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)ab$$
$$+ \left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\,c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2$$
$$+ a^2x$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))/c^2))*b^2 + a^2*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(cx))^2\,dx = \int (a + b\,\text{asinh}(c\,x))^2\,dx$$

input
```
int((a + b*asinh(c*x))^2,x)
```

output
```
int((a + b*asinh(c*x))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int (a + b\text{arcsinh}(cx))^2\, dx$$
$$= \frac{asinh(cx)^2 b^2cx - 2\sqrt{c^2x^2+1}\, asinh(cx)\, b^2 + 2asinh(cx)\, abcx - 2\sqrt{c^2x^2+1}\, ab + a^2cx + 2b^2cx}{c}$$

input
```
int((a+b*asinh(c*x))^2,x)
```

output
```
(asinh(c*x)**2*b**2*c*x - 2*sqrt(c**2*x**2 + 1)*asinh(c*x)*b**2 + 2*asinh(
c*x)*a*b*c*x - 2*sqrt(c**2*x**2 + 1)*a*b + a**2*c*x + 2*b**2*c*x)/c
```

3.16 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = &-\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be}\\ &+\frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

output
```
-1/3*(a+b*arcsinh(c*x))^3/b/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x^2+1)
^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*x))*polylog
(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsin
h(c*x))*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-
2*b^2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*
b^2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = \frac{-\frac{(a+b\operatorname{arcsinh}(cx))^3}{b}+3(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)+3(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)+\cdots}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

output
```
(-((a + b*ArcSinh[c*x])^3/b) + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcSi
nh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (
e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*ArcSinh[c*x])*
PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b
*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])
)] - 6*b^2*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] -
 6*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(3*e
)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx$$

$\Big\downarrow$ 6242

$$\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{cd+cex}d\text{arcsinh}(cx)$$

$\Big\downarrow$ 6095

$$\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)+\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$\Big\downarrow$ 2620

$$-\frac{2b\int (a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{2b\int (a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$\Big\downarrow$ 3011

$$-\frac{2b\left(b\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2b\left(b\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$\Big\downarrow$ 2720

$$-\frac{2b\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2b\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$$\Big\downarrow \; 7143$$

$$-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])^3/(b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^A
rcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log
[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e - (2*b*(-((a + b*A
rcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]
) + b*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]))/e -
(2*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c
^2*d^2 + e^2]))]) + b*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))]))/e
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(xc))^2}{ex + d} dx$$

input
```
int((a+b*arcsinh(x*c))^2/(e*x+d),x)
```

output
```
int((a+b*arcsinh(x*c))^2/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex} dx = \int \frac{(b \operatorname{arsinh}(cx) + a)^2}{ex + d} dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex} dx = \int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex} dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x+d),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{ex + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x + d
) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{ex + d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{d + ex}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))^2}{d + e\,x}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = \frac{2\left(\int \frac{asinh(cx)}{ex+d}dx\right)abe + \left(\int \frac{asinh(cx)^2}{ex+d}dx\right)b^2e + \log(ex+d)\,a^2}{e}$$

input
```
int((a+b*asinh(c*x))^2/(e*x+d),x)
```

output
```
(2*int(asinh(c*x)/(d + e*x),x)*a*b*e + int(asinh(c*x)**2/(d + e*x),x)*b**2
*e + log(d + e*x)*a**2)/e
```

3.17 $\int \frac{(a+b\,\mathbf{arcsinh}(cx))^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2}\,dx =& -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} \\ &+ \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e e^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} \\ &- \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e e^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} \\ &+ \frac{2b^2c\operatorname{PolyLog}\left(2,-\frac{e e^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} \\ &- \frac{2b^2c\operatorname{PolyLog}\left(2,-\frac{e e^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*
x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b*c*(a+b*
arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/
(c^2*d^2+e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2
*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2
+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$$
$$=\frac{-\frac{(a+b\text{arcsinh}(cx))^2}{d+ex}+\frac{2bc\left((a+b\text{arcsinh}(cx))\left(\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)+b\,\text{PolyLog}\left(2,\frac{ee^{\text{arcsinh}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-b}{\sqrt{c^2d^2+e^2}}}{e}$$

input

```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

output

```
(-((a + b*ArcSinh[c*x])^2/(d + e*x)) + (2*b*c*((a + b*ArcSinh[c*x])*(Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[
c*x])/(c*d + Sqrt[c^2*d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c
*d) + Sqrt[c^2*d^2 + e^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2]))]))/Sqrt[c^2*d^2 + e^2])/e
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6243, 6258, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$$
$$\Big\downarrow \text{ 6243}$$
$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)\sqrt{c^2x^2+1}}dx}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$
$$\Big\downarrow \text{ 6258}$$
$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{cd+cex}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{e}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{4bc\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4bc\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{4bc\left(\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}-\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4bc\left(\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}-\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{4bc\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+\cdots\right.}{e}\right.}{2\sqrt{\cdots}}\cdots\right.}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{4bc\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee}{cd}\right.}{e}\right.}{}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\big\downarrow$ 2838

$$-\frac{4bc\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{b\,\text{Poly}}{}\right.}{2\sqrt{c^2d^2+e^2}}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) - (4*b*c*(-1/2*(e*(((a + b*ArcSinh
[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + (b*Pol
yLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e))/Sqrt[c^2*d
^2 + e^2] + (e*(((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sq
rt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^
2*d^2 + e^2]))])/e))/(2*Sqrt[c^2*d^2 + e^2])))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.00

method	result
derivativedivides	$-\frac{a^2c^2}{(cex+cd)e}+b^2c^2\left(-\frac{\operatorname{arcsinh}(xc)^2}{e(cex+cd)}+\frac{2\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc+\sqrt{c^2\cdots}\right.}{cd+\cdots}\right.}{e\sqrt{c^2d^2+e\cdots}}\right.$
default	$-\frac{a^2c^2}{(cex+cd)e}+b^2c^2\left(-\frac{\operatorname{arcsinh}(xc)^2}{e(cex+cd)}+\frac{2\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(xc+\sqrt{c^2\cdots}\right.}{cd+\cdots}\right.}{e\sqrt{c^2d^2+e\cdots}}\right.$
parts	$-\frac{a^2}{(ex+d)e}+\frac{b^2\left(-\frac{c^2\operatorname{arcsinh}(xc)^2}{e(cex+cd)}+\frac{2c^2\operatorname{arcsinh}(xc)\ln\left(\frac{-cd-e\left(xc+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2c^2\operatorname{arcsinh}(xc)\ln\left(\frac{cd+e\left(x\cdots\right.}{\cdots}\right.}{e\sqrt{c^2\cdots}}\right.}{\cdots}$

input
```
int((a+b*arcsinh(x*c))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a^2*c^2/(c*e*x+c*d)/e+b^2*c^2*(-arcsinh(x*c)^2/e/(c*e*x+c*d)+2/e*arc
sinh(x*c)/(c^2*d^2+e^2)^(1/2)*ln((-c*d-e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+
e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2/e*arcsinh(x*c)/(c^2*d^2+e^2)^(1/
2)*ln((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^
2)^(1/2)))+2/e/(c^2*d^2+e^2)^(1/2)*dilog((-c*d-e*(x*c+(c^2*x^2+1)^(1/2))+(
c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2/e/(c^2*d^2+e^2)^(1/2)*di
log((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)
^(1/2))))+2*a*b*c^2*(-1/(c*e*x+c*d)/e*arcsinh(x*c)-1/e^2/((c^2*d^2+e^2)/e^
2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(x*c+d*c/e)+2*((c^2*d^2+e^2)/e^2)
^(1/2)*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(x*c+d
*c/e))))
```

Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + ex)^2}\, dx = \int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(ex + d)^2}\, dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^2*x^2 + 2*d*e*
x + d^2), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(cx))^2}{(d + ex)^2}\, dx = \int \frac{(a + b\,\text{asinh}\,(cx))^2}{(d + ex)^2}\, dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-b^2*(log(c*x + sqrt(c^2*x^2 + 1))^2/(e^2*x + d*e) - integrate(2*(c^3*x^2
+ sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^2*x^4 +
 c^3*d*e*x^3 + c*e^2*x^2 + c*d*e*x + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x +
d*e)*sqrt(c^2*x^2 + 1)), x)) - 2*a*b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsi
nh(c*d*sqrt(e^4)*x/(e*abs(e^2*x + d*e)) - sqrt(e^4)/(c*abs(e^2*x + d*e)))/
(sqrt(c^2*d^2/e^2 + 1)*e^2)) - a^2/(e^2*x + d*e)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+e\,x)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(d + ex)^2} \, dx$$
$$= \frac{2\left(\int \frac{asinh(cx)}{e^2x^2+2dex+d^2}dx\right)ab\,d^2 + 2\left(\int \frac{asinh(cx)}{e^2x^2+2dex+d^2}dx\right)abdex + \left(\int \frac{asinh(cx)^2}{e^2x^2+2dex+d^2}dx\right)b^2d^2 + \left(\int \frac{asinh(cx)^2}{e^2x^2+2dex+d^2}dx\right)}{d\,(ex + d)}$$

input
```
int((a+b*asinh(c*x))^2/(e*x+d)^2,x)
```

output
```
(2*int(asinh(c*x)/(d**2 + 2*d*e*x + e**2*x**2),x)*a*b*d**2 + 2*int(asinh(c
*x)/(d**2 + 2*d*e*x + e**2*x**2),x)*a*b*d*e*x + int(asinh(c*x)**2/(d**2 +
2*d*e*x + e**2*x**2),x)*b**2*d**2 + int(asinh(c*x)**2/(d**2 + 2*d*e*x + e*
*2*x**2),x)*b**2*d*e*x + a**2*x)/(d*(d + e*x))
```

3.18 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 349

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = & -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2} \\ & + \frac{bc^3d(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} \\ & - \frac{bc^3d(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} \\ & + \frac{b^2c^2\log(d+ex)}{e(c^2d^2+e^2)} + \frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} \\ & - \frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}}\end{aligned}$$

output

```
-b*c*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))/(c^2*d^2+e^2)/(e*x+d)-1/2*(a+b*a
rcsinh(c*x))^2/e/(e*x+d)^2+b*c^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2
+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c^3*d*(a+b*a
rcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(
c^2*d^2+e^2)^(3/2)+b^2*c^2*ln(e*x+d)/e/(c^2*d^2+e^2)+b^2*c^3*d*polylog(2,-
e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)
-b^2*c^3*d*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))
/e/(c^2*d^2+e^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \frac{-\frac{2bce\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2} + \frac{2b^2c^2\log(d+ex)}{c^2d^2+e^2} + \frac{2bc^3d\left((a+b\text{arcsinh}(cx))\left(\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\cdots\right)\right)}{\cdots}}{2e}$$

input

```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

output

```
((-2*b*c*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e
*x)) - (a + b*ArcSinh[c*x])^2/(d + e*x)^2 + (2*b^2*c^2*Log[d + e*x])/(c^2*
d^2 + e^2) + (2*b*c^3*d*((a + b*ArcSinh[c*x])*(Log[1 + (e*E^ArcSinh[c*x])/
(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*
d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e
^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]))/
(c^2*d^2 + e^2)^(3/2))/(2*e)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6243, 6258, 3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{6258}$$

$$\frac{bc^2\int \frac{a+b\text{arcsinh}(cx)}{(cd+cex)^2}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\int \frac{a+b\text{arcsinh}(cx)}{(cd-ie\sin(i\text{arcsinh}(cx)))^2}d\text{arcsinh}(cx)}{e}$$

$$\Big\downarrow \text{3805}$$

$$\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd+cex}d\text{arcsinh}(cx)}{c^2d^2+e^2}+\frac{be\int \frac{\sqrt{c^2x^2+1}}{cd+cex}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}+\frac{be\int \frac{\cos(i\text{arcsinh}(cx))}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}\right)}{e}$$

$$\Big\downarrow \text{3147}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}+\frac{b\int \frac{1}{cd+cex}d(cex)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}\right)}{e}$$

$$\Big\downarrow \; 16$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}+\frac{b\log(cd+cex)}{c^2d^2+e^2}\right)}{e}$$

$$\Big\downarrow \; 3803$$

$$\frac{bc^2\left(\frac{2cd\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}+\frac{b\log(cd+cex)}{c^2d^2+e^2}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{bc^2\left(-\frac{2cd\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}+\frac{b\log(cd+cex)}{c^2d^2+e^2}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2694$$

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}-\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}\right)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+ba\cdots}{(c^2d^2+e^2)(\cdots}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}-\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}\right)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsin}\ldots}{(c^2d^2+e^2)(cd+ce\ldots}\ldots\right.}{e}\ldots$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2620$$

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\ldots\right.}{e}\ldots\right.}{2\ldots}\ldots\right)}{c^2d^2+e^2}\ldots\right.}{e}\ldots$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\ldots\right.}{e}\ldots\right.}{\ldots}\ldots\right)}{c^2d^2+e^2}\ldots\right.}{e}\ldots$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{b\,\text{Po}}{}\right.}{2\sqrt{c^2d^2+e^2}}\right.}{c^2d^2+e^2}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(e*(d + e*x)^2) + (b*c^2*(-((e*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(c*d + c*e*x))) + (b*Log[c*d + c
*e*x])/(c^2*d^2 + e^2) - (2*c*d*(-1/2*(e*(((a + b*ArcSinh[c*x])*Log[1 + (e
*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^Ar
cSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e))/Sqrt[c^2*d^2 + e^2] + (e*((
(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]
)])/e + (b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/
e))/(2*Sqrt[c^2*d^2 + e^2])))/(c^2*d^2 + e^2)))/e
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(365) = 730$.

Time = 6.43 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.34

method	result
derivativedivides	$\frac{-\frac{a^2c^3}{2(cex+cd)^2e}+b^2c^3\left(-\frac{\operatorname{arcsinh}(xc)\left(e^2\operatorname{arcsinh}(xc)+2dce\sqrt{c^2x^2+1}-4c^2dex+c^2d^2\operatorname{arcsinh}(xc)-2c^2d^2+2\sqrt{c^2x^2+1}\,e^2xc-2c^2e^2x\right.}{2e(cex+cd)^2\left(c^2d^2+e^2\right)}\right.}{}$
default	$\frac{-\frac{a^2c^3}{2(cex+cd)^2e}+b^2c^3\left(-\frac{\operatorname{arcsinh}(xc)\left(e^2\operatorname{arcsinh}(xc)+2dce\sqrt{c^2x^2+1}-4c^2dex+c^2d^2\operatorname{arcsinh}(xc)-2c^2d^2+2\sqrt{c^2x^2+1}\,e^2xc-2c^2e^2x\right.}{2e(cex+cd)^2\left(c^2d^2+e^2\right)}\right.}{}$
parts	$-\frac{a^2}{2(ex+d)^2e}+\frac{b^2\left(-\frac{c^3\operatorname{arcsinh}(xc)\left(e^2\operatorname{arcsinh}(xc)+2dce\sqrt{c^2x^2+1}-4c^2dex+c^2d^2\operatorname{arcsinh}(xc)-2c^2d^2+2\sqrt{c^2x^2+1}\,e^2xc-2c^2\right.}{2e\left(c^2d^2+e^2\right)(cex+cd)^2}\right.}{}$

input
```
int((a+b*arcsinh(x*c))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*a^2*c^3/(c*e*x+c*d)^2/e+b^2*c^3*(-1/2*arcsinh(x*c)*(e^2*arcsinh(
x*c)+2*d*c*e*(c^2*x^2+1)^(1/2)-4*c^2*d*e*x+c^2*d^2*arcsinh(x*c)-2*c^2*d^2+
2*(c^2*x^2+1)^(1/2)*e^2*x*c-2*c^2*e^2*x^2)/e/(c*e*x+c*d)^2/(c^2*d^2+e^2)+1
/e/(c^2*d^2+e^2)*ln(2*d*c*(x*c+(c^2*x^2+1)^(1/2))+e*(x*c+(c^2*x^2+1)^(1/2)
)^2-e)-2/e/(c^2*d^2+e^2)*ln(x*c+(c^2*x^2+1)^(1/2))+1/e/(c^2*d^2+e^2)^(3/2)
*d*c*arcsinh(x*c)*ln((-c*d-e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/
(-c*d+(c^2*d^2+e^2)^(1/2)))-1/e/(c^2*d^2+e^2)^(3/2)*d*c*arcsinh(x*c)*ln((c
*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)
))+1/e/(c^2*d^2+e^2)^(3/2)*d*c*dilog((-c*d-e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*
d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-1/e/(c^2*d^2+e^2)^(3/2)*d*c*di
log((c*d+e*(x*c+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)
^(1/2))))-a*b*c^3/(c*e*x+c*d)^2/e*arcsinh(x*c)-a*b*c^3/e/(c^2*d^2+e^2)/(x*
c+d*c/e)*((x*c+d*c/e)^2-2*d*c/e*(x*c+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2)-a*b*c
^4/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2
*d*c/e*(x*c+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((x*c+d*c/e)^2-2*d*c/e*(x*c
+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(x*c+d*c/e)))
```

Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(ex+d\right)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^3*x^3 + 3*d*e^
2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{\left(a+b\,\text{asinh}\left(cx\right)\right)^2}{\left(d+ex\right)^3}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x+d)**3,x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(ex+d\right)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d*arc
sinh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(3/2)
*e^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*a*b - 1/2*b^2*(log(c*
x + sqrt(c^2*x^2 + 1))^2/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 2*integrate((c^3*
x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^3*x
^5 + 2*c^3*d*e^2*x^4 + 2*c*d*e^2*x^2 + c*d^2*e*x + (c^3*d^2*e + c*e^3)*x^3
 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*
x^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx$$
$$= \frac{4\left(\int \frac{asinh(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)ab\,d^2e+8\left(\int \frac{asinh(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)abd\,e^2x+4\left(\int \frac{asinh(cx)}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)}{}$$

input
```
int((a+b*asinh(c*x))^2/(e*x+d)^3,x)
```

output
```
(4*int(asinh(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)*a*b*d
**2*e + 8*int(asinh(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x
)*a*b*d*e**2*x + 4*int(asinh(c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**
3*x**3),x)*a*b*e**3*x**2 + 2*int(asinh(c*x)**2/(d**3 + 3*d**2*e*x + 3*d*e*
*2*x**2 + e**3*x**3),x)*b**2*d**2*e + 4*int(asinh(c*x)**2/(d**3 + 3*d**2*e
*x + 3*d*e**2*x**2 + e**3*x**3),x)*b**2*d*e**2*x + 2*int(asinh(c*x)**2/(d*
*3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)*b**2*e**3*x**2 - a**2)/(2*
e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.19 $\int \frac{(d+ex)^3}{a+b\,\mathbf{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 394

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx = \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} - \frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3} + \frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3} - \frac{3d^2e\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} + \frac{e^3\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{4bc^4} - \frac{e^3\text{Chi}\left(\frac{4a}{b}+4\text{arcsinh}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{8bc^4} - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} + \frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3} + \frac{3d^2e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2bc^2} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{4bc^4} - \frac{3de^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\text{arcsinh}(cx)\right)}{8bc^4}$$

output
```
d^3*cosh(a/b)*Chi(a/b+arcsinh(c*x))/b/c-3/4*d*e^2*cosh(a/b)*Chi(a/b+arcsin
h(c*x))/b/c^3+3/4*d*e^2*cosh(3*a/b)*Chi(3*a/b+3*arcsinh(c*x))/b/c^3-3/2*d^
2*e*Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^2+1/4*e^3*Chi(2*a/b+2*arcsin
h(c*x))*sinh(2*a/b)/b/c^4-1/8*e^3*Chi(4*a/b+4*arcsinh(c*x))*sinh(4*a/b)/b/
c^4-d^3*sinh(a/b)*Shi(a/b+arcsinh(c*x))/b/c+3/4*d*e^2*sinh(a/b)*Shi(a/b+ar
csinh(c*x))/b/c^3+3/2*d^2*e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-1/
4*e^3*cosh(2*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^4-3/4*d*e^2*sinh(3*a/b)*Sh
i(3*a/b+3*arcsinh(c*x))/b/c^3+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arcsinh(c*x)
)/b/c^4
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{d^3\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{bc}$$
$$+\frac{3de^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsi}\right.\right.}{4bc^3}$$
$$+\frac{e^3\left(2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\right.\right.\right.}{8bc^4}$$
$$-\frac{3d^2e\left(\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\right)}{2bc^2}$$

input
```
Integrate[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]]))/(b*c) + (3*d*e^2*(-(Cosh[a/b]*CoshIntegral[a/b + Arc
Sinh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/
b]*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
ArcSinh[c*x])]))/(4*b*c^3) + (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c*x])]*
Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIntegr
al[4*(a/b + ArcSinh[c*x])]))/(8*b*c^4) - (3*d^2*e*(CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*Arc
Sinh[c*x]]))/(2*b*c^2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx$$

$\downarrow$ 6245

$$\frac{\int \frac{(cd+cex)^3\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}d\text{arcsinh}(cx)}{c^4}$$

$\downarrow$ 7293

$$\frac{\int \left(\frac{d^3\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}+\frac{e^3x^3\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}+\frac{3de^2x^2\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}+\frac{3d^2ex\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)}{c^4}$$

$\downarrow$ 2009

$$\frac{c^3d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{c^3d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{3c^2d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2b}+\frac{3c^2d^2e\cosh}{}$$

input
```
Int[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
((c^3*d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/b - (3*c*d*e^2*Cosh[
a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b) + (3*c*d*e^2*Cosh[(3*a)/b]*Co
shIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b) - (3*c^2*d^2*e*CoshIntegral[(2
*a)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b) + (e^3*CoshIntegral[(2*a)/b +
 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b) - (e^3*CoshIntegral[(4*a)/b + 4*ArcS
inh[c*x]]*Sinh[(4*a)/b])/(8*b) - (c^3*d^3*Sinh[a/b]*SinhIntegral[a/b + Arc
Sinh[c*x]])/b + (3*c*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*
b) + (3*c^2*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2
*b) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b) - (
3*c*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b) + (e
^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b))/c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{e^3 e^{\frac{4a}{b}} \operatorname{expIntegral}_1\left(4 \operatorname{arcsinh}(xc)+\frac{4a}{b}\right)}{16c^3 b}-\frac{e^3 e^{-\frac{4a}{b}} \operatorname{expIntegral}_1\left(-4 \operatorname{arcsinh}(xc)-\frac{4a}{b}\right)}{16c^3 b}+\frac{3e\, e^{\frac{2a}{b}} \operatorname{expIntegral}_1\left(2 \operatorname{arcsinh}(xc)+\frac{2a}{b}\right)\cdots}{4cb}$
default	$\frac{e^3 e^{\frac{4a}{b}} \operatorname{expIntegral}_1\left(4 \operatorname{arcsinh}(xc)+\frac{4a}{b}\right)}{16c^3 b}-\frac{e^3 e^{-\frac{4a}{b}} \operatorname{expIntegral}_1\left(-4 \operatorname{arcsinh}(xc)-\frac{4a}{b}\right)}{16c^3 b}+\frac{3e\, e^{\frac{2a}{b}} \operatorname{expIntegral}_1\left(2 \operatorname{arcsinh}(xc)+\frac{2a}{b}\right)\cdots}{4cb}$

input
```
int((e*x+d)^3/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/16/c^3*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(x*c)+4*a/b)-1/16/c^3*e^3/b*e
xp(-4*a/b)*Ei(1,-4*arcsinh(x*c)-4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arcsinh
(x*c)+2*a/b)*d^2-1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arcsinh(x*c)+2*a/b)-3/4/c
*e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(x*c)-2*a/b)*d^2+1/8/c^3*e^3/b*exp(-2*a/b)
*Ei(1,-2*arcsinh(x*c)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(x
*c)-3*a/b)-3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)-1/2*d^3/b
*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)+3/8/c^2*d/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/
b)*e^2-1/2*d^3/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)+3/8/c^2*d/b*exp(-a/b)*E
i(1,-arcsinh(x*c)-a/b)*e^2)
```

Fricas [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arcsinh(c*x) + a), x
)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+ex)^3}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)**3/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)**3/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+e\,x)^3}{a+b\operatorname{asinh}(c\,x)}\,dx$$

input
```
int((d + e*x)^3/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)^3/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\left(\int \frac{x^3}{asinh(cx)\,b+a}dx\right)e^3+3\left(\int \frac{x^2}{asinh(cx)\,b+a}dx\right)d\,e^2+3\left(\int \frac{x}{asinh(cx)\,b+a}dx\right)d^2e+\left(\int \frac{1}{asinh(cx)\,b+a}dx\right)d^3$$

input
```
int((e*x+d)^3/(a+b*asinh(c*x)),x)
```

output
```
int(x**3/(asinh(c*x)*b + a),x)*e**3 + 3*int(x**2/(asinh(c*x)*b + a),x)*d*e
**2 + 3*int(x/(asinh(c*x)*b + a),x)*d**2*e + int(1/(asinh(c*x)*b + a),x)*d
**3
```

3.20 $\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 245

$$\begin{aligned}\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx &= \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\
&-\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\
&+\frac{e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\\
&-\frac{de\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2}\\
&-\frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\
&+\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\
&+\frac{de\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{bc^2}\\
&-\frac{e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\end{aligned}$$

output
```
d^2*cosh(a/b)*Chi(a/b+arcsinh(c*x))/b/c-1/4*e^2*cosh(a/b)*Chi(a/b+arcsinh(
c*x))/b/c^3+1/4*e^2*cosh(3*a/b)*Chi(3*a/b+3*arcsinh(c*x))/b/c^3-d*e*Chi(2*
a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^2-d^2*sinh(a/b)*Shi(a/b+arcsinh(c*x))/
b/c+1/4*e^2*sinh(a/b)*Shi(a/b+arcsinh(c*x))/b/c^3+d*e*cosh(2*a/b)*Shi(2*a/
b+2*arcsinh(c*x))/b/c^2-1/4*e^2*sinh(3*a/b)*Shi(3*a/b+3*arcsinh(c*x))/b/c^
3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx$$
$$= \frac{(4c^2d^2-e^2)\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)-4cde\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{a}\right.\right.}{}$$

input
```
Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
((4*c^2*d^2 - e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[(
3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c*d*e*CoshIntegral[2*(a/b
 + ArcSinh[c*x])]*Sinh[(2*a)/b] - 4*c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + A
rcSinh[c*x]] + e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 4*c*d*e*Co
sh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - e^2*Sinh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \; 6245$$

$$\frac{\int \frac{(cd+cex)^2\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}d\text{arcsinh}(cx)}{c^3}$$

$$\Big\downarrow \; 7293$$

$$\frac{\int \left(\frac{c^2\sqrt{c^2x^2+1}d^2}{a+b\text{arcsinh}(cx)}+\frac{ce\sinh(2\text{arcsinh}(cx))d}{a+b\text{arcsinh}(cx)}+\frac{c^2e^2x^2\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)}{c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{c^2d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{cde\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{b}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}}{}$$

input
```
Int[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
((c^2*d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/b - (e^2*Cosh[a/b]*C
oshIntegral[a/b + ArcSinh[c*x]])/(4*b) + (e^2*Cosh[(3*a)/b]*CoshIntegral[(
3*a)/b + 3*ArcSinh[c*x]])/(4*b) - (c*d*e*CoshIntegral[(2*a)/b + 2*ArcSinh[
c*x]]*Sinh[(2*a)/b])/b - (c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x
]])/b + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b) + (c*d*e*Co
sh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/b - (e^2*Sinh[(3*a)/b]
*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b))/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

method	result
derivativedivides	$-\frac{e^2 e^{\frac{3a}{b}} \text{expIntegral}_1\left(3\,\text{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2 b}-\frac{e^2 e^{-\frac{3a}{b}} \text{expIntegral}_1\left(-3\,\text{arcsinh}(xc)-\frac{3a}{b}\right)}{8c^2 b}-\frac{e^{\frac{a}{b}} \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right) d^2}{2b}+$
default	$-\frac{e^2 e^{\frac{3a}{b}} \text{expIntegral}_1\left(3\,\text{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2 b}-\frac{e^2 e^{-\frac{3a}{b}} \text{expIntegral}_1\left(-3\,\text{arcsinh}(xc)-\frac{3a}{b}\right)}{8c^2 b}-\frac{e^{\frac{a}{b}} \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right) d^2}{2b}+$

input
```
int((e*x+d)^2/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/8/c^2*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(x*c)+3*a/b)-1/8/c^2*e^2/b*ex
p(-3*a/b)*Ei(1,-3*arcsinh(x*c)-3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b
)*d^2+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-
arcsinh(x*c)-a/b)*d^2+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)*e^2+1/2/
c*d*e/b*exp(2*a/b)*Ei(1,2*arcsinh(x*c)+2*a/b)-1/2/c*d*e/b*exp(-2*a/b)*Ei(1
,-2*arcsinh(x*c)-2*a/b))
```

Fricas [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+ex)^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)**2/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+e\,x)^2}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x)^2/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)^2/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \left(\int \frac{x^2}{asinh\,(cx)\,b+a}dx\right)e^2+2\left(\int \frac{x}{asinh\,(cx)\,b+a}dx\right)de + \left(\int \frac{1}{asinh\,(cx)\,b+a}dx\right)d^2$$

input
```
int((e*x+d)^2/(a+b*asinh(c*x)),x)
```

output
```
int(x**2/(asinh(c*x)*b + a),x)*e**2 + 2*int(x/(asinh(c*x)*b + a),x)*d*e +
int(1/(asinh(c*x)*b + a),x)*d**2
```

3.21 $\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} - \frac{e\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2bc^2}$$

output
```
d*cosh(a/b)*Chi(a/b+arcsinh(c*x))/b/c-1/2*e*Chi(2*a/b+2*arcsinh(c*x))*sinh
(2*a/b)/b/c^2-d*sinh(a/b)*Shi(a/b+arcsinh(c*x))/b/c+1/2*e*cosh(2*a/b)*Shi(
2*a/b+2*arcsinh(c*x))/b/c^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{2cd\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-e\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2cd\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsin}\right.}{2bc^2}$$

input
```
Integrate[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

output
```
(2*c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - e*CoshIntegral[2*(a/b
+ ArcSinh[c*x])]*Sinh[(2*a)/b] - 2*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcSin
h[c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx$$
$$\Big\downarrow \; 6245$$
$$\frac{\int \frac{(cd+cex)\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}d\text{arcsinh}(cx)}{c^2}$$
$$\Big\downarrow \; 7293$$
$$\frac{\int \left(\frac{c\sqrt{c^2x^2+1}d}{a+b\text{arcsinh}(cx)}+\frac{cex\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)}{c^2}$$
$$\Big\downarrow \; 2009$$

$$\frac{\frac{cd\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{b}-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2b}-\frac{cd\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{b}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\right.}{2b}}{c^2}$$

input
```
Int[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

output
```
((c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/b - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b) - (c*d*Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]])/b + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[
c*x]])/(2*b))/c^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{d\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)}{2b}-\frac{d\,\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}_1\left(-\operatorname{arcsinh}(xc)-\frac{a}{b}\right)}{2b}+\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}_1\left(2\operatorname{arcsinh}(xc)+\frac{2a}{b}\right)}{4cb}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}}{}}{c}$
default	$\frac{-\frac{d\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}_1\left(\operatorname{arcsinh}(xc)+\frac{a}{b}\right)}{2b}-\frac{d\,\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}_1\left(-\operatorname{arcsinh}(xc)-\frac{a}{b}\right)}{2b}+\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}_1\left(2\operatorname{arcsinh}(xc)+\frac{2a}{b}\right)}{4cb}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}}{}}{c}$

input
```
int((e*x+d)/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*d/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/2*d/b*exp(-a/b)*Ei(1,-arcs
inh(x*c)-a/b)+1/4*e/c/b*exp(2*a/b)*Ei(1,2*arcsinh(x*c)+2*a/b)-1/4*e/c/b*ex
p(-2*a/b)*Ei(1,-2*arcsinh(x*c)-2*a/b))
```

Fricas [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{ex+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{d+ex}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{d+e\,x}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x)/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \left(\int \frac{x}{asinh\left(cx\right)b+a}dx\right)e+\left(\int \frac{1}{asinh\left(cx\right)b+a}dx\right)d$$

input
```
int((e*x+d)/(a+b*asinh(c*x)),x)
```

output
```
int(x/(asinh(c*x)*b + a),x)*e + int(1/(asinh(c*x)*b + a),x)*d
```

3.22 $\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

output
```
cosh(a/b)*Chi((a+b*arcsinh(c*x))/b)/b/c-sinh(a/b)*Shi((a+b*arcsinh(c*x))/b
)/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(b*c)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b] - Sinh[a/b]*SinhIntegral[(
a + b*ArcSinh[c*x])/b])/(b*c)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}} \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \text{expIntegral}_1\left(-\text{arcsinh}(xc)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{-\frac{e^{\frac{a}{b}} \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \text{expIntegral}_1\left(-\text{arcsinh}(xc)-\frac{a}{b}\right)}{2b}}{c}$	56

input
```
int(1/(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(
x*c)-a/b))
```

Fricas [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)} dx = \int \frac{1}{b \operatorname{arsinh}(cx) + a} dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a + b\operatorname{arcsinh}(cx)}\,dx = \int \frac{1}{a + b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a + b\operatorname{arcsinh}(cx)}\,dx = \int \frac{1}{b\operatorname{arsinh}(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{a + b\operatorname{arcsinh}(cx)}\,dx = \int \frac{1}{b\operatorname{arsinh}(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

input
```
int(1/(a + b*asinh(c*x)),x)
```

output
```
int(1/(a + b*asinh(c*x)), x)
```

Reduce [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{1}{asinh\,(cx)\,b+a}dx$$

input
```
int(1/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b + a),x)
```

3.23 $\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex)(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx$$

$$\Big\downarrow \; 6272$$

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\,(a+b\,\text{arcsinh}\,(xc))}dx$$

input
```
int(1/(e*x+d)/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x+d)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x + a*d + (b*e*x + b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,(d+e\,x)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{asinh\,(cx)\,bd + asinh\,(cx)\,bex + ad + aex}dx$$

input
```
int(1/(e*x+d)/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b*d + asinh(c*x)*b*e*x + a*d + a*e*x),x)
```

3.24 $\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))} \, dx$$

$$\Big\downarrow \text{ 6272}$$

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(ex+d\right)^{2} \left(a+b \,\operatorname{arcsinh}\left(xc\right)\right)}dx$$

input
```
int(1/(e*x+d)^2/(a+b*arcsinh(x*c)),x)
```

output
```
int(1/(e*x+d)^2/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2
)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,(d+e\,x)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.06

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx$$
$$=\int \frac{1}{asinh\,(cx)\,b\,d^2+2asinh\,(cx)\,bdex+asinh\,(cx)\,b\,e^2x^2+a\,d^2+2adex+a\,e^2x^2}dx$$

input
```
int(1/(e*x+d)^2/(a+b*asinh(c*x)),x)
```

output
```
int(1/(asinh(c*x)*b*d**2 + 2*asinh(c*x)*b*d*e*x + asinh(c*x)*b*e**2*x**2 +
 a*d**2 + 2*a*d*e*x + a*e**2*x**2),x)
```

3.25 $\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 359

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} + \frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{d^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{e^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3} - \frac{3e^2\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3} + \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} - \frac{2de\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2} + \frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}$$

output

```
-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-2*d*e*x*(c^2*x^2+1)^(1/2)/b/
c/(a+b*arcsinh(c*x))-e^2*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))+2*d*
e*cosh(2*a/b)*Chi(2*(a+b*arcsinh(c*x))/b)/b^2/c^2-d^2*Chi((a+b*arcsinh(c*x
))/b)*sinh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^3-
3/4*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3+d^2*cosh(a/b)*Shi(
(a+b*arcsinh(c*x))/b)/b^2/c-1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^
2/c^3-2*d*e*sinh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^2+3/4*e^2*cosh(3
*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx =$$

$$-\frac{\frac{4bc^2d^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{8bc^2dex\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{4bc^2e^2x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}-8cde\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+(4c^2d^2-$$

input
```
Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/4*((4*b*c^2*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (8*b*c^2*d*e*
x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e^2*x^2*Sqrt[1 + c^2*
x^2])/(a + b*ArcSinh[c*x]) - 8*c*d*e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + A
rcSinh[c*x])] + (4*c^2*d^2 - e^2)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/
b] + 3*e^2*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d^2*
Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/
b + ArcSinh[c*x]] + 8*c*d*e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*
x])] - 3*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^3)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{6244}$$

$$\int \left(\frac{d^2}{(a+b\text{arcsinh}(cx))^2}+\frac{2dex}{(a+b\text{arcsinh}(cx))^2}+\frac{e^2x^2}{(a+b\text{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2 \sinh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} - \frac{3e^2 \sinh\left(\frac{3a}{b}\right) \text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} - \frac{e^2 \cosh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3} + \frac{3e^2 \cosh\left(\frac{3a}{b}\right) \text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3} + \frac{2de \cosh\left(\frac{2a}{b}\right) \text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{2de \sinh\left(\frac{2a}{b}\right) \text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{d^2 \sinh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} + \frac{d^2 \cosh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{d^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))} - \frac{2dex\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))} - \frac{e^2x^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 +
c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[
c*x]))/b])/(b^2*c^2) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b]
)/(b^2*c) + (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b^2*c^
3) - (3*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b^2
*c^3) + (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e^
2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) - (2*d*e*Sin
h[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2) + (3*e^2*Co
sh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{\left(-4x^2c^2\sqrt{c^2x^2+1}+4x^3c^3-\sqrt{c^2x^2+1}+3xc\right)e^2}{8c^2b(a+b\ \mathrm{arcsinh}(xc))}+\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\ \mathrm{expIntegral}_1\left(3\ \mathrm{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4x^3c^3+3xc+4x^2c^2\sqrt{c^2x^2+1}+\sqrt{}\right.}{8b\,c^2(a+b\ \mathrm{arcsinh}(xc))}}{}$
default	$\frac{\frac{\left(-4x^2c^2\sqrt{c^2x^2+1}+4x^3c^3-\sqrt{c^2x^2+1}+3xc\right)e^2}{8c^2b(a+b\ \mathrm{arcsinh}(xc))}+\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\ \mathrm{expIntegral}_1\left(3\ \mathrm{arcsinh}(xc)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4x^3c^3+3xc+4x^2c^2\sqrt{c^2x^2+1}+\sqrt{}\right.}{8b\,c^2(a+b\ \mathrm{arcsinh}(xc))}}{}$

input
```
int((e*x+d)^2/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/8*(-4*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x^3*c^3-(c^2*x^2+1)^(1/2)+3*x*c)*
e^2/c^2/b/(a+b*arcsinh(x*c))+3/8*e^2/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(x*c
)+3*a/b)-1/8/b*e^2/c^2*(4*x^3*c^3+3*x*c+4*x^2*c^2*(c^2*x^2+1)^(1/2)+(c^2*x
^2+1)^(1/2))/(a+b*arcsinh(x*c))-3/8/b^2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arcsin
h(x*c)-3*a/b)+1/2*(x*c-(c^2*x^2+1)^(1/2))*d^2/b/(a+b*arcsinh(x*c))+1/2*d^2
/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/8*(x*c-(c^2*x^2+1)^(1/2))*e^2/c^2/b
/(a+b*arcsinh(x*c))-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arcsinh(x*c)+a/b)-1/2/b*
d^2*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-1/2/b^2*d^2*exp(-a/b)*Ei(1,
-arcsinh(x*c)-a/b)+1/8/c^2/b*e^2*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c)
)+1/8/c^2/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)+1/2*(-2*(c^2*x^2+1)^(1
/2)*x*c+2*c^2*x^2+1)*d*e/c/b/(a+b*arcsinh(x*c))-e*d/c/b^2*exp(2*a/b)*Ei(1,
2*arcsinh(x*c)+2*a/b)-1/2/b*e*d/c*(2*c^2*x^2+2*(c^2*x^2+1)^(1/2)*x*c+1)/(a
+b*arcsinh(x*c))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arcsinh(x*c)-2*a/b))
```

Fricas [F]

$$\int \frac{(d+ex)^2}{(a+b\mathrm{arcsinh}(cx))^2}\,dx=\int \frac{(ex+d)^2}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x
) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

input
```
integrate((e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x)**2/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e^2*x^5 + 2*c^3*d*e*x^4 + 2*c*d*e*x^2 + c*d^2*x + (c^3*d^2 + c*e^2)*
x^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 + 2*d*e*x + (c^2*d^2 + e^2)*x^2 + d^2)*
sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
 1))) + integrate((3*c^5*e^2*x^6 + 4*c^5*d*e*x^5 + 8*c^3*d*e*x^3 + (c^5*d^
2 + 6*c^3*e^2)*x^4 + 4*c*d*e*x + c*d^2 + (2*c^3*d^2 + 3*c*e^2)*x^2 + (3*c^
3*e^2*x^4 + 4*c^3*d*e*x^3 - c*d^2 + (c^3*d^2 + c*e^2)*x^2)*(c^2*x^2 + 1) +
 (6*c^4*e^2*x^5 + 8*c^4*d*e*x^4 + 8*c^2*d*e*x^2 + (2*c^4*d^2 + 7*c^2*e^2)*
x^3 + 2*d*e + (c^2*d^2 + 2*e^2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*
x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1
)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c
^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sq
rt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+e\,x)^2}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x)^2/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)^2/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx =& \left(\int \frac{x^2}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)e^2\\ &+2\left(\int \frac{x}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)de\\ &+\left(\int \frac{1}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx\right)d^2\end{aligned}$$

input
```
int((e*x+d)^2/(a+b*asinh(c*x))^2,x)
```

output
```
int(x**2/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*e**2 + 2*int(x/
(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*d*e + int(1/(asinh(c*x)*
*2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*d**2
```

3.26 $\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 180

$$\begin{aligned}\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx &= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}\\ &\quad+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}\\ &\quad-\frac{d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}\\ &\quad+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}\\ &\quad-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}\end{aligned}$$

output
```
-d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b
*arcsinh(c*x))+e*cosh(2*a/b)*Chi(2*(a+b*arcsinh(c*x))/b)/b^2/c^2-d*Chi((a+
b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b
^2/c-e*sinh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx =$$
$$-\frac{\frac{bcd\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{bcex\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}-e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+cd\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\text{sinh}}{b^2c^2}$$

input
```
Integrate[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-(((b*c*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (b*c*e*x*Sqrt[1 + c^2*
x^2])/(a + b*ArcSinh[c*x]) - e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh
[c*x])] + c*d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - c*d*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] + e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cSinh[c*x])])/(b^2*c^2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx$$
$$\Big\downarrow \ 6244$$
$$\int \left(\frac{d}{(a+b\text{arcsinh}(cx))^2}+\frac{ex}{(a+b\text{arcsinh}(cx))^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input

```
Int[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

output

```
-((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^
2])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*A
rcSinh[c*x]))/b])/(b^2*c^2) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh
[a/b])/(b^2*c) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\left(xc-\sqrt{c^2x^2+1}\right)d}{2b(a+b\ \text{arcsinh}(xc))}+\frac{d\,e^{\frac{a}{b}}\ \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right)}{2b^2}-\frac{d\left(xc+\sqrt{c^2x^2+1}\right)}{2b(a+b\ \text{arcsinh}(xc))}-\frac{d\,e^{-\frac{a}{b}}\ \text{expIntegral}_1\left(-\text{arcsinh}(xc)-\frac{a}{b}\right)}{2b^2}+\frac{(-2}{}$
default	$\frac{\left(xc-\sqrt{c^2x^2+1}\right)d}{2b(a+b\ \text{arcsinh}(xc))}+\frac{d\,e^{\frac{a}{b}}\ \text{expIntegral}_1\left(\text{arcsinh}(xc)+\frac{a}{b}\right)}{2b^2}-\frac{d\left(xc+\sqrt{c^2x^2+1}\right)}{2b(a+b\ \text{arcsinh}(xc))}-\frac{d\,e^{-\frac{a}{b}}\ \text{expIntegral}_1\left(-\text{arcsinh}(xc)-\frac{a}{b}\right)}{2b^2}+\frac{(-2}{}$

input
```
int((e*x+d)/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(x*c-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(x*c))+1/2*d/b^2*exp(a/b)
*Ei(1,arcsinh(x*c)+a/b)-1/2/b*d*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))
-1/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b)+1/4*(-2*(c^2*x^2+1)^(1/2)*x*c
+2*c^2*x^2+1)*e/c/b/(a+b*arcsinh(x*c))-1/2*e/c/b^2*exp(2*a/b)*Ei(1,2*arcsi
nh(x*c)+2*a/b)-1/4*e/c/b*(2*c^2*x^2+2*(c^2*x^2+1)^(1/2)*x*c+1)/(a+b*arcsin
h(x*c))-1/2*e/c/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(x*c)-2*a/b))
```

Fricas [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{d+ex}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x)/(a + b*asinh(c*x))**2, x)
```

Maxima [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e*x^4 + c^3*d*x^3 + c*e*x^2 + c*d*x + (c^2*e*x^3 + c^2*d*x^2 + e*x +
 d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x
^2 + 1))) + integrate((2*c^5*e*x^5 + c^5*d*x^4 + 4*c^3*e*x^3 + 2*c^3*d*x^2
 + 2*c*e*x + (2*c^3*e*x^3 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (4*c^4*
e*x^4 + 2*c^4*d*x^3 + 4*c^2*e*x^2 + c^2*d*x + e)*sqrt(c^2*x^2 + 1))/(a*b*c
^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4
+ (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2
*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 +
 a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{d+e\,x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((d + e*x)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\left(\int \frac{x}{asinh\left(cx\right)^{2}b^{2}+2asinh\left(cx\right)ab+a^{2}}dx\right)e+\left(\int \frac{1}{asinh\left(cx\right)^{2}b^{2}+2asinh\left(cx\right)ab+a^{2}}dx\right)d$$

input
```
int((e*x+d)/(a+b*asinh(c*x))^2,x)
```

output
```
int(x/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*e + int(1/(asinh(c
*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)*d
```

3.27 $\int \frac{1}{(a+b\,\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{(a+b\,\text{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc(a+b\,\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)}{b^2c}$$

output
```
-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-Chi((a+b*arcsinh(c*x))/b)*sinh(a
/b)/b^2/c+cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\,\text{arcsinh}(cx))^2}\,dx = \frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\,\text{arcsinh}(cx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
(-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSin
h[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6188, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{c\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx}{b}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$\downarrow$ 3784

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 3042

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 26

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$\downarrow$ 3779

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
-(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(a +
b*ArcSinh[c*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x
])/b]))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{\frac{xc-\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}_1\left(\mathrm{arcsinh}(xc)+\frac{a}{b}\right)}{2b^2}-\frac{xc+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}_1\left(-\,\mathrm{arcsinh}(xc)-\frac{a}{b}\right)}{2b^2}}{c}$	118
default	$\frac{\frac{xc-\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}_1\left(\mathrm{arcsinh}(xc)+\frac{a}{b}\right)}{2b^2}-\frac{xc+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(xc))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}_1\left(-\,\mathrm{arcsinh}(xc)-\frac{a}{b}\right)}{2b^2}}{c}$	118

input
```
int(1/(a+b*arcsinh(x*c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(x*c-(c^2*x^2+1)^(1/2))/b/(a+b*arcsinh(x*c))+1/2/b^2*exp(a/b)*Ei(
1,arcsinh(x*c)+a/b)-1/2/b*(x*c+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(x*c))-1/2/b
^2*exp(-a/b)*Ei(1,-arcsinh(x*c)-a/b))
```

Fricas [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2} \, dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(a + b \operatorname{asinh}(cx))^2} \, dx$$

input
```
integrate(1/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} \, dx = \int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2} \, dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(
c*x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)
*(c^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (
c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 +
 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c
^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^2,x)
```

output
```
int(1/(a + b*asinh(c*x))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{asinh\,(cx)^2\,b^2+2asinh\,(cx)\,ab+a^2}dx$$

input
```
int(1/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)
```

3.28 $\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \; 6272$$
$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\left(a+b\,\text{arcsinh}\left(xc\right)\right)^{2}}dx$$

input
```
int(1/(e*x+d)/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x+d)/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*e*
x + a*b*d)*arcsinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 746, normalized size of antiderivative = 41.44

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^3 + a*b*c^3*d*x^2 + a*
b*c*e*x + a*b*c*d + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d +
 (b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) + integrate((c^5*d
*x^4 + 2*c^3*d*x^2 + (c^3*d*x^2 - 2*c*e*x - c*d)*(c^2*x^2 + 1) + c*d + (2*
c^4*d*x^3 - 2*c^2*e*x^2 + c^2*d*x - e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^6
 + 2*a*b*c^5*d*e*x^5 + 4*a*b*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*a*b*x^4 +
 2*a*b*c*d*e*x + a*b*c*d^2 + (2*c^3*d^2 + c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^
4 + 2*a*b*c^3*d*e*x^3 + a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^6
+ 2*b^2*c^5*d*e*x^5 + 4*b^2*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*b^2*x^4 +
2*b^2*c*d*e*x + b^2*c*d^2 + (2*c^3*d^2 + c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4
 + 2*b^2*c^3*d*e*x^3 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^2*x^5
 + 2*b^2*c^4*d*e*x^4 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x + (c^4*d^2 + c^2*
e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*e^2*x^5 + 2*a*b*c^4*d*e*x^4 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x + (c^4*d^
2 + c^2*e^2)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx$$
$$= \int \frac{1}{asinh\left(cx\right)^2b^2d+asinh\left(cx\right)^2b^2ex+2asinh\left(cx\right)abd+2asinh\left(cx\right)abex+a^2d+a^2ex}dx$$

input
```
int(1/(e*x+d)/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2*d + asinh(c*x)**2*b**2*e*x + 2*asinh(c*x)*a*b*d
+ 2*asinh(c*x)*a*b*e*x + a**2*d + a**2*e*x),x)
```

3.29 $\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$$
$$\Big\downarrow \text{ 6272}$$
$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(ex+d\right)^{2} \left(a+b\,\mathrm{arcsinh}\left(xc\right)\right)^{2}}dx$$

input
```
int(1/(e*x+d)^2/(a+b*arcsinh(x*c))^2,x)
```

output
```
int(1/(e*x+d)^2/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 5.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e
*x + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arc
sinh(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 7.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 1050, normalized size of antiderivative = 58.33

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^4 + 2*a*b*c^3*d*e*x^
3 + 2*a*b*c*d*e*x + a*b*c*d^2 + (c^3*d^2 + c*e^2)*a*b*x^2 + (b^2*c^3*e^2*x
^4 + 2*b^2*c^3*d*e*x^3 + 2*b^2*c*d*e*x + b^2*c*d^2 + (c^3*d^2 + c*e^2)*b^2
*x^2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x)*sqrt(c^2*x^2
+ 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2
+ a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^5 - c^5*d*x^4 + 2
*c^3*e*x^3 - 2*c^3*d*x^2 + c*e*x + (c^3*e*x^3 - c^3*d*x^2 + 3*c*e*x + c*d)
*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^4 - 2*c^4*d*x^3 + 5*c^2*e*x^2 - c^2*d*x
+ 2*e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*
d^2*e + 2*c^3*e^3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*a*b
*x^4 + a*b*c*d^3 + (6*c^3*d^2*e + c*e^3)*a*b*x^3 + (2*c^3*d^3 + 3*c*d*e^2)
*a*b*x^2 + (a*b*c^3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 +
a*b*c^3*d^3*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^5*d*e^2*x^6 +
(3*c^5*d^2*e + 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e
^2)*b^2*x^4 + b^2*c*d^3 + (6*c^3*d^2*e + c*e^3)*b^2*x^3 + (2*c^3*d^3 + 3*c
*d*e^2)*b^2*x^2 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^2*c^3*d^2*e
*x^3 + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^3*x^6 + 3*b^2*c^4*d*e
^2*x^5 + 3*b^2*c^2*d^2*e*x^2 + b^2*c^2*d^3*x + (3*c^4*d^2*e + c^2*e^3)*b^2
*x^4 + (c^4*d^3 + 3*c^2*d*e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 + 3*a*b*c^2*d^...
```

Giac [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 6.06

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$$
$$= \int \frac{1}{asinh\left(cx\right)^2 b^2 d^2 + 2asinh\left(cx\right)^2 b^2 dex + asinh\left(cx\right)^2 b^2 e^2 x^2 + 2asinh\left(cx\right) ab\, d^2 + 4asinh\left(cx\right) abdex + 2}$$

input
```
int(1/(e*x+d)^2/(a+b*asinh(c*x))^2,x)
```

output
```
int(1/(asinh(c*x)**2*b**2*d**2 + 2*asinh(c*x)**2*b**2*d*e*x + asinh(c*x)**
2*b**2*e**2*x**2 + 2*asinh(c*x)*a*b*d**2 + 4*asinh(c*x)*a*b*d*e*x + 2*asin
h(c*x)*a*b*e**2*x**2 + a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2),x)
```

3.30 $\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx = \frac{(d+ex)^{1+m}(a+b\text{arcsinh}(cx))^2}{e(1+m)} - \frac{2bc\text{Int}\left(\frac{(d+ex)^{1+m}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}},x\right)}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(a+b*arcsinh(c*x))^2/e/(1+m)-2*b*c*Defer(Int)((e*x+d)^(1+m)*
(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)/e/(1+m)
```

Mathematica [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx = \int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2 \, dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))^2}{e(m+1)}-\frac{2bc\int \frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{e(m+1)}$$

$$\Big\downarrow \text{ 6272}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))^2}{e(m+1)}-\frac{2bc\int \frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex+d)^m (a+b\,\mathrm{arcsinh}\,(xc))^2\,dx$$

input
```
int((e*x+d)^m*(a+b*arcsinh(x*c))^2,x)
```

output
```
int((e*x+d)^m*(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int (d+ex)^m(a+b\mathrm{arcsinh}(cx))^2\,dx = \int (b\,\mathrm{arsinh}\,(cx)+a)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*(e*x + d)^m, x)
```

Sympy [N/A]

Not integrable

Time = 6.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (d+ex)^m(a+b\mathrm{arcsinh}(cx))^2\,dx = \int (a+b\,\mathrm{asinh}\,(cx))^2\,(d+ex)^m\,dx$$

input
```
integrate((e*x+d)**m*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**2*(d + e*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 271, normalized size of antiderivative = 15.06

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx = \int (b\,\text{arsinh}\,(cx)+a)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
(b^2*e*x + b^2*d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*(m + 1)) +
 (e*x + d)^(m + 1)*a^2/(e*(m + 1)) + integrate(-2*((b^2*c^2*d*x - a*b*e*(m
 + 1) - (a*b*c^2*e*(m + 1) - b^2*c^2*e)*x^2)*sqrt(c^2*x^2 + 1)*(e*x + d)^m
 + (b^2*c^3*d*x^2 + b^2*c*d - (a*b*c^3*e*(m + 1) - b^2*c^3*e)*x^3 - (a*b*c
*e*(m + 1) - b^2*c*e)*x)*(e*x + d)^m)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e*
(m + 1)*x^3 + c*e*(m + 1)*x + (c^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2
 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx = \int (b\,\text{arsinh}\,(cx)+a)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*(e*x + d)^m, x)
```

Mupad [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx=\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d+e\,x\right)}^m \,d x$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x)^m,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 6.50

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{(ex+d)^m a^2 d+(ex+d)^m a^2 ex+2\left(\int (ex+d)^m \mathit{asinh}(cx)\,dx\right) abem+2\left(\int (ex+d)^m \mathit{asinh}(cx)\,dx\right) abe}{e\,(m+1)}$$

input
```
int((e*x+d)^m*(a+b*asinh(c*x))^2,x)
```

output
```
((d + e*x)**m*a**2*d + (d + e*x)**m*a**2*e*x + 2*int((d + e*x)**m*asinh(c*
x),x)*a*b*e*m + 2*int((d + e*x)**m*asinh(c*x),x)*a*b*e + int((d + e*x)**m*
asinh(c*x)**2,x)*b**2*e*m + int((d + e*x)**m*asinh(c*x)**2,x)*b**2*e)/(e*(
m + 1))
```

3.31 $\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 179

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = -\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,\text{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}} + \frac{(d+ex)^{1+m}(a+b\text{arcsinh}(cx))}{e(1+m)}$$

output
```
-b*c*(e*x+d)^(2+m)*(1-(e*x+d)/(d-e/(-c^2)^(1/2)))^(1/2)*(1-(e*x+d)/(d+e/(-
c^2)^(1/2)))^(1/2)*AppellF1(2+m,1/2,1/2,3+m,(e*x+d)/(d-e/(-c^2)^(1/2)),(e*
x+d)/(d+e/(-c^2)^(1/2)))/e^2/(1+m)/(2+m)/(c^2*x^2+1)^(1/2)+(e*x+d)^(1+m)*(
a+b*arcsinh(c*x))/e/(1+m)
```

Mathematica [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx=\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx$$

input
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]), x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6243, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{e(m+1)}-\frac{bc\int \frac{(d+ex)^{m+1}}{\sqrt{c^2x^2+1}}dx}{e(m+1)}$$

$$\Big\downarrow \text{ 514}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\int \frac{(d+ex)^{m+1}}{\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}d(d+ex)}{e^2(m+1)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{(d+ex)^{m+1}(a+b\operatorname{arcsinh}(cx))}{e(m+1)} - \frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}\operatorname{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

input

```
Int[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

output

```
-((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (
d + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d
 - e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt
[1 + c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

Defintions of rubi rules used

rule 150

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 6243

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

Maple [F]

$$\int (ex + d)^m (a + b \operatorname{arcsinh}(xc)) \, dx$$

input
```
int((e*x+d)^m*(a+b*arcsinh(x*c)),x)
```

output
```
int((e*x+d)^m*(a+b*arcsinh(x*c)),x)
```

Fricas [F]

$$\int (d + ex)^m (a + b\text{arcsinh}(cx)) \, dx = \int (b \operatorname{arsinh}(cx) + a)(ex + d)^m \, dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d + ex)^m (a + b\text{arcsinh}(cx)) \, dx = \int (a + b \operatorname{asinh}(cx)) (d + ex)^m \, dx$$

input
```
integrate((e*x+d)**m*(a+b*asinh(c*x)),x)
```

output
```
Integral((a + b*asinh(c*x))*(d + e*x)**m, x)
```

Maxima [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
b*((e*x + d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))/(e*(m + 1)) - integr
ate((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + e*(m + 1)), x)
- integrate((c*e*x + c*d)*(e*x + d)^m/(c^3*e*(m + 1)*x^3 + c*e*(m + 1)*x +
 (c^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2 + 1)), x)) + (e*x + d)^(m +
1)*a/(e*(m + 1))
```

Giac [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d+e\,x\right)^m\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x)^m,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{(ex+d)^m ad + (ex+d)^m aex + \left(\int (ex+d)^m\, asinh(cx)\,dx\right) bem + \left(\int (ex+d)^m\, asinh(cx)\,dx\right) be}{e\,(m+1)}$$

input
```
int((e*x+d)^m*(a+b*asinh(c*x)),x)
```

output
```
((d + e*x)**m*a*d + (d + e*x)**m*a*e*x + int((d + e*x)**m*asinh(c*x),x)*b*
e*m + int((d + e*x)**m*asinh(c*x),x)*b*e)/(e*(m + 1))
```

3.32 $\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)} dx = \text{Int}\left(\frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}, x\right)$$

output
```
Defer(Int)((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)} dx = \int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)} dx$$

input
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{6272}$$

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m}{a+b\,\text{arcsinh}\,(xc)}dx$$

input
```
int((e*x+d)^m/(a+b*arcsinh(x*c)),x)
```

output
```
int((e*x+d)^m/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+ex)^m}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)**m/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)**m/(a + b*asinh(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+e\,x)^m}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x)^m/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)^m/(a + b*asinh(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^m}{asinh\,(cx)\,b+a}dx$$

input
```
int((e*x+d)^m/(a+b*asinh(c*x)),x)
```

output
```
int((e*x+d)^m/(a+b*asinh(c*x)),x)
```

3.33 $\int \frac{(d+ex)^m}{(a+b\,\mathbf{arcsinh}(cx))^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2} \, dx = \text{Int}\left(\frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}, x\right)$$

output
```
Defer(Int)((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2} \, dx$$

input
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2} \, dx$$

$$\Big\downarrow \text{ 6272}$$

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m}{(a+b \operatorname{arcsinh}(xc))^2}dx$$

input
```
int((e*x+d)^m/(a+b*arcsinh(x*c))^2,x)
```

output
```
int((e*x+d)^m/(a+b*arcsinh(x*c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^m}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((e*x+d)**m/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x)**m/(a + b*asinh(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 607, normalized size of antiderivative = 33.72

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 + 1)^(3/2)*(e*x + d)^m + (c^3*x^3 + c*x)*(e*x + d)^m)/(a*b*c^3*
x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 +
1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*e*(m
 + 1)*x^3 + c^3*d*x^2 + c*e*(m - 1)*x - c*d)*(c^2*x^2 + 1)*(e*x + d)^m + (
2*c^4*e*(m + 1)*x^4 + 2*c^4*d*x^3 + c^2*e*(3*m + 1)*x^2 + c^2*d*x + e*m)*s
qrt(c^2*x^2 + 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 + 2*c^3*e*(m
 + 1)*x^3 + 2*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5
 + a*b*c^5*d*x^4 + 2*a*b*c^3*e*x^3 + 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d
 + (a*b*c^3*e*x^3 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*x^5 + b^2*c^
5*d*x^4 + 2*b^2*c^3*e*x^3 + 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (b^2*c
^3*e*x^3 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^4 + b^2*c^4*d*x^3
 + b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*e*x^4 + a*b*c^4*d*x^3 + a*b*c^2*e*x^2 + a*b*c^2*d*x)*sq
rt(c^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+e\,x)^m}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x)^m/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)^m/(a + b*asinh(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2} \, dx = \int \frac{(ex+d)^m}{asinh\left(cx\right)^2 b^2+2asinh\left(cx\right)ab+a^2} dx$$

input
```
int((e*x+d)^m/(a+b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)**m/(asinh(c*x)**2*b**2 + 2*asinh(c*x)*a*b + a**2),x)
```

3.34 $\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 620

$$\begin{aligned}
&\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}+\frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} \\
&\quad-\frac{3bfg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} \\
&\quad-\frac{bg^3x^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{3bcfg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bcg^3x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\
&\quad+\frac{1}{2}f^3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{3fg^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2} \\
&\quad+\frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{f^2g(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{c^2d} \\
&\quad-\frac{g^3(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^4d}+\frac{g^3(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^4d^2} \\
&\quad+\frac{f^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}-\frac{3fg^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-b*f^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)+2/15*b*g^3*x*(c^2*d*x^2
+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/4*b*c*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)-3/16*b*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/3*b
*c*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(c^2*d*x
^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/16*b*c*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/25*b*c*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*
f^3*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+3/8*f*g^2*x*(c^2*d*x^2+d)^(1/
2)*(a+b*arcsinh(c*x))/c^2+3/4*f*g^2*x^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c
*x))+f^2*g*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^2/d-1/3*g^3*(c^2*d*x^2
+d)^(3/2)*(a+b*arcsinh(c*x))/c^4/d+1/5*g^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsin
h(c*x))/c^4/d^2+1/4*f^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*
x^2+1)^(1/2)-3/16*f*g^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c^3/(c^
2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.66

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{240ad(1+c^2x^2)^{3/2}\left(-16g^3+c^2g(120f^2+45fgx+8g^2x^2)+6c^4x(10f^3+20f^2gx+15fg^2x^2+4g^3x^3)\right)}{}$$

input
```
Integrate[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(240*a*d*(1 + c^2*x^2)^(3/2)*(-16*g^3 + c^2*g*(120*f^2 + 45*f*g*x + 8*g^2*
x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x^3)) - 9600*b*
c^2*d*f^2*g*(3*c*x + 4*c^3*x^3 + c^5*x^5 - 3*(1 + c^2*x^2)^(5/2)*ArcSinh[c
*x]) + 128*b*d*g^3*(1 + c^2*x^2)*(30*c*x - 5*c^3*x^3 - 9*c^5*x^5 + 15*Sqrt
[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) + 3600*a*c*Sqrt[d]*
f*(4*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sq
rt[d]*Sqrt[d + c^2*d*x^2]] - 3600*b*c^3*d*f^3*(1 + c^2*x^2)*(Cosh[2*ArcSin
h[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 675*b*c*
d*f*g^2*(1 + c^2*x^2)*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh
[c*x]*Sinh[4*ArcSinh[c*x]]))/(28800*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x
^2])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.59, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(f+gx)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6260$$

$$\frac{\sqrt{c^2dx^2+d}\int (f+gx)^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6253$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))f^3+3gx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))f^2+3g^2x^2\sqrt{c^2x^2+1}(a+bar\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{3fg^2(a+b\text{arcsinh}(cx))^2}{16bc^3}+\frac{1}{2}f^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{f^2g(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{c^2}+\frac{3fg^2x\right.}{}$$

input
```
Int[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-((b*f^2*g*x)/c) + (2*b*g^3*x)/(15*c^3) - (b*c*f^3*x
^2)/4 - (3*b*f*g^2*x^2)/(16*c) - (b*c*f^2*g*x^3)/3 - (b*g^3*x^3)/(45*c) -
(3*b*c*f*g^2*x^4)/16 - (b*c*g^3*x^5)/25 + (f^3*x*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x]))/2 + (3*f*g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c
^2) + (3*f*g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 +
 c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)^(3/2)*(a +
b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/
(5*c^4) + (f^3*(a + b*ArcSinh[c*x])^2)/(4*b*c) - (3*f*g^2*(a + b*ArcSinh[c
*x])^2)/(16*b*c^3)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. $2(540) = 1080$.

Time = 1.32 (sec) , antiderivative size = 1309, normalized size of antiderivative = 2.11

method	result	size
default	Expression too large to display	1309
parts	Expression too large to display	1309

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBO
SE)
```

output

```
a*(f^3*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2))+g^3*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c
^4*(c^2*d*x^2+d)^(3/2))+3*f*g^2*(1/4*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/4/c^2*(
1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)))+f^2*g*(c^2*d*x^2+d)^(3/2)/c^2/d)+b*(1/16*(d*(c^2*x^2+1
))^(1/2)*f*arcsinh(x*c)^2*(4*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)/c^3+1/800*(d
*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20
*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*g^3*(-1+5
*arcsinh(x*c))/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*x^
4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*
x^2+1)^(1/2))*f*g^2*(-1+4*arcsinh(x*c))/c^3/(c^2*x^2+1)+1/288*(d*(c^2*x^2+
1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(
1/2)*x*c+1)*g*(36*arcsinh(x*c)*c^2*f^2-12*c^2*f^2-3*arcsinh(x*c)*g^2+g^2)/
c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1
)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*f^3*(-1+2*arcsinh(x*c))/c/(c^2*x^2+1)+1/1
6*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*g*(6*arcsinh(x*c
)*c^2*f^2-6*c^2*f^2-arcsinh(x*c)*g^2+g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2
+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*g*(6*arcsinh(x*c)*c^2*f^2+6*c
^2*f^2-arcsinh(x*c)*g^2-g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2
*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*f^3*(1+2*...
```

Fricas [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\int \sqrt{c^2dx^2+d}(gx+f)^3(b\operatorname{arsinh}(cx)+a)\,dx$$

input

```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output

```
integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\int \sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))\,(f+gx)^3\,dx$$

input
```
integrate((g*x+f)**3*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(60\sqrt{c^2x^2+1}\,a\,c^4f^3x+120\sqrt{c^2x^2+1}\,a\,c^4f^2g\,x^2+90\sqrt{c^2x^2+1}\,a\,c^4f\,g^2x^3+24\sqrt{c^2x^2+1}\,a\,c^4g^3x^4\right.}{}$$

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x)),x)
```

output

```
(sqrt(d)*(60*sqrt(c**2*x**2 + 1)*a*c**4*f**3*x + 120*sqrt(c**2*x**2 + 1)*a
*c**4*f**2*g*x**2 + 90*sqrt(c**2*x**2 + 1)*a*c**4*f*g**2*x**3 + 24*sqrt(c*
*2*x**2 + 1)*a*c**4*g**3*x**4 + 120*sqrt(c**2*x**2 + 1)*a*c**2*f**2*g + 45
*sqrt(c**2*x**2 + 1)*a*c**2*f*g**2*x + 8*sqrt(c**2*x**2 + 1)*a*c**2*g**3*x
**2 - 16*sqrt(c**2*x**2 + 1)*a*g**3 + 120*int(sqrt(c**2*x**2 + 1)*asinh(c*
x)*x**3,x)*b*c**4*g**3 + 360*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*
c**4*f*g**2 + 360*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x,x)*b*c**4*f**2*g +
120*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c**4*f**3 + 60*log(sqrt(c**2*x
**2 + 1) + c*x)*a*c**3*f**3 - 45*log(sqrt(c**2*x**2 + 1) + c*x)*a*c*f*g**2
))/(120*c**4)
```

3.35 $\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 425

$$\begin{aligned}
&\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx\\
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}\\
&\quad-\frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}-\frac{bcg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}\\
&\quad+\frac{1}{2}f^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{g^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2}\\
&\quad+\frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{2fg(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}\\
&\quad+\frac{f^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}-\frac{g^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-2/3*b*f*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/4*b*c*f^2*x^2*(c^2*
d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2
*x^2+1)^(1/2)-2/9*b*c*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b
*c*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*f^2*x*(c^2*d*x^2+d)^(
1/2)*(a+b*arcsinh(c*x))+1/8*g^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c
^2+1/4*g^2*x^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+2/3*f*g*(c^2*d*x^2+d
)^(3/2)*(a+b*arcsinh(c*x))/c^2/d+1/4*f^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(
c*x))^2/b/c/(c^2*x^2+1)^(1/2)-1/16*g^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*
x))^2/b/c^3/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.71

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\frac{48ac\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(12c^2f^2x+3g^2x(1+2c^2x^2)+16f(g+c^2gx^2))-256bcfg\sqrt{d+c^2dx^2}\Big(3cx+$$

input
```
Integrate[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(48*a*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(12*c^2*f^2*x + 3*g^2*x*(1 +
 2*c^2*x^2) + 16*f*(g + c^2*g*x^2)) - 256*b*c*f*g*Sqrt[d + c^2*d*x^2]*(3*c
*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 144*a*Sqrt[d]*(2*c*f
- g)*(2*c*f + g)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] - 144*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*
x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 9*b*g^2*Sqrt[d + c^2*d*x^2]*(8
*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x
]]))/(1152*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(f+gx)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int (f+gx)^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))f^2+2gx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))f+g^2x^2\sqrt{c^2x^2+1}(a+b\text{arcs}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{g^2(a+b\text{arcsinh}(cx))^2}{16bc^3}+\frac{1}{2}f^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{2fg(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}+\frac{g^2x\sqrt{c^2}}{}\right.}{}$$

input
```
Int[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*((-2*b*f*g*x)/(3*c) - (b*c*f^2*x^2)/4 - (b*g^2*x^2)/(
16*c) - (2*b*c*f*g*x^3)/9 - (b*c*g^2*x^4)/16 + (f^2*x*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*
c^2) + (g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 + (2*f*g*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2) + (f^2*(a + b*ArcSinh[c*x])^2)/
(4*b*c) - (g^2*(a + b*ArcSinh[c*x])^2)/(16*b*c^3)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(367) = 734.

Time = 1.29 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.15

method	result
default	$a\left(f^2\left(\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}\right)+g^2\left(\frac{x(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}}{4c^2}\right)+\right.$
parts	$a\left(f^2\left(\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}\right)+g^2\left(\frac{x(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}}{4c^2}\right)+\right.$

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2))+g^2*(1/4*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/4/c^2*(1
/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2
))/(c^2*d)^(1/2)))+2/3*f*g*(c^2*d*x^2+d)^(3/2)/c^2/d)+b*(1/16*(d*(c^2*x^2+
1))^(1/2)*arcsinh(x*c)^2*(4*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3+1/256*(d*(c
^2*x^2+1))^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c
^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*g^2*(-1+4*arcsinh(x*c))/c^3/
(c^2*x^2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*
x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(-1+3*arcsinh(x*c))/c^2/(c^2*
x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2
*x*c+(c^2*x^2+1)^(1/2))*f^2*(-1+2*arcsinh(x*c))/c/(c^2*x^2+1)+1/4*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(arcsinh(x*c)-1)/c^2/(
c^2*x^2+1)+1/4*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*f*g
*(arcsinh(x*c)+1)/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*
x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*f^2*(1+2*arcsinh(x*c))/
c/(c^2*x^2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^
3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(1+3*arcsinh(x*c))/c^2/(c^2
*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)
+12*x^3*c^3-8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*g^2*(1+4*
arcsinh(x*c))/c^3/(c^2*x^2+1))
```

Fricas [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2+d}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arcsinh(c*x)), x)
```

Sympy [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\int \sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\int (f+g\,x)^2\,(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(12\sqrt{c^2x^2+1}\,a\,c^3f^2x+16\sqrt{c^2x^2+1}\,a\,c^3fg\,x^2+6\sqrt{c^2x^2+1}\,a\,c^3g^2x^3+16\sqrt{c^2x^2+1}\,acfg+3\sqrt{c^2x}\right.}{}$$

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*(12*sqrt(c**2*x**2 + 1)*a*c**3*f**2*x + 16*sqrt(c**2*x**2 + 1)*a*
c**3*f*g*x**2 + 6*sqrt(c**2*x**2 + 1)*a*c**3*g**2*x**3 + 16*sqrt(c**2*x**2
 + 1)*a*c*f*g + 3*sqrt(c**2*x**2 + 1)*a*c*g**2*x + 24*int(sqrt(c**2*x**2 +
 1)*asinh(c*x)*x**2,x)*b*c**3*g**2 + 48*int(sqrt(c**2*x**2 + 1)*asinh(c*x)
*x,x)*b*c**3*f*g + 24*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c**3*f**2 +
12*log(sqrt(c**2*x**2 + 1) + c*x)*a*c**2*f**2 - 3*log(sqrt(c**2*x**2 + 1)
+ c*x)*a*g**2))/(24*c**3)
```

3.36 $\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 221

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}fx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{g(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2d}+\frac{f\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output

```
-1/3*b*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/4*b*c*f*x^2*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+1/2*f*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+1/3*g*(c^2*d*x^2+d)^
(3/2)*(a+b*arcsinh(c*x))/c^2/d+1/4*f*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)
)^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.94

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx=\frac{1}{6}a\sqrt{d+c^2dx^2}\left(\frac{2g}{c^2}+x(3f+2gx)\right)$$
$$-\frac{bg\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\operatorname{arcsinh}(cx)\right)}{9c^2\sqrt{1+c^2x^2}}$$
$$+\frac{a\sqrt{d}f\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{2c}$$
$$+\frac{bf\sqrt{d+c^2dx^2}(-\cosh(2\operatorname{arcsinh}(cx))+2\operatorname{arcsinh}(cx)(\operatorname{arcsinh}(cx)+\sinh(2\operatorname{arcsinh}(cx))))}{8c\sqrt{1+c^2x^2}}$$

input
```
Integrate[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(a*Sqrt[d + c^2*d*x^2]*((2*g)/c^2 + x*(3*f + 2*g*x)))/6 - (b*g*Sqrt[d + c^
2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*c^2*Sq
rt[1 + c^2*x^2]) + (a*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/
(2*c) + (b*f*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(
ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(f+gx)(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int (f+gx)\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(f\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+gx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{1}{2}fx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{g(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}+\frac{f(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcfx^2-\frac{1}{9}b\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-1/3*(b*g*x)/c - (b*c*f*x^2)/4 - (b*c*g*x^3)/9 + (f*
x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1 + c^2*x^2)^(3/2)*(a +
b*ArcSinh[c*x]))/(3*c^2) + (f*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/Sqrt[1 + c
^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(189) = 378$.

Time = 1.62 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.63

method	result
default	$\frac{afx\sqrt{c^2dx^2+d}}{2}+\frac{afd\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+\frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\,\mathrm{arcsinh}(xc)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(4c^4x^4}{}\right.$
parts	$\frac{afx\sqrt{c^2dx^2+d}}{2}+\frac{afd\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+\frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\,\mathrm{arcsinh}(xc)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(4c^4x^4}{}\right.$

input
```
int((g*x+f)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*a*f*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2)+1/3*a*g*(c^2*d*x^2+d)^(3/2)/c^2/d+b*(1/4*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(x*c)^2+1/72*(d*(c^2*x^2+1))^(
1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*
x*c+1)*g*(-1+3*arcsinh(x*c))/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2
*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*f*(-1+2*arcs
inh(x*c))/(c^2*x^2+1)/c+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/
2)*x*c+1)*g*(arcsinh(x*c)-1)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^
2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*g*(arcsinh(x*c)+1)/c^2/(c^2*x^2+1)+1/16*(d*
(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1
)^(1/2))*f*(1+2*arcsinh(x*c))/(c^2*x^2+1)/c+1/72*(d*(c^2*x^2+1))^(1/2)*(4*
c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*g
*(1+3*arcsinh(x*c))/c^2/(c^2*x^2+1))
```

Fricas [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))\,dx$$
$$=\int \sqrt{c^2dx^2+d}(gx+f)(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x)), x
)
```

Sympy [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$=\int \sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))\,(f+gx)\;dx$$

input
```
integrate((g*x+f)*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$= \int (f+g\,x)\,\left(a+b\,\text{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$
$$= \frac{\sqrt{d}\,\left(3\sqrt{c^2x^2+1}\,a\,c^2fx+2\sqrt{c^2x^2+1}\,a\,c^2g\,x^2+2\sqrt{c^2x^2+1}\,ag+6\left(\int\sqrt{c^2x^2+1}\,asinh(cx)\,xdx\right)b\,c^2g+\right.}{6c^2}$$

input
```
int((g*x+f)*(c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*(3*sqrt(c**2*x**2 + 1)*a*c**2*f*x + 2*sqrt(c**2*x**2 + 1)*a*c**2*
g*x**2 + 2*sqrt(c**2*x**2 + 1)*a*g + 6*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*
x,x)*b*c**2*g + 6*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c**2*f + 3*log(s
qrt(c**2*x**2 + 1) + c*x)*a*c*f))/(6*c**2)
```

3.37 $\int \frac{\sqrt{d+c^2dx^2}(a+b\,\mathbf{arcsinh}(cx))}{f+gx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 664

$$\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx \\
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
a*(c^2*d*x^2+d)^(1/2)/g-b*c*x*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)+b*(c
^2*d*x^2+d)^(1/2)*arcsinh(c*x)/g-1/2*c*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(
c*x))^2/b/g/(c^2*x^2+1)^(1/2)-1/2*(1+c^2*f^2/g^2)*(c^2*d*x^2+d)^(1/2)*(a+b
*arcsinh(c*x))^2/b/c/(g*x+f)/(c^2*x^2+1)^(1/2)+1/2*(c^2*x^2+1)^(1/2)*(c^2*
d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(g*x+f)-a*(c^2*f^2+g^2)^(1/2)*(c^2
*d*x^2+d)^(1/2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)
)/g^2/(c^2*x^2+1)^(1/2)+b*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)*arcsinh(
c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/g^2/(c^2*x^
2+1)^(1/2)-b*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/g^2/(c^2*x^2+1)^(1/2)+b*
(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))
*g/(c*f-(c^2*f^2+g^2)^(1/2)))/g^2/(c^2*x^2+1)^(1/2)-b*(c^2*f^2+g^2)^(1/2)*
(c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2
)^(1/2)))/g^2/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.50 (sec) , antiderivative size = 1358, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(2*a*g*Sqrt[d + c^2*d*x^2] + 2*a*Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Log[f + g*x]
- 2*a*c*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 2*a*Sqrt[d]*S
qrt[c^2*f^2 + g^2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[
d + c^2*d*x^2]] + b*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*
g*ArcSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*((-I)*c*f + g
)*(I*c*f + g)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f
^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f +
I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*
I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sq
rt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh
[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*L
og[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*S
qrt[c*(f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*C
ot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f -
 I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/
2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(
f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c
*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4...
```

Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6260, 6254, 25, 6250, 25, 6271, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{f+gx}dx$$
$$\Big\downarrow \text{6260}$$
$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{f+gx}dx}{\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{6254}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)}-\frac{\int -\frac{\left(-gx^2c^2-2fxc^2+g\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^2}dx}{2bc}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\int \frac{\left(-gx^2c^2-2fxc^2+g\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^2}dx}{2bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6250$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{-2bc\int -\frac{\left(\frac{xc^2}{g}+\frac{\frac{c^2f^2}{g^2}+1}{f+gx}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{\left(\frac{c^2f^2}{g^2}+1\right)(a+b\text{arcsinh}(cx))^2}{f+gx}-\frac{c^2x(a+b\text{arcsinh}(cx))^2}{g}}{2bc}+\frac{(c^2x^2+1)(a+b\text{a}\cdots}{2bc(f\cdots}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{2bc\int \frac{\left(\frac{xc^2}{g}+\frac{\frac{c^2f^2}{g^2}+1}{f+gx}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{\left(\frac{c^2f^2}{g^2}+1\right)(a+b\text{arcsinh}(cx))^2}{f+gx}-\frac{c^2x(a+b\text{arcsinh}(cx))^2}{g}}{2bc}+\frac{(c^2x^2+1)(a+b\text{arcs}\cdots}{2bc(f+gx\cdots}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6271$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{2bc\int \left(\frac{b\text{arcsinh}(cx)\left(f^2c^2+g^2x^2c^2+fgxc^2+g^2\right)}{g^2(f+gx)\sqrt{c^2x^2+1}}+\frac{a\left(f^2c^2+g^2x^2c^2+fgxc^2+g^2\right)}{g^2(f+gx)\sqrt{c^2x^2+1}}\right)dx-\frac{\left(\frac{c^2f^2}{g^2}+1\right)(a+b\text{arcsinh}(cx))^2}{f+gx}-\frac{c^2x(a+b\text{ar}\cdots}{\cdots}}{2bc}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2009$$

$$\sqrt{c^2dx^2+d}\left(\frac{2bc\left(-\frac{a\sqrt{c^2f^2+g^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\right)}{g^2}+\frac{a\sqrt{c^2x^2+1}}{g}+\frac{b\sqrt{c^2f^2+g^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2}-\frac{b\sqrt{c^2f^2+g^2}\operatorname{PolyLo}}{}\right.}{}\right.$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g
*x)) + (-((c^2*x*(a + b*ArcSinh[c*x])^2)/g) - ((1 + (c^2*f^2)/g^2)*(a + b*
ArcSinh[c*x])^2)/(f + g*x) + 2*b*c*(-((b*c*x)/g) + (a*Sqrt[1 + c^2*x^2])/g
 + (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/g - (a*Sqrt[c^2*f^2 + g^2]*ArcTanh[(
g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/g^2 + (b*Sqrt[c^2*f
^2 + g^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^
2])])/g^2 - (b*Sqrt[c^2*f^2 + g^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)
/(c*f + Sqrt[c^2*f^2 + g^2])])/g^2 + (b*Sqrt[c^2*f^2 + g^2]*PolyLog[2, -((
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^2 - (b*Sqrt[c^2*f^2 + g
^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/g^2))/(
2*b*c)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6250
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Simp[(a + b*ArcSinh[c*x])^n   u, x] - Simp[b
*c*n   Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x
^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IG
tQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

rule 6254
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[1/(b*c*Sqrt[d]*(n +
1))   Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcS
inh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*
d] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

rule 6271
```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.12

method	result
default	$\frac{a\left(\sqrt{\left(x+\frac{f}{g}\right)^{2}c^{2}d-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}-\frac{c^{2}df\ln\left(\frac{-\frac{c^{2}df}{g}+\left(x+\frac{f}{g}\right)c^{2}d}{\sqrt{c^{2}d}}+\sqrt{\left(x+\frac{f}{g}\right)^{2}c^{2}d-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}\right)}{g\sqrt{c^{2}d}}-\frac{d\left(c^{2}f^{2}+\cdots\right.}{\cdots}\right.}{g}$
parts	$\frac{a\left(\sqrt{\left(x+\frac{f}{g}\right)^{2}c^{2}d-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}-\frac{c^{2}df\ln\left(\frac{-\frac{c^{2}df}{g}+\left(x+\frac{f}{g}\right)c^{2}d}{\sqrt{c^{2}d}}+\sqrt{\left(x+\frac{f}{g}\right)^{2}c^{2}d-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}\right)}{g\sqrt{c^{2}d}}-\frac{d\left(c^{2}f^{2}+\cdots\right.}{\cdots}\right.}{g}$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-c^2*d
*f/g*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*
f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g^2)/g^2/
(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+
2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*
f^2+g^2)/g^2)^(1/2))/(x+f/g)))+b*(-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*f*arcsinh(x*c)^2*c/g^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)
^(1/2)*x*c+1)*(arcsinh(x*c)-1)/(c^2*x^2+1)/g+1/2*(d*(c^2*x^2+1))^(1/2)*(c^
2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*(arcsinh(x*c)+1)/(c^2*x^2+1)/g+(d*(c^2*x^2+
1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(x*c)*ln((-(x*c+(c
^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-ar
csinh(x*c)*ln(((x*c+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^
2*f^2+g^2)^(1/2)))+dilog((-(x*c+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/
2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-dilog(((x*c+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*
f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))))/g^2)
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))}{f+gx}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
-(c*sqrt(d)*f*arcsinh(c*x)/g^2 - sqrt(c^2*d*f^2/g^2 + d)*arcsinh(c*f*x/abs
(g*x + f) - g/(c*abs(g*x + f)))/g - sqrt(c^2*d*x^2 + d)/g)*a + b*integrate
(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(g*x + f), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{f+g\,x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx$$
$$= \frac{\sqrt{d}\left(2\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)ai+\sqrt{c^2x^2+1}\,ag+\left(\int\frac{\sqrt{c^2x^2+1}\,asinh(cx)}{gx+f}dx\right)b\,g^2-\log\left(\sqrt{c^2x^2\right.\right.}{g^2}$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
(sqrt(d)*(2*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*i +
 c*g*i*x)/sqrt(c**2*f**2 + g**2))*a*i + sqrt(c**2*x**2 + 1)*a*g + int((sqr
t(c**2*x**2 + 1)*asinh(c*x))/(f + g*x),x)*b*g**2 - log(sqrt(c**2*x**2 + 1)
 + c*x)*a*c*f))/g**2
```

3.38 $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 781

$$
\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2}\,dx \\
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)}-\frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)}+\frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2(c^2f^2+g^2)\sqrt{1+c^2x^2}} \\
&\quad+\frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2(c^2f^2+g^2)\sqrt{1+c^2x^2}}-\frac{(g-c^2fx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(c^2f^2+g^2)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad+\frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad+\frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad-\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad+\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad+\frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}}-\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad+\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
-a*(c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)/g/(g*x
+f)+a*c^3*f^2*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)/g^2/(c^2*f^2+g^2)/(c^2*x^2+
1)^(1/2)+1/2*b*c^3*f^2*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)^2/g^2/(c^2*f^2+g^2
)/(c^2*x^2+1)^(1/2)-1/2*(-c^2*f*x+g)^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*
x))^2/b/c/(c^2*f^2+g^2)/(g*x+f)^2/(c^2*x^2+1)^(1/2)+1/2*(c^2*x^2+1)^(1/2)*
(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(g*x+f)^2+a*c^2*f*(c^2*d*x^2+
d)^(1/2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))/g^2/(
c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*f*(c^2*d*x^2+d)^(1/2)*arcsinh(c
*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/g^2/(c^2*f^2
+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+b*c^2*f*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)*ln(
1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/g^2/(c^2*f^2+g^2)^(
1/2)/(c^2*x^2+1)^(1/2)+b*c*(c^2*d*x^2+d)^(1/2)*ln(g*x+f)/g^2/(c^2*x^2+1)^(
1/2)-b*c^2*f*(c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
-(c^2*f^2+g^2)^(1/2)))/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+b*c^2*f*(
c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)
^(1/2)))/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.44 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

output
```
((-2*a*g*Sqrt[d + c^2*d*x^2])/(f + g*x) - (2*a*c^2*Sqrt[d]*f*Log[f + g*x])
/Sqrt[c^2*f^2 + g^2] + 2*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] + (2*a*c^2*Sqrt[d]*f*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]
*Sqrt[d + c^2*d*x^2]])/Sqrt[c^2*f^2 + g^2] + b*c*Sqrt[d + c^2*d*x^2]*((-2*
g*ArcSinh[c*x])/(c*f + c*g*x) + ArcSinh[c*x]^2/Sqrt[1 + c^2*x^2] + ((2*I)*
c*f*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/(Sqrt
[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (2*Log[1 + (g*x)/f])/Sqrt[1 + c^2*x^2
] + (2*c*f*(2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*Ar
cSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTan
h[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] +
 (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSin
h[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(
c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (A
rcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[
c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*A
rcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x
]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[(
(-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]
)/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*...
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6260, 6254, 27, 6249, 27, 6271, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{(f+gx)^2}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(f+gx)^2}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6254}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}-\frac{\int -\frac{2\left(g-c^2fx\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^3}dx}{2bc}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\int \frac{\left(g-c^2fx\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^3}dx}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6249}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{-2bc\int -\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))}{2\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{c^2x^2+1}}dx-\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))^2}{2\left(c^2f^2+g^2\right)(f+gx)^2}}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\frac{bc\int \frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))}{(f+gx)^2\sqrt{c^2x^2+1}}dx}{c^2f^2+g^2}-\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))^2}{2\left(c^2f^2+g^2\right)(f+gx)^2}}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6271}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\frac{bc\int \left(\frac{b\text{arcsinh}(cx)\left(c^2fx-g\right)^2}{(f+gx)^2\sqrt{c^2x^2+1}}+\frac{a\left(c^2fx-g\right)^2}{(f+gx)^2\sqrt{c^2x^2+1}}\right)dx}{c^2f^2+g^2}-\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))^2}{2\left(c^2f^2+g^2\right)(f+gx)^2}}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{bc\left(\frac{ac^3f^2\operatorname{arcsinh}(cx)}{g^2}+\frac{ac^2f\sqrt{c^2f^2+g^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\right)}{g^2}-\frac{a\sqrt{c^2x^2+1}\left(c^2f^2+g^2\right)}{g(f+gx)}+\frac{bc^3f^2\operatorname{arcsinh}(cx)^2}{2g^2}-\frac{bc^2f\sqrt{c\cdots}}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g
*x)^2) + (-1/2*((g - c^2*f*x)^2*(a + b*ArcSinh[c*x])^2)/((c^2*f^2 + g^2)*(
f + g*x)^2) + (b*c*(-((a*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2])/(g*(f + g*x)))
 + (a*c^3*f^2*ArcSinh[c*x])/g^2 - (b*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Arc
Sinh[c*x])/(g*(f + g*x)) + (b*c^3*f^2*ArcSinh[c*x]^2)/(2*g^2) + (a*c^2*f*S
qrt[c^2*f^2 + g^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2
*x^2])])/g^2 - (b*c^2*f*Sqrt[c^2*f^2 + g^2]*ArcSinh[c*x]*Log[1 + (E^ArcSin
h[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/g^2 + (b*c^2*f*Sqrt[c^2*f^2 + g^2]
*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g^2
 + (b*c*(c^2*f^2 + g^2)*Log[f + g*x])/g^2 - (b*c^2*f*Sqrt[c^2*f^2 + g^2]*P
olyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^2 + (b*c^2
*f*Sqrt[c^2*f^2 + g^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/g^2))/(c^2*f^2 + g^2))/(b*c)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6249
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Simp[(a + b*ArcSinh[c*x])^n   u, x] - Simp[b*c*n   Int[SimplifyInt
egrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x], x]] /; F
reeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] &&
 LtQ[m + p + 1, 0]
```

rule 6254
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[1/(b*c*Sqrt[d]*(n +
1))   Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcS
inh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*
d] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

rule 6271
```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1700 vs. $2(745) = 1490$.

Time = 1.60 (sec) , antiderivative size = 1701, normalized size of antiderivative = 2.18

method	result	size
default	Expression too large to display	1701
parts	Expression too large to display	1701

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(x*c))/(g*x+f)^2,x,method=_RETURNVERBO
SE)
```

output
```
a/g^2*(-1/d/(c^2*f^2+g^2)*g^2/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)
+d*(c^2*f^2+g^2)/g^2)^(3/2)-c^2*f*g/(c^2*f^2+g^2)*(((x+f/g)^2*c^2*d-2*c^2*
d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-c^2*d*f/g*ln((-c^2*d*f/g+(x+f/g)*
c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/
g^2)^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*
ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2
)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g)
))+2*c^2/(c^2*f^2+g^2)*g^2*(1/4*(2*(x+f/g)*c^2*d-2*c^2*d*f/g)/c^2/d*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(
c^2*f^2+g^2)/g^2-4*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c
^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2
))/(c^2*d)^(1/2)))+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(x
*c)^2*c/g^2+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/(c^2*x^2+1)/g^2/(g*x+f)*x
^3*c^4*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/g^2/(g*x+f)*x*c^2*f-b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(x*c)/(c^2*x^2+1)/g/(g*x+f)*x^2*c^2+b*(d*(c^2*x^2+1
))^(1/2)*arcsinh(x*c)/(c^2*x^2+1)^(1/2)/g/(g*x+f)*x*c+b*(d*(c^2*x^2+1))^(1
/2)*arcsinh(x*c)/(c^2*x^2+1)/g^2/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*a
rcsinh(x*c)/(c^2*x^2+1)^(1/2)/g^2/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcs
inh(x*c)/(c^2*x^2+1)/g/(g*x+f)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g
^2/(c^2*f^2+g^2)^(1/2)*c^2*ln((-(x*c+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+...
```

Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{(gx+f)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(g*x+f)^2,x, algorithm="f
ricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2
), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}\,(a+b\,\text{asinh}\,(cx))}{(f+gx)^2}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))/(g*x+f)**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{(gx+f)^2}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(g*x+f)^2,x, algorithm="m
axima")
```

output
```
-(c^2*d*f*arcsinh(c*f*x/(g*abs(x + f/g)) - 1/(c*abs(x + f/g)))/(sqrt(c^2*d
*f^2/g^2 + d)*g^3) - c*sqrt(d)*arcsinh(c*x)/g^2 + sqrt(c^2*d*x^2 + d)/(g^2
*x + f*g))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)
)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/(g*x+f)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{(f+g\,x)^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx$$
$$= \frac{\sqrt{d}\left(-2\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)a\,c^2f^2i-2\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)a\,c^2fgix-\right.}{}$$

input
```
int((c^2*d*x^2+d)^(1/2)*(a+b*asinh(c*x))/(g*x+f)^2,x)
```

output

```
(sqrt(d)*( - 2*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*
i + c*g*i*x)/sqrt(c**2*f**2 + g**2))*a*c**2*f**2*i - 2*sqrt(c**2*f**2 + g*
*2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*i + c*g*i*x)/sqrt(c**2*f**2 + g**2
))*a*c**2*f*g*i*x - sqrt(c**2*x**2 + 1)*a*c**2*f**2*g - sqrt(c**2*x**2 + 1
)*a*g**3 + int((sqrt(c**2*x**2 + 1)*asinh(c*x))/(f**2 + 2*f*g*x + g**2*x**
2),x)*b*c**2*f**3*g**2 + int((sqrt(c**2*x**2 + 1)*asinh(c*x))/(f**2 + 2*f*
g*x + g**2*x**2),x)*b*c**2*f**2*g**3*x + int((sqrt(c**2*x**2 + 1)*asinh(c*
x))/(f**2 + 2*f*g*x + g**2*x**2),x)*b*f*g**4 + int((sqrt(c**2*x**2 + 1)*as
inh(c*x))/(f**2 + 2*f*g*x + g**2*x**2),x)*b*g**5*x + log(sqrt(c**2*x**2 +
1) + c*x)*a*c**3*f**3 + log(sqrt(c**2*x**2 + 1) + c*x)*a*c**3*f**2*g*x + l
og(sqrt(c**2*x**2 + 1) + c*x)*a*c*f*g**2 + log(sqrt(c**2*x**2 + 1) + c*x)*
a*c*g**3*x))/(g**2*(c**2*f**3 + c**2*f**2*g*x + f*g**2 + g**3*x))
```

3.39 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 868

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx)) \, dx = -\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}+\frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{3bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} -\frac{3bdfg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{2bcdf^2gx^3\sqrt{d+c^2dx^2}}{5\sqrt{1+c^2x^2}} -\frac{bdg^3x^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{7bcdfg^2x^4\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}}-\frac{3bc^3df^2gx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} -\frac{8bcdg^3x^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3dfg^2x^6\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x^2}}-\frac{bc^3dg^3x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} -\frac{bdf^3(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{16c}+\frac{3}{8}df^3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) +\frac{3dfg^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c^2}+\frac{3}{8}dfg^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) +\frac{1}{4}f^3x\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{2}fg^2x^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3f^2g(d+c^2dx^2)^{5/2}(a+}{5c^2d}$$

output
```
-3/5*b*d*f^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)+2/35*b*d*g^3*x*(c
^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-3/16*b*c*d*f^3*x^2*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-3/32*b*d*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1
)^(1/2)-2/5*b*c*d*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/105*b*
d*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/32*b*c*d*f*g^2*x^4*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/25*b*c^3*d*f^2*g*x^5*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)-8/175*b*c*d*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-1/12*b*c^3*d*f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*
c^3*d*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*d*f^3*(c^2*x^2+
1)^(3/2)*(c^2*d*x^2+d)^(1/2)/c+3/8*d*f^3*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsin
h(c*x))+3/16*d*f*g^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c^2+3/8*d*f*
g^2*x^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+1/4*f^3*x*(c^2*d*x^2+d)^(3/
2)*(a+b*arcsinh(c*x))+1/2*f*g^2*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))
+3/5*f^2*g*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^2/d-1/5*g^3*(c^2*d*x^2
+d)^(5/2)*(a+b*arcsinh(c*x))/c^4/d+1/7*g^3*(c^2*d*x^2+d)^(7/2)*(a+b*arcsin
h(c*x))/c^4/d^2+3/16*d*f^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c
^2*x^2+1)^(1/2)-3/32*d*f*g^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c^
3/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.62

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(-1680a\sqrt{1+c^2x^2}(-32g^3+c^2g(336f^2+105fgx+16g^2x^2)+4c^6x^3(3\cdots\right.}{\cdots}$$

input
```
Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^2*(1 + c^2*x^2)*(-1680*a*Sqrt[1 + c^2*x^2]*(-32*g^3 + c^2*g*(336*f^2
+ 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*x + 70*f*g^2*x^2
+ 20*g^3*x^3) + 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2*x^2 + 64*g^3*x^
3)) + b*(-35*c*g^2*(245*f + 1536*g*x) + 70*c^3*(1785*f^3 + 8064*f^2*g*x +
1260*f*g^2*x^2 + 128*g^3*x^3) + 168*c^5*x^2*(1750*f^3 + 2240*f^2*g*x + 122
5*f*g^2*x^2 + 256*g^3*x^3) + 16*c^7*x^4*(3675*f^3 + 7056*f^2*g*x + 4900*f*
g^2*x^2 + 1200*g^3*x^3)))) + 88200*b*c*d^2*f*(2*c^2*f^2 - g^2)*(1 + c^2*x^
2)*ArcSinh[c*x]^2 + 176400*a*c*d^(3/2)*f*(2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^
2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 420*b*d^
2*(1 + c^2*x^2)*ArcSinh[c*x]*(35*c*f*(16*c^2*f^2 - 3*g^2)*Sinh[2*ArcSinh[c
*x]] + 35*c*f*(2*c^2*f^2 + 3*g^2)*Sinh[4*ArcSinh[c*x]] + g*(64*(1 + c^2*x^
2)^(5/2)*(-2*g^2 + c^2*(21*f^2 + 5*g^2*x^2)) + 35*c*f*g*Sinh[6*ArcSinh[c*x
]])))/(940800*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(f+gx)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (f+gx)^3\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f^3+3gx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f^2+3g^2x^2\left(c^2x^2+1\right)\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$d\sqrt{c^2dx^2+d}\left(-\frac{3fg^2(a+b\text{arcsinh}(cx))^2}{32bc^3}+\frac{1}{4}f^3x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}f^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx$$

input
```
Int[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((-3*b*f^2*g*x)/(5*c) + (2*b*g^3*x)/(35*c^3) - (5*b
*c*f^3*x^2)/16 - (3*b*f*g^2*x^2)/(32*c) - (2*b*c*f^2*g*x^3)/5 - (b*g^3*x^3
)/(105*c) - (b*c^3*f^3*x^4)/16 - (7*b*c*f*g^2*x^4)/32 - (3*b*c^3*f^2*g*x^5
)/25 - (8*b*c*g^3*x^5)/175 - (b*c^3*f*g^2*x^6)/12 - (b*c^3*g^3*x^7)/49 + (
3*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*f*g^2*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*f*g^2*x^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/8 + (f^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4
+ (f*g^2*x^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/2 + (3*f^2*g*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2) - (g^3*(1 + c^2*x^2)^(5/2)*(a
+ b*ArcSinh[c*x]))/(5*c^4) + (g^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x])
)/(7*c^4) + (3*f^3*(a + b*ArcSinh[c*x])^2)/(16*b*c) - (3*f*g^2*(a + b*ArcS
inh[c*x])^2)/(32*b*c^3)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. $2(752) = 1504$.

Time = 1.43 (sec) , antiderivative size = 2079, normalized size of antiderivative = 2.40

method	result	size
default	Expression too large to display	2079
parts	Expression too large to display	2079

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)))+g^3*(1/7*x^2*
(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+3*f*g^2*(1/6*x*(
c^2*d*x^2+d)^(5/2)/c^2/d-1/6/c^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(
c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^
2*d)^(1/2))))+3/5*f^2*g*(c^2*d*x^2+d)^(5/2)/c^2/d)+b*(3/32*(d*(c^2*x^2+1))
^(1/2)*f*arcsinh(x*c)^2*(2*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/c^3+1/6272*(d*
(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*x^7*c^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+11
2*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*c^3*x^3+25*c^
2*x^2+7*(c^2*x^2+1)^(1/2)*x*c+1)*g^3*(-1+7*arcsinh(x*c))*d/c^4/(c^2*x^2+1)
+1/768*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x
^5*c^5+48*x^4*c^4*(c^2*x^2+1)^(1/2)+38*x^3*c^3+18*x^2*c^2*(c^2*x^2+1)^(1/2
)+6*x*c+(c^2*x^2+1)^(1/2))*f*g^2*(-1+6*arcsinh(x*c))*d/c^3/(c^2*x^2+1)+1/3
200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*
x^4+20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*g*(
60*arcsinh(x*c)*c^2*f^2-12*c^2*f^2+5*arcsinh(x*c)*g^2-g^2)*d/c^4/(c^2*x^2+
1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x
^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*f*(8*arcsinh(x
*c)*c^2*f^2-2*c^2*f^2+12*arcsinh(x*c)*g^2-3*g^2)*d/c^3/(c^2*x^2+1)+1/384*(
d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3...
```

Fricas [F]

$$\int (f+gx)^3 \left(d + c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)^3(b\,\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 + 3*a*d*f^2*g*x + a*d*f^3
+ (3*a*c^2*d*f^2*g + a*d*g^3)*x^3 + (a*c^2*d*f^3 + 3*a*d*f*g^2)*x^2 + (b*c
^2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 + 3*b*d*f^2*g*x + b*d*f^3 + (3*b*c^2*d*
f^2*g + b*d*g^3)*x^3 + (b*c^2*d*f^3 + 3*b*d*f*g^2)*x^2)*arcsinh(c*x))*sqrt
(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))\,(f+gx)^3\,dx$$

input
```
integrate((g*x+f)**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{\sqrt{d}\,d\left(140\sqrt{c^2x^2+1}\,a\,c^6f^3x^3+336\sqrt{c^2x^2+1}\,a\,c^6f^2g\,x^4+280\sqrt{c^2x^2+1}\,a\,c^6f\,g^2x^5+\right.}{}$$

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x)),x)
```

output

```
(sqrt(d)*d*(140*sqrt(c**2*x**2 + 1)*a*c**6*f**3*x**3 + 336*sqrt(c**2*x**2
+ 1)*a*c**6*f**2*g*x**4 + 280*sqrt(c**2*x**2 + 1)*a*c**6*f*g**2*x**5 + 80*
sqrt(c**2*x**2 + 1)*a*c**6*g**3*x**6 + 350*sqrt(c**2*x**2 + 1)*a*c**4*f**3
*x + 672*sqrt(c**2*x**2 + 1)*a*c**4*f**2*g*x**2 + 490*sqrt(c**2*x**2 + 1)*
a*c**4*f*g**2*x**3 + 128*sqrt(c**2*x**2 + 1)*a*c**4*g**3*x**4 + 336*sqrt(c
**2*x**2 + 1)*a*c**2*f**2*g + 105*sqrt(c**2*x**2 + 1)*a*c**2*f*g**2*x + 16
*sqrt(c**2*x**2 + 1)*a*c**2*g**3*x**2 - 32*sqrt(c**2*x**2 + 1)*a*g**3 + 56
0*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**5,x)*b*c**6*g**3 + 1680*int(sqrt(c
**2*x**2 + 1)*asinh(c*x)*x**4,x)*b*c**6*f*g**2 + 1680*int(sqrt(c**2*x**2 +
 1)*asinh(c*x)*x**3,x)*b*c**6*f**2*g + 560*int(sqrt(c**2*x**2 + 1)*asinh(c
*x)*x**3,x)*b*c**4*g**3 + 560*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b
*c**6*f**3 + 1680*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*c**4*f*g**2
 + 1680*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x,x)*b*c**4*f**2*g + 560*int(sq
rt(c**2*x**2 + 1)*asinh(c*x),x)*b*c**4*f**3 + 210*log(sqrt(c**2*x**2 + 1)
+ c*x)*a*c**3*f**3 - 105*log(sqrt(c**2*x**2 + 1) + c*x)*a*c*f*g**2))/(560*
c**4)
```

3.40 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 619

$$\begin{aligned}\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx &= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} \\ &- \frac{3bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bdg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{4bcdfgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} \\ &- \frac{7bcdg^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bc^3dfgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{bc^3dg^2x^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} \\ &- \frac{bdf^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}}{16c} + \frac{3}{8}df^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\ &+ \frac{dg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\ &+ \frac{1}{4}f^2x\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{6}g^2x^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx)) + \frac{2fg\left(d+c^2dx^2\right)^{5/2}(a+b\text{a}}{5c^2d}\end{aligned}$$

output
```
-2/5*b*d*f*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/16*b*c*d*f^2*x^2*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(c^2*d*x^2+d)^(1/2)
/c/(c^2*x^2+1)^(1/2)-4/15*b*c*d*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-7/96*b*c*d*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*
f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/36*b*c^3*d*g^2*x^6*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*d*f^2*(c^2*x^2+1)^(3/2)*(c^2*d*x^2+d
)^(1/2)/c+3/8*d*f^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+1/16*d*g^2*x*
(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c^2+1/8*d*g^2*x^3*(c^2*d*x^2+d)^(1/
2)*(a+b*arcsinh(c*x))+1/4*f^2*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6
*g^2*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+2/5*f*g*(c^2*d*x^2+d)^(5/2
)*(a+b*arcsinh(c*x))/c^2/d+3/16*d*f^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x
))^2/b/c/(c^2*x^2+1)^(1/2)-1/32*d*g^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x
))^2/b/c^3/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.67

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\Big(b(-175g^2+90c^2(85f^2+256fgx+20g^2x^2)+120c^4x^2(150f^2+128f}{}$$

input
```
Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^2*(1 + c^2*x^2)*(b*(-175*g^2 + 90*c^2*(85*f^2 + 256*f*g*x + 20*g^2*x^
2) + 120*c^4*x^2*(150*f^2 + 128*f*g*x + 35*g^2*x^2) + 16*c^6*x^4*(225*f^2
+ 288*f*g*x + 100*g^2*x^2)) - 240*a*c*Sqrt[1 + c^2*x^2]*(96*f*g*(1 + c^2*x
^2)^2 + 30*c^2*f^2*x*(5 + 2*c^2*x^2) + 5*g^2*x*(3 + 14*c^2*x^2 + 8*c^4*x^4
)))) + 1800*b*d^2*(6*c^2*f^2 - g^2)*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 3600*a*
d^(3/2)*(6*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 60*b*d^2*(1 + c^2*x^2)*ArcSinh[c*x]*(15*(
16*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 15*(2*c^2*f^2 + g^2)*Sinh[4*ArcSi
nh[c*x]] + g*(384*c*f*(1 + c^2*x^2)^(5/2) + 5*g*Sinh[6*ArcSinh[c*x]])))/(5
7600*c^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(f+gx)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (f+gx)^2\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f^2+2gx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f+g^2x^2\left(c^2x^2+1\right)^{3/}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\sqrt{c^2dx^2+d}\left(-\frac{g^2(a+b\text{arcsinh}(cx))^2}{32bc^3}+\frac{1}{4}f^2x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}f^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)\right)}{}$$

input
```
Int[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((-2*b*f*g*x)/(5*c) - (5*b*c*f^2*x^2)/16 - (b*g^2*x
^2)/(32*c) - (4*b*c*f*g*x^3)/15 - (b*c^3*f^2*x^4)/16 - (7*b*c*g^2*x^4)/96
- (2*b*c^3*f*g*x^5)/25 - (b*c^3*g^2*x^6)/36 + (3*f^2*x*Sqrt[1 + c^2*x^2]*(
a + b*ArcSinh[c*x]))/8 + (g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(1
6*c^2) + (g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/8 + (f^2*x*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (g^2*x^3*(1 + c^2*x^2)^(3/2)*(a +
 b*ArcSinh[c*x]))/6 + (2*f*g*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*
c^2) + (3*f^2*(a + b*ArcSinh[c*x])^2)/(16*b*c) - (g^2*(a + b*ArcSinh[c*x])
^2)/(32*b*c^3)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. 2(535) = 1070.

Time = 1.37 (sec) , antiderivative size = 1568, normalized size of antiderivative = 2.53

method	result	size
default	Expression too large to display	1568
parts	Expression too large to display	1568

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)))+g^2*(1/6*x*(c
^2*d*x^2+d)^(5/2)/c^2/d-1/6/c^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c
^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2
*d)^(1/2))))+2/5*f*g*(c^2*d*x^2+d)^(5/2)/c^2/d)+b*(1/32*(d*(c^2*x^2+1))^(1
/2)*arcsinh(x*c)^2*(6*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/c^3+1/2304*(d*(c^2*
x^2+1))^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x^5*c^5+48*x^4*c
^4*(c^2*x^2+1)^(1/2)+38*x^3*c^3+18*x^2*c^2*(c^2*x^2+1)^(1/2)+6*x*c+(c^2*x^
2+1)^(1/2))*g^2*(-1+6*arcsinh(x*c))*d/c^3/(c^2*x^2+1)+1/400*(d*(c^2*x^2+1)
)^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)
^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(-1+5*arcsinh(x*c
))*d/c^2/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2
*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/
2))*(8*arcsinh(x*c)*c^2*f^2-2*c^2*f^2+4*arcsinh(x*c)*g^2-g^2)*d/c^3/(c^2*x
^2+1)+1/48*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*
c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(-1+3*arcsinh(x*c))*d/c^2/(c^2*x^2+
1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*
c+(c^2*x^2+1)^(1/2))*(32*arcsinh(x*c)*c^2*f^2-16*c^2*f^2-2*arcsinh(x*c)*g^
2+g^2)*d/c^3/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1
/2)*x*c+1)*f*g*(arcsinh(x*c)-1)*d/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(...
```

Fricas [F]

$$\int (f+gx)^2 \left(d + c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 + 2*a*d*f*g*x + a*d*f^2 + (a
*c^2*d*f^2 + a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 + 2*b*d*f
*g*x + b*d*f^2 + (b*c^2*d*f^2 + b*d*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2
 + d), x)
```

Sympy [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \frac{\sqrt{d}\,d\left(60\sqrt{c^2x^2+1}\,a\,c^5f^2x^3+96\sqrt{c^2x^2+1}\,a\,c^5fg\,x^4+40\sqrt{c^2x^2+1}\,a\,c^5g^2x^5+150\sqrt{c}\right.}{}$$

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x)),x)
```

output

```
(sqrt(d)*d*(60*sqrt(c**2*x**2 + 1)*a*c**5*f**2*x**3 + 96*sqrt(c**2*x**2 +
1)*a*c**5*f*g*x**4 + 40*sqrt(c**2*x**2 + 1)*a*c**5*g**2*x**5 + 150*sqrt(c*
*2*x**2 + 1)*a*c**3*f**2*x + 192*sqrt(c**2*x**2 + 1)*a*c**3*f*g*x**2 + 70*
sqrt(c**2*x**2 + 1)*a*c**3*g**2*x**3 + 96*sqrt(c**2*x**2 + 1)*a*c*f*g + 15
*sqrt(c**2*x**2 + 1)*a*c*g**2*x + 240*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x
**4,x)*b*c**5*g**2 + 480*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**3,x)*b*c**5
*f*g + 240*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*c**5*f**2 + 240*in
t(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*c**3*g**2 + 480*int(sqrt(c**2*x
**2 + 1)*asinh(c*x)*x,x)*b*c**3*f*g + 240*int(sqrt(c**2*x**2 + 1)*asinh(c*
x),x)*b*c**3*f**2 + 90*log(sqrt(c**2*x**2 + 1) + c*x)*a*c**2*f**2 - 15*log
(sqrt(c**2*x**2 + 1) + c*x)*a*g**2))/(240*c**3)
```

3.41 $\int (f+gx) \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx)) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 331

$$\int (f+gx) \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx)) \, dx = -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{3bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{2bcdgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{bc^3dgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{bdf(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{16c} + \frac{3}{8}dfx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{1}{4}fx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx)) + \frac{g(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{5c^2d} + \frac{3df\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(\ldots}{16bc\sqrt{1+c^2x^2}}$$

output
```
-1/5*b*d*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/16*b*c*d*f*x^2*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/15*b*c*d*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/
16*b*d*f*(c^2*x^2+1)^(3/2)*(c^2*d*x^2+d)^(1/2)/c+3/8*d*f*x*(c^2*d*x^2+d)^(
1/2)*(a+b*arcsinh(c*x))+1/4*f*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/5
*g*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^2/d+3/16*d*f*(c^2*d*x^2+d)^(1/
2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(-240a\sqrt{1+c^2x^2}\left(8g(1+c^2x^2)^2+5c^2fx(5+2c^2x^2)\right)+bc(128gx(15\right.}{}$$

input
```
Integrate[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^2*(1 + c^2*x^2)*(-240*a*Sqrt[1 + c^2*x^2]*(8*g*(1 + c^2*x^2)^2 + 5*c^
2*f*x*(5 + 2*c^2*x^2)) + b*c*(128*g*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 75*f
*(17 + 40*c^2*x^2 + 8*c^4*x^4)))) + 1800*b*c*d^2*f*(1 + c^2*x^2)*ArcSinh[c
*x]^2 + 3600*a*c*d^(3/2)*f*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
 + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 60*b*d^2*(1 + c^2*x^2)*ArcSinh[c*x]*(32*
g*(1 + c^2*x^2)^(5/2) + 40*c*f*Sinh[2*ArcSinh[c*x]] + 5*c*f*Sinh[4*ArcSinh
[c*x]]))/(9600*c^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.57, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(f+gx)(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (f+gx)\left(c^2x^2+1\right)^{3/2}(a+b\operatorname{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{c^2dx^2+d}\int\left(f(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}+gx(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}\right)dx}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$\frac{d\sqrt{c^2dx^2+d}\left(\frac{1}{4}fx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}fx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{g\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}}{5c^2}\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(-1/5*(b*g*x)/c - (5*b*c*f*x^2)/16 - (2*b*c*g*x^3)/
15 - (b*c^3*f*x^4)/16 - (b*c^3*g*x^5)/25 + (3*f*x*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/8 + (f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (g*
(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*f*(a + b*ArcSinh[c*
x])^2)/(16*b*c)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
 + b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(283) = 566.

Time = 1.87 (sec) , antiderivative size = 1065, normalized size of antiderivative = 3.22

method	result	size
default	Expression too large to display	1065
parts	Expression too large to display	1065

input
```
int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE
)
```

output
```
1/4*a*f*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2*
ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/5*a*g*(c^2*d
*x^2+d)^(5/2)/c^2/d+b*(3/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*ar
csinh(x*c)^2*d+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2
)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)
^(1/2)*x*c+1)*g*(-1+5*arcsinh(x*c))*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1)
)^(1/2)*(8*x^5*c^5+8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x
^2+1)^(1/2)+4*x*c+(c^2*x^2+1)^(1/2))*f*(-1+4*arcsinh(x*c))*d/(c^2*x^2+1)/c
+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x
^2+3*(c^2*x^2+1)^(1/2)*x*c+1)*g*(-1+3*arcsinh(x*c))*d/c^2/(c^2*x^2+1)+1/16
*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x
^2+1)^(1/2))*f*(-1+2*arcsinh(x*c))*d/(c^2*x^2+1)/c+1/16*(d*(c^2*x^2+1))^(1
/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*g*(arcsinh(x*c)-1)*d/c^2/(c^2*x^2+1)
+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*g*(arcsinh(x
*c)+1)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(
c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*f*(1+2*arcsinh(x*c))*d/(c^2*x^2+
1)/c+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c
^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*g*(1+3*arcsinh(x*c))*d/c^2/(c^2*x^2+1)+1
/256*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5-8*x^4*c^4*(c^2*x^2+1)^(1/2)+12*x^3*c
^3-8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c-(c^2*x^2+1)^(1/2))*f*(1+4*arcsinh(...
```

Fricas [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fri
cas")
```

output
```
integral((a*c^2*d*g*x^3 + a*c^2*d*f*x^2 + a*d*g*x + a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 + b*d*g*x + b*d*f)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))\,(f+gx)\,dx$$

input
```
integrate((g*x+f)*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\,(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (f+g\,x)\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{\sqrt{d}\,d\left(10\sqrt{c^2x^2+1}\,a\,c^4f\,x^3+8\sqrt{c^2x^2+1}\,a\,c^4g\,x^4+25\sqrt{c^2x^2+1}\,a\,c^2f\,x+16\sqrt{c^2x^2+\right.}{}$$

input
```
int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*d*(10*sqrt(c**2*x**2 + 1)*a*c**4*f*x**3 + 8*sqrt(c**2*x**2 + 1)*a
*c**4*g*x**4 + 25*sqrt(c**2*x**2 + 1)*a*c**2*f*x + 16*sqrt(c**2*x**2 + 1)*
a*c**2*g*x**2 + 8*sqrt(c**2*x**2 + 1)*a*g + 40*int(sqrt(c**2*x**2 + 1)*asi
nh(c*x)*x**3,x)*b*c**4*g + 40*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b
*c**4*f + 40*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x,x)*b*c**2*g + 40*int(sqr
t(c**2*x**2 + 1)*asinh(c*x),x)*b*c**2*f + 15*log(sqrt(c**2*x**2 + 1) + c*x
)*a*c*f))/(40*c**2)
```

3.42 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 974

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx = \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} + \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} - \frac{d(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)\sqrt{1+c^2x^2}} + \frac{d(c^2f^2+g^2)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^2(f+gx)} - \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}}$$

output
```
a*d*(c^2*f^2+g^2)*(c^2*d*x^2+d)^(1/2)/g^3-1/3*b*c*d*x*(c^2*d*x^2+d)^(1/2)/
g/(c^2*x^2+1)^(1/2)-b*c*d*(c^2*f^2+g^2)*x*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*x^2
+1)^(1/2)+1/4*b*c^3*d*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-1/9*
b*c^3*d*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)+b*d*(c^2*f^2+g^2)*(c^2
*d*x^2+d)^(1/2)*arcsinh(c*x)/g^3-1/2*c^2*d*f*x*(c^2*d*x^2+d)^(1/2)*(a+b*ar
csinh(c*x))/g^2+1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/g-1/4*c*d*f*(c^
2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/g^2/(c^2*x^2+1)^(1/2)-1/2*c*d*(c^2
*f^2+g^2)*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/g^3/(c^2*x^2+1)^(1/
2)-1/2*d*(c^2*f^2+g^2)^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/g^4/
(g*x+f)/(c^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2+g^2)*(c^2*x^2+1)^(1/2)*(c^2*d*x^2
+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/g^2/(g*x+f)-a*d*(c^2*f^2+g^2)^(3/2)*(c^
2*d*x^2+d)^(1/2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2
))/g^4/(c^2*x^2+1)^(1/2)+b*d*(c^2*f^2+g^2)^(3/2)*(c^2*d*x^2+d)^(1/2)*arcsi
nh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/g^4/(c^2
*x^2+1)^(1/2)-b*d*(c^2*f^2+g^2)^(3/2)*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)*ln(
1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/g^4/(c^2*x^2+1)^(1/
2)+b*d*(c^2*f^2+g^2)^(3/2)*(c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)
^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/g^4/(c^2*x^2+1)^(1/2)-b*d*(c^2*f^2+g^
2)^(3/2)*(c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^
2*f^2+g^2)^(1/2)))/g^4/(c^2*x^2+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.88 (sec) , antiderivative size = 2869, normalized size of antiderivative = 2.95

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(a*d*Sqrt[d + c^2*d*x^2]*(8*g^2 + c^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)))/(6*g
^3) + (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*c*d^(3/2)*f*
(2*c^2*f^2 + 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*g^4) - (a
*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2
+ g^2]*Sqrt[d + c^2*d*x^2]])/g^4 + (b*d*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sq
rt[1 + c^2*x^2] + 2*g*ArcSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2
] + (2*((-I)*c*f + g)*(I*c*f + g)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh
[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)
/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
 - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*
ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*
ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g
^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-
(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*
x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(Ar
cTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2
]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2
) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(S
qrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((
c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*L...
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6255, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6255}$$

$$\frac{d\sqrt{c^2dx^2+d}\int\left(\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^2}{g}-\frac{f\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^2}{g^2}+\frac{(c^2f^2+g^2)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{g^2(f+gx)}\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)(c^2f^2+g^2)(a+b\text{arcsinh}(cx))^2}{2bcg^2(f+gx)}-\frac{(c^2f^2+g^2)^2(a+b\text{arcsinh}(cx))^2}{2bcg^4(f+gx)}-\frac{cx(c^2f^2+g^2)(a+b\text{arcsinh}(cx))^2}{2bg^3}-\frac{c^2j}{}\right.}{}$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(-1/3*(b*c*x)/g - (b*c*(c^2*f^2 + g^2)*x)/g^3 + (b*
c^3*f*x^2)/(4*g^2) - (b*c^3*x^3)/(9*g) + (a*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x
^2])/g^3 + (b*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/g^3 - (c^2*f
*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + ((1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x]))/(3*g) - (c*f*(a + b*ArcSinh[c*x])^2)/(4*b*g^2) - (c*
(c^2*f^2 + g^2)*x*(a + b*ArcSinh[c*x])^2)/(2*b*g^3) - ((c^2*f^2 + g^2)^2*(
a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) + ((c^2*f^2 + g^2)*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*(c^2*f^2 + g^2)^(3/
2)*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/g^4 + (
b*(c^2*f^2 + g^2)^(3/2)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqr
t[c^2*f^2 + g^2])])/g^4 - (b*(c^2*f^2 + g^2)^(3/2)*ArcSinh[c*x]*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g^4 + (b*(c^2*f^2 + g^2)^(3
/2)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^4 - (
b*(c^2*f^2 + g^2)^(3/2)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))])/g^4))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6255
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 1557, normalized size of antiderivative = 1.60

method	result	size
default	Expression too large to display	1557
parts	Expression too large to display	1557

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(x*c))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*(1/3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)-c
^2*d*f/g*(1/4*(2*(x+f/g)*c^2*d-2*c^2*d*f/g)/c^2/d*((x+f/g)^2*c^2*d-2*c^2*d
*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(c^2*f^2+g^2)/g^2-4
*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2))+
d*(c^2*f^2+g^2)/g^2*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/
g^2)^(1/2)-c^2*d*f/g*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^
2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-d*(c
^2*f^2+g^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^
2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g
*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))))+b*(d*(c^2*x^2+1))^(1/2)*d/
(c^2*x^2+1)/g^3*arcsinh(x*c)*x^2*c^4*f^2-b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^
2+1)^(1/2)/g^3*x*c^3*f^2-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^4*d/(c^2*x^2+1)/g
^2*arcsinh(x*c)*x^3+1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c^3*d/(c^2*x^2+1)^(1/2)/
g^2*x^2-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^2*d/(c^2*x^2+1)/g^2*arcsinh(x*c)*x
+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsi
nh(x*c)*ln((-(x*c+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2
*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/g^4*arcsinh(x*c)*ln(((x*c+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(
1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2...
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{f + gx} \, dx = \int \frac{(d(c^2x^2 + 1))^{\frac{3}{2}} (a + b \operatorname{asinh}(cx))}{f + gx} \, dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{f + gx} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{f + gx} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^{3/2}}{f+g\,x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \frac{\sqrt{d}\,d\left(12\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)a\,c^2f^2i+12\sqrt{c^2f^2}\right.}{}$$

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
(sqrt(d)*d*(12*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*
i + c*g*i*x)/sqrt(c**2*f**2 + g**2))*a*c**2*f**2*i + 12*sqrt(c**2*f**2 + g
**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*i + c*g*i*x)/sqrt(c**2*f**2 + g**
2))*a*g**2*i + 6*sqrt(c**2*x**2 + 1)*a*c**2*f**2*g - 3*sqrt(c**2*x**2 + 1)
*a*c**2*f*g**2*x + 2*sqrt(c**2*x**2 + 1)*a*c**2*g**3*x**2 + 8*sqrt(c**2*x*
*2 + 1)*a*g**3 + 6*int((sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2)/(f + g*x),x)*
b*c**2*g**4 + 6*int((sqrt(c**2*x**2 + 1)*asinh(c*x))/(f + g*x),x)*b*g**4 -
 6*log(sqrt(c**2*x**2 + 1) + c*x)*a*c**3*f**3 - 9*log(sqrt(c**2*x**2 + 1)
+ c*x)*a*c*f*g**2))/(6*g**4)
```

3.43 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\mathbf{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 1142

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Too large to display}$$

output
```
2/63*b*d^2*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-5/32*b*c*d^2*f^
3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/189*b*d^2*g^3*x^3*(c^2*d*x^2
+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/21*b*c*d^2*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-19/441*b*c^3*d^2*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-1/81*b*c^5*d^2*g^3*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+15/128*d^
2*f*g^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c^2+5/32*d^2*f^3*(c^2*d*x
^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)+15/64*d^2*f*g^2*x^3
*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))-5/96*b*d^2*f^3*(c^2*x^2+1)^(3/2)*(
c^2*d*x^2+d)^(1/2)/c-1/36*b*d^2*f^3*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/
c+5/24*d*f^3*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+3/8*f*g^2*x^3*(c^2*d
*x^2+d)^(5/2)*(a+b*arcsinh(c*x))-1/7*g^3*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(
c*x))/c^4/d+1/9*g^3*(c^2*d*x^2+d)^(9/2)*(a+b*arcsinh(c*x))/c^4/d^2+1/6*f^3
*x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))+5/16*d^2*f^3*x*(c^2*d*x^2+d)^(1/
2)*(a+b*arcsinh(c*x))+3/7*f^2*g*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2
/d+5/16*d*f*g^2*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-59/256*b*c*d^2*
f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-15/256*d^2*f*g^2*(c^2*d*x^
2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c^3/(c^2*x^2+1)^(1/2)-9/35*b*c^3*d^2*f^2
*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-17/96*b*c^3*d^2*f*g^2*x^6*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/49*b*c^5*d^2*f^2*g*x^7*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)-3/64*b*c^5*d^2*f*g^2*x^8*(c^2*d*x^2+d)^(1/2)/(c...
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 810, normalized size of antiderivative = 0.71

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(-20160a\sqrt{1+c^2x^2}(-256g^3+c^2g(3456f^2+945fgx+128g^2x^2)+16\right.}{}$$

input
```
Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^3*(1 + c^2*x^2)*(-20160*a*Sqrt[1 + c^2*x^2]*(-256*g^3 + c^2*g*(3456*f
^2 + 945*f*g*x + 128*g^2*x^2) + 16*c^8*x^5*(84*f^3 + 216*f^2*g*x + 189*f*g
^2*x^2 + 56*g^3*x^3) + 8*c^6*x^3*(546*f^3 + 1296*f^2*g*x + 1071*f*g^2*x^2
+ 304*g^3*x^3) + 6*c^4*x*(924*f^3 + 1728*f^2*g*x + 1239*f*g^2*x^2 + 320*g^
3*x^3)) + b*(-315*c*g^2*(7539*f + 16384*g*x) + 30240*c^5*x^2*(1848*f^3 + 2
304*f^2*g*x + 1239*f*g^2*x^2 + 256*g^3*x^3) + 3360*c^3*(6279*f^3 + 20736*f
^2*g*x + 2835*f*g^2*x^2 + 256*g^3*x^3) + 2304*c^7*x^4*(9555*f^3 + 18144*f^
2*g*x + 12495*f*g^2*x^2 + 3040*g^3*x^3) + 640*c^9*x^6*(7056*f^3 + 15552*f^
2*g*x + 11907*f*g^2*x^2 + 3136*g^3*x^3)))) + 3175200*b*c*d^3*f*(8*c^2*f^2
- 3*g^2)*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 6350400*a*c*d^(5/2)*f*(8*c^2*f^2 -
 3*g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
 c^2*d*x^2]] + 2520*b*d^3*(1 + c^2*x^2)*ArcSinh[c*x]*(27648*c^2*f^2*g*Sqrt
[1 + c^2*x^2] - 2048*g^3*Sqrt[1 + c^2*x^2] + 82944*c^4*f^2*g*x^2*Sqrt[1 +
c^2*x^2] + 1024*c^2*g^3*x^2*Sqrt[1 + c^2*x^2] + 82944*c^6*f^2*g*x^4*Sqrt[1
 + c^2*x^2] + 15360*c^4*g^3*x^4*Sqrt[1 + c^2*x^2] + 27648*c^8*f^2*g*x^6*Sq
rt[1 + c^2*x^2] + 19456*c^6*g^3*x^6*Sqrt[1 + c^2*x^2] + 7168*c^8*g^3*x^8*S
qrt[1 + c^2*x^2] + 3024*c*f*(5*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 1512*
c*f*(2*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 336*c^3*f^3*Sinh[6*ArcSinh[c*
x]] + 1008*c*f*g^2*Sinh[6*ArcSinh[c*x]] + 189*c*f*g^2*Sinh[8*ArcSinh[c*x]]
))/(162570240*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.54, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(f+gx)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (f+gx)^3\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int\left(\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))f^3+3gx\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))f^2+3g^2x^2\left(c^2x^2+1\right.\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\left(-\frac{15fg^2(a+b\text{arcsinh}(cx))^2}{256bc^3}+\frac{1}{6}f^3x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{24}f^3x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcs}\right.}{}$$

input
```
Int[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((-3*b*f^2*g*x)/(7*c) + (2*b*g^3*x)/(63*c^3) - (2
5*b*c*f^3*x^2)/96 - (15*b*f*g^2*x^2)/(256*c) - (3*b*c*f^2*g*x^3)/7 - (b*g^
3*x^3)/(189*c) - (5*b*c^3*f^3*x^4)/96 - (59*b*c*f*g^2*x^4)/256 - (9*b*c^3*
f^2*g*x^5)/35 - (b*c*g^3*x^5)/21 - (17*b*c^3*f*g^2*x^6)/96 - (3*b*c^5*f^2*
g*x^7)/49 - (19*b*c^3*g^3*x^7)/441 - (3*b*c^5*f*g^2*x^8)/64 - (b*c^5*g^3*x
^9)/81 - (b*f^3*(1 + c^2*x^2)^3)/(36*c) + (5*f^3*x*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/16 + (15*f*g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
(128*c^2) + (15*f*g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*
f^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (5*f*g^2*x^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/16 + (f^3*x*(1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/6 + (3*f*g^2*x^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8
 + (3*f^2*g*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2) - (g^3*(1 +
c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4) + (g^3*(1 + c^2*x^2)^(9/2)*(a
 + b*ArcSinh[c*x]))/(9*c^4) + (5*f^3*(a + b*ArcSinh[c*x])^2)/(32*b*c) - (1
5*f*g^2*(a + b*ArcSinh[c*x])^2)/(256*b*c^3)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2883 vs. $2(994) = 1988$.

Time = 1.56 (sec) , antiderivative size = 2884, normalized size of antiderivative = 2.53

method	result	size
default	Expression too large to display	2884
parts	Expression too large to display	2884

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/6*x*(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(
1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2))))+g^3*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^
2*d*x^2+d)^(7/2))+3*f*g^2*(1/8*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/8/c^2*(1/6*x*
(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x
^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1
/2)))))+3/7*f^2*g*(c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(d*(c^2*x^2+1))^(1/2
)*f*arcsinh(x*c)^2*(8*c^2*f^2-3*g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/41472*(d*
(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*(c^2*x^2+1)^(1/2)*x^9*c^9+704*c^8*x^
8+576*x^7*c^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*(c^2*x^2+1)^(1/2)*x^5*c^5+
280*c^4*x^4+120*(c^2*x^2+1)^(1/2)*c^3*x^3+41*c^2*x^2+9*(c^2*x^2+1)^(1/2)*x
*c+1)*g^3*(-1+9*arcsinh(x*c))*d^2/c^4/(c^2*x^2+1)+3/16384*(d*(c^2*x^2+1))^
(1/2)*(128*c^9*x^9+128*(c^2*x^2+1)^(1/2)*x^8*c^8+320*x^7*c^7+256*x^6*c^6*(
c^2*x^2+1)^(1/2)+272*x^5*c^5+160*x^4*c^4*(c^2*x^2+1)^(1/2)+88*x^3*c^3+32*x
^2*c^2*(c^2*x^2+1)^(1/2)+8*x*c+(c^2*x^2+1)^(1/2))*f*g^2*(-1+8*arcsinh(x*c)
)*d^2/c^3/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*x^7*c^7
*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+5
6*(c^2*x^2+1)^(1/2)*c^3*x^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*x*c+1)*g*(28*ar
csinh(x*c)*c^2*f^2-4*c^2*f^2+7*arcsinh(x*c)*g^2-g^2)*d^2/c^4/(c^2*x^2+1)+1
/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64...
```

Fricas [F]

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\, dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (gx+f)^3 (b\,\text{arsinh}\,(cx)+a)\, dx$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*
d^2*f^3 + (3*a*c^4*d^2*f^2*g + 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 + 6*a
*c^2*d^2*f*g^2)*x^4 + (6*a*c^2*d^2*f^2*g + a*d^2*g^3)*x^3 + (2*a*c^2*d^2*f
^3 + 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b
*d^2*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g + 2*b*c^2*d^2*g^3)*x^5 + (b*
c^4*d^2*f^3 + 6*b*c^2*d^2*f*g^2)*x^4 + (6*b*c^2*d^2*f^2*g + b*d^2*g^3)*x^3
 + (2*b*c^2*d^2*f^3 + 3*b*d^2*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d
), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*asinh(c*x)),x)
```

output

```
(sqrt(d)*d**2*(1344*sqrt(c**2*x**2 + 1)*a*c**8*f**3*x**5 + 3456*sqrt(c**2*
x**2 + 1)*a*c**8*f**2*g*x**6 + 3024*sqrt(c**2*x**2 + 1)*a*c**8*f*g**2*x**7
 + 896*sqrt(c**2*x**2 + 1)*a*c**8*g**3*x**8 + 4368*sqrt(c**2*x**2 + 1)*a*c
**6*f**3*x**3 + 10368*sqrt(c**2*x**2 + 1)*a*c**6*f**2*g*x**4 + 8568*sqrt(c
**2*x**2 + 1)*a*c**6*f*g**2*x**5 + 2432*sqrt(c**2*x**2 + 1)*a*c**6*g**3*x*
*6 + 5544*sqrt(c**2*x**2 + 1)*a*c**4*f**3*x + 10368*sqrt(c**2*x**2 + 1)*a*
c**4*f**2*g*x**2 + 7434*sqrt(c**2*x**2 + 1)*a*c**4*f*g**2*x**3 + 1920*sqrt
(c**2*x**2 + 1)*a*c**4*g**3*x**4 + 3456*sqrt(c**2*x**2 + 1)*a*c**2*f**2*g
+ 945*sqrt(c**2*x**2 + 1)*a*c**2*f*g**2*x + 128*sqrt(c**2*x**2 + 1)*a*c**2
*g**3*x**2 - 256*sqrt(c**2*x**2 + 1)*a*g**3 + 8064*int(sqrt(c**2*x**2 + 1)
*asinh(c*x)*x**7,x)*b*c**8*g**3 + 24192*int(sqrt(c**2*x**2 + 1)*asinh(c*x)
*x**6,x)*b*c**8*f*g**2 + 24192*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**5,x)*
b*c**8*f**2*g + 16128*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**5,x)*b*c**6*g*
*3 + 8064*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**4,x)*b*c**8*f**3 + 48384*i
nt(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**4,x)*b*c**6*f*g**2 + 48384*int(sqrt(c
**2*x**2 + 1)*asinh(c*x)*x**3,x)*b*c**6*f**2*g + 8064*int(sqrt(c**2*x**2 +
 1)*asinh(c*x)*x**3,x)*b*c**4*g**3 + 16128*int(sqrt(c**2*x**2 + 1)*asinh(c
*x)*x**2,x)*b*c**6*f**3 + 24192*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)
*b*c**4*f*g**2 + 24192*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x,x)*b*c**4*f**2
*g + 8064*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c**4*f**3 + 2520*log(...
```

3.44 $\int (f+gx)^2 (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 837

$$\begin{aligned}
\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = &-\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} \\
&-\frac{5bcd^2f^2x^2\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}}-\frac{5bd^2g^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} \\
&-\frac{59bcd^2g^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{6bc^3d^2fgx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{17bc^3d^2g^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}} \\
&-\frac{2bc^5d^2fgx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{bc^5d^2g^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} \\
&-\frac{5bd^2f^2(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{96c}-\frac{bd^2f^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&+\frac{5}{16}d^2f^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{5d^2g^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{128c^2}+\frac{5}{64}d^2g^2x^3\sqrt{d+c^2dx^2}(a+
\end{aligned}$$

output
```
-2/7*b*d^2*f*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/32*b*c*d^2*f^2*
x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/256*b*d^2*g^2*x^2*(c^2*d*x^2+d
)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/7*b*c*d^2*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)-59/768*b*c*d^2*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-6
/35*b*c^3*d^2*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-17/288*b*c^3*d
^2*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/49*b*c^5*d^2*f*g*x^7*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/64*b*c^5*d^2*g^2*x^8*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-5/96*b*d^2*f^2*(c^2*x^2+1)^(3/2)*(c^2*d*x^2+d)^(1/2
)/c-1/36*b*d^2*f^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*
(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+5/128*d^2*g^2*x*(c^2*d*x^2+d)^(1/2)
*(a+b*arcsinh(c*x))/c^2+5/64*d^2*g^2*x^3*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(
c*x))+5/24*d*f^2*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+5/48*d*g^2*x^3*(
c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*f^2*x*(c^2*d*x^2+d)^(5/2)*(a+b*a
rcsinh(c*x))+1/8*g^2*x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))+2/7*f*g*(c
^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2/d+5/32*d^2*f^2*(c^2*d*x^2+d)^(1/2
)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)-5/256*d^2*g^2*(c^2*d*x^2+d)^(
1/2)*(a+b*arcsinh(c*x))^2/b/c^3/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.66

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx)) \, dx = \frac{-d^3(1+c^2x^2)\left(b(-87955g^2+1120c^2(2093f^2+4608fgx+315g^2x^2)+3360c^4x^2(184\right.}{}$$

input
```
Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^3*(1 + c^2*x^2)*(b*(-87955*g^2 + 1120*c^2*(2093*f^2 + 4608*f*g*x + 31
5*g^2*x^2) + 3360*c^4*x^2*(1848*f^2 + 1536*f*g*x + 413*g^2*x^2) + 640*c^8*
x^6*(784*f^2 + 1152*f*g*x + 441*g^2*x^2) + 1792*c^6*x^4*(1365*f^2 + 1728*f
*g*x + 595*g^2*x^2)) - 6720*a*c*Sqrt[1 + c^2*x^2]*(768*f*g*(1 + c^2*x^2)^3
 + 56*c^2*f^2*x*(33 + 26*c^2*x^2 + 8*c^4*x^4) + 7*g^2*x*(15 + 118*c^2*x^2
+ 136*c^4*x^4 + 48*c^6*x^6)))) + 352800*b*d^3*(8*c^2*f^2 - g^2)*(1 + c^2*x
^2)*ArcSinh[c*x]^2 + 705600*a*d^(5/2)*(8*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*
Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 840*b*d^3*(
1 + c^2*x^2)*ArcSinh[c*x]*(6144*c*f*g*Sqrt[1 + c^2*x^2] + 18432*c^3*f*g*x^
2*Sqrt[1 + c^2*x^2] + 18432*c^5*f*g*x^4*Sqrt[1 + c^2*x^2] + 6144*c^7*f*g*x
^6*Sqrt[1 + c^2*x^2] + 336*(15*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 168*(
6*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 112*c^2*f^2*Sinh[6*ArcSinh[c*x]] +
 112*g^2*Sinh[6*ArcSinh[c*x]] + 21*g^2*Sinh[8*ArcSinh[c*x]]))/(18063360*c^
3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(f+gx)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (f+gx)^2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(f^2(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}+g^2x^2(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}+2fgx(a+b\text{arcsin}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{c^2dx^2+d}\left(-\frac{5g^2(a+b\text{arcsinh}(cx))^2}{256bc^3}+\frac{1}{6}f^2x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{24}f^2x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsin}\right.$$

input
```
Int[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((-2*b*f*g*x)/(7*c) - (25*b*c*f^2*x^2)/96 - (5*b*
g^2*x^2)/(256*c) - (2*b*c*f*g*x^3)/7 - (5*b*c^3*f^2*x^4)/96 - (59*b*c*g^2*
x^4)/768 - (6*b*c^3*f*g*x^5)/35 - (17*b*c^3*g^2*x^6)/288 - (2*b*c^5*f*g*x^
7)/49 - (b*c^5*g^2*x^8)/64 - (b*f^2*(1 + c^2*x^2)^3)/(36*c) + (5*f^2*x*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*g^2*x*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/(128*c^2) + (5*g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*f^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (5*g^2*
x^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (f^2*x*(1 + c^2*x^2)^(5
/2)*(a + b*ArcSinh[c*x]))/6 + (g^2*x^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/8 + (2*f*g*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*f
^2*(a + b*ArcSinh[c*x])^2)/(32*b*c) - (5*g^2*(a + b*ArcSinh[c*x])^2)/(256*
b*c^3)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2308 vs. $2(727) = 1454$.

Time = 1.45 (sec) , antiderivative size = 2309, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	2309
parts	Expression too large to display	2309

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/6*x*(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(
1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2))))+g^2*(1/8*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/8/c^2*(1/6*x*(
c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^
2+d)^(1/2)+1/2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/
2)))))+2/7*f*g*(c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(d*(c^2*x^2+1))^(1/2)*a
rcsinh(x*c)^2*(8*c^2*f^2-g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/16384*(d*(c^2*x^
2+1))^(1/2)*(128*c^9*x^9+128*(c^2*x^2+1)^(1/2)*x^8*c^8+320*x^7*c^7+256*x^6
*c^6*(c^2*x^2+1)^(1/2)+272*x^5*c^5+160*x^4*c^4*(c^2*x^2+1)^(1/2)+88*x^3*c^
3+32*x^2*c^2*(c^2*x^2+1)^(1/2)+8*x*c+(c^2*x^2+1)^(1/2))*g^2*(-1+8*arcsinh(
x*c))*d^2/c^3/(c^2*x^2+1)+1/3136*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*x^7*
c^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^
4+56*(c^2*x^2+1)^(1/2)*c^3*x^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(
-1+7*arcsinh(x*c))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*x^
7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x^5*c^5+48*x^4*c^4*(c^2*x^2+1)^(1/2)
+38*x^3*c^3+18*x^2*c^2*(c^2*x^2+1)^(1/2)+6*x*c+(c^2*x^2+1)^(1/2))*(6*arcsi
nh(x*c)*c^2*f^2-c^2*f^2+6*arcsinh(x*c)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/320*
(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+
20*(c^2*x^2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(-1
+5*arcsinh(x*c))*d^2/c^2/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*x^...
```

Fricas [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}} (gx+f)^2 (b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 + 4*a*c^2*d^2*f*g*x^3 +
2*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 + 2*a*c^2*d^2*g^2)*x^4 + (2*a*c
^2*d^2*f^2 + a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 + 4
*b*c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 + 2*b*c^2*
d^2*g^2)*x^4 + (2*b*c^2*d^2*f^2 + b*d^2*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d), x)
```

Sympy [F]

$$\int (f+gx)^2 (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(d(c^2x^2+1)\right)^{\frac{5}{2}} (a+b\,\text{asinh}\,(cx))\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \int (f+g\,x)^2\,(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \frac{\sqrt{d}\,d^2\left(448\sqrt{c^2x^2+1}\,a\,c^7f^2x^5+768\sqrt{c^2x^2+1}\,a\,c^7fg\,x^6+336\sqrt{c^2x^2+1}\,a\,c^7g^2x^7+1\right.}{}$$

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*asinh(c*x)),x)
```

output

```
(sqrt(d)*d**2*(448*sqrt(c**2*x**2 + 1)*a*c**7*f**2*x**5 + 768*sqrt(c**2*x*
*2 + 1)*a*c**7*f*g*x**6 + 336*sqrt(c**2*x**2 + 1)*a*c**7*g**2*x**7 + 1456*
sqrt(c**2*x**2 + 1)*a*c**5*f**2*x**3 + 2304*sqrt(c**2*x**2 + 1)*a*c**5*f*g
*x**4 + 952*sqrt(c**2*x**2 + 1)*a*c**5*g**2*x**5 + 1848*sqrt(c**2*x**2 + 1
)*a*c**3*f**2*x + 2304*sqrt(c**2*x**2 + 1)*a*c**3*f*g*x**2 + 826*sqrt(c**2
*x**2 + 1)*a*c**3*g**2*x**3 + 768*sqrt(c**2*x**2 + 1)*a*c*f*g + 105*sqrt(c
**2*x**2 + 1)*a*c*g**2*x + 2688*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**6,x)
*b*c**7*g**2 + 5376*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**5,x)*b*c**7*f*g
+ 2688*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**4,x)*b*c**7*f**2 + 5376*int(s
qrt(c**2*x**2 + 1)*asinh(c*x)*x**4,x)*b*c**5*g**2 + 10752*int(sqrt(c**2*x*
*2 + 1)*asinh(c*x)*x**3,x)*b*c**5*f*g + 5376*int(sqrt(c**2*x**2 + 1)*asinh
(c*x)*x**2,x)*b*c**5*f**2 + 2688*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x
)*b*c**3*g**2 + 5376*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x,x)*b*c**3*f*g +
2688*int(sqrt(c**2*x**2 + 1)*asinh(c*x),x)*b*c**3*f**2 + 840*log(sqrt(c**2
*x**2 + 1) + c*x)*a*c**2*f**2 - 105*log(sqrt(c**2*x**2 + 1) + c*x)*a*g**2)
)/(2688*c**3)
```

3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 455

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}$$

$$-\frac{5bcd^2fx^2\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}}-\frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}$$

$$-\frac{bc^5d^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{5bd^2f(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}}{96c}-\frac{bd^2f(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}$$

$$+\frac{5}{16}d^2fx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{5}{24}dfx\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{1}{6}fx\left(d+c^2dx^2\right)^{5/2}(a+ba$$

output
```
-1/7*b*d^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/32*b*c*d^2*f*x^2*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*g*x^3*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-3/35*b*c^3*d^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-1/49*b*c^5*d^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*d^2
*f*(c^2*x^2+1)^(3/2)*(c^2*d*x^2+d)^(1/2)/c-1/36*b*d^2*f*(c^2*x^2+1)^(5/2)*
(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))+
5/24*d*f*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*f*x*(c^2*d*x^2+d)^(5
/2)*(a+b*arcsinh(c*x))+1/7*g*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2/d+
5/32*d^2*f*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.86

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(-1680a\sqrt{1+c^2x^2}\left(48g(1+c^2x^2)^3+7c^2fx(33+26c^2x^2+8c^4x^4)\right)+\right.}{}$$

input
```
Integrate[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^3*(1 + c^2*x^2)*(-1680*a*Sqrt[1 + c^2*x^2]*(48*g*(1 + c^2*x^2)^3 + 7*
c^2*f*x*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + b*c*(2304*g*x*(35 + 35*c^2*x^2 +
21*c^4*x^4 + 5*c^6*x^6) + 245*f*(299 + 792*c^2*x^2 + 312*c^4*x^4 + 64*c^6*
x^6)))) + 88200*b*c*d^3*f*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 176400*a*c*d^(5/2
)*f*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2
*d*x^2]] + 420*b*d^3*(1 + c^2*x^2)*ArcSinh[c*x]*(192*g*Sqrt[1 + c^2*x^2] +
 576*c^2*g*x^2*Sqrt[1 + c^2*x^2] + 576*c^4*g*x^4*Sqrt[1 + c^2*x^2] + 192*c
^6*g*x^6*Sqrt[1 + c^2*x^2] + 315*c*f*Sinh[2*ArcSinh[c*x]] + 63*c*f*Sinh[4*
ArcSinh[c*x]] + 7*c*f*Sinh[6*ArcSinh[c*x]]))/(564480*c^2*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(f+gx)(a+b\operatorname{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6260$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (f+gx)\left(c^2x^2+1\right)^{5/2}(a+b\operatorname{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(f(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}+gx(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{c^2dx^2+d}\left(\frac{1}{6}fx\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{24}fx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{5}{16}fx\sqrt{c^2x^2+1}(\right.$$

input
```
Int[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(-1/7*(b*g*x)/c - (25*b*c*f*x^2)/96 - (b*c*g*x^3)
/7 - (5*b*c^3*f*x^4)/96 - (3*b*c^3*g*x^5)/35 - (b*c^5*g*x^7)/49 - (b*f*(1
+ c^2*x^2)^3)/(36*c) + (5*f*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (5*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (f*x*(1 + c^2*x^2)^
(5/2)*(a + b*ArcSinh[c*x]))/6 + (g*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*f*(a + b*ArcSinh[c*x])^2)/(32*b*c)))/Sqrt[1 + c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1678 vs. $2(391) = 782$.

Time = 2.01 (sec) , antiderivative size = 1679, normalized size of antiderivative = 3.69

method	result	size
default	Expression too large to display	1679
parts	Expression too large to display	1679

input
```
int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c)),x,method=_RETURNVERBOSE
)
```

output
```
1/6*a*f*x*(c^2*d*x^2+d)^(5/2)+5/24*a*f*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f*d^
2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d
)^(1/2))/(c^2*d)^(1/2)+1/7*a*g*(c^2*d*x^2+d)^(7/2)/c^2/d+b*(5/32*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(x*c)^2*d^2+1/6272*(d*(c^2*x^2+1
))^(1/2)*(64*c^8*x^8+64*x^7*c^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*(c^2*x^2
+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*c^3*x^3+25*c^2*x^2+7*(c
^2*x^2+1)^(1/2)*x*c+1)*g*(-1+7*arcsinh(x*c))*d^2/c^2/(c^2*x^2+1)+1/2304*(d
*(c^2*x^2+1))^(1/2)*(32*x^7*c^7+32*x^6*c^6*(c^2*x^2+1)^(1/2)+64*x^5*c^5+48
*x^4*c^4*(c^2*x^2+1)^(1/2)+38*x^3*c^3+18*x^2*c^2*(c^2*x^2+1)^(1/2)+6*x*c+(
c^2*x^2+1)^(1/2))*f*(-1+6*arcsinh(x*c))*d^2/(c^2*x^2+1)/c+1/640*(d*(c^2*x^
2+1))^(1/2)*(16*c^6*x^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^
2+1)^(1/2)*c^3*x^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*x*c+1)*g*(-1+5*arcsinh(x
*c))*d^2/c^2/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*x^5*c^5+8*x^4*c^4*
(c^2*x^2+1)^(1/2)+12*x^3*c^3+8*x^2*c^2*(c^2*x^2+1)^(1/2)+4*x*c+(c^2*x^2+1)
^(1/2))*f*(-1+4*arcsinh(x*c))*d^2/(c^2*x^2+1)/c+1/128*(d*(c^2*x^2+1))^(1/2
)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*x*c
+1)*g*(-1+3*arcsinh(x*c))*d^2/c^2/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)
*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2+1)^(1/2))*f*(-1+2*a
rcsinh(x*c))*d^2/(c^2*x^2+1)/c+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x
^2+1)^(1/2)*x*c+1)*g*(arcsinh(x*c)-1)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2...
```

Fricas [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fri
cas")
```

output
```
integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 + 2*a*c^2*d^2*g*x^3 + 2*a*c^2*
d^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 + 2*b
*c^2*d^2*g*x^3 + 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arcsinh(c*x))*sq
rt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int (f+g\,x)\,\left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\frac{\sqrt{d}\,d^2\left(56\sqrt{c^2x^2+1}\,a\,c^6f\,x^5+48\sqrt{c^2x^2+1}\,a\,c^6g\,x^6+182\sqrt{c^2x^2+1}\,a\,c^4f\,x^3+144\sqrt{c^2x^2+1}\right.}{}$$

input
```
int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*asinh(c*x)),x)
```

output
```
(sqrt(d)*d**2*(56*sqrt(c**2*x**2 + 1)*a*c**6*f*x**5 + 48*sqrt(c**2*x**2 +
1)*a*c**6*g*x**6 + 182*sqrt(c**2*x**2 + 1)*a*c**4*f*x**3 + 144*sqrt(c**2*x
**2 + 1)*a*c**4*g*x**4 + 231*sqrt(c**2*x**2 + 1)*a*c**2*f*x + 144*sqrt(c**
2*x**2 + 1)*a*c**2*g*x**2 + 48*sqrt(c**2*x**2 + 1)*a*g + 336*int(sqrt(c**2
*x**2 + 1)*asinh(c*x)*x**5,x)*b*c**6*g + 336*int(sqrt(c**2*x**2 + 1)*asinh
(c*x)*x**4,x)*b*c**6*f + 672*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**3,x)*b*
c**4*g + 672*int(sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2,x)*b*c**4*f + 336*int
(sqrt(c**2*x**2 + 1)*asinh(c*x)*x,x)*b*c**2*g + 336*int(sqrt(c**2*x**2 + 1
)*asinh(c*x),x)*b*c**2*f + 105*log(sqrt(c**2*x**2 + 1) + c*x)*a*c*f))/(336
*c**2)
```

3.46 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 1500

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx} \, dx = \text{Too large to display}$$

output
```
-1/4*c*d^2*f*(c^2*f^2+2*g^2)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b/g^
4/(c^2*x^2+1)^(1/2)-1/2*c*d^2*(c^2*f^2+g^2)^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*a
rcsinh(c*x))^2/b/g^5/(c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2+g^2)^3*(c^2*d*x^2+
d)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/g^6/(g*x+f)/(c^2*x^2+1)^(1/2)+1/2*d^2*(c
^2*f^2+g^2)^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b
/c/g^4/(g*x+f)+1/4*b*c^3*d^2*f*(c^2*f^2+2*g^2)*x^2*(c^2*d*x^2+d)^(1/2)/g^4
/(c^2*x^2+1)^(1/2)+1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/g-b*c*d^2*(c
^2*f^2+g^2)^2*x*(c^2*d*x^2+d)^(1/2)/g^5/(c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g
^2)^(5/2)*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f+(c^2*f^2+g^2)^(1/2)))/g^6/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^(5/2)
*(c^2*d*x^2+d)^(1/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2
*f^2+g^2)^(1/2)))/g^6/(c^2*x^2+1)^(1/2)-1/3*b*c*d^2*(c^2*f^2+2*g^2)*x*(c^2
*d*x^2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/16*b*c^3*d^2*f*x^2*(c^2*d*x^2+d)^(
1/2)/g^2/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d^2*(c^2*f^2+2*g^2)*x^3*(c^2*d*x^2+d)
^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/16*b*c^5*d^2*f*x^4*(c^2*d*x^2+d)^(1/2)/g^2/
(c^2*x^2+1)^(1/2)-1/2*c^2*d^2*f*(c^2*f^2+2*g^2)*x*(c^2*d*x^2+d)^(1/2)*(a+b
*arcsinh(c*x))/g^4+1/16*c*d^2*f*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2/b
/g^2/(c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*(c^2*d*x^2+d)^(1/2)*polyl
og(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/g^6/(c^2*x^2+1)
^(1/2)+b*d^2*(c^2*f^2+g^2)^(5/2)*(c^2*d*x^2+d)^(1/2)*polylog(2,-(c*x+(c...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.88 (sec) , antiderivative size = 7168, normalized size of antiderivative = 4.78

$$\int \frac{(d+c^2dx^2)^{5/2}\,(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 945, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6255, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6255}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^4}{g}-\frac{fx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^4}{g^2}+\frac{\left(c^2f^2+2g^2\right)x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{g^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{c^2dx^2+d}\left(-\frac{bx^5c^5}{25g}+\frac{bfx^4c^5}{16g^2}-\frac{fx^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^4}{4g^2}-\frac{b\left(c^2f^2+2g^2\right)x^3c^3}{9g^3}-\frac{bx^3c^3}{45g}+\frac{bf\left(c^2f^2+2g^2\right)x^2c^3}{4g^4}+\frac{bfx}{16g}\right.$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((2*b*c*x)/(15*g) - (b*c*(c^2*f^2 + g^2)^2*x)/g^5
 - (b*c*(c^2*f^2 + 2*g^2)*x)/(3*g^3) + (b*c^3*f*x^2)/(16*g^2) + (b*c^3*f*(
c^2*f^2 + 2*g^2)*x^2)/(4*g^4) - (b*c^3*x^3)/(45*g) - (b*c^3*(c^2*f^2 + 2*g
^2)*x^3)/(9*g^3) + (b*c^5*f*x^4)/(16*g^2) - (b*c^5*x^5)/(25*g) + (a*(c^2*f
^2 + g^2)^2*Sqrt[1 + c^2*x^2])/g^5 + (b*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/g^5 - (c^2*f*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*
g^2) - (c^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
(2*g^4) - (c^4*f*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - ((1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*g) + ((c^2*f^2 + 2*g^2)*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*g^3) + ((1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/(5*g) + (c*f*(a + b*ArcSinh[c*x])^2)/(16*b*g^2) - (c*f*(c^2*
f^2 + 2*g^2)*(a + b*ArcSinh[c*x])^2)/(4*b*g^4) - (c*(c^2*f^2 + g^2)^2*x*(a
 + b*ArcSinh[c*x])^2)/(2*b*g^5) - ((c^2*f^2 + g^2)^3*(a + b*ArcSinh[c*x])^
2)/(2*b*c*g^6*(f + g*x)) + ((c^2*f^2 + g^2)^2*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*(c^2*f^2 + g^2)^(5/2)*ArcTanh[(g - c^
2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/g^6 + (b*(c^2*f^2 + g^2)^
(5/2)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])]
)/g^6 - (b*(c^2*f^2 + g^2)^(5/2)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(
c*f + Sqrt[c^2*f^2 + g^2])])/g^6 + (b*(c^2*f^2 + g^2)^(5/2)*PolyLog[2, -((
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^6 - (b*(c^2*f^2 + g^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6255
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] && !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3498 vs. 2(1384) = 2768.

Time = 1.64 (sec) , antiderivative size = 3499, normalized size of antiderivative = 2.33

method	result	size
default	Expression too large to display	3499
parts	Expression too large to display	3499

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(x*c))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(x*c)*x^4*c^4+34/15
*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(x*c)*x^2*c^2+b*(d*(c^2*
x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^5*arcsinh(x*c)*c^4*f^4-1/2*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*f^5*arcsinh(x*c)^2*d^2*c^5/g^6-5/4*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f^3*arcsinh(x*c)^2*d^2*c^3/g^4+1/16*b*(d*(c^
2*x^2+1))^(1/2)*f*d^2*c^5/(c^2*x^2+1)^(1/2)/g^2*x^4+9/16*b*(d*(c^2*x^2+1))
^(1/2)*f*d^2*c^3/(c^2*x^2+1)^(1/2)/g^2*x^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*f^3
*d^2*c^5/(c^2*x^2+1)^(1/2)/g^4*x^2-b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)
^(1/2)/g^5*x*c^5*f^4-7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^3
*x*c^3*f^2-1/9*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^3*x^3*c^5*f
^2+7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(x*c)*c^2*f^2+b*
d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsin
h(x*c)*ln((-(x*c+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*
f^2+g^2)^(1/2)))-15/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh
(x*c)^2*d^2*c/g^2-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2
+1)^(1/2)/g^2*arcsinh(x*c)*ln(((x*c+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)
^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^
2+1)/g*arcsinh(x*c)*x^6*c^6+23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/
g*arcsinh(x*c)+33/128*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c/(c^2*x^2+1)^(1/2)/g^
2-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^6/(c^2*x^2+1)/g^2*arcsinh(x*c)*x^...
```

Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(cx))}{f+gx}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,(d\,c^2\,x^2+d)^{5/2}}{f+g\,x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \frac{\sqrt{d}\,d^2\left(240\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)a\,c^4f^4i+480\sqrt{c}\right.}{}$$

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output

```
(sqrt(d)*d**2*(240*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i +
c*f*i + c*g*i*x)/sqrt(c**2*f**2 + g**2))*a*c**4*f**4*i + 480*sqrt(c**2*f**
2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*i + c*g*i*x)/sqrt(c**2*f**2
+ g**2))*a*c**2*f**2*g**2*i + 240*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x
**2 + 1)*g*i + c*f*i + c*g*i*x)/sqrt(c**2*f**2 + g**2))*a*g**4*i + 120*sqr
t(c**2*x**2 + 1)*a*c**4*f**4*g - 60*sqrt(c**2*x**2 + 1)*a*c**4*f**3*g**2*x
 + 40*sqrt(c**2*x**2 + 1)*a*c**4*f**2*g**3*x**2 - 30*sqrt(c**2*x**2 + 1)*a
*c**4*f*g**4*x**3 + 24*sqrt(c**2*x**2 + 1)*a*c**4*g**5*x**4 + 280*sqrt(c**
2*x**2 + 1)*a*c**2*f**2*g**3 - 135*sqrt(c**2*x**2 + 1)*a*c**2*f*g**4*x + 8
8*sqrt(c**2*x**2 + 1)*a*c**2*g**5*x**2 + 184*sqrt(c**2*x**2 + 1)*a*g**5 +
120*int((sqrt(c**2*x**2 + 1)*asinh(c*x)*x**4)/(f + g*x),x)*b*c**4*g**6 + 2
40*int((sqrt(c**2*x**2 + 1)*asinh(c*x)*x**2)/(f + g*x),x)*b*c**2*g**6 + 12
0*int((sqrt(c**2*x**2 + 1)*asinh(c*x))/(f + g*x),x)*b*g**6 - 120*log(sqrt(
c**2*x**2 + 1) + c*x)*a*c**5*f**5 - 300*log(sqrt(c**2*x**2 + 1) + c*x)*a*c
**3*f**3*g**2 - 225*log(sqrt(c**2*x**2 + 1) + c*x)*a*c*f*g**4))/(120*g**6)
```

3.47 $\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 406

$$\begin{aligned}\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = &-\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}\\ &-\frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}-\frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}\\ &+\frac{3f^2g\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^2d}\\ &-\frac{2g^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^4d}\\ &+\frac{3fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2c^2d}\\ &+\frac{g^3x^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2d}\\ &+\frac{f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}\\ &-\frac{3fg^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-3*b*f^2*g*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+2/3*b*g^3*x*(c^2*x^2+
1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-3/4*b*f*g^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*
d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+3*f^2
*g*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c^2/d-2/3*g^3*(c^2*d*x^2+d)^(1/2
)*(a+b*arcsinh(c*x))/c^4/d+3/2*f*g^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*
x))/c^2/d+1/3*g^3*x^2*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c^2/d+1/2*f^3
*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*d*x^2+d)^(1/2)-3/4*f*g^2*
(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c^3/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.75

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{4\sqrt{d}g\left(-2bcx\sqrt{1+c^2x^2}(-6g^2+c^2(27f^2+g^2x^2))+3a(1+c^2x^2)\left(-4g^2+c^2(18f^2+9fgx+2g^2x^2)\right)\right)+}{}$$

input
```
Integrate[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(4*Sqrt[d]*g*(-2*b*c*x*Sqrt[1 + c^2*x^2]*(-6*g^2 + c^2*(27*f^2 + g^2*x^2))
 + 3*a*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 12*b
*Sqrt[d]*g*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))*Arc
Sinh[c*x] + 18*b*c*Sqrt[d]*f*(2*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]^2 - 27*b*c*Sqrt[d]*f*g^2*Sqrt[1 + c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 36
*a*c*f*(2*c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d
+ c^2*d*x^2]])/(72*c^4*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2x^2+1}\int \left(\frac{(a+b\text{arcsinh}(cx))f^3}{\sqrt{c^2x^2+1}}+\frac{3gx(a+b\text{arcsinh}(cx))f^2}{\sqrt{c^2x^2+1}}+\frac{3g^2x^2(a+b\text{arcsinh}(cx))f}{\sqrt{c^2x^2+1}}+\frac{g^3x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2x^2+1}\left(-\frac{3fg^2(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{3f^2g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{3fg^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}+\frac{g^3x^2\sqrt{c^2x^2+1}(a+}{3c^2}\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*((-3*b*f^2*g*x)/c + (2*b*g^3*x)/(3*c^3) - (3*b*f*g^2*x^
2)/(4*c) - (b*g^3*x^3)/(9*c) + (3*f^2*g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/c^2 - (2*g^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (3*f*
g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) + (g^3*x^2*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f^3*(a + b*ArcSinh[c*x])^2)/(2*b
*c) - (3*f*g^2*(a + b*ArcSinh[c*x])^2)/(4*b*c^3)))/Sqrt[d + c^2*d*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(358) = 716.

Time = 1.47 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.93

method	result
default	$a\left(\frac{f^{3}\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{\sqrt{c^{2}d}}+g^{3}\left(\frac{x^{2}\sqrt{c^{2}d\,x^{2}+d}}{3c^{2}d}-\frac{2\sqrt{c^{2}d\,x^{2}+d}}{3d\,c^{4}}\right)+3f\,g^{2}\left(\frac{x\sqrt{c^{2}d\,x^{2}+d}}{2c^{2}d}-\frac{\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{2c^{2}\sqrt{c^{2}d}}\right.\right.$
parts	$a\left(\frac{f^{3}\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{\sqrt{c^{2}d}}+g^{3}\left(\frac{x^{2}\sqrt{c^{2}d\,x^{2}+d}}{3c^{2}d}-\frac{2\sqrt{c^{2}d\,x^{2}+d}}{3d\,c^{4}}\right)+3f\,g^{2}\left(\frac{x\sqrt{c^{2}d\,x^{2}+d}}{2c^{2}d}-\frac{\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{2c^{2}\sqrt{c^{2}d}}\right.\right.$

input
```
int((g*x+f)^3*(a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+g^3*(1/
3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+3*f*g^2*(1/
2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2))+3*f^2*g/c^2/d*(c^2*d*x^2+d)^(1/2))+b*(1/4*(d*(c^2
*x^2+1))^(1/2)*f*arcsinh(x*c)^2*(2*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)/d/c^3+
1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*c^3*x^3+5*c^2*x^
2+3*(c^2*x^2+1)^(1/2)*x*c+1)*g^3*(-1+3*arcsinh(x*c))/c^4/d/(c^2*x^2+1)+3/1
6*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*
x^2+1)^(1/2))*f*g^2*(-1+2*arcsinh(x*c))/d/c^3/(c^2*x^2+1)+3/8*(d*(c^2*x^2+
1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*g*(4*arcsinh(x*c)*c^2*f^2-4*c^
2*f^2-arcsinh(x*c)*g^2+g^2)/c^4/d/(c^2*x^2+1)+3/8*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*g*(4*arcsinh(x*c)*c^2*f^2+4*c^2*f^2-arcsin
h(x*c)*g^2-g^2)/c^4/d/(c^2*x^2+1)+3/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*
x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*f*g^2*(1+2*arcsinh(x*c)
)/d/c^3/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1
/2)*c^3*x^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*x*c+1)*g^3*(1+3*arcsinh(x*c))/c^
4/d/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)^3(b\operatorname{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))\,(f+gx)^3}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((g*x+f)**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*(f + g*x)**3/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)^3(b\,\text{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((g*x + f)^3*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(f+g\,x)^3\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{3asinh(cx)^2\,b\,c^3f^3+18\sqrt{c^2x^2+1}\,asinh(cx)\,b\,c^2f^2g+18\sqrt{c^2x^2+1}\,a\,c^2f^2g+9\sqrt{c^2x^2+1}\,a\,c^2f\,g^2x+2\sqrt{}}{}$$

input
```
int((g*x+f)^3*(a+b*asinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(3*asinh(c*x)**2*b*c**3*f**3 + 18*sqrt(c**2*x**2 + 1)*asinh(c*x)*b*c**2*f*
*2*g + 18*sqrt(c**2*x**2 + 1)*a*c**2*f**2*g + 9*sqrt(c**2*x**2 + 1)*a*c**2
*f*g**2*x + 2*sqrt(c**2*x**2 + 1)*a*c**2*g**3*x**2 - 4*sqrt(c**2*x**2 + 1)
*a*g**3 + 6*int((asinh(c*x)*x**3)/sqrt(c**2*x**2 + 1),x)*b*c**4*g**3 + 18*
int((asinh(c*x)*x**2)/sqrt(c**2*x**2 + 1),x)*b*c**4*f*g**2 + 6*log(sqrt(c*
*2*x**2 + 1) + c*x)*a*c**3*f**3 - 9*log(sqrt(c**2*x**2 + 1) + c*x)*a*c*f*g
**2 - 18*b*c**3*f**2*g*x)/(6*sqrt(d)*c**4)
```

3.48 $\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 246

$$\begin{aligned}\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx &= -\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} \\ &\quad + \frac{2fg\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2d} \\ &\quad + \frac{g^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{2c^2d} \\ &\quad + \frac{f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} \\ &\quad - \frac{g^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-2*b*f*g*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c^2*x^2+
1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+2*f*g*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)
)/c^2/d+1/2*g^2*x*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/c^2/d+1/2*f^2*(c^
2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*d*x^2+d)^(1/2)-1/4*g^2*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c^3/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{4c\sqrt{d}g\left(-4bcfx\sqrt{1+c^2x^2}+a(4f+gx)\left(1+c^2x^2\right)\right)+4bc\sqrt{d}g(4f+gx)\left(1+c^2x^2\right)\text{arcsinh}(cx)+2b\sqrt{d}(\ldots}{\ldots}$$

input
```
Integrate[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(4*c*Sqrt[d]*g*(-4*b*c*f*x*Sqrt[1 + c^2*x^2] + a*(4*f + g*x)*(1 + c^2*x^2)
) + 4*b*c*Sqrt[d]*g*(4*f + g*x)*(1 + c^2*x^2)*ArcSinh[c*x] + 2*b*Sqrt[d]*(
2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[d]*g^2*Sqrt[1 +
 c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 4*a*(2*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]
*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c^3*Sqrt[d]*Sqrt[d + c^2*d*x
^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2x^2+1}\int \left(\frac{(a+b\text{arcsinh}(cx))f^2}{\sqrt{c^2x^2+1}}+\frac{2gx(a+b\text{arcsinh}(cx))f}{\sqrt{c^2x^2+1}}+\frac{g^2x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2009

$$\frac{\sqrt{c^2x^2+1}\left(-\frac{g^2(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{2fg\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{g^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}+\frac{f^2(a+b\text{arcsinh}(cx))^2}{2bc}\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*((-2*b*f*g*x)/c - (b*g^2*x^2)/(4*c) + (2*f*g*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 + (g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(2*c^2) + (f^2*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (g^2*(a + b*ArcS
inh[c*x])^2)/(4*b*c^3)))/Sqrt[d + c^2*d*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(218) = 436$.

Time = 1.43 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.97

method	result
default	$a\left(\frac{f^2\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{\sqrt{c^2 d}}+g^2\left(\frac{x\sqrt{c^2 d x^2+d}}{2c^2 d}-\frac{\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{2c^2\sqrt{c^2 d}}\right)+\frac{2fg\sqrt{c^2 d x^2+d}}{c^2 d}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}}{4}\right.$
parts	$a\left(\frac{f^2\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{\sqrt{c^2 d}}+g^2\left(\frac{x\sqrt{c^2 d x^2+d}}{2c^2 d}-\frac{\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{2c^2\sqrt{c^2 d}}\right)+\frac{2fg\sqrt{c^2 d x^2+d}}{c^2 d}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}}{4}\right.$

input
```
int((g*x+f)^2*(a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+g^2*(1/
2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2))+2*f*g/c^2/d*(c^2*d*x^2+d)^(1/2))+b*(1/4*(d*(c^2*x
^2+1))^(1/2)*arcsinh(x*c)^2*(2*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/d/c^3+1/16*(
d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3+2*x^2*c^2*(c^2*x^2+1)^(1/2)+2*x*c+(c^2*x^2
+1)^(1/2))*g^2*(-1+2*arcsinh(x*c))/d/c^3/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(arcsinh(x*c)-1)/c^2/d/(c^2*x^2+1)+
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*x*c+1)*f*g*(arcsinh(x*c)+
1)/c^2/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*x^3*c^3-2*x^2*c^2*(c^2*
x^2+1)^(1/2)+2*x*c-(c^2*x^2+1)^(1/2))*g^2*(1+2*arcsinh(x*c))/d/c^3/(c^2*x^
2+1))
```

Fricas [F]

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)^2(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*
arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))\,(f+gx)^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((g*x+f)**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*(f + g*x)**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)^2(b\operatorname{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((g*x + f)^2*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(f+g\,x)^2\,(a+b\operatorname{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{asinh(cx)^2\,b\,c^2f^2+4\sqrt{c^2x^2+1}\,asinh(cx)\,bcfg+4\sqrt{c^2x^2+1}\,acfg+\sqrt{c^2x^2+1}\,ac\,g^2x+2\left(\int \frac{asinh(cx)x^2}{\sqrt{c^2x^2+1}}d\right.}{2\sqrt{d}\,c^3}$$

input
```
int((g*x+f)^2*(a+b*asinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(asinh(c*x)**2*b*c**2*f**2 + 4*sqrt(c**2*x**2 + 1)*asinh(c*x)*b*c*f*g + 4*
sqrt(c**2*x**2 + 1)*a*c*f*g + sqrt(c**2*x**2 + 1)*a*c*g**2*x + 2*int((asin
h(c*x)*x**2)/sqrt(c**2*x**2 + 1),x)*b*c**3*g**2 + 2*log(sqrt(c**2*x**2 + 1
) + c*x)*a*c**2*f**2 - log(sqrt(c**2*x**2 + 1) + c*x)*a*g**2 - 4*b*c**2*f*
g*x)/(2*sqrt(d)*c**3)
```

3.49 $\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{g\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2d}+\frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

output
```
-b*g*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+g*(c^2*d*x^2+d)^(1/2)*(a+b*
arcsinh(c*x))/c^2/d+1/2*f*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*
d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.39

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{2\sqrt{d}g\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\right)+2b\sqrt{d}g(1+c^2x^2)\,\text{arcsinh}(cx)+bc\sqrt{d}f\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2+2a}{2c^2\sqrt{d}\sqrt{d+c^2dx^2}}$$

input
```
Integrate[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(2*Sqrt[d]*g*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]) + 2*b*Sqrt[d]*g*(1
+ c^2*x^2)*ArcSinh[c*x] + b*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 +
 2*a*c*f*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*
c^2*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2x^2+1}\int \left(\frac{f(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}+\frac{gx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2x^2+1}\left(\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{f(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{bgx}{c}\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(-((b*g*x)/c) + (g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/c^2 + (f*(a + b*ArcSinh[c*x])^2)/(2*b*c)))/Sqrt[d + c^2*d*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 1.85 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.99

method	result
default	$\frac{af\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{ag\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\,\mathrm{arcsinh}(xc)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+\sqrt{c^2x^2+1}\,xc+1\right)g(\mathrm{arc}}{2c^2d(c^2x^2+1)}\right.$
parts	$\frac{af\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{ag\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\,\mathrm{arcsinh}(xc)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+\sqrt{c^2x^2+1}\,xc+1\right)g(\mathrm{arc}}{2c^2d(c^2x^2+1)}\right.$

input
```
int((g*x+f)*(a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
a*f*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+a*g/c^2/d*
(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*f*a
rcsinh(x*c)^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*x*c+1)*
g*(arcsinh(x*c)-1)/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c
^2*x^2+1)^(1/2)*x*c+1)*g*(arcsinh(x*c)+1)/c^2/d/(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)(b\operatorname{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x
)
```

Sympy [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))(f+gx)}{\sqrt{d(c^2x^2+1)}}\,dx$$

input
```
integrate((g*x+f)*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*(f + g*x)/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{bf\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}} - \frac{bgx}{c\sqrt{d}} + \frac{af\,\text{arsinh}\,(cx)}{c\sqrt{d}} + \frac{\sqrt{c^2dx^2+d}bg\,\text{arsinh}\,(cx)}{c^2d} + \frac{\sqrt{c^2dx^2+d}ag}{c^2d}$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
1/2*b*f*arcsinh(c*x)^2/(c*sqrt(d)) - b*g*x/(c*sqrt(d)) + a*f*arcsinh(c*x)/
(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*g*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^
2 + d)*a*g/(c^2*d)
```

Giac [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((g*x + f)*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\text{asinh}(c\,x)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{\sqrt{d}\left(asinh(cx)^2bcf+2\sqrt{c^2x^2+1}\,asinh(cx)\,bg+2\sqrt{c^2x^2+1}\,ag+2\log\left(\sqrt{c^2x^2+1}+cx\right)acf-2bcgx\right)}{2c^2d}$$

input
```
int((g*x+f)*(a+b*asinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*(asinh(c*x)**2*b*c*f + 2*sqrt(c**2*x**2 + 1)*asinh(c*x)*b*g + 2*s
qrt(c**2*x**2 + 1)*a*g + 2*log(sqrt(c**2*x**2 + 1) + c*x)*a*c*f - 2*b*c*g*
x))/(2*c**2*d)
```

3.50 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

output
```
1/2*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d(1+c^2x^2)}}+\frac{a\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{c\sqrt{d}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d*(1 + c^2*x^2)]) + (a*ArcT
anh[(c*Sqrt[d]*x)/Sqrt[d + c^2*d*x^2]])/(c*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \text{6198}$$
$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{a\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(xc)^2}{2\sqrt{c^2x^2+1}\,dc}$	77
parts	$\frac{a\ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{\sqrt{c^2 d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(xc)^2}{2\sqrt{c^2x^2+1}\,dc}$	77

input
```
int((a+b*arcsinh(x*c))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d/c*arcsinh(x*c)^2
```

Fricas [F]

$$\int \frac{a + b\mathrm{arcsinh}(cx)}{\sqrt{d + c^2 dx^2}} \, dx = \int \frac{b \,\mathrm{arsinh}\,(cx) + a}{\sqrt{c^2 dx^2 + d}} \, dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\mathrm{arcsinh}(cx)}{\sqrt{d + c^2 dx^2}} \, dx = \int \frac{a + b \,\mathrm{asinh}\,(cx)}{\sqrt{d\,(c^2 x^2 + 1)}} \, dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \frac{b\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}} + \frac{a\,\text{arsinh}\,(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{d}\left(asinh(cx)^2 b+2\log\left(\sqrt{c^2x^2+1}+cx\right)a\right)}{2cd}$$

input
```
int((a+b*asinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*(asinh(c*x)**2*b + 2*log(sqrt(c**2*x**2 + 1) + c*x)*a))/(2*c*d)
```

3.51 $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 325

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\\
&\quad+\frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\\
&\quad-\frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(
c^2*f^2+g^2)^(1/2)))/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)-(c^2*x^2+1)^(
1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^
(1/2)))/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)+b*(c^2*x^2+1)^(1/2)*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/(c^2*f^2+g^2)^(1
/2)/(c^2*d*x^2+d)^(1/2)-b*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.74

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-\frac{a\text{arctanh}\left(\frac{\sqrt{d}(g-c^2fx)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\right)}{\sqrt{d}}+\frac{b\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)\left(\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)+\text{PolyLog}\left(2,\frac{e^{\text{a}}}{-cf}\right.}{\sqrt{d+c^2dx^2}}}{\sqrt{c^2f^2+g^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(-((a*ArcTanh[(Sqrt[d]*(g - c^2*f*x))/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*
x^2])])/Sqrt[d]) + (b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + S
qrt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f
^2 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))
]))/Sqrt[d + c^2*d*x^2])/Sqrt[c^2*f^2 + g^2]
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6260, 6258, 3042, 3803, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}(f+gx)}\,dx$$
$$\Big\downarrow \; 6260$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \; 6258$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cf+cgx}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{2\sqrt{c^2x^2+1}\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2ce^{\text{arcsinh}(cx)}f+e^{2\text{arcsinh}(cx)}g-g}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}-\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}-\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{ar}\cdots}}{\sqrt{c^2f\cdots}}\right.}{g}\right.}{\cdots}\right.}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(\cdots}{\cdots}\right.}{\cdots}\right.}{\sqrt{c^2dx^2+d}}$$

$\big\downarrow$ 2838

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{b\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+\cdots\right.}{g}\right.}{2\sqrt{c^2f^2\cdots}}\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*((g*(((a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2])])/g + (b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f
 - Sqrt[c^2*f^2 + g^2]))])/g))/(2*Sqrt[c^2*f^2 + g^2]) - (g*(((a + b*ArcSi
nh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g + (b*P
olyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/g))/(2*Sqrt[
c^2*f^2 + g^2])))/Sqrt[d + c^2*d*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.63

method	result
default	$-\frac{a \ln\left(\frac{\frac{2d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}-\frac{2c^{2} d f \left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}\sqrt{\left(x+\frac{f}{g}\right)^{2} c^{2} d-\frac{2c^{2} d f\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}}+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,\sqrt{c^{2} f^{2}+g^{2}}}{\dots}\right.$
parts	$-\frac{a \ln\left(\frac{\frac{2d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}-\frac{2c^{2} d f \left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}\sqrt{\left(x+\frac{f}{g}\right)^{2} c^{2} d-\frac{2c^{2} d f\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}}+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\,\sqrt{c^{2} f^{2}+g^{2}}}{\dots}\right.$

input
```
int((a+b*arcsinh(x*c))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-a/g/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+
f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*
(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*((d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^
(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(x*
c)*(ln((-(x*c+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2
+g^2)^(1/2)))-ln(((x*c+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+
(c^2*f^2+g^2)^(1/2))))+(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+
1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*(dilog((-(x*c+(c^2*x^2+1)
^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-dilog(((x*c
+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2))))
)
```

Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\operatorname{arsinh}\left(cx\right) + a}{\sqrt{c^2dx^2 + d}(gx + f)}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x
^2 + d*g*x + d*f), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(cx\right)}{\sqrt{d\left(c^2x^2 + 1\right)}\left(f + gx\right)}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(g*x+f)/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}(gx + f)}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}(gx + f)}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}(c\,x)}{(f + g\,x)\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

input
```
int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)\sqrt{d + c^2dx^2}}\,dx$$
$$= \frac{2\sqrt{c^2f^2 + g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)ai + \left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,f+\sqrt{c^2x^2+1}\,gx}dx\right)b\,c^2f^2 + \left(\int \frac{asinh(cx)}{\sqrt{c^2x^2+1}\,f+\sqrt{c^2x^2+1}\,gx}dx\right.}{\sqrt{d}\,(c^2f^2 + g^2)}$$

input
```
int((a+b*asinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(2*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*i + c*g*i*x)
/sqrt(c**2*f**2 + g**2))*a*i + int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*f + sqr
t(c**2*x**2 + 1)*g*x),x)*b*c**2*f**2 + int(asinh(c*x)/(sqrt(c**2*x**2 + 1)
*f + sqrt(c**2*x**2 + 1)*g*x),x)*b*g**2)/(sqrt(d)*(c**2*f**2 + g**2))
```

3.52 $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 438

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = -\frac{g\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{d\left(c^2f^2+g^2\right)(f+gx)} + \frac{c^2f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} - \frac{c^2f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} + \frac{bc^2f\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} - \frac{bc^2f\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

output
```
-g*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))/d/(c^2*f^2+g^2)/(g*x+f)+c^2*f*(c
^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^
2*f^2+g^2)^(1/2)))/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)-c^2*f*(c^2*x^2+
1)^(1/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g
^2)^(1/2)))/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)+b*c*(c^2*x^2+1)^(1/2)*
ln(g*x+f)/(c^2*f^2+g^2)/(c^2*d*x^2+d)^(1/2)+b*c^2*f*(c^2*x^2+1)^(1/2)*poly
log(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/(c^2*f^2+g^2)^
(3/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*f*(c^2*x^2+1)^(1/2)*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.57

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$$

$$= \frac{c\sqrt{1+c^2x^2}\left(-\frac{g\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{cf+cgx}+b\log(f+gx)+\frac{cf\left((a+b\text{arcsinh}(cx))\left(\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\text{a}\cdots}}{cf\cdots}\right.\right.\right.}{\cdots}\right.}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(c*Sqrt[1 + c^2*x^2]*(-((g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c*f +
c*g*x)) + b*Log[f + g*x] + (c*f*((a + b*ArcSinh[c*x])*(Log[1 + (E^ArcSinh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + S
qrt[c^2*f^2 + g^2])]) + b*PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2
*f^2 + g^2])] - b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))]))/Sqrt[c^2*f^2 + g^2]))/((c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {6260, 6258, 3042, 3805, 3042, 3147, 16, 3803, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}(f+gx)^2}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6258}$$

$$\frac{c\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(cf+cgx)^2}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(cf-ig\sin(i\text{arcsinh}(cx)))^2}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3805}$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf+cgx}d\text{arcsinh}(cx)}{c^2f^2+g^2}+\frac{bg\int \frac{\sqrt{c^2x^2+1}}{cf+cgx}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}+\frac{bg\int \frac{\cos(i\text{arcsinh}(cx))}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)}{(c^2f^2+g^2)(cf+cgx)}\right.}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{3147}$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}+\frac{b\int\frac{1}{cf+cgx}d(cgx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 16$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}+\frac{b\log(cf+cgx)}{c^2f^2+g^2}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 3803$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2ce^{\text{arcsinh}(cx)}f+e^{2\text{arcsinh}(cx)}g-g}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}+\frac{b\log(cf+cgx)}{c^2f^2+g^2}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \; 2694$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}-\frac{g\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}\right)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}\cdots}{(c^2f^2\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}-\frac{g\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}\right)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+\cdots}{(c^2f^2+g^2\cdots}$$

$$\Big\downarrow \; 2620$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\cdots}}{\sqrt{c\cdots}}\cdots\right)}{g}\cdots\right)}{\cdots}\right)}{c^2f^2+g^2}\cdots\right)}{\sqrt{c^2dx^2+d}\cdots}$$

$\downarrow$ 2715

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}\cdots}{\cdots}\right)}{\cdots}\right)}{c^2f^2+g^2}\cdots\right)}{\sqrt{c^2d\cdots}}$$

$\downarrow$ 2838

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\left(\frac{(a+b\,\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{b\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\,\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\cdots\right.}{g}\cdots\right.}{2\sqrt{c^2f^2\cdots}}\right)}{c^2f^2+g^2}\cdots\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(c*Sqrt[1 + c^2*x^2]*(-((g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*f
^2 + g^2)*(c*f + c*g*x))) + (b*Log[c*f + c*g*x])/(c^2*f^2 + g^2) + (2*c*f*
((g*(((a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
+ g^2])])/g + (b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2
]))])/g))/(2*Sqrt[c^2*f^2 + g^2]) - (g*(((a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g + (b*PolyLog[2, -((E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/g))/(2*Sqrt[c^2*f^2 + g^2])))/(c^
2*f^2 + g^2)))/Sqrt[d + c^2*d*x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(436) = 872$.

Time = 1.54 (sec) , antiderivative size = 1770, normalized size of antiderivative = 4.04

method	result	size
default	Expression too large to display	1770
parts	Expression too large to display	1770

input
```
int((a+b*arcsinh(x*c))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2
+g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2+g^2)/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2
*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x
+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(
d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^3*
c^4*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d/(c^2*f^2+g^2)/(g*x+f)*x*c^2*f
-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*
x^2*c^2*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d/(c^2*x^2+1)^(1/2)/(c^2*f^
2+g^2)/(g*x+f)*x*c*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d/(c^2*x^2+1)/(c
^2*f^2+g^2)/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/d/(c^2*x^
2+1)^(1/2)/(c^2*f^2+g^2)/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(x*c)/
d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*g-b*c^2*(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+
1))^(1/2)/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2
+g^4)/d*ln(((x*c+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f
^2+g^2)^(1/2)))*arcsinh(x*c)*(c^2*f^2+g^2)^(1/2)*f-2*b*c^3*(c^2*x^2+1)^(1/
2)*(d*(c^2*x^2+1))^(1/2)/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^
2+2*c^2*f^2*g^2+g^4)/d*ln(x*c+(c^2*x^2+1)^(1/2))*f^2+b*c^3*(c^2*x^2+1)^(1/
2)*(d*(c^2*x^2+1))^(1/2)/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^
2+2*c^2*f^2*g^2+g^4)/d*ln((x*c+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(x*c+(c^2*x^2+
1)^(1/2))-g)*f^2+b*c^2*(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/(c^6*f^4...
```

Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d
*f*g*x^3 + 2*d*f*g*x + d*f^2 + (c^2*d*f^2 + d*g^2)*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2 + 1)}\,(f + gx)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(g*x+f)**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}(gx + f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}(gx + f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{(f+g\,x)^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \frac{2\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)a\,c^2f^2i+2\sqrt{c^2f^2+g^2}\,atan\left(\frac{\sqrt{c^2x^2+1}\,gi+cfi+cgix}{\sqrt{c^2f^2+g^2}}\right)a\,c^2fgix-\sqrt{c^2x^2}}{}$$

input
```
int((a+b*asinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x)
```

output
```
(2*sqrt(c**2*f**2 + g**2)*atan((sqrt(c**2*x**2 + 1)*g*i + c*f*i + c*g*i*x)
/sqrt(c**2*f**2 + g**2))*a*c**2*f**2*i + 2*sqrt(c**2*f**2 + g**2)*atan((sq
rt(c**2*x**2 + 1)*g*i + c*f*i + c*g*i*x)/sqrt(c**2*f**2 + g**2))*a*c**2*f*
g*i*x - sqrt(c**2*x**2 + 1)*a*c**2*f**2*g - sqrt(c**2*x**2 + 1)*a*g**3 + i
nt(asinh(c*x)/(sqrt(c**2*x**2 + 1)*f**2 + 2*sqrt(c**2*x**2 + 1)*f*g*x + sq
rt(c**2*x**2 + 1)*g**2*x**2),x)*b*c**4*f**5 + int(asinh(c*x)/(sqrt(c**2*x*
*2 + 1)*f**2 + 2*sqrt(c**2*x**2 + 1)*f*g*x + sqrt(c**2*x**2 + 1)*g**2*x**2
),x)*b*c**4*f**4*g*x + 2*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*f**2 + 2*sqrt
(c**2*x**2 + 1)*f*g*x + sqrt(c**2*x**2 + 1)*g**2*x**2),x)*b*c**2*f**3*g**2
 + 2*int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*f**2 + 2*sqrt(c**2*x**2 + 1)*f*g*
x + sqrt(c**2*x**2 + 1)*g**2*x**2),x)*b*c**2*f**2*g**3*x + int(asinh(c*x)/
(sqrt(c**2*x**2 + 1)*f**2 + 2*sqrt(c**2*x**2 + 1)*f*g*x + sqrt(c**2*x**2 +
 1)*g**2*x**2),x)*b*f*g**4 + int(asinh(c*x)/(sqrt(c**2*x**2 + 1)*f**2 + 2*
sqrt(c**2*x**2 + 1)*f*g*x + sqrt(c**2*x**2 + 1)*g**2*x**2),x)*b*g**5*x)/(s
qrt(d)*(c**4*f**5 + c**4*f**4*g*x + 2*c**2*f**3*g**2 + 2*c**2*f**2*g**3*x
+ f*g**4 + g**5*x))
```

3.53 $\int \frac{(a+b\text{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}} dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}, x\right)$$

output
```
Defer(Int)((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} dx = \int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} dx$$

input
```
Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x
]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$
$$\Big\downarrow \text{ 6272}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
Int[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 8.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\,\text{arcsinh}\,(xc))^n \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
int((a+b*arcsinh(x*c))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^n\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^n\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**n*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^n \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 48.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^n \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,\left(a+b\,\text{asinh}(c\,x)\right)^n}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(asinh(cx)\,b+a)^n \log((gx+f)^m\,h)}{\sqrt{c^2x^2+1}}dx$$

input
```
int((a+b*asinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int(((asinh(c*x)*b + a)**n*log((f + g*x)**m*h))/sqrt(c**2*x**2 + 1),x)
```

3.54 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 438

$$\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{2b^2m\operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{2b^2m\operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}$$

output
```
1/12*m*(a+b*arcsinh(c*x))^4/b^2/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/b/c-1/3*m*(a+b*arcsinh(c*x))^
3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c+1/3*(a+b*a
rcsinh(c*x))^3*ln(h*(g*x+f)^m)/b/c-m*(a+b*arcsinh(c*x))^2*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))^2*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+2*b*m*(a+
b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1
/2)))/c+2*b*m*(a+b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
+(c^2*f^2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
-(c^2*f^2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
+(c^2*f^2+g^2)^(1/2)))/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx =$$
$$-\frac{-\frac{m(a+b\text{arcsinh}(cx))^4}{4b}+m(a+b\text{arcsinh}(cx))^3\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)+m(a+b\text{arcsinh}(cx))^3\log\left(1+\frac{}{c}\right.}{}$$

input
```
Integrate[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
-1/3*(-1/4*(m*(a + b*ArcSinh[c*x])^4)/b + m*(a + b*ArcSinh[c*x])^3*Log[1 +
 (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^
3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh
[c*x])^3*Log[h*(f + g*x)^m] + 3*b*m*((a + b*ArcSinh[c*x])^2*PolyLog[2, (E^
ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - 2*b*(a + b*ArcSinh[c*x])
*PolyLog[3, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] + 2*b^2*Pol
yLog[4, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 3*b*m*((a +
b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))] - 2*b*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sq
rt[c^2*f^2 + g^2]))] + 2*b^2*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))]))/(b*c)
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6261, 6242, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6261}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\int \frac{(a+b\text{arcsinh}(cx))^3}{f+gx}dx}{3bc}$$

$$\Big\downarrow \text{ 6242}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{cf+cgx}d\text{arcsinh}(cx)}{3bc}$$

$$\Big\downarrow \text{ 6095}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\left(\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^3}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)+\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^3}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^4}{4bg}\right)}{3bc}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\left(-\frac{3b\int (a+b\text{arcsinh}(cx))^2\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\frac{3b\int (a+b\text{arcsinh}(cx))^2\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\right.}{3bc}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-$$

$$gm\left(-\frac{3b\left(2b\int (a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))^2\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{3b}{}\right.$$

$\Big\downarrow$ 7163

$$\frac{(a+b\text{arcsinh}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$
$$gm\left(-\frac{3b\left(2b\left((a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-b\int\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)d\text{arcsinh}(cx)\right)-(a+b\text{arcsinh}(cx))^2\,\text{P}\right.}{g}\right.$$

$\Big\downarrow$ 2720

$$\frac{(a+b\text{arcsinh}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$
$$gm\left(-\frac{3b\left(2b\left((a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)de^{\text{arcsinh}(cx)}\right)-(a+b\text{arcs}\right.}{g}\right.$$

$\Big\downarrow$ 7143

$$\frac{(a+b\text{arcsinh}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$
$$gm\left(-\frac{3b\left(2b\left((a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-b\,\text{PolyLog}\left(4,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)-(a+b\text{arcsinh}(cx))^2\,\text{PolyLog}\left(2,-\frac{e^{\text{ar}}}{cf-}\right.\right.}{g}\right.$$

input
```
Int[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (g*m*(-1/4*(a + b*Ar
cSinh[c*x])^4/(b*g) + ((a + b*ArcSinh[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(
c*f - Sqrt[c^2*f^2 + g^2])])/g + ((a + b*ArcSinh[c*x])^3*Log[1 + (E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g - (3*b*(-((a + b*ArcSinh[c*x])^2
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]) + 2*b*((a
+ b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^
2]))] - b*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])))
/g - (3*b*(-((a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f +
 Sqrt[c^2*f^2 + g^2]))]) + 2*b*((a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSi
nh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] - b*PolyLog[4, -((E^ArcSinh[c*x]*
g)/(c*f + Sqrt[c^2*f^2 + g^2]))])))/g))/(3*b*c)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6261
```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[g*(m/(b*c*S
qrt[d]*(n + 1)))   Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n
, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arcsinh}\,(xc))^2 \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
int((a+b*arcsinh(x*c))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*log((g*x + f)^m*h
)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2\log(h(f+gx)^m)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2\log((gx+f)^m h)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,1,0,0]%%%}+%%%{-1,[0,0,1,1,0,0]%%%} / %%%{1,[0,0,
0,0,1,1]%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\ln\left(h\,(f+g\,x)^m\right)\,\left(a+b\,\text{asinh}(c\,x)\right)^2}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\left(\int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}dx\right)a^2+2\left(\int \frac{asinh(cx)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}dx\right)ab+\left(\int \frac{asinh(cx)^2\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}dx\right)b^2$$

input
```
int((a+b*asinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int(log((f + g*x)**m*h)/sqrt(c**2*x**2 + 1),x)*a**2 + 2*int((asinh(c*x)*lo
g((f + g*x)**m*h))/sqrt(c**2*x**2 + 1),x)*a*b + int((asinh(c*x)**2*log((f
 + g*x)**m*h))/sqrt(c**2*x**2 + 1),x)*b**2
```

3.55 $\int \frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 332

$$\begin{aligned}
&\int \frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\text{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad + \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{bm\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{bm\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}$$

output
```
1/6*m*(a+b*arcsinh(c*x))^3/b^2/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2
*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/b/c-1/2*m*(a+b*arcsinh(c*x))^2
*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c+1/2*(a+b*ar
csinh(c*x))^2*ln(h*(g*x+f)^m)/b/c-m*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,
-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*
x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$
$$=\frac{\frac{m(a+b\text{arcsinh}(cx))^3}{3b}-m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsi}}}{cf+\sqrt{}}\right.}{}$$

input
```
Integrate[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
((m*(a + b*ArcSinh[c*x])^3)/(3*b) - m*(a + b*ArcSinh[c*x])^2*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - m*(a + b*ArcSinh[c*x])^2*Log[
1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] + (a + b*ArcSinh[c*x])
^2*Log[h*(f + g*x)^m] + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, (E^ArcSin
h[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + b*PolyLog[3, (E^ArcSinh[c*x]*
g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLo
g[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[3, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/(2*b*c)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6261, 6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6261}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\int\frac{(a+b\text{arcsinh}(cx))^2}{f+gx}dx}{2bc}$$

$$\Big\downarrow \text{6242}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{cf+cgx}d\text{arcsinh}(cx)}{2bc}$$

$$\Big\downarrow \text{6095}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)+\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^3}{3bg}\right)}{2bc}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(-\frac{2b\int(a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\frac{2b\int(a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}+\cdots\right.}{2bc}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(-\frac{2b\left(b\int\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{2b\left(b\int\text{PolyLog}\left(2,-\frac{e^{\cdots}}{cf\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \; 2720$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-$$

$$gm\left(-\frac{2b\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)de^{\text{arcsinh}(cx)}-(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{2b\left(b\int e^{-a}\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-$$

$$gm\left(-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)-(a-\right.}{}\right.$$

input
```
Int[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (g*m*(-1/3*(a + b*Ar
cSinh[c*x])^3/(b*g) + ((a + b*ArcSinh[c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(
c*f - Sqrt[c^2*f^2 + g^2])])/g + ((a + b*ArcSinh[c*x])^2*Log[1 + (E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g - (2*b*(-((a + b*ArcSinh[c*x])*P
olyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[
3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]))/g - (2*b*(-((a + b
*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
)]) + b*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/g)
)/(2*b*c)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6261
```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[g*(m/(b*c*S
qrt[d]*(n + 1)))   Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n
, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(xc))\ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
int((a+b*arcsinh(x*c))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arcsinh(x*c))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}\,(cx))\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,1,0,0]%%%}+%%%{-1,[0,0,1,1,0,0]%%%} / %%%{1,[0,0,
0,0,1,1]%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,\left(a+b\,\text{asinh}(c\,x)\right)}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \left(\int \frac{\log((gx+f)^m h)}{\sqrt{c^2x^2+1}}dx\right)a + \left(\int \frac{asinh(cx)\log((gx+f)^m h)}{\sqrt{c^2x^2+1}}dx\right)b$$

input
```
int((a+b*asinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int(log((f + g*x)**m*h)/sqrt(c**2*x**2 + 1),x)*a + int((asinh(c*x)*log((f
+ g*x)**m*h))/sqrt(c**2*x**2 + 1),x)*b
```

3.56 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{m\,\text{arcsinh}(cx)^2}{2c} - \frac{m\,\text{arcsinh}(cx)\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\,\text{arcsinh}(cx)\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - \frac{m\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

output

```
1/2*m*arcsinh(c*x)^2/c-m*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-
(c^2*f^2+g^2)^(1/2)))/c-m*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
+(c^2*f^2+g^2)^(1/2)))/c+arcsinh(c*x)*ln(h*(g*x+f)^m)/c-m*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}} dx = \frac{m\text{arcsinh}(cx)^2}{2c} - \frac{m\text{arcsinh}(cx)\log \left(1+\frac{ce^{\text{arcsinh}(cx)}g}{c^2f-c\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\text{arcsinh}(cx)\log \left(1+\frac{ce^{\text{arcsinh}(cx)}g}{c^2f+c\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\text{arcsinh}(cx)\log (h(f+gx)^m)}{c} - \frac{m\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

input
```
Integrate[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

output
```
(m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]*g)/(c
^2*f - c*Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c
*x]*g)/(c^2*f + c*Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)
^m])/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2851, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{c^2x^2+1}} dx$$

$$\Big\downarrow \text{2851}$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-gm\int\frac{\text{arcsinh}(cx)}{c(f+gx)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\int\frac{\text{arcsinh}(cx)}{f+gx}dx}{c}$$

$$\Big\downarrow \ 6242$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\int\frac{\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{cf+cgx}d\text{arcsinh}(cx)}{c}$$

$$\Big\downarrow \ 6095$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(\int\frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)+\int\frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)-\frac{\text{arcsinh}(cx)^2}{2g}\right)}{c}$$

$$\Big\downarrow \ 2620$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(-\frac{\int\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\frac{\int\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}+\frac{\text{arcsinh}(cx)\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\ldots\right.}{c}$$

$$\Big\downarrow \ 2715$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}+\frac{\text{arcsinh}(cx)\log\left(\ldots\right.}{\ldots}\right.}{c}$$

$$\Big\downarrow \ 2838$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(\frac{\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g}+\frac{\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g}+\frac{\text{arcsinh}(cx)\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{\text{arcsinh}(cx)\log\left(\frac{ge^{\ldots}}{\sqrt{c^2f^2\ldots}}\right.}{g}\right.}{c}$$

input
```
Int[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

output
```
(ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (g*m*(-1/2*ArcSinh[c*x]^2/g + (ArcSi
nh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/g + (ArcS
inh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g + Poly
Log[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]/g + PolyLog[2, -
((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]/g))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\ln \left(h(gx+f)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}dx$$

input
```
int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log \left(h(f+gx)^{m}\right)}{\sqrt{1+c^{2}x^{2}}}\,dx=\int \frac{\log \left((gx+f)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln(h\,(f+g\,x)^m)}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\log((gx+f)^m\,h)}{\sqrt{c^2x^2+1}}dx$$

input
```
int(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int(log((f + g*x)**m*h)/sqrt(c**2*x**2 + 1),x)
```

3.57 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Defer(Int)(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{ 6272}$$
$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\ln \left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}\ (a+b\ \text{arcsinh}\ (xc))}dx$$

input
```
int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(x*c)),x)
```

output
```
int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(x*c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 + b)
*arcsinh(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 8.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log(h(f+gx)^m)}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

output
```
Integral(log(h*(f + g*x)**m)/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{\ln \left(h\,(f+g\,x)^m\right)}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}} \, dx$$

input
```
int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}\,asinh\,(cx)\,b+\sqrt{c^2x^2+1}\,a}dx$$

input
```
int(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2)/(a+b*asinh(c*x)),x)
```

output
```
int(log((f + g*x)**m*h)/(sqrt(c**2*x**2 + 1)*asinh(c*x)*b + sqrt(c**2*x**2
 + 1)*a),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0])  #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:])  #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file