Computer Algebra
Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/330-7.2

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**190**]. This is test number [330].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (190)	0.00 (0)
Mathematica	97.89 (186)	2.11 (4)
Maple	61.05 (116)	38.95 (74)
Fricas	38.95 (74)	61.05 (116)
Giac	22.63 (43)	77.37 (147)
Maxima	22.11 (42)	77.89 (148)
Reduce	21.58 (41)	78.42 (149)
Mupad	17.37 (33)	82.63 (157)
Sympy	11.58 (22)	88.42 (168)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

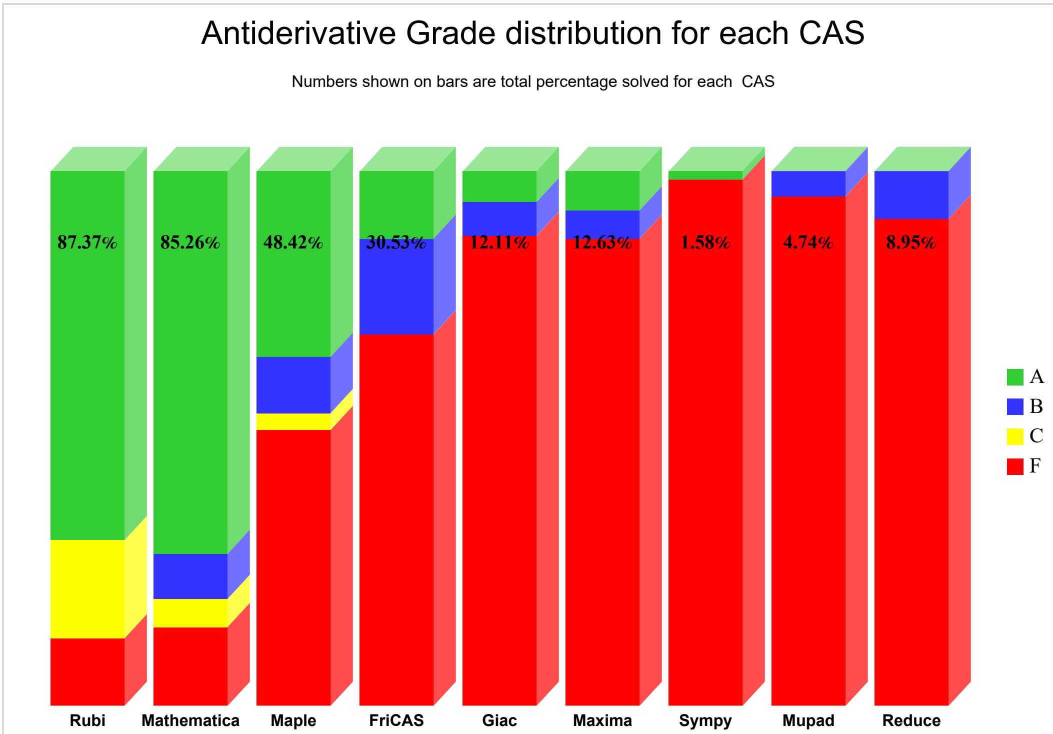
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

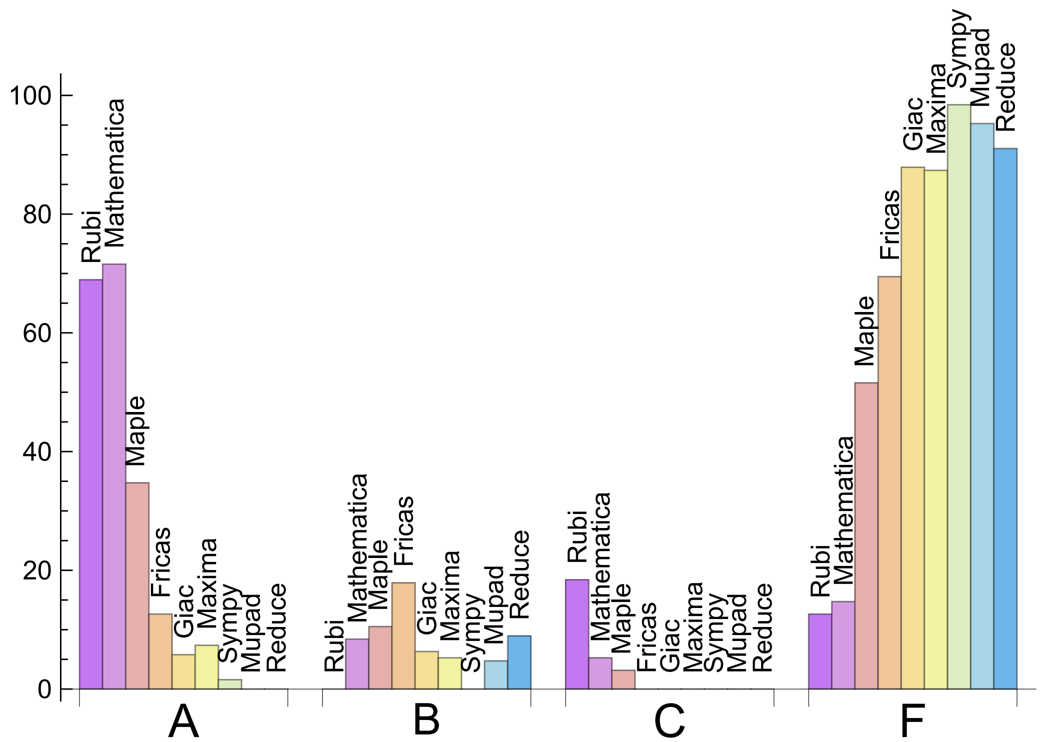
System	% A grade	% B grade	% C grade	% F grade
Mathematica	71.579	8.421	5.263	14.737
Rubi	68.947	0.000	18.421	12.632
Maple	34.737	10.526	3.158	51.579
Fricas	12.632	17.895	0.000	69.474
Maxima	7.368	5.263	0.000	87.368
Giac	5.789	6.316	0.000	87.895
Sympy	1.579	0.000	0.000	98.421
Mupad	0.000	4.737	0.000	95.263
Reduce	0.000	8.947	0.000	91.053

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Maple	74	100.00	0.00	0.00
Fricas	116	46.55	0.00	53.45
Maxima	148	79.05	4.05	16.89
Giac	147	79.59	5.44	14.97
Reduce	149	100.00	0.00	0.00
Mupad	157	0.00	100.00	0.00
Sympy	168	84.52	15.48	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Reduce	0.28
Maple	0.36
Rubi	0.95
Mathematica	1.94
Giac	2.88
Mupad	3.36
Maxima	4.20
Sympy	21.45

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	50.41	1.94	35.00	1.00
Mupad	59.33	1.59	25.00	1.00
Giac	106.49	1.57	45.00	1.09
Reduce	129.54	3.28	102.00	2.28
Rubi	167.98	1.01	147.50	1.00
Mathematica	208.42	1.22	144.50	1.06
Maple	223.23	1.61	132.50	1.33
Fricas	223.38	2.16	131.00	2.04
Maxima	398.71	13.06	44.00	1.04

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

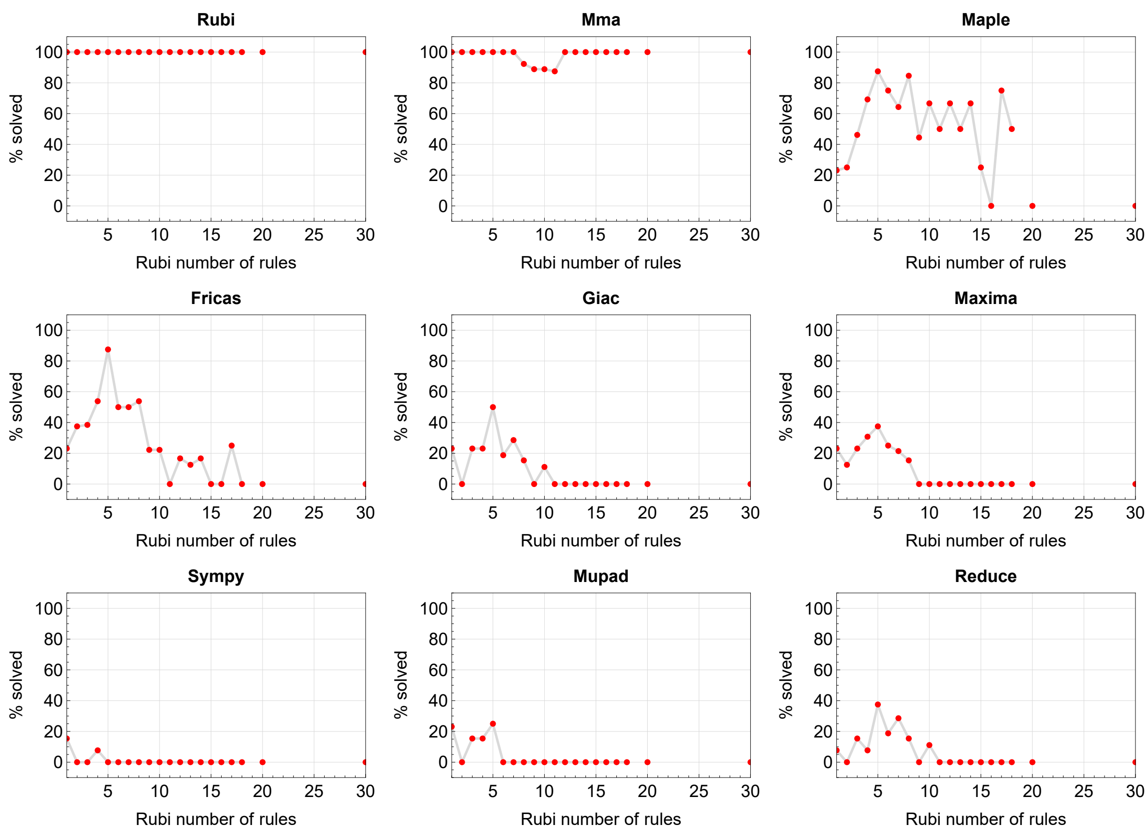


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

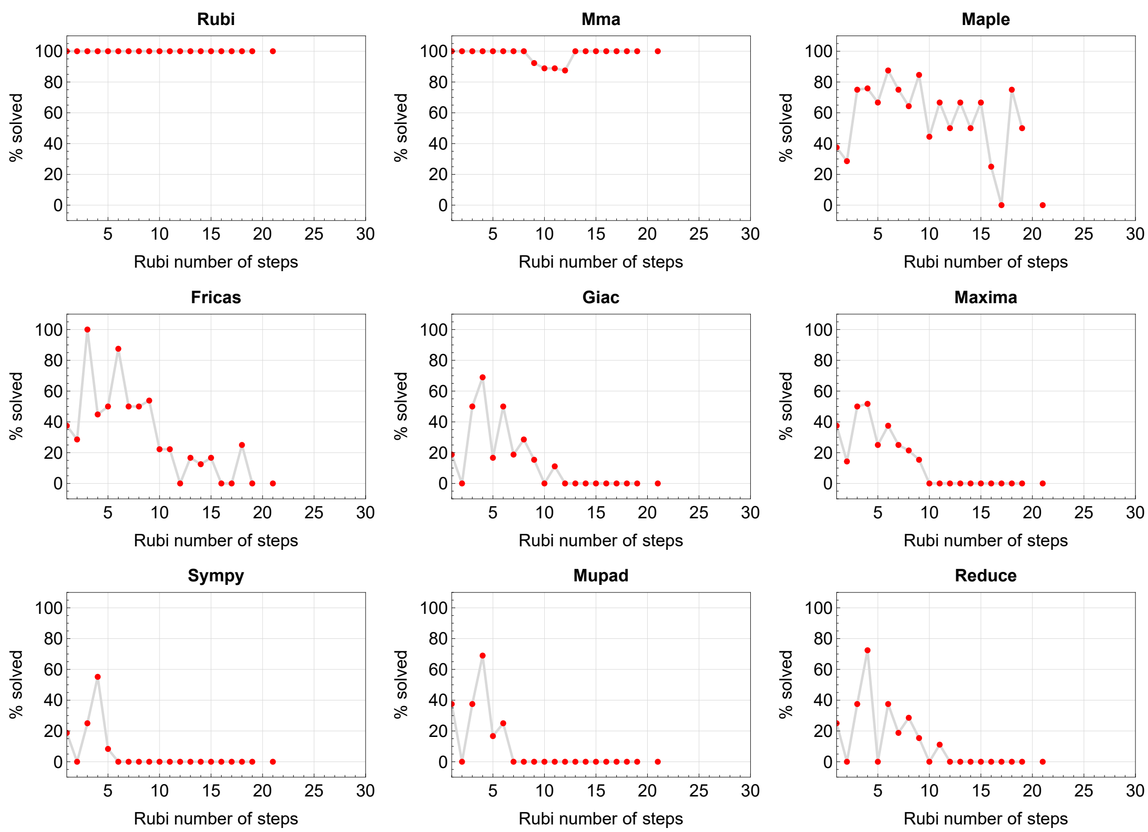


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

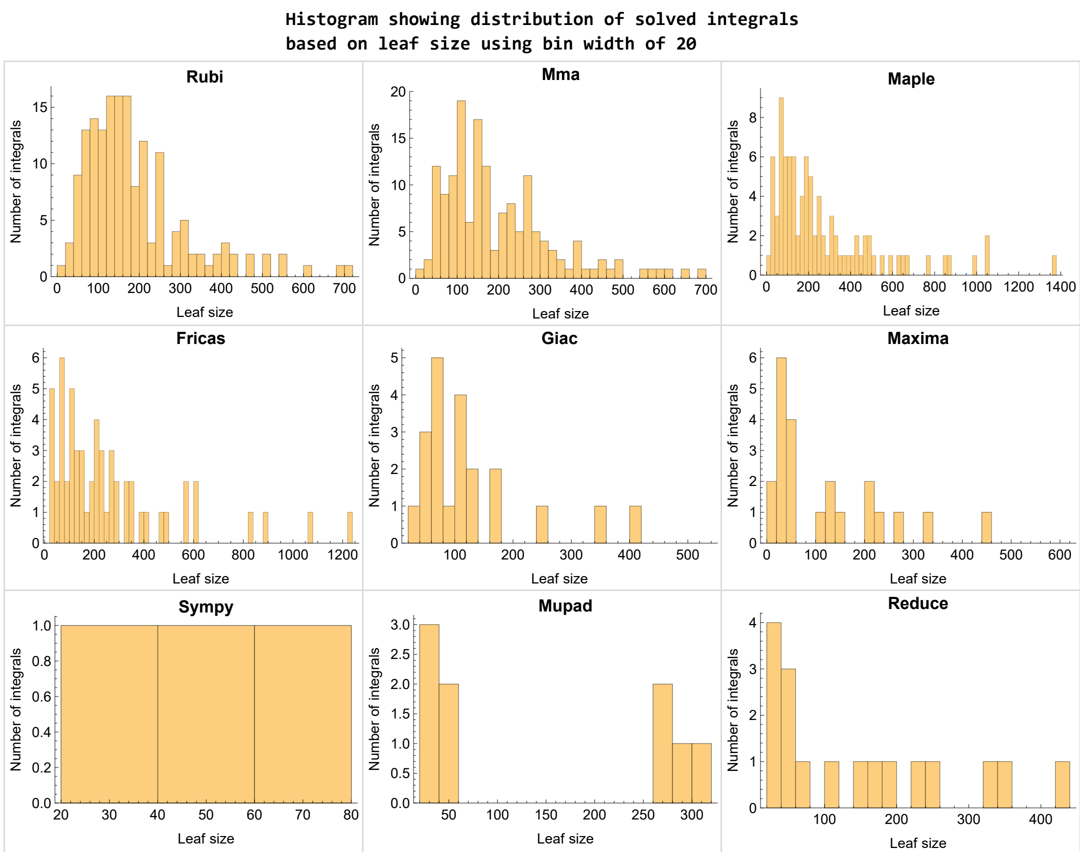


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

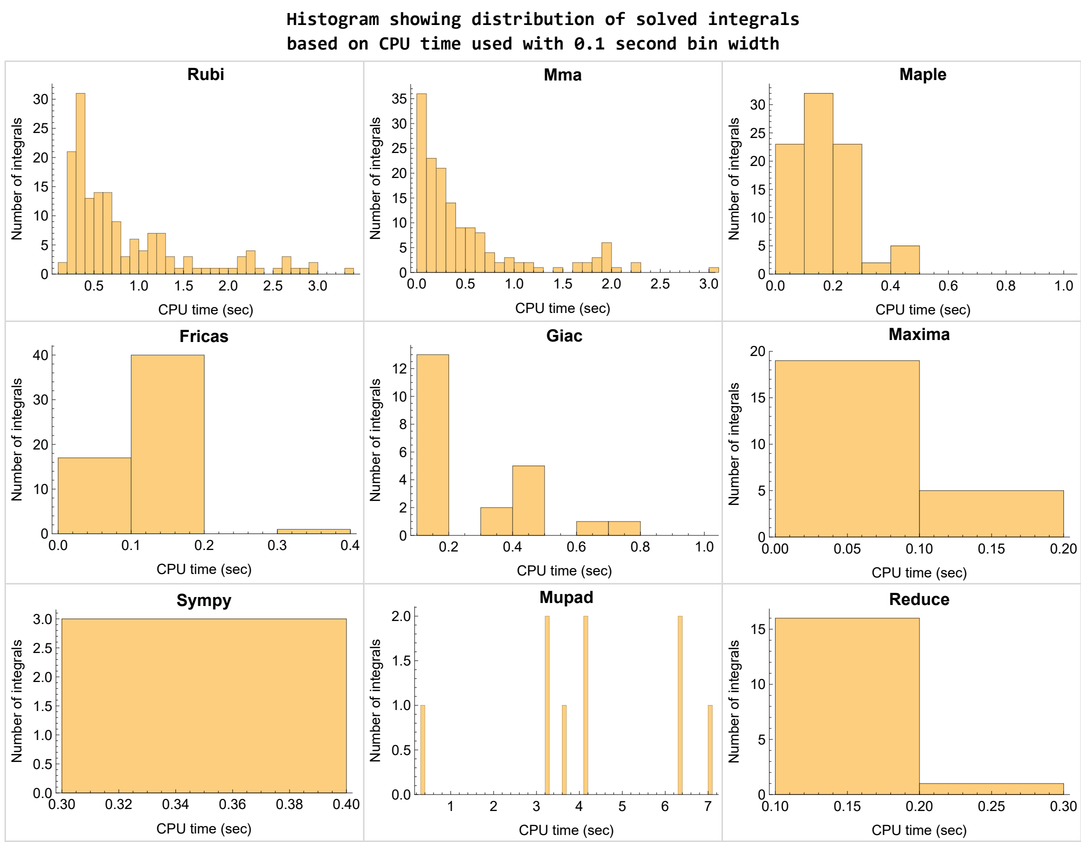


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

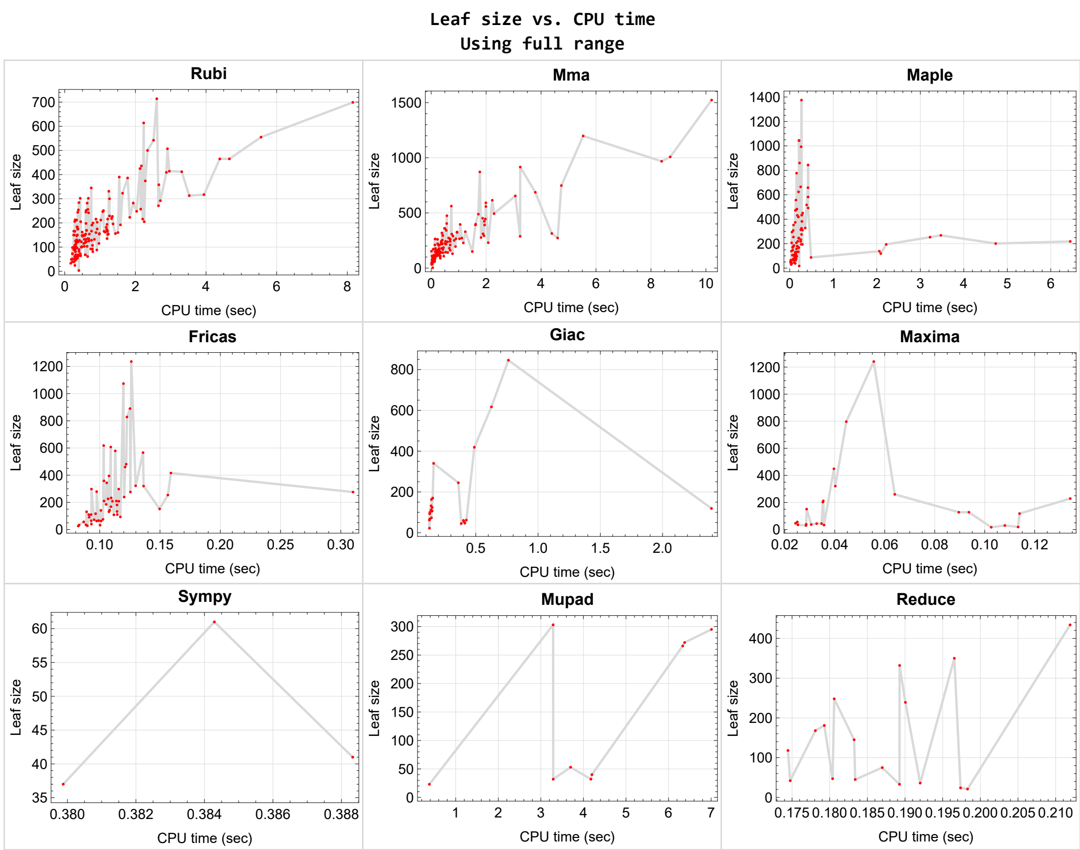


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{53, 59, 65, 71, 77, 82, 87, 91, 97, 103, 109, 115, 129, 130, 131, 132, 133, 134, 135, 138, 139, 177, 181, 182\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{16, 27, 36, 38, 44, 46, 178, 179, 180, 183, 184\}$

Mathematica $\{4, 15, 30, 38, 39, 46, 47, 54, 55, 56, 57, 58, 72, 78, 79, 81, 84, 86, 88, 90, 98, 99, 100, 102, 104, 108, 110, 112, 113, 114, 116, 141, 142, 161, 169, 188\}$

Maple {185}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

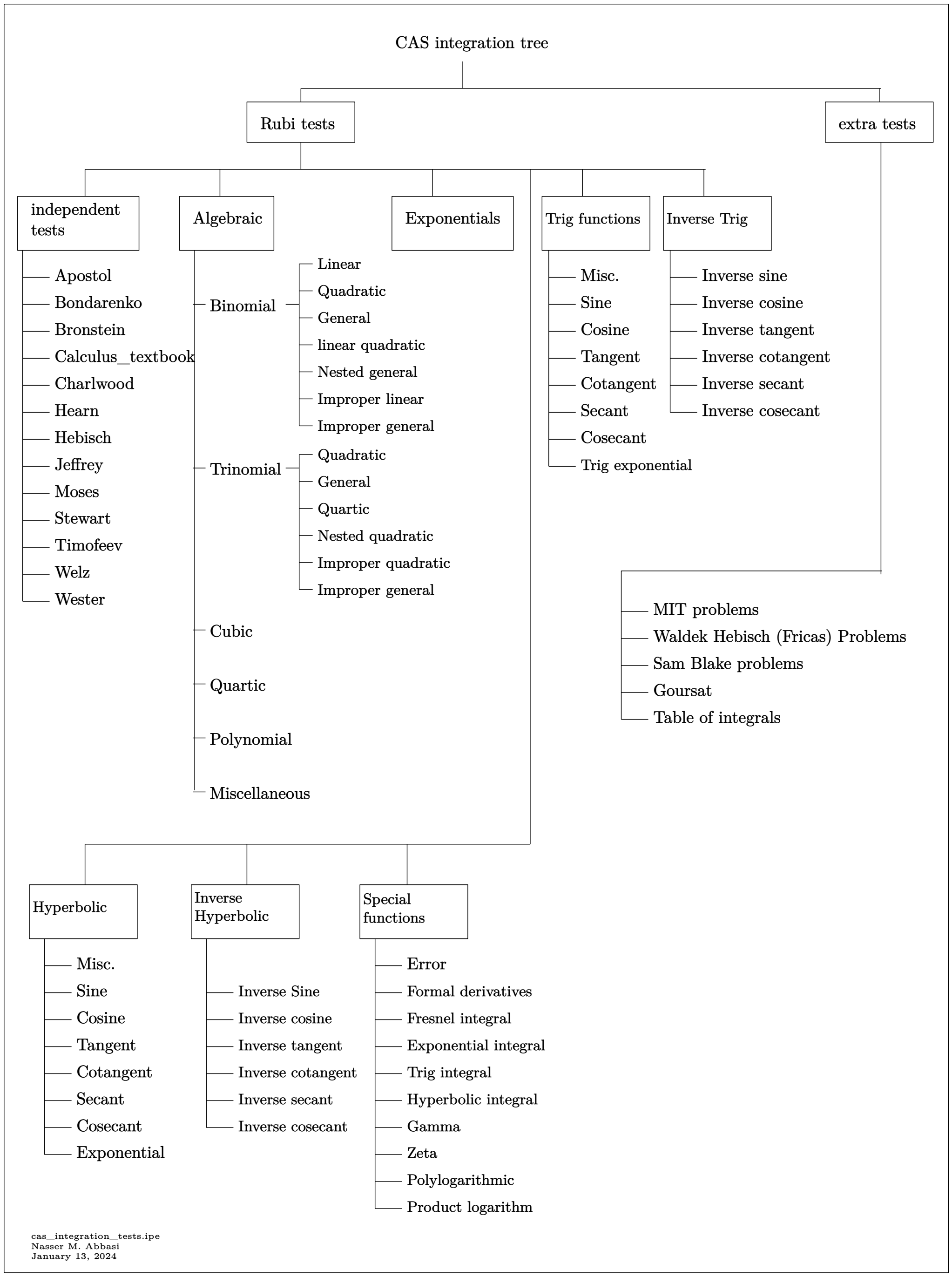
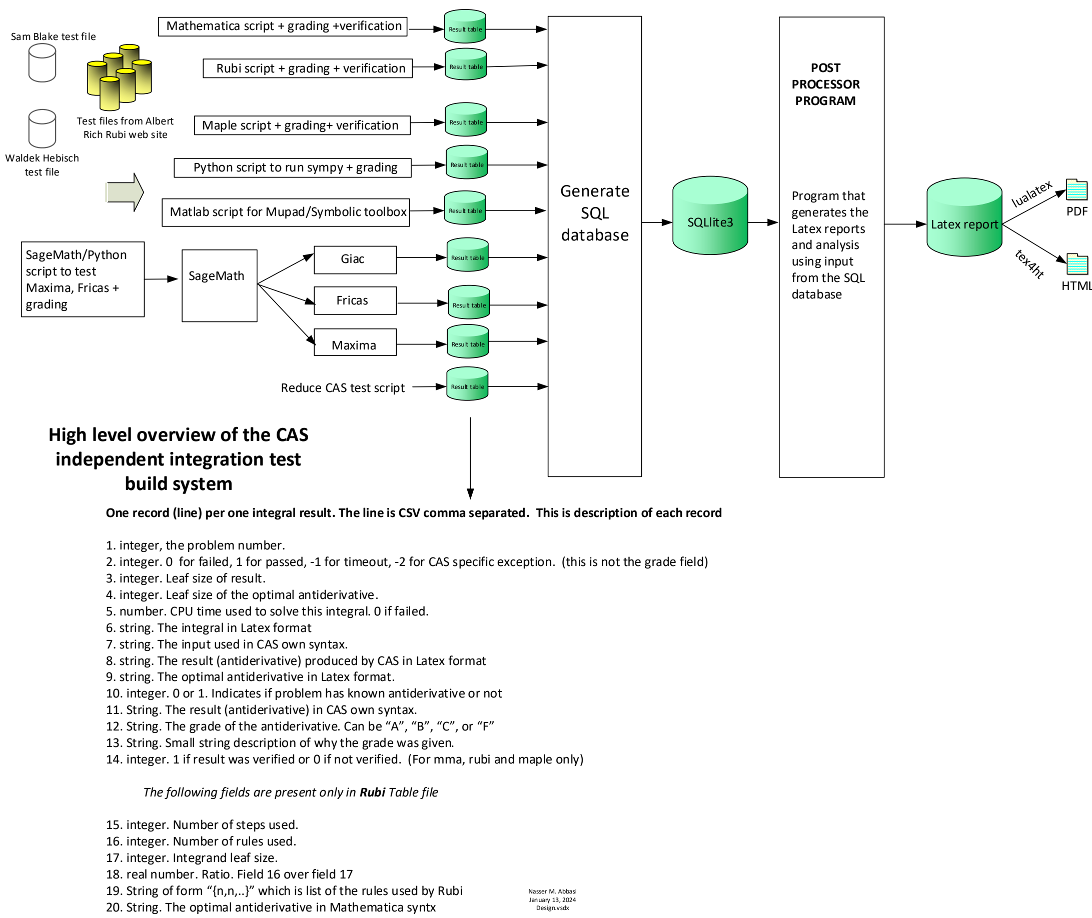


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 54, 55, 56, 57, 58, 60, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 83, 84, 85, 86, 92, 93, 94, 98, 99, 100, 101, 102, 104, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 136, 137, 141, 142, 143, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 185, 186, 187, 188, 189, 190 }

B grade { }

C grade { 9, 10, 16, 27, 36, 38, 44, 46, 51, 52, 61, 62, 63, 64, 78, 79, 80, 81, 88, 89, 90, 95, 96, 105, 106, 107, 108, 140, 144, 148, 178, 179, 180, 183, 184 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 78, 79, 80, 81, 85, 89, 92, 93, 94, 95, 96, 98, 99, 100, 102, 104, 105, 106, 108, 110, 111, 112, 114, 123, 124, 125, 126, 127, 128, 136, 137, 140, 141, 142, 144, 145, 146, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 184, 185, 186, 187, 189, 190 }

B grade { 43, 45, 46, 47, 83, 84, 86, 88, 90, 101, 107, 113, 143, 147, 148, 188 }

C grade { 6, 7, 8, 116, 117, 118, 119, 120, 121, 122 }

F normal fail { 178, 179, 180, 183 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 40, 41, 42, 48, 49, 50, 51, 52, 57, 58, 63, 64, 69, 70, 116, 118, 120, 122, 141, 142, 143, 144, 145, 146, 147, 148, 152, 159, 179, 180, 183, 184, 186, 188 }

B grade { 5, 43, 44, 46, 54, 55, 56, 60, 61, 62, 66, 67, 68, 149, 150, 151, 156, 157, 158, 178 }

C grade { 7, 8, 117, 119, 121, 185 }

F normal fail { 9, 10, 37, 39, 45, 47, 72, 73, 74, 75, 76, 78, 79, 80, 81, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 136, 137, 140, 153, 154, 155, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 187, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 14, 15, 116, 117, 118, 119, 120, 121, 122, 141, 142, 143, 145, 146, 150, 151, 152, 158, 186, 190 }

B grade { 6, 7, 8, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 31, 32, 33, 34, 35, 40, 41, 42, 43, 147, 149, 156, 157, 159, 187, 188, 189 }

C grade { }

F normal fail { 5, 16, 27, 28, 30, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 123, 124, 125, 126, 127, 128, 136, 137, 140, 144, 153, 154, 155, 160, 161, 162, 178, 179, 180, 184, 185 }

F(-1) timedout fail { }

F(-2) exception fail { 9, 10, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 148, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 183 }

Maxima

A grade { 2, 4, 15, 141, 142, 143, 145, 146, 151, 152, 158, 159, 186, 187 }

B grade { 1, 3, 11, 12, 13, 14, 18, 20, 29, 147 }

C grade { }

F normal fail { 5, 9, 10, 16, 19, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 72, 73, 74, 75, 76, 78, 79, 80, 81, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 136, 137, 140, 144, 148, 149, 150, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 183, 184, 185, 188, 189, 190 }

F(-1) timedout fail { 66, 67, 68, 69, 70, 71 }

F(-2) exception fail { 6, 7, 8, 17, 28, 37, 45, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133 }

Giac

A grade { 1, 2, 3, 6, 7, 141, 142, 143, 145, 146, 152 }

B grade { 4, 8, 11, 12, 13, 14, 15, 147, 159, 186, 187, 188 }

C grade { }

F normal fail { 5, 9, 10, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 78, 79, 80, 81, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 136, 137, 140, 144, 148, 153, 154, 155, 160, 161, 162, 167, 168, 169, 174, 175, 176, 183, 184, 185 }

F(-1) timedout fail { 177, 178, 179, 180, 181, 182, 189, 190 }

F(-2) exception fail { 17, 20, 28, 37, 45, 121, 128, 133, 149, 150, 151, 156, 157, 158, 163, 164, 165, 166, 170, 171, 172, 173 }

Mupad

A grade { }

B grade { 4, 15, 143, 147, 152, 159, 186, 187, 188 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 78, 79, 80, 81, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 136, 137, 140, 141, 142, 144, 145, 146, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 183, 184, 185, 189, 190 }

F(-2) exception fail { }

Sympy

A grade { 152, 159, 186 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 78, 79, 80, 81, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 124, 125, 126, 127, 128, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 183, 184, 185, 187, 188, 189, 190 }

F(-1) timedout fail { 83, 84, 85, 86, 87, 88, 89, 90, 91, 110, 111, 112, 113, 114, 115, 116, 122, 123, 163, 168, 169, 170, 175, 176, 177, 182 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 11, 12, 13, 14, 15, 141, 142, 143, 145, 146 }

C grade { }

F normal fail { 5, 9, 10, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 78, 79, 80, 81, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 136, 137, 140, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	169	121	194	321	110	0	163	248	0
N.S.	1	0.96	0.69	1.10	1.82	0.62	0.00	0.93	1.41	0.00
time (sec)	N/A	0.615	0.159	0.265	0.040	0.093	0.000	0.144	0.181	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	121	101	147	212	91	0	132	168	0
N.S.	1	0.93	0.78	1.13	1.63	0.70	0.00	1.02	1.29	0.00
time (sec)	N/A	0.351	0.119	0.070	0.035	0.091	0.000	0.141	0.178	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	87	106	151	75	0	112	118	0
N.S.	1	1.06	0.97	1.18	1.68	0.83	0.00	1.24	1.31	0.00
time (sec)	N/A	0.278	0.068	0.059	0.029	0.103	0.000	0.141	0.174	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	39	59	36	30	57	0	93	42	266
N.S.	1	0.95	1.44	0.88	0.73	1.39	0.00	2.27	1.02	6.49
time (sec)	N/A	0.220	0.097	0.040	0.028	0.087	0.000	0.129	0.175	6.333

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	153	431	0	0	0	0	12	0
N.S.	1	1.00	1.17	3.29	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.590	0.011	0.174	0.000	0.000	0.000	0.000	0.190	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	83	101	0	322	0	73	75	0
N.S.	1	1.06	1.30	1.58	0.00	5.03	0.00	1.14	1.17	0.00
time (sec)	N/A	0.309	0.096	0.106	0.000	0.130	0.000	0.144	0.187	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	136	172	0	460	0	170	145	0
N.S.	1	1.09	1.28	1.62	0.00	4.34	0.00	1.60	1.37	0.00
time (sec)	N/A	0.320	0.203	0.072	0.000	0.121	0.000	0.154	0.183	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	175	162	308	0	566	0	340	332	0
N.S.	1	1.14	1.05	2.00	0.00	3.68	0.00	2.21	2.16	0.00
time (sec)	N/A	0.373	0.231	0.078	0.000	0.136	0.000	0.161	0.189	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	100	110	0	0	0	0	0	26	0
N.S.	1	1.09	1.20	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.487	0.098	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	102	111	0	0	0	0	0	31	0
N.S.	1	1.09	1.18	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.485	0.092	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	126	74	78	1241	279	0	846	434	0
N.S.	1	0.93	0.55	0.58	9.19	2.07	0.00	6.27	3.21	0.00
time (sec)	N/A	0.306	0.096	0.147	0.056	0.097	0.000	0.762	0.212	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	110	113	127	797	226	0	617	350	0
N.S.	1	0.92	0.95	1.07	6.70	1.90	0.00	5.18	2.94	0.00
time (sec)	N/A	0.306	0.092	0.135	0.045	0.107	0.000	0.626	0.197	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	90	71	67	449	168	0	419	239	0
N.S.	1	0.93	0.73	0.69	4.63	1.73	0.00	4.32	2.46	0.00
time (sec)	N/A	0.403	0.060	0.125	0.040	0.109	0.000	0.489	0.190	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	72	81	100	203	110	0	245	181	0
N.S.	1	0.96	1.08	1.33	2.71	1.47	0.00	3.27	2.41	0.00
time (sec)	N/A	0.439	0.126	0.104	0.035	0.091	0.000	0.360	0.179	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	41	35	65	0	100	45	272
N.S.	1	1.00	1.39	0.89	0.76	1.41	0.00	2.17	0.98	5.91
time (sec)	N/A	0.303	0.102	0.039	0.025	0.093	0.000	0.131	0.183	6.378

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	88	69	97	0	0	0	0	35	0
N.S.	1	1.09	0.85	1.20	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.908	0.085	0.161	0.000	0.000	0.000	0.000	0.191	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	51	78	81	0	133	0	0	82	0
N.S.	1	0.91	1.39	1.45	0.00	2.38	0.00	0.00	1.46	0.00
time (sec)	N/A	0.280	0.093	0.141	0.000	0.114	0.000	0.000	0.188	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	61	55	65	118	117	0	0	168	0
N.S.	1	0.92	0.83	0.98	1.79	1.77	0.00	0.00	2.55	0.00
time (sec)	N/A	0.280	0.040	0.137	0.114	0.097	0.000	0.000	0.196	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	92	115	110	0	276	0	0	273	0
N.S.	1	0.93	1.16	1.11	0.00	2.79	0.00	0.00	2.76	0.00
time (sec)	N/A	0.297	0.090	0.142	0.000	0.125	0.000	0.000	0.200	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	97	86	76	260	208	0	0	400	0
N.S.	1	0.93	0.83	0.73	2.50	2.00	0.00	0.00	3.85	0.00
time (sec)	N/A	0.344	0.053	0.144	0.064	0.111	0.000	0.000	0.201	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	128	147	131	0	416	0	0	549	0
N.S.	1	0.93	1.07	0.96	0.00	3.04	0.00	0.00	4.01	0.00
time (sec)	N/A	0.415	0.144	0.152	0.000	0.159	0.000	0.000	0.212	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	192	220	218	0	618	0	0	573	0
N.S.	1	0.88	1.01	1.00	0.00	2.83	0.00	0.00	2.63	0.00
time (sec)	N/A	1.577	0.365	0.250	0.000	0.103	0.000	0.000	0.265	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	165	212	244	0	481	0	0	459	0
N.S.	1	0.89	1.14	1.31	0.00	2.59	0.00	0.00	2.47	0.00
time (sec)	N/A	1.110	0.317	0.210	0.000	0.122	0.000	0.000	0.235	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	134	168	167	0	358	0	0	317	0
N.S.	1	0.89	1.12	1.11	0.00	2.39	0.00	0.00	2.11	0.00
time (sec)	N/A	0.681	0.279	0.208	0.000	0.104	0.000	0.000	0.214	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	103	167	157	0	233	0	0	229	0
N.S.	1	0.94	1.52	1.43	0.00	2.12	0.00	0.00	2.08	0.00
time (sec)	N/A	0.632	0.279	0.155	0.000	0.109	0.000	0.000	0.228	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	105	98	0	141	0	0	67	0
N.S.	1	1.03	1.64	1.53	0.00	2.20	0.00	0.00	1.05	0.00
time (sec)	N/A	0.385	0.091	0.107	0.000	0.101	0.000	0.000	0.195	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	128	140	220	0	0	0	0	62	0
N.S.	1	1.09	1.20	1.88	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.756	0.427	0.188	0.000	0.000	0.000	0.000	0.200	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	92	161	246	0	0	0	0	160	0
N.S.	1	0.84	1.46	2.24	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.780	0.692	0.202	0.000	0.000	0.000	0.000	0.217	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	81	81	168	229	320	0	0	323	0
N.S.	1	0.88	0.88	1.83	2.49	3.48	0.00	0.00	3.51	0.00
time (sec)	N/A	0.795	0.139	0.204	0.134	0.136	0.000	0.000	0.214	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	154	251	314	0	0	0	0	527	0
N.S.	1	0.83	1.35	1.69	0.00	0.00	0.00	0.00	2.83	0.00
time (sec)	N/A	1.233	0.839	0.253	0.000	0.000	0.000	0.000	0.217	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	425	404	450	0	1074	0	0	724	0
N.S.	1	1.14	1.08	1.20	0.00	2.87	0.00	0.00	1.94	0.00
time (sec)	N/A	2.131	0.564	0.296	0.000	0.120	0.000	0.000	0.312	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	323	359	433	0	828	0	0	578	0
N.S.	1	1.05	1.17	1.41	0.00	2.70	0.00	0.00	1.88	0.00
time (sec)	N/A	1.636	0.505	0.273	0.000	0.123	0.000	0.000	0.294	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	251	296	326	0	607	0	0	401	0
N.S.	1	0.99	1.17	1.28	0.00	2.39	0.00	0.00	1.58	0.00
time (sec)	N/A	1.095	0.422	0.269	0.000	0.109	0.000	0.000	0.269	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	165	244	233	0	395	0	0	285	0
N.S.	1	0.94	1.39	1.33	0.00	2.26	0.00	0.00	1.63	0.00
time (sec)	N/A	1.208	0.371	0.254	0.000	0.108	0.000	0.000	0.237	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	107	168	180	0	239	0	0	90	0
N.S.	1	0.96	1.51	1.62	0.00	2.15	0.00	0.00	0.81	0.00
time (sec)	N/A	0.783	0.163	0.172	0.000	0.120	0.000	0.000	0.205	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	172	217	382	0	0	0	0	89	0
N.S.	1	1.08	1.36	2.40	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	1.186	0.539	0.279	0.000	0.000	0.000	0.000	0.227	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	152	327	0	0	0	0	0	240	0
N.S.	1	0.82	1.76	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	1.215	0.940	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	156	266	311	0	0	0	0	482	0
N.S.	1	0.95	1.62	1.90	0.00	0.00	0.00	0.00	2.94	0.00
time (sec)	N/A	0.982	0.901	0.290	0.000	0.000	0.000	0.000	0.225	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	248	489	0	0	0	0	0	787	0
N.S.	1	0.84	1.65	0.00	0.00	0.00	0.00	0.00	2.65	0.00
time (sec)	N/A	2.035	1.717	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	409	562	658	0	1236	0	0	677	0
N.S.	1	1.08	1.49	1.75	0.00	3.28	0.00	0.00	1.80	0.00
time (sec)	N/A	2.878	0.734	0.418	0.000	0.126	0.000	0.000	0.344	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	313	475	517	0	890	0	0	473	0
N.S.	1	1.01	1.54	1.67	0.00	2.88	0.00	0.00	1.53	0.00
time (sec)	N/A	3.524	0.573	0.388	0.000	0.125	0.000	0.000	0.277	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	196	360	328	0	579	0	0	327	0
N.S.	1	0.94	1.72	1.57	0.00	2.77	0.00	0.00	1.56	0.00
time (sec)	N/A	1.366	0.484	0.355	0.000	0.113	0.000	0.000	0.234	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	125	261	275	0	344	0	0	109	0
N.S.	1	0.97	2.02	2.13	0.00	2.67	0.00	0.00	0.84	0.00
time (sec)	N/A	0.680	0.235	0.270	0.000	0.106	0.000	0.000	0.196	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	212	308	582	0	0	0	0	116	0
N.S.	1	1.10	1.60	3.03	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	1.020	0.750	0.420	0.000	0.000	0.000	0.000	0.225	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	216	872	0	0	0	0	0	320	0
N.S.	1	0.82	3.30	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	1.214	1.771	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	194	398	492	0	0	0	0	641	0
N.S.	1	0.99	2.04	2.52	0.00	0.00	0.00	0.00	3.29	0.00
time (sec)	N/A	1.197	1.620	0.422	0.000	0.000	0.000	0.000	0.231	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	432	358	1198	0	0	0	0	0	1047	0
N.S.	1	0.83	2.77	0.00	0.00	0.00	0.00	0.00	2.42	0.00
time (sec)	N/A	2.664	5.533	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	169	151	194	0	0	0	0	117	0
N.S.	1	0.79	0.71	0.91	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.616	0.215	0.290	0.000	0.000	0.000	0.000	0.267	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	119	109	134	0	0	0	0	91	0
N.S.	1	0.82	0.75	0.92	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.579	0.159	0.197	0.000	0.000	0.000	0.000	0.246	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	115	102	130	0	0	0	0	65	0
N.S.	1	0.82	0.72	0.92	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.537	0.134	0.125	0.000	0.000	0.000	0.000	0.235	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	61	66	0	0	0	0	37	0
N.S.	1	0.99	0.88	0.96	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.578	0.060	0.067	0.000	0.000	0.000	0.000	0.206	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	61	50	60	0	0	0	0	14	0
N.S.	1	1.05	0.86	1.03	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.647	0.051	0.027	0.000	0.000	0.000	0.000	0.199	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	31	34	25	35	25
N.S.	1	1.00	1.09	1.00	1.09	1.35	1.48	1.09	1.52	1.09
time (sec)	N/A	0.484	0.680	0.068	0.138	0.091	1.284	0.182	0.225	2.926

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	216	293	665	0	0	0	0	197	0
N.S.	1	0.82	1.11	2.53	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.882	1.923	0.250	0.000	0.000	0.000	0.000	0.272	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	166	230	418	0	0	0	0	155	0
N.S.	1	0.85	1.18	2.14	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.757	2.077	0.198	0.000	0.000	0.000	0.000	0.252	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	162	150	374	0	0	0	0	113	0
N.S.	1	0.85	0.79	1.96	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.493	1.498	0.125	0.000	0.000	0.000	0.000	0.236	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	103	108	170	0	0	0	0	69	0
N.S.	1	0.94	0.98	1.55	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.553	0.459	0.066	0.000	0.000	0.000	0.000	0.211	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	92	89	139	0	0	0	0	30	0
N.S.	1	0.94	0.91	1.42	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.735	0.334	0.039	0.000	0.000	0.000	0.000	0.180	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	1077	61	73	25	70	25
N.S.	1	1.00	1.09	1.00	46.83	2.65	3.17	1.09	3.04	1.09
time (sec)	N/A	0.284	3.042	0.058	2.599	0.098	3.309	0.204	0.193	2.834

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	390	323	993	0	0	0	0	277	0
N.S.	1	1.19	0.99	3.04	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	1.543	0.943	0.260	0.000	0.000	0.000	0.000	0.303	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	292	186	624	0	0	0	0	219	0
N.S.	1	1.15	0.73	2.46	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	2.707	0.449	0.201	0.000	0.000	0.000	0.000	0.253	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	282	223	557	0	0	0	0	161	0
N.S.	1	1.12	0.88	2.21	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	1.939	0.505	0.141	0.000	0.000	0.000	0.000	0.242	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	160	127	254	0	0	0	0	101	0
N.S.	1	0.98	0.78	1.56	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.505	0.252	0.083	0.000	0.000	0.000	0.000	0.221	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	109	207	0	0	0	0	46	0
N.S.	1	1.00	0.83	1.57	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.839	0.306	0.043	0.000	0.000	0.000	0.000	0.183	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	6669	91	112	25	105	25
N.S.	1	1.00	1.09	1.00	289.96	3.96	4.87	1.09	4.57	1.09
time (sec)	N/A	0.486	1.309	0.059	122.568	0.134	10.323	0.223	0.198	2.960

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	500	424	1375	0	0	0	0	357	0
N.S.	1	1.16	0.98	3.19	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	2.342	1.934	0.269	0.000	0.000	0.000	0.000	0.288	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	386	330	860	0	0	0	0	283	0
N.S.	1	1.07	0.92	2.39	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	1.779	1.242	0.224	0.000	0.000	0.000	0.000	0.255	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	374	272	777	0	0	0	0	209	0
N.S.	1	1.06	0.77	2.21	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	2.280	1.114	0.156	0.000	0.000	0.000	0.000	0.230	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	205	195	353	0	0	0	0	133	0
N.S.	1	0.94	0.89	1.62	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	2.250	0.885	0.086	0.000	0.000	0.000	0.000	0.210	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	171	144	295	0	0	0	0	62	0
N.S.	1	0.98	0.83	1.70	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.200	0.525	0.050	0.000	0.000	0.000	0.000	0.180	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	0	121	151	25	140	25
N.S.	1	1.00	1.09	1.00	0.00	5.26	6.57	1.09	6.09	1.09
time (sec)	N/A	0.293	5.387	0.060	0.000	0.161	33.789	0.228	0.203	2.806

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	361	331	342	0	0	0	0	0	112	0
N.S.	1	0.92	0.95	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.247	0.550	0.000	0.000	0.000	0.000	0.000	0.989	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	248	223	0	0	0	0	0	87	0
N.S.	1	0.91	0.82	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.084	0.406	0.000	0.000	0.000	0.000	0.000	0.756	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	227	237	0	0	0	0	0	62	0
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.348	0.361	0.000	0.000	0.000	0.000	0.000	0.570	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	156	146	0	0	0	0	0	35	0
N.S.	1	0.95	0.89	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.433	0.342	0.000	0.000	0.000	0.000	0.000	0.401	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	0	0	0	0	0	13	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.967	0.184	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	20	25	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.80	1.00	1.00	1.00
time (sec)	N/A	0.313	1.413	0.065	0.603	0.000	0.561	11.230	0.286	2.831

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	465	558	0	0	0	0	0	202	0
N.S.	1	1.24	1.49	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	4.388	1.992	0.000	0.000	0.000	0.000	0.000	1.523	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	414	592	0	0	0	0	0	144	0
N.S.	1	1.21	1.73	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	2.961	1.991	0.000	0.000	0.000	0.000	0.000	1.053	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	216	180	0	0	0	0	0	84	0
N.S.	1	1.02	0.85	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	2.205	0.662	0.000	0.000	0.000	0.000	0.000	0.703	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	162	295	0	0	0	0	0	38	0
N.S.	1	1.03	1.88	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.145	0.423	0.000	0.000	0.000	0.000	0.000	0.396	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	51	25	57	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	2.04	1.00	2.28	1.00
time (sec)	N/A	0.508	0.209	0.073	0.784	0.000	8.420	17.019	0.420	2.753

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	555	968	0	0	0	0	0	343	0
N.S.	1	1.18	2.06	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	5.558	8.388	0.000	0.000	0.000	0.000	0.000	2.533	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	408	465	1008	0	0	0	0	0	246	0
N.S.	1	1.14	2.47	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	4.662	8.700	0.000	0.000	0.000	0.000	0.000	1.866	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	257	228	0	0	0	0	0	147	0
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	2.150	1.172	0.000	0.000	0.000	0.000	0.000	1.072	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	182	494	0	0	0	0	0	68	0
N.S.	1	0.98	2.66	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	1.173	2.289	0.000	0.000	0.000	0.000	0.000	0.567	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	0	25	94	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	3.76	1.00
time (sec)	N/A	0.464	0.244	0.073	1.160	0.000	0.000	28.514	0.572	2.921

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	509	699	1523	0	0	0	0	0	348	0
N.S.	1	1.37	2.99	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	8.158	10.208	0.000	0.000	0.000	0.000	0.000	2.352	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	317	288	0	0	0	0	0	210	0
N.S.	1	0.99	0.90	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	3.941	3.238	0.000	0.000	0.000	0.000	0.000	1.372	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	229	748	0	0	0	0	0	98	0
N.S.	1	1.00	3.25	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.267	4.738	0.000	0.000	0.000	0.000	0.000	0.676	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	0	25	131	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	5.24	1.00
time (sec)	N/A	0.323	0.243	0.069	2.156	0.000	0.000	39.323	1.078	2.924

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	300	319	0	0	0	0	0	173	0
N.S.	1	0.92	0.98	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.267	0.391	0.000	0.000	0.000	0.000	0.000	0.769	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	203	205	0	0	0	0	0	136	0
N.S.	1	0.94	0.94	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.919	0.278	0.000	0.000	0.000	0.000	0.000	0.585	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	200	216	0	0	0	0	0	99	0
N.S.	1	0.93	1.01	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.799	0.249	0.000	0.000	0.000	0.000	0.000	0.453	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	122	109	0	0	0	0	0	60	0
N.S.	1	1.08	0.96	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.607	0.198	0.000	0.000	0.000	0.000	0.000	0.336	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	100	110	0	0	0	0	0	26	0
N.S.	1	1.09	1.20	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.464	0.033	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	36	25	47	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	1.44	1.00	1.88	1.00
time (sec)	N/A	0.320	0.109	0.074	0.659	0.000	0.969	11.692	0.305	2.851

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	345	396	0	0	0	0	0	0	0
N.S.	1	0.92	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.750	1.056	0.000	0.000	0.000	0.000	0.000	3.900	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	252	265	0	0	0	0	0	0	0
N.S.	1	0.94	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.604	0.683	0.000	0.000	0.000	0.000	0.000	3.233	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	245	265	0	0	0	0	0	0	0
N.S.	1	0.94	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	1.030	0.000	0.000	0.000	0.000	0.000	2.706	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	160	314	0	0	0	0	0	0	0
N.S.	1	1.03	2.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.883	4.392	0.000	0.000	0.000	0.000	0.000	2.201	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	135	145	0	0	0	0	0	42	0
N.S.	1	1.05	1.13	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.197	0.207	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	88	25	82	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	3.52	1.00	3.28	1.00
time (sec)	N/A	0.549	0.136	0.081	0.658	0.000	4.145	0.334	0.292	2.840

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	444	614	615	0	0	0	0	0	0	0
N.S.	1	1.38	1.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.234	2.224	0.000	0.000	0.000	0.000	0.000	4.479	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	436	391	0	0	0	0	0	0	0
N.S.	1	1.31	1.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.169	1.612	0.000	0.000	0.000	0.000	0.000	3.702	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	412	391	0	0	0	0	0	0	0
N.S.	1	1.26	1.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.312	1.946	0.000	0.000	0.000	0.000	0.000	3.204	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	223	687	0	0	0	0	0	0	0
N.S.	1	1.03	3.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.841	3.791	0.000	0.000	0.000	0.000	0.000	2.499	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	175	219	0	0	0	0	0	58	0
N.S.	1	1.06	1.33	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.917	0.399	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	155	25	117	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	6.20	1.00	4.68	1.00
time (sec)	N/A	0.326	0.145	0.079	0.650	0.000	67.087	0.379	0.338	3.001

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	714	654	0	0	0	0	0	0	0
N.S.	1	1.29	1.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.605	3.063	0.000	0.000	0.000	0.000	0.000	5.198	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	441	542	445	0	0	0	0	0	0	0
N.S.	1	1.23	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.513	1.974	0.000	0.000	0.000	0.000	0.000	4.218	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	431	507	452	0	0	0	0	0	0	0
N.S.	1	1.18	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.908	1.874	0.000	0.000	0.000	0.000	0.000	3.445	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	271	916	0	0	0	0	0	0	0
N.S.	1	1.02	3.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.653	3.242	0.000	0.000	0.000	0.000	0.000	2.726	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	218	243	0	0	0	0	0	74	0
N.S.	1	1.04	1.16	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.326	0.523	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	0	25	152	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	6.08	1.00
time (sec)	N/A	0.330	0.148	0.085	0.694	0.000	0.000	0.433	0.336	3.039

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	174	149	218	0	254	0	0	137	0
N.S.	1	1.03	0.88	1.29	0.00	1.50	0.00	0.00	0.81	0.00
time (sec)	N/A	0.361	0.254	6.447	0.000	0.157	0.000	0.000	0.434	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	150	109	253	0	185	0	0	90	0
N.S.	1	1.03	0.75	1.74	0.00	1.28	0.00	0.00	0.62	0.00
time (sec)	N/A	0.357	0.338	3.224	0.000	0.105	0.000	0.000	0.387	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	130	131	194	0	142	0	0	48	0
N.S.	1	1.02	1.03	1.53	0.00	1.12	0.00	0.00	0.38	0.00
time (sec)	N/A	0.338	0.300	2.217	0.000	0.108	0.000	0.000	0.280	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	102	94	138	0	93	0	0	38	0
N.S.	1	0.98	0.90	1.33	0.00	0.89	0.00	0.00	0.37	0.00
time (sec)	N/A	0.312	0.141	2.059	0.000	0.117	0.000	0.000	0.253	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	82	92	119	0	110	0	0	61	0
N.S.	1	0.98	1.10	1.42	0.00	1.31	0.00	0.00	0.73	0.00
time (sec)	N/A	0.299	0.110	2.092	0.000	0.115	0.000	0.000	0.279	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	148	94	268	0	181	0	0	147	0
N.S.	1	0.99	0.63	1.79	0.00	1.21	0.00	0.00	0.98	0.00
time (sec)	N/A	0.490	0.097	3.473	0.000	0.114	0.000	0.000	0.280	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	132	94	201	0	208	0	0	273	0
N.S.	1	1.02	0.72	1.55	0.00	1.60	0.00	0.00	2.10	0.00
time (sec)	N/A	0.552	0.106	4.733	0.000	0.116	0.000	0.000	0.281	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	154	140	0	0	0	0	0	229	0
N.S.	1	1.01	0.92	0.00	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.977	0.355	0.000	0.000	0.000	0.000	0.000	0.630	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	154	140	0	0	0	0	0	148	0
N.S.	1	1.01	0.92	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.696	0.334	0.000	0.000	0.000	0.000	0.000	0.464	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	154	140	0	0	0	0	0	76	0
N.S.	1	1.01	0.92	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.591	0.290	0.000	0.000	0.000	0.000	0.000	0.327	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	150	140	0	0	0	0	0	66	0
N.S.	1	0.99	0.93	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.546	0.203	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	148	140	0	0	0	0	0	107	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.562	0.204	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	150	140	0	0	0	0	0	275	0
N.S.	1	0.98	0.92	0.00	0.00	0.00	0.00	0.00	1.80	0.00
time (sec)	N/A	0.572	0.239	0.000	0.000	0.000	0.000	0.000	0.310	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	100	22	25	204	25
N.S.	1	1.00	1.08	0.92	0.00	4.00	0.88	1.00	8.16	1.00
time (sec)	N/A	0.616	32.548	0.096	0.000	0.100	72.517	1.392	0.638	3.169

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	55	22	25	102	25
N.S.	1	1.00	1.08	0.92	0.00	2.20	0.88	1.00	4.08	1.00
time (sec)	N/A	0.622	94.850	0.079	0.000	0.102	6.739	0.994	0.399	3.095

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	55	22	25	94	25
N.S.	1	1.00	1.08	0.92	0.00	2.20	0.88	1.00	3.76	1.00
time (sec)	N/A	0.828	44.071	0.094	0.000	0.116	4.231	0.475	0.314	3.130

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	83	22	25	153	25
N.S.	1	1.00	1.08	0.92	0.00	3.32	0.88	1.00	6.12	1.00
time (sec)	N/A	0.986	17.681	0.087	0.000	0.103	8.153	0.614	0.335	3.039

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	97	22	0	405	25
N.S.	1	1.00	1.08	0.92	0.00	3.88	0.88	0.00	16.20	1.00
time (sec)	N/A	0.832	15.475	0.095	0.000	0.113	28.755	0.000	0.375	3.047

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	935	71	20	25	265	25
N.S.	1	1.00	1.09	1.00	40.65	3.09	0.87	1.09	11.52	1.09
time (sec)	N/A	0.807	2.473	0.572	6.372	0.117	94.446	1.056	0.286	3.190

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	713	55	20	25	204	25
N.S.	1	1.00	1.09	1.00	31.00	2.39	0.87	1.09	8.87	1.09
time (sec)	N/A	0.649	1.091	0.485	4.680	0.113	36.490	0.714	0.277	3.152

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	198	178	0	0	0	0	0	143	0
N.S.	1	0.96	0.86	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.600	0.286	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	122	106	0	0	0	0	0	82	0
N.S.	1	1.09	0.95	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.380	0.132	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	19	25	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.83	1.09	1.09	1.09
time (sec)	N/A	0.287	0.440	0.338	0.112	0.083	1.302	0.183	0.255	3.091

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	1180	41	20	25	41	25
N.S.	1	1.00	1.09	1.00	51.30	1.78	0.87	1.09	1.78	1.09
time (sec)	N/A	0.280	0.418	0.349	3.205	0.106	14.667	0.201	0.212	3.086

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	67	50	0	0	0	0	0	12	0
N.S.	1	1.24	0.93	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.364	0.039	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	125	79	75	56	40	0	60	47	0
N.S.	1	1.07	0.68	0.64	0.48	0.34	0.00	0.51	0.40	0.00
time (sec)	N/A	0.306	0.040	0.079	0.025	0.093	0.000	0.401	0.180	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	89	74	65	46	35	0	55	36	0
N.S.	1	1.03	0.86	0.76	0.53	0.41	0.00	0.64	0.42	0.00
time (sec)	N/A	0.257	0.032	0.024	0.024	0.089	0.000	0.404	0.192	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	51	273	49	33	28	0	47	24	40
N.S.	1	1.02	5.46	0.98	0.66	0.56	0.00	0.94	0.48	0.80
time (sec)	N/A	0.216	4.603	0.015	0.036	0.090	0.000	0.412	0.197	4.200

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	65	46	65	0	0	0	0	9	0
N.S.	1	1.41	1.00	1.41	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.344	0.033	0.108	0.000	0.000	0.000	0.000	0.205	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	29	19	26	0	45	21	0
N.S.	1	1.00	1.00	0.72	0.48	0.65	0.00	1.12	0.52	0.00
time (sec)	N/A	0.260	0.016	0.033	0.113	0.082	0.000	0.383	0.198	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	49	35	30	32	0	62	33	0
N.S.	1	1.07	0.64	0.46	0.39	0.42	0.00	0.82	0.43	0.00
time (sec)	N/A	0.305	0.019	0.030	0.108	0.100	0.000	0.424	0.189	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	59	38	17	72	0	22	6	23
N.S.	1	1.00	2.46	1.58	0.71	3.00	0.00	0.92	0.25	0.96
time (sec)	N/A	0.287	0.128	0.126	0.103	0.095	0.000	0.128	0.208	0.380

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	68	179	86	0	0	0	0	12	0
N.S.	1	1.13	2.98	1.43	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.583	0.378	0.118	0.000	0.000	0.000	0.000	0.186	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	264	1044	0	298	0	0	76	0
N.S.	1	1.00	1.82	7.20	0.00	2.06	0.00	0.00	0.52	0.00
time (sec)	N/A	0.503	0.140	0.211	0.000	0.093	0.000	0.000	0.197	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	171	473	0	210	0	0	56	0
N.S.	1	1.02	1.42	3.94	0.00	1.75	0.00	0.00	0.47	0.00
time (sec)	N/A	0.468	0.077	0.129	0.000	0.114	0.000	0.000	0.180	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	104	184	128	131	0	0	36	0
N.S.	1	1.00	1.44	2.56	1.78	1.82	0.00	0.00	0.50	0.00
time (sec)	N/A	0.218	0.042	0.095	0.094	0.089	0.000	0.000	0.209	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	37	36	44	63	37	62	16	32
N.S.	1	1.11	0.84	0.82	1.00	1.43	0.84	1.41	0.36	0.73
time (sec)	N/A	0.185	0.040	0.035	0.035	0.101	0.380	0.127	0.186	3.291

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	118	0	0	0	0	0	16	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.243	0.124	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	130	0	0	0	0	0	34	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.264	0.776	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	183	152	0	0	0	0	0	52	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.356	0.306	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	264	1044	0	298	0	0	76	0
N.S.	1	1.00	1.80	7.10	0.00	2.03	0.00	0.00	0.52	0.00
time (sec)	N/A	0.336	0.150	0.214	0.000	0.116	0.000	0.000	0.199	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	107	171	473	0	210	0	0	56	0
N.S.	1	0.97	1.55	4.30	0.00	1.91	0.00	0.00	0.51	0.00
time (sec)	N/A	0.281	0.079	0.135	0.000	0.104	0.000	0.000	0.192	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	104	184	128	131	0	0	36	0
N.S.	1	1.00	1.42	2.52	1.75	1.79	0.00	0.00	0.49	0.00
time (sec)	N/A	0.217	0.042	0.089	0.090	0.108	0.000	0.000	0.192	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	36	44	63	41	67	16	32
N.S.	1	1.00	1.00	1.09	1.33	1.91	1.24	2.03	0.48	0.97
time (sec)	N/A	0.169	0.016	0.039	0.033	0.097	0.388	0.130	0.182	4.175

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	0	0	0	0	0	16	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.218	0.108	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	141	0	0	0	0	0	34	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.250	0.602	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	184	168	0	0	0	0	0	52	0
N.S.	1	1.02	0.93	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.345	0.426	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	282	311	0	0	0	0	0	78	0
N.S.	1	1.01	1.11	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.633	1.887	0.000	0.000	0.000	0.000	0.000	0.350	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	242	254	0	0	0	0	0	44	0
N.S.	1	1.02	1.07	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.675	0.406	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	210	0	0	0	0	0	15	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.451	0.134	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	166	0	0	0	0	0	30	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.285	0.280	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	242	0	0	0	0	0	997	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	4.68	0.00
time (sec)	N/A	0.294	0.689	0.000	0.000	0.000	0.000	0.000	0.362	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	254	273	0	0	0	0	0	0	0
N.S.	1	1.01	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.635	0.000	0.000	0.000	0.000	0.000	0.393	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	301	302	291	0	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.795	0.000	0.000	0.000	0.000	0.000	0.419	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	284	277	0	0	0	0	0	78	0
N.S.	1	1.01	0.99	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.400	1.816	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	244	221	0	0	0	0	0	44	0
N.S.	1	1.02	0.92	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.367	0.237	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	178	0	0	0	0	0	15	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.280	0.156	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0	30	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.255	0.264	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	209	0	0	0	0	0	997	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	4.70	0.00
time (sec)	N/A	0.334	0.692	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	256	238	0	0	0	0	0	0	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.655	0.594	0.000	0.000	0.000	0.000	0.000	0.310	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	260	0	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.668	0.691	0.000	0.000	0.000	0.000	0.000	0.352	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	0	0	39	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.00	0.00	0.98	0.98
time (sec)	N/A	0.423	0.288	0.145	23.473	0.125	0.000	0.000	0.253	2.971

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	247	0	844	0	0	0	0	150	0
N.S.	1	0.93	0.00	3.18	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	1.075	0.000	0.420	0.000	0.000	0.000	0.000	0.259	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	191	0	483	0	0	0	0	108	0
N.S.	1	0.97	0.00	2.45	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.767	0.000	0.164	0.000	0.000	0.000	0.000	0.251	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	135	0	207	0	0	0	0	64	0
N.S.	1	1.02	0.00	1.56	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.645	0.000	0.152	0.000	0.000	0.000	0.000	0.216	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	0	63	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.00	1.58	0.98
time (sec)	N/A	0.254	0.466	0.088	1.710	0.161	73.551	0.000	0.247	2.996

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	1177	91	0	0	123	39
N.S.	1	1.00	1.05	0.90	29.42	2.28	0.00	0.00	3.08	0.98
time (sec)	N/A	0.250	3.287	0.082	2.705	0.186	0.000	0.000	0.250	3.614

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	0	110	0	0	0	0	12	0
N.S.	1	1.08	0.00	1.45	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.402	0.000	0.105	0.000	0.000	0.000	0.000	0.171	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	53	92	0	0	0	0	21	0
N.S.	1	1.07	0.88	1.53	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.449	0.058	0.144	0.000	0.000	0.000	0.000	0.180	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	17	0	0	0	0	20	0
N.S.	1	1.00	1.00	5.67	0.00	0.00	0.00	0.00	6.67	0.00
time (sec)	N/A	0.398	0.056	0.216	0.000	0.000	0.000	0.000	0.179	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	50	50	45	37	66	61	106	14	295
N.S.	1	0.93	0.93	0.83	0.69	1.22	1.13	1.96	0.26	5.46
time (sec)	N/A	0.324	0.051	0.101	0.031	0.099	0.384	0.149	0.267	7.009

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	50	0	39	152	0	124	17	303
N.S.	1	0.91	0.91	0.00	0.71	2.76	0.00	2.25	0.31	5.51
time (sec)	N/A	0.309	0.031	0.000	0.029	0.150	0.000	0.149	0.183	3.288

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	58	116	87	0	276	0	119	12	53
N.S.	1	1.23	2.47	1.85	0.00	5.87	0.00	2.53	0.26	1.13
time (sec)	N/A	0.331	0.233	0.489	0.000	0.310	0.000	2.393	0.195	3.699

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	108	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	1.74	0.00	0.00	0.39	0.00
time (sec)	N/A	0.375	0.122	0.000	0.000	0.112	0.000	0.000	0.186	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	33	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.61	0.00	0.00	0.44	0.00
time (sec)	N/A	0.358	0.071	0.000	0.000	0.083	0.000	0.000	0.178	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [88] had the largest ratio of [1.19999999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	0.96	10	0.800
2	A	7	6	0.93	10	0.600
3	A	8	7	1.06	8	0.875
4	A	4	3	0.95	6	0.500
5	A	9	8	1.00	10	0.800
6	A	6	5	1.06	10	0.500
7	A	8	7	1.09	10	0.700
8	A	11	10	1.14	10	1.000
9	C	10	9	1.09	14	0.643
10	C	9	8	1.09	15	0.533
11	A	9	8	0.93	21	0.381
12	A	8	7	0.92	21	0.333
13	A	7	6	0.93	21	0.286
14	A	6	5	0.96	19	0.263
15	A	1	1	1.00	10	0.100
16	C	11	10	1.09	21	0.476
17	A	6	5	0.91	21	0.238
18	A	5	4	0.92	21	0.190
19	A	7	6	0.93	21	0.286
20	A	7	6	0.93	21	0.286
21	A	9	8	0.93	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	10	9	0.88	23	0.391
23	A	9	8	0.89	23	0.348
24	A	8	7	0.89	23	0.304
25	A	7	6	0.94	21	0.286
26	A	5	4	1.03	12	0.333
27	C	12	11	1.09	23	0.478
28	A	9	8	0.84	23	0.348
29	A	6	5	0.88	23	0.217
30	A	11	10	0.83	23	0.435
31	A	18	17	1.14	23	0.739
32	A	15	14	1.05	23	0.609
33	A	11	10	0.99	23	0.435
34	A	9	8	0.94	21	0.381
35	A	5	4	0.96	12	0.333
36	C	13	12	1.08	23	0.522
37	A	10	9	0.82	23	0.391
38	C	13	12	0.95	23	0.522
39	A	14	13	0.84	23	0.565
40	A	13	12	1.08	23	0.522
41	A	14	13	1.01	23	0.565
42	A	10	9	0.94	21	0.429
43	A	7	6	0.97	12	0.500
44	C	14	13	1.10	23	0.565
45	A	11	10	0.82	23	0.435
46	C	14	13	0.99	23	0.565
47	A	15	14	0.83	23	0.609
48	A	7	6	0.79	23	0.261
49	A	7	6	0.82	23	0.261
50	A	7	6	0.82	23	0.261
51	C	15	14	0.99	21	0.667
52	C	12	11	1.05	12	0.917
53	N/A	4	0	1.00	23	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	0.82	23	0.174
55	A	5	4	0.85	23	0.174
56	A	5	4	0.85	23	0.174
57	A	12	11	0.94	21	0.524
58	A	11	10	0.94	12	0.833
59	N/A	4	0	1.00	23	0.000
60	A	9	8	1.19	23	0.348
61	C	18	17	1.15	23	0.739
62	C	19	18	1.12	23	0.783
63	C	18	17	0.98	21	0.810
64	C	14	13	1.00	12	1.083
65	N/A	4	0	1.00	23	0.000
66	A	7	6	1.16	23	0.261
67	A	15	14	1.07	23	0.609
68	A	16	15	1.06	23	0.652
69	A	15	14	0.94	21	0.667
70	A	13	12	0.98	12	1.000
71	N/A	4	0	1.00	23	0.000
72	A	8	7	0.92	25	0.280
73	A	8	7	0.91	25	0.280
74	A	8	7	0.93	25	0.280
75	A	8	7	0.95	23	0.304
76	A	10	9	1.00	14	0.643
77	N/A	4	0	1.00	25	0.000
78	C	21	20	1.24	25	0.800
79	C	18	17	1.21	25	0.680
80	C	16	15	1.02	23	0.652
81	C	12	11	1.03	14	0.786
82	N/A	4	0	1.00	25	0.000
83	A	13	12	1.18	25	0.480
84	A	17	16	1.14	25	0.640
85	A	11	10	0.96	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	12	11	0.98	14	0.786
87	N/A	4	0	1.00	25	0.000
88	C	31	30	1.37	25	1.200
89	C	19	18	0.99	23	0.783
90	C	14	13	1.00	14	0.929
91	N/A	4	0	1.00	25	0.000
92	A	7	6	0.92	25	0.240
93	A	7	6	0.94	25	0.240
94	A	7	6	0.93	25	0.240
95	C	13	12	1.08	23	0.522
96	C	10	9	1.09	14	0.643
97	N/A	4	0	1.00	25	0.000
98	A	5	4	0.92	25	0.160
99	A	5	4	0.94	25	0.160
100	A	5	4	0.94	25	0.160
101	A	11	10	1.03	23	0.435
102	A	10	9	1.05	14	0.643
103	N/A	4	0	1.00	25	0.000
104	A	9	8	1.38	25	0.320
105	C	16	15	1.31	25	0.600
106	C	17	16	1.26	25	0.640
107	C	16	15	1.03	23	0.652
108	C	12	11	1.06	14	0.786
109	N/A	4	0	1.00	25	0.000
110	A	7	6	1.29	25	0.240
111	A	14	13	1.23	25	0.520
112	A	15	14	1.18	25	0.560
113	A	14	13	1.02	23	0.565
114	A	12	11	1.04	14	0.786
115	N/A	4	0	1.00	25	0.000
116	A	9	8	1.03	23	0.348
117	A	8	7	1.03	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.02	23	0.261
119	A	6	5	0.98	23	0.217
120	A	5	4	0.98	23	0.174
121	A	9	8	0.99	23	0.348
122	A	8	7	1.02	23	0.304
123	A	4	3	1.01	25	0.120
124	A	4	3	1.01	25	0.120
125	A	4	3	1.01	25	0.120
126	A	4	3	0.99	25	0.120
127	A	4	3	0.99	25	0.120
128	A	4	3	0.98	25	0.120
129	N/A	4	0	1.00	25	0.000
130	N/A	4	0	1.00	25	0.000
131	N/A	4	0	1.00	25	0.000
132	N/A	4	0	1.00	25	0.000
133	N/A	4	0	1.00	25	0.000
134	N/A	4	0	1.00	23	0.000
135	N/A	4	0	1.00	23	0.000
136	A	4	3	0.96	23	0.130
137	A	6	5	1.09	21	0.238
138	N/A	3	0	1.00	23	0.000
139	N/A	3	0	1.00	23	0.000
140	C	8	7	1.24	10	0.700
141	A	8	7	1.07	10	0.700
142	A	7	6	1.03	8	0.750
143	A	6	5	1.02	6	0.833
144	C	8	7	1.41	10	0.700
145	A	3	3	1.00	10	0.300
146	A	4	4	1.07	10	0.400
147	A	3	3	1.00	4	0.750
148	C	8	7	1.13	10	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	A	3	3	1.00	14	0.214
150	A	2	2	1.02	14	0.143
151	A	2	2	1.00	14	0.143
152	A	1	1	1.11	12	0.083
153	A	1	1	1.00	14	0.071
154	A	1	1	1.00	14	0.071
155	A	2	2	1.02	14	0.143
156	A	3	3	1.00	14	0.214
157	A	2	2	0.97	14	0.143
158	A	2	2	1.00	14	0.143
159	A	1	1	1.00	12	0.083
160	A	1	1	1.00	14	0.071
161	A	1	1	1.00	14	0.071
162	A	2	2	1.02	14	0.143
163	A	2	2	1.01	16	0.125
164	A	2	2	1.02	16	0.125
165	A	1	1	1.00	16	0.062
166	A	1	1	1.00	16	0.062
167	A	1	1	1.00	16	0.062
168	A	2	2	1.01	16	0.125
169	A	2	2	1.00	16	0.125
170	A	2	2	1.01	16	0.125
171	A	2	2	1.02	16	0.125
172	A	1	1	1.00	16	0.062
173	A	1	1	1.00	16	0.062
174	A	1	1	1.00	16	0.062
175	A	2	2	1.01	16	0.125
176	A	2	2	1.00	16	0.125
177	N/A	1	0	1.00	40	0.000
178	C	12	11	0.93	40	0.275
179	C	11	10	0.97	40	0.250
180	C	10	9	1.02	38	0.237

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
181	N/A	1	0	1.00	40	0.000
182	N/A	1	0	1.00	40	0.000
183	C	9	8	1.08	10	0.800
184	C	10	9	1.07	19	0.474
185	A	4	3	1.00	20	0.150
186	A	5	4	0.93	12	0.333
187	A	5	4	0.91	14	0.286
188	A	6	5	1.23	10	0.500
189	A	3	2	1.00	26	0.077
190	A	3	2	1.00	26	0.077

CHAPTER 3

Listing of integrals

3.1 $\int x^3 \operatorname{arccosh}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 176

$$\begin{aligned}\int x^3\operatorname{arccosh}(a+bx)\,dx = &-\frac{(9-64a+26a^2-76a^3)\sqrt{-1+a+bx}\sqrt{1+a+bx}}{96b^4}\\&+\frac{7ax^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{48b^2}\\&-\frac{x^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}{16b}\\&-\frac{(9+26a^2)(-1+a+bx)^{3/2}\sqrt{1+a+bx}}{96b^4}\\&-\frac{(3+24a^2+8a^4)\operatorname{arccosh}(a+bx)}{32b^4}+\frac{1}{4}x^4\operatorname{arccosh}(a+bx)\end{aligned}$$

output
```
-1/96*(-76*a^3+26*a^2-64*a+9)*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^4+7/48*a*x
^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/16*x^3*(b*x+a-1)^(1/2)*(b*x+a+1)^
(1/2)/b-1/96*(26*a^2+9)*(b*x+a-1)^(3/2)*(b*x+a+1)^(1/2)/b^4-1/32*(8*a^4+24
*a^2+3)*arccosh(b*x+a)/b^4+1/4*x^4*arccosh(b*x+a)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int x^3 \text{arccosh}(a+bx)\,dx$$
$$= \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}(55a+50a^3-9bx-26a^2bx+14ab^2x^2-6b^3x^3)+24b^4x^4\text{arccosh}(a+bx)-}{96b^4}$$

input
```
Integrate[x^3*ArcCosh[a + b*x],x]
```

output
```
(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x
+ 14*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCosh[a + b*x] - 3*(3 + 24*a^2
+ 8*a^4)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(96*b^4)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6411, 25, 27, 6378, 111, 170, 164, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arccosh}(a+bx)\,dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int x^3 \text{arccosh}(a+bx)d(a+bx)}{b}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -x^3 \text{arccosh}(a+bx)d(a+bx)}{b}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int -b^3x^3 \text{arccosh}(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{6378}$$

$$-\frac{\frac{1}{4}\int \frac{b^4x^4}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{1}{4}b^4x^4\text{arccosh}(a+bx)}{b^4}$$

$$\Big\downarrow \text{111}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\int \frac{b^2x^2(4a^2-7(a+bx)a+3)}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)+\frac{1}{4}b^3x^3\sqrt{a+bx-1}\sqrt{a+bx+1}\right)-\frac{1}{4}b^4x^4\text{arccosh}(a+bx)}{b^4}$$

$$\Big\downarrow \text{170}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\int -\frac{bx(a(12a^2+23)-(26a^2+9)(a+bx))}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{7}{3}ab^2x^2\sqrt{a+bx-1}\sqrt{a+bx+1}\right)+\frac{1}{4}b^3x^3\sqrt{a+bx-1}\sqrt{a}\right.}{b^4}$$

$$\Big\downarrow \text{164}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}(8a^4+24a^2+3)\int \frac{1}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{1}{2}\sqrt{a+bx-1}\sqrt{a+bx+1}(4a(19a^2+16)-(26a^2\right.\right.\right.}{b}$$

$$\Big\downarrow \text{43}$$

$$-\frac{\frac{1}{4}(\frac{1}{4}(\frac{1}{3}(\frac{3}{2}(8a^4+24a^2+3)\,\text{arccosh}(a+bx)-\frac{1}{2}\sqrt{a+bx-1}\sqrt{a+bx+1}(4a(19a^2+16)-(26a^2+9)\,(a+bx)}{b^4}$$

input
```
Int[x^3*ArcCosh[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcCosh[a + b*x]) + ((b^3*x^3*Sqrt[-1 + a + b*x]*Sqrt[1 +
 a + b*x])/4 + ((-7*a*b^2*x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/3 + (-
1/2*(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4*a*(16 + 19*a^2) - (9 + 26*a^2
)*(a + b*x))) + (3*(3 + 24*a^2 + 8*a^4)*ArcCosh[a + b*x])/2)/3)/4)/4)/b^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 6378
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(
n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10

method	result
orering	$-\frac{\left(-14b^5x^5+2a\,b^4x^4-4a^2b^3x^3+12a^3b^2x^2+82a^4bx-3b^3x^3+50a^5+23a\,b^2x^2+151a^2bx+5a^3+12bx-55a\right)\operatorname{arccosh}(bx+}{32x\,b^5}$
derivativedivides	$\frac{\frac{\operatorname{arccosh}(bx+a)a^4}{4}-\operatorname{arccosh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccosh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccosh}(bx+a)a(bx+a)^3+\frac{\operatorname{arccosh}(bx+a)(bx+a)}{4}}{}$
default	$\frac{\frac{\operatorname{arccosh}(bx+a)a^4}{4}-\operatorname{arccosh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccosh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccosh}(bx+a)a(bx+a)^3+\frac{\operatorname{arccosh}(bx+a)(bx+a)}{4}}{}$
parts	$\frac{x^4\operatorname{arccosh}(bx+a)}{4}+\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-6\operatorname{csgn}(b)b^3x^3\sqrt{b^2x^2+2abx+a^2-1}+14\operatorname{csgn}(b)a\,b^2x^2\sqrt{b^2x^2+2abx+a^2}\right.}{}$

input
```
int(x^3*arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-14*b^5*x^5+2*a*b^4*x^4-4*a^2*b^3*x^3+12*a^3*b^2*x^2+82*a^4*b*x-3*b
^3*x^3+50*a^5+23*a*b^2*x^2+151*a^2*b*x+5*a^3+12*b*x-55*a)/x/b^5*arccosh(b*
x+a)+1/96*(-6*b^3*x^3+14*a*b^2*x^2-26*a^2*b*x+50*a^3-9*b*x+55*a)/b^5*(b*x+
a-1)*(b*x+a+1)/x^3*(3*x^2*arccosh(b*x+a)+x^3*b/(b*x+a-1)^(1/2)/(b*x+a+1)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$\int x^3 \text{arccosh}(a+bx)\,dx = \frac{3\,(8\,b^4x^4 - 8\,a^4 - 24\,a^2 - 3)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 - 1}\right) - (6\,b^3x^3 - 14\,ab^2x^2 - 50\,a^3 + (26\,a}{96\,b^4}$$

input
```
integrate(x^3*arccosh(b*x+a),x, algorithm="fricas")
```

output
```
1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 - 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x -
55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^4
```

Sympy [F]

$$\int x^3 \text{arccosh}(a+bx)\,dx = \int x^3 \operatorname{acosh}(a+bx)\,dx$$

input
```
integrate(x**3*acosh(b*x+a),x)
```

output
```
Integral(x**3*acosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(148) = 296$.

Time = 0.04 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.82

$$\int x^3 \operatorname{arccosh}(a+bx)\,dx = \frac{1}{4}x^4 \operatorname{arcosh}(bx+a) - \frac{1}{96}\left(\frac{6\sqrt{b^2x^2+2abx+a^2-1}x^3}{b^2} - \frac{14\sqrt{b^2x^2+2abx+a^2-1}ax^2}{b^3} + \frac{105a^4\log\left(2b^2x+2ab+2\sqrt{b^2x}\right.}{b^5}\right.$$

input
```
integrate(x^3*arccosh(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arccosh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x^3/b
^2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x^2/b^3 + 105*a^4*log(2*b^2*x
+ 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 35*sqrt(b^2*x^2 + 2
*a*b*x + a^2 - 1)*a^2*x/b^4 - 90*(a^2 - 1)*a^2*log(2*b^2*x + 2*a*b + 2*sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*a^3/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*x/b^4 + 9*(a^2
 - 1)^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 +
 55*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*a/b^5)*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int x^3 \operatorname{arccosh}(a+bx)\,dx = \frac{1}{4}x^4\log\left(bx+a+\sqrt{(bx+a)^2-1}\right) - \frac{1}{96}\left(\sqrt{b^2x^2+2abx+a^2-1}\left(\left(2x\left(\frac{3x}{b^2}-\frac{7a}{b^3}\right)+\frac{26a^2b^3+9b^3}{b^7}\right)x - \frac{5(10a^3b^2+11ab^2)}{b^7}\right) - \frac{3(8a^{\cdot}}{}\right.$$

input
```
integrate(x^3*arccosh(b*x+a),x, algorithm="giac")
```

output
```
1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*
x + a^2 - 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 + 9*b^3)/b^7)*x - 5*(
10*a^3*b^2 + 11*a*b^2)/b^7) - 3*(8*a^4 + 24*a^2 + 3)*log(abs(-a*b - (x*abs
(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^4*abs(b)))*b
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arccosh}(a+bx)\,dx = \int x^3\,\mathrm{acosh}(a+b\,x)\,dx$$

input
```
int(x^3*acosh(a + b*x),x)
```

output
```
int(x^3*acosh(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.41

$$\int x^3 \operatorname{arccosh}(a+bx)\,dx = \frac{24acosh(bx+a)\,b^4x^4 + 50\sqrt{b^2x^2+2abx+a^2-1}\,a^3 - 26\sqrt{b^2x^2+2abx+a^2-1}\,a^2bx + 14\sqrt{b^2x^2+2ab}}{}$$

input
```
int(x^3*acosh(b*x+a),x)
```

output
```
(24*acosh(a + b*x)*b**4*x**4 + 50*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a**
3 - 26*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a**2*b*x + 14*sqrt(a**2 + 2*a*
b*x + b**2*x**2 - 1)*a*b**2*x**2 + 55*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)
*a - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*b**3*x**3 - 9*sqrt(a**2 + 2*a*
b*x + b**2*x**2 - 1)*b*x - 24*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + a
 + b*x)*a**4 - 72*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + a + b*x)*a**2
 - 9*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + a + b*x))/(96*b**4)
```

3.2 $\int x^2 \textbf{arccosh}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int x^2\operatorname{arccosh}(a+bx)\,dx = -\frac{(4-5a+16a^2)\sqrt{-1+a+bx}\sqrt{1+a+bx}}{18b^3} - \frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b} + \frac{5a(-1+a+bx)^{3/2}\sqrt{1+a+bx}}{18b^3} + \frac{a(3+2a^2)\operatorname{arccosh}(a+bx)}{6b^3} + \frac{1}{3}x^3\operatorname{arccosh}(a+bx)$$

output
```
-1/18*(16*a^2-5*a+4)*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^3-1/9*x^2*(b*x+a-1)
^(1/2)*(b*x+a+1)^(1/2)/b+5/18*a*(b*x+a-1)^(3/2)*(b*x+a+1)^(1/2)/b^3+1/6*a*
(2*a^2+3)*arccosh(b*x+a)/b^3+1/3*x^3*arccosh(b*x+a)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\int x^2 \text{arccosh}(a+bx)\,dx$$
$$= \frac{-\sqrt{-1+a+bx}\sqrt{1+a+bx}(4+11a^2-5abx+2b^2x^2)+6b^3x^3\text{arccosh}(a+bx)+(9a+6a^3)\log\left(a+b\right.}{18b^3}$$

input
```
Integrate[x^2*ArcCosh[a + b*x],x]
```

output
```
(-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)
) + 6*b^3*x^3*ArcCosh[a + b*x] + (9*a + 6*a^3)*Log[a + b*x + Sqrt[-1 + a +
 b*x]*Sqrt[1 + a + b*x]])/(18*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6411, 27, 6378, 111, 164, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arccosh}(a+bx)\,dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int x^2 \text{arccosh}(a+bx)d(a+bx)}{b}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int b^2x^2 \text{arccosh}(a+bx)d(a+bx)}{b^3}$$
$$\Big\downarrow \text{ 6378}$$
$$\frac{\frac{1}{3}\int -\frac{b^3x^3}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)+\frac{1}{3}b^3x^3\text{arccosh}(a+bx)}{b^3}$$

$$\Big\downarrow \text{111}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\int -\frac{bx\left(3a^2-5(a+bx)a+2\right)}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{1}{3}b^2x^2\sqrt{a+bx-1}\sqrt{a+bx+1}\right)+\frac{1}{3}b^3x^3\text{arccosh}(a+bx)}{b^3}$$

$$\Big\downarrow \text{164}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}a\left(2a^2+3\right)\int \frac{1}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{1}{2}\sqrt{a+bx-1}\sqrt{a+bx+1}\left(4\left(4a^2+1\right)-5a(a+bx)\right)\right)-\frac{1}{3}b^2x^2 \right.}{b^3}$$

$$\Big\downarrow \text{43}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}a\left(2a^2+3\right)\text{arccosh}(a+bx)-\frac{1}{2}\sqrt{a+bx-1}\sqrt{a+bx+1}\left(4\left(4a^2+1\right)-5a(a+bx)\right)\right)-\frac{1}{3}b^2x^2\sqrt{a+bx-1} \right.}{b^3}$$

input

```
Int[x^2*ArcCosh[a + b*x],x]
```

output

```
((b^3*x^3*ArcCosh[a + b*x])/3 + (-1/3*(b^2*x^2*Sqrt[-1 + a + b*x]*Sqrt[1 +
 a + b*x]) + (-1/2*(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4*(1 + 4*a^2) -
5*a*(a + b*x))) + (3*a*(3 + 2*a^2)*ArcCosh[a + b*x])/2)/3)/3)/b^3
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 6378
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(
n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

method	result
orering	$\frac{(10b^4x^4-2a\,b^3x^3+6a^2b^2x^2+40a^3bx+22a^4+4b^2x^2+35abx-14a^2-8)\operatorname{arccosh}(bx+a)}{18b^4x}-\frac{(2b^2x^2-5abx+11a^2+4)(bx\cdots}{}$
derivativedivides	$\frac{-\frac{\operatorname{arccosh}(bx+a)a^3}{3}+\operatorname{arccosh}(bx+a)a^2(bx+a)-\operatorname{arccosh}(bx+a)a(bx+a)^2+\frac{\operatorname{arccosh}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,(6\cdots}{}}{}$
default	$\frac{-\frac{\operatorname{arccosh}(bx+a)a^3}{3}+\operatorname{arccosh}(bx+a)a^2(bx+a)-\operatorname{arccosh}(bx+a)a(bx+a)^2+\frac{\operatorname{arccosh}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,(6\cdots}{}}{}$
parts	$\frac{x^3\operatorname{arccosh}(bx+a)}{3}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(2\operatorname{csgn}(b)b^2x^2\sqrt{b^2x^2+2abx+a^2-1}-5\operatorname{csgn}(b)\sqrt{b^2x^2+2abx+a^2-1}\,abx+\cdots\right.}{}$

input
```
int(x^2*arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/18*(10*b^4*x^4-2*a*b^3*x^3+6*a^2*b^2*x^2+40*a^3*b*x+22*a^4+4*b^2*x^2+35*
a*b*x-14*a^2-8)/b^4/x*arccosh(b*x+a)-1/18*(2*b^2*x^2-5*a*b*x+11*a^2+4)/b^4
*(b*x+a-1)*(b*x+a+1)/x^2*(2*x*arccosh(b*x+a)+x^2*b/(b*x+a-1)^(1/2)/(b*x+a+
1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.70

$$\int x^2\operatorname{arccosh}(a+bx)\,dx$$
$$=\frac{3\,(2\,b^3x^3+2\,a^3+3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-(2\,b^2x^2-5\,abx+11\,a^2+4)\sqrt{b^2x^2+2\,a\cdots}}{18\,b^3}$$

input
```
integrate(x^2*arccosh(b*x+a),x, algorithm="fricas")
```

output
```
1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a
^2 - 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 + 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2
 - 1))/b^3
```

Sympy [F]

$$\int x^2 \text{arccosh}(a+bx)\,dx = \int x^2 \operatorname{acosh}(a+bx)\,dx$$

input
```
integrate(x**2*acosh(b*x+a),x)
```

output
```
Integral(x**2*acosh(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.63

$$\int x^2 \text{arccosh}(a+bx)\,dx = \frac{1}{3}\,x^3 \operatorname{arcosh}(bx+a) - \frac{1}{18}\,b\left(\frac{2\,\sqrt{b^2x^2+2\,abx+a^2-1}x^2}{b^2} - \frac{15\,a^3\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}b\right)}{b^4} - \frac{5\,\sqrt{b^2x^2 -}}{}\right.$$

input
```
integrate(x^2*arccosh(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arccosh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x^2
/b^2 - 15*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)
/b^4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x/b^3 + 9*(a^2 - 1)*a*log(2*b
^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^4 + 15*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2
- 1)/b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int x^2 \text{arccosh}(a+bx)\,dx = \frac{1}{3}\,x^3\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)$$
$$-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b+4\,b}{b^5}\right)+\frac{3\,(2\,a^3+3\,a)\log\left(\left|-ab-(x|b|-\sqrt{b\right.\right.}{b^3|b|}$$

input
```
integrate(x^2*arccosh(b*x+a),x, algorithm="giac")
```

output
```
1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*
x + a^2 - 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b + 4*b)/b^5) + 3*(2*a^3 + 3
*a)*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))
/(b^3*abs(b)))*b
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arccosh}(a+bx)\,dx = \int x^2\,\mathrm{acosh}(a+b\,x)\;dx$$

input
```
int(x^2*acosh(a + b*x),x)
```

output
```
int(x^2*acosh(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.29

$$\int x^2 \text{arccosh}(a+bx)\,dx$$
$$= \frac{6acosh(bx+a)\,b^3x^3-11\sqrt{b^2x^2+2abx+a^2-1}\,a^2+5\sqrt{b^2x^2+2abx+a^2-1}\,abx-2\sqrt{b^2x^2+2abx+a}}{}$$

input
```
int(x^2*acosh(b*x+a),x)
```

output
```
(6*acosh(a + b*x)*b**3*x**3 - 11*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a**2
 + 5*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a*b*x - 2*sqrt(a**2 + 2*a*b*x +
b**2*x**2 - 1)*b**2*x**2 - 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + 6*log(
sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + a + b*x)*a**3 + 9*log(sqrt(a**2 + 2
*a*b*x + b**2*x**2 - 1) + a + b*x)*a)/(18*b**3)
```

3.3 $\int x\text{arccosh}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 90

$$\int x\text{arccosh}(a+bx)\,dx=\frac{3a\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b^2}-\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}-\frac{(1+2a^2)\,\text{arccosh}(a+bx)}{4b^2}+\frac{1}{2}x^2\text{arccosh}(a+bx)$$

output
```
3/4*a*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/4*x*(b*x+a-1)^(1/2)*(b*x+a+1)^
(1/2)/b-1/4*(2*a^2+1)*arccosh(b*x+a)/b^2+1/2*x^2*arccosh(b*x+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int x\text{arccosh}(a+bx)\,dx=\frac{(3a-bx)\sqrt{-1+a+bx}\sqrt{1+a+bx}+2b^2x^2\text{arccosh}(a+bx)-(1+2a^2)\log\left(a+bx+\sqrt{-1+a+bx}\sqrt{\ldots}\right.}{4b^2}$$

input
```
Integrate[x*ArcCosh[a + b*x],x]
```

output
```
((3*a - b*x)*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*b^2*x^2*ArcCosh[a +
b*x] - (1 + 2*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(4
*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6411, 25, 27, 6378, 101, 90, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arccosh}(a+bx)\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int x\text{arccosh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\text{arccosh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\text{arccosh}(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6378}$$

$$-\frac{\frac{1}{2}\int \frac{b^2x^2}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{1}{2}b^2x^2\text{arccosh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{101}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{2a^2-3(a+bx)a+1}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)+\frac{1}{2}bx\sqrt{a+bx-1}\sqrt{a+bx+1}\right)-\frac{1}{2}b^2x^2\text{arccosh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\left(\left(2a^2+1\right)\int \frac{1}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-3a\sqrt{a+bx-1}\sqrt{a+bx+1}\right)+\frac{1}{2}bx\sqrt{a+bx-1}\sqrt{a+bx+1}\right)-}{b^2}$$

$$\Big\downarrow \; 43$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\left(\left(2a^2+1\right)\operatorname{arccosh}(a+bx)-3a\sqrt{a+bx-1}\sqrt{a+bx+1}\right)+\frac{1}{2}bx\sqrt{a+bx-1}\sqrt{a+bx+1}\right)-\frac{1}{2}b^2x^2\text{arccos}}{b^2}$$

input
```
Int[x*ArcCosh[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcCosh[a + b*x]) + ((b*x*Sqrt[-1 + a + b*x]*Sqrt[1 + a +
 b*x])/2 + (-3*a*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + (1 + 2*a^2)*ArcCos
h[a + b*x])/2)/2)/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6378
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(
n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

method	result
orering	$-\frac{\left(-3b^{3}x^{3}+a\,b^{2}x^{2}+7a^{2}bx+3a^{3}+2bx-3a\right)\operatorname{arccosh}(bx+a)}{4b^{3}x}+\frac{(-bx+3a)(bx+a-1)(bx+a+1)\left(\operatorname{arccosh}(bx+a)+\frac{\cdots}{\sqrt{bx+\cdots}}\right.}{4b^{3}x}$
derivativedivides	$\frac{\frac{\operatorname{arccosh}(bx+a)(bx+a)^{2}}{2}-\operatorname{arccosh}(bx+a)a(bx+a)-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-4a\sqrt{(bx+a)^{2}-1}+(bx+a)\sqrt{(bx+a)^{2}-1}+\ln\left(bx+a+\sqrt{\cdots}\right.\right.}{4\sqrt{(bx+a)^{2}-1}}}{b^{2}}$
default	$\frac{\frac{\operatorname{arccosh}(bx+a)(bx+a)^{2}}{2}-\operatorname{arccosh}(bx+a)a(bx+a)-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-4a\sqrt{(bx+a)^{2}-1}+(bx+a)\sqrt{(bx+a)^{2}-1}+\ln\left(bx+a+\sqrt{\cdots}\right.\right.}{4\sqrt{(bx+a)^{2}-1}}}{b^{2}}$
parts	$\frac{x^{2}\operatorname{arccosh}(bx+a)}{2}+\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-\operatorname{csgn}(b)\sqrt{b^{2}x^{2}+2abx+a^{2}-1}\,bx+3\operatorname{csgn}(b)\sqrt{b^{2}x^{2}+2abx+a^{2}-1}\,a-2\ln\cdots\right.}{4b^{2}\cdots}$

input
```
int(x*arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-3*b^3*x^3+a*b^2*x^2+7*a^2*b*x+3*a^3+2*b*x-3*a)/b^3/x*arccosh(b*x+a)
+1/4*(-b*x+3*a)/b^3*(b*x+a-1)*(b*x+a+1)/x*(arccosh(b*x+a)+x*b/(b*x+a-1)^(1
/2)/(b*x+a+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int x\text{arccosh}(a+bx)\,dx = \frac{(2\,b^2x^2-2\,a^2-1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}(bx-3\,a)}{4\,b^2}$$

input
```
integrate(x*arccosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((2*b^2*x^2 - 2*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 -
1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 3*a))/b^2
```

Sympy [F]

$$\int x\text{arccosh}(a+bx)\,dx = \int x\,\text{acosh}\,(a+bx)\,dx$$

input
```
integrate(x*acosh(b*x+a),x)
```

output
```
Integral(x*acosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.68

$$\int x\text{arccosh}(a+bx)\,dx = \frac{1}{2}x^2\,\text{arcosh}\left(bx+a\right) - \frac{1}{4}b\left(\frac{3a^2\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}b\right)}{b^3} + \frac{\sqrt{b^2x^2+2abx+a^2-1}x}{b^2} - \frac{(a^2-1)\log\left(2\right.}{}\right.$$

input
```
integrate(x*arccosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arccosh(b*x + a) - 1/4*b*(3*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x
^2 + 2*a*b*x + a^2 - 1)*b)/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x/b^2 -
 (a^2 - 1)*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^
3 - 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a/b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x\text{arccosh}(a+bx)\,dx = \frac{1}{2}x^2\log\left(bx+a+\sqrt{(bx+a)^2-1}\right) - \frac{1}{4}\left(\sqrt{b^2x^2+2abx+a^2-1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right) - \frac{(2a^2+1)\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)|b|\right|\right)}{b^2|b|}\right.$$

input
```
integrate(x*arccosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*(x/b^2 - 3*a/b^3) - (2*a^2 + 1)*log(abs(-a*b - (x*abs(b) - sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^2*abs(b)))*b
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arccosh}(a+bx)\,dx=\int x\,\text{acosh}(a+b\,x)\;dx$$

input
```
int(x*acosh(a + b*x),x)
```

output
```
int(x*acosh(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int x\text{arccosh}(a+bx)\,dx$$
$$=\frac{2acosh(bx+a)\,b^2x^2+3\sqrt{b^2x^2+2abx+a^2-1}\,a-\sqrt{b^2x^2+2abx+a^2-1}\,bx-2\log\left(\sqrt{b^2x^2+2abx+a}\right.}{4b^2}$$

input
```
int(x*acosh(b*x+a),x)
```

output
```
(2*acosh(a + b*x)*b**2*x**2 + 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a - s
qrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*b*x - 2*log(sqrt(a**2 + 2*a*b*x + b**2
*x**2 - 1) + a + b*x)*a**2 - log(sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + a
+ b*x))/(4*b**2)
```

3.4 $\int \operatorname{arccosh}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 41

$$\int \operatorname{arccosh}(a+bx)\,dx=-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{b}+\frac{(a+bx)\operatorname{arccosh}(a+bx)}{b}$$

output
```
-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b+(b*x+a)*arccosh(b*x+a)/b
```

Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \operatorname{arccosh}(a+bx)\,dx=x\operatorname{arccosh}(a+bx)-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}-2a\operatorname{arctanh}\left(\sqrt{\frac{-1+a+bx}{1+a+bx}}\right)}{b}$$

input
```
Integrate[ArcCosh[a + b*x],x]
```

output
```
x*ArcCosh[a + b*x] - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] - 2*a*ArcTanh[S
qrt[(-1 + a + b*x)/(1 + a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6410, 6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arccosh}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6410}$$

$$\frac{\int \text{arccosh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6294}$$

$$\frac{(a+bx)\text{arccosh}(a+bx)-\int \frac{a+bx}{\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{(a+bx)\text{arccosh}(a+bx)-\sqrt{a+bx-1}\sqrt{a+bx+1}}{b}$$

input
```
Int[ArcCosh[a + b*x],x]
```

output
```
(-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) + (a + b*x)*ArcCosh[a + b*x])/b
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{(bx+a)\operatorname{arccosh}(bx+a)-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{b}$
default	$\frac{(bx+a)\operatorname{arccosh}(bx+a)-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{b}$
orering	$-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{b}+\frac{(bx+a)\operatorname{arccosh}(bx+a)}{b}$
parts	$x\ \operatorname{arccosh}\left(bx+a\right)-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\left(\operatorname{csgn}(b)\sqrt{b^2x^2+2abx+a^2-1}-\ln\left(\left(\operatorname{csgn}(b)\sqrt{(bx+a-1)(bx+a+1)}\right.\right.\right.}{b\sqrt{b^2x^2+2abx+a^2-1}}$

input
```
int(arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arccosh(b*x+a)-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \operatorname{arccosh}(a+bx)\,dx = \frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}}{b}$$

input
```
integrate(arccosh(b*x+a),x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2
 + 2*a*b*x + a^2 - 1))/b
```

Sympy [F]

$$\int \operatorname{arccosh}(a+bx)\,dx=\int \operatorname{acosh}\left(a+bx\right)dx$$

input
```
integrate(acosh(b*x+a),x)
```

output
```
Integral(acosh(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \operatorname{arccosh}(a+bx)\,dx=\frac{(bx+a)\operatorname{arcosh}\left(bx+a\right)-\sqrt{\left(bx+a\right)^2-1}}{b}$$

input
```
integrate(arccosh(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*arccosh(b*x + a) - sqrt((b*x + a)^2 - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \operatorname{arccosh}(a+bx)\,dx = -b\left(\frac{a\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2-1}}{b^2}\right) + x\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)$$

input
```
integrate(arccosh(b*x+a),x, algorithm="giac")
```

output
```
-b*(a*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)
))/(b*abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)/b^2) + x*log(b*x + a + s
qrt((b*x + a)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 266, normalized size of antiderivative = 6.49

$$\int \operatorname{arccosh}(a+bx)\,dx = x\,\mathrm{acosh}(a+b\,x) - \frac{\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}+\frac{4\,a\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^3}{b\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^3}-\frac{8\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^2\,\sqrt{a-1}\,\sqrt{a+1}}{b\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^2}}{\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^4}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^4}-\frac{2\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^2}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^2}+1} + \frac{4\,a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x-1}}{\sqrt{a+1}-\sqrt{a+b\,x+1}}\right)}{b}$$

input
```
int(acosh(a + b*x),x)
```

output
```
x*acosh(a + b*x) - ((4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)))/(b*((a + 1
)^(1/2) - (a + b*x + 1)^(1/2))) + (4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2
))^3)/(b*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) - (8*((a - 1)^(1/2) - (a
 + b*x - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^2))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4/((a + 1)^(1/
2) - (a + b*x + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/
((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 + 1) + (4*a*atanh(((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \operatorname{arccosh}(a+bx)\,dx = \frac{acosh(bx+a)\,a + acosh(bx+a)\,bx - \sqrt{b^2x^2+2abx+a^2-1}}{b}$$

input
```
int(acosh(b*x+a),x)
```

output
```
(acosh(a + b*x)*a + acosh(a + b*x)*b*x - sqrt(a**2 + 2*a*b*x + b**2*x**2 -
 1))/b
```

3.5 $\int \frac{\operatorname{arccosh}(a+bx)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 131

$$\int \frac{\operatorname{arccosh}(a+bx)}{x} dx = -\frac{1}{2}\operatorname{arccosh}(a+bx)^2 + \operatorname{arccosh}(a+bx)\log\left(1-\frac{e^{\operatorname{arccosh}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \operatorname{arccosh}(a+bx)\log\left(1-\frac{e^{\operatorname{arccosh}(a+bx)}}{a+\sqrt{-1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arccosh}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arccosh}(a+bx)}}{a+\sqrt{-1+a^2}}\right)$$

output
```
-1/2*arccosh(b*x+a)^2+arccosh(b*x+a)*ln(1-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)
^(1/2))/(a-(a^2-1)^(1/2)))+arccosh(b*x+a)*ln(1-(b*x+a+(b*x+a-1)^(1/2)*(b*x
+a+1)^(1/2))/(a+(a^2-1)^(1/2)))+polylog(2,(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)
^(1/2))/(a-(a^2-1)^(1/2)))+polylog(2,(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2
))/(a+(a^2-1)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = -\frac{1}{2}\operatorname{arccosh}(a+bx)^2 + \operatorname{arccosh}(a+bx)\log\left(1+\frac{e^{\operatorname{arccosh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{-1+a^2}}{b}\right)b}\right) + \operatorname{arccosh}(a+bx)\log\left(1+\frac{e^{\operatorname{arccosh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{-1+a^2}}{b}\right)b}\right) + \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arccosh}(a+bx)}}{-a+\sqrt{-1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arccosh}(a+bx)}}{a+\sqrt{-1+a^2}}\right)$$

input
```
Integrate[ArcCosh[a + b*x]/x,x]
```

output
```
-1/2*ArcCosh[a + b*x]^2 + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/((-(
a/b) - Sqrt[-1 + a^2]/b)*b)] + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]
/((-(a/b) + Sqrt[-1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcCosh[a + b*x]/(-a +
Sqrt[-1 + a^2]))] + PolyLog[2, E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6411, 25, 27, 6377, 6096, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx$$

$$\Big\downarrow \ 6411$$

$$\frac{\int \frac{\text{arccosh}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{\text{arccosh}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$-\int -\frac{\text{arccosh}(a+bx)}{bx}d(a+bx)$$

$$\Big\downarrow \ 6377$$

$$-\int -\frac{\sqrt{\frac{a+bx-1}{a+bx+1}}(a+bx+1)\text{arccosh}(a+bx)}{bx}d\text{arccosh}(a+bx)$$

$$\Big\downarrow \ 6096$$

$$-\int \frac{e^{\text{arccosh}(a+bx)}\text{arccosh}(a+bx)}{a-e^{\text{arccosh}(a+bx)}-\sqrt{a^2-1}}d\text{arccosh}(a+bx)-\int \frac{e^{\text{arccosh}(a+bx)}\text{arccosh}(a+bx)}{a-e^{\text{arccosh}(a+bx)}+\sqrt{a^2-1}}d\text{arccosh}(a+bx)-\frac{1}{2}\text{arccosh}(a+bx)^2$$

$$\Big\downarrow \ 2620$$

$$-\int \log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a-\sqrt{a^2-1}}\right)d\text{arccosh}(a+bx)-\int \log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a+\sqrt{a^2-1}}\right)d\text{arccosh}(a+bx)+\text{arccosh}(a+bx)\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{arccosh}(a+bx)\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{\sqrt{a^2-1}+a}\right)-\frac{1}{2}\text{arccosh}(a+bx)^2$$

$$\Big\downarrow \ 2715$$

$$-\int e^{-\text{arccosh}(a+bx)}\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a-\sqrt{a^2-1}}\right)de^{\text{arccosh}(a+bx)}-\int e^{-\text{arccosh}(a+bx)}\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a+\sqrt{a^2-1}}\right)de^{\text{arccosh}(a+bx)}+\text{arccosh}(a+bx)\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{arccosh}(a+bx)\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{\sqrt{a^2-1}+a}\right)-\frac{1}{2}\text{arccosh}(a+bx)^2$$

$$\Big\downarrow \ 2838$$

$$\text{PolyLog}\left(2,\frac{e^{\text{arccosh}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{PolyLog}\left(2,\frac{e^{\text{arccosh}(a+bx)}}{a+\sqrt{a^2-1}}\right)+\text{arccosh}(a+bx)\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{arccosh}(a+bx)\log\left(1-\frac{e^{\text{arccosh}(a+bx)}}{\sqrt{a^2-1}+a}\right)-\frac{1}{2}\text{arccosh}(a+bx)^2$$

input
```
Int[ArcCosh[a + b*x]/x,x]
```

output
```
-1/2*ArcCosh[a + b*x]^2 + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a -
 Sqrt[-1 + a^2])] + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a + Sqrt[
-1 + a^2])] + PolyLog[2, E^ArcCosh[a + b*x]/(a - Sqrt[-1 + a^2])] + PolyLo
g[2, E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 6377
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(177) = 354$.

Time = 0.17 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.29

method	result
derivativedivides	$-\frac{\operatorname{arccosh}(bx+a)^2}{2}+\frac{a \operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{a+\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{a \operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}+bx+}{-a+}\right.}{\sqrt{a^2-1}}$
default	$-\frac{\operatorname{arccosh}(bx+a)^2}{2}+\frac{a \operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{a+\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{a \operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}+bx+}{-a+}\right.}{\sqrt{a^2-1}}$

input
```
int(arccosh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccosh(b*x+a)^2+a*arccosh(b*x+a)/(a^2-1)^(1/2)*ln(((a^2-1)^(1/2)-b*x
-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))-a*arccosh(b*x+a)/(a^2
-1)^(1/2)*ln(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-
1)^(1/2)))+(a^2-1+a*(a^2-1)^(1/2))/(a^2-1)*arccosh(b*x+a)*(ln(((a^2-1)^(1/
2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^(1/2)))-2*ln(((a^2-1)^
(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))*a^2+ln(((a^2
-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))+2*a*(a^2
-1)^(1/2)*ln(((a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1
)^(1/2))))+dilog(((a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a
^2-1)^(1/2)))+dilog(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-
a+(a^2-1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcosh}(bx+a)}{x}\,dx$$

input
```
integrate(arccosh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arccosh(b*x + a)/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = \int \frac{\operatorname{acosh}(a+bx)}{x}\,dx$$

input
```
integrate(acosh(b*x+a)/x,x)
```

output
```
Integral(acosh(a + b*x)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcosh}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(arccosh(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(arccosh(b*x + a)/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcosh}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(arccosh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arccosh(b*x + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = \int \frac{\operatorname{acosh}(a+b\,x)}{x}\,dx$$

input
```
int(acosh(a + b*x)/x,x)
```

output
```
int(acosh(a + b*x)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x}\,dx = \int \frac{acosh(bx+a)}{x}dx$$

input
```
int(acosh(b*x+a)/x,x)
```

output
```
int(acosh(a + b*x)/x,x)
```

3.6 $\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx=-\frac{\operatorname{arccosh}(a+bx)}{x}-\frac{2b\arctan\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}$$

output
```
-arccosh(b*x+a)/x-2*b*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+
a-1)^(1/2))/(-a^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx=-\frac{\operatorname{arccosh}(a+bx)}{x}-\frac{ib\log\left(\frac{2\left(\sqrt{-1+a+bx}\sqrt{1+a+bx}+\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}$$

input
```
Integrate[ArcCosh[a + b*x]/x^2,x]
```

output
```
-(ArcCosh[a + b*x]/x) - (I*b*Log[(2*(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]
+ (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(b*x)])/Sqrt[1 - a^2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6411, 27, 6378, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{\text{arccosh}(a+bx)}{x^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\text{arccosh}(a+bx)}{b^2x^2}d(a+bx)$$

$$\Big\downarrow \text{6378}$$

$$b\left(-\int -\frac{1}{bx\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{\text{arccosh}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{104}$$

$$b\left(-2\int \frac{1}{a+\frac{(1-a)(a+bx+1)}{a+bx-1}+1}d\frac{\sqrt{a+bx+1}}{\sqrt{a+bx-1}}-\frac{\text{arccosh}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{218}$$

$$b\left(-\frac{2\arctan\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\text{arccosh}(a+bx)}{bx}\right)$$

input
```
Int[ArcCosh[a + b*x]/x^2,x]
```

output
```
b*(-(ArcCosh[a + b*x]/(b*x)) - (2*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(
Sqrt[1 + a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 6378
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(
n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$b\left(-\frac{\operatorname{arccosh}(bx+a)}{bx}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)}{\sqrt{(bx+a)^2-1}\,(a-1)(1+a)}\right)$	101
default	$b\left(-\frac{\operatorname{arccosh}(bx+a)}{bx}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)}{\sqrt{(bx+a)^2-1}\,(a-1)(1+a)}\right)$	101
parts	$-\frac{\operatorname{arccosh}(bx+a)}{x}+\frac{b\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\operatorname{csgn}(b)^2\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)}{(1-a)(1+a)\sqrt{b^2x^2+2abx+a^2-1}}$	115

input
```
int(arccosh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-arccosh(b*x+a)/b/x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(a^2-1)^(1/2)*ln(2*
((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)/((b*x+a)^2-1)^(1/2)/(
a-1)/(1+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 5.03

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx$$
$$= \left[\frac{\sqrt{a^2-1}bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)+(a^2-1)x\log\left(-bx-a+\right.}{(a^2-1)x}\right.$$

input
```
integrate(arccosh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)
*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (
a^2 - 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2
 - 1)*x - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*
x), (2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a
*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + (a^2 - 1)*x*log(-b*x - a + sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 - 1)*x - 1)*log(b*x + a + s
qrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x)]
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acosh}(a+bx)}{x^2}\,dx$$

input
```
integrate(acosh(b*x+a)/x**2,x)
```

output
```
Integral(acosh(a + b*x)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccosh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx = \frac{2\,b\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}} - \frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{x}$$

input
```
integrate(arccosh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
2*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1))
/sqrt(-a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 - 1))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acosh}(a+b\,x)}{x^2}\,dx$$

input
```
int(acosh(a + b*x)/x^2,x)
```

output
```
int(acosh(a + b*x)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^2}\,dx = \frac{-acosh(bx+a)\,a^2+acosh(bx+a)+2\sqrt{-a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2-1}+bx}{\sqrt{-a^2+1}}\right)bx}{x\,(a^2-1)}$$

input
```
int(acosh(b*x+a)/x^2,x)
```

output
```
( - acosh(a + b*x)*a**2 + acosh(a + b*x) + 2*sqrt( - a**2 + 1)*atan((sqrt(
a**2 + 2*a*b*x + b**2*x**2 - 1) + b*x)/sqrt( - a**2 + 1))*b*x)/(x*(a**2 -
1))
```

3.7 $\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx = \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)\,x} - \frac{\operatorname{arccosh}(a+bx)}{2x^2} - \frac{ab^2\arctan\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{(1-a^2)^{3/2}}$$

output
```
1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)/x-1/2*arccosh(b*x+a)/x^2-a*
b^2*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+
1)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx = \frac{-\operatorname{arccosh}(a+bx) + \frac{bx\left(-\sqrt{-1+a+bx}\sqrt{1+a+bx}+\frac{iabx\log\left(\frac{4i\sqrt{1-a^2}\left(-1+a^2+abx-i\sqrt{1-a^2}\sqrt{-1+a+bx}\sqrt{1+a+bx}\right)}{ab^2x}\right)}{\sqrt{1-a^2}}\right)}{-1+a^2}}{2x^2}$$

input
```
Integrate[ArcCosh[a + b*x]/x^3,x]
```

output
```
(-ArcCosh[a + b*x] + (b*x*(-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) + (I*a*
b*x*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 +
 a + b*x]*Sqrt[1 + a + b*x]))/(a*b^2*x)])/Sqrt[1 - a^2]))/(-1 + a^2))/(2*x
^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6411, 25, 27, 6378, 107, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{\operatorname{arccosh}(a+bx)}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\operatorname{arccosh}(a+bx)}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-b^2\int -\frac{\operatorname{arccosh}(a+bx)}{b^3x^3}d(a+bx)$$

$$\Big\downarrow \text{6378}$$

$$-b^2\left(\frac{\operatorname{arccosh}(a+bx)}{2b^2x^2}-\frac{1}{2}\int \frac{1}{b^2x^2\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)\right)$$

$$\Big\downarrow \text{107}$$

$$-b^2\left(\frac{1}{2}\left(\frac{a\int -\frac{1}{bx\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{1-a^2}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)\,bx}\right)+\frac{\operatorname{arccosh}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \; 104$$

$$-b^2\left(\frac{1}{2}\left(\frac{2a\int \frac{1}{a+\frac{(1-a)(a+bx+1)}{a+bx-1}+1}d\frac{\sqrt{a+bx+1}}{\sqrt{a+bx-1}}}{1-a^2}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)\,bx}\right)+\frac{\operatorname{arccosh}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \; 218$$

$$-b^2\left(\frac{1}{2}\left(\frac{2a\arctan\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{(1-a^2)^{3/2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)\,bx}\right)+\frac{\operatorname{arccosh}(a+bx)}{2b^2x^2}\right)$$

input

```
Int[ArcCosh[a + b*x]/x^3,x]
```

output

```
-(b^2*(ArcCosh[a + b*x]/(2*b^2*x^2) + (-((Sqrt[-1 + a + b*x]*Sqrt[1 + a +
b*x])/((1 - a^2)*b*x)) + (2*a*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt
[1 + a]*Sqrt[-1 + a + b*x])])/(1 - a^2)^(3/2))/2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 6378
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(
n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.62

method	result
parts	$-\frac{\operatorname{arccosh}(bx+a)}{2x^2}+\frac{b\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\operatorname{csgn}(b)^2\left(\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)abx-a^2\cdots\right)}{2x(a^2-1)(1+a)(a-1)\sqrt{b^2x^2+2abx+a^2-1}\cdots}$
derivativedivides	$b^2\left(-\frac{\operatorname{arccosh}(bx+a)}{2b^2x^2}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left(\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)a^2-\sqrt{a^2-1}\,\ln\left(\frac{2\cdots}{}\right)\cdots\right)}{2bx(a^2-1)(1+a)(a-1)\sqrt{\cdots}}\cdots\right)$
default	$b^2\left(-\frac{\operatorname{arccosh}(bx+a)}{2b^2x^2}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left(\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)a^2-\sqrt{a^2-1}\,\ln\left(\frac{2\cdots}{}\right)\cdots\right)}{2bx(a^2-1)(1+a)(a-1)\sqrt{\cdots}}\cdots\right)$

input
```
int(arccosh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccosh(b*x+a)/x^2+1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*csgn(b)^2*((
a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1
)/x)*a*b*x-a^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
)/x/(a^2-1)/(1+a)/(a-1)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(86) = 172$.

Time = 0.12 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.34

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx$$

$$= \left[\frac{\sqrt{a^2-1}ab^2x^2 \log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2+\sqrt{a^2-1}a-1\right)+(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right) - (a^2-1)b^2x^2 + (a^4-2}{} \right.$$

$$-\frac{2\sqrt{-a^2+1}ab^2x^2 \arctan\left(-\frac{\sqrt{-a^2+1}bx-\sqrt{b^2x^2+2\,abx+a^2-1}\sqrt{-a^2+1}}{a^2-1}\right) + (a^2-1)b^2x^2 - (a^4-2\,a^2+1)x^2 \log}{}$$

input
```
integrate(arccosh(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - 1)*a*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) -
 a)/x) - (a^2 - 1)*b^2*x^2 + (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*b
*x - (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*a*b
^2*x^2*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqr
t(-a^2 + 1))/(a^2 - 1)) + (a^2 - 1)*b^2*x^2 - (a^4 - 2*a^2 + 1)*x^2*log(-b
*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2
 - 1)*(a^2 - 1)*b*x + (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2)]
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{acosh}\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(acosh(b*x+a)/x**3,x)
```

output
```
Integral(acosh(a + b*x)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccosh(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx = -\left(\frac{ab\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{(a^2-1)\sqrt{-a^2+1}} - \frac{\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)ab+a^2|b|-|b|}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)^2-a^2+1\right)(a^2-1)}\right)b - \frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{2\,x^2}$$

input
```
integrate(arccosh(b*x+a)/x^3,x, algorithm="giac")
```

output
```
-(a*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1
))/((a^2 - 1)*sqrt(-a^2 + 1)) - ((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
- 1))*a*b + a^2*abs(b) - abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^
2 - 1))^2 - a^2 + 1)*(a^2 - 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 -
1))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx=\int \frac{\operatorname{acosh}(a+b\,x)}{x^3}\,dx$$

input
```
int(acosh(a + b*x)/x^3,x)
```

output
```
int(acosh(a + b*x)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.37

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^3}\,dx$$
$$=\frac{-acosh(bx+a)\,a^4+2acosh(bx+a)\,a^2-acosh(bx+a)-2\sqrt{-a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2-1}+bx}{\sqrt{-a^2+1}}\right)a\,b^2x^2}{2x^2\,(a^4-2a^2+1)}$$

input
```
int(acosh(b*x+a)/x^3,x)
```

output
```
( - acosh(a + b*x)*a**4 + 2*acosh(a + b*x)*a**2 - acosh(a + b*x) - 2*sqrt(
 - a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + b*x)/sqrt( - a**
2 + 1))*a*b**2*x**2 + sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a**2*b*x - sqrt
(a**2 + 2*a*b*x + b**2*x**2 - 1)*b*x)/(2*x**2*(a**4 - 2*a**2 + 1))
```

3.8 $\int \frac{\text{arccosh}(a+bx)}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 154

$$\int \frac{\text{arccosh}(a+bx)}{x^4} dx = \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)^2 x} - \frac{\text{arccosh}(a+bx)}{3x^3} - \frac{(1+2a^2)b^3 \arctan\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{3(1-a^2)^{5/2}}$$

output
```
1/6*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)/x^2+1/2*a*b^2*(b*x+a-1)^(1/
2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/x-1/3*arccosh(b*x+a)/x^3-1/3*(2*a^2+1)*b^3*a
rctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(5
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^4}\,dx = \frac{1}{6}\left(\frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}(1-a^2+3abx)}{(-1+a^2)^2x^2} - \frac{2\operatorname{arccosh}(a+bx)}{x^3} - \frac{i(1+2a^2)b^3\log\left(\frac{12(1-a^2)^{3/2}\left(-i+ia^2+iabx+\sqrt{1-a^2}\sqrt{-1+a+bx}\sqrt{1+a+bx}\right)}{b^3(x+2a^2x)}\right)}{(1-a^2)^{5/2}}\right)$$

input
```
Integrate[ArcCosh[a + b*x]/x^4,x]
```

output
```
((b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(1 - a^2 + 3*a*b*x))/((-1 + a^2)^
2*x^2) - (2*ArcCosh[a + b*x])/x^3 - (I*(1 + 2*a^2)*b^3*Log[(12*(1 - a^2)^(
3/2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a +
 b*x]))/(b^3*(x + 2*a^2*x))])/(1 - a^2)^(5/2))/6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6411, 27, 6378, 114, 25, 168, 25, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{\operatorname{arccosh}(a+bx)}{x^4}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b^3 \int \frac{\text{arccosh}(a+bx)}{b^4x^4}d(a+bx)$$

$$\Big\downarrow \; 6378$$

$$b^3 \left(-\frac{1}{3}\int -\frac{1}{b^3x^3\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 114$$

$$b^3 \left(\frac{1}{3}\left(\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}-\frac{\int -\frac{3a+bx}{b^2x^2\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{2\left(1-a^2\right)}\right)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 25$$

$$b^3 \left(\frac{1}{3}\left(\frac{\int \frac{3a+bx}{b^2x^2\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{2\left(1-a^2\right)}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 168$$

$$b^3 \left(\frac{1}{3}\left(\frac{\frac{\int \frac{2a^2+1}{bx\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{1-a^2}+\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)bx}}{2\left(1-a^2\right)}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 25$$

$$b^3 \left(\frac{1}{3}\left(\frac{\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)bx}-\frac{\int -\frac{2a^2+1}{bx\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{1-a^2}}{2\left(1-a^2\right)}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 27$$

$$b^3 \left(\frac{1}{3}\left(\frac{\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)bx}-\frac{(2a^2+1)\int -\frac{1}{bx\sqrt{a+bx-1}\sqrt{a+bx+1}}d(a+bx)}{1-a^2}}{2\left(1-a^2\right)}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 104$$

$$b^3 \left(\frac{1}{3}\left(\frac{\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)bx}-\frac{2(2a^2+1)\int \frac{1}{a+\frac{(1-a)(a+bx+1)}{a+bx-1}+1}d\frac{\sqrt{a+bx+1}}{\sqrt{a+bx-1}}}{1-a^2}}{2\left(1-a^2\right)}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\text{arccosh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 218$$

$$b^3\left(\frac{1}{3}\left(\frac{\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{(1-a^2)bx}-\frac{2(2a^2+1)\arctan\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{(1-a^2)^{3/2}}}{2\left(1-a^2\right)}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\operatorname{arccosh}(a+bx)}{3b^3x^3}\right)$$

input
```
Int[ArcCosh[a + b*x]/x^4,x]
```

output
```
b^3*(-1/3*ArcCosh[a + b*x]/(b^3*x^3) + ((Sqrt[-1 + a + b*x]*Sqrt[1 + a + b
*x])/(2*(1 - a^2)*b^2*x^2) + ((3*a*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(
(1 - a^2)*b*x) - (2*(1 + 2*a^2)*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sq
rt[1 + a]*Sqrt[-1 + a + b*x])])/(1 - a^2)^(3/2))/(2*(1 - a^2)))/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 6378
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(
n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.00

method	result
parts	$-\frac{\operatorname{arccosh}(bx+a)}{3x^3}-\frac{b\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\operatorname{csgn}(b)^2\left(2\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)a^2b^2x^2\right.}{}$
derivativedivides	$b^3\left(-\frac{\operatorname{arccosh}(bx+a)}{3b^3x^3}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left(2\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)a^4-4\sqrt{a^2-1}\,\ln\left(\right.\right.}{}\right.$
default	$b^3\left(-\frac{\operatorname{arccosh}(bx+a)}{3b^3x^3}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left(2\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)a^4-4\sqrt{a^2-1}\,\ln\left(\right.\right.}{}\right.$

input
```
int(arccosh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccosh(b*x+a)/x^3-1/6*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*csgn(b)^2*(2
*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2
-1)/x)*a^2*b^2*x^2+(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2-1)^(1/2)+a^2-1)/x)*b^2*x^2-3*a^3*b*x*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a
^4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+3*a*b*x*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-2*a
^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/x^2/(a^2-1
)^2/(1+a)/(a-1)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(124) = 248$.

Time = 0.14 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.68

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^4}\,dx$$
$$=\left[\frac{(2\,a^2+1)\sqrt{a^2-1}b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)+3\,(a^3-a)b^3x^3}{}\right.$$

input
```
integrate(arccosh(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
[1/6*((2*a^2 + 1)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(
a^2 - 1) - a)/x) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*l
og(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*a^4 - (a^6 -
 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), 1/6*(2*(2*a^2 + 1)*s
qrt(-a^2 + 1)*b^3*x^3*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*
a^4 + 3*a^2 - 1)*x^3*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2
*(a^6 - 3*a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + s
qrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 +
1)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3)
]
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{acosh}(a+bx)}{x^4}\,dx$$

input
```
integrate(acosh(b*x+a)/x**4,x)
```

output
```
Integral(acosh(a + b*x)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccosh(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(124) = 248$.

Time = 0.16 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{arccosh}(a+bx)}{x^4}\,dx$$
$$=\frac{1}{3}b\left(\frac{(2\,a^2b^2+b^2)\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{(a^4-2\,a^2+1)\sqrt{-a^2+1}}-\frac{2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)^3a^2b^2-6\left(x|b|-\right.}{}\right.$$
$$-\frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{3\,x^3}$$

input
```
integrate(arccosh(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/3*b*((2*a^2*b^2 + b^2)*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
- 1))/sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - (2*(x*abs(b) -
sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1))*a^4*b^2 - 4*a^5*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1))^3*b^2 + 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))*a^2*b^2 + 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))*b^2 - 4*a*b*abs(b))/((a^4 - 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1))^2 - a^2 + 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 -
1))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arccosh}(a+bx)}{x^4}\,dx=\int \frac{\text{acosh}(a+b\,x)}{x^4}\,dx$$

input
```
int(acosh(a + b*x)/x^4,x)
```

output
```
int(acosh(a + b*x)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.16

$$\int \frac{\text{arccosh}(a+bx)}{x^4}\,dx = \frac{-4acosh(bx+a)\,a^7+12acosh(bx+a)\,a^5-12acosh(bx+a)\,a^3+4acosh(bx+a)\,a+8\sqrt{-a^2+1}\,atan\Big($$

input
```
int(acosh(b*x+a)/x^4,x)
```

output
```
( - 4*acosh(a + b*x)*a**7 + 12*acosh(a + b*x)*a**5 - 12*acosh(a + b*x)*a**
3 + 4*acosh(a + b*x)*a + 8*sqrt( - a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b
**2*x**2 - 1) + b*x)/sqrt( - a**2 + 1))*a**3*b**3*x**3 + 4*sqrt( - a**2 +
1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1) + b*x)/sqrt( - a**2 + 1))*a*
b**3*x**3 + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a**5*b*x - 6*sqrt(a**2
+ 2*a*b*x + b**2*x**2 - 1)*a**4*b**2*x**2 - 4*sqrt(a**2 + 2*a*b*x + b**2*x
**2 - 1)*a**3*b*x + 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a**2*b**2*x**2
+ 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)*a*b*x + 4*a**4*b**3*x**3 - 5*a**2
*b**3*x**3 + b**3*x**3)/(12*a*x**3*(a**6 - 3*a**4 + 3*a**2 - 1))
```

3.9 $\int \frac{1}{\sqrt{a+b\,\mathbf{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

output
```
-1/2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d+1
/2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx$$
$$= \frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)+\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\text{arccosh}(c+da}{b}\right.\right.}{2d\sqrt{a+b\text{arccosh}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x
]] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d
*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6410, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6296}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \; 26$$

$$\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \; 3789$$

$$\frac{i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{bd}$$

$$\Big\downarrow \; 2611$$

$$\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{bd}$$

$$\Big\downarrow \; 2633$$

$$\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{bd}$$

$$\Big\downarrow \; 2634$$

$$\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{bd}$$

input
```
Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(I*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b
]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E
^(a/b)))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b \operatorname{arccosh}(dx + c)}}dx$$

input
```
int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\operatorname{arccosh}(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\operatorname{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arcosh}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arcosh}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + b\,\text{acosh}\,(c + d\,x)}}\,dx$$

input
```
int(1/(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + b\mathrm{arccosh}(c + dx)}}\,dx = \int \frac{\sqrt{acosh\left(dx+c\right)b+a}}{acosh\left(dx+c\right)b+a}dx$$

input

```
int(1/(a+b*acosh(d*x+c))^(1/2),x)
```

output

```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b + a),x)
```

3.10 $\int \frac{1}{\sqrt{a-b\,\mathrm{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{1}{\sqrt{a-b\,\mathrm{arccosh}(c+dx)}}\,dx = -\frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a-b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

output
```
-1/2*exp(a/b)*Pi^(1/2)*erf((a-b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d+1
/2*Pi^(1/2)*erfi((a-b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{a-b\,\mathrm{arccosh}(c+dx)}}\,dx = \frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}-\mathrm{arccosh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}-\mathrm{arccosh}(c+dx)\right)+\sqrt{-\frac{a}{b}+\mathrm{arccosh}(c+dx)}\,\Gamma\left(\frac{1}{2},-\frac{a}{b}+\mathrm{arccosh}\right.\right.}{2d\sqrt{a-b\,\mathrm{arccosh}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

output
```
(E^((2*a)/b)*Sqrt[a/b - ArcCosh[c + d*x]]*Gamma[1/2, a/b - ArcCosh[c + d*x
]] + Sqrt[-(a/b) + ArcCosh[c + d*x]]*Gamma[1/2, -(a/b) + ArcCosh[c + d*x]]
)/(2*d*E^(a/b)*Sqrt[a - b*ArcCosh[c + d*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6410, 6296, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-b\text{arccosh}(c+dx)}}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{\sqrt{a-b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6296}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a-b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a-b\text{arccosh}(c+dx)}}d(a-b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a-b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a-b\text{arccosh}(c+dx)}}d(a-b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a-b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a-b\text{arccosh}(c+dx)}}d(a-b\text{arccosh}(c+dx))}{bd}$$

$\downarrow$ 3789

$$\frac{i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a-b\text{arccosh}(c+dx)}}d(a-b\text{arccosh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a-b\text{arccosh}(c+dx)}}d(a-b\text{arccosh}(c+dx))\right)}{bd}$$

$\downarrow$ 2611

$$\frac{i\left(i\int e^{\frac{a}{b}-\frac{a-b\text{arccosh}(c+dx)}{b}}d\sqrt{a-b\text{arccosh}(c+dx)}-i\int e^{\frac{a-b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a-b\text{arccosh}(c+dx)}\right)}{bd}$$

$\downarrow$ 2633

$$\frac{i\left(i\int e^{\frac{a}{b}-\frac{a-b\text{arccosh}(c+dx)}{b}}d\sqrt{a-b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a-b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{bd}$$

$\downarrow$ 2634

$$\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a-b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a-b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{bd}$$

input
```
Int[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

output
```
(I*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b
]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/E
^(a/b)))/(b*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a - b\,\text{arccosh}\,(dx + c)}}dx$$

input
```
int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

output
```
int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - b\text{arccosh}(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a - b\,\text{acosh}\,(c + dx)}}\,dx$$

input
```
integrate(1/(a-b*acosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a - b*acosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a - b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-b\,\text{arcosh}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a - b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-b\,\text{arcosh}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - b\text{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a - b\,\text{acosh}\,(c + d\,x)}}\,dx$$

input
```
int(1/(a - b*acosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a - b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a - b\text{arccosh}(c + dx)}}\,dx = -\left(\int \frac{\sqrt{-acosh\,(dx + c)\,b + a}}{acosh\,(dx + c)\,b - a}dx\right)$$

input
```
int(1/(a-b*acosh(d*x+c))^(1/2),x)
```

output
```
- int(sqrt( - acosh(c + d*x)*b + a)/(acosh(c + d*x)*b - a),x)
```

3.11 $\int (ce+dex)^4(a+b\text{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))\,dx = -\frac{8be^4\sqrt{-1+c+dx}\sqrt{1+c+dx}}{75d} - \frac{4be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5(a+b\text{arccosh}(c+dx))}{5d}$$

output

```
-8/75*b*e^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-4/75*b*e^4*(d*x+c-1)^(1/2)*(
d*x+c)^2*(d*x+c+1)^(1/2)/d-1/25*b*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^
(1/2)/d+1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.55

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))\,dx$$
$$=\frac{e^4\left(-\frac{1}{75}b\sqrt{-1+c+dx}\sqrt{1+c+dx}(8+4(c+dx)^2+3(c+dx)^4)+\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))\right)}{d}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^4*(-1/75*(b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(8 + 4*(c + d*x)^2 + 3
*(c + d*x)^4)) + ((c + d*x)^5*(a + b*ArcCosh[c + d*x]))/5))/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6411, 27, 6298, 111, 27, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{111}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{1}{5}\int\frac{4(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4\right.\right.}{d}$$

$\big\downarrow$ 27

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{4}{5}\int\frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4\right.\right.}{d}$$

$\big\downarrow$ 111

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{4}{5}\left(\frac{1}{3}\int\frac{2(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+d\right.\right.\right.}{d}$$

$\big\downarrow$ 27

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+d\right.\right.\right.}{d}$$

$\big\downarrow$ 83

$$\frac{e^4(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b(\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4+\frac{4}{5}(\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c}{d}$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^4*(-1/5*(b*((Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/5 + (4*(
(2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/3 + (Sqrt[-1 + c + d*x]*(c + d*x)
^2*Sqrt[1 + c + d*x])/3))/5)) + ((c + d*x)^5*(a + b*ArcCosh[c + d*x]))/5))
/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{\frac{e^4 a (dx+c)^5}{5}+e^4 b\left(\frac{(dx+c)^5 \operatorname{arccosh}(dx+c)}{5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(3(dx+c)^4+4(dx+c)^2+8\right)}{75}\right)}{d}$
default	$\frac{\frac{e^4 a (dx+c)^5}{5}+e^4 b\left(\frac{(dx+c)^5 \operatorname{arccosh}(dx+c)}{5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(3(dx+c)^4+4(dx+c)^2+8\right)}{75}\right)}{d}$
parts	$\frac{e^4 a (dx+c)^5}{5d}+\frac{e^4 b\left(\frac{(dx+c)^5 \operatorname{arccosh}(dx+c)}{5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(3(dx+c)^4+4(dx+c)^2+8\right)}{75}\right)}{d}$
orering	$\frac{(27d^6x^6+162c\,d^5x^5+405c^2d^4x^4+540c^3d^3x^3+405c^4d^2x^2+4d^4x^4+162c^5dx+16c\,d^3x^3+27c^6+24c^2d^2x^2+16c^3dx+4c^4}{75(dx+c)^5 d}$

input
```
int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*e^4*a*(d*x+c)^5+e^4*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(115) = 230.

Time = 0.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.07

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))\,dx$$
$$=\frac{15\,ad^5e^4x^5+75\,acd^4e^4x^4+150\,ac^2d^3e^4x^3+150\,ac^3d^2e^4x^2+75\,ac^4de^4x+15\,(bd^5e^4x^5+5\,bcd^4e^4x^4+10}{}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*
a*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x
^4 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5
*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (3*b*d^4*e^4*x^4
+ 12*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 + 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 + 2*b*c)
*d*e^4*x + (3*b*c^4 + 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
))/d
```

Sympy [F]

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))\,dx = e^4\left(\int ac^4\,dx+\int ad^4x^4\,dx+\int bc^4\operatorname{acosh}(c+dx)\,dx+\int 4acd^3x^3\,dx+\int 6ac^2d^2x^2\,dx+\int 4ac^3dx\,dx+\int bd^4x^4\operatorname{acosh}(c+dx)\,dx+\int 4bcd^3x^3\operatorname{acosh}(c+dx)\,dx+\int 6bc^2d^2x^2\operatorname{acosh}(c+dx)\,dx+\int 4bc^3dx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c)),x)
```

output
```
e**4*(Integral(a*c**4, x) + Integral(a*d**4*x**4, x) + Integral(b*c**4*aco
sh(c + d*x), x) + Integral(4*a*c*d**3*x**3, x) + Integral(6*a*c**2*d**2*x*
*2, x) + Integral(4*a*c**3*d*x, x) + Integral(b*d**4*x**4*acosh(c + d*x),
x) + Integral(4*b*c*d**3*x**3*acosh(c + d*x), x) + Integral(6*b*c**2*d**2*
x**2*acosh(c + d*x), x) + Integral(4*b*c**3*d*x*acosh(c + d*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(115) = 230$.

Time = 0.06 (sec) , antiderivative size = 1241, normalized size of antiderivative = 9.19

$$\int (ce + dex)^4(a + b\text{arccosh}(c + dx))\, dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*
x^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2
- (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d
^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*
arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3
*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d
+ 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*
c^2*d^2*e^4 + 1/24*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^
4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x +
 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*
x/d^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*b*
c*d^3*e^4 + 1/600*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 12...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(115) = 230$.

Time = 0.76 (sec) , antiderivative size = 846, normalized size of antiderivative = 6.27

$$\int (ce + dex)^4 (a + b\text{arccosh}(c + dx))\, dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*
x^2 - (d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*
abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^4*e^4 + (2*x^2*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x
/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*c^3*d*e^4 + 1/3*(6*x^3*log(d
*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*lo
g(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*
abs(d)))*d)*b*c^2*d^2*e^4 + 1/24*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*x*(3*x/d^2 - 7*c/
d^3) + (26*c^2*d^3 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2 + 11*c*d^2)/d^7) - 3*(8
*c^4 + 24*c^2 + 3)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1))*abs(d)))/(d^4*abs(d)))*d)*b*c*d^3*e^4 + 1/600*(120*x^5*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*
((2*(3*x*(4*x/d^2 - 9*c/d^3) + (47*c^2*d^5 + 16*d^5)/d^9)*x - 7*(22*c^3*d^
4 + 23*c*d^4)/d^9)*x + (274*c^4*d^3 + 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c
^5 + 40*c^3 + 15*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))*abs(d)))/(d^5*abs(d)))*d)*b*d^4*e^4 + a*c^4*e^4*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^4 (a + b\operatorname{arccosh}(c + dx))\, dx = \int (c\, e + d\, e\, x)^4\, (a + b\operatorname{acosh}(c + d\, x))\, dx$$

input
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.21

$$\int (ce + dex)^4 (a + b\operatorname{arccosh}(c + dx))\, dx$$
$$= \frac{e^4 (75 acosh(dx + c)\, b\, c^5 + 75 acosh(dx + c)\, b\, c^4 dx + 150 acosh(dx + c)\, b\, c^3 d^2 x^2 + 150 acosh(dx + c)\, b\, c^2 d^3}{}$$

input
```
int((d*e*x+c*e)^4*(a+b*acosh(d*x+c)),x)
```

output
```
(e**4*(75*acosh(c + d*x)*b*c**5 + 75*acosh(c + d*x)*b*c**4*d*x + 150*acosh
(c + d*x)*b*c**3*d**2*x**2 + 150*acosh(c + d*x)*b*c**2*d**3*x**3 + 75*acos
h(c + d*x)*b*c*d**4*x**4 + 15*acosh(c + d*x)*b*d**5*x**5 + 72*sqrt(c**2 +
2*c*d*x + d**2*x**2 - 1)*b*c**4 - 12*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*
b*c**3*d*x - 18*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c**2*d**2*x**2 - 4*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c**2 - 12*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*b*c*d**3*x**3 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c*d*x
 - 3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*d**4*x**4 - 4*sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)*b*d**2*x**2 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*
b - 75*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*b*c**4 - 60*log(sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1) + c + d*x)*b*c**5 + 75*a*c**4*d*x + 150*a*c**3*d**2*
x**2 + 150*a*c**2*d**3*x**3 + 75*a*c*d**4*x**4 + 15*a*d**5*x**5))/(75*d)
```

3.12 $\int (ce+dex)^3(a+b\text{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))\,dx = -\frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{16d} - \frac{3be^3\text{arccosh}(c+dx)}{32d} + \frac{e^3(c+dx)^4(a+b\text{arccosh}(c+dx))}{4d}$$

output
```
-3/32*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/d-1/16*b*e^3*(d*x+c-1)
^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/d-3/32*b*e^3*arccosh(d*x+c)/d+1/4*e^3*(d*
x+c)^4*(a+b*arccosh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))\,dx$$
$$=\frac{e^3\left((c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{8}b\left(3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}+2\sqrt{-1+c+dx}(c+d\right.\right.}{4d}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^3*((c + d*x)^4*(a + b*ArcCosh[c + d*x]) - (b*(3*Sqrt[-1 + c + d*x]*(c +
 d*x)*Sqrt[1 + c + d*x] + 2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*
x] + 6*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/8))/(4*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6411, 27, 6298, 111, 27, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))\,dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int e^3(c+dx)^3(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{e^3\int (c+dx)^3(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$
$$\Big\downarrow \text{ 6298}$$
$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\int \frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\downarrow$ 111

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}\int\frac{3(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\right.\right.}{d}$$

$\downarrow$ 27

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{3}{4}\int\frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\right.\right.}{d}$$

$\downarrow$ 101

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+d\right.\right.\right.}{d}$$

$\downarrow$ 43

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\text{arccosh}(c+dx)+\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\right)+\frac{1}{4}\sqrt{c+\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^3*(-1/4*(b*((Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/4 + (3*(
(Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/2 + ArcCosh[c + d*x]/2))/
4)) + ((c + d*x)^4*(a + b*ArcCosh[c + d*x]))/4))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{e^3 a(dx+c)^4}{4}+e^3 b\left(\frac{(dx+c)^4 \operatorname{arccosh}(dx+c)}{4}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^3\sqrt{(dx+c)^2-1}+3(dx+c)\sqrt{(dx+c)^2-1}+3\ln\left(dx+c+\sqrt{\right.\right.}{32\sqrt{(dx+c)^2-1}}}{d}$
default	$\frac{\frac{e^3 a(dx+c)^4}{4}+e^3 b\left(\frac{(dx+c)^4 \operatorname{arccosh}(dx+c)}{4}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^3\sqrt{(dx+c)^2-1}+3(dx+c)\sqrt{(dx+c)^2-1}+3\ln\left(dx+c+\sqrt{\right.\right.}{32\sqrt{(dx+c)^2-1}}}{d}$
parts	$\frac{e^3 a(dx+c)^4}{4d}+\frac{e^3 b\left(\frac{(dx+c)^4 \operatorname{arccosh}(dx+c)}{4}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^3\sqrt{(dx+c)^2-1}+3(dx+c)\sqrt{(dx+c)^2-1}+3\ln\left(dx+c\right.\right.}{32\sqrt{(dx+c)^2-1}}}{d}$
orering	$\frac{(14d^4x^4+56c\,d^3x^3+84c^2d^2x^2+56c^3dx+14c^4+3d^2x^2+6cdx+3c^2-12)(dex+ce)^3(a+b\operatorname{arccosh}(dx+c))}{32d(dx+c)^3}-\frac{(dx+c+1)}{}$

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a*(d*x+c)^4+e^3*b*(1/4*(d*x+c)^4*arccosh(d*x+c)-1/32*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^3*((d*x+c)^2-1)^(1/2)+3*(d*x+c)*((d*x+c
)^2-1)^(1/2)+3*ln(d*x+c+((d*x+c)^2-1)^(1/2)))/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.90

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))\,dx$$
$$=\frac{8\,ad^4e^3x^4+32\,acd^3e^3x^3+48\,ac^2d^2e^3x^2+32\,ac^3de^3x+\left(8\,bd^4e^3x^4+32\,bcd^3e^3x^3+48\,bc^2d^2e^3x^2+32\,bc^3\right.}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c
^3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2
+ 32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 + b)*
d*e^3*x + (2*b*c^3 + 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))\,dx = e^3\left(\int ac^3\,dx+\int ad^3x^3\,dx + \int bc^3\operatorname{acosh}(c+dx)\,dx+\int 3acd^2x^2\,dx + \int 3ac^2dx\,dx+\int bd^3x^3\operatorname{acosh}(c+dx)\,dx + \int 3bcd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 3bc^2dx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c)),x)
```

output
```
e**3*(Integral(a*c**3, x) + Integral(a*d**3*x**3, x) + Integral(b*c**3*aco
sh(c + d*x), x) + Integral(3*a*c*d**2*x**2, x) + Integral(3*a*c**2*d*x, x)
 + Integral(b*d**3*x**3*acosh(c + d*x), x) + Integral(3*b*c*d**2*x**2*acos
h(c + d*x), x) + Integral(3*b*c**2*d*x*acosh(c + d*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(103) = 206.

Time = 0.04 (sec) , antiderivative size = 797, normalized size of antiderivative = 6.70

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/4*(2*x^2*arc
cosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(
2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arccosh(d*x + c)
 - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2
*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^
4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*c*d^2*e^3 + 1/96
*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2 -
 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*
c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*
c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1
)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*e^
3*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c^3*e^3/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(103) = 206$.

Time = 0.63 (sec) , antiderivative size = 617, normalized size of antiderivative = 5.18

$$\int (ce + dex)^3(a + b\operatorname{arccosh}(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 - (d*(c*log(abs(
-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d))
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)))*b*c^3*e^3 + 3/4*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3)
- (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
)*abs(d)))/(d^2*abs(d)))*d)*b*c^2*d*e^3 + 1/6*(6*x^3*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x*(2*x/d
^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*d - (x*
abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*d)*b*c*
d^2*e^3 + 1/96*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) -
(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3
 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2 + 3)
*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d
^4*abs(d)))*d)*b*d^3*e^3 + a*c^3*e^3*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.94

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))\,dx = \frac{e^3\left(32acosh(dx+c)\,b\,c^4+32acosh(dx+c)\,b\,c^3dx+48acosh(dx+c)\,b\,c^2d^2x^2+32acosh(dx+c)\,bc\,d^3x^3\right.}{}$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c)),x)
```

output
```
(e**3*(32*acosh(c + d*x)*b*c**4 + 32*acosh(c + d*x)*b*c**3*d*x + 48*acosh(
c + d*x)*b*c**2*d**2*x**2 + 32*acosh(c + d*x)*b*c*d**3*x**3 + 8*acosh(c +
d*x)*b*d**4*x**4 + 30*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c**3 - 6*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c**2*d*x - 6*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*b*c*d**2*x**2 - 3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c - 2*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*d**3*x**3 - 3*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*b*d*x - 32*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*b*c**3 - 24
*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*b*c**4 - 3*log(sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*b + 32*a*c**3*d*x + 48*a*c**2*d*
*2*x**2 + 32*a*c*d**3*x**3 + 8*a*d**4*x**4))/(32*d)
```

3.13 $\int (ce+dex)^2(a+b\text{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))\,dx = -\frac{2be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{9d} - \frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{9d} + \frac{e^2(c+dx)^3(a+b\text{arccosh}(c+dx))}{3d}$$

output
```
-2/9*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/9*b*e^2*(d*x+c-1)^(1/2)*(d*
x+c)^2*(d*x+c+1)^(1/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))\,dx = \frac{e^2\left(-\frac{1}{9}b\sqrt{-1+c+dx}\sqrt{1+c+dx}(2+c^2+2cdx+d^2x^2)+\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))\right)}{d}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^2*(-1/9*(b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^
2*x^2)) + ((c + d*x)^3*(a + b*ArcCosh[c + d*x]))/3))/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6411, 27, 6298, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{111}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{1}{3}\int \frac{2(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{2}{3}\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2\right.}{d}$$

$$\Big\downarrow \text{83}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2+\frac{2}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^2*(-1/3*(b*((2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/3 + (Sqrt[-1 + c +
 d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/3)) + ((c + d*x)^3*(a + b*ArcCosh[c +
 d*x]))/3))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\frac{e^2 a (dx+c)^3}{3}+e^2 b \left(\frac{(dx+c)^3 \operatorname{arccosh}(dx+c)}{3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left((dx+c)^2+2\right)}{9}\right)}{d}$
default	$\frac{\frac{e^2 a (dx+c)^3}{3}+e^2 b \left(\frac{(dx+c)^3 \operatorname{arccosh}(dx+c)}{3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left((dx+c)^2+2\right)}{9}\right)}{d}$
parts	$\frac{e^2 a (dx+c)^3}{3d}+\frac{e^2 b \left(\frac{(dx+c)^3 \operatorname{arccosh}(dx+c)}{3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left((dx+c)^2+2\right)}{9}\right)}{d}$
orering	$\frac{(5d^4x^4+20c\,d^3x^3+30c^2d^2x^2+20c^3dx+5c^4+2d^2x^2+4cdx+2c^2-4)(dex+ce)^2(a+b \operatorname{arccosh}(dx+c))}{9d(dx+c)^3}-\frac{(d^2x^2+2cdx+$

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a*(d*x+c)^3+e^2*b*(1/3*(d*x+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(83) = 166.

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))\,dx$$
$$=\frac{3\,ad^3e^2x^3+9\,acd^2e^2x^2+9\,ac^2de^2x+3\,(bd^3e^2x^3+3\,bcd^2e^2x^2+3\,bc^2de^2x+bc^3e^2)\log\left(dx+c+\sqrt{d^2x^2+\right.}{9\,d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*
x^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 +
2*b)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))\,dx = e^2\left(\int ac^2\,dx+\int ad^2x^2\,dx+\int bc^2\operatorname{acosh}(c+dx)\,dx+\int 2acdx\,dx+\int bd^2x^2\operatorname{acosh}(c+dx)\,dx+\int 2bcdx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c)),x)
```

output
```
e**2*(Integral(a*c**2, x) + Integral(a*d**2*x**2, x) + Integral(b*c**2*aco
sh(c + d*x), x) + Integral(2*a*c*d*x, x) + Integral(b*d**2*x**2*acosh(c +
d*x), x) + Integral(2*b*c*d*x*acosh(c + d*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(83) = 166$.

Time = 0.04 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.63

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))\,dx = \frac{1}{3}ad^2e^2x^3 + acde^2x^2 + \frac{1}{2}\left(2x^2\,\text{arcosh}\,(dx+c) - d\left(\frac{3c^2\log\left(2d^2x+2cd+2\sqrt{d^2x^2+2cdx+c^2-1}d\right)}{d^3} + \frac{\sqrt{d^2x^2+2cdx+}}{d^2}\right.\right.$$
$$+\frac{1}{18}\left(6x^3\,\text{arcosh}\,(dx+c) - d\left(\frac{2\sqrt{d^2x^2+2cdx+c^2-1}x^2}{d^2} - \frac{15c^3\log\left(2d^2x+2cd+2\sqrt{d^2x^2+2cd}\right.}{d^4}\right.\right.$$
$$+ac^2e^2x + \frac{\left((dx+c)\,\text{arcosh}\,(dx+c) - \sqrt{(dx+c)^2-1}\right)bc^2e^2}{d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + 1/2*(2*x^2*arccosh(d*x + c) - d*(3*c^2
*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c/d^3))*b*c*d*e^2 + 1/18*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9
*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/
d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + ((d*x + c)*arccos
h(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c^2*e^2/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(83) = 166$.

Time = 0.49 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.32

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))\,dx = \frac{1}{3}ad^2e^2x^3 + acde^2x^2$$
$$-\left(d\left(\frac{c\log\left(\left|-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2-1}\right)|d|\right|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}}{d^2}\right)-x\log\left(dx+c\right.\right.$$
$$+\frac{1}{2}\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2-1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right)-\frac{(2\,c^2+1)\,\right.\right.$$
$$+\frac{1}{18}\left(6\,x^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2-1}\left(x\left(\frac{2\,x}{d^2}-\frac{5\,c}{d^3}\right)+\frac{11\,c^2}{}\right.\right.\right.$$
$$+ac^2e^2x$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 - (d*(c*log(abs(-c*d - (x*abs(d) - sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c
^2*e^2 + 1/2*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c
*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))
*d)*b*c*d*e^2 + 1/18*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d
 + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*d)*b*d^2*e^2 + a*c^2*e^2*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.46

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))\,dx$$
$$= \frac{e^2\big(9acosh(dx+c)\,b\,c^3+9acosh(dx+c)\,b\,c^2dx+9acosh(dx+c)\,bc\,d^2x^2+3acosh(dx+c)\,b\,d^3x^3+8\sqrt{d}}{}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c)),x)
```

output
```
(e**2*(9*acosh(c + d*x)*b*c**3 + 9*acosh(c + d*x)*b*c**2*d*x + 9*acosh(c +
 d*x)*b*c*d**2*x**2 + 3*acosh(c + d*x)*b*d**3*x**3 + 8*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*b*c**2 - 2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c*d*x
- sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*d**2*x**2 - 2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*b - 9*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*b*c**2 - 6*log
(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*b*c**3 + 9*a*c**2*d*x + 9
*a*c*d**2*x**2 + 3*a*d**3*x**3))/(9*d)
```

3.14 $\int (ce + dex)(a + b\text{arccosh}(c + dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))\,dx = -\frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{4d} - \frac{be\text{arccosh}(c+dx)}{4d} + \frac{e(c+dx)^2(a+b\text{arccosh}(c+dx))}{2d}$$

output
```
-1/4*b*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/d-1/4*b*e*arccosh(d*x+c)/
d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))\,dx = \frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx)) - \frac{1}{4}b\left(\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx} + 2\text{arctanh}\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}\right)\right.\right.}{d}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x]))/2 - (b*(Sqrt[-1 + c + d*x]*(c +
 d*x)*Sqrt[1 + c + d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/
4))/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6411, 27, 6298, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\operatorname{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e(c+dx)(a+b\operatorname{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\operatorname{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\operatorname{arccosh}(c+dx))-\frac{1}{2}b\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{101}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\operatorname{arccosh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}\int \frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{43}$$

$$\frac{e(\frac{1}{2}(c+dx)^2(a+b\operatorname{arccosh}(c+dx))-\frac{1}{2}b(\frac{1}{2}\operatorname{arccosh}(c+dx)+\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)))}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e*(-1/2*(b*((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/2 + ArcCosh[
c + d*x]/2)) + ((c + d*x)^2*(a + b*ArcCosh[c + d*x]))/2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{ea(dx+c)^2}{2}+eb\left(\frac{(dx+c)^2\,\mathrm{arccosh}(dx+c)}{2}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left((dx+c)\sqrt{(dx+c)^2-1}+\ln\left(dx+c+\sqrt{(dx+c)^2-1}\right)\right)}{4\sqrt{(dx+c)^2-1}}\right)}{d}$
default	$\frac{\frac{ea(dx+c)^2}{2}+eb\left(\frac{(dx+c)^2\,\mathrm{arccosh}(dx+c)}{2}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left((dx+c)\sqrt{(dx+c)^2-1}+\ln\left(dx+c+\sqrt{(dx+c)^2-1}\right)\right)}{4\sqrt{(dx+c)^2-1}}\right)}{d}$
parts	$ea\left(\frac{1}{2}d\,x^2+cx\right)+\frac{eb\left(\frac{(dx+c)^2\,\mathrm{arccosh}(dx+c)}{2}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left((dx+c)\sqrt{(dx+c)^2-1}+\ln\left(dx+c+\sqrt{(dx+c)^2-1}\right)\right)}{4\sqrt{(dx+c)^2-1}}\right.}{d}$
orering	$\frac{(3d^2x^2+6cdx+3c^2-2)(dex+ce)(a+b\,\mathrm{arccosh}(dx+c))}{4(dx+c)d}-\frac{(dx+c-1)(dx+c+1)\left(de(a+b\,\mathrm{arccosh}(dx+c))+\frac{(dex+c}{\sqrt{dx+c-1}\,\sqrt{}}\right.}{4d^2}$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a*(d*x+c)^2+e*b*(1/2*(d*x+c)^2*arccosh(d*x+c)-1/4*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*((d*x+c)*((d*x+c)^2-1)^(1/2)+ln(d*x+c+((d*x+c)^2-1)^(1/
2)))/((d*x+c)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int (ce+dex)(a+b\mathrm{arccosh}(c+dx))\,dx$$
$$=\frac{2\,ad^2ex^2+4\,acdex+(2\,bd^2ex^2+4\,bcdex+(2\,bc^2-b)e)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-(bdex}{4\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
 - b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (b*d*e*x + b*c
*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))\,dx = e\left(\int ac\,dx + \int adx\,dx + \int bc\operatorname{acosh}(c + dx)\,dx + \int bdx\operatorname{acosh}(c + dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c)),x)
```

output
```
e*(Integral(a*c, x) + Integral(a*d*x, x) + Integral(b*c*acosh(c + d*x), x)
 + Integral(b*d*x*acosh(c + d*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(65) = 130$.

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.71

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))\,dx = \frac{1}{2}adex^2 + \frac{1}{4}\left(2x^2\operatorname{arcosh}(dx + c) - d\left(\frac{3c^2\log\left(2d^2x + 2cd + 2\sqrt{d^2x^2 + 2cdx + c^2 - 1}d\right)}{d^3} + \frac{\sqrt{d^2x^2 + 2cdx +}}{d^2}\right.\right. + acex + \frac{\left((dx + c)\operatorname{arcosh}(dx + c) - \sqrt{(dx + c)^2 - 1}\right)bce}{d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*d*e*x^2 + 1/4*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d
 + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*d*e + a*c
*e*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c*e/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(65) = 130$.

Time = 0.36 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.27

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))\,dx = \frac{1}{2}adex^2$$
$$-\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}}{d^2}\right) - x\log\left(dx + c\right.\right.$$
$$+\frac{1}{4}\left(2\,x^2\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - \left(\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\left(\frac{x}{d^2} - \frac{3\,c}{d^3}\right) - \frac{(2\,c^2 + 1)}{}\right.\right.$$
$$+ acex$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*a*d*e*x^2 - (d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c*e + 1/4*(2*x^2*log(
d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*d*e + a*c*e*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))\,dx = \int (c\,e + d\,e\,x)\,\left(a + b\,\operatorname{acosh}(c + d\,x)\right)\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.41

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))\,dx$$
$$= \frac{e\left(4acosh(dx + c)\,b\,c^2 + 4acosh(dx + c)\,bcdx + 2acosh(dx + c)\,b\,d^2x^2 + 3\sqrt{d^2x^2 + 2cdx + c^2 - 1}\,bc - \sqrt{\right.}{}$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c)),x)
```

output
```
(e*(4*acosh(c + d*x)*b*c**2 + 4*acosh(c + d*x)*b*c*d*x + 2*acosh(c + d*x)*
b*d**2*x**2 + 3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*b*c - sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*b*d*x - 4*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*b*c -
2*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*b*c**2 - log(sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*b + 4*a*c*d*x + 2*a*d**2*x**2))/(
4*d)
```

3.15 $\int (a+b\,\text{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a+b\,\text{arccosh}(c+dx))\,dx = ax-\frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d}+\frac{b(c+dx)\text{arccosh}(c+dx)}{d}$$

output
```
a*x-b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d+b*(d*x+c)*arccosh(d*x+c)/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int (a+b\,\text{arccosh}(c+dx))\,dx = ax+bx\,\text{arccosh}(c+dx)-\frac{b\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}-2c\,\text{arctanh}\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}\right)\right)}{d}$$

input
```
Integrate[a + b*ArcCosh[c + d*x],x]
```

output
```
a*x + b*x*ArcCosh[c + d*x] - (b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*
c*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\text{arccosh}(c + dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax + \frac{b(c + dx)\text{arccosh}(c + dx)}{d} - \frac{b\sqrt{c + dx - 1}\sqrt{c + dx + 1}}{d}$$

input
```
Int[a + b*ArcCosh[c + d*x],x]
```

output
```
a*x - (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (b*(c + d*x)*ArcCosh[c
+ d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
default	$ax+\frac{b\left((dx+c)\operatorname{arccosh}(dx+c)-\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)}{d}$	41
parts	$ax+\frac{b\left((dx+c)\operatorname{arccosh}(dx+c)-\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)}{d}$	41
orering	$\frac{(dx+c)(a+b\ \operatorname{arccosh}(dx+c))}{d}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{d}$	43
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\operatorname{arccosh}(dx+c)-\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)}{d}$	46

input
```
int(a+b*arccosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int (a+b\operatorname{arccosh}(c+dx))\,dx = \frac{adx+(bdx+bc)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-\sqrt{d^2x^2+2\,cdx+c^2-1}b}{d}$$

input
```
integrate(a+b*arccosh(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) -
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b)/d
```

Sympy [F]

$$\int (a+b\text{arccosh}(c+dx))\,dx=\int (a+b\operatorname{acosh}(c+dx))\ dx$$

input
```
integrate(a+b*acosh(d*x+c),x)
```

output
```
Integral(a + b*acosh(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int (a+b\text{arccosh}(c+dx))\,dx=ax+\frac{\left((dx+c)\operatorname{arcosh}(dx+c)-\sqrt{(dx+c)^2-1}\right)b}{d}$$

input
```
integrate(a+b*arccosh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(42) = 84.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.17

$$\int (a+b\text{arccosh}(c+dx))\,dx= -\left(d\left(\frac{c\log\left(\left|-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2-1}\right)|d|\right|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}}{d^2}\right)-x\log\left(dx+c\right.\right.$$
$$+ax$$

input
```
integrate(a+b*arccosh(d*x+c),x, algorithm="giac")
```

output
```
-(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d
)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c +
sqrt((d*x + c)^2 - 1)))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.91

$$\int (a+b\text{arccosh}(c+dx))\,dx$$
$$= a\,x+b\,x\,\text{acosh}(c+d\,x)$$
$$-\frac{b\left(\frac{4\,c\,\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}{d\,\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}+\frac{4\,c\,{\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}^3}{d\,{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}^3}-\frac{8\,{\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}^2\,\sqrt{c-1}\,\sqrt{c+1}}{d\,{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}^2}\right)}{\frac{{\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}^4}{{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}^4}-\frac{2\,{\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}^2}{{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}^2}+1}$$
$$+\frac{4\,b\,c\,\text{atanh}\left(\frac{\sqrt{c-1}-\sqrt{c+d\,x-1}}{\sqrt{c+1}-\sqrt{c+d\,x+1}}\right)}{d}$$

input
```
int(a + b*acosh(c + d*x),x)
```

output
```
a*x + b*x*acosh(c + d*x) - (b*((4*c*((c - 1)^(1/2) - (c + d*x - 1)^(1/2)))
/(d*((c + 1)^(1/2) - (c + d*x + 1)^(1/2))) + (4*c*((c - 1)^(1/2) - (c + d*
x - 1)^(1/2))^3)/(d*((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^3) - (8*((c - 1)
^(1/2) - (c + d*x - 1)^(1/2))^2*(c - 1)^(1/2)*(c + 1)^(1/2))/(d*((c + 1)^(
1/2) - (c + d*x + 1)^(1/2))^2)))/(((c - 1)^(1/2) - (c + d*x - 1)^(1/2))^4/
((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^4 - (2*((c - 1)^(1/2) - (c + d*x - 1
)^(1/2))^2)/((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^2 + 1) + (4*b*c*atanh(((
c - 1)^(1/2) - (c + d*x - 1)^(1/2))/((c + 1)^(1/2) - (c + d*x + 1)^(1/2)))
)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int (a+b\text{arccosh}(c+dx))\,dx$$
$$=\frac{acosh(dx+c)\,bc+acosh(dx+c)\,bdx-\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,b+adx}{d}$$

input
```
int(a+b*acosh(d*x+c),x)
```

output
```
(acosh(c + d*x)*b*c + acosh(c + d*x)*b*d*x - sqrt(c + d*x + 1)*sqrt(c + d*
x - 1)*b + a*d*x)/d
```

3.16 $\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}\,dx=-\frac{(a+b\text{arccosh}(c+dx))^2}{2bde}+\frac{(a+b\text{arccosh}(c+dx))\log\left(1+e^{2\text{arccosh}(c+dx)}\right)}{de}+\frac{b\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(c+dx)}\right)}{2de}$$

output
```
-1/2*(a+b*arccosh(d*x+c))^2/b/d/e+(a+b*arccosh(d*x+c))*ln(1+(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e+1/2*b*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2))^2)/d/e
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}\,dx=\frac{b\text{arccosh}(c+dx)^2+2b\text{arccosh}(c+dx)\log\left(1+e^{-2\text{arccosh}(c+dx)}\right)+2a\log(c+dx)-b\,\text{PolyLog}\left(2,-e^{-2\text{ar}}\right.}{2de}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

output
```
(b*ArcCosh[c + d*x]^2 + 2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x
])] + 2*a*Log[c + d*x] - b*PolyLog[2, -E^(-2*ArcCosh[c + d*x])])/(2*d*e)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6411, 27, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{c+dx}d(c+dx)}{de}$$
$$\Big\downarrow \text{6297}$$
$$\frac{\int -\left((a+b\text{arccosh}(c+dx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)d(a+b\text{arccosh}(c+dx))}{bde}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int (a+b\text{arccosh}(c+dx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int -i(a+b\text{arccosh}(c+dx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \; 26$$

$$\frac{i\int (a+b\text{arccosh}(c+dx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \; 4201$$

$$\frac{i\left(2i\int \frac{e^{\frac{2(a-c-dx)}{b}}(a+b\text{arccosh}(c+dx))}{1+e^{\frac{2(a-c-dx)}{b}}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i(a+b\text{arccosh}(c+dx))^2\right)}{bde}$$

$$\Big\downarrow \; 2620$$

$$\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}(c+dx))-\frac{1}{2}b(a+b\text{arccosh}(c+dx))\log\left(e^{\frac{2(a-c-dx)}{b}}+1\right)\right)-\frac{1}{2}i(a\right.}{bde}$$

$$\Big\downarrow \; 2715$$

$$\frac{i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2(a-c-dx)}{b}}\log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)de^{\frac{2(a-c-dx)}{b}}-\frac{1}{2}b(a+b\text{arccosh}(c+dx))\log\left(e^{\frac{2(a-c-dx)}{b}}+1\right)\right)-\frac{1}{2}i(\right.}{bde}$$

$$\Big\downarrow \; 2838$$

$$\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-c-dx)-\frac{1}{2}b(a+b\text{arccosh}(c+dx))\log\left(e^{\frac{2(a-c-dx)}{b}}+1\right)\right)-\frac{1}{2}i(a+b\text{arccosh}(c+dx))^2\right)}{bde}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

output
```
(I*((-1/2*I)*(a + b*ArcCosh[c + d*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c
+ d*x])*Log[1 + E^((2*(a - c - d*x))/b)]) + (b^2*PolyLog[2, -c - d*x])/4))
)/(b*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)^2}{2}+\operatorname{arccosh}(dx+c)\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+\right.}{2}\right.}{e}}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)^2}{2}+\operatorname{arccosh}(dx+c)\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+\right.}{2}\right.}{e}}{d}$
parts	$\frac{a\ln(dx+c)}{ed}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)^2}{2}+\operatorname{arccosh}(dx+c)\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-(dx+c+\sqrt{dx+\right.}{2}\right.}{ed}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a/e*ln(d*x+c)+b/e*(-1/2*arccosh(d*x+c)^2+arccosh(d*x+c)*ln(1+(d*x+c+(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\text{arcosh}\left(dx+c\right)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}\,dx=\frac{\int \frac{a}{c+dx}\,dx+\int \frac{b\,\text{acosh}\,(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e),x)
```

output
```
(Integral(a/(c + d*x), x) + Integral(b*acosh(c + d*x)/(c + d*x), x))/e
```

Maxima [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\text{arcosh}\left(dx+c\right)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
b*integrate(log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d*e*x + c*
e), x) + a*log(d*e*x + c*e)/(d*e)
```

Giac [F]

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{ce + dex}\, dx = \int \frac{b\operatorname{arcosh}(dx + c) + a}{dex + ce}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{ce + dex}\, dx = \int \frac{a + b\operatorname{acosh}(c + d\,x)}{c\,e + d\,e\,x}\, dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x), x)
```

Reduce [F]

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{ce + dex}\, dx = \frac{\left(\int \frac{a\cosh(dx+c)}{dx+c} dx\right) bd + \log(dx + c)\, a}{de}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e),x)
```

output
```
(int(acosh(c + d*x)/(c + d*x),x)*b*d + log(c + d*x)*a)/(d*e)
```

3.17 $\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^2}\,dx=-\frac{a+b\text{arccosh}(c+dx)}{de^2(c+dx)}+\frac{b\arctan\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{de^2}$$

output
```
-(a+b*arccosh(d*x+c))/d/e^2/(d*x+c)+b*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2))/d/e^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^2}\,dx=\frac{\frac{-a-b\text{arccosh}(c+dx)}{c+dx}+\frac{b\sqrt{-1+(c+dx)^2}\arctan\left(\sqrt{-1+(c+dx)^2}\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}}{de^2}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
((-a - b*ArcCosh[c + d*x])/(c + d*x) + (b*Sqrt[-1 + (c + d*x)^2]*ArcTan[Sq
rt[-1 + (c + d*x)^2]])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(d*e^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6411, 27, 6298, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{b\int \frac{1}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{103}$$

$$\frac{b\int \frac{1}{(c+dx-1)(c+dx+1)+1}d\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\arctan\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}}{de^2}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcCosh[c + d*x])/(c + d*x)) + b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]])/(d*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{dx+c}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{\sqrt{(dx+c)^2-1}}\right)}{e^2}}{d}$	81
default	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{dx+c}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{\sqrt{(dx+c)^2-1}}\right)}{e^2}}{d}$	81
parts	$-\frac{a}{e^2(dx+c)d}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{dx+c}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{\sqrt{(dx+c)^2-1}}\right)}{e^2d}$	83

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(52) = 104.

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.38

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^2}\,dx$$
$$=\frac{bdx\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-ac+2\left(bcdx+bc^2\right)\arctan\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2}\right.}{cd^2e^2x+c^2de^2}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
(b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - a*c + 2*(b*c*d*x
 + b*c^2)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (b*d*x +
b*c)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))/(c*d^2*e^2*x + c^2
*d*e^2)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^2}\,dx = \frac{\int \frac{a}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*acosh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^2} dx = \int \frac{a + b\,\text{acosh}(c + d\,x)}{(c\,e + d\,e\,x)^2} \,dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^2, x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^2} dx = \frac{\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right) b\,c^2 + \left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right) bcdx + ax}{c\,e^2\,(dx + c)}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^2,x)
```

output
```
(int(acosh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c**2 + int(acosh(c +
 d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c*d*x + a*x)/(c*e**2*(c + d*x))
```

3.18 $\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx=\frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2de^3(c+dx)}-\frac{a+b\text{arccosh}(c+dx)}{2de^3(c+dx)^2}$$

output
```
1/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^3/(d*x+c)-1/2*(a+b*arccosh(d*x+c
))/d/e^3/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx = -\frac{a-b\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}+b\text{arccosh}(c+dx)}{2de^3(c+dx)^2}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(a - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] + b*ArcCosh[c +
 d*x])/(d*e^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6411, 27, 6298, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3} \, dx$$
$$\Big\downarrow \; 6411$$
$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{e^3(c+dx)^3}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(c+dx)^3}d(c+dx)}{de^3}$$
$$\Big\downarrow \; 6298$$
$$\frac{\frac{1}{2}b\int \frac{1}{\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}d(c+dx)-\frac{a+b\text{arccosh}(c+dx)}{2(c+dx)^2}}{de^3}$$
$$\Big\downarrow \; 106$$
$$\frac{\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)}-\frac{a+b\text{arccosh}(c+dx)}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*(c + d*x)) - (a + b*ArcCosh[c
 + d*x])/(2*(c + d*x)^2))/(d*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{2(dx+c)^2}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2dx+2c}\right)}{e^3}}{d}$
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{2(dx+c)^2}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2dx+2c}\right)}{e^3}}{d}$
parts	$-\frac{a}{2e^3(dx+c)^2d}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{2(dx+c)^2}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2dx+2c}\right)}{e^3d}$
orering	$\frac{(dx+c)(3d^2x^2+6cdx+3c^2-4)(a+b\operatorname{arccosh}(dx+c))}{2d(dex+ce)^3}+\frac{(dx+c-1)(dx+c+1)(dx+c)^2\left(\frac{bd}{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dex+ce)^3}-\right.}{2d^2}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx = \frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+(bc^2dx+bc^3)\sqrt{d^2x^2+2\,cdx+c^2-1}}{2\,(c^2d^3e^3x^2+2\,c^3d^2e^3x+c^4de^3)}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)) + (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^2*d^
3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx = \frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b\,\text{acosh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral
(b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**
3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.79

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^3}\,dx$$
$$= \frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}d}{d^3e^3x + cd^2e^3} - \frac{\operatorname{arcosh}\left(dx + c\right)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3}\right)$$
$$- \frac{a}{2\left(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3\right)}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*d^2*e^3) - arcco
sh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x^
2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^3}\,dx = \int \frac{b\operatorname{arcosh}\left(dx + c\right) + a}{\left(dex + ce\right)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx=\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^3}\,dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^3}\,dx = \frac{2\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b\,c^2d+4\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)bc\,d^2x+2\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b\,d}{2d\,e^3\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^3,x)
```

output
```
(2*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b
*c**2*d + 4*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3),x)*b*c*d**2*x + 2*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3),x)*b*d**3*x**2 - a)/(2*d*e**3*(c**2 + 2*c*d*x + d**2*x**2)
)
```

3.19 $\int \frac{a+b\,\mathrm{arccosh}(c+dx)}{(ce+dex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{a+b\,\mathrm{arccosh}(c+dx)}{(ce+dex)^4}\,dx = \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\,\mathrm{arccosh}(c+dx)}{3de^4(c+dx)^3} + \frac{b\arctan\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{6de^4}$$

output
```
1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2-1/3*(a+b*arccosh(d*x
+c))/d/e^4/(d*x+c)^3+1/6*b*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16

$$\int \frac{a+b\,\mathrm{arccosh}(c+dx)}{(ce+dex)^4}\,dx = \frac{-\frac{2a}{(c+dx)^3} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{(c+dx)^2} - \frac{2b\,\mathrm{arccosh}(c+dx)}{(c+dx)^3} + \frac{b\sqrt{-1+(c+dx)^2}\arctan\left(\sqrt{-1+(c+dx)^2}\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}}{6de^4}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

output
```
((-2*a)/(c + d*x)^3 + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c + d*x)^2
 - (2*b*ArcCosh[c + d*x])/(c + d*x)^3 + (b*Sqrt[-1 + (c + d*x)^2]*ArcTan[S
qrt[-1 + (c + d*x)^2]])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(6*d*e^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6411, 27, 6298, 114, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^4}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{1}{3}b\int \frac{1}{\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}}d(c+dx)-\frac{a+b\text{arccosh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{114}$$

$$\frac{\frac{1}{3}b\left(\frac{1}{2}\int \frac{1}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)^2}\right)-\frac{a+b\text{arccosh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{103}$$

$$\frac{\frac{1}{3}b\left(\frac{1}{2}\int \frac{1}{(c+dx-1)(c+dx+1)+1}d\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)^2}\right)-\frac{a+b\text{arccosh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{3}b\left(\frac{1}{2}\arctan\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)^2}\right)-\frac{a+b\text{arccosh}(c+dx)}{3(c+dx)^3}}{de^4}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcCosh[c + d*x])/(c + d*x)^3 + (b*((Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x])/(2*(c + d*x)^2) + ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x
]]/2))/3)/(d*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left(\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)(dx+c)^2-\sqrt{(dx+c)^2-1}\right)}{6(dx+c)^2\sqrt{(dx+c)^2-1}}\right)}{e^4}}{d}$	110
default	$\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left(\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)(dx+c)^2-\sqrt{(dx+c)^2-1}\right)}{6(dx+c)^2\sqrt{(dx+c)^2-1}}\right)}{e^4}}{d}$	110
parts	$-\frac{a}{3e^4(dx+c)^3d}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left(\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)(dx+c)^2-\sqrt{(dx+c)^2-1}\right)}{6(dx+c)^2\sqrt{(dx+c)^2-1}}\right)}{e^4d}$	112

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a/e^4/(d*x+c)^3+b/e^4*(-1/3/(d*x+c)^3*arccosh(d*x+c)-1/6*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*(arctan(1/((d*x+c)^2-1)^(1/2))*(d*x+c)^2-((d*x+c)
^2-1)^(1/2))/(d*x+c)^2/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(85) = 170$.

Time = 0.13 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.79

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^4}\,dx = -\frac{2\,ac^3 - 2\left(bc^3d^3x^3 + 3\,bc^4d^2x^2 + 3\,bc^5dx + bc^6\right)\arctan\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - 2\,(bd^3x^3}{}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a*c^3 - 2*(b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x + b*c^6)*
arctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)) - (b*c^3*d*x + b*c^4)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1))/(c^3*d^4*e^4*x^3 + 3*c^4*d^3*e^4*x^2 + 3*c^5*d^2*e^4*x + c^6*
d*e^4)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^4}\,dx = \frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x
**4), x) + Integral(b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2
 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Maxima [F]

$$\int \frac{a + b\,\text{arccosh}(c + dx)}{(ce + dex)^4}\,dx = \int \frac{b\,\text{arcosh}\,(dx + c) + a}{(dex + ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
1/6*b*((2*d^2*x^2 + 4*c*d*x + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3)*log(d*x + c + 1) + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*log(d*x
 + c - 1) - 2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^4*e^4
*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 6*integrate(1/3/(d
^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e
^4)*x^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2
*e^4)*x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x + (d^5*e^4*x^5 + 5*c*d^4*e^4*x
^4 + c^5*e^4 - c^3*e^4 + (10*c^2*d^3*e^4 - d^3*e^4)*x^3 + (10*c^3*d^2*e^4
- 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 - 3*c^2*d*e^4)*x)*e^(1/2*log(d*x + c + 1
) + 1/2*log(d*x + c - 1))), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3
*c^2*d^2*e^4*x + c^3*d*e^4)
```

Giac [F]

$$\int \frac{a + b\,\text{arccosh}(c + dx)}{(ce + dex)^4}\,dx = \int \frac{b\,\text{arcosh}\,(dx + c) + a}{(dex + ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^4}\,dx=\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^4, x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^4}\,dx$$
$$=\frac{3\left(\int \frac{acosh(dx+c)}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)b\,c^3d+9\left(\int \frac{acosh(dx+c)}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)b\,c^2d^2x+9\left(\int \frac{aco}{d^4x^4+4c\,d^3x^3}\right.}{3d\,e^4\left(d^3x^3+3c\,d^2x^2+3c^2dx+\right.}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^4,x)
```

output
```
(3*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**
3 + d**4*x**4),x)*b*c**3*d + 9*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c
**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b*c**2*d**2*x + 9*int(acosh(
c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4
),x)*b*c*d**3*x**2 + 3*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2
*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b*d**4*x**3 - a)/(3*d*e**4*(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))
```

3.20 $\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^5}\,dx = \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{12de^5(c+dx)^3} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6de^5(c+dx)} - \frac{a+b\text{arccosh}(c+dx)}{4de^5(c+dx)^4}$$

output
```
1/12*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^5/(d*x+c)^3+1/6*b*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/d/e^5/(d*x+c)-1/4*(a+b*arccosh(d*x+c))/d/e^5/(d*x+c)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^5}\,dx = \frac{-3a+b\sqrt{-1+c+dx}\sqrt{1+c+dx}(c+2c^3+dx+6c^2dx+6cd^2x^2+2d^3x^3)-3b\text{arccosh}(c+dx)}{12de^5(c+dx)^4}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

output
```
(-3*a + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(c + 2*c^3 + d*x + 6*c^2*d*
x + 6*c*d^2*x^2 + 2*d^3*x^3) - 3*b*ArcCosh[c + d*x])/(12*d*e^5*(c + d*x)^4
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6411, 27, 6298, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^5}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{e^5(c+dx)^5}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(c+dx)^5}d(c+dx)}{de^5}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{1}{4}b\int \frac{1}{\sqrt{c+dx-1}(c+dx)^4\sqrt{c+dx+1}}d(c+dx)-\frac{a+b\text{arccosh}(c+dx)}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{114}$$

$$\frac{\frac{1}{4}b\left(\frac{1}{3}\int \frac{2}{\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3(c+dx)^3}\right)-\frac{a+b\text{arccosh}(c+dx)}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}b\left(\frac{2}{3}\int \frac{1}{\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3(c+dx)^3}\right)-\frac{a+b\text{arccosh}(c+dx)}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{106}$$

$$\frac{\frac{1}{4}b\left(\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3(c+dx)}+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3(c+dx)^3}\right)-\frac{a+b\text{arccosh}(c+dx)}{4(c+dx)^4}}{de^5}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

output
```
((b*((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*(c + d*x)^3) + (2*Sqrt[-1 +
 c + d*x]*Sqrt[1 + c + d*x])/(3*(c + d*x))))/4 - (a + b*ArcCosh[c + d*x])/
(4*(c + d*x)^4))/(d*e^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^2+1\right)}{12(dx+c)^3}\right)}{e^5}}{d}$
default	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^2+1\right)}{12(dx+c)^3}\right)}{e^5}}{d}$
parts	$-\frac{a}{4e^5(dx+c)^4d}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^2+1\right)}{12(dx+c)^3}\right)}{e^5d}$
orering	$\frac{(dx+c)\left(10d^4x^4+40c\,d^3x^3+60c^2d^2x^2+40c^3dx+10c^4-5d^2x^2-10cdx-5c^2-8\right)(a+b\operatorname{arccosh}(dx+c))}{12d(dex+ce)^5}+\frac{(2d^2x^2+4cdx}{}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arccosh(d*x+c)+1/12*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^2+1)/(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(90) = 180$.

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.00

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^5}\,dx = \frac{3\,ad^4x^4+12\,acd^3x^3+18\,ac^2d^2x^2+12\,ac^3dx-3\,bc^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\left(2\,bc^4d^3x^3\right.}{12\left(c^4d^5e^5x^4+4\,c^5d^4e^5x^3+6\,c^6d^3e^5x^2+4\,c^7d^2e\right.}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

output
```
1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b
*c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (2*b*c^4*d^3*x^3 +
 6*b*c^5*d^2*x^2 + 2*b*c^7 + b*c^5 + (6*b*c^6 + b*c^4)*d*x)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*
x^2 + 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^5}\,dx = \frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**5,x)
```

output
```
(Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5
*c*d**4*x**4 + d**5*x**5), x) + Integral(b*acosh(c + d*x)/(c**5 + 5*c**4*d
*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x
))/e**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(90) = 180$.

Time = 0.06 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.50

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^5}\,dx = \frac{1}{12}b\left(\frac{(2\,d^4x^4 + 8\,cd^3x^3 + 2\,c^4 + (12\,c^2d^2 - d^2)x^2 - c^2 + 2\,(4\,c^3d - cd)x - 1)d}{(d^5e^5x^3 + 3\,cd^4e^5x^2 + 3\,c^2d^3e^5x + c^3d^2e^5)\sqrt{dx + c + 1}\sqrt{dx + c - 1}} - \frac{3\operatorname{arc}}{d^5e^5x^4 + 4\,cd^4e^5x^3 + 6} \right. - \frac{a}{4\,(d^5e^5x^4 + 4\,cd^4e^5x^3 + 6\,c^2d^3e^5x^2 + 4\,c^3d^2e^5x + c^4de^5)}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

output
```
1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 - d^2)*x^2 - c^2 +
2*(4*c^3*d - c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5
*x + c^3*d^2*e^5)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)) - 3*arccosh(d*x + c
)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c
^4*d*e^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*
c^3*d^2*e^5*x + c^4*d*e^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^5}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^5}\, dx = \int \frac{a + b\,\text{acosh}(c + d\,x)}{(c\,e + d\,e\,x)^5}\, dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^5,x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^5, x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^5}\,dx$$
$$= \frac{4\left(\int \frac{acosh(dx+c)}{d^5x^5+5c\,d^4x^4+10c^2d^3x^3+10c^3d^2x^2+5c^4dx+c^5}dx\right)b\,c^4d + 16\left(\int \frac{acosh(dx+c)}{d^5x^5+5c\,d^4x^4+10c^2d^3x^3+10c^3d^2x^2+5c^4dx+c^5}dx\right)b\,c^3d^2}{}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^5,x)
```

output
```
(4*int(acosh(c + d*x)/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**
3*x**3 + 5*c*d**4*x**4 + d**5*x**5),x)*b*c**4*d + 16*int(acosh(c + d*x)/(c
**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 +
 d**5*x**5),x)*b*c**3*d**2*x + 24*int(acosh(c + d*x)/(c**5 + 5*c**4*d*x +
10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5),x)*b*c*
*2*d**3*x**2 + 16*int(acosh(c + d*x)/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**
2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5),x)*b*c*d**4*x**3 + 4*in
t(acosh(c + d*x)/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**
3 + 5*c*d**4*x**4 + d**5*x**5),x)*b*d**5*x**4 - a)/(4*d*e**5*(c**4 + 4*c**
3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4))
```

3.21 $\int \frac{a+b\,\mathrm{arccosh}(c+dx)}{(ce+dex)^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{a+b\,\mathrm{arccosh}(c+dx)}{(ce+dex)^6}\,dx = \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\,\mathrm{arccosh}(c+dx)}{5de^6(c+dx)^5} + \frac{3b\arctan\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{40de^6}$$

output
```
1/20*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^4+3/40*b*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^2-1/5*(a+b*arccosh(d*x+c))/d/e^6/(d*x+c)
^5+3/40*b*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{a+b\,\mathrm{arccosh}(c+dx)}{(ce+dex)^6}\,dx = \frac{-\frac{8a}{(c+dx)^5} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{(c+dx)^2} - \frac{8b\,\mathrm{arccosh}(c+dx)}{(c+dx)^5} + \frac{3b\sqrt{-1+(c+dx)^2}\arctan\left(\sqrt{-1+(c+dx)^2}\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}}{40de^6}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

output
```
((-8*a)/(c + d*x)^5 + (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c + d*x)
^4 + (3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c + d*x)^2 - (8*b*ArcCosh
[c + d*x])/(c + d*x)^5 + (3*b*Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c +
 d*x)^2]])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(40*d*e^6)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6411, 27, 6298, 114, 27, 114, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^6}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{e^6(c+dx)^6}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(c+dx)^6}d(c+dx)}{de^6}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{1}{5}b\int \frac{1}{\sqrt{c+dx-1}(c+dx)^5\sqrt{c+dx+1}}d(c+dx)-\frac{a+b\text{arccosh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \text{114}$$

$$\frac{\frac{1}{5}b\left(\frac{1}{4}\int \frac{3}{\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{4(c+dx)^4}\right)-\frac{a+b\text{arccosh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}b\left(\frac{3}{4}\int \frac{1}{\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{4(c+dx)^4}\right)-\frac{a+b\text{arccosh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \text{114}$$

$$\frac{\frac{1}{5}b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)^2}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{4(c+dx)^4}\right)-\frac{a+b\text{arccosh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \text{103}$$

$$\frac{\frac{1}{5}b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{(c+dx-1)(c+dx+1)+1}d\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)^2}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{4(c+dx)^4}\right)-\frac{a+b\text{arccos}}{5(c+d}}{de^6}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{5}b\left(\frac{3}{4}\left(\frac{1}{2}\arctan\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2(c+dx)^2}\right)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{4(c+dx)^4}\right)-\frac{a+b\text{arccosh}(c+dx)}{5(c+dx)^5}}{de^6}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

output
```
(-1/5*(a + b*ArcCosh[c + d*x])/(c + d*x)^5 + (b*((Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x])/(4*(c + d*x)^4) + (3*((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])
/(2*(c + d*x)^2) + ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]/2))/4))/5)
/(d*e^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left(3\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)(dx+c)^4-3(dx+c)^2\sqrt{(dx+c)^2-1}-2\sqrt{(dx+}\right.}{40\sqrt{(dx+c)^2-1}\,(dx+c)^4}\right.}{e^6}}{d}$
default	$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left(3\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)(dx+c)^4-3(dx+c)^2\sqrt{(dx+c)^2-1}-2\sqrt{(dx+}\right.}{40\sqrt{(dx+c)^2-1}\,(dx+c)^4}\right.}{e^6}}{d}$
parts	$-\frac{a}{5e^6(dx+c)^5d}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\left(3\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)(dx+c)^4-3(dx+c)^2\sqrt{(dx+c)^2-1}-2\right.}{40\sqrt{(dx+c)^2-1}\,(dx+c)^4}\right.}{e^6d}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*a/e^6/(d*x+c)^5+b/e^6*(-1/5/(d*x+c)^5*arccosh(d*x+c)-1/40*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*(3*arctan(1/((d*x+c)^2-1)^(1/2))*(d*x+c)^4-3*(d*
x+c)^2*((d*x+c)^2-1)^(1/2)-2*((d*x+c)^2-1)^(1/2))/((d*x+c)^2-1)^(1/2)/(d*x
+c)^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(117) = 234$.

Time = 0.16 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.04

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^6}\,dx =$$
$$-\frac{8\,ac^5-6\left(bc^5d^5x^5+5\,bc^6d^4x^4+10\,bc^7d^3x^3+10\,bc^8d^2x^2+5\,bc^9dx+bc^{10}\right)\arctan\left(-dx-c+\sqrt{d^2x^2-}\right.}{}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

output
```
-1/40*(8*a*c^5 - 6*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 1
0*b*c^8*d^2*x^2 + 5*b*c^9*d*x + b*c^10)*arctan(-d*x - c + sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)) - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*
b*c^3*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)) - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 +
 5*b*c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) -
(3*b*c^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 + 2*b*c^6 + (9*b*c^7 + 2*b*c^
5)*d*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^5*d^6*e^6*x^5 + 5*c^6*d^5*e^
6*x^4 + 10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x + c^10*d
*e^6)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^6}\,dx = \frac{\int \frac{a}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx}{e^6}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**6,x)
```

output
```
(Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 1
5*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*acosh(c + d
*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d
**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

Maxima [F]

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^6}\,dx = \int \frac{b\operatorname{arcosh}(dx+c)+a}{(dex+ce)^6}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

output
```
1/30*b*((6*d^4*x^4 + 24*c*d^3*x^3 + 6*c^4 + 2*(18*c^2*d^2 + d^2)*x^2 + 2*c
^2 + 4*(6*c^3*d + c*d)*x - 3*(d^5*x^5 + 5*c*d^4*x^4 + 10*c^2*d^3*x^3 + 10*
c^3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c + 1) + 3*(d^5*x^5 + 5*c*d^4*x^4
 + 10*c^2*d^3*x^3 + 10*c^3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c - 1) - 6
*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^6*e^6*x^5 + 5*c*d^
5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 + 5*c^4*d^2*e^6*x + c^
5*d*e^6) - 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*e^6*x^7 + c^8*e^6 - c^6
*e^6 + (28*c^2*d^6*e^6 - d^6*e^6)*x^6 + 2*(28*c^3*d^5*e^6 - 3*c*d^5*e^6)*x
^5 + 5*(14*c^4*d^4*e^6 - 3*c^2*d^4*e^6)*x^4 + 4*(14*c^5*d^3*e^6 - 5*c^3*d^
3*e^6)*x^3 + (28*c^6*d^2*e^6 - 15*c^4*d^2*e^6)*x^2 + 2*(4*c^7*d*e^6 - 3*c^
5*d*e^6)*x + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e^6 - c^5*e^6 + (21*c^2*
d^5*e^6 - d^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 - c*d^4*e^6)*x^4 + 5*(7*c^4*d^3*
e^6 - 2*c^2*d^3*e^6)*x^3 + (21*c^5*d^2*e^6 - 10*c^3*d^2*e^6)*x^2 + (7*c^6*
d*e^6 - 5*c^4*d*e^6)*x)*e^(1/2*log(d*x + c + 1) + 1/2*log(d*x + c - 1))),
x)) - 1/5*a/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d
^3*e^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^6)
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^6} \, dx = \int \frac{b \operatorname{arcosh}(dx + c) + a}{(dex + ce)^6} \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^6} \, dx = \int \frac{a + b \operatorname{acosh}(c + d\,x)}{(c\,e + d\,e\,x)^6} \, dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^6,x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^6, x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^6} \, dx$$
$$= \frac{5\left(\int \frac{acosh(dx+c)}{d^6x^6+6c\,d^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6}dx\right) b\,c^5d + 25\left(\int \frac{acosh(dx+c)}{d^6x^6+6c\,d^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c}\right.}{}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^6,x)
```

output
```
(5*int(acosh(c + d*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**
3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6),x)*b*c**5*d + 25*i
nt(acosh(c + d*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x*
*3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6),x)*b*c**4*d**2*x + 50*
int(acosh(c + d*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x
**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6),x)*b*c**3*d**3*x**2 +
 50*int(acosh(c + d*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d*
*3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6),x)*b*c**2*d**4*x*
*3 + 25*int(acosh(c + d*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**
3*d**3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6),x)*b*c*d**5*x
**4 + 5*int(acosh(c + d*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**
3*d**3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6),x)*b*d**6*x**
5 - a)/(5*d*e**6*(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**
3 + 5*c*d**4*x**4 + d**5*x**5))
```

3.22 $\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 218

$$\begin{aligned}\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^2\,dx &= \frac{16}{75}b^2e^4x+\frac{8b^2e^4(c+dx)^3}{225d}+\frac{2b^2e^4(c+dx)^5}{125d}\\ &\quad-\frac{16be^4\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{75d}\\ &\quad-\frac{8be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{75d}\\ &\quad-\frac{2be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{25d}\\ &\quad+\frac{e^4(c+dx)^5(a+b\text{arccosh}(c+dx))^2}{5d}\end{aligned}$$

output
```
16/75*b^2*e^4*x+8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d-16/75*
b*e^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-8/75*b*e^4*(d
*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-2/25*b*e^4*
(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d+1/5*e^4*(
d*x+c)^5*(a+b*arccosh(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.01

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^2\,dx$$
$$=\frac{e^4\left(240b^2(c+dx)+40b^2(c+dx)^3+9(25a^2+2b^2)\,(c+dx)^5+30ab\sqrt{-1+c+dx}\sqrt{1+c+dx}(-8-4(c+dx)^2-3(c+dx)^4)+\ldots\right)}{1125d}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^4*(240*b^2*(c + d*x) + 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x
)^5 + 30*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8 - 4*(c + d*x)^2 - 3*
(c + d*x)^4) + 30*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x] - 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 3*b*Sqrt[-
1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 225*b^2*(c
+ d*x)^5*ArcCosh[c + d*x]^2))/(1125*d)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6411, 27, 6298, 6354, 15, 6354, 15, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 6298$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b\int\frac{(c+dx)^5(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\big\downarrow$ 6354

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b\left(\frac{4}{5}\int\frac{(c+dx)^3(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{5}b\int(c+dx)^4d(c+dx)+\right.\right.}{d}$$

$\big\downarrow$ 15

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b\left(\frac{4}{5}\int\frac{(c+dx)^3(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+1}\right.\right.}{d}$$

$\big\downarrow$ 6354

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{3}b\int(c+dx)^2d(c+dx)\right.\right.\right.$$

$\big\downarrow$ 15

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+\right.\right.\right.$$

$\big\downarrow$ 6330

$$e^4(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b(\frac{4}{5}(\frac{2}{3}(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))-b\int 1d(c+d$$

$\big\downarrow$ 24

$$e^4(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b(\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4(a+b\text{arccosh}(c+dx))+\frac{4}{5}(\frac{1}{3}\sqrt{$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^4*(((c + d*x)^5*(a + b*ArcCosh[c + d*x])^2)/5 - (2*b*(-1/25*(b*(c + d*x
)^5) + (Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c
+ d*x]))/5 + (4*(-1/9*(b*(c + d*x)^3) + (Sqrt[-1 + c + d*x]*(c + d*x)^2*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/3 + (2*(-(b*(c + d*x)) + Sqrt[-1
 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])))/3))/5))/5))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{e^4a^2(dx+c)^5}{5}+e^4b^2\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^2}{5}-\frac{16\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{75}-\frac{2(dx+c)^4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx\cdots}}{25}\right.}{}$
default	$\frac{\frac{e^4a^2(dx+c)^5}{5}+e^4b^2\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^2}{5}-\frac{16\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{75}-\frac{2(dx+c)^4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx\cdots}}{25}\right.}{}$
parts	$\frac{e^4a^2(dx+c)^5}{5d}+\frac{e^4b^2\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^2}{5}-\frac{16\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{75}-\frac{2(dx+c)^4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\cdots}{25}\right.}{d}$
orering	$\frac{(549d^7x^7+3843c\,d^6x^6+11529c^2d^5x^5+19215c^3d^4x^4+19215c^4d^3x^3+200d^5x^5+11385c^5d^2x^2+1000c\,d^4x^4+3555c^6dx+\cdots}{}$

input
```
int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*e^4*a^2*(d*x+c)^5+e^4*b^2*(1/5*(d*x+c)^5*arccosh(d*x+c)^2-16/75*a
rccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/25*(d*x+c)^4*arccosh(d*x+c
)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-8/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)*(d*x+c)^2+16/75*d*x+16/75*c+2/125*(d*x+c)^5+8/225*(d*x+c)^3)+2
*e^4*a*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(192) = 384$.

Time = 0.10 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.83

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^2\,dx$$

$$= \frac{9\,(25\,a^2+2\,b^2)d^5e^4x^5+45\,(25\,a^2+2\,b^2)cd^4e^4x^4+10\,(9\,(25\,a^2+2\,b^2)c^2+4\,b^2)d^3e^4x^3+30\,(3\,(25\,a^2+2$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
 + 10*(9*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2
)*c^3 + 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16
*b^2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^
3*e^4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(
d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 30*(15*a*b*d^5*e^4*x^5 +
75*a*b*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 +
 75*a*b*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e
^4*x^3 + 2*(9*b^2*c^2 + 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*e^4
*x + (3*b^2*c^4 + 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 30*(3*a*b*d^4*e^4*x^
4 + 12*a*b*c*d^3*e^4*x^3 + 2*(9*a*b*c^2 + 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^
3 + 2*a*b*c)*d*e^4*x + (3*a*b*c^4 + 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$= e^4\left(\int a^2c^4\,dx + \int a^2d^4x^4\,dx + \int b^2c^4\operatorname{acosh}^2(c+dx)\,dx + \int 2abc^4\operatorname{acosh}(c+dx)\,dx + \int 4a^2cd^3x^3\,dx + \int 6a^2c^2d^2x^2\,dx + \int 4a^2c^3dx\,dx + \int b^2d^4x^4\operatorname{acosh}^2(c+dx)\,dx + \int 2abd^4x^4\operatorname{acosh}(c+dx)\,dx + \int 4b^2cd^3x^3\operatorname{acosh}^2(c+dx)\,dx + \int 6b^2c^2d^2x^2\operatorname{acosh}^2(c+dx)\,dx + \int 4b^2c^3dx\operatorname{acosh}^2(c+dx)\,dx + \int 8abcd^3x^3\operatorname{acosh}(c+dx)\,dx + \int 12abc^2d^2x^2\operatorname{acosh}(c+dx)\,dx + \int 8abc^3dx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**2,x)
```

output
```
e**4*(Integral(a**2*c**4, x) + Integral(a**2*d**4*x**4, x) + Integral(b**2
*c**4*acosh(c + d*x)**2, x) + Integral(2*a*b*c**4*acosh(c + d*x), x) + Int
egral(4*a**2*c*d**3*x**3, x) + Integral(6*a**2*c**2*d**2*x**2, x) + Integr
al(4*a**2*c**3*d*x, x) + Integral(b**2*d**4*x**4*acosh(c + d*x)**2, x) + I
ntegral(2*a*b*d**4*x**4*acosh(c + d*x), x) + Integral(4*b**2*c*d**3*x**3*a
cosh(c + d*x)**2, x) + Integral(6*b**2*c**2*d**2*x**2*acosh(c + d*x)**2, x
) + Integral(4*b**2*c**3*d*x*acosh(c + d*x)**2, x) + Integral(8*a*b*c*d**3
*x**3*acosh(c + d*x), x) + Integral(12*a*b*c**2*d**2*x**2*acosh(c + d*x),
x) + Integral(8*a*b*c**3*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^2\,dx=\int (dex+ce)^4(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*d^4*e^4*x^5 + a^2*c*d^3*e^4*x^4 + 2*a^2*c^2*d^2*e^4*x^3 + 2*a^2*c^
3*d*e^4*x^2 + 2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*c^3*d*e^4 +
 2/3*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/
d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/
d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^
2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 -
 1)/d^4))*a*b*c^2*d^2*e^4 + 1/12*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^
2/d^3 + 105*c^4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*
d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2
*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1
)*c/d^5)*d)*a*b*c*d^3*e^4 + 1/300*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1...
```

Giac [F]

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^2\,dx=\int (dex+ce)^4(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{acosh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$=\frac{e^4\left(75\left(\int acosh(dx+c)^2\,x^4dx\right)b^2d^5-16\sqrt{d^2x^2+2cdx+c^2-1}\,ab+15a^2d^5x^5+150acosh(dx+c)\,ab\,c^5\right.}{}$$

input
```
int((d*e*x+c*e)^4*(a+b*acosh(d*x+c))^2,x)
```

output
```
(e**4*(150*acosh(c + d*x)*a*b*c**5 + 150*acosh(c + d*x)*a*b*c**4*d*x + 300
*acosh(c + d*x)*a*b*c**3*d**2*x**2 + 300*acosh(c + d*x)*a*b*c**2*d**3*x**3
 + 150*acosh(c + d*x)*a*b*c*d**4*x**4 + 30*acosh(c + d*x)*a*b*d**5*x**5 +
144*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c**4 - 24*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*a*b*c**3*d*x - 36*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a
*b*c**2*d**2*x**2 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c**2 - 24*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c*d**3*x**3 - 16*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*a*b*c*d*x - 6*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b
*d**4*x**4 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*d**2*x**2 - 16*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b - 150*sqrt(c + d*x + 1)*sqrt(c + d*x
 - 1)*a*b*c**4 + 75*int(acosh(c + d*x)**2,x)*b**2*c**4*d + 75*int(acosh(c
+ d*x)**2*x**4,x)*b**2*d**5 + 300*int(acosh(c + d*x)**2*x**3,x)*b**2*c*d**
4 + 450*int(acosh(c + d*x)**2*x**2,x)*b**2*c**2*d**3 + 300*int(acosh(c + d
*x)**2*x,x)*b**2*c**3*d**2 - 120*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
+ c + d*x)*a*b*c**5 + 75*a**2*c**4*d*x + 150*a**2*c**3*d**2*x**2 + 150*a**
2*c**2*d**3*x**3 + 75*a**2*c*d**4*x**4 + 15*a**2*d**5*x**5))/(75*d)
```

3.23 $\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 186

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx \\
&= \frac{3b^2e^3(c+dx)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d} \\
&\quad -\frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{16d} \\
&\quad -\frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{8d} \\
&\quad -\frac{3e^3(a+b\text{arccosh}(c+dx))^2}{32d}+\frac{e^3(c+dx)^4(a+b\text{arccosh}(c+dx))^2}{4d}
\end{aligned}
$$

output

```
3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/16*b*e^3*(d*x+c-1)^(1/
2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-1/8*b*e^3*(d*x+c-1)^(1/2
)*(d*x+c)^3*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-3/32*e^3*(a+b*arccosh(d
*x+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.14

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx$$
$$= \frac{e^3\left(3b^2(c+dx)^2+(8a^2+b^2)(c+dx)^4+2ab\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(-3-2(c+dx)^2)+2b\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^3*(3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*Sqrt[-1 + c +
d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3 - 2*(c + d*x)^2) + 2*b*(c + d*x)*(8*a
*(c + d*x)^3 - 3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-3 + 8*(c +
d*x)^4)*ArcCosh[c + d*x]^2 - 6*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1
 + c + d*x]]))/(32*d)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6411, 27, 6298, 6354, 15, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int e^3(c+dx)^3(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^3\int (c+dx)^3(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$
$$\Big\downarrow \text{6298}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\int\frac{(c+dx)^4(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\Big\downarrow$ 6354

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{4}b\int(c+dx)^3d(c+dx)+\right.\right.}{d}$$

$\Big\downarrow$ 15

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\sqrt{c+dx-1}\sqrt{c+dx+1}\right.\right.}{d}$$

$\Big\downarrow$ 6354

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{2}b\int(c+dx)d(c+dx)+\frac{1}{2}\sqrt{c+}\right.\right.\right.}{}$$

$\Big\downarrow$ 15

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+\right.\right.\right.}{}$$

$\Big\downarrow$ 6308

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{1}{4}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3(a+b\text{arccosh}(c+dx))+\frac{3}{4}\left(\frac{1}{2}\sqrt{}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcCosh[c + d*x])^2)/4 - (b*(-1/16*(b*(c + d*x)^
4) + (Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
d*x]))/4 + (3*(-1/4*(b*(c + d*x)^2) + (Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x]))/2 + (a + b*ArcCosh[c + d*x])^2/(4*b)
))/4))/2))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\,\mathrm{arccosh}(dx+c)^2}{4}-\frac{(dx+c)^3\,\mathrm{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{8}-\frac{3\,\mathrm{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c}}{16}\right.}{}$
default	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\,\mathrm{arccosh}(dx+c)^2}{4}-\frac{(dx+c)^3\,\mathrm{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{8}-\frac{3\,\mathrm{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c}}{16}\right.}{}$
parts	$\frac{e^3a^2(dx+c)^4}{4d}+\frac{e^3b^2\left(\frac{(dx+c)^4\,\mathrm{arccosh}(dx+c)^2}{4}-\frac{(dx+c)^3\,\mathrm{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{8}-\frac{3\,\mathrm{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{d}}{16}\right.}{d}$
orering	$\frac{(37d^6x^6+222c\,d^5x^5+555c^2d^4x^4+740c^3d^3x^3+546c^4d^2x^2+21d^4x^4+204c^5dx+84c\,d^3x^3+28c^6+99c^2d^2x^2+30c^3dx+6c}{64d(dx+c)^5}$

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^2*(d*x+c)^4+e^3*b^2*(1/4*(d*x+c)^4*arccosh(d*x+c)^2-1/8*(d*
x+c)^3*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/16*arccosh(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-3/32*arccosh(d*x+c)^2+1/32*(d*x+c)
^4+3/32*(d*x+c)^2)+2*e^3*a*b*(1/4*(d*x+c)^4*arccosh(d*x+c)-1/32*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^3*((d*x+c)^2-1)^(1/2)+3*(d*x+c)*((d*x+c)^
2-1)^(1/2)+3*ln(d*x+c+((d*x+c)^2-1)^(1/2)))/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(166) = 332$.

Time = 0.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.59

$$\int (ce+dex)^3(a+b\,\mathrm{arccosh}(c+dx))^2\,dx$$
$$=\frac{(8\,a^2+b^2)d^4e^3x^4+4\,(8\,a^2+b^2)cd^3e^3x^3+3\,(2\,(8\,a^2+b^2)c^2+b^2)d^2e^3x^2+2\,(2\,(8\,a^2+b^2)c^3+3\,b^2c)de^3x}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*
a^2 + b^2)*c^2 + b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*e^
3*x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 +
 32*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48
*a*b*c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b
^2*d^3*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 + b^2)*d*e^3*x + (2*b^
2*c^3 + 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x
^2 + 3*(2*a*b*c^2 + a*b)*d*e^3*x + (2*a*b*c^3 + 3*a*b*c)*e^3)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^2\,dx = e^3\left(\int a^2c^3\,dx + \int a^2d^3x^3\,dx + \int b^2c^3\operatorname{acosh}^2(c+dx)\,dx + \int 2abc^3\operatorname{acosh}(c+dx)\,dx + \int 3a^2cd^2x^2\,dx + \int 3a^2c^2dx\,dx + \int b^2d^3x^3\operatorname{acosh}^2(c+dx)\,dx + \int 2abd^3x^3\operatorname{acosh}(c+dx)\,dx + \int 3b^2cd^2x^2\operatorname{acosh}^2(c+dx)\,dx + \int 3b^2c^2dx\operatorname{acosh}^2(c+dx)\,dx + \int 6abcd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 6abc^2dx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**2,x)
```

output
```
e**3*(Integral(a**2*c**3, x) + Integral(a**2*d**3*x**3, x) + Integral(b**2
*c**3*acosh(c + d*x)**2, x) + Integral(2*a*b*c**3*acosh(c + d*x), x) + Int
egral(3*a**2*c*d**2*x**2, x) + Integral(3*a**2*c**2*d*x, x) + Integral(b**
2*d**3*x**3*acosh(c + d*x)**2, x) + Integral(2*a*b*d**3*x**3*acosh(c + d*x
), x) + Integral(3*b**2*c*d**2*x**2*acosh(c + d*x)**2, x) + Integral(3*b**
2*c**2*d*x*acosh(c + d*x)**2, x) + Integral(6*a*b*c*d**2*x**2*acosh(c + d*
x), x) + Integral(6*a*b*c**2*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3/2*(2*x
^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1
)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arccosh(
d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d
^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a*b*c*d^2*
e^3 + 1/48*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*
d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d +
 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 +
9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a*b*d^3*e^
3 + a^2*c^3*e^3*x + 2*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))
*a*b*c^3*e^3/d + 1/4*(b^2*d^3*e^3*x^4 + 4*b^2*c*d^2*e^3*x^3 + 6*b^2*c^2...
```

Giac [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^3\,\left(a+b\,\text{acosh}(c+d\,x)\right)^2\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^2\,dx$$
$$=\frac{e^3\left(32acosh(dx+c)\,ab\,c^4+32acosh(dx+c)\,ab\,c^3dx+48acosh(dx+c)\,ab\,c^2d^2x^2+32acosh(dx+c)\,abc\right.}{}$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c))^2,x)
```

output

```
(e**3*(32*acosh(c + d*x)*a*b*c**4 + 32*acosh(c + d*x)*a*b*c**3*d*x + 48*ac
osh(c + d*x)*a*b*c**2*d**2*x**2 + 32*acosh(c + d*x)*a*b*c*d**3*x**3 + 8*ac
osh(c + d*x)*a*b*d**4*x**4 + 30*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c
**3 - 6*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c**2*d*x - 6*sqrt(c**2 +
2*c*d*x + d**2*x**2 - 1)*a*b*c*d**2*x**2 - 3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*a*b*c - 2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*d**3*x**3 - 3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*d*x - 32*sqrt(c + d*x + 1)*sqrt(c
+ d*x - 1)*a*b*c**3 + 16*int(acosh(c + d*x)**2,x)*b**2*c**3*d + 16*int(aco
sh(c + d*x)**2*x**3,x)*b**2*d**4 + 48*int(acosh(c + d*x)**2*x**2,x)*b**2*c
*d**3 + 48*int(acosh(c + d*x)**2*x,x)*b**2*c**2*d**2 - 24*log(sqrt(c**2 +
2*c*d*x + d**2*x**2 - 1) + c + d*x)*a*b*c**4 - 3*log(sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1) + c + d*x)*a*b + 16*a**2*c**3*d*x + 24*a**2*c**2*d**2*x**2
 + 16*a**2*c*d**3*x**3 + 4*a**2*d**4*x**4))/(16*d)
```

3.24 $\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^2\,dx = \frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}-\frac{4be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{9d} - \frac{2be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{9d} + \frac{e^2(c+dx)^3(a+b\text{arccosh}(c+dx))^2}{3d}$$

output
```
4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d-4/9*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*(a+b*arccosh(d*x+c))/d-2/9*b*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1
)^(1/2)*(a+b*arccosh(d*x+c))/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.12

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$= \frac{e^2\left(12b^2(c+dx)+\left(9a^2+2b^2\right)(c+dx)^3+6ab\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(-2-(c+dx)^2\right)+6b\big(3a(c+\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^2*(12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x]*(-2 - (c + d*x)^2) + 6*b*(3*a*(c + d*x)^3 - 2*b*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x] - b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt
[1 + c + d*x])*ArcCosh[c + d*x] + 9*b^2*(c + d*x)^3*ArcCosh[c + d*x]^2))/(
27*d)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6411, 27, 6298, 6354, 15, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^2(c+dx)^2(a+b\operatorname{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\operatorname{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\int\frac{(c+dx)^3(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\Big\downarrow$ 6354

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{3}b\int(c+dx)^2d(c+dx)+\frac{1}{3}\right.}{d}$$

$\Big\downarrow$ 15

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(\right.}{d}$$

$\Big\downarrow$ 6330

$$\frac{e^2(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b(\frac{2}{3}(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))-b\int 1d(c+dx)}{d}$$

$\Big\downarrow$ 24

$$\frac{e^2(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b(\frac{1}{3}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2(a+b\text{arccosh}(c+dx))+\frac{2}{3}(\sqrt{c}}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/3 - (2*b*(-1/9*(b*(c + d*x)
^3) + (Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
 d*x]))/3 + (2*(-(b*(c + d*x)) + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a +
 b*ArcCosh[c + d*x])))/3))/3))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{e^2a^2(dx+c)^3}{3}+e^2b^2\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^2}{3}-\frac{4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\sqrt{dx+c+1}}{9}-\frac{2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\sqrt{dx+c+1}(dx}{9}\right.}{d}$
default	$\frac{\frac{e^2a^2(dx+c)^3}{3}+e^2b^2\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^2}{3}-\frac{4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\sqrt{dx+c+1}}{9}-\frac{2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\sqrt{dx+c+1}(dx}{9}\right.}{d}$
parts	$\frac{e^2a^2(dx+c)^3}{3d}+\frac{e^2b^2\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^2}{3}-\frac{4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\sqrt{dx+c+1}}{9}-\frac{2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\sqrt{dx+c+1}}{9}\right.}{d}$
orering	$\frac{(19d^5x^5+95c\,d^4x^4+190c^2d^3x^3+186c^3d^2x^2+87c^4dx+24d^3x^3+15c^5+48c\,d^2x^2+24c^2dx+6c^3-48dx-12c)(dex+ce)^2(}{27(dx+c)^4d}$

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^2*(d*x+c)^3+e^2*b^2*(1/3*(d*x+c)^3*arccosh(d*x+c)^2-4/9*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/9*arccosh(d*x+c)*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+4/9*d*x+4/9*c+2/27*(d*x+c)^3)+2*e^2*a*b*(1/
3*(d*x+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(132) = 264.

Time = 0.10 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.39

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$=\frac{(9\,a^2+2\,b^2)d^3e^2x^3+3\,(9\,a^2+2\,b^2)cd^2e^2x^2+3\,((9\,a^2+2\,b^2)c^2+4\,b^2)de^2x+9\,(b^2d^3e^2x^3+3\,b^2cd^2e^2x^2+}{}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((
9*a^2 + 2*b^2)*c^2 + 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2
*x^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*
d*e^2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 +
2*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 +
2*a*b)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$= e^2\left(\int a^2c^2\,dx+\int a^2d^2x^2\,dx+\int b^2c^2\operatorname{acosh}^2(c+dx)\,dx+\int 2abc^2\operatorname{acosh}(c+dx)\,dx + \int 2a^2cdx\,dx+\int b^2d^2x^2\operatorname{acosh}^2(c+dx)\,dx+\int 2abd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 2b^2cdx\operatorname{acosh}^2(c+dx)\,dx+\int 4abcdx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**2,x)
```

output
```
e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2
*c**2*acosh(c + d*x)**2, x) + Integral(2*a*b*c**2*acosh(c + d*x), x) + Int
egral(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*acosh(c + d*x)**2, x) + I
ntegral(2*a*b*d**2*x**2*acosh(c + d*x), x) + Integral(2*b**2*c*d*x*acosh(c
 + d*x)**2, x) + Integral(4*a*b*c*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^2\,dx=\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*e^2*x^3 + a^2*c*d*e^2*x^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2
*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c/d^3))*a*b*c*d*e^2 + 1/9*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 +
9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 2*((d*x + c)
*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*e^
2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c)^2 - integrate(2/3*(b^2*d^5*e^2*x^5 + 5*b^2*c*d^4*e^2
*x^4 + (10*c^2*d^3*e^2 - d^3*e^2)*b^2*x^3 + 3*(3*c^3*d^2*e^2 - c*d^2*e^2)*
b^2*x^2 + 3*(c^4*d*e^2 - c^2*d*e^2)*b^2*x + (b^2*d^4*e^2*x^4 + 4*b^2*c*d^3
*e^2*x^3 + 6*b^2*c^2*d^2*e^2*x^2 + 3*b^2*c^3*d*e^2*x)*sqrt(d*x + c + 1)*sq
rt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x
^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Giac [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^2\,dx=\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{acosh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^2\,dx = \frac{e^2(18acosh(dx+c)\,ab\,c^3+18acosh(dx+c)\,ab\,c^2dx+18acosh(dx+c)\,abc\,d^2x^2+6acosh(dx+c)\,ab\,d^3x}{}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^2,x)
```

output
```
(e**2*(18*acosh(c + d*x)*a*b*c**3 + 18*acosh(c + d*x)*a*b*c**2*d*x + 18*ac
osh(c + d*x)*a*b*c*d**2*x**2 + 6*acosh(c + d*x)*a*b*d**3*x**3 + 16*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c**2 - 4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*a*b*c*d*x - 2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*d**2*x**2 - 4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b - 18*sqrt(c + d*x + 1)*sqrt(c +
d*x - 1)*a*b*c**2 + 9*int(acosh(c + d*x)**2,x)*b**2*c**2*d + 9*int(acosh(c
 + d*x)**2*x**2,x)*b**2*d**3 + 18*int(acosh(c + d*x)**2*x,x)*b**2*c*d**2 -
 12*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a*b*c**3 + 9*a**2*
c**2*d*x + 9*a**2*c*d**2*x**2 + 3*a**2*d**3*x**3))/(9*d)
```

3.25 $\int (ce + dex)(a + b\text{arccosh}(c + dx))^2\, dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^2\, dx = \frac{b^2e(c+dx)^2}{4d} - \frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{2d} - \frac{e(a+b\text{arccosh}(c+dx))^2}{4d} + \frac{e(c+dx)^2(a+b\text{arccosh}(c+dx))^2}{2d}$$

output
```
1/4*b^2*e*(d*x+c)^2/d-1/2*b*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b
*arccosh(d*x+c))/d-1/4*e*(a+b*arccosh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*arc
cosh(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.52

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^2\, dx = \frac{e\left((c+dx)\left(2a^2(c+dx) + b^2(c+dx) - 2ab\sqrt{-1+c+dx}\sqrt{1+c+dx}\right) - 2b(c+dx)\left(-2a(c+dx) + b\ldots\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e*((c + d*x)*(2*a^2*(c + d*x) + b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*
Sqrt[1 + c + d*x]) - 2*b*(c + d*x)*(-2*a*(c + d*x) + b*Sqrt[-1 + c + d*x]*
Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-1 + 2*c^2 + 4*c*d*x + 2*d^2*x^
2)*ArcCosh[c + d*x]^2 - 2*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]]))/(4*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6411, 27, 6298, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e(c+dx)(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\int \frac{(c+dx)^2(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{2}b\int (c+dx)d(c+dx)+\frac{1}{2}\sqrt{c+dx-}\right.\right.}{d}$$

$$\Big\downarrow \text{15}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)(c\right.\right.}{d}$$

$\downarrow$ 6308

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\left(\frac{1}{2}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)(a+b\text{arccosh}(c+dx))+\frac{(a+b\text{arccc}}{}\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/2 - b*(-1/4*(b*(c + d*x)^2) +
 (Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))
/2 + (a + b*ArcCosh[c + d*x])^2/(4*b))))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{e\,a^2(dx+c)^2}{2}+e\,b^2\left(\frac{\cosh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^2}{4}-\frac{\sinh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)}{4}+\frac{\cosh(2\,\operatorname{arccosh}(dx+c))}{8}\right)+2}{d}$
default	$\frac{\frac{e\,a^2(dx+c)^2}{2}+e\,b^2\left(\frac{\cosh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^2}{4}-\frac{\sinh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)}{4}+\frac{\cosh(2\,\operatorname{arccosh}(dx+c))}{8}\right)+2}{d}$
parts	$e\,a^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^2\left(\frac{\cosh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^2}{4}-\frac{\sinh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)}{4}+\frac{\cosh(2\,\operatorname{arccos}}{8}}{d}$
orering	$\frac{(7d^4x^4+28c\,d^3x^3+41c^2d^2x^2+26c^3dx+6c^4-6d^2x^2-12cdx-4c^2)(dex+ce)(a+b\,\operatorname{arccosh}(dx+c))^2}{8d(dx+c)^3}-\frac{(3d^4x^4+12c\,d^3}{}$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^2*(d*x+c)^2+e*b^2*(1/4*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^
2-1/4*sinh(2*arccosh(d*x+c))*arccosh(d*x+c)+1/8*cosh(2*arccosh(d*x+c)))+2*
e*a*b*(1/2*(d*x+c)^2*arccosh(d*x+c)-1/4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((
d*x+c)*((d*x+c)^2-1)^(1/2)+ln(d*x+c+((d*x+c)^2-1)^(1/2)))/((d*x+c)^2-1)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(98) = 196$.

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.12

$$\int (ce+dex)(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$= \frac{(2\,a^2+b^2)d^2ex^2+2\,(2\,a^2+b^2)cdex+(2\,b^2d^2ex^2+4\,b^2cdex+(2\,b^2c^2-b^2)e)\log\left(dx+c+\sqrt{d^2x^2+2\,c}\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2
+ 4*b^2*c*d*e*x + (2*b^2*c^2 - b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 - a*b)*e
 - (b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^2\,dx = e\left(\int a^2c\,dx + \int a^2dx\,dx + \int b^2c\,\text{acosh}^2\,(c+dx)\,dx + \int 2abc\,\text{acosh}\,(c+dx)\,dx + \int b^2dx\,\text{acosh}^2\,(c+dx)\,dx + \int 2abdx\,\text{acosh}\,(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**2,x)
```

output
```
e*(Integral(a**2*c, x) + Integral(a**2*d*x, x) + Integral(b**2*c*acosh(c +
 d*x)**2, x) + Integral(2*a*b*c*acosh(c + d*x), x) + Integral(b**2*d*x*aco
sh(c + d*x)**2, x) + Integral(2*a*b*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^2\,dx = \int (dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*e*x^2 + 1/2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c
*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*d*e +
 a^2*c*e*x + 2*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c*
e/d + 1/2*(b^2*d*e*x^2 + 2*b^2*c*e*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + c)^2 - integrate((b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + (5*c^2*d
^2*e - d^2*e)*b^2*x^2 + 2*(c^3*d*e - c*d*e)*b^2*x + (b^2*d^3*e*x^3 + 3*b^2
*c*d^2*e*x^2 + 2*b^2*c^2*d*e*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d
*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3
 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*
c^2*d - d)*x - c), x)
```

Giac [F]

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^2 \, dx = \int (dex + ce)(b \,\text{arcosh}\,(dx + c) + a)^2 \, dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^2 \, dx = \int (c\,e + d\,e\,x)\,\left(a + b\,\text{acosh}(c + d\,x)\right)^2 \, dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^2 \, dx$$
$$= \frac{e\left(4acosh(dx + c)\, ab\, c^2 + 4acosh(dx + c)\, abcdx + 2acosh(dx + c)\, ab\, d^2x^2 + 3\sqrt{d^2x^2 + 2cdx + c^2 - 1}\, abc\right.}{}$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^2,x)
```

output
```
(e*(4*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*acosh(c + d
*x)*a*b*d**2*x**2 + 3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a*b*c - sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)*a*b*d*x - 4*sqrt(c + d*x + 1)*sqrt(c + d*x -
 1)*a*b*c + 2*int(acosh(c + d*x)**2,x)*b**2*c*d + 2*int(acosh(c + d*x)**2*
x,x)*b**2*d**2 - 2*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a*b
*c**2 - log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a*b + 2*a**2*c
*d*x + a**2*d**2*x**2))/(2*d)
```

3.26 $\int (a+b\,\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int (a+b\,\text{arccosh}(c+dx))^2\,dx = 2b^2x - \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))}{d} + \frac{(c+dx)(a+b\,\text{arccosh}(c+dx))^2}{d}$$

output
```
2*b^2*x-2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d+(d*x+c)
*(a+b*arccosh(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.64

$$\int (a+b\,\text{arccosh}(c+dx))^2\,dx = \frac{a^2(c+dx)+2b^2(c+dx)-2ab\sqrt{-1+c+dx}\sqrt{1+c+dx}-2b\left(-a(c+dx)+b\sqrt{-1+c+dx}\sqrt{1+c+\cdots\right.}{d}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d
*x] - 2*b*(-(a*(c + d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCos
h[c + d*x] + b^2*(c + d*x)*ArcCosh[c + d*x]^2)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6410, 6294, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int (a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6294}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^2-2b\int \frac{(c+dx)(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6330}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^2-2b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))-b\int 1d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^2-2b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))-b(c+dx)\right)}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcCosh[c + d*x])^2 - 2*b*(-(b*(c + d*x)) + Sqrt[-1 + c
+ d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])))/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

method	result
parts	$x\,a^2 + \frac{b^2\left((dx+c)\operatorname{arccosh}(dx+c)^2-2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2dx+2c\right)}{d} + \frac{2ab((dx+c)\operatorname{arccosh}(dx+\cdots}{}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left((dx+c)\operatorname{arccosh}(dx+c)^2-2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2dx+2c\right)+2ab((dx+c)\operatorname{arccosh}(dx+c\cdots}{d}$
default	$\frac{(dx+c)a^2+b^2\left((dx+c)\operatorname{arccosh}(dx+c)^2-2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2dx+2c\right)+2ab((dx+c)\operatorname{arccosh}(dx+c\cdots}{d}$
orering	$\frac{(dx+c)(a+b\operatorname{arccosh}(dx+c))^2}{d} - \frac{2(cdx+c^2-1)(a+b\operatorname{arccosh}(dx+c))b}{d\sqrt{dx+c-1}\,\sqrt{dx+c+1}} + \frac{x(dx+c-1)(dx+c+1)\left(\frac{2b^2d^2}{(dx+c-1)(dx+c+1\cdots}\cdots\right.}{\cdots}$

input
```
int((a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+b^2/d*((d*x+c)*arccosh(d*x+c)^2-2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)+2*d*x+2*c)+2*a*b/d*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int (a + b\text{arccosh}(c + dx))^2 \, dx$$
$$= \frac{(a^2 + 2\,b^2)dx + (b^2dx + b^2c)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)^2 - 2\,\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}ab + 2\,(}{d}$$

input
```
integrate((a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b + 2*(a*b*d*x + a
*b*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)))/d
```

Sympy [F]

$$\int (a + b\text{arccosh}(c + dx))^2 \, dx = \int (a + b\operatorname{acosh}(c + dx))^2 \, dx$$

input
```
integrate((a+b*acosh(d*x+c))**2,x)
```

output
```
Integral((a + b*acosh(c + d*x))**2, x)
```

Maxima [F]

$$\int (a+b\text{arccosh}(c+dx))^2 \, dx = \int (b \operatorname{arcosh}(dx+c)+a)^2 \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
(x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(2*(d^3
*x^3 + 2*c*d^2*x^2 + (d^2*x^2 + c*d*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
 + (c^2*d - d)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*
x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x))*b^2 + a^2*x + 2*((d*x + c)*a
rccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b/d
```

Giac [F]

$$\int (a+b\text{arccosh}(c+dx))^2 \, dx = \int (b \operatorname{arcosh}(dx+c)+a)^2 \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arccosh}(c+dx))^2 \, dx = \int (a+b \operatorname{acosh}(c+d\,x))^2 \, dx$$

input
```
int((a + b*acosh(c + d*x))^2,x)
```

output
```
int((a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (a+b\text{arccosh}(c+dx))^2\,dx$$
$$=\frac{2acosh(dx+c)\,abc+2acosh(dx+c)\,abdx-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,ab+\left(\int acosh(dx+c)^2\,dx\right)b^2d}{d}$$

input
```
int((a+b*acosh(d*x+c))^2,x)
```

output
```
(2*acosh(c + d*x)*a*b*c + 2*acosh(c + d*x)*a*b*d*x - 2*sqrt(c + d*x + 1)*s
qrt(c + d*x - 1)*a*b + int(acosh(c + d*x)**2,x)*b**2*d + a**2*d*x)/d
```

3.27 $\int \frac{(a+b\text{arccosh}(c+dx))^2}{ce+dex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{ce+dex}\,dx = -\frac{(a+b\text{arccosh}(c+dx))^3}{3bde} + \frac{(a+b\text{arccosh}(c+dx))^2\log\left(1+e^{2\text{arccosh}(c+dx)}\right)}{de} + \frac{b(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(c+dx)}\right)}{de} - \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arccosh}(c+dx)}\right)}{2de}$$

output
```
-1/3*(a+b*arccosh(d*x+c))^3/b/d/e+(a+b*arccosh(d*x+c))^2*ln(1+(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e+b*(a+b*arccosh(d*x+c))*polylog(2,-(d*x+
c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-1/2*b^2*polylog(3,-(d*x+c+(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{ce+dex}\,dx$$
$$= \frac{ab\text{arccosh}(c+dx)^2+\frac{1}{3}b^2\text{arccosh}(c+dx)^3+2ab\text{arccosh}(c+dx)\log\left(1+e^{-2\text{arccosh}(c+dx)}\right)+b^2\text{arccosh}(c$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(a*b*ArcCosh[c + d*x]^2 + (b^2*ArcCosh[c + d*x]^3)/3 + 2*a*b*ArcCosh[c + d
*x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-
2*ArcCosh[c + d*x])] + a^2*Log[c + d*x] - b*(a + b*ArcCosh[c + d*x])*PolyL
og[2, -E^(-2*ArcCosh[c + d*x])] - (b^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])
])/2)/(d*e)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6411, 27, 6297, 25, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{ce+dex}\,dx$$
$$\Big\downarrow \ 6411$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{ 6297}$$

$$\frac{\int -(a+b\text{arccosh}(c+dx))^2\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int (a+b\text{arccosh}(c+dx))^2\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -i(a+b\text{arccosh}(c+dx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (a+b\text{arccosh}(c+dx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{i\left(2i\int \frac{e^{\frac{2(a-c-dx)}{b}}(a+b\text{arccosh}(c+dx))^2}{1+e^{\frac{2(a-c-dx)}{b}}}d(a+b\text{arccosh}(c+dx))-\frac{1}{3}i(a+b\text{arccosh}(c+dx))^3\right)}{bde}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{i\left(2i\left(b\int (a+b\text{arccosh}(c+dx))\log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}(c+dx))-\frac{1}{2}b(a+b\text{arccosh}(c+dx))^2\log\left(e\right.\right.\right.}{bde}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}(c+\right.\right.\right.}{bde}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2(a-c-dx)}{b}}\text{PolyLog}(2,-c-dx)de^{\frac{2(a-c-dx)}{b}}+\frac{1}{2}b(a+b\text{arccosh}(c+dx))\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)\right)\right.\right.}{bde}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)+\frac{1}{4}b^2\text{PolyLog}(3,-c-dx)\right)-\frac{1}{2}b(a+b\text{arccosh}(c+\right.\right.}{bde}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(I*((-1/3*I)*(a + b*ArcCosh[c + d*x])^3 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c
+ d*x])^2*Log[1 + E^((2*(a - c - d*x))/b)]) + b*((b*(a + b*ArcCosh[c + d*x
])*PolyLog[2, -E^((2*(a - c - d*x))/b)])/2 + (b^2*PolyLog[3, -c - d*x])/4)
)))/(b*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(-\frac{\operatorname{arccosh}(dx+c)^3}{3}+\operatorname{arccosh}(dx+c)^2\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\operatorname{arccosh}(dx+c)\operatorname{polylog}\left(2,-\left(dx+c+\right.\right.}{e}$
default	$\frac{\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(-\frac{\operatorname{arccosh}(dx+c)^3}{3}+\operatorname{arccosh}(dx+c)^2\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\operatorname{arccosh}(dx+c)\operatorname{polylog}\left(2,-\left(dx+c+\right.\right.}{e}$
parts	$\frac{a^2\ln(dx+c)}{ed}+\frac{b^2\left(-\frac{\operatorname{arccosh}(dx+c)^3}{3}+\operatorname{arccosh}(dx+c)^2\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\operatorname{arccosh}(dx+c)\operatorname{polyl}\right.}{ed}$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2/e*ln(d*x+c)+b^2/e*(-1/3*arccosh(d*x+c)^3+arccosh(d*x+c)^2*ln(1+(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+arccosh(d*x+c)*polylog(2,-(d*x+c+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/2*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2))+2*a*b/e*(-1/2*arccosh(d*x+c)^2+arccosh(d*x+c)*ln(1+(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/2*polylog(2,-(d*x+c+(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{ce+dex}\,dx=\int \frac{\left(b\operatorname{arcosh}\left(dx+c\right)+a\right)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d*e*x +
c*e), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{ce + dex}\,dx = \frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e),x)
```

output
```
(Integral(a**2/(c + d*x), x) + Integral(b**2*acosh(c + d*x)**2/(c + d*x),
x) + Integral(2*a*b*acosh(c + d*x)/(c + d*x), x))/e
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{ce + dex}\,dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^2}{dex + ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^2*log(d*e*x + c*e)/(d*e) + integrate(b^2*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^2/(d*e*x + c*e) + 2*a*b*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)/(d*e*x + c*e), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{ce + dex}\,dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^2}{dex + ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(a+b\,\text{acosh}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{ce+dex}\,dx = \frac{2\left(\int \frac{acosh(dx+c)}{dx+c}dx\right)abd+\left(\int \frac{acosh(dx+c)^2}{dx+c}dx\right)b^2d+\log(dx+c)\,a^2}{de}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e),x)
```

output
```
(2*int(acosh(c + d*x)/(c + d*x),x)*a*b*d + int(acosh(c + d*x)**2/(c + d*x)
,x)*b**2*d + log(c + d*x)*a**2)/(d*e)
```

3.28 $\int \frac{(a+b\,\mathrm{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{(a+b\,\mathrm{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{(a+b\,\mathrm{arccosh}(c+dx))^2}{de^2(c+dx)} + \frac{4b(a+b\,\mathrm{arccosh}(c+dx))\arctan\left(e^{\mathrm{arccosh}(c+dx)}\right)}{de^2} - \frac{2ib^2\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2} + \frac{2ib^2\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2}$$

output
```
-(a+b*arccosh(d*x+c))^2/d/e^2/(d*x+c)+4*b*(a+b*arccosh(d*x+c))*arctan(d*x+
c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-2*I*b^2*polylog(2,-I*(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+2*I*b^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)))/d/e^2
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx$$
$$= \frac{-\frac{a^2}{c+dx}+2ab\left(-\frac{\text{arccosh}(c+dx)}{c+dx}+2\arctan\left(\tanh\left(\frac{1}{2}\text{arccosh}(c+dx)\right)\right)\right)-ib^2\left(\text{arccosh}(c+dx)\left(-\frac{i\text{arccosh}}{c+d}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(-(a^2/(c + d*x)) + 2*a*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[A
rcCosh[c + d*x]/2]]) - I*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c + d*x])/(c
 + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c + d*x]
]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCosh[c +
d*x]]))/(d*e^2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6411, 27, 6298, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx$$
$$\Big\downarrow \; 6411$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{e^2(c+dx)^2}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{6298}$$

$$\frac{2b\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{6362}$$

$$\frac{2b\int \frac{a+b\text{arccosh}(c+dx)}{c+dx}d\text{arccosh}(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\int (a+b\text{arccosh}(c+dx))\csc\left(i\text{arccosh}(c+dx)+\frac{\pi}{2}\right)d\text{arccosh}(c+dx)}{de^2}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\big(-ib\int \log\left(1-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)+ib\int \log\left(1+ie^{\text{arccosh}(c+dx)}\right)d\text{arccos}}{de^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\big(-ib\int e^{-\text{arccosh}(c+dx)}\log\left(1-ie^{\text{arccosh}(c+dx)}\right)de^{\text{arccosh}(c+dx)}+ib\int e^{-\text{arccosh}(c+dx)}\log(1}{de^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\big(2\arctan\left(e^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)+ib}{de^2}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcCosh[c + d*x])^2/(c + d*x)) + 2*b*(2*(a + b*ArcCosh[c + d*x])
*ArcTan[E^ArcCosh[c + d*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + I*
b*PolyLog[2, I*E^ArcCosh[c + d*x]]))/(d*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.24

method	result
derivativedivides	$\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arccosh}(dx+c)^2}{dx+c}-2i\operatorname{arccosh}(dx+c)\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)\right)+2i\operatorname{arccosh}(dx+c)\ln\left(1-i\left(dx+c+\sqrt{dx\cdots}\right.\right.\right.}{e^2}\cdots}{\cdots}$
default	$\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arccosh}(dx+c)^2}{dx+c}-2i\operatorname{arccosh}(dx+c)\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)\right)+2i\operatorname{arccosh}(dx+c)\ln\left(1-i\left(dx+c+\sqrt{dx\cdots}\right.\right.\right.}{e^2}\cdots}{\cdots}$
parts	$-\frac{a^2}{e^2(dx+c)d}+\frac{b^2\left(-\frac{\operatorname{arccosh}(dx+c)^2}{dx+c}-2i\operatorname{arccosh}(dx+c)\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)\right)+2i\operatorname{arccosh}(dx+c)\ln\cdots\right.}{\cdots}$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2/e^2/(d*x+c)+b^2/e^2*(-1/(d*x+c)*arccosh(d*x+c)^2-2*I*arccosh(d*x
+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I*arccosh(d*x+c)*ln(
1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2*I*dilog(1+I*(d*x+c+(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)))+2*I*dilog(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2))))+2*a*b/e^2*(-1/(d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d^2*e^2*
x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^2\,\text{acosh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2ab\,\text{acosh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*acosh(c +
d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*acosh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{(ce + dex)^2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{(ce + dex)^2}\, dx = \int \frac{(a + b\,\text{acosh}(c + d\,x))^2}{(c\,e + d\,e\,x)^2}\, dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^2}\,dx$$
$$=\frac{2\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right)ab\,c^2+2\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right)abcdx+\left(\int \frac{acosh(dx+c)^2}{d^2x^2+2cdx+c^2}dx\right)b^2c^2+\left(\int \frac{acosh(dx+c)^2}{d^2x^2+2cdx+c^2}dx\right)}{c\,e^2\,(dx+c)}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e)^2,x)
```

output
```
(2*int(acosh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c**2 + 2*int(aco
sh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c*d*x + int(acosh(c + d*x)
**2/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c**2 + int(acosh(c + d*x)**2/(c**
2 + 2*c*d*x + d**2*x**2),x)*b**2*c*d*x + a**2*x)/(c*e**2*(c + d*x))
```

3.29 $\int \frac{(a+b\,\mathbf{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{(a+b\,\text{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))}{de^3(c+dx)} - \frac{(a+b\,\text{arccosh}(c+dx))^2}{2de^3(c+dx)^2} - \frac{b^2\log(c+dx)}{de^3}$$

output
```
b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d/e^3/(d*x+c)-1/2*(
a+b*arccosh(d*x+c))^2/d/e^3/(d*x+c)^2-b^2*ln(d*x+c)/d/e^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\,\text{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{-\frac{(a+b\,\text{arccosh}(c+dx))^2}{2(c+dx)^2} + b\left(\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))}{c+dx} - b\log(c+dx)\right)}{de^3}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcCosh[c + d*x])^2/(c + d*x)^2 + b*((Sqrt[-1 + c + d*x]*Sqrt
[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(c + d*x) - b*Log[c + d*x]))/(d*e^
3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6411, 27, 6298, 6333, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^3}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{6298}$$

$$\frac{b\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6333}$$

$$\frac{b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}{c+dx}-b\int \frac{1}{c+dx}d(c+dx)\right)-\frac{(a+b\text{arccosh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}{c+dx}-b\log(c+dx)\right)-\frac{(a+b\text{arccosh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcCosh[c + d*x])^2/(c + d*x)^2 + b*((Sqrt[-1 + c + d*x]*Sqrt
[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(c + d*x) - b*Log[c + d*x]))/(d*e^
3)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6333
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
 && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(2\operatorname{arccosh}(dx+c)-\frac{\operatorname{arccosh}(dx+c)\left(2(dx+c)^2-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}-\ln\left(1+(dx+c+\sqrt{\right.\right.}{e^3}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(2\operatorname{arccosh}(dx+c)-\frac{\operatorname{arccosh}(dx+c)\left(2(dx+c)^2-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}-\ln\left(1+(dx+c+\sqrt{\right.\right.}{e^3}}{d}$
parts	$-\frac{a^2}{2e^3(dx+c)^2d}+\frac{b^2\left(2\operatorname{arccosh}(dx+c)-\frac{\operatorname{arccosh}(dx+c)\left(2(dx+c)^2-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}-\ln\left(1+\right.\right.}{e^3d}$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*(2*arccosh(d*x+c)-1/2*arccosh(d*x+c)*(
2*(d*x+c)^2-2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+arccosh(d*x+c))/(d*x
+c)^2-ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))+2*a*b/e^3*(-1/2/(d*
x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(86) = 172$.

Time = 0.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.48

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx$$
$$=\frac{2\,abc^2d^2x^2+4\,abc^3dx+2\,abc^4-b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2-a^2c^2+2\left(abd^2x^2+2\,ab\right.}{}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 - b^2*c^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - a^2*c^2 + 2*(a*b*d^2*x^2 + 2*a*b*c
*d*x + (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*
d*x + b^2*c^4)*log(d*x + c) + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(
-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^
4*d*e^3)
```

Sympy [F]

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^2}{(ce + dex)^3}\,dx = \frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ab\operatorname{acosh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(2*a*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3), x))/e**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(86) = 172$.

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.49

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx = \left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}d\,\text{arcosh}\,(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log\,(dx+c)}{de^3}\right)b^2 + ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}d}{d^3e^3x+cd^2e^3}-\frac{\text{arcosh}\,(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right) - \frac{b^2\,\text{arcosh}\,(dx+c)^2}{2\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)}-\frac{a^2}{2\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*e^3*x + c*d^2*e
^3) - log(d*x + c)/(d*e^3))*b^2 + a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d
/(d^3*e^3*x + c*d^2*e^3) - arccosh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x +
 c^2*d*e^3)) - 1/2*b^2*arccosh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c
^2*d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arcosh}\,(dx+c)+a)^2}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx=\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^2}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{4\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)ab\,c^2d+8\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)abc\,d^2x+4\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)}{2c}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e)^3,x)
```

output
```
(4*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a
*b*c**2*d + 8*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3),x)*a*b*c*d**2*x + 4*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3),x)*a*b*d**3*x**2 + 2*int(acosh(c + d*x)**2/(c**3 + 3*c
**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*c**2*d + 4*int(acosh(c + d*x)
**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*c*d**2*x + 2*i
nt(acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b*
*2*d**3*x**2 - a**2)/(2*d*e**3*(c**2 + 2*c*d*x + d**2*x**2))
```

3.30 $\int \frac{(a+b\,\text{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx &= \frac{b^2}{3de^4(c+dx)}\\ &+\frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))}{3de^4(c+dx)^2}\\ &-\frac{(a+b\operatorname{arccosh}(c+dx))^2}{3de^4(c+dx)^3}\\ &+\frac{2b(a+b\operatorname{arccosh}(c+dx))\arctan\left(e^{\operatorname{arccosh}(c+dx)}\right)}{3de^4}\\ &-\frac{ib^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(c+dx)}\right)}{3de^4}\\ &+\frac{ib^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arccosh}(c+dx)}\right)}{3de^4}\end{aligned}$$

output
```
1/3*b^2/d/e^4/(d*x+c)+1/3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d
*x+c))/d/e^4/(d*x+c)^2-1/3*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a
+b*arccosh(d*x+c))*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4-1/3
*I*b^2*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/3*I*b
^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx$$

$$= \frac{-\frac{a^2}{(c+dx)^3}+ab\left(\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{(c+dx)^2}-\frac{2\text{arccosh}(c+dx)}{(c+dx)^3}+2\arctan\left(\tanh\left(\frac{1}{2}\text{arccosh}(c+dx)\right)\right)\right)+b^2\left(\frac{1}{c+dx}+\cdots\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

output
```
(-(a^2/(c + d*x)^3) + a*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*
x))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[ArcCosh
[c + d*x]/2]]) + b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]
*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c + d*x
)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c + d*x
]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] +
I*PolyLog[2, I/E^ArcCosh[c + d*x]]))/(3*d*e^4)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6411, 27, 6298, 6348, 15, 6362, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{(c+dx)^4}d(c+dx)}{de^4}$$

$\Big\downarrow$ 6298

$$\frac{\frac{2}{3}b\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\Big\downarrow$ 6348

$$\frac{\frac{2}{3}b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{1}{2}b\int \frac{1}{(c+dx)^2}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}{2(c+dx)^2}\right)-\frac{(a+b\text{arc}}{3}}{de^4}$$

$\Big\downarrow$ 15

$$\frac{\frac{2}{3}b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}{2(c+dx)^2}+\frac{b}{2(c+dx)}\right)-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\Big\downarrow$ 6362

$$\frac{\frac{2}{3}b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{c+dx}d\text{arccosh}(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}{2(c+dx)^2}+\frac{b}{2(c+dx)}\right)-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(\frac{1}{2}\int (a+b\text{arccosh}(c+dx))\csc\left(i\text{arccosh}(c+dx)+\frac{\pi}{2}\right)d\text{arccosh}(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c}}{}\right.}{de^4}$$

$\Big\downarrow$ 4668

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(\frac{1}{2}\left(-ib\int \log\left(1-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)+ib\int \log\left(1+ie^{\text{arccosh}(c+dx)}\right)d\text{arc}\right.\right.}{de}$$

$\Big\downarrow$ 2715

$$-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(\frac{1}{2}\left(-ib\int e^{-\text{arccosh}(c+dx)}\log\left(1-ie^{\text{arccosh}(c+dx)}\right)de^{\text{arccosh}(c+dx)}+ib\int e^{-\text{arccosh}(c+dx)}\text{lo}\right.\right.$$

$\Big\downarrow$ 2838

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(\frac{1}{2}\left(2\arctan\left(e^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-ib\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)-\right.\right.}{de^4}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcCosh[c + d*x])^2/(c + d*x)^3 + (2*b*(b/(2*(c + d*x)) + (Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(2*(c + d*x)^
2) + (2*(a + b*ArcCosh[c + d*x])*ArcTan[E^ArcCosh[c + d*x]] - I*b*PolyLog[
2, (-I)*E^ArcCosh[c + d*x]] + I*b*PolyLog[2, I*E^ArcCosh[c + d*x]])/2))/3)
/(d*e^4)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6348
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp[b*c*(n/(f
*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]
   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{-\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)^2-(dx+c)^2}{3(dx+c)^3}-\frac{i\operatorname{arccosh}(dx+c)\ln(1+i(dx+c+\sqrt{}}{3}\right.}{}}{}$
default	$\frac{-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{-\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)^2-(dx+c)^2}{3(dx+c)^3}-\frac{i\operatorname{arccosh}(dx+c)\ln(1+i(dx+c+\sqrt{}}{3}\right.}{}}{}$
parts	$-\frac{a^2}{3e^4(dx+c)^3d}+\frac{b^2\left(-\frac{-\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)^2-(dx+c)^2}{3(dx+c)^3}-\frac{i\operatorname{arccosh}(dx+c)\ln(1+i(dx}{}\right.}{}$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2/e^4/(d*x+c)^3+b^2/e^4*(-1/3*(-arccosh(d*x+c)*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*(d*x+c)+arccosh(d*x+c)^2-(d*x+c)^2)/(d*x+c)^3-1/3*I*arcco
sh(d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+1/3*I*arccosh(d*
x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-1/3*I*dilog(1+I*(d*x+
c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+1/3*I*dilog(1-I*(d*x+c+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2))))+2*a*b/e^4*(-1/3/(d*x+c)^3*arccosh(d*x+c)-1/6*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*(arctan(1/((d*x+c)^2-1)^(1/2))*(d*x+c)^2-((d*x+c)^2
-1)^(1/2))/(d*x+c)^2/((d*x+c)^2-1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{arcosh}(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d^4*e^4*
x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{(ce + dex)^4} \, dx$$
$$= \frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} \, dx + \int \frac{b^2 \operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} \, dx + \int \frac{2ab \operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} \, dx}{e^4}$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*acosh(c + d*x)
/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e
**4
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{(ce + dex)^4} \, dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^2}{(dex + ce)^4} \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*b^2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^4*e^4*x^3
 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 +
 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^
3 + b^2*d^3)*x^3 + 3*(c^3 - c)*a*b + (c^3 - c)*b^2 + 3*(3*a*b*c*d^2 + b^2*
c*d^2)*x^2 + (b^2*c^2 + 3*(c^2 - 1)*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3
*a*b*c*d + b^2*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*d -
 d)*a*b + (3*c^2*d - d)*b^2)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*
e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4
- 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^
6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 +
 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2
 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
(7*c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{arcosh}(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^2}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^4}\,dx$$
$$= \frac{6\left(\int \frac{acosh(dx+c)}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)ab\,c^3d+18\left(\int \frac{acosh(dx+c)}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)ab\,c^2d^2x+18\left(\int \frac{}{d^4x^4+4}\right.}{}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e)^4,x)
```

output
```
(6*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**
3 + d**4*x**4),x)*a*b*c**3*d + 18*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x +
6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a*b*c**2*d**2*x + 18*int(
acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4),x)*a*b*c*d**3*x**2 + 6*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c
**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a*b*d**4*x**3 + 3*int(acosh(
c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x
**4),x)*b**2*c**3*d + 9*int(acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**2*c**2*d**2*x + 9*int(acosh(c
 + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4),x)*b**2*c*d**3*x**2 + 3*int(acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c
**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**2*d**4*x**3 - a**2)/(3*d*
e**4*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))
```

3.31 $\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 374

$$\begin{aligned}\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx &= -\frac{4144b^3e^4\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5625d}-\frac{272b^3e^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5625d}\\ &\quad-\frac{6b^3e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{625d}+\frac{16b^2e^4(c+dx)(a+b\text{arccosh}(c+dx))}{25d}\\ &\quad+\frac{8b^2e^4(c+dx)^3(a+b\text{arccosh}(c+dx))}{75d}+\frac{6b^2e^4(c+dx)^5(a+b\text{arccosh}(c+dx))}{125d}\\ &\quad-\frac{8be^4\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{25d}\\ &\quad-\frac{4be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{25d}\\ &\quad-\frac{3be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{25d}\\ &\quad+\frac{e^4(c+dx)^5(a+b\text{arccosh}(c+dx))^3}{5d}\end{aligned}$$

output
```
-4144/5625*b^3*e^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-272/5625*b^3*e^4*(d*x
+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/d-6/625*b^3*e^4*(d*x+c-1)^(1/2)*(d*x
+c)^4*(d*x+c+1)^(1/2)/d+16/25*b^2*e^4*(d*x+c)*(a+b*arccosh(d*x+c))/d+8/75*
b^2*e^4*(d*x+c)^3*(a+b*arccosh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arcc
osh(d*x+c))/d-8/25*b*e^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+
c))^2/d-4/25*b*e^4*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)*(a+b*arccosh(
d*x+c))^2/d-3/25*b*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)*(a+b*arcc
osh(d*x+c))^2/d+1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.08

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{e^4\left(240ab^2(c+dx)+40ab^2(c+dx)^3+3a(25a^2+6b^2)\,(c+dx)^5+\frac{1}{15}b\sqrt{-1+c+dx}\sqrt{1+c+dx}(-8(22$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^4*(240*a*b^2*(c + d*x) + 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
 + d*x)^5 + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8*(225*a^2 + 518*b^2
) - 4*(225*a^2 + 68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/1
5 - b*(-240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*
b^2*(c + d*x)^5 + 240*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 120*a*b*S
qrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] + 90*a*b*Sqrt[-1 + c + d*x
]*(c + d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 15*b^2*(-15*a*(c + d*x
)^5 + 8*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*b*Sqrt[-1 + c + d*x]*(c
 + d*x)^2*Sqrt[1 + c + d*x] + 3*b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 +
c + d*x])*ArcCosh[c + d*x]^2 + 75*b^3*(c + d*x)^5*ArcCosh[c + d*x]^3))/(37
5*d)
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {6411, 27, 6298, 6354, 6298, 111, 27, 111, 27, 83, 6354, 6298, 111, 27, 83, 6330, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\int \frac{(c+dx)^5(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\int (c+dx)^4(a+b\text{arccosh}(c+dx))d(c+dx)+\frac{4}{5}\int \frac{(c+dx)^3(a+b\text{ar}}{\sqrt{c+dx-1}}\right.\right.}{d}$$

$$\Big\downarrow \text{6298}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c\right.\right.\right.$$

$$\Big\downarrow \text{111}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{1}{5}\int \frac{4(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}c\right.\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{4}{5}\int\frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d\right.\right.\right.\right.$$

$\downarrow$ 111

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{4}{5}\left(\frac{1}{3}\int\frac{2(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.\right.\right.\right.$$

$\downarrow$ 27

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))-\frac{1}{5}b\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.\right.\right.\right.$$

$\downarrow$ 83

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(\frac{4}{5}\int\frac{(c+dx)^3(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+}\right.\right.$$

$\downarrow$ 6354

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(\frac{4}{5}\left(-\frac{2}{3}b\int(c+dx)^2(a+b\text{arccosh}(c+dx))d(c+dx)+\frac{2}{3}\int\frac{(c+dx)(a+b}{\sqrt{c+dx}}\right.\right.\right.$$

$\downarrow$ 6298

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(\frac{4}{5}\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\int\frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d\right.\right.\right.\right.$$

$\downarrow$ 111

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(\frac{4}{5}\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{1}{3}\int\frac{2(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.\right.\right.\right.$$

$\downarrow$ 27

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(\frac{4}{5}\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{2}{3}\int\frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.\right.\right.\right.$$

$\downarrow$ 83

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}(c+dx)^2\right.\right.\right.$$

$\downarrow$ 6330

$$e^4(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b(\frac{4}{5}(\frac{2}{3}(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2-2b\int(a+b$$

$\downarrow$ 2009

$$e^4(\frac{1}{5}(c+dx)^5(a+b\text{arccosh}(c+dx))^3-\frac{3}{5}b(\frac{1}{5}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{2}{5}b(\frac{1}{5}$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^4*(((c + d*x)^5*(a + b*ArcCosh[c + d*x])^3)/5 - (3*b*((Sqrt[-1 + c + d*
x]*(c + d*x)^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/5 - (2*b*(-1/
5*(b*((Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/5 + (4*((2*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/3 + (Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
 + c + d*x])/3))/5)) + ((c + d*x)^5*(a + b*ArcCosh[c + d*x]))/5))/5 + (4*(
(Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])
^2)/3 - (2*b*(-1/3*(b*((2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/3 + (Sqrt[
-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/3)) + ((c + d*x)^3*(a + b*Arc
Cosh[c + d*x]))/3))/3 + (2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x])^2 - 2*b*(a*(c + d*x) - b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d
*x] + b*(c + d*x)*ArcCosh[c + d*x])))/3))/5))/5))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{e^4a^3(dx+c)^5}{5}+e^4b^3\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^3}{5}-\frac{8\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{25}-\frac{3(dx+c)^4\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx}}{25}\right.}{}$
default	$\frac{\frac{e^4a^3(dx+c)^5}{5}+e^4b^3\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^3}{5}-\frac{8\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{25}-\frac{3(dx+c)^4\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx}}{25}\right.}{}$
parts	$\frac{e^4a^3(dx+c)^5}{5d}+\frac{e^4b^3\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^3}{5}-\frac{8\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{25}-\frac{3(dx+c)^4\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}}{25}\right.}{}$
orering	Expression too large to display

input
```
int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*e^4*a^3*(d*x+c)^5+e^4*b^3*(1/5*(d*x+c)^5*arccosh(d*x+c)^3-8/25*ar
ccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/25*(d*x+c)^4*arccosh(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4/25*(d*x+c)^2*arccosh(d*x+c)^2*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)+16/25*(d*x+c)*arccosh(d*x+c)-4144/5625*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+6/125*(d*x+c)^5*arccosh(d*x+c)-6/625*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*(d*x+c)^4-272/5625*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x
+c)^2+8/75*(d*x+c)^3*arccosh(d*x+c))+3*e^4*a*b^2*(1/5*(d*x+c)^5*arccosh(d*
x+c)^2-16/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/25*(d*x+c)^4
*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-8/75*arccosh(d*x+c)*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+16/75*d*x+16/75*c+2/125*(d*x+c)^5+8/22
5*(d*x+c)^3)+3*e^4*a^2*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(330) = 660$.

Time = 0.12 (sec) , antiderivative size = 1074, normalized size of antiderivative = 2.87

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e
^4*x^4 + 150*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 + 450*(4*a*b
^2*c + (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 + 225*(8*a*b^2*c^2 + (25*a^3 +
6*a*b^2)*c^4 + 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4
*x^4 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x
 + b^3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 225*(
15*a*b^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3
+ 150*a*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3
*b^3*d^4*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 + 2*b^3)*d^2*e^4*x^
2 + 4*(3*b^3*c^3 + 2*b^3*c)*d*e^4*x + (3*b^3*c^4 + 4*b^3*c^2 + 8*b^3)*e^4)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1))^2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3
)*c*d^4*e^4*x^4 + 10*(4*b^3 + 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 + 30*(
4*b^3*c + 3*(25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 + 15*(8*b^3*c^2 + 3*(25*a^
2*b + 2*b^3)*c^4 + 16*b^3)*d*e^4*x + (40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^
5 + 240*b^3*c)*e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*
(9*a*b^2*c^2 + 2*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 + 2*a*b^2*c)*d*e^4*x
+ (3*a*b^2*c^4 + 4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (27*(25*a^2*b + 2
*b^3)*d^4*e^4*x^4 + 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 + 2*(450*a^2*b...
```

Sympy [F]

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^3\,dx$$
$$= e^4\left(\int a^3c^4\,dx + \int a^3d^4x^4\,dx + \int b^3c^4\operatorname{acosh}^3(c+dx)\,dx + \int 3ab^2c^4\operatorname{acosh}^2(c+dx)\,dx + \int 3a^2bc^4\operatorname{acosh}(c+dx)\,dx + \int 4a^3cd^3x^3\,dx + \int 6a^3c^2d^2x^2\,dx + \int 4a^3c^3dx\,dx + \int b^3d^4x^4\operatorname{acosh}^3(c+dx)\,dx + \int 3ab^2d^4x^4\operatorname{acosh}^2(c+dx)\,dx + \int 3a^2bd^4x^4\operatorname{acosh}(c+dx)\,dx + \int 4b^3cd^3x^3\operatorname{acosh}^3(c+dx)\,dx + \int 6b^3c^2d^2x^2\operatorname{acosh}^3(c+dx)\,dx + \int 4b^3c^3dx\operatorname{acosh}^3(c+dx)\,dx + \int 12ab^2cd^3x^3\operatorname{acosh}^2(c+dx)\,dx + \int 18ab^2c^2d^2x^2\operatorname{acosh}^2(c+dx)\,dx + \int 12ab^2c^3dx\operatorname{acosh}^2(c+dx)\,dx + \int 12a^2bcd^3x^3\operatorname{acosh}(c+dx)\,dx + \int 18a^2bc^2d^2x^2\operatorname{acosh}(c+dx)\,dx + \int 12a^2bc^3dx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**3,x)
```

output
```
e**4*(Integral(a**3*c**4, x) + Integral(a**3*d**4*x**4, x) + Integral(b**3
*c**4*acosh(c + d*x)**3, x) + Integral(3*a*b**2*c**4*acosh(c + d*x)**2, x)
 + Integral(3*a**2*b*c**4*acosh(c + d*x), x) + Integral(4*a**3*c*d**3*x**3
, x) + Integral(6*a**3*c**2*d**2*x**2, x) + Integral(4*a**3*c**3*d*x, x) +
 Integral(b**3*d**4*x**4*acosh(c + d*x)**3, x) + Integral(3*a*b**2*d**4*x*
*4*acosh(c + d*x)**2, x) + Integral(3*a**2*b*d**4*x**4*acosh(c + d*x), x)
+ Integral(4*b**3*c*d**3*x**3*acosh(c + d*x)**3, x) + Integral(6*b**3*c**2
*d**2*x**2*acosh(c + d*x)**3, x) + Integral(4*b**3*c**3*d*x*acosh(c + d*x)
**3, x) + Integral(12*a*b**2*c*d**3*x**3*acosh(c + d*x)**2, x) + Integral(
18*a*b**2*c**2*d**2*x**2*acosh(c + d*x)**2, x) + Integral(12*a*b**2*c**3*d
*x*acosh(c + d*x)**2, x) + Integral(12*a**2*b*c*d**3*x**3*acosh(c + d*x),
x) + Integral(18*a**2*b*c**2*d**2*x**2*acosh(c + d*x), x) + Integral(12*a*
*2*b*c**3*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^3\,dx=\int (dex+ce)^4(b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/5*a^3*d^4*e^4*x^5 + a^3*c*d^3*e^4*x^4 + 2*a^3*c^2*d^2*e^4*x^3 + 2*a^3*c^
3*d*e^4*x^2 + 3*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*c^3*d*e^4
 + (6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^
2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^
4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*
x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1
)/d^4))*a^2*b*c^2*d^2*e^4 + 1/8*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2
/d^3 + 105*c^4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d
)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)
*c/d^5)*d)*a^2*b*c*d^3*e^4 + 1/200*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - ...
```

Giac [F]

$$\int (ce+dex)^4(a+b\operatorname{arccosh}(c+dx))^3\,dx=\int (dex+ce)^4(b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\text{acosh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int (ce+dex)^4(a+b\text{arccosh}(c+dx))^3\,dx=\text{Too large to display}$$

input
```
int((d*e*x+c*e)^4*(a+b*acosh(d*x+c))^3,x)
```

output
```
(e**4*(75*acosh(c + d*x)*a**2*b*c**5 + 75*acosh(c + d*x)*a**2*b*c**4*d*x +
 150*acosh(c + d*x)*a**2*b*c**3*d**2*x**2 + 150*acosh(c + d*x)*a**2*b*c**2
*d**3*x**3 + 75*acosh(c + d*x)*a**2*b*c*d**4*x**4 + 15*acosh(c + d*x)*a**2
*b*d**5*x**5 + 72*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c**4 - 12*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c**3*d*x - 18*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1)*a**2*b*c**2*d**2*x**2 - 4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*a**2*b*c**2 - 12*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c*d**
3*x**3 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c*d*x - 3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*d**4*x**4 - 4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*a**2*b*d**2*x**2 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2
*b - 75*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*a**2*b*c**4 + 25*int(acosh(c +
 d*x)**3,x)*b**3*c**4*d + 75*int(acosh(c + d*x)**2,x)*a*b**2*c**4*d + 25*i
nt(acosh(c + d*x)**3*x**4,x)*b**3*d**5 + 100*int(acosh(c + d*x)**3*x**3,x)
*b**3*c*d**4 + 150*int(acosh(c + d*x)**3*x**2,x)*b**3*c**2*d**3 + 100*int(
acosh(c + d*x)**3*x,x)*b**3*c**3*d**2 + 75*int(acosh(c + d*x)**2*x**4,x)*a
*b**2*d**5 + 300*int(acosh(c + d*x)**2*x**3,x)*a*b**2*c*d**4 + 450*int(aco
sh(c + d*x)**2*x**2,x)*a*b**2*c**2*d**3 + 300*int(acosh(c + d*x)**2*x,x)*a
*b**2*c**3*d**2 - 60*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a
**2*b*c**5 + 25*a**3*c**4*d*x + 50*a**3*c**3*d**2*x**2 + 50*a**3*c**2*d**3
*x**3 + 25*a**3*c*d**4*x**4 + 5*a**3*d**5*x**5))/(25*d)
```

3.32 $\int (ce+dex)^3(a+b\,\mathrm{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 307

$$\begin{aligned}\int (ce+dex)^3(a+b\,\mathrm{arccosh}(c+dx))^3\,dx &= -\frac{45b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{256d} \\ &\quad -\frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{128d}-\frac{45b^3e^3\mathrm{arccosh}(c+dx)}{256d} \\ &\quad +\frac{9b^2e^3(c+dx)^2(a+b\,\mathrm{arccosh}(c+dx))}{32d}+\frac{3b^2e^3(c+dx)^4(a+b\,\mathrm{arccosh}(c+dx))}{32d} \\ &\quad -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\,\mathrm{arccosh}(c+dx))^2}{32d} \\ &\quad -\frac{3be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}(a+b\,\mathrm{arccosh}(c+dx))^2}{16d} \\ &\quad -\frac{3e^3(a+b\,\mathrm{arccosh}(c+dx))^3}{32d}+\frac{e^3(c+dx)^4(a+b\,\mathrm{arccosh}(c+dx))^3}{4d}\end{aligned}$$

output

```
-45/256*b^3*e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/d-3/128*b^3*e^3*(d
*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/d-45/256*b^3*e^3*arccosh(d*x+c)/d+
9/32*b^2*e^3*(d*x+c)^2*(a+b*arccosh(d*x+c))/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*
arccosh(d*x+c))/d-9/32*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*
arccosh(d*x+c))^2/d-3/16*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)*(
a+b*arccosh(d*x+c))^2/d-3/32*e^3*(a+b*arccosh(d*x+c))^3/d+1/4*e^3*(d*x+c)^
4*(a+b*arccosh(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.17

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx$$
$$= \frac{e^3\left(72ab^2(c+dx)^2+8a(8a^2+3b^2)(c+dx)^4+3b\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(-3(8a^2+5b^2)-2\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^3*(72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*Sqrt[-1
 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b
^2)*(c + d*x)^2) - 24*b*(c + d*x)*(-3*b^2*(c + d*x) - 8*a^2*(c + d*x)^3 -
b^2*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*a*b*Sqrt[
-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 24*b^2*(-3
*a + 8*a*(c + d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]
- 2*b*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2
 + 8*b^3*(-3 + 8*(c + d*x)^4)*ArcCosh[c + d*x]^3 - 9*b*(8*a^2 + 5*b^2)*Log
[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(256*d)
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {6411, 27, 6298, 6354, 6298, 111, 27, 101, 43, 6354, 6298, 101, 43, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int e^3(c+dx)^3(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int(c+dx)^3(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$\downarrow$ 6298

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\int\frac{(c+dx)^4(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\downarrow$ 6354

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\int(c+dx)^3(a+b\text{arccosh}(c+dx))d(c+dx)+\frac{3}{4}\int\frac{(c+dx)^2(a+bar}{\sqrt{c+dx-1}}\right.\right.}{d}$$

$\downarrow$ 6298

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\int\frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c\right.\right.\right.$$

$\downarrow$ 111

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}\int\frac{3(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.\right.$$

$\downarrow$ 27

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{3}{4}\int\frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.\right.$$

$\downarrow$ 101

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))-\frac{1}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.\right.$$

$\downarrow$ 43

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\sqrt{c+dx-1}\sqrt{c+dx+}\right.\right.$$

$\downarrow$ 6354

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{3}{4}\left(-b\int(c+dx)(a+b\text{arccosh}(c+dx))d(c+dx)+\frac{1}{2}\int\frac{(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c}}\right.\right.\right.$$

$\Big\downarrow$ 6298

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{3}{4}\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))-\frac{1}{2}b\int\frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(\right.\right.\right.\right.$$

$\Big\downarrow$ 101

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{3}{4}\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}\int\frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 43

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{2}\sqrt{c+dx-1}(c+dx)\sqrt{c+c}\right.\right.\right.$$

$\Big\downarrow$ 6308

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{1}{4}\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{1}{4}\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcCosh[c + d*x])^3)/4 - (3*b*((Sqrt[-1 + c + d*
x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/4 - (b*(-1/4*
(b*((Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/4 + (3*((Sqrt[-1 +
c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/2 + ArcCosh[c + d*x]/2))/4)) + ((c +
 d*x)^4*(a + b*ArcCosh[c + d*x]))/4))/2 + (3*((Sqrt[-1 + c + d*x]*(c + d*x
)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/2 + (a + b*ArcCosh[c + d*x
])^3/(6*b) - b*(-1/2*(b*((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/
2 + ArcCosh[c + d*x]/2)) + ((c + d*x)^2*(a + b*ArcCosh[c + d*x]))/2)))/4))
/4))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{e^3a^3(dx+c)^4}{4}+e^3b^3\left(\frac{(dx+c)^4\operatorname{arccosh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{16}-\frac{9\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx}}{32}\right.$
default	$\frac{e^3a^3(dx+c)^4}{4}+e^3b^3\left(\frac{(dx+c)^4\operatorname{arccosh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{16}-\frac{9\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx}}{32}\right.$
parts	$\frac{e^3a^3(dx+c)^4}{4d}+\frac{e^3b^3\left(\frac{(dx+c)^4\operatorname{arccosh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{16}-\frac{9\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}}{32}\right.}{}$
orering	Expression too large to display

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^3*(d*x+c)^4+e^3*b^3*(1/4*(d*x+c)^4*arccosh(d*x+c)^3-3/16*(d
*x+c)^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-9/32*arccosh(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-3/32*arccosh(d*x+c)^3+3/32*(d
*x+c)^4*arccosh(d*x+c)-3/128*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-45/
256*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-45/256*arccosh(d*x+c)+9/32*(d*
x+c)^2*arccosh(d*x+c))+3*e^3*a*b^2*(1/4*(d*x+c)^4*arccosh(d*x+c)^2-1/8*(d*
x+c)^3*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/16*arccosh(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-3/32*arccosh(d*x+c)^2+1/32*(d*x+c)
^4+3/32*(d*x+c)^2)+3*e^3*a^2*b*(1/4*(d*x+c)^4*arccosh(d*x+c)-1/32*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^3*((d*x+c)^2-1)^(1/2)+3*(d*x+c)*((d*x+c
)^2-1)^(1/2)+3*ln(d*x+c+((d*x+c)^2-1)^(1/2)))/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(273) = 546$.

Time = 0.12 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.70

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^
3 + 24*(3*a*b^2 + 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 + 16*(9*a*b^2*c + 2
*(8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*
x^3 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^
3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 24*(8*a*b^2*d^4*e^
3*x^4 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d
*e^3*x + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^
3*x^2 + 3*(2*b^3*c^2 + b^3)*d*e^3*x + (2*b^3*c^3 + 3*b^3*c)*e^3)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
^2 + 3*(8*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 +
 24*(b^3 + 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 + 16*(3*b^3*c + 2*(8*a^2*b +
 b^3)*c^3)*d*e^3*x + (24*b^3*c^2 + 8*(8*a^2*b + b^3)*c^4 - 24*a^2*b - 15*b
^3)*e^3 - 16*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2
 + a*b^2)*d*e^3*x + (2*a*b^2*c^3 + 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*(2*(8*a^2
*b + b^3)*d^3*e^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 + 3*(8*a^2*b + 5*b
^3 + 2*(8*a^2*b + b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 + 3*(8*a^2*b
+ 5*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^3\,dx = e^3\left(\int a^3c^3\,dx + \int a^3d^3x^3\,dx + \int b^3c^3\operatorname{acosh}^3(c+dx)\,dx + \int 3ab^2c^3\operatorname{acosh}^2(c+dx)\,dx + \int 3a^2bc^3\operatorname{acosh}(c+dx)\,dx + \int 3a^3cd^2x^2\,dx + \int 3a^3c^2dx\,dx + \int b^3d^3x^3\operatorname{acosh}^3(c+dx)\,dx + \int 3ab^2d^3x^3\operatorname{acosh}^2(c+dx)\,dx + \int 3a^2bd^3x^3\operatorname{acosh}(c+dx)\,dx + \int 3b^3cd^2x^2\operatorname{acosh}^3(c+dx)\,dx + \int 3b^3c^2dx\operatorname{acosh}^3(c+dx)\,dx + \int 9ab^2cd^2x^2\operatorname{acosh}^2(c+dx)\,dx + \int 9ab^2c^2dx\operatorname{acosh}^2(c+dx)\,dx + \int 9a^2bcd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 9a^2bc^2dx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**3,x)
```

output
```
e**3*(Integral(a**3*c**3, x) + Integral(a**3*d**3*x**3, x) + Integral(b**3
*c**3*acosh(c + d*x)**3, x) + Integral(3*a*b**2*c**3*acosh(c + d*x)**2, x)
 + Integral(3*a**2*b*c**3*acosh(c + d*x), x) + Integral(3*a**3*c*d**2*x**2
, x) + Integral(3*a**3*c**2*d*x, x) + Integral(b**3*d**3*x**3*acosh(c + d*
x)**3, x) + Integral(3*a*b**2*d**3*x**3*acosh(c + d*x)**2, x) + Integral(3
*a**2*b*d**3*x**3*acosh(c + d*x), x) + Integral(3*b**3*c*d**2*x**2*acosh(c
 + d*x)**3, x) + Integral(3*b**3*c**2*d*x*acosh(c + d*x)**3, x) + Integral
(9*a*b**2*c*d**2*x**2*acosh(c + d*x)**2, x) + Integral(9*a*b**2*c**2*d*x*a
cosh(c + d*x)**2, x) + Integral(9*a**2*b*c*d**2*x**2*acosh(c + d*x), x) +
Integral(9*a**2*b*c**2*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*d^3*e^3*x^4 + a^3*c*d^2*e^3*x^3 + 3/2*a^3*c^2*d*e^3*x^2 + 9/4*(2*x
^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1
)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arccos
h(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2
*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^2*b*c*
d^2*e^3 + 1/32*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*lo
g(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c
*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^
4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^2*b*
d^3*e^3 + a^3*c^3*e^3*x + 3*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2
 - 1))*a^2*b*c^3*e^3/d + 1/4*(b^3*d^3*e^3*x^4 + 4*b^3*c*d^2*e^3*x^3 + 6...
```

Giac [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx=\int (dex+ce)^3(b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\text{acosh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{e^3\left(-9\log\left(\sqrt{d^2x^2+2cdx+c^2-1}+c+dx\right)a^2b+8a^3d^4x^4+24acosh(dx+c)\,a^2b\,d^4x^4-6\sqrt{d^2x^2+2cd}\right.}{}$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c))^3,x)
```

output
```
(e**3*(96*acosh(c + d*x)*a**2*b*c**4 + 96*acosh(c + d*x)*a**2*b*c**3*d*x +
 144*acosh(c + d*x)*a**2*b*c**2*d**2*x**2 + 96*acosh(c + d*x)*a**2*b*c*d**
3*x**3 + 24*acosh(c + d*x)*a**2*b*d**4*x**4 + 90*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*a**2*b*c**3 - 18*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c
**2*d*x - 18*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c*d**2*x**2 - 9*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c - 6*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*a**2*b*d**3*x**3 - 9*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2
*b*d*x - 96*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*a**2*b*c**3 + 32*int(acosh
(c + d*x)**3,x)*b**3*c**3*d + 96*int(acosh(c + d*x)**2,x)*a*b**2*c**3*d +
32*int(acosh(c + d*x)**3*x**3,x)*b**3*d**4 + 96*int(acosh(c + d*x)**3*x**2
,x)*b**3*c*d**3 + 96*int(acosh(c + d*x)**3*x,x)*b**3*c**2*d**2 + 96*int(ac
osh(c + d*x)**2*x**3,x)*a*b**2*d**4 + 288*int(acosh(c + d*x)**2*x**2,x)*a*
b**2*c*d**3 + 288*int(acosh(c + d*x)**2*x,x)*a*b**2*c**2*d**2 - 72*log(sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a**2*b*c**4 - 9*log(sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a**2*b + 32*a**3*c**3*d*x + 48*a**3
*c**2*d**2*x**2 + 32*a**3*c*d**3*x**3 + 8*a**3*d**4*x**4))/(32*d)
```

3.33 $\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 254

$$\begin{aligned}\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx &= -\frac{40b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{27d}-\frac{2b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{27d}\\&+\frac{4b^2e^2(c+dx)(a+b\text{arccosh}(c+dx))}{3d}+\frac{2b^2e^2(c+dx)^3(a+b\text{arccosh}(c+dx))}{9d}\\&-\frac{2be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{3d}\\&-\frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{3d}\\&+\frac{e^2(c+dx)^3(a+b\text{arccosh}(c+dx))^3}{3d}\end{aligned}$$

output
```
-40/27*b^3*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/27*b^3*e^2*(d*x+c-1)^(1
/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/d+4/3*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))/d
+2/9*b^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))/d-2/3*b*e^2*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^2/d-1/3*b*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2
*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d
*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.17

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{e^2\left(12ab^2(c+dx)+a(3a^2+2b^2)(c+dx)^3+\frac{1}{3}b\sqrt{-1+c+dx}\sqrt{1+c+dx}(-2(9a^2+20b^2)-(9a^2+2b^2\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^2*(12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[-1 + c
+ d*x]*Sqrt[1 + c + d*x]*(-2*(9*a^2 + 20*b^2) - (9*a^2 + 2*b^2)*(c + d*x)^
2))/3 - b*(-12*b^2*(c + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 + 12*
a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*a*b*Sqrt[-1 + c + d*x]*(c + d
*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-3*a*(c + d*x)^3 + 2*b*
Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sq
rt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcCosh[c + d*x]^3
))/(9*d)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6411, 27, 6298, 6354, 6298, 111, 27, 83, 6330, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int e^2(c+dx)^2(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^2\int (c+dx)^2(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \; 6298$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\int\frac{(c+dx)^3(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \; 6354$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\left(-\frac{2}{3}b\int(c+dx)^2(a+b\text{arccosh}(c+dx))d(c+dx)+\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcc}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.}{d}$$

$$\Big\downarrow \; 6298$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\int\frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+\right.\right.\right.$$

$$\Big\downarrow \; 111$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{1}{3}\int\frac{2(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(\right.\right.\right.\right.$$

$$\Big\downarrow \; 27$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))-\frac{1}{3}b\left(\frac{2}{3}\int\frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(\right.\right.\right.\right.$$

$$\Big\downarrow \; 83$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{3}\sqrt{c+dx-1}(c+dx)^2\sqrt{c-}\right.\right.$$

$$\Big\downarrow \; 6330$$

$$e^2\Big(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\Big(\frac{2}{3}\Big(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2-2b\int(a+\text{barcc}$$

$$\Big\downarrow \; 2009$$

$$e^2\Big(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^3-b\Big(\frac{1}{3}\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\Big(\frac{1}{3}(c$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^3)/3 - b*((Sqrt[-1 + c + d*x]*
(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/3 - (2*b*(-1/3*(
b*((2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/3 + (Sqrt[-1 + c + d*x]*(c + d
*x)^2*Sqrt[1 + c + d*x])/3)) + ((c + d*x)^3*(a + b*ArcCosh[c + d*x]))/3))/
3 + (2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2 -
2*b*(a*(c + d*x) - b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*(c + d*x)*Ar
cCosh[c + d*x])))/3)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{e^2a^3(dx+c)^3}{3}+e^2b^3\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^3}{3}-\frac{2\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{3}-\frac{(dx+c)^2\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+}}{3}\right.}{}$
default	$\frac{\frac{e^2a^3(dx+c)^3}{3}+e^2b^3\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^3}{3}-\frac{2\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{3}-\frac{(dx+c)^2\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+}}{3}\right.}{}$
parts	$\frac{e^2a^3(dx+c)^3}{3d}+\frac{e^2b^3\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^3}{3}-\frac{2\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{3}-\frac{(dx+c)^2\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{}}{3}\right.}{}$
orering	Expression too large to display

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^3*(d*x+c)^3+e^2*b^3*(1/3*(d*x+c)^3*arccosh(d*x+c)^3-2/3*arc
cosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/3*(d*x+c)^2*arccosh(d*x+c)
^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4/3*(d*x+c)*arccosh(d*x+c)-40/27*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+2/9*(d*x+c)^3*arccosh(d*x+c)-2/27*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*(d*x+c)^2)+3*e^2*a*b^2*(1/3*(d*x+c)^3*arccosh(d*x+c)^2-4
/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/9*arccosh(d*x+c)*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+4/9*d*x+4/9*c+2/27*(d*x+c)^3)+3*e^2*a
^2*b*(1/3*(d*x+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d
*x+c)^2+2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(224) = 448$.

Time = 0.11 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.39

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^3\,dx$$

$$= \frac{3\left(3a^3+2ab^2\right)d^3e^2x^3+9\left(3a^3+2ab^2\right)cd^2e^2x^2+9\left(4ab^2+\left(3a^3+2ab^2\right)c^2\right)de^2x+9\left(b^3d^3e^2x^3+3b^3cd^2\right.}{}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2
+ 9*(4*a*b^2 + (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3
*c*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^
2 + 9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e
^2*x + (b^3*c^2 + 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x
^3 + 3*(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 + 3*(4*b^3 + (9*a^2*b + 2*b^3)*c^2)
*d*e^2*x + (12*b^3*c + (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 +
 2*a*b^2*c*d*e^2*x + (a*b^2*c^2 + 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - ((9*a^2*b + 2*
b^3)*d^2*e^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x + (18*a^2*b + 40*b^3 + (9
*a^2*b + 2*b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^3\,dx$$
$$= e^2\left(\int a^3c^2\,dx + \int a^3d^2x^2\,dx + \int b^3c^2\operatorname{acosh}^3(c+dx)\,dx + \int 3ab^2c^2\operatorname{acosh}^2(c+dx)\,dx + \int 3a^2bc^2\operatorname{acosh}(c+dx)\,dx + \int 2a^3cdx\,dx + \int b^3d^2x^2\operatorname{acosh}^3(c+dx)\,dx + \int 3ab^2d^2x^2\operatorname{acosh}^2(c+dx)\,dx + \int 3a^2bd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 2b^3cdx\operatorname{acosh}^3(c+dx)\,dx + \int 6ab^2cdx\operatorname{acosh}^2(c+dx)\,dx + \int 6a^2bcdx\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**3,x)
```

output
```
e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3
*c**2*acosh(c + d*x)**3, x) + Integral(3*a*b**2*c**2*acosh(c + d*x)**2, x)
 + Integral(3*a**2*b*c**2*acosh(c + d*x), x) + Integral(2*a**3*c*d*x, x) +
 Integral(b**3*d**2*x**2*acosh(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x*
*2*acosh(c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*acosh(c + d*x), x)
+ Integral(2*b**3*c*d*x*acosh(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*ac
osh(c + d*x)**2, x) + Integral(6*a**2*b*c*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx=\int (dex+ce)^2(b\,\text{arcosh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arccosh(d*x + c) - d*(3
*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1)*c/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/
d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3*((
d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^2*e^2/d + 1/3*(
b^3*d^2*e^2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*log(d*x + sqrt(d*x
+ c + 1)*sqrt(d*x + c - 1) + c)^3 + integrate(((3*a*b^2*d^5*e^2 - b^3*d^5*
e^2)*x^5 + 5*(3*a*b^2*c*d^4*e^2 - b^3*c*d^4*e^2)*x^4 + 3*(c^5*e^2 - c^3*e^
2)*a*b^2 + (3*(10*c^2*d^3*e^2 - d^3*e^2)*a*b^2 - (10*c^2*d^3*e^2 - d^3*e^2
)*b^3)*x^3 + 3*((10*c^3*d^2*e^2 - 3*c*d^2*e^2)*a*b^2 - (3*c^3*d^2*e^2 - c*
d^2*e^2)*b^3)*x^2 + ((3*a*b^2*d^4*e^2 - b^3*d^4*e^2)*x^4 + 3*(c^4*e^2 - c^
2*e^2)*a*b^2 + 4*(3*a*b^2*c*d^3*e^2 - b^3*c*d^3*e^2)*x^3 - 3*(2*b^3*c^2*d^
2*e^2 - (6*c^2*d^2*e^2 - d^2*e^2)*a*b^2)*x^2 - 3*(b^3*c^3*d*e^2 - 2*(2*c^3
*d*e^2 - c*d*e^2)*a*b^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 3*((5...
```

Giac [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx=\int (dex+ce)^2(b\,\text{arcosh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{acosh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{e^2\left(9acosh(dx+c)\,a^2b\,c^3+9acosh(dx+c)\,a^2b\,c^2dx+9acosh(dx+c)\,a^2bc\,d^2x^2+3acosh(dx+c)\,a^2b\,d^3x\right.}{}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^3,x)
```

output
```
(e**2*(9*acosh(c + d*x)*a**2*b*c**3 + 9*acosh(c + d*x)*a**2*b*c**2*d*x + 9
*acosh(c + d*x)*a**2*b*c*d**2*x**2 + 3*acosh(c + d*x)*a**2*b*d**3*x**3 + 8
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*c**2 - 2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*a**2*b*c*d*x - sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*
b*d**2*x**2 - 2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b - 9*sqrt(c + d
*x + 1)*sqrt(c + d*x - 1)*a**2*b*c**2 + 3*int(acosh(c + d*x)**3,x)*b**3*c*
*2*d + 9*int(acosh(c + d*x)**2,x)*a*b**2*c**2*d + 3*int(acosh(c + d*x)**3*
x**2,x)*b**3*d**3 + 6*int(acosh(c + d*x)**3*x,x)*b**3*c*d**2 + 9*int(acosh
(c + d*x)**2*x**2,x)*a*b**2*d**3 + 18*int(acosh(c + d*x)**2*x,x)*a*b**2*c*
d**2 - 6*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a**2*b*c**3 +
 3*a**3*c**2*d*x + 3*a**3*c*d**2*x**2 + a**3*d**3*x**3))/(3*d)
```

3.34 $\int (ce+dex)(a+b\text{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 175

$$\begin{aligned}\int (ce+dex)(a+b\text{arccosh}(c+dx))^3\,dx &= -\frac{3b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{8d} \\ &\quad -\frac{3b^3e\text{arccosh}(c+dx)}{8d}+\frac{3b^2e(c+dx)^2(a+b\text{arccosh}(c+dx))}{4d} \\ &\quad -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{4d} \\ &\quad -\frac{e(a+b\text{arccosh}(c+dx))^3}{4d}+\frac{e(c+dx)^2(a+b\text{arccosh}(c+dx))^3}{2d}\end{aligned}$$

output
```
-3/8*b^3*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/d-3/8*b^3*e*arccosh(d*x
+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))/d-3/4*b*e*(d*x+c-1)^(1/2)*(
d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^2/d-1/4*e*(a+b*arccosh(d*x+c))
^3/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.39

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^3 \, dx$$
$$= \frac{e\left(2a(2a^2 + 3b^2)(c + dx)^2 - 3b(2a^2 + b^2)\sqrt{-1 + c + dx}(c + dx)\sqrt{1 + c + dx} - 6b(c + dx)\left(-2a^2(c + dx\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*Sqrt[-1 + c + d*x]
*(c + d*x)*Sqrt[1 + c + d*x] - 6*b*(c + d*x)*(-2*a^2*(c + d*x) - b^2*(c +
d*x) + 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 6*b^
2*(-a + 2*a*(c + d*x)^2 - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]
)*ArcCosh[c + d*x]^2 + 2*b^3*(-1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^3 - 3*b
*(2*a^2 + b^2)*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(8*d)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6411, 27, 6298, 6354, 6298, 101, 43, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^3 \, dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e(c + dx)(a + b\operatorname{arccosh}(c + dx))^3 d(c + dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c + dx)(a + b\operatorname{arccosh}(c + dx))^3 d(c + dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^3-\frac{3}{2}b\int\frac{(c+dx)^2(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\downarrow$ 6354

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^3-\frac{3}{2}b\left(-b\int(c+dx)(a+b\text{arccosh}(c+dx))d(c+dx)+\frac{1}{2}\int\frac{(a+b\text{arccosh}(c+d}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.}{d}$$

$\downarrow$ 6298

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^3-\frac{3}{2}b\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))-\frac{1}{2}b\int\frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+\right.\right.\right.}{d}$$

$\downarrow$ 101

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^3-\frac{3}{2}b\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}\int\frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c\right.\right.\right.\right.}{}$$

$\downarrow$ 43

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^3-\frac{3}{2}b\left(\frac{1}{2}\int\frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{2}\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+\right.\right.}{}$$

$\downarrow$ 6308

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^3-\frac{3}{2}b\left(\frac{(a+b\text{arccosh}(c+dx))^3}{6b}+\frac{1}{2}\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}(a+b\text{arcco}\right.\right.}{}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^3)/2 - (3*b*((Sqrt[-1 + c + d*x]
*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/2 + (a + b*ArcCos
h[c + d*x])^3/(6*b) - b*(-1/2*(b*((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c
 + d*x])/2 + ArcCosh[c + d*x]/2)) + ((c + d*x)^2*(a + b*ArcCosh[c + d*x]))
/2)))/2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{e a^3 (dx+c)^2}{2} + e b^3 \left(\frac{\cosh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^3}{4} - \frac{3\sinh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^2}{8} + \frac{3\cosh(2\,\operatorname{arccosh}(dx+c))\,\mathrm{a}}{8}\right.}{}$
default	$\frac{\frac{e a^3 (dx+c)^2}{2} + e b^3 \left(\frac{\cosh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^3}{4} - \frac{3\sinh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^2}{8} + \frac{3\cosh(2\,\operatorname{arccosh}(dx+c))\,\mathrm{a}}{8}\right.}{}$
parts	$e a^3 \left(\frac{1}{2} d x^2 + cx\right) + \frac{e b^3 \left(\frac{\cosh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^3}{4} - \frac{3\sinh(2\,\operatorname{arccosh}(dx+c))\operatorname{arccosh}(dx+c)^2}{8} + \frac{3\cosh(2\,\mathrm{ar}}{}\right.}{d}$
orering	$\frac{(15d^4x^4+60c\,d^3x^3+90c^2d^2x^2+60c^3dx+15c^4-20d^2x^2-40cdx-20c^2+8)(dex+ce)(a+b\operatorname{arccosh}(dx+c))^3}{16d(dx+c)^3} - \frac{(7d^4x^4}{}$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^3*(d*x+c)^2+e*b^3*(1/4*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^
3-3/8*sinh(2*arccosh(d*x+c))*arccosh(d*x+c)^2+3/8*cosh(2*arccosh(d*x+c))*a
rccosh(d*x+c)-3/16*sinh(2*arccosh(d*x+c)))+3*e*a*b^2*(1/4*cosh(2*arccosh(d
*x+c))*arccosh(d*x+c)^2-1/4*sinh(2*arccosh(d*x+c))*arccosh(d*x+c)+1/8*cosh
(2*arccosh(d*x+c)))+3*e*a^2*b*(1/2*(d*x+c)^2*arccosh(d*x+c)-1/4*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*((d*x+c)*((d*x+c)^2-1)^(1/2)+ln(d*x+c+((d*x+c)^2-1)^
(1/2)))/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(155) = 310$.

Time = 0.11 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.26

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^3\,dx$$
$$= \frac{2\,(2\,a^3 + 3\,ab^2)d^2ex^2 + 4\,(2\,a^3 + 3\,ab^2)cdex + 2\,(2\,b^3d^2ex^2 + 4\,b^3cdex + (2\,b^3c^2 - b^3)e)\log\left(dx + c + \sqrt{d}\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^
3*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 - b^3)*e)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*
a*b^2*c^2 - a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*(2*(2*a^2*b +
 b^3)*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x - (2*a^2*b + b^3 - 2*(2*a^2*b
+ b^3)*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*((2*a^2*b + b^3)
*d*e*x + (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^3\,dx = e\left(\int a^3c\,dx + \int a^3dx\,dx + \int b^3c\operatorname{acosh}^3(c + dx)\,dx + \int 3ab^2c\operatorname{acosh}^2(c + dx)\,dx + \int 3a^2bc\operatorname{acosh}(c + dx)\,dx + \int b^3dx\operatorname{acosh}^3(c + dx)\,dx + \int 3ab^2dx\operatorname{acosh}^2(c + dx)\,dx + \int 3a^2bdx\operatorname{acosh}(c + dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**3,x)
```

output
```
e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*acosh(c +
 d*x)**3, x) + Integral(3*a*b**2*c*acosh(c + d*x)**2, x) + Integral(3*a**2
*b*c*acosh(c + d*x), x) + Integral(b**3*d*x*acosh(c + d*x)**3, x) + Integr
al(3*a*b**2*d*x*acosh(c + d*x)**2, x) + Integral(3*a**2*b*d*x*acosh(c + d*
x), x))
```

Maxima [F]

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^3\,dx = \int (dex + ce)(b\operatorname{arcosh}(dx + c) + a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*e*x^2 + 3/4*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c
*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*d*e
 + a^3*c*e*x + 3*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*
b*c*e/d + 1/2*(b^3*d*e*x^2 + 2*b^3*c*e*x)*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3 + integrate(3/2*((2*a*b^2*d^4*e - b^3*d^4*e)*x^4 + 2*
(c^4*e - c^2*e)*a*b^2 + 4*(2*a*b^2*c*d^3*e - b^3*c*d^3*e)*x^3 + (2*(6*c^2*
d^2*e - d^2*e)*a*b^2 - (5*c^2*d^2*e - d^2*e)*b^3)*x^2 + (2*(c^3*e - c*e)*a
*b^2 + (2*a*b^2*d^3*e - b^3*d^3*e)*x^3 + 3*(2*a*b^2*c*d^2*e - b^3*c*d^2*e)
*x^2 - 2*(b^3*c^2*d*e - (3*c^2*d*e - d*e)*a*b^2)*x)*sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + 2*(2*(2*c^3*d*e - c*d*e)*a*b^2 - (c^3*d*e - c*d*e)*b^3)*x)
*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^3 + 3*c*d^2*x
^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (3*c^2*d - d)*x - c), x)
```

Giac [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^3\,dx = \int (dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}(c+d\,x)\right)^3\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{e\left(12acosh(dx+c)\,a^2b\,c^2+12acosh(dx+c)\,a^2bcdx+6acosh(dx+c)\,a^2b\,d^2x^2+9\sqrt{d^2x^2+2cdx+c^2-}\right.}{}$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^3,x)
```

output
```
(e*(12*acosh(c + d*x)*a**2*b*c**2 + 12*acosh(c + d*x)*a**2*b*c*d*x + 6*aco
sh(c + d*x)*a**2*b*d**2*x**2 + 9*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2
*b*c - 3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**2*b*d*x - 12*sqrt(c + d*x
 + 1)*sqrt(c + d*x - 1)*a**2*b*c + 4*int(acosh(c + d*x)**3,x)*b**3*c*d + 1
2*int(acosh(c + d*x)**2,x)*a*b**2*c*d + 4*int(acosh(c + d*x)**3*x,x)*b**3*
d**2 + 12*int(acosh(c + d*x)**2*x,x)*a*b**2*d**2 - 6*log(sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1) + c + d*x)*a**2*b*c**2 - 3*log(sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1) + c + d*x)*a**2*b + 4*a**3*c*d*x + 2*a**3*d**2*x**2))/(4*d)
```

3.35 $\int (a+b\operatorname{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 111

$$\begin{aligned}\int (a+b\operatorname{arccosh}(c+dx))^3\,dx = &-\frac{6b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d}\\&+\frac{6b^2(c+dx)(a+b\operatorname{arccosh}(c+dx))}{d}\\&-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^2}{d}\\&+\frac{(c+dx)(a+b\operatorname{arccosh}(c+dx))^3}{d}\end{aligned}$$

output
```
-6*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d+6*b^2*(d*x+c)*(a+b*arccosh(d*x+c)
)/d-3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^2/d+(d*x+c)*(
a+b*arccosh(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.51

$$\int (a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{a(a^2+6b^2)(c+dx)-3b(a^2+2b^2)\sqrt{-1+c+dx}\sqrt{1+c+dx}-3b\left(-a^2(c+dx)-2b^2(c+dx)+2ab\sqrt{}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[-1 + c + d*x]*Sqrt[1 +
 c + d*x] - 3*b*(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[
-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + b^3*(c + d*x)*ArcCos
h[c + d*x]^3)/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6410, 6294, 6330, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{6410}$$
$$\frac{\int (a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{6294}$$
$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^3-3b\int \frac{(c+dx)(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{d}$$
$$\Big\downarrow \text{6330}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^3-3b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2-2b\int(a+b\text{arccosh}(c+\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^3-3b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2-2b(a(c+dx)+b(c+\right.}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcCosh[c + d*x])^3 - 3*b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c
 + d*x]*(a + b*ArcCosh[c + d*x])^2 - 2*b*(a*(c + d*x) - b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x] + b*(c + d*x)*ArcCosh[c + d*x])))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left((dx+c)\operatorname{arccosh}(dx+c)^3-3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}+6(dx+c)\operatorname{arccosh}(dx+c)-6\sqrt{dx+c-}\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left((dx+c)\operatorname{arccosh}(dx+c)^3-3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}+6(dx+c)\operatorname{arccosh}(dx+c)-6\sqrt{dx+c-}\right.}{}$
parts	$x\,a^3+\frac{b^3\left((dx+c)\operatorname{arccosh}(dx+c)^3-3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}+6(dx+c)\operatorname{arccosh}(dx+c)-6\sqrt{dx+c-1}\right.}{d}$
orering	$\frac{(dx+c)(a+b\operatorname{arccosh}(dx+c))^3}{d}-\frac{3(d^2x^2+2cdx+c^2-2)(a+b\operatorname{arccosh}(dx+c))^2b}{d\sqrt{dx+c-1}\,\sqrt{dx+c+1}}-\frac{2(dx+c+1)(dx+c)(dx+c-1)\left(\right.}{}$

input
```
int((a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*((d*x+c)*arccosh(d*x+c)^3-3*arccosh(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))+3*a*b^2*((d*x+c)*arccosh(d*x+c)^2-2*arccosh(d*x+c)*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+3*a^2*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(103) = 206.

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.15

$$\int (a+b\operatorname{arccosh}(c+dx))^3\,dx$$
$$=\frac{(b^3dx+b^3c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^3+(a^3+6\,ab^2)dx+3\left(ab^2dx+ab^2c-\sqrt{d^2x^2+2\,c}\right.}{}$$

input
```
integrate((a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a
^3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - 3*(2*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*
c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*(a^2*b + 2*b^3))/d
```

Sympy [F]

$$\int (a+b\text{arccosh}(c+dx))^3\,dx=\int (a+b\operatorname{acosh}(c+dx))^3\;dx$$

input
```
integrate((a+b*acosh(d*x+c))**3,x)
```

output
```
Integral((a + b*acosh(c + d*x))**3, x)
```

Maxima [F]

$$\int (a+b\text{arccosh}(c+dx))^3\,dx=\int (b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
b^3*x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + a^3*x + 3*((d
*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b/d + integrate(3*((
c^3 - c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)
*x^2 + ((c^2 - 1)*a*b^2 + (a*b^2*d^2 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c
*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d - d)*a*b^2 - (c^2*d
 - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Giac [F]

$$\int (a+b\text{arccosh}(c+dx))^3\,dx=\int (b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arccosh}(c+dx))^3\,dx=\int (a+b\,\mathrm{acosh}(c+d\,x))^3\,dx$$

input
```
int((a + b*acosh(c + d*x))^3,x)
```

output
```
int((a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int (a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{3acosh(dx+c)\,a^2bc+3acosh(dx+c)\,a^2bdx-3\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,a^2b+\left(\int acosh(dx+c)^3\,dx\right)b}{d}$$

input
```
int((a+b*acosh(d*x+c))^3,x)
```

output
```
(3*acosh(c + d*x)*a**2*b*c + 3*acosh(c + d*x)*a**2*b*d*x - 3*sqrt(c + d*x
+ 1)*sqrt(c + d*x - 1)*a**2*b + int(acosh(c + d*x)**3,x)*b**3*d + 3*int(ac
osh(c + d*x)**2,x)*a*b**2*d + a**3*d*x)/d
```

3.36 $\int \frac{(a+b\mathbf{arccosh}(c+dx))^3}{ce+dex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{ce+dex}\,dx = -\frac{(a+b\text{arccosh}(c+dx))^4}{4bde}+\frac{(a+b\text{arccosh}(c+dx))^3\log\left(1+e^{2\text{arccosh}(c+dx)}\right)}{de} + \frac{3b(a+b\text{arccosh}(c+dx))^2\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(c+dx)}\right)}{2de} - \frac{3b^2(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(3,-e^{2\text{arccosh}(c+dx)}\right)}{2de} + \frac{3b^3\,\text{PolyLog}\left(4,-e^{2\text{arccosh}(c+dx)}\right)}{4de}$$

output
```
-1/4*(a+b*arccosh(d*x+c))^4/b/d/e+(a+b*arccosh(d*x+c))^3*ln(1+(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e+3/2*b*(a+b*arccosh(d*x+c))^2*polylog(2,
-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/2*b^2*(a+b*arccosh(d*x+c
))*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e+3/4*b^3*polyl
og(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{ce+dex}\,dx$$
$$= \frac{6a^2b\text{arccosh}(c+dx)^2+4ab^2\text{arccosh}(c+dx)^3+b^3\text{arccosh}(c+dx)^4+12a^2b\text{arccosh}(c+dx)\log\left(1+e^{-2\ldots}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(6*a^2*b*ArcCosh[c + d*x]^2 + 4*a*b^2*ArcCosh[c + d*x]^3 + b^3*ArcCosh[c +
 d*x]^4 + 12*a^2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + 12*
a*b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*b^3*ArcCosh[
c + d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*a^3*Log[c + d*x] - 6*b*(a
+ b*ArcCosh[c + d*x])^2*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] - 6*b^2*(a +
b*ArcCosh[c + d*x])*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*b^3*PolyLog[4
, -E^(-2*ArcCosh[c + d*x])])/(4*d*e)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6411, 27, 6297, 25, 3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{ce+dex}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6297}$$

$$\frac{\int -(a+b\text{arccosh}(c+dx))^3\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (a+b\text{arccosh}(c+dx))^3\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(a+b\text{arccosh}(c+dx))^3\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (a+b\text{arccosh}(c+dx))^3\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$$\Big\downarrow \text{4201}$$

$$\frac{i\left(2i\int \frac{e^{\frac{2(a-c-dx)}{b}}(a+b\text{arccosh}(c+dx))^3}{1+e^{\frac{2(a-c-dx)}{b}}}d(a+b\text{arccosh}(c+dx))-\frac{1}{4}i(a+b\text{arccosh}(c+dx))^4\right)}{bde}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(2i\left(\frac{3}{2}b\int (a+b\text{arccosh}(c+dx))^2\log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}(c+dx))-\frac{1}{2}b(a+b\text{arccosh}(c+dx))^3\log\right.\right.}{bde}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^2\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-b\int (a+b\text{arccosh}(c+dx))\text{PolyLog}\left(2,-e^{\frac{2(a-c-}{b}}\right.\right.\right.\right.}{bd}$$

$$\Big\downarrow \text{7163}$$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^2\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-b\left(\frac{1}{2}b\int\text{PolyLog}\left(3,-e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^2\,\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-b\left(-\frac{1}{4}b^2\int e^{-\frac{2(a-c-dx)}{b}}\,\text{PolyLog}(3,-c-dx)de^{\frac{2(a-}{}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^2\,\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-b\left(-\frac{1}{2}b(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(3,-e^{\frac{2(a}{}}\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(I*((-1/4*I)*(a + b*ArcCosh[c + d*x])^4 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c
+ d*x])^3*Log[1 + E^((2*(a - c - d*x))/b)]) + (3*b*((b*(a + b*ArcCosh[c +
d*x])^2*PolyLog[2, -E^((2*(a - c - d*x))/b)])/2 - b*(-1/2*(b*(a + b*ArcCos
h[c + d*x])*PolyLog[3, -E^((2*(a - c - d*x))/b)]) - (b^2*PolyLog[4, -c - d
*x])/4)))/2)))/(b*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{\frac{a^3\ln(dx+c)}{e}+\frac{b^3\left(-\frac{\operatorname{arccosh}(dx+c)^4}{4}+\operatorname{arccosh}(dx+c)^3\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\frac{3\operatorname{arccosh}(dx+c)^2\operatorname{polylog}\left(2,-(dx+\ldots\right.}{2}\ldots\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{a^3\ln(dx+c)}{e}+\frac{b^3\left(-\frac{\operatorname{arccosh}(dx+c)^4}{4}+\operatorname{arccosh}(dx+c)^3\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\frac{3\operatorname{arccosh}(dx+c)^2\operatorname{polylog}\left(2,-(dx+\ldots\right.}{2}\ldots\right.}{\ldots}}{\ldots}$
parts	$\frac{a^3\ln(dx+c)}{ed}+\frac{b^3\left(-\frac{\operatorname{arccosh}(dx+c)^4}{4}+\operatorname{arccosh}(dx+c)^3\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)+\frac{3\operatorname{arccosh}(dx+c)^2\operatorname{poly}\ldots}{\ldots}\ldots\right.}{\ldots}$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3/e*ln(d*x+c)+b^3/e*(-1/4*arccosh(d*x+c)^4+arccosh(d*x+c)^3*ln(1+(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/2*arccosh(d*x+c)^2*polylog(2,-(
d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*arccosh(d*x+c)*polylog(3,-(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/4*polylog(4,-(d*x+c+(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2))^2))+3*a*b^2/e*(-1/3*arccosh(d*x+c)^3+arccosh(d*x+c)^
2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+arccosh(d*x+c)*polylog(2
,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/2*polylog(3,-(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))^2))+3*a^2*b/e*(-1/2*arccosh(d*x+c)^2+arccosh(d*x
+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/2*polylog(2,-(d*x+c+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{ce + dex} dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^3}{dex + ce} dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{ce + dex} dx = \frac{\int \frac{a^3}{c+dx} dx + \int \frac{b^3 \operatorname{acosh}^3(c+dx)}{c+dx} dx + \int \frac{3ab^2 \operatorname{acosh}^2(c+dx)}{c+dx} dx + \int \frac{3a^2 b \operatorname{acosh}(c+dx)}{c+dx} dx}{e}$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e),x)
```

output
```
(Integral(a**3/(c + d*x), x) + Integral(b**3*acosh(c + d*x)**3/(c + d*x),
x) + Integral(3*a*b**2*acosh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b
*acosh(c + d*x)/(c + d*x), x))/e
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{ce + dex} dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^3}{dex + ce} dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^3*log(d*e*x + c*e)/(d*e) + integrate(b^3*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^3/(d*e*x + c*e) + 3*a*b^2*log(d*x + sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + c)^2/(d*e*x + c*e) + 3*a^2*b*log(d*x + sqrt(d*x + c +
1)*sqrt(d*x + c - 1) + c)/(d*e*x + c*e), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(a+b\,\text{acosh}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{ce + dex}\,dx$$
$$= \frac{3\left(\int \frac{acosh(dx+c)}{dx+c}dx\right)a^2bd + \left(\int \frac{acosh(dx+c)^3}{dx+c}dx\right)b^3d + 3\left(\int \frac{acosh(dx+c)^2}{dx+c}dx\right)a\,b^2d + \log(dx + c)\,a^3}{de}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e),x)
```

output
```
(3*int(acosh(c + d*x)/(c + d*x),x)*a**2*b*d + int(acosh(c + d*x)**3/(c + d
*x),x)*b**3*d + 3*int(acosh(c + d*x)**2/(c + d*x),x)*a*b**2*d + log(c + d*
x)*a**3)/(d*e)
```

3.37 $\int \frac{(a+b\,\mathrm{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx &= -\frac{(a+b\,\mathrm{arccosh}(c+dx))^3}{de^2(c+dx)}+\frac{6b(a+b\,\mathrm{arccosh}(c+dx))^2\arctan\left(e^{\mathrm{arccosh}(c+dx)}\right)}{de^2}\\&\quad-\frac{6ib^2(a+b\,\mathrm{arccosh}(c+dx))\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2}\\&\quad+\frac{6ib^2(a+b\,\mathrm{arccosh}(c+dx))\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2}\\&\quad+\frac{6ib^3\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2}-\frac{6ib^3\,\mathrm{PolyLog}\left(3,ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2}\end{aligned}$$

output

```
-(a+b*arccosh(d*x+c))^3/d/e^2/(d*x+c)+6*b*(a+b*arccosh(d*x+c))^2*arctan(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-6*I*b^2*(a+b*arccosh(d*x+c))*po
lylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+6*I*b^2*(a+b*arc
cosh(d*x+c))*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+6*
I*b^3*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2-6*I*b^3*
polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.76

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{(ce + dex)^2} dx =$$
$$-\frac{\frac{a^3}{c+dx} + \frac{3a^2b\text{arccosh}(c+dx)}{c+dx} + 3a^2b\arctan\left(\frac{1}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right) + 3iab^2\left(\text{arccosh}(c+dx)\left(-\frac{i\text{arccosh}(c+dx)}{c+dx}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
-((a^3/(c + d*x) + (3*a^2*b*ArcCosh[c + d*x])/(c + d*x) + 3*a^2*b*ArcTan[1
/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])] + (3*I)*a*b^2*(ArcCosh[c + d*x]*(
((-I)*ArcCosh[c + d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Lo
g[1 + I/E^ArcCosh[c + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*P
olyLog[2, I/E^ArcCosh[c + d*x]]) + b^3*(ArcCosh[c + d*x]^3/(c + d*x) - (3*
I)*(-(ArcCosh[c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcC
osh[c + d*x]])) - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]]
- PolyLog[2, I/E^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]
] + 2*PolyLog[3, I/E^ArcCosh[c + d*x]])))/(d*e^2))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6411, 27, 6298, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{(ce + dex)^2} dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{e^2(c+dx)^2}d(c+dx)}{d}$$

$\downarrow$ 27

$$\frac{\int \frac{(a+b\operatorname{arccosh}(c+dx))^3}{(c+dx)^2}d(c+dx)}{de^2}$$

$\downarrow$ 6298

$$\frac{3b\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}}{de^2}$$

$\downarrow$ 6362

$$\frac{3b\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{c+dx}d\operatorname{arccosh}(c+dx)-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}}{de^2}$$

$\downarrow$ 3042

$$\frac{-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}+3b\int (a+b\operatorname{arccosh}(c+dx))^2\csc\left(i\operatorname{arccosh}(c+dx)+\frac{\pi}{2}\right)d\operatorname{arccosh}(c+dx)}{de^2}$$

$\downarrow$ 4668

$$\frac{-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}+3b\left(-2ib\int (a+b\operatorname{arccosh}(c+dx))\log\left(1-ie^{\operatorname{arccosh}(c+dx)}\right)d\operatorname{arccosh}(c+dx)+2ib\int (a+b\right.}{de^2}$$

$\downarrow$ 3011

$$-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}+3b\left(2ib\left(b\int \operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(c+dx)}\right)d\operatorname{arccosh}(c+dx)-\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(c+dx)}\right)\right.\right.$$

$\downarrow$ 2720

$$-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}+3b\left(2ib\left(b\int e^{-\operatorname{arccosh}(c+dx)}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arccosh}(c+dx)}\right)de^{\operatorname{arccosh}(c+dx)}-\operatorname{PolyLog}\left(2,-ie^{a}\right.\right.\right.$$

$\downarrow$ 7143

$$-\frac{(a+b\operatorname{arccosh}(c+dx))^3}{c+dx}+3b\left(2\arctan\left(e^{\operatorname{arccosh}(c+dx)}\right)(a+b\operatorname{arccosh}(c+dx))^2+2ib\left(b\operatorname{PolyLog}\left(3,-ie^{\operatorname{arccosh}(c+dx)}\right)\right.\right.$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcCosh[c + d*x])^3/(c + d*x)) + 3*b*(2*(a + b*ArcCosh[c + d*x])
^2*ArcTan[E^ArcCosh[c + d*x]] + (2*I)*b*(-((a + b*ArcCosh[c + d*x])*PolyLo
g[2, (-I)*E^ArcCosh[c + d*x]]) + b*PolyLog[3, (-I)*E^ArcCosh[c + d*x]]) -
(2*I)*b*(-((a + b*ArcCosh[c + d*x])*PolyLog[2, I*E^ArcCosh[c + d*x]]) + b*
PolyLog[3, I*E^ArcCosh[c + d*x]])))/(d*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arccosh}\,(dx+c))^3}{(dex+ce)^2}dx$$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

output
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^3\operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3a^2b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*acosh(c +
d*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*acosh(c + d*x
)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*acosh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\text{acosh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

Reduce [F]

$$\int \frac{(a + b\,\mathrm{arccosh}(c + dx))^3}{(ce + dex)^2}\,dx$$
$$= \frac{3\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right)a^2b\,c^2 + 3\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right)a^2bcdx + \left(\int \frac{acosh(dx+c)^3}{d^2x^2+2cdx+c^2}dx\right)b^3c^2 + \left(\int \frac{acosh(dx+c)^3}{d^2x^2+2cdx+c^2}dx\right.}{c\,e^2\,(dx + c)}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

output
```
(3*int(acosh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b*c**2 + 3*int(
acosh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b*c*d*x + int(acosh(c
+ d*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)*b**3*c**2 + int(acosh(c + d*x)**
3/(c**2 + 2*c*d*x + d**2*x**2),x)*b**3*c*d*x + 3*int(acosh(c + d*x)**2/(c*
*2 + 2*c*d*x + d**2*x**2),x)*a*b**2*c**2 + 3*int(acosh(c + d*x)**2/(c**2 +
 2*c*d*x + d**2*x**2),x)*a*b**2*c*d*x + a**3*x)/(c*e**2*(c + d*x))
```

3.38 $\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3} dx = \frac{3b(a+b\text{arccosh}(c+dx))^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\text{arccosh}(c+dx))^3}{2de^3(c+dx)^2} - \frac{3b^2(a+b\text{arccosh}(c+dx))\log\left(1+e^{2\text{arccosh}(c+dx)}\right)}{de^3} - \frac{3b^3\operatorname{PolyLog}\left(2,-e^{2\text{arccosh}(c+dx)}\right)}{2de^3}$$

output
```
3/2*b*(a+b*arccosh(d*x+c))^2/d/e^3+3/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(
a+b*arccosh(d*x+c))^2/d/e^3/(d*x+c)-1/2*(a+b*arccosh(d*x+c))^3/d/e^3/(d*x+
c)^2-3*b^2*(a+b*arccosh(d*x+c))*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))^2)/d/e^3-3/2*b^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/
d/e^3
```

Mathematica [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.62

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$= \frac{-\frac{a^3}{(c+dx)^2} + \frac{3a^2b\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}(c+c^2+2cdx+dx(1+dx))-\text{arccosh}(c+dx)\right)}{(c+dx)^2} - \frac{b^3\text{arccosh}(c+dx)^3}{(c+dx)^2} + 6ab^2\left(\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\text{a}}{c+dx}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
(-(a^3/(c + d*x)^2) + (3*a^2*b*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^
2 + 2*c*d*x + d*x*(1 + d*x)) - ArcCosh[c + d*x]))/(c + d*x)^2 - (b^3*ArcCo
sh[c + d*x]^3)/(c + d*x)^2 + 6*a*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*
(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x
)^2) - Log[c + d*x]) + 3*b^3*(ArcCosh[c + d*x]*(-ArcCosh[c + d*x] + (Sqrt[
(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) -
2*Log[1 + E^(-2*ArcCosh[c + d*x])]) + PolyLog[2, -E^(-2*ArcCosh[c + d*x])]
))/(2*d*e^3)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6411, 27, 6298, 6333, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{3}{2}b\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6333}$$

$$\frac{\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2b\int \frac{a+b\text{arccosh}(c+dx)}{c+dx}d(c+dx)\right)-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6297}$$

$$\frac{\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2\int -\left((a+b\text{arccosh}(c+dx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)d(a+b\text{ar}}{de^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}b\left(2\int (a+b\text{arccosh}(c+dx))\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arcc}}{c+dx}\right.}{de^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}+2\int -i(a+b\text{arccosh}(c+dx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{ar}}{}\right.\right.}{de^3}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2i\int (a+b\text{arccosh}(c+dx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcc}}{}\right.\right.}{de^3}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2i\left(2i\int \frac{e^{\frac{2(a-c-dx)}{b}}(a+b\text{arccosh}(c+dx))}{1+e^{\frac{2(a-c-dx)}{b}}}d(a+b\text{arc}\right.\right.}{de^3}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}\right.\right.\right.}{de^3}$$

$\Big\downarrow$ 2715

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2i\left(2i\left(-\frac{1}{4}b^2\int e^{-\frac{2(a-c-dx)}{b}}\log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)\right.\right.\right.}{de^3}$$

$\Big\downarrow$ 2838

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{c+dx}-2i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-c-dx)-\frac{1}{2}b(a+\text{barc}\right.\right.\right.}{de^3}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcCosh[c + d*x])^3/(c + d*x)^2 + (3*b*((Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(c + d*x) - (2*I)*((-1/2*I)*(
a + b*ArcCosh[c + d*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c + d*x])*Log[1
+ E^((2*(a - c - d*x))/b)]) + (b^2*PolyLog[2, -c - d*x])/4))))/2)/(d*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6333
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
 && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\operatorname{arccosh}(dx+c)^2\left(-3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+3(dx+c)^2+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}+3\operatorname{arccosh}(dx+c)^2-3\operatorname{arccosh}(}{e^3}$
default	$\frac{-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\operatorname{arccosh}(dx+c)^2\left(-3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+3(dx+c)^2+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}+3\operatorname{arccosh}(dx+c)^2-3\operatorname{arccosh}(}{e^3}$
parts	$-\frac{a^3}{2e^3(dx+c)^2d}+\frac{b^3\left(-\frac{\operatorname{arccosh}(dx+c)^2\left(-3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+3(dx+c)^2+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}+3\operatorname{arccosh}(dx+c)^2-3}{}$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^3/e^3/(d*x+c)^2+b^3/e^3*(-1/2*arccosh(d*x+c)^2*(-3*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)*(d*x+c)+3*(d*x+c)^2+arccosh(d*x+c))/(d*x+c)^2+3*arcco
sh(d*x+c)^2-3*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^
2)-3/2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))+3*a*b^2/e^3*
(2*arccosh(d*x+c)-1/2*arccosh(d*x+c)*(2*(d*x+c)^2-2*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)*(d*x+c)+arccosh(d*x+c))/(d*x+c)^2-ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))^2))+3*a^2*b/e^3*(-1/2/(d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3
*e^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$=\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^3\,\text{acosh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3ab^2\,\text{acosh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3a^2b\,\text{acosh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(3*a*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*e^3*x + c*d^2
*e^3) - log(d*x + c)/(d*e^3))*a*b^2 - 1/2*(log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2*integr
ate(3/2*(d^2*x^2 + 2*c*d*x + sqrt(d*x + c + 1)*(d*x + c)*sqrt(d*x + c - 1)
 + c^2 - 1)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^5*e^3*
x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^2*d^3*e^3 - d^3*e^3)*x^3
 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (d^4*e^3*x^4 + 4*c*d^3*e^3*x^3 + c
^4*e^3 - c^2*e^3 + (6*c^2*d^2*e^3 - d^2*e^3)*x^2 + 2*(2*c^3*d*e^3 - c*d*e^
3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^3 - 3*c^2*d*e^3)*x)
, x))*b^3 + 3/2*a^2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*
d^2*e^3) - arccosh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3
/2*a*b^2*arccosh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/
2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^3}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{(ce + dex)^3} \, dx = \int \frac{(a + b\,\text{acosh}(c + d\,x))^3}{(c\,e + d\,e\,x)^3} \, dx$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

Reduce [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{(ce + dex)^3} \, dx = \frac{6\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)a^2b\,c^2d + 12\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)a^2bc\,d^2x + 6\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}d\right.}{}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e)^3,x)
```

output
```
(6*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a
**2*b*c**2*d + 12*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3),x)*a**2*b*c*d**2*x + 6*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3),x)*a**2*b*d**3*x**2 + 2*int(acosh(c + d*x)**3/(
c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**3*c**2*d + 4*int(acos
h(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**3*c*d*
*2*x + 2*int(acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3),x)*b**3*d**3*x**2 + 6*int(acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*a*b**2*c**2*d + 12*int(acosh(c + d*x)**2/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**2*c*d**2*x + 6*int(acosh(
c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**2*d**3
*x**2 - a**3)/(2*d*e**3*(c**2 + 2*c*d*x + d**2*x**2))
```

3.39 $\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 297

$$\begin{aligned}
&\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^4} dx \\
&= \frac{b^2(a+b\text{arccosh}(c+dx))}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^2}{2de^4(c+dx)^2} \\
&\quad - \frac{(a+b\text{arccosh}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b(a+b\text{arccosh}(c+dx))^2 \arctan\left(e^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3 \arctan\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{de^4} \\
&\quad - \frac{ib^2(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{ib^2(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(2,ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{ib^3\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right)}{de^4} - \frac{ib^3\,\text{PolyLog}\left(3,ie^{\text{arccosh}(c+dx)}\right)}{de^4}
\end{aligned}$$

output
```
b^2*(a+b*arccosh(d*x+c))/d/e^4/(d*x+c)+1/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)^2-1/3*(a+b*arccosh(d*x+c))^3/d/e^4
/(d*x+c)^3+b*(a+b*arccosh(d*x+c))^2*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))/d/e^4-b^3*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4-I*b^2*(a+b
*arccosh(d*x+c))*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e
^4+I*b^2*(a+b*arccosh(d*x+c))*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)))/d/e^4+I*b^3*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
/d/e^4-I*b^3*polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4
```

Mathematica [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^3}{(ce+dex)^4}\,dx$$

$$= \frac{-\frac{2a^3}{(c+dx)^3} + \frac{3a^2b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{(c+dx)^2} - \frac{6a^2b\operatorname{arccosh}(c+dx)}{(c+dx)^3} - 3a^2b\arctan\left(\frac{1}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right) + 6ab^2\left(\frac{1}{c+dx} + \sqrt{\ }\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
((-2*a^3)/(c + d*x)^3 + (3*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c
+ d*x)^2 - (6*a^2*b*ArcCosh[c + d*x])/(c + d*x)^3 - 3*a^2*b*ArcTan[1/(Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])] + 6*a*b^2*((c + d*x)^(-1) + (Sqrt[(-1 +
 c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - Arc
Cosh[c + d*x]^2/(c + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d
*x]] + I*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I
)/E^ArcCosh[c + d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + b^3*((6*ArcC
osh[c + d*x])/(c + d*x) + (3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d
*x)*ArcCosh[c + d*x]^2)/(c + d*x)^2 - (2*ArcCosh[c + d*x]^3)/(c + d*x)^3 +
 (3*I)*((4*I)*ArcTan[E^ArcCosh[c + d*x]] + ArcCosh[c + d*x]^2*Log[1 - I*E^
ArcCosh[c + d*x]] - ArcCosh[c + d*x]^2*Log[1 + I*E^ArcCosh[c + d*x]] - 2*A
rcCosh[c + d*x]*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + 2*ArcCosh[c + d*x]*P
olyLog[2, I*E^ArcCosh[c + d*x]] + 2*PolyLog[3, (-I)*E^ArcCosh[c + d*x]] -
2*PolyLog[3, I*E^ArcCosh[c + d*x]])))/(6*d*e^4)
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6411, 27, 6298, 6348, 6298, 103, 216, 6362, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^4}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{b\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{ 6348}$$

$$\frac{b\left(-b\int \frac{a+b\text{arccosh}(c+dx)}{(c+dx)^2}d(c+dx)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}{2(c+dx)^2}\right)}{de^4}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{b\left(-b\left(b\int \frac{1}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}\right)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-}}{}\right.}{de^4}$$

$$\Big\downarrow \text{ 103}$$

$$\frac{b\left(-b\left(b\int \frac{1}{(c+dx-1)(c+dx+1)+1}d\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}\right)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+\right.}{de^4}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{b\left(\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-b\left(b\arctan\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}\right)+\frac{\sqrt{c+dx-1}\sqrt{}}{}\right.}{de^4}$$

$$\Big\downarrow \; 6362$$

$$\frac{b\left(\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}d\text{arccosh}(c+dx)-b\left(b\arctan\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)-\frac{a+b\text{arccosh}(c+dx)}{c+dx}\right)+\frac{\sqrt{c+d}}{}\right.}{de^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^3}{3(c+dx)^3}+b\left(\frac{1}{2}\int (a+b\text{arccosh}(c+dx))^2\csc\left(i\text{arccosh}(c+dx)+\frac{\pi}{2}\right)d\text{arccosh}(c+dx)-b\left(b\arctan\right.\right.}{de^4}$$

$$\Big\downarrow \; 4668$$

$$-\frac{(a+b\text{arccosh}(c+dx))^3}{3(c+dx)^3}+b\left(\frac{1}{2}\left(-2ib\int (a+b\text{arccosh}(c+dx))\log\left(1-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)+2ib\int (a+\right.\right.$$

$$\Big\downarrow \; 3011$$

$$-\frac{(a+b\text{arccosh}(c+dx))^3}{3(c+dx)^3}+b\left(\frac{1}{2}\left(2ib\left(b\int \text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)-\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$-\frac{(a+b\text{arccosh}(c+dx))^3}{3(c+dx)^3}+b\left(\frac{1}{2}\left(2ib\left(b\int e^{-\text{arccosh}(c+dx)}\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)de^{\text{arccosh}(c+dx)}-\text{PolyLog}\left(2,-ie\right.\right.\right.\right.$$

$$\Big\downarrow \; 7143$$

$$-\frac{(a+b\text{arccosh}(c+dx))^3}{3(c+dx)^3}+b\left(\frac{1}{2}\left(2\arctan\left(e^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))^2+2ib\left(b\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcCosh[c + d*x])^3/(c + d*x)^3 + b*((Sqrt[-1 + c + d*x]*Sqrt
[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*(c + d*x)^2) - b*(-((a + b*Ar
cCosh[c + d*x])/(c + d*x)) + b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]
]) + (2*(a + b*ArcCosh[c + d*x])^2*ArcTan[E^ArcCosh[c + d*x]] + (2*I)*b*(-
((a + b*ArcCosh[c + d*x])*PolyLog[2, (-I)*E^ArcCosh[c + d*x]]) + b*PolyLog
[3, (-I)*E^ArcCosh[c + d*x]]) - (2*I)*b*(-((a + b*ArcCosh[c + d*x])*PolyLo
g[2, I*E^ArcCosh[c + d*x]]) + b*PolyLog[3, I*E^ArcCosh[c + d*x]]))/2))/(d*
e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6348
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp[b*c*(n/(f
*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]
   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^3}{(dex+ce)^4}dx$$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

output
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 +
 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^3}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int}{e^4}$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**3*acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*acosh(c + d
*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),
 x) + Integral(3*a**2*b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x*
*2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^4*e^4*x^3
 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 +
 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 - c)*
a*b^2 + (c^3 - c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b
^3*c*d^2)*x^2 + (b^3*c^2 + 3*(c^2 - 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2
 + 2*(3*a*b^2*c*d + b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(
3*c^2*d - d)*a*b^2 + (3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d -
d)*a^2*b*x + (c^3 - c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 - 1)*
a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 +
 (21*c^2*d^5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7
*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2
+ (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d
^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2
*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) + (7*c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(a+b\,\text{acosh}(c+d\,x))^3}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

Reduce [F]

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^3}{(ce + dex)^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

output
```
(9*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**
3 + d**4*x**4),x)*a**2*b*c**3*d + 27*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x
 + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a**2*b*c**2*d**2*x + 2
7*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3
 + d**4*x**4),x)*a**2*b*c*d**3*x**2 + 9*int(acosh(c + d*x)/(c**4 + 4*c**3*
d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a**2*b*d**4*x**3 +
3*int(acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x
**3 + d**4*x**4),x)*b**3*c**3*d + 9*int(acosh(c + d*x)**3/(c**4 + 4*c**3*d
*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**3*c**2*d**2*x + 9
*int(acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x*
*3 + d**4*x**4),x)*b**3*c*d**3*x**2 + 3*int(acosh(c + d*x)**3/(c**4 + 4*c*
*3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**3*d**4*x**3 +
 9*int(acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*
x**3 + d**4*x**4),x)*a*b**2*c**3*d + 27*int(acosh(c + d*x)**2/(c**4 + 4*c*
*3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a*b**2*c**2*d**2
*x + 27*int(acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*
d**3*x**3 + d**4*x**4),x)*a*b**2*c*d**3*x**2 + 9*int(acosh(c + d*x)**2/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a*b**2*
d**4*x**3 - a**3)/(3*d*e**4*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
))
```

3.40 $\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 377

$$\begin{aligned}
\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^4\,dx
&= \frac{45b^4e^3(c+dx)^2}{128d}+\frac{3b^4e^3(c+dx)^4}{128d}\\
&\quad-\frac{45b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))}{64d}\\
&\quad-\frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))}{32d}\\
&\quad-\frac{45b^2e^3(a+b\operatorname{arccosh}(c+dx))^2}{128d}+\frac{9b^2e^3(c+dx)^2(a+b\operatorname{arccosh}(c+dx))^2}{16d}\\
&\quad+\frac{3b^2e^3(c+dx)^4(a+b\operatorname{arccosh}(c+dx))^2}{16d}\\
&\quad-\frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^3}{8d}\\
&\quad-\frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^3}{4d}\\
&\quad-\frac{3e^3(a+b\operatorname{arccosh}(c+dx))^4}{32d}+\frac{e^3(c+dx)^4(a+b\operatorname{arccosh}(c+dx))^4}{4d}
\end{aligned}$$

output
```
45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/64*b^3*e^3*(d*x+c-
1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-3/32*b^3*e^3*(d*x+
c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-45/128*b^2*e^3
*(a+b*arccosh(d*x+c))^2/d+9/16*b^2*e^3*(d*x+c)^2*(a+b*arccosh(d*x+c))^2/d+
3/16*b^2*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^2/d-3/8*b*e^3*(d*x+c-1)^(1/2)*
(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^3/d-1/4*b*e^3*(d*x+c-1)^(1/2)
*(d*x+c)^3*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^3/d-3/32*e^3*(a+b*arccosh(
d*x+c))^4/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.49

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^4\,dx$$
$$= \frac{e^3\left(9b^2(8a^2+5b^2)(c+dx)^2+(32a^4+24a^2b^2+3b^4)(c+dx)^4+2ab\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^3*(9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
 + d*x)^4 + 2*a*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^
2 + 15*b^2) - 2*(8*a^2 + 3*b^2)*(c + d*x)^2) + 2*b*(c + d*x)*(72*a*b^2*(c
+ d*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 - 72*a^2*b*Sqrt[-1 + c
+ d*x]*Sqrt[1 + c + d*x] - 45*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 4
8*a^2*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 6*b^3*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-24*a^2
 - 15*b^2 + 24*b^2*(c + d*x)^2 + 64*a^2*(c + d*x)^4 + 8*b^2*(c + d*x)^4 -
48*a*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 32*a*b*Sqrt[-1 + c
 + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 16*b^3*(-3*a +
 8*a*(c + d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*
b*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 4
*b^4*(-3 + 8*(c + d*x)^4)*ArcCosh[c + d*x]^4 - 6*a*b*(8*a^2 + 15*b^2)*Log[
c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(128*d)
```

Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6411, 27, 6298, 6354, 6298, 6354, 15, 6298, 6308, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\int \frac{(c+dx)^4(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6354}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\int (c+dx)^3(a+b\text{arccosh}(c+dx))^2d(c+dx)+\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{ar}}{\sqrt{c+dx-1}}\right.\right.}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\int \frac{(c+dx)^4(a+b\text{arccosh}(c+}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.$$

$$\Big\downarrow \text{ 6354}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 15}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 6298

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 6308

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 6354

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(c+d}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.\right.\right.\right.$$

$\downarrow$ 15

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^2-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\text{arccosh}(c+d}{\sqrt{c+dx-1}\sqrt{c+dx-}}\right.\right.\right.\right.\right.$$

$\downarrow$ 6308

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^4-b\left(\frac{1}{4}\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-\frac{3}{4}b\left(\frac{1}{4}(\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcCosh[c + d*x])^4)/4 - b*((Sqrt[-1 + c + d*x]*
(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/4 - (3*b*(((c +
d*x)^4*(a + b*ArcCosh[c + d*x])^2)/4 - (b*(-1/16*(b*(c + d*x)^4) + (Sqrt[-
1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/4 + (
3*(-1/4*(b*(c + d*x)^2) + (Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*
(a + b*ArcCosh[c + d*x]))/2 + (a + b*ArcCosh[c + d*x])^2/(4*b)))/4))/2))/4
 + (3*((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
d*x])^3)/2 + (a + b*ArcCosh[c + d*x])^4/(8*b) - (3*b*(((c + d*x)^2*(a + b*
ArcCosh[c + d*x])^2)/2 - b*(-1/4*(b*(c + d*x)^2) + (Sqrt[-1 + c + d*x]*(c
+ d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/2 + (a + b*ArcCosh[c +
d*x])^2/(4*b))))/2))/4)))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{\frac{e^3a^4(dx+c)^4}{4}+e^3b^4\left(\frac{(dx+c)^4\operatorname{arccosh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4}-\frac{3\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+}}{8}\right.}{}$
default	$\frac{\frac{e^3a^4(dx+c)^4}{4}+e^3b^4\left(\frac{(dx+c)^4\operatorname{arccosh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4}-\frac{3\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+}}{8}\right.}{}$
parts	$\frac{e^3a^4(dx+c)^4}{4d}+\frac{e^3b^4\left(\frac{(dx+c)^4\operatorname{arccosh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4}-\frac{3\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{}}{8}\right.}{}$
orering	Expression too large to display

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^4*(d*x+c)^4+e^3*b^4*(1/4*(d*x+c)^4*arccosh(d*x+c)^4-1/4*(d*
x+c)^3*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/8*arccosh(d*x+c)
^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-3/32*arccosh(d*x+c)^4+3/16*(d*x
+c)^4*arccosh(d*x+c)^2-3/32*(d*x+c)^3*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)-45/64*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-45
/128*arccosh(d*x+c)^2+3/128*(d*x+c)^4+45/128*(d*x+c)^2+9/16*(d*x+c)^2*arcc
osh(d*x+c)^2)+4*e^3*a*b^3*(1/4*(d*x+c)^4*arccosh(d*x+c)^3-3/16*(d*x+c)^3*a
rccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-9/32*arccosh(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-3/32*arccosh(d*x+c)^3+3/32*(d*x+c)^4*a
rccosh(d*x+c)-3/128*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-45/256*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-45/256*arccosh(d*x+c)+9/32*(d*x+c)^2*ar
ccosh(d*x+c))+6*e^3*a^2*b^2*(1/4*(d*x+c)^4*arccosh(d*x+c)^2-1/8*(d*x+c)^3*
arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/16*arccosh(d*x+c)*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)-3/32*arccosh(d*x+c)^2+1/32*(d*x+c)^4+3/32
*(d*x+c)^2)+4*e^3*a^3*b*(1/4*(d*x+c)^4*arccosh(d*x+c)-1/32*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*(2*(d*x+c)^3*((d*x+c)^2-1)^(1/2)+3*(d*x+c)*((d*x+c)^2-1)^
(1/2)+3*ln(d*x+c+((d*x+c)^2-1)^(1/2)))/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(339) = 678$.

Time = 0.13 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.28

$$\int (ce + dex)^3 (a + b\operatorname{arccosh}(c + dx))^4 \, dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2
+ 3*b^4)*c*d^3*e^3*x^3 + 3*(24*a^2*b^2 + 15*b^4 + 2*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 + 9*(8*a
^2*b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 +
 48*b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4
 + 32*a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*
x + (8*a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2
 + 3*(2*b^4*c^2 + b^4)*d*e^3*x + (2*b^4*c^3 + 3*b^4*c)*e^3)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 +
3*(8*(8*a^2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 +
24*(b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 + 16*(3*b^4*c + 2*(8*a^2*b^
2 + b^4)*c^3)*d*e^3*x + (24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*a^2*b^2
 - 15*b^4)*e^3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*
b^3*c^2 + a*b^3)*d*e^3*x + (2*a*b^3*c^3 + 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 2*
(8*(8*a^3*b + 3*a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3
+ 24*(3*a*b^3 + 2*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 + 16*(9*a*b^3*c + 2
*(8*a^3*b + 3*a*b^3)*c^3)*d*e^3*x + (72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*
c^4 - 24*a^3*b - 45*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6...
```

Sympy [F]

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^4\,dx$$
$$= e^3\Bigg(\int a^4c^3\,dx + \int a^4d^3x^3\,dx + \int b^4c^3\operatorname{acosh}^4(c+dx)\,dx + \int 4ab^3c^3\operatorname{acosh}^3(c+dx)\,dx + \int 6a^2b^2c^3\operatorname{acosh}^2(c+dx)\,dx + \int 4a^3bc^3\operatorname{acosh}(c+dx)\,dx + \int 3a^4cd^2x^2\,dx + \int 3a^4c^2dx\,dx + \int b^4d^3x^3\operatorname{acosh}^4(c+dx)\,dx + \int 4ab^3d^3x^3\operatorname{acosh}^3(c+dx)\,dx + \int 6a^2b^2d^3x^3\operatorname{acosh}^2(c+dx)\,dx + \int 4a^3bd^3x^3\operatorname{acosh}(c+dx)\,dx + \int 3b^4cd^2x^2\operatorname{acosh}^4(c+dx)\,dx + \int 3b^4c^2dx\operatorname{acosh}^4(c+dx)\,dx + \int 12ab^3cd^2x^2\operatorname{acosh}^3(c+dx)\,dx + \int 12ab^3c^2dx\operatorname{acosh}^3(c+dx)\,dx + \int 18a^2b^2cd^2x^2\operatorname{acosh}^2(c+dx)\,dx + \int 18a^2b^2c^2dx\operatorname{acosh}^2(c+dx)\,dx + \int 12a^3bcd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 12a^3bc^2dx\operatorname{acosh}(c+dx)\,dx\Bigg)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**4,x)
```

output
```
e**3*(Integral(a**4*c**3, x) + Integral(a**4*d**3*x**3, x) + Integral(b**4
*c**3*acosh(c + d*x)**4, x) + Integral(4*a*b**3*c**3*acosh(c + d*x)**3, x)
 + Integral(6*a**2*b**2*c**3*acosh(c + d*x)**2, x) + Integral(4*a**3*b*c**
3*acosh(c + d*x), x) + Integral(3*a**4*c*d**2*x**2, x) + Integral(3*a**4*c
**2*d*x, x) + Integral(b**4*d**3*x**3*acosh(c + d*x)**4, x) + Integral(4*a
*b**3*d**3*x**3*acosh(c + d*x)**3, x) + Integral(6*a**2*b**2*d**3*x**3*aco
sh(c + d*x)**2, x) + Integral(4*a**3*b*d**3*x**3*acosh(c + d*x), x) + Inte
gral(3*b**4*c*d**2*x**2*acosh(c + d*x)**4, x) + Integral(3*b**4*c**2*d*x*a
cosh(c + d*x)**4, x) + Integral(12*a*b**3*c*d**2*x**2*acosh(c + d*x)**3, x
) + Integral(12*a*b**3*c**2*d*x*acosh(c + d*x)**3, x) + Integral(18*a**2*b
**2*c*d**2*x**2*acosh(c + d*x)**2, x) + Integral(18*a**2*b**2*c**2*d*x*aco
sh(c + d*x)**2, x) + Integral(12*a**3*b*c*d**2*x**2*acosh(c + d*x), x) + I
ntegral(12*a**3*b*c**2*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^4\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*a^4*d^3*e^3*x^4 + a^4*c*d^2*e^3*x^3 + 3/2*a^4*c^2*d*e^3*x^2 + 3*(2*x^2
*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arccosh(
d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d
^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^3*b*c*d^
2*e^3 + 1/24*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(
2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d
 + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4
+ 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*
d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^3*b*d^
3*e^3 + a^4*c^3*e^3*x + 4*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 -
 1))*a^3*b*c^3*e^3/d + 1/4*(b^4*d^3*e^3*x^4 + 4*b^4*c*d^2*e^3*x^3 + 6*b...
```

Giac [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^4\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^4\,dx=\text{Too large to display}$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c))^4,x)
```

output
```
(e**3*(32*acosh(c + d*x)*a**3*b*c**4 + 32*acosh(c + d*x)*a**3*b*c**3*d*x +
 48*acosh(c + d*x)*a**3*b*c**2*d**2*x**2 + 32*acosh(c + d*x)*a**3*b*c*d**3
*x**3 + 8*acosh(c + d*x)*a**3*b*d**4*x**4 + 30*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)*a**3*b*c**3 - 6*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b*c**2
*d*x - 6*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b*c*d**2*x**2 - 3*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b*c - 2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*a**3*b*d**3*x**3 - 3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b*d
*x - 32*sqrt(c + d*x + 1)*sqrt(c + d*x - 1)*a**3*b*c**3 + 8*int(acosh(c +
d*x)**4,x)*b**4*c**3*d + 32*int(acosh(c + d*x)**3,x)*a*b**3*c**3*d + 48*in
t(acosh(c + d*x)**2,x)*a**2*b**2*c**3*d + 8*int(acosh(c + d*x)**4*x**3,x)*
b**4*d**4 + 24*int(acosh(c + d*x)**4*x**2,x)*b**4*c*d**3 + 24*int(acosh(c
+ d*x)**4*x,x)*b**4*c**2*d**2 + 32*int(acosh(c + d*x)**3*x**3,x)*a*b**3*d*
*4 + 96*int(acosh(c + d*x)**3*x**2,x)*a*b**3*c*d**3 + 96*int(acosh(c + d*x
)**3*x,x)*a*b**3*c**2*d**2 + 48*int(acosh(c + d*x)**2*x**3,x)*a**2*b**2*d*
*4 + 144*int(acosh(c + d*x)**2*x**2,x)*a**2*b**2*c*d**3 + 144*int(acosh(c
+ d*x)**2*x,x)*a**2*b**2*c**2*d**2 - 24*log(sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1) + c + d*x)*a**3*b*c**4 - 3*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
 + c + d*x)*a**3*b + 8*a**4*c**3*d*x + 12*a**4*c**2*d**2*x**2 + 8*a**4*c*d
**3*x**3 + 2*a**4*d**4*x**4))/(8*d)
```

3.41 $\int (ce+dex)^2(a+b\,\text{arccosh}(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$
\begin{aligned}
\int (ce+dex)^2(a+b\,\text{arccosh}(c+dx))^4\,dx
&= \frac{160}{27}b^4e^2x+\frac{8b^4e^2(c+dx)^3}{81d} \\
&\quad -\frac{160b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))}{27d} \\
&\quad -\frac{8b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))}{27d} \\
&\quad +\frac{8b^2e^2(c+dx)(a+b\,\text{arccosh}(c+dx))^2}{3d}+\frac{4b^2e^2(c+dx)^3(a+b\,\text{arccosh}(c+dx))^2}{9d} \\
&\quad -\frac{8be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))^3}{9d} \\
&\quad -\frac{4be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))^3}{9d} \\
&\quad +\frac{e^2(c+dx)^3(a+b\,\text{arccosh}(c+dx))^4}{3d}
\end{aligned}
$$

output
```
160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d-160/27*b^3*e^2*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d-8/27*b^3*e^2*(d*x+c-1)^(1/2)*(d*x+c)
^2*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d+8/3*b^2*e^2*(d*x+c)*(a+b*arccosh
(d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^2/d-8/9*b*e^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^3/d-4/9*b*e^2*(d*x+c-1)^(
1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^3/d+1/3*e^2*(d*x+c)^3*
(a+b*arccosh(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.54

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^4\,dx$$
$$=\frac{e^2\left(24b^2(9a^2+20b^2)(c+dx)+(27a^4+36a^2b^2+8b^4)(c+dx)^3+12ab\sqrt{-1+c+dx}\sqrt{1+c+dx}(-6a^2\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^2*(24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
 + d*x)^3 + 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-6*a^2 - 40*b^2 -
 (3*a^2 + 2*b^2)*(c + d*x)^2) + 12*b*(36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)
^3 + 6*a*b^2*(c + d*x)^3 - 18*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] -
 40*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 9*a^2*b*Sqrt[-1 + c + d*x]*
(c + d*x)^2*Sqrt[1 + c + d*x] - 2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[
1 + c + d*x])*ArcCosh[c + d*x] + 18*b^2*(12*b^2*(c + d*x) + 9*a^2*(c + d*x
)^3 + 2*b^2*(c + d*x)^3 - 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 6*
a*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 -
 36*b^3*(-3*a*(c + d*x)^3 + 2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*S
qrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 27*b
^4*(c + d*x)^3*ArcCosh[c + d*x]^4))/(81*d)
```

Rubi [A] (verified)

Time = 3.52 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6411, 27, 6298, 6354, 6298, 6330, 6294, 6330, 24, 6354, 15, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\int \frac{(c+dx)^3(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(-b\int (c+dx)^2(a+b\text{arccosh}(c+dx))^2d(c+dx)+\frac{2}{3}\int \frac{(c+dx)(a+b\text{arc}}{\sqrt{c+dx-1}}\right.\right.}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\int \frac{(c+dx)^3(a+b\text{arccosh}(c}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{6330}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\int \frac{(c+dx)^3(a+b\text{arccosh}(c}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{6294}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-3b\left((c+dx)(\right.\right.\right.\right.$$

$\downarrow$ 6330

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(\frac{2}{3}(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-3b((c+dx)(\right.\right.$$

$\downarrow$ 24

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\int\frac{(c+dx)^3(a+b\text{arccosh}(c+}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.$$

$\downarrow$ 6354

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 15

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b\left(\frac{2}{3}\int\frac{(c+dx)(a+b\text{arccosh}}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 6330

$$e^2(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b(-b(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^2-\frac{2}{3}b(\frac{2}{3}(\sqrt{c+dx-1}\sqrt{c+dx}$$

$\downarrow$ 24

$$e^2(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^4-\frac{4}{3}b(\frac{1}{3}\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-b(\frac{1}{3}(c$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^4)/3 - (4*b*((Sqrt[-1 + c + d*
x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/3 - b*(((c +
d*x)^3*(a + b*ArcCosh[c + d*x])^2)/3 - (2*b*(-1/9*(b*(c + d*x)^3) + (Sqrt[
-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/3 +
(2*(-(b*(c + d*x)) + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c
 + d*x])))/3))/3) + (2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCos
h[c + d*x])^3 - 3*b*((c + d*x)*(a + b*ArcCosh[c + d*x])^2 - 2*b*(-(b*(c +
d*x)) + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])))))/
3))/3))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{\frac{e^2a^4(dx+c)^3}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^4}{3}-\frac{8\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{9}-\frac{4(dx+c)^2\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx}}{9}\right.}{}$
default	$\frac{\frac{e^2a^4(dx+c)^3}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^4}{3}-\frac{8\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{9}-\frac{4(dx+c)^2\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx}}{9}\right.}{}$
parts	$\frac{e^2a^4(dx+c)^3}{3d}+\frac{e^2b^4\left(\frac{(dx+c)^3\operatorname{arccosh}(dx+c)^4}{3}-\frac{8\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{9}-\frac{4(dx+c)^2\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}}{9}\right.}{}$
orering	Expression too large to display

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^4*(d*x+c)^3+e^2*b^4*(1/3*(d*x+c)^3*arccosh(d*x+c)^4-8/9*arc
cosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4/9*(d*x+c)^2*arccosh(d*x+c)
^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+8/3*(d*x+c)*arccosh(d*x+c)^2-160/27*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+160/27*d*x+160/27*c+4/9*(d*x+c
)^3*arccosh(d*x+c)^2-8/27*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(
d*x+c)^2+8/81*(d*x+c)^3)+4*e^2*a*b^3*(1/3*(d*x+c)^3*arccosh(d*x+c)^3-2/3*a
rccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/3*(d*x+c)^2*arccosh(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4/3*(d*x+c)*arccosh(d*x+c)-40/27*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)+2/9*(d*x+c)^3*arccosh(d*x+c)-2/27*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)*(d*x+c)^2)+6*e^2*a^2*b^2*(1/3*(d*x+c)^3*arccosh(d*x+c)
^2-4/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/9*arccosh(d*x+c)*(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+4/9*d*x+4/9*c+2/27*(d*x+c)^3)+4*e
^2*a^3*b*(1/3*(d*x+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
*((d*x+c)^2+2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(275) = 550$.

Time = 0.13 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.88

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
 8*b^4)*c*d^2*e^2*x^2 + 3*(72*a^2*b^2 + 160*b^4 + (27*a^4 + 36*a^2*b^2 + 8
*b^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2
*d*e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4
 + 36*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x +
 3*a*b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 + 2*b^4)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*
b^4)*c*d^2*e^2*x^2 + 3*(4*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x + (12*b^4
*c + (9*a^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2
*x + (a*b^3*c^2 + 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^
3*e^2*x^3 + 9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 + 9*(4*a*b^3 + (3*a^3*b +
2*a*b^3)*c^2)*d*e^2*x + 3*(12*a*b^3*c + (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9
*a^2*b^2 + 2*b^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x + (18*a^2*
b^2 + 40*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 12*((3*a^3*b + 2*
a*b^3)*d^2*e^2*x^2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x + (6*a^3*b + 40*a*b^3
 + (3*a^3*b + 2*a*b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$
\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^4\,dx \\
&= e^2\left(\int a^4c^2\,dx + \int a^4d^2x^2\,dx + \int b^4c^2\operatorname{acosh}^4(c+dx)\,dx \right.\\
&\quad + \int 4ab^3c^2\operatorname{acosh}^3(c+dx)\,dx + \int 6a^2b^2c^2\operatorname{acosh}^2(c+dx)\,dx \\
&\quad + \int 4a^3bc^2\operatorname{acosh}(c+dx)\,dx + \int 2a^4cdx\,dx + \int b^4d^2x^2\operatorname{acosh}^4(c+dx)\,dx \\
&\quad + \int 4ab^3d^2x^2\operatorname{acosh}^3(c+dx)\,dx + \int 6a^2b^2d^2x^2\operatorname{acosh}^2(c+dx)\,dx \\
&\quad + \int 4a^3bd^2x^2\operatorname{acosh}(c+dx)\,dx + \int 2b^4cdx\operatorname{acosh}^4(c+dx)\,dx \\
&\quad + \int 8ab^3cdx\operatorname{acosh}^3(c+dx)\,dx + \int 12a^2b^2cdx\operatorname{acosh}^2(c+dx)\,dx \\
&\quad \left. + \int 8a^3bcdx\operatorname{acosh}(c+dx)\,dx\right)
\end{aligned}
$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**4,x)
```

output
```
e**2*(Integral(a**4*c**2, x) + Integral(a**4*d**2*x**2, x) + Integral(b**4
*c**2*acosh(c + d*x)**4, x) + Integral(4*a*b**3*c**2*acosh(c + d*x)**3, x)
 + Integral(6*a**2*b**2*c**2*acosh(c + d*x)**2, x) + Integral(4*a**3*b*c**
2*acosh(c + d*x), x) + Integral(2*a**4*c*d*x, x) + Integral(b**4*d**2*x**2
*acosh(c + d*x)**4, x) + Integral(4*a*b**3*d**2*x**2*acosh(c + d*x)**3, x)
 + Integral(6*a**2*b**2*d**2*x**2*acosh(c + d*x)**2, x) + Integral(4*a**3*
b*d**2*x**2*acosh(c + d*x), x) + Integral(2*b**4*c*d*x*acosh(c + d*x)**4,
x) + Integral(8*a*b**3*c*d*x*acosh(c + d*x)**3, x) + Integral(12*a**2*b**2
*c*d*x*acosh(c + d*x)**2, x) + Integral(8*a**3*b*c*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^4\,dx=\int (dex+ce)^2(b\,\text{arcosh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*a^4*d^2*e^2*x^3 + a^4*c*d*e^2*x^2 + 2*(2*x^2*arccosh(d*x + c) - d*(3*c
^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*c/d^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^
3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*e^2*x + 4*((d*
x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c^2*e^2/d + 1/3*(b^
4*d^2*e^2*x^3 + 3*b^4*c*d*e^2*x^2 + 3*b^4*c^2*e^2*x)*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c)^4 + integrate(2/3*(2*((3*a*b^3*d^5*e^2 - b^4
*d^5*e^2)*x^5 + 3*(c^5*e^2 - c^3*e^2)*a*b^3 + 5*(3*a*b^3*c*d^4*e^2 - b^4*c
*d^4*e^2)*x^4 + (3*(10*c^2*d^3*e^2 - d^3*e^2)*a*b^3 - (10*c^2*d^3*e^2 - d^
3*e^2)*b^4)*x^3 + 3*((10*c^3*d^2*e^2 - 3*c*d^2*e^2)*a*b^3 - (3*c^3*d^2*e^2
 - c*d^2*e^2)*b^4)*x^2 + (3*(c^4*e^2 - c^2*e^2)*a*b^3 + (3*a*b^3*d^4*e^2 -
 b^4*d^4*e^2)*x^4 + 4*(3*a*b^3*c*d^3*e^2 - b^4*c*d^3*e^2)*x^3 - 3*(2*b^4*c
^2*d^2*e^2 - (6*c^2*d^2*e^2 - d^2*e^2)*a*b^3)*x^2 - 3*(b^4*c^3*d*e^2 - 2*(
2*c^3*d*e^2 - c*d*e^2)*a*b^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ...
```

Giac [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^4\,dx=\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{acosh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^4\,dx$$
$$=\frac{e^2\left(36acosh(dx+c)\,a^3b\,c^3+36acosh(dx+c)\,a^3b\,c^2dx+36acosh(dx+c)\,a^3bc\,d^2x^2+12acosh(dx+c)\,a^3\right.}{}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^4,x)
```

output
```
(e**2*(36*acosh(c + d*x)*a**3*b*c**3 + 36*acosh(c + d*x)*a**3*b*c**2*d*x +
 36*acosh(c + d*x)*a**3*b*c*d**2*x**2 + 12*acosh(c + d*x)*a**3*b*d**3*x**3
 + 32*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b*c**2 - 8*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*a**3*b*c*d*x - 4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1
)*a**3*b*d**2*x**2 - 8*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b - 36*sq
rt(c + d*x + 1)*sqrt(c + d*x - 1)*a**3*b*c**2 + 9*int(acosh(c + d*x)**4,x)
*b**4*c**2*d + 36*int(acosh(c + d*x)**3,x)*a*b**3*c**2*d + 54*int(acosh(c
+ d*x)**2,x)*a**2*b**2*c**2*d + 9*int(acosh(c + d*x)**4*x**2,x)*b**4*d**3
+ 18*int(acosh(c + d*x)**4*x,x)*b**4*c*d**2 + 36*int(acosh(c + d*x)**3*x**
2,x)*a*b**3*d**3 + 72*int(acosh(c + d*x)**3*x,x)*a*b**3*c*d**2 + 54*int(ac
osh(c + d*x)**2*x**2,x)*a**2*b**2*d**3 + 108*int(acosh(c + d*x)**2*x,x)*a*
*2*b**2*c*d**2 - 24*log(sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a*
*3*b*c**3 + 9*a**4*c**2*d*x + 9*a**4*c*d**2*x**2 + 3*a**4*d**3*x**3))/(9*d
)
```

3.42 $\int (ce+dex)(a+b\,\text{arccosh}(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 209

$$\int (ce+dex)(a+b\,\text{arccosh}(c+dx))^4\,dx = \frac{3b^4e(c+dx)^2}{4d} - \frac{3b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))}{2d} - \frac{3b^2e(a+b\,\text{arccosh}(c+dx))^2}{4d} + \frac{3b^2e(c+dx)^2(a+b\,\text{arccosh}(c+dx))^2}{2d} - \frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\,\text{arccosh}(c+dx))^3}{d} - \frac{e(a+b\,\text{arccosh}(c+dx))^4}{4d} + \frac{e(c+dx)^2(a+b\,\text{arccosh}(c+dx))^4}{2d}$$

output
```
3/4*b^4*e*(d*x+c)^2/d-3/2*b^3*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a
+b*arccosh(d*x+c))/d-3/4*b^2*e*(a+b*arccosh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)^
2*(a+b*arccosh(d*x+c))^2/d-b*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+
b*arccosh(d*x+c))^3/d-1/4*e*(a+b*arccosh(d*x+c))^4/d+1/2*e*(d*x+c)^2*(a+b*
arccosh(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.72

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^4\,dx$$
$$=\frac{e\left((2a^4+6a^2b^2+3b^4)(c+dx)^2-2ab(2a^2+3b^2)\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}-2b(c+dx)\left(-4a\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*Sqrt[-
1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*(c + d*x)*(-4*a^3*(c + d*x)
 - 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 3*b^
3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-2*a^2 -
 b^2 + 4*a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*Sqrt[-1 + c + d*x]*(c
 + d*x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 4*b^3*(-a + 2*a*(c + d*x)^
2 - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 +
 b^4*(-1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^4 - 2*a*b*(2*a^2 + 3*b^2)*Log[c
 + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(4*d)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {6411, 27, 6298, 6354, 6298, 6308, 6354, 15, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int e(c+dx)(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int (c+dx)(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$\downarrow$ 6298

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\int \frac{(c+dx)^2(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\downarrow$ 6354

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\left(-\frac{3}{2}b\int (c+dx)(a+b\text{arccosh}(c+dx))^2d(c+dx)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c}{\sqrt{c+dx-1}\sqrt{c+c}}\right.\right.}{d}$$

$\downarrow$ 6298

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\int \frac{(c+dx)^2(a+b\text{arccosh}(c+c}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.$$

$\downarrow$ 6308

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\int \frac{(c+dx)^2(a+b\text{arccosh}(c+c}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{d}$$

$\downarrow$ 6354

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(\right.\right.\right.\right.$$

$\downarrow$ 15

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^2-b\left(\frac{1}{2}\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(\right.\right.\right.\right.$$

$\downarrow$ 6308

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^4-2b\left(\frac{(a+b\text{arccosh}(c+dx))^4}{8b}+\frac{1}{2}\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}(a+\text{barcco}\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^4)/2 - 2*b*((Sqrt[-1 + c + d*x]*
(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/2 + (a + b*ArcCosh
[c + d*x])^4/(8*b) - (3*b*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/2 - b*
(-1/4*(b*(c + d*x)^2) + (Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a
 + b*ArcCosh[c + d*x]))/2 + (a + b*ArcCosh[c + d*x])^2/(4*b))))/2)))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{\frac{e\,a^{4}(dx+c)^{2}}{2}+e\,b^{4}\left(\frac{\cosh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{arccosh}(dx+c)^{4}}{4}-\frac{\sinh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{arccosh}(dx+c)^{3}}{2}+\frac{3\cosh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{ar}}{4}\right.}{}$
default	$\frac{\frac{e\,a^{4}(dx+c)^{2}}{2}+e\,b^{4}\left(\frac{\cosh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{arccosh}(dx+c)^{4}}{4}-\frac{\sinh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{arccosh}(dx+c)^{3}}{2}+\frac{3\cosh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{ar}}{4}\right.}{}$
parts	$e\,a^{4}\left(\frac{1}{2}d\,x^{2}+cx\right)+\frac{e\,b^{4}\left(\frac{\cosh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{arccosh}(dx+c)^{4}}{4}-\frac{\sinh(2\ \mathrm{arccosh}(dx+c))\ \mathrm{arccosh}(dx+c)^{3}}{2}+\frac{3\cosh(2\ \mathrm{arc}}{}\right.}{d}$
orering	Expression too large to display

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^4*(d*x+c)^2+e*b^4*(1/4*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^
4-1/2*sinh(2*arccosh(d*x+c))*arccosh(d*x+c)^3+3/4*cosh(2*arccosh(d*x+c))*a
rccosh(d*x+c)^2-3/4*sinh(2*arccosh(d*x+c))*arccosh(d*x+c)+3/8*cosh(2*arcco
sh(d*x+c)))+4*e*a*b^3*(1/4*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^3-3/8*sin
h(2*arccosh(d*x+c))*arccosh(d*x+c)^2+3/8*cosh(2*arccosh(d*x+c))*arccosh(d*
x+c)-3/16*sinh(2*arccosh(d*x+c)))+6*e*a^2*b^2*(1/4*cosh(2*arccosh(d*x+c))*
arccosh(d*x+c)^2-1/4*sinh(2*arccosh(d*x+c))*arccosh(d*x+c)+1/8*cosh(2*arcc
osh(d*x+c)))+4*e*a^3*b*(1/2*(d*x+c)^2*arccosh(d*x+c)-1/4*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*((d*x+c)*((d*x+c)^2-1)^(1/2)+ln(d*x+c+((d*x+c)^2-1)^(1/2)))
/((d*x+c)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(189) = 378$.

Time = 0.11 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.77

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^4 \, dx$$
$$= \frac{(2\,a^4 + 6\,a^2b^2 + 3\,b^4)d^2ex^2 + 2\,(2\,a^4 + 6\,a^2b^2 + 3\,b^4)cdex + (2\,b^4d^2ex^2 + 4\,b^4cdex + (2\,b^4c^2 - b^4)e)\log\left(d\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)
*c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 - b^4)*e)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b
^3*c*d*e*x + (2*a*b^3*c^2 - a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 +
 3*(2*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x - (2*a^2*b
^2 + b^4 - 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1))^2 + 2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x
 - (2*a^3*b + 3*a*b^3 - 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4
)*d*e*x + (2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x
+ (2*a^3*b + 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (ce+dex)(a+b\operatorname{arccosh}(c+dx))^4\,dx = e\bigg(\int a^4c\,dx + \int a^4dx\,dx + \int b^4c\operatorname{acosh}^4(c+dx)\,dx + \int 4ab^3c\operatorname{acosh}^3(c+dx)\,dx + \int 6a^2b^2c\operatorname{acosh}^2(c+dx)\,dx + \int 4a^3bc\operatorname{acosh}(c+dx)\,dx + \int b^4dx\operatorname{acosh}^4(c+dx)\,dx + \int 4ab^3dx\operatorname{acosh}^3(c+dx)\,dx + \int 6a^2b^2dx\operatorname{acosh}^2(c+dx)\,dx + \int 4a^3bdx\operatorname{acosh}(c+dx)\,dx\bigg)$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**4,x)
```

output
```
e*(Integral(a**4*c, x) + Integral(a**4*d*x, x) + Integral(b**4*c*acosh(c +
 d*x)**4, x) + Integral(4*a*b**3*c*acosh(c + d*x)**3, x) + Integral(6*a**2
*b**2*c*acosh(c + d*x)**2, x) + Integral(4*a**3*b*c*acosh(c + d*x), x) + I
ntegral(b**4*d*x*acosh(c + d*x)**4, x) + Integral(4*a*b**3*d*x*acosh(c + d
*x)**3, x) + Integral(6*a**2*b**2*d*x*acosh(c + d*x)**2, x) + Integral(4*a
**3*b*d*x*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^4\,dx=\int (dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2*a^4*d*e*x^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d +
 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^3*b*d*e + a
^4*c*e*x + 4*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c*
e/d + 1/2*(b^4*d*e*x^2 + 2*b^4*c*e*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + c)^4 + integrate(2*((2*(c^4*e - c^2*e)*a*b^3 + (2*a*b^3*d^4*e
- b^4*d^4*e)*x^4 + 4*(2*a*b^3*c*d^3*e - b^4*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e
 - d^2*e)*a*b^3 - (5*c^2*d^2*e - d^2*e)*b^4)*x^2 + (2*(c^3*e - c*e)*a*b^3
+ (2*a*b^3*d^3*e - b^4*d^3*e)*x^3 + 3*(2*a*b^3*c*d^2*e - b^4*c*d^2*e)*x^2
- 2*(b^4*c^2*d*e - (3*c^2*d*e - d*e)*a*b^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + 2*(2*(2*c^3*d*e - c*d*e)*a*b^3 - (c^3*d*e - c*d*e)*b^4)*x)*log(
d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*(a^2*b^2*d^4*e*x^4 +
4*a^2*b^2*c*d^3*e*x^3 + (6*c^2*d^2*e - d^2*e)*a^2*b^2*x^2 + 2*(2*c^3*d*e -
 c*d*e)*a^2*b^2*x + (c^4*e - c^2*e)*a^2*b^2 + (a^2*b^2*d^3*e*x^3 + 3*a^2*b
^2*c*d^2*e*x^2 + (3*c^2*d*e - d*e)*a^2*b^2*x + (c^3*e - c*e)*a^2*b^2)*sqrt
(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + c)^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Giac [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^4\,dx=\int (dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}(c+d\,x)\right)^4\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^4\,dx = \frac{e\big(8acosh(dx+c)\,a^3b\,c^2+8acosh(dx+c)\,a^3bcdx+4acosh(dx+c)\,a^3b\,d^2x^2+6\sqrt{d^2x^2+2cdx+c^2-1}\,a}{}$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^4,x)
```

output
```
(e*(8*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c + d*x)*a**3*b*c*d*x + 4*acosh
(c + d*x)*a**3*b*d**2*x**2 + 6*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b
*c - 2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*a**3*b*d*x - 8*sqrt(c + d*x +
1)*sqrt(c + d*x - 1)*a**3*b*c + 2*int(acosh(c + d*x)**4,x)*b**4*c*d + 8*in
t(acosh(c + d*x)**3,x)*a*b**3*c*d + 12*int(acosh(c + d*x)**2,x)*a**2*b**2*
c*d + 2*int(acosh(c + d*x)**4*x,x)*b**4*d**2 + 8*int(acosh(c + d*x)**3*x,x
)*a*b**3*d**2 + 12*int(acosh(c + d*x)**2*x,x)*a**2*b**2*d**2 - 4*log(sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a**3*b*c**2 - 2*log(sqrt(c**2 +
 2*c*d*x + d**2*x**2 - 1) + c + d*x)*a**3*b + 2*a**4*c*d*x + a**4*d**2*x**
2))/(2*d)
```

3.43 $\int (a+b\text{arccosh}(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 129

$$\int (a+b\text{arccosh}(c+dx))^4\,dx = 24b^4x - \frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))}{d} + \frac{12b^2(c+dx)(a+b\text{arccosh}(c+dx))^2}{d} - \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^3}{d} + \frac{(c+dx)(a+b\text{arccosh}(c+dx))^4}{d}$$

output
```
24*b^4*x-24*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))/d+12*
b^2*(d*x+c)*(a+b*arccosh(d*x+c))^2/d-4*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(
a+b*arccosh(d*x+c))^3/d+(d*x+c)*(a+b*arccosh(d*x+c))^4/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. $2(129) = 258$.

Time = 0.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.02

$$\int (a + b\text{arccosh}(c + dx))^4 \, dx = \frac{(a^4 + 12a^2b^2 + 24b^4)(c + dx) - 4ab(a^2 + 6b^2)\sqrt{-1 + c + dx}\sqrt{1 + c + dx} - 4b\left(-a^3(c + dx) - 6ab^2(c +\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^
2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1
 + c + d*x])*ArcCosh[c + d*x] + 6*b^2*(a^2*(c + d*x) + 2*b^2*(c + d*x) - 2
*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 4*b^3*(-(a
*(c + d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 +
 b^4*(c + d*x)*ArcCosh[c + d*x]^4)/d
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6410, 6294, 6330, 6294, 6330, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\text{arccosh}(c + dx))^4 \, dx$$

$$\Big\downarrow \; 6410$$

$$\frac{\int (a + b\text{arccosh}(c + dx))^4 d(c + dx)}{d}$$

$$\Big\downarrow \; 6294$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^4-4b\int \frac{(c+dx)(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{d}$$

$\downarrow$ 6330

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^4-4b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-3b\int (a+b\text{arccosh}(c+\right.}{d}$$

$\downarrow$ 6294

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^4-4b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-3b\left((c+dx)(a+\text{barc}\right.\right.}{d}$$

$\downarrow$ 6330

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^4-4b(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-3b((c+dx)(a+\text{barcc}}{d}$$

$\downarrow$ 24

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^4-4b(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3-3b((c+dx)(a+\text{barcc}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
((c + d*x)*(a + b*ArcCosh[c + d*x])^4 - 4*b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c
 + d*x]*(a + b*ArcCosh[c + d*x])^3 - 3*b*((c + d*x)*(a + b*ArcCosh[c + d*x
])^2 - 2*b*(-(b*(c + d*x)) + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])))))/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(121) = 242$.

Time = 0.27 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.13

method	result
derivativedivides	$\frac{(dx+c)a^4+b^4\left((dx+c)\operatorname{arccosh}(dx+c)^4-4\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+12(dx+c)\operatorname{arccosh}(dx+c)^2-24\operatorname{arcco}\right.}{}$
default	$\frac{(dx+c)a^4+b^4\left((dx+c)\operatorname{arccosh}(dx+c)^4-4\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+12(dx+c)\operatorname{arccosh}(dx+c)^2-24\operatorname{arcco}\right.}{}$
parts	$x\,a^4+\frac{b^4\left((dx+c)\operatorname{arccosh}(dx+c)^4-4\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+12(dx+c)\operatorname{arccosh}(dx+c)^2-24\operatorname{arccosh}\right.}{d}$
orering	Expression too large to display

input
```
int((a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^4+b^4*((d*x+c)*arccosh(d*x+c)^4-4*arccosh(d*x+c)^3*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)*arccosh(d*x+c)^2-24*arccosh(d*x+c)*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)+24*d*x+24*c)+4*a*b^3*((d*x+c)*arccosh(d*x+c)^3
-3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+6*(d*x+c)*arccosh(d*x+
c)-6*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))+6*a^2*b^2*((d*x+c)*arccosh(d*x+c)^2-
2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+4*a^3*b*((d*x+
c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(121) = 242$.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.67

$$\int (a + b\text{arccosh}(c + dx))^4 \, dx$$
$$= \frac{(b^4dx + b^4c)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)^4 + 4\left(ab^3dx + ab^3c - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}b^4\right)\log}{}$$

input
```
integrate((a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*
(a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^4)*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x -
 6*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a
^2*b^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 4*
((a^3*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)) - 4*(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [F]

$$\int (a + b\text{arccosh}(c + dx))^4 \, dx = \int (a + b\operatorname{acosh}(c + dx))^4 \, dx$$

input
```
integrate((a+b*acosh(d*x+c))**4,x)
```

output
```
Integral((a + b*acosh(c + d*x))**4, x)
```

Maxima [F]

$$\int (a + b\text{arccosh}(c + dx))^4 \, dx = \int (b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4 + a^4*x + 4*((d
*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b/d + integrate(2*(2
*((c^3 - c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d
^2)*x^2 + ((c^2 - 1)*a*b^3 + (a*b^3*d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^
4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d - d)*a*b^3 - (c^
2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*
(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d - d)*a^2*b^2*x + (c^3 -
c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 - 1)*a^2*b^2)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*s
qrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Giac [F]

$$\int (a + b\text{arccosh}(c + dx))^4 \, dx = \int (b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arccosh}(c+dx))^4\,dx=\int (a+b\,\text{acosh}(c+d\,x))^4\,dx$$

input
```
int((a + b*acosh(c + d*x))^4,x)
```

output
```
int((a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int (a+b\text{arccosh}(c+dx))^4\,dx$$
$$=\frac{4acosh(dx+c)\,a^3bc+4acosh(dx+c)\,a^3bdx-4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,a^3b+\left(\int acosh(dx+c)^4\,dx\right)b}{d}$$

input
```
int((a+b*acosh(d*x+c))^4,x)
```

output
```
(4*acosh(c + d*x)*a**3*b*c + 4*acosh(c + d*x)*a**3*b*d*x - 4*sqrt(c + d*x
+ 1)*sqrt(c + d*x - 1)*a**3*b + int(acosh(c + d*x)**4,x)*b**4*d + 4*int(ac
osh(c + d*x)**3,x)*a*b**3*d + 6*int(acosh(c + d*x)**2,x)*a**2*b**2*d + a**
4*d*x)/d
```

3.44 $\int \frac{(a+b\operatorname{arccosh}(c+dx))^4}{ce+dex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\begin{aligned}\int \frac{(a+b\operatorname{arccosh}(c+dx))^4}{ce+dex}\,dx &= -\frac{(a+b\operatorname{arccosh}(c+dx))^5}{5bde}+\frac{(a+b\operatorname{arccosh}(c+dx))^4\log\left(1+e^{2\operatorname{arccosh}(c+dx)}\right)}{de}\\&+\frac{2b(a+b\operatorname{arccosh}(c+dx))^3\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(c+dx)}\right)}{de}\\&-\frac{3b^2(a+b\operatorname{arccosh}(c+dx))^2\operatorname{PolyLog}\left(3,-e^{2\operatorname{arccosh}(c+dx)}\right)}{de}\\&+\frac{3b^3(a+b\operatorname{arccosh}(c+dx))\operatorname{PolyLog}\left(4,-e^{2\operatorname{arccosh}(c+dx)}\right)}{de}\\&-\frac{3b^4\operatorname{PolyLog}\left(5,-e^{2\operatorname{arccosh}(c+dx)}\right)}{2de}\end{aligned}$$

output
```
-1/5*(a+b*arccosh(d*x+c))^5/b/d/e+(a+b*arccosh(d*x+c))^4*ln(1+(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e+2*b*(a+b*arccosh(d*x+c))^3*polylog(2,-(
d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3*b^2*(a+b*arccosh(d*x+c))^2
*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e+3*b^3*(a+b*arcc
osh(d*x+c))*polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/2*
b^4*polylog(5,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{ce+dex}\,dx$$
$$= \frac{2a^3b\text{arccosh}(c+dx)^2+2a^2b^2\text{arccosh}(c+dx)^3+ab^3\text{arccosh}(c+dx)^4+\frac{1}{5}b^4\text{arccosh}(c+dx)^5+4a^3b\text{arcc}}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

output
```
(2*a^3*b*ArcCosh[c + d*x]^2 + 2*a^2*b^2*ArcCosh[c + d*x]^3 + a*b^3*ArcCosh
[c + d*x]^4 + (b^4*ArcCosh[c + d*x]^5)/5 + 4*a^3*b*ArcCosh[c + d*x]*Log[1
+ E^(-2*ArcCosh[c + d*x])] + 6*a^2*b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*Ar
cCosh[c + d*x])] + 4*a*b^3*ArcCosh[c + d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*
x])] + b^4*ArcCosh[c + d*x]^4*Log[1 + E^(-2*ArcCosh[c + d*x])] + a^4*Log[c
 + d*x] - 2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2, -E^(-2*ArcCosh[c + d*x
])] - 3*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])
] - 3*a*b^3*PolyLog[4, -E^(-2*ArcCosh[c + d*x])] - 3*b^4*ArcCosh[c + d*x]*
PolyLog[4, -E^(-2*ArcCosh[c + d*x])] - (3*b^4*PolyLog[5, -E^(-2*ArcCosh[c
+ d*x])])/2)/(d*e)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6411, 27, 6297, 25, 3042, 26, 4201, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{ce+dex}\,dx$$
$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{e(c+dx)}d(c+dx)}{d}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}d(c+dx)}{de}$$

$\Big\downarrow$ 6297

$$\frac{\int -(a+b\text{arccosh}(c+dx))^4 \tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$\Big\downarrow$ 25

$$-\frac{\int (a+b\text{arccosh}(c+dx))^4 \tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$\Big\downarrow$ 3042

$$-\frac{\int -i(a+b\text{arccosh}(c+dx))^4 \tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$\Big\downarrow$ 26

$$\frac{i\int (a+b\text{arccosh}(c+dx))^4 \tan\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)d(a+b\text{arccosh}(c+dx))}{bde}$$

$\Big\downarrow$ 4201

$$\frac{i\left(2i\int \frac{e^{\frac{2(a-c-dx)}{b}}(a+b\text{arccosh}(c+dx))^4}{1+e^{\frac{2(a-c-dx)}{b}}}d(a+b\text{arccosh}(c+dx))-\frac{1}{5}i(a+b\text{arccosh}(c+dx))^5\right)}{bde}$$

$\Big\downarrow$ 2620

$$\frac{i\left(2i\left(2b\int (a+b\text{arccosh}(c+dx))^3 \log\left(1+e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arccosh}(c+dx))-\frac{1}{2}b(a+b\text{arccosh}(c+dx))^4 \log\left(\right.\right.\right.}{bde}$$

$\Big\downarrow$ 3011

$$\frac{i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^3 \text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-\frac{3}{2}b\int (a+b\text{arccosh}(c+dx))^2 \text{PolyLog}\left(2,-e^{\frac{2(a-}{}}\right.\right.\right.\right.}{b}$$

$\Big\downarrow$ 7163

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^3\,\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-\frac{3}{2}b\left(b\int(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(3,-e^{\frac{2(a\cdots}{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 7163

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^3\,\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-\frac{3}{2}b\left(b\left(\frac{1}{2}b\int\text{PolyLog}\left(4,-e^{\frac{2(a-c-dx)}{b}}\right)d(a+b\text{arcc}\cdots\right.\right.\right.\right.\right.$$

$\downarrow$ 2720

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^3\,\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-\frac{3}{2}b\left(b\left(-\frac{1}{4}b^2\int e^{-\frac{2(a-c-dx)}{b}}\,\text{PolyLog}(4,-c-dx)de\cdots\right.\right.\right.\right.\right.$$

$\downarrow$ 7143

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))^3\,\text{PolyLog}\left(2,-e^{\frac{2(a-c-dx)}{b}}\right)-\frac{3}{2}b\left(b\left(-\frac{1}{2}b(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(4,-\cdots\right.\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

output
```
(I*((-1/5*I)*(a + b*ArcCosh[c + d*x])^5 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c
+ d*x])^4*Log[1 + E^((2*(a - c - d*x))/b)]) + 2*b*((b*(a + b*ArcCosh[c + d
*x])^3*PolyLog[2, -E^((2*(a - c - d*x))/b)])/2 - (3*b*(-1/2*(b*(a + b*ArcC
osh[c + d*x])^2*PolyLog[3, -E^((2*(a - c - d*x))/b)]) + b*(-1/2*(b*(a + b*
ArcCosh[c + d*x])*PolyLog[4, -E^((2*(a - c - d*x))/b)]) - (b^2*PolyLog[5,
-c - d*x])/4)))/2))))/(b*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(258) = 516$.

Time = 0.42 (sec) , antiderivative size = 582, normalized size of antiderivative = 3.03

method	result
derivativedivides	$\frac{\frac{a^4 \ln(dx+c)}{e} + \frac{b^4\left(-\frac{\operatorname{arccosh}(dx+c)^5}{5} + \operatorname{arccosh}(dx+c)^4 \ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right) + 2\operatorname{arccosh}(dx+c)^3 \operatorname{polylog}\left(2, -(dx+c\right.\right.}{}}{}$
default	$\frac{\frac{a^4 \ln(dx+c)}{e} + \frac{b^4\left(-\frac{\operatorname{arccosh}(dx+c)^5}{5} + \operatorname{arccosh}(dx+c)^4 \ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right) + 2\operatorname{arccosh}(dx+c)^3 \operatorname{polylog}\left(2, -(dx+c\right.\right.}{}}{}$
parts	$\frac{a^4 \ln(dx+c)}{ed} + \frac{b^4\left(-\frac{\operatorname{arccosh}(dx+c)^5}{5} + \operatorname{arccosh}(dx+c)^4 \ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right) + 2\operatorname{arccosh}(dx+c)^3 \operatorname{po}\right.}{}$

input
```
int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4/e*ln(d*x+c)+b^4/e*(-1/5*arccosh(d*x+c)^5+arccosh(d*x+c)^4*ln(1+(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+2*arccosh(d*x+c)^3*polylog(2,-(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3*arccosh(d*x+c)^2*polylog(3,-(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3*arccosh(d*x+c)*polylog(4,-(d*x+c+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*polylog(5,-(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2))+4*a*b^3/e*(-1/4*arccosh(d*x+c)^4+arccosh(d*x+c)^3*ln(
1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/2*arccosh(d*x+c)^2*polylog(
2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*arccosh(d*x+c)*polylog(3
,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/4*polylog(4,-(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))^2))+6*a^2*b^2/e*(-1/3*arccosh(d*x+c)^3+arccosh(d
*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+arccosh(d*x+c)*pol
ylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/2*polylog(3,-(d*x+c+(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))+4*a^3*b/e*(-1/2*arccosh(d*x+c)^2+arcco
sh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/2*polylog(2,-(
d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{ce + dex}\, dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^4}{dex + ce}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*
arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{ce + dex}\, dx = \frac{\int \frac{a^4}{c+dx}\, dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c+dx}\, dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c+dx}\, dx + \int \frac{6a^2b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\, dx + \int \frac{4a^3 b \operatorname{acosh}(c+dx)}{c+dx}\, dx}{e}$$

input
```
integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e),x)
```

output
```
(Integral(a**4/(c + d*x), x) + Integral(b**4*acosh(c + d*x)**4/(c + d*x),
x) + Integral(4*a*b**3*acosh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b
**2*acosh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*acosh(c + d*x)/(c
+ d*x), x))/e
```

Maxima [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{ce + dex}\, dx = \int \frac{(b\,\text{arcosh}\,(dx + c) + a)^4}{dex + ce}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^4*log(d*e*x + c*e)/(d*e) + integrate(b^4*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^4/(d*e*x + c*e) + 4*a*b^3*log(d*x + sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + c)^3/(d*e*x + c*e) + 6*a^2*b^2*log(d*x + sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + c)^2/(d*e*x + c*e) + 4*a^3*b*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)/(d*e*x + c*e), x)
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{ce + dex}\, dx = \int \frac{(b\,\text{arcosh}\,(dx + c) + a)^4}{dex + ce}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^4}{ce+dex}\,dx=\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^4}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^4}{ce+dex}\,dx = \frac{4\left(\int \frac{acosh(dx+c)}{dx+c}dx\right)a^3bd+\left(\int \frac{acosh(dx+c)^4}{dx+c}dx\right)b^4d+4\left(\int \frac{acosh(dx+c)^3}{dx+c}dx\right)a\,b^3d+6\left(\int \frac{acosh(dx+c)^2}{dx+c}dx\right)a^2b^2d}{de}$$

input
```
int((a+b*acosh(d*x+c))^4/(d*e*x+c*e),x)
```

output
```
(4*int(acosh(c + d*x)/(c + d*x),x)*a**3*b*d + int(acosh(c + d*x)**4/(c + d
*x),x)*b**4*d + 4*int(acosh(c + d*x)**3/(c + d*x),x)*a*b**3*d + 6*int(acos
h(c + d*x)**2/(c + d*x),x)*a**2*b**2*d + log(c + d*x)*a**4)/(d*e)
```

3.45 $\int \frac{(a+b\,\mathrm{arccosh}(c+dx))^4}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 264

$$
\begin{aligned}
\int \frac{(a+b\,\mathrm{arccosh}(c+dx))^4}{(ce+dex)^2}\,dx
&= -\frac{(a+b\,\mathrm{arccosh}(c+dx))^4}{de^2(c+dx)} + \frac{8b(a+b\,\mathrm{arccosh}(c+dx))^3\arctan\left(e^{\mathrm{arccosh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{12ib^2(a+b\,\mathrm{arccosh}(c+dx))^2\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{12ib^2(a+b\,\mathrm{arccosh}(c+dx))^2\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{24ib^3(a+b\,\mathrm{arccosh}(c+dx))\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{24ib^3(a+b\,\mathrm{arccosh}(c+dx))\,\mathrm{PolyLog}\left(3,ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{24ib^4\,\mathrm{PolyLog}\left(4,-ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2} + \frac{24ib^4\,\mathrm{PolyLog}\left(4,ie^{\mathrm{arccosh}(c+dx)}\right)}{de^2}
\end{aligned}
$$

output
```
-(a+b*arccosh(d*x+c))^4/d/e^2/(d*x+c)+8*b*(a+b*arccosh(d*x+c))^3*arctan(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-12*I*b^2*(a+b*arccosh(d*x+c))^2
*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+12*I*b^2*(a+b
*arccosh(d*x+c))^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/
e^2+24*I*b^3*(a+b*arccosh(d*x+c))*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)))/d/e^2-24*I*b^3*(a+b*arccosh(d*x+c))*polylog(3,I*(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2-24*I*b^4*polylog(4,-I*(d*x+c+(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)))/d/e^2+24*I*b^4*polylog(4,I*(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)))/d/e^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. $2(264) = 528$.

Time = 1.77 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.30

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{(ce + dex)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

output
```
(-(a^4/(c + d*x)) + 4*a^3*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh
[ArcCosh[c + d*x]/2]]) - (6*I)*a^2*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c
+ d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCos
h[c + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^Ar
cCosh[c + d*x]]) + 4*a*b^3*(-(ArcCosh[c + d*x]^3/(c + d*x)) + (3*I)*(-(Arc
Cosh[c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c + d
*x]])) - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyLog
[2, I/E^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*Pol
yLog[3, I/E^ArcCosh[c + d*x]])) + b^4*(((-7*I)/16)*Pi^4 + (Pi^3*ArcCosh[c
+ d*x])/2 - ((3*I)/2)*Pi^2*ArcCosh[c + d*x]^2 - 2*Pi*ArcCosh[c + d*x]^3 +
I*ArcCosh[c + d*x]^4 - ArcCosh[c + d*x]^4/(c + d*x) + (Pi^3*Log[1 + I/E^Ar
cCosh[c + d*x]])/2 - (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d
*x]] - 6*Pi*ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + (4*I)*ArcCo
sh[c + d*x]^3*Log[1 + I/E^ArcCosh[c + d*x]] + (3*I)*Pi^2*ArcCosh[c + d*x]*
Log[1 - I*E^ArcCosh[c + d*x]] + 6*Pi*ArcCosh[c + d*x]^2*Log[1 - I*E^ArcCos
h[c + d*x]] - (Pi^3*Log[1 + I*E^ArcCosh[c + d*x]])/2 - (4*I)*ArcCosh[c + d
*x]^3*Log[1 + I*E^ArcCosh[c + d*x]] + (Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[c
+ d*x])/4]])/2 + (3*I)*(Pi - (2*I)*ArcCosh[c + d*x])^2*PolyLog[2, (-I)/E^A
rcCosh[c + d*x]] - (12*I)*ArcCosh[c + d*x]^2*PolyLog[2, (-I)*E^ArcCosh[c +
 d*x]] + (3*I)*Pi^2*PolyLog[2, I*E^ArcCosh[c + d*x]] + 12*Pi*ArcCosh[c ...
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6411, 27, 6298, 6362, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{(ce + dex)^2} \, dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{e^2(c+dx)^2} d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{(c+dx)^2}d(c+dx)}{de^2}$$

$\downarrow$ 6298

$$\frac{4b\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}}{de^2}$$

$\downarrow$ 6362

$$\frac{4b\int \frac{(a+b\text{arccosh}(c+dx))^3}{c+dx}d\text{arccosh}(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}}{de^2}$$

$\downarrow$ 3042

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}+4b\int (a+b\text{arccosh}(c+dx))^3\csc\left(i\text{arccosh}(c+dx)+\frac{\pi}{2}\right)d\text{arccosh}(c+dx)}{de^2}$$

$\downarrow$ 4668

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}+4b\Big(-3ib\int (a+b\text{arccosh}(c+dx))^2\log\left(1-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)+3ib\int (a+$$

de

$\downarrow$ 3011

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}+4b\Big(3ib\Big(2b\int (a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)-\text{Poly}$$

$\downarrow$ 7163

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}+4b\Big(3ib\Big(2b\Big(\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-b\int \text{PolyLog}\left(3,-ie^{\text{arcc}}\right.$$

$\downarrow$ 2720

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}+4b\Big(3ib\Big(2b\Big(\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-b\int e^{-\text{arccosh}(c+dx)}\text{Poly}$$

$\downarrow$ 7143

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{c+dx}+4b\Big(2\arctan\left(e^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))^3+3ib\Big(2b\Big(\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx}\right.$$

input
```
Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcCosh[c + d*x])^4/(c + d*x)) + 4*b*(2*(a + b*ArcCosh[c + d*x])
^3*ArcTan[E^ArcCosh[c + d*x]] + (3*I)*b*(-((a + b*ArcCosh[c + d*x])^2*Poly
Log[2, (-I)*E^ArcCosh[c + d*x]]) + 2*b*((a + b*ArcCosh[c + d*x])*PolyLog[3
, (-I)*E^ArcCosh[c + d*x]] - b*PolyLog[4, (-I)*E^ArcCosh[c + d*x]])) - (3*
I)*b*(-((a + b*ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh[c + d*x]]) + 2*b
*((a + b*ArcCosh[c + d*x])*PolyLog[3, I*E^ArcCosh[c + d*x]] - b*PolyLog[4,
 I*E^ArcCosh[c + d*x]]))))/(d*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^4}{(dex+ce)^2} dx$$

input
```
int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

output
```
int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^2}\, dx = \int \frac{(b \operatorname{arcosh}(dx+c)+a)^4}{(dex+ce)^2}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*
arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*
e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^2}\, dx$$
$$= \frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\, dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c^2+2cdx+d^2x^2}\, dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\, dx + \int \frac{6a^2b^2 \operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\, dx + \int \frac{4a^3b \operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\, dx}{e^2}$$

input
```
integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*acosh(c +
d*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*acosh(c + d*x
)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*acosh(c + d*x
)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*acosh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{(ce + dex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{(ce + dex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^2}\,dx=\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

input
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^2}\,dx = \frac{4\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right)a^3b\,c^2+4\left(\int \frac{acosh(dx+c)}{d^2x^2+2cdx+c^2}dx\right)a^3bcdx+\left(\int \frac{acosh(dx+c)^4}{d^2x^2+2cdx+c^2}dx\right)b^4c^2+\left(\int \frac{acosh(dx+c)^4}{d^2x^2+2cdx+c^2}dx\right.}{}$$

input
```
int((a+b*acosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

output
```
(4*int(acosh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b*c**2 + 4*int(
acosh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b*c*d*x + int(acosh(c
+ d*x)**4/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**2 + int(acosh(c + d*x)**
4/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c*d*x + 4*int(acosh(c + d*x)**3/(c*
*2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c**2 + 4*int(acosh(c + d*x)**3/(c**2 +
 2*c*d*x + d**2*x**2),x)*a*b**3*c*d*x + 6*int(acosh(c + d*x)**2/(c**2 + 2*
c*d*x + d**2*x**2),x)*a**2*b**2*c**2 + 6*int(acosh(c + d*x)**2/(c**2 + 2*c
*d*x + d**2*x**2),x)*a**2*b**2*c*d*x + a**4*x)/(c*e**2*(c + d*x))
```

3.46 $\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx &= \frac{2b(a+b\text{arccosh}(c+dx))^3}{de^3}+\frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^3}{de^3(c+dx)} \\ &\quad -\frac{(a+b\text{arccosh}(c+dx))^4}{2de^3(c+dx)^2}-\frac{6b^2(a+b\text{arccosh}(c+dx))^2\log\left(1+e^{2\text{arccosh}(c+dx)}\right)}{de^3} \\ &\quad -\frac{6b^3(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(2,-e^{2\text{arccosh}(c+dx)}\right)}{de^3} \\ &\quad +\frac{3b^4\,\text{PolyLog}\left(3,-e^{2\text{arccosh}(c+dx)}\right)}{de^3}\end{aligned}$$

output
```
2*b*(a+b*arccosh(d*x+c))^3/d/e^3+2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*
arccosh(d*x+c))^3/d/e^3/(d*x+c)-1/2*(a+b*arccosh(d*x+c))^4/d/e^3/(d*x+c)^2
-6*b^2*(a+b*arccosh(d*x+c))^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
^2)/d/e^3-6*b^3*(a+b*arccosh(d*x+c))*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2))^2)/d/e^3+3*b^4*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2))^2)/d/e^3
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 398 vs. $2(195) = 390$.

Time = 1.62 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.04

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^4}{(ce + dex)^3}\,dx$$
$$= \frac{-\frac{a^4}{(c+dx)^2} + \frac{4a^3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{c+dx} - \frac{4a^3b\operatorname{arccosh}(c+dx)}{(c+dx)^2} - \frac{b^4\operatorname{arccosh}(c+dx)^4}{(c+dx)^2} + 12a^2b^2\left(\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\operatorname{arccos}}{c+dx}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

output
```
(-(a^4/(c + d*x)^2) + (4*a^3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c +
d*x) - (4*a^3*b*ArcCosh[c + d*x])/(c + d*x)^2 - (b^4*ArcCosh[c + d*x]^4)/(
c + d*x)^2 + 12*a^2*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)
*ArcCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2) - Log[c
+ d*x]) + 4*a*b^3*(-(ArcCosh[c + d*x]*(3*ArcCosh[c + d*x] - (3*Sqrt[(-1 +
c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) + ArcCos
h[c + d*x]^2/(c + d*x)^2 + 6*Log[1 + E^(-2*ArcCosh[c + d*x])])) + 3*PolyLo
g[2, -E^(-2*ArcCosh[c + d*x])]) + 2*b^4*(2*ArcCosh[c + d*x]^2*(-ArcCosh[c
+ d*x] + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x
])/(c + d*x) - 3*Log[1 + E^(-2*ArcCosh[c + d*x])]) + 6*ArcCosh[c + d*x]*Po
lyLog[2, -E^(-2*ArcCosh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcCosh[c + d*x])
]))/(2*d*e^3)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6411, 27, 6298, 6333, 6297, 25, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}dx$$

$\Big\downarrow$ 6411

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{e^3(c+dx)^3}d(c+dx)}{d}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{(c+dx)^3}d(c+dx)}{de^3}$$

$\Big\downarrow$ 6298

$$\frac{2b\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 6333

$$\frac{2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3b\int \frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}d(c+dx)\right)-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 6297

$$\frac{2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3\int -(a+b\text{arccosh}(c+dx))^2\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arcc}\right.}{de^3}$$

$\Big\downarrow$ 25

$$\frac{2b\left(3\int (a+b\text{arccosh}(c+dx))^2\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)d(a+b\text{arccosh}(c+dx))+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arcc}}{c+dx}\right.}{de^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}+3\int -i(a+b\text{arccosh}(c+dx))^2\tan\left(\frac{ia}{b}-\frac{i(a+ba}{}\right.\right.}{de^3}$$

$\Big\downarrow$ 26

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3i\int (a+b\text{arccosh}(c+dx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arc}}{}\right.\right.}{de^3}$$

$\Big\downarrow$ 4201

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3i\left(2i\int\frac{e^{\frac{2(a-c-dx)}{b}}(a+b\text{arccosh}(c+dx))^2}{1+e^{\frac{2(a-c-dx)}{b}}}d(a+bar\right.\right.}{de^3}$$

$\downarrow$ 2620

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3i\left(2i\left(b\int(a+b\text{arccosh}(c+dx))\log\left(1+e^{\frac{2(a-}{}}\right.\right.\right.\right.}{d}$$

$\downarrow$ 3011

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))\text{PolyLog}\left(2\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 2720

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3i\left(2i\left(b\left(\frac{1}{4}b^2\int e^{-\frac{2(a-c-dx)}{b}}\text{PolyLog}(2,-c-dx\right.\right.\right.\right.}{}$$

$\downarrow$ 7143

$$\frac{-\frac{(a+b\text{arccosh}(c+dx))^4}{2(c+dx)^2}+2b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}{c+dx}-3i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arccosh}(c+dx))\text{PolyLog}\left(2\right.\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcCosh[c + d*x])^4/(c + d*x)^2 + 2*b*((Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(c + d*x) - (3*I)*((-1/3*I)*(a
 + b*ArcCosh[c + d*x])^3 + (2*I)*(-1/2*(b*(a + b*ArcCosh[c + d*x])^2*Log[1
 + E^((2*(a - c - d*x))/b)]) + b*((b*(a + b*ArcCosh[c + d*x])*PolyLog[2, -
E^((2*(a - c - d*x))/b)])/2 + (b^2*PolyLog[3, -c - d*x])/4)))))/(d*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6333
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p]   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
 && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(231) = 462$.

Time = 0.42 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.52

method	result
derivativedivides	$-\frac{a^4}{2e^3(dx+c)^2}+\frac{b^4\left(-\frac{\operatorname{arccosh}(dx+c)^3\left(4(dx+c)^2-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}+4\operatorname{arccosh}(dx+c)^3-6\operatorname{arccosh}(d$
default	$-\frac{a^4}{2e^3(dx+c)^2}+\frac{b^4\left(-\frac{\operatorname{arccosh}(dx+c)^3\left(4(dx+c)^2-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}+4\operatorname{arccosh}(dx+c)^3-6\operatorname{arccosh}(d$
parts	$-\frac{a^4}{2e^3(dx+c)^2d}+\frac{b^4\left(-\frac{\operatorname{arccosh}(dx+c)^3\left(4(dx+c)^2-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+\operatorname{arccosh}(dx+c)\right)}{2(dx+c)^2}+4\operatorname{arccosh}(dx+c)^3-6$

input

```
int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2*a^4/e^3/(d*x+c)^2+b^4/e^3*(-1/2*arccosh(d*x+c)^3*(4*(d*x+c)^2-4*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+arccosh(d*x+c))/(d*x+c)^2+4*arccos
h(d*x+c)^3-6*arccosh(d*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
^2)-6*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
+3*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))+4*a*b^3/e^3*(-1/
2*arccosh(d*x+c)^2*(-3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+3*(d*x+c)^2
+arccosh(d*x+c))/(d*x+c)^2+3*arccosh(d*x+c)^2-3*arccosh(d*x+c)*ln(1+(d*x+c
+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2))^2))+6*a^2*b^2/e^3*(2*arccosh(d*x+c)-1/2*arccosh(d*x+c)*(
2*(d*x+c)^2-2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+arccosh(d*x+c))/(d*x
+c)^2-ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))+4*a^3*b/e^3*(-1/2/(
d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Fricas [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*
arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^
2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx$$
$$=\frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^4\,\text{acosh}^4\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{4ab^3\,\text{acosh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{6a^2b^2\,\text{acosh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**4*acosh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(4*a*b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3), x) + Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*acosh(c + d*x)
/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Maxima [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arcosh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-1/2*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^3*e^3*x^2
 + 2*c*d^2*e^3*x + c^2*d*e^3) + 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arc
cosh(d*x + c)/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c)/(d*e^3))*a^2*b^2 + 2*
a^3*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*d^2*e^3) - arcco
sh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3*a^2*b^2*arccosh
(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^4/(d^3*e^3*x
^2 + 2*c*d^2*e^3*x + c^2*d*e^3) + integrate(2*(2*(c^3 - c)*a*b^3 + (c^3 -
c)*b^4 + (2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 +
 (b^4*c^2 + 2*(c^2 - 1)*a*b^3 + (2*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c
*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (2*(3*c^2*d - d)*a*
b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^3/(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 - c^4*e^3 + (15*c^2*d^4*e^
3 - d^4*e^3)*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 -
2*c^2*d^2*e^3)*x^2 + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 +
(10*c^2*d^3*e^3 - d^3*e^3)*x^3 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (5*c
^4*d*e^3 - 3*c^2*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(3*c^5*
d*e^3 - 2*c^3*d*e^3)*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arcosh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\text{acosh}(c+d\,x))^4}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^3}\,dx$$
$$= \frac{8\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)a^3b\,c^2d + 16\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)a^3bc\,d^2x + 8\left(\int \frac{acosh(dx+c)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}d\right.}{}$$

input
```
int((a+b*acosh(d*x+c))^4/(d*e*x+c*e)^3,x)
```

output
```
(8*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a
**3*b*c**2*d + 16*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3),x)*a**3*b*c*d**2*x + 8*int(acosh(c + d*x)/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3),x)*a**3*b*d**3*x**2 + 2*int(acosh(c + d*x)**4/(
c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**4*c**2*d + 4*int(acos
h(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**4*c*d*
*2*x + 2*int(acosh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3),x)*b**4*d**3*x**2 + 8*int(acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*a*b**3*c**2*d + 16*int(acosh(c + d*x)**3/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**3*c*d**2*x + 8*int(acosh(
c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**3*d**3
*x**2 + 12*int(acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3),x)*a**2*b**2*c**2*d + 24*int(acosh(c + d*x)**2/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**2*c*d**2*x + 12*int(acosh(c + d*x)*
*2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**2*d**3*x**2
- a**4)/(2*d*e**3*(c**2 + 2*c*d*x + d**2*x**2))
```

3.47 $\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 432

$$
\begin{aligned}
&\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx \\
&= \frac{2b^2(a+b\text{arccosh}(c+dx))^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\text{arccosh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{8b^3(a+b\text{arccosh}(c+dx))\arctan\left(e^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b(a+b\text{arccosh}(c+dx))^3\arctan\left(e^{\text{arccosh}(c+dx)}\right)}{3de^4} \\
&\quad + \frac{4ib^4\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{2ib^2(a+b\text{arccosh}(c+dx))^2\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{4ib^4\,\text{PolyLog}\left(2,ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{2ib^2(a+b\text{arccosh}(c+dx))^2\,\text{PolyLog}\left(2,ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4ib^3(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{4ib^3(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(3,ie^{\text{arccosh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{4ib^4\,\text{PolyLog}\left(4,-ie^{\text{arccosh}(c+dx)}\right)}{de^4} + \frac{4ib^4\,\text{PolyLog}\left(4,ie^{\text{arccosh}(c+dx)}\right)}{de^4}
\end{aligned}
$$

output
```
2*b^2*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)+2/3*b*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*(a+b*arccosh(d*x+c))^3/d/e^4/(d*x+c)^2-1/3*(a+b*arccosh(d*x+c))^4/d
/e^4/(d*x+c)^3-8*b^3*(a+b*arccosh(d*x+c))*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2))/d/e^4+4/3*b*(a+b*arccosh(d*x+c))^3*arctan(d*x+c+(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2))/d/e^4+4*I*b^4*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)))/d/e^4-2*I*b^2*(a+b*arccosh(d*x+c))^2*polylog(2,-I*(d*x+c+(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^4*polylog(2,I*(d*x+c+(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+2*I*b^2*(a+b*arccosh(d*x+c))^2*polylog(2,I*
(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+4*I*b^3*(a+b*arccosh(d*x+c)
)*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^3*(a+b
*arccosh(d*x+c))*polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^
4-4*I*b^4*polylog(4,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+4*I*
b^4*polylog(4,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1198 vs. $2(432) = 864$.

Time = 5.53 (sec) , antiderivative size = 1198, normalized size of antiderivative = 2.77

$$\int \frac{(a + b\text{arccosh}(c + dx))^4}{(ce + dex)^4}\, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

output
```
(-(a^4/(c + d*x)^3) + 2*a^3*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c
+ d*x))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[Arc
Cosh[c + d*x]/2]]) + 6*a^2*b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 +
 c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^
2/(c + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCo
sh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c
+ d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + 2*a*b^3*((6*ArcCosh[c + d*
x])/(c + d*x) + (3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCos
h[c + d*x]^2)/(c + d*x)^2 - (2*ArcCosh[c + d*x]^3)/(c + d*x)^3 + (3*I)*((4
*I)*ArcTan[E^ArcCosh[c + d*x]] + ArcCosh[c + d*x]^2*Log[1 - I*E^ArcCosh[c
+ d*x]] - ArcCosh[c + d*x]^2*Log[1 + I*E^ArcCosh[c + d*x]] - 2*ArcCosh[c +
 d*x]*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + 2*ArcCosh[c + d*x]*PolyLog[2,
I*E^ArcCosh[c + d*x]] + 2*PolyLog[3, (-I)*E^ArcCosh[c + d*x]] - 2*PolyLog[
3, I*E^ArcCosh[c + d*x]])) + 3*b^4*(((-7*I)/96)*Pi^4 + (Pi^3*ArcCosh[c + d
*x])/12 - (I/4)*Pi^2*ArcCosh[c + d*x]^2 + (2*ArcCosh[c + d*x]^2)/(c + d*x)
 - (Pi*ArcCosh[c + d*x]^3)/3 + (2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
c + d*x)*ArcCosh[c + d*x]^3)/(3*(c + d*x)^2) + (I/6)*ArcCosh[c + d*x]^4 -
ArcCosh[c + d*x]^4/(3*(c + d*x)^3) + (4*I)*ArcCosh[c + d*x]*Log[1 - I/E^Ar
cCosh[c + d*x]] + (Pi^3*Log[1 + I/E^ArcCosh[c + d*x]])/12 - (4*I)*ArcCosh[
c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - (I/2)*Pi^2*ArcCosh[c + d*x]*Lo...
```

Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {6411, 27, 6298, 6348, 6298, 6362, 3042, 4668, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^4}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^4}{(c+dx)^4}d(c+dx)}{de^4}$$

$\Big\downarrow$ 6298

$$\frac{\frac{4}{3}b\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}}{de^4}$$

$\Big\downarrow$ 6348

$$\frac{\frac{4}{3}b\left(-\frac{3}{2}b\int \frac{(a+b\text{arccosh}(c+dx))^2}{(c+dx)^2}d(c+dx)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+a}{2(c+dx)^2}\right.}{de^4}$$

$\Big\downarrow$ 6298

$$\frac{\frac{4}{3}b\left(-\frac{3}{2}b\left(2b\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}\right)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}d(c+dx)+\right.}{de^4}$$

$\Big\downarrow$ 6362

$$\frac{\frac{4}{3}b\left(-\frac{3}{2}b\left(2b\int \frac{a+b\text{arccosh}(c+dx)}{c+dx}d\text{arccosh}(c+dx)-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}\right)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(c+dx))^3}{c+dx}d\text{arccosh}(c+\right.}{de^4}$$

$\Big\downarrow$ 3042

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{3}{2}b\left(-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\int (a+b\text{arccosh}(c+dx))\csc\left(i\text{arccosh}(c+dx)+\frac{\pi}{2}\right)\right.\right.$$

$\Big\downarrow$ 4668

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{3}{2}b\left(-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\left(-ib\int \log\left(1-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)+ib\right.\right.\right.$$

$\Big\downarrow$ 2715

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{3}{2}b\left(-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\left(-ib\int e^{-\text{arccosh}(c+dx)}\log\left(1-ie^{\text{arccosh}(c+dx)}\right)de^{\text{arccos}}\right.\right.\right.$$

$\Big\downarrow$ 2838

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(\frac{1}{2}\left(-3ib\int (a+b\text{arccosh}(c+dx))^2\log\left(1-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)+3ib\int (a\right.\right.$$

$\Big\downarrow$ 3011

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\bigg(\frac{1}{2}\Big(3ib\Big(2b\int(a+b\text{arccosh}(c+dx))\,\text{PolyLog}\left(2,-ie^{\text{arccosh}(c+dx)}\right)d\text{arccosh}(c+dx)-\text{P}$$

$\Big\downarrow$ 7163

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\bigg(\frac{1}{2}\Big(3ib\Big(2b\Big(\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-b\int\text{PolyLog}\left(3,-ie^{\text{a}}\right.$$

$\Big\downarrow$ 2720

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\bigg(\frac{1}{2}\Big(3ib\Big(2b\Big(\text{PolyLog}\left(3,-ie^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-b\int e^{-\text{arccosh}(c+dx)}\text{P}$$

$\Big\downarrow$ 7143

$$-\frac{(a+b\text{arccosh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\bigg(-\frac{3}{2}b\bigg(-\frac{(a+b\text{arccosh}(c+dx))^2}{c+dx}+2b\Big(2\arctan\left(e^{\text{arccosh}(c+dx)}\right)(a+b\text{arccosh}(c+dx))-ib$$

input
```
Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcCosh[c + d*x])^4/(c + d*x)^3 + (4*b*((Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(2*(c + d*x)^2) - (3*b*(-((a
+ b*ArcCosh[c + d*x])^2/(c + d*x)) + 2*b*(2*(a + b*ArcCosh[c + d*x])*ArcTa
n[E^ArcCosh[c + d*x]] - I*b*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + I*b*Poly
Log[2, I*E^ArcCosh[c + d*x]])))/2 + (2*(a + b*ArcCosh[c + d*x])^3*ArcTan[E
^ArcCosh[c + d*x]] + (3*I)*b*(-((a + b*ArcCosh[c + d*x])^2*PolyLog[2, (-I)
*E^ArcCosh[c + d*x]]) + 2*b*((a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E^Ar
cCosh[c + d*x]] - b*PolyLog[4, (-I)*E^ArcCosh[c + d*x]])) - (3*I)*b*(-((a
+ b*ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh[c + d*x]]) + 2*b*((a + b*Ar
cCosh[c + d*x])*PolyLog[3, I*E^ArcCosh[c + d*x]] - b*PolyLog[4, I*E^ArcCos
h[c + d*x]])))/2))/3)/(d*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6348
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Simp[b*c*(n/(f
*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]
   Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6362
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Subst
[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arccosh}\,(dx+c))^4}{(dex+ce)^4}dx$$

input
```
int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

output
```
int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*
arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^
3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4\operatorname{acosh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3\operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int}{e^4}$$

input
```
integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**4*acosh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*acosh(c + d
*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),
 x) + Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d
**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*acosh(c + d*
x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))
/e**4
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{(b\,\operatorname{arcosh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^4*e^4*x^3
 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 +
 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3
 - c)*a*b^3 + (c^3 - c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d
^2 + b^4*c*d^2)*x^2 + (b^4*c^2 + 3*(c^2 - 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^
2)*x^2 + 2*(3*a*b^3*c*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ (3*(3*c^2*d - d)*a*b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c +
1)*sqrt(d*x + c - 1) + c)^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (
3*c^2*d - d)*a^2*b^2*x + (c^3 - c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*
c*d*x + (c^2 - 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x +
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*
d^2*x^2 + (3*c^2*d - d)*a^3*b*x + (c^3 - c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3
*b*c*d*x + (c^2 - 1)*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x +
 sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6
+ c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 -
c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 -
 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4
+ (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(
5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt
(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^4}{\left(c\,e+d\,e\,x\right)^4}\,dx$$

input
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^4}{(ce+dex)^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*acosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

output

```
(12*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x*
*3 + d**4*x**4),x)*a**3*b*c**3*d + 36*int(acosh(c + d*x)/(c**4 + 4*c**3*d*
x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a**3*b*c**2*d**2*x +
36*int(acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**
3 + d**4*x**4),x)*a**3*b*c*d**3*x**2 + 12*int(acosh(c + d*x)/(c**4 + 4*c**
3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a**3*b*d**4*x**3
+ 3*int(acosh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3
*x**3 + d**4*x**4),x)*b**4*c**3*d + 9*int(acosh(c + d*x)**4/(c**4 + 4*c**3
*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**4*c**2*d**2*x +
 9*int(acosh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*
x**3 + d**4*x**4),x)*b**4*c*d**3*x**2 + 3*int(acosh(c + d*x)**4/(c**4 + 4*
c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**4*d**4*x**3
 + 12*int(acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d*
*3*x**3 + d**4*x**4),x)*a*b**3*c**3*d + 36*int(acosh(c + d*x)**3/(c**4 + 4
*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a*b**3*c**2*d
**2*x + 36*int(acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4
*c*d**3*x**3 + d**4*x**4),x)*a*b**3*c*d**3*x**2 + 12*int(acosh(c + d*x)**3
/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a*b
**3*d**4*x**3 + 18*int(acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*
x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*a**2*b**2*c**3*d + 54*int(acosh(c ...
```

3.48 $\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx = -\frac{e^4\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bd} - \frac{3e^4\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bd} - \frac{e^4\text{Chi}\left(\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16bd} + \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8bd} + \frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16bd} + \frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16bd}$$

output
```
-1/8*e^4*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b/d-3/16*e^4*Chi(3*(a+b*arc
cosh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Chi(5*(a+b*arccosh(d*x+c))/b)*sin
h(5*a/b)/b/d+1/8*e^4*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d+3/16*e^4*co
sh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b/d+1/16*e^4*cosh(5*a/b)*Shi(5*(a+
b*arccosh(d*x+c))/b)/b/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx$$
$$= \frac{e^4\left(-2\text{Chi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-3\text{Chi}\left(3\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\text{arcco}\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^4*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3
*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c
 + d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]
 + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b
]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])]))/(16*b*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6411, 27, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{e^4(c+dx)^4}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{e^4\int \frac{(c+dx)^4}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 6302}$$

$$\frac{e^4\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\downarrow$ 25

$$-\frac{e^4\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\downarrow$ 5971

$$-\frac{e^4\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16(a+b\text{arccosh}(c+dx))}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16(a+b\text{arccosh}(c+dx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccos}$$

$\downarrow$ 2009

$$e^4\left(-\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\frac{3}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)-\frac{1}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arccos}}{b}\right.\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^4*(-1/8*(CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b]) - (3*CoshI
ntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/16 - (CoshIntegral[
(5*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*SinhIntegra
l[(a + b*ArcCosh[c + d*x])/b])/8 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b
*ArcCosh[c + d*x]))/b])/16 + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh
[c + d*x]))/b])/16))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{e^4 e^{\frac{5a}{b}} \operatorname{expIntegral}_1\left(5 \operatorname{arccosh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3 \operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{32b}+\frac{e^4 e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\right.}{16b}$
default	$\frac{e^4 e^{\frac{5a}{b}} \operatorname{expIntegral}_1\left(5 \operatorname{arccosh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3 \operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{32b}+\frac{e^4 e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\right.}{16b}$

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/32*e^4/b*exp(5*a/b)*Ei(1,5*arccosh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a
/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*e^4/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a
/b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-3/32*e^4/b*exp(-3*a/b)*
Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c
)-5*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^4}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b*arccosh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx=e^4\left(\int \frac{c^4}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{d^4x^4}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{4cd^3x^3}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{6c^2d^2x^2}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{4c^3dx}{a+b\,\text{acosh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c)),x)
```

output
```
e**4*(Integral(c**4/(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/(a + b
*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*acosh(c + d*x)), x) +
 Integral(6*c**2*d**2*x**2/(a + b*acosh(c + d*x)), x) + Integral(4*c**3*d*
x/(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{a+b\text{arccosh}(c+dx)}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{a+b\operatorname{acosh}(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(ce + dex)^4}{a + b\text{arccosh}(c + dx)} dx = e^4 \left(\left(\int \frac{x^4}{acosh\,(dx + c)\,b + a} dx \right) d^4 + 4 \left(\int \frac{x^3}{acosh\,(dx + c)\,b + a} dx \right) c\,d^3 + 6 \left(\int \frac{x^2}{acosh\,(dx + c)\,b + a} dx \right) c^2 d^2 + 4 \left(\int \frac{x}{acosh\,(dx + c)\,b + a} dx \right) c^3 d + \left(\int \frac{1}{acosh\,(dx + c)\,b + a} dx \right) c^4 \right)$$

input
```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c)),x)
```

output
```
e**4*(int(x**4/(acosh(c + d*x)*b + a),x)*d**4 + 4*int(x**3/(acosh(c + d*x)
*b + a),x)*c*d**3 + 6*int(x**2/(acosh(c + d*x)*b + a),x)*c**2*d**2 + 4*int
(x/(acosh(c + d*x)*b + a),x)*c**3*d + int(1/(acosh(c + d*x)*b + a),x)*c**4
)
```

3.49 $\int \frac{(ce+dex)^3}{a+b\,\mathrm{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{(ce+dex)^3}{a+b\,\mathrm{arccosh}(c+dx)}\,dx = -\frac{e^3\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{4bd} - \frac{e^3\mathrm{Chi}\left(\frac{4(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8bd} + \frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{4bd} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{8bd}$$

output
```
-1/4*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*sinh(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*
arccosh(d*x+c))/b)*sinh(4*a/b)/b/d+1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arccosh(
d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)*Shi(4*(a+b*arccosh(d*x+c))/b)/b/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \frac{(ce+dex)^3}{a+b\text{arccosh}(c+dx)}\,dx$$
$$=\frac{e^3\left(-2\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\text{Chi}\left(4\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right)+2\cosh\left(\frac{2a}{b}\right)\text{Sh}\right.}{8bd}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^3*(-2*CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - CoshInte
gral[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + 2*Cosh[(2*a)/b]*SinhInteg
ral[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcC
osh[c + d*x])]))/(8*b*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6411, 27, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{a+b\text{arccosh}(c+dx)}\,dx$$
$$\Big\downarrow \; 6411$$
$$\frac{\int \frac{e^3(c+dx)^3}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{e^3\int \frac{(c+dx)^3}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \; 6302$$

$$\frac{e^3 \int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 25

$$-\frac{e^3 \int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 5971

$$-\frac{e^3 \int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{8(a+b\text{arccosh}(c+dx))}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{4(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 2009

$$\frac{e^3\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcco}}{b}\right.\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^3*(-1/4*(CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b]) -
(CoshIntegral[(4*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(4*a)/b])/8 + (Cosh[(2*
a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/4 + (Cosh[(4*a)/b]*Sin
hIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/8))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{e^3 \mathrm{e}^{\frac{4a}{b}} \operatorname{expIntegral}_1\left(4 \operatorname{arccosh}(dx+c)+\frac{4a}{b}\right)}{16b}+\frac{e^3 \mathrm{e}^{\frac{2a}{b}} \operatorname{expIntegral}_1\left(2 \operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{8b}-\frac{e^3 \mathrm{e}^{-\frac{2a}{b}} \operatorname{expIntegral}_1\left(-2 \operatorname{arccosh}(dx+\right.}{8b}}{d}$
default	$\frac{\frac{e^3 \mathrm{e}^{\frac{4a}{b}} \operatorname{expIntegral}_1\left(4 \operatorname{arccosh}(dx+c)+\frac{4a}{b}\right)}{16b}+\frac{e^3 \mathrm{e}^{\frac{2a}{b}} \operatorname{expIntegral}_1\left(2 \operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{8b}-\frac{e^3 \mathrm{e}^{-\frac{2a}{b}} \operatorname{expIntegral}_1\left(-2 \operatorname{arccosh}(dx+\right.}{8b}}{d}$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*e^3/b*exp(2*a/
b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+
c)-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^3}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcc
osh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arccosh}(c+dx)}\,dx=e^3\left(\int \frac{c^3}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{d^3x^3}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{3cd^2x^2}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{3c^2dx}{a+b\,\text{acosh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c)),x)
```

output
```
e**3*(Integral(c**3/(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/(a + b
*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*acosh(c + d*x)), x) +
 Integral(3*c**2*d*x/(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(ce + dex)^3}{a + b\text{arccosh}(c + dx)}\,dx = \int \frac{(dex + ce)^3}{b\,\text{arcosh}\,(dx + c) + a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(ce + dex)^3}{a + b\text{arccosh}(c + dx)}\,dx = \int \frac{(dex + ce)^3}{b\,\text{arcosh}\,(dx + c) + a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce + dex)^3}{a + b\text{arccosh}(c + dx)}\,dx = \int \frac{(c\,e + d\,e\,x)^3}{a + b\,\text{acosh}\,(c + d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arccosh}(c+dx)}\,dx = e^3\left(\left(\int \frac{x^3}{acosh\,(dx+c)\,b+a}dx\right)d^3 + 3\left(\int \frac{x^2}{acosh\,(dx+c)\,b+a}dx\right)c\,d^2 + 3\left(\int \frac{x}{acosh\,(dx+c)\,b+a}dx\right)c^2d + \left(\int \frac{1}{acosh\,(dx+c)\,b+a}dx\right)c^3\right)$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c)),x)
```

output
```
e**3*(int(x**3/(acosh(c + d*x)*b + a),x)*d**3 + 3*int(x**2/(acosh(c + d*x)
*b + a),x)*c*d**2 + 3*int(x/(acosh(c + d*x)*b + a),x)*c**2*d + int(1/(acos
h(c + d*x)*b + a),x)*c**3)
```

3.50 $\int \frac{(ce+dex)^2}{a+b\operatorname{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arccosh}(c+dx)}\,dx = -\frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bd} - \frac{e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bd} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(c+dx)}{b}\right)}{4bd} + \frac{e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{4bd}$$

output
```
-1/4*e^2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b/d-1/4*e^2*Chi(3*(a+b*arcc
osh(d*x+c))/b)*sinh(3*a/b)/b/d+1/4*e^2*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/
b)/b/d+1/4*e^2*cosh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arccosh}(c+dx)}\,dx$$
$$= \frac{e^2\left(-\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+a\right.}{4bd}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^2*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) - CoshIntegral[3*(
a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcC
osh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]))/(
4*b*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6411, 27, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arccosh}(c+dx)}\,dx$$
$$\Big\downarrow \; 6411$$
$$\frac{\int \frac{e^2(c+dx)^2}{a+b\operatorname{arccosh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{e^2\int \frac{(c+dx)^2}{a+b\operatorname{arccosh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \; 6302$$

$$\frac{e^2 \int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{e^2 \int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{e^2 \int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4(a+b\text{arccosh}(c+dx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right.\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

output
```
(e^2*(-1/4*(CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b]) - (CoshInt
egral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/4 + (Cosh[a/b]*SinhIn
tegral[(a + b*ArcCosh[c + d*x])/b])/4 + (Cosh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcCosh[c + d*x]))/b])/4))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{e^2 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3\operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{8b}-\frac{e^2 e^{-\frac{a}{b}} \operatorname{expIntegral}_1\left(-\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)}{8b}}{d}$
default	$\frac{\frac{e^2 e^{\frac{3a}{b}} \operatorname{expIntegral}_1\left(3\operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{8b}-\frac{e^2 e^{-\frac{a}{b}} \operatorname{expIntegral}_1\left(-\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)}{8b}}{d}$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*
Ei(1,arccosh(d*x+c)+a/b)-1/8*e^2/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8
*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^2}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arccosh(d*x + c) + a), x
)
```

Sympy [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arccosh}(c+dx)}\,dx=e^2\left(\int \frac{c^2}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{d^2x^2}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{2cdx}{a+b\,\text{acosh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c)),x)
```

output
```
e**2*(Integral(c**2/(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/(a + b
*acosh(c + d*x)), x) + Integral(2*c*d*x/(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^2}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^2}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{a+b\,\text{acosh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arccosh}(c+dx)}\,dx=e^2\left(\left(\int \frac{x^2}{acosh\,(dx+c)\,b+a}dx\right)d^2+2\left(\int \frac{x}{acosh\,(dx+c)\,b+a}dx\right)cd+\left(\int \frac{1}{acosh\,(dx+c)\,b+a}dx\right)c^2\right)$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c)),x)
```

output
```
e**2*(int(x**2/(acosh(c + d*x)*b + a),x)*d**2 + 2*int(x/(acosh(c + d*x)*b
+ a),x)*c*d + int(1/(acosh(c + d*x)*b + a),x)*c**2)
```

3.51 $\int \frac{ce+dex}{a+b\,\mathrm{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{ce+dex}{a+b\,\mathrm{arccosh}(c+dx)}\,dx=-\frac{e\,\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2bd}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{2bd}$$

output
```
-1/2*e*Chi(2*(a+b*arccosh(d*x+c))/b)*sinh(2*a/b)/b/d+1/2*e*cosh(2*a/b)*Shi
(2*(a+b*arccosh(d*x+c))/b)/b/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{ce+dex}{a+b\,\mathrm{arccosh}(c+dx)}\,dx=-\frac{e\left(\mathrm{Chi}\left(\frac{2a}{b}+2\,\mathrm{arccosh}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\,\mathrm{arccosh}(c+dx)\right)\right)}{2bd}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

output
```
-1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(
2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]))/(b*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6411, 27, 6302, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{a+b\text{arccosh}(c+dx)}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e(c+dx)}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6302}$$

$$\frac{e\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{e\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2(a+b\text{arccosh}(c+dx))}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 27

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{2bd}$$

$\Big\downarrow$ 3042

$$-\frac{e\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{2bd}$$

$\Big\downarrow$ 26

$$\frac{ie\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{2bd}$$

$\Big\downarrow$ 3784

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+\right.}{2bd}$$

$\Big\downarrow$ 26

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+\right.}{2bd}$$

$\Big\downarrow$ 3042

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(\right.}{2bd}$$

$\Big\downarrow$ 26

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+\right.}{2bd}$$

$\Big\downarrow$ 3779

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)\right)}{2bd}$$

$$\Big\downarrow \text{3782}$$

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)\right)}{2bd}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

output
```
((I/2)*e*(I*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b] - I
*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{e\,{\mathrm e}^{\frac{2a}{b}}\operatorname{expIntegral}_{1}\left(2\,\operatorname{arccosh}\left(dx+c\right)+\frac{2a}{b}\right)}{4b}-\frac{e\,{\mathrm e}^{-\frac{2a}{b}}\operatorname{expIntegral}_{1}\left(-2\,\operatorname{arccosh}\left(dx+c\right)-\frac{2a}{b}\right)}{4b}}{d}$	66
default	$\frac{\frac{e\,{\mathrm e}^{\frac{2a}{b}}\operatorname{expIntegral}_{1}\left(2\,\operatorname{arccosh}\left(dx+c\right)+\frac{2a}{b}\right)}{4b}-\frac{e\,{\mathrm e}^{-\frac{2a}{b}}\operatorname{expIntegral}_{1}\left(-2\,\operatorname{arccosh}\left(dx+c\right)-\frac{2a}{b}\right)}{4b}}{d}$	66

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*E
i(1,-2*arccosh(d*x+c)-2*a/b))
```

Fricas [F]

$$\int \frac{ce+dex}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{dex+ce}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{ce+dex}{a+b\text{arccosh}(c+dx)}\,dx=e\left(\int \frac{c}{a+b\,\text{acosh}\,(c+dx)}\,dx+\int \frac{dx}{a+b\,\text{acosh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

output
```
e*(Integral(c/(a + b*acosh(c + d*x)), x) + Integral(d*x/(a + b*acosh(c + d
*x)), x))
```

Maxima [F]

$$\int \frac{ce+dex}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{dex+ce}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{ce + dex}{a + b\text{arccosh}(c + dx)}\,dx = \int \frac{dex + ce}{b\,\text{arcosh}\,(dx + c) + a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce + dex}{a + b\text{arccosh}(c + dx)}\,dx = \int \frac{c\,e + d\,e\,x}{a + b\,\text{acosh}\,(c + d\,x)}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{ce + dex}{a + b\text{arccosh}(c + dx)}\,dx = e\left(\left(\int \frac{x}{acosh\,(dx + c)\,b + a}dx\right)d + \left(\int \frac{1}{acosh\,(dx + c)\,b + a}dx\right)c\right)$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

output
```
e*(int(x/(acosh(c + d*x)*b + a),x)*d + int(1/(acosh(c + d*x)*b + a),x)*c)
```

3.52 $\int \frac{1}{a+b\text{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{a+b\text{arccosh}(c+dx)}\,dx = -\frac{\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bd} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{bd}$$

output
```
-Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b/d+cosh(a/b)*Shi((a+b*arccosh(d*x+
c))/b)/b/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+b\text{arccosh}(c+dx)}\,dx = -\frac{\text{Chi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)}{bd}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-1),x]
```

output
```
-((CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c + d*x]])/(b*d))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6410, 6296, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arccosh}(c+dx)}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6296}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3784}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int-\frac{i\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arcco}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccos}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arc}$$

$$\Big\downarrow \text{3779}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)}{bd}$$

$$\Big\downarrow \text{3782}$$

$$\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)}{bd}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(-1),x]
```

output
```
(I*(I*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b] - I*Cosh[a/b]*Sin
hIntegral[(a + b*ArcCosh[c + d*x])/b]))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\frac{e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{expIntegral}_1\left(-\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60
default	$\frac{\frac{e^{\frac{a}{b}} \operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{expIntegral}_1\left(-\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60

input
```
int(1/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh
(d*x+c)-a/b))
```

Fricas [F]

$$\int \frac{1}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{1}{b\,\operatorname{arcosh}\left(dx+c\right)+a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccosh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{a + b\text{arccosh}(c + dx)} \, dx = \int \frac{1}{a + b \operatorname{acosh}(c + dx)} \, dx$$

input
```
integrate(1/(a+b*acosh(d*x+c)),x)
```

output
```
Integral(1/(a + b*acosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{a + b\text{arccosh}(c + dx)} \, dx = \int \frac{1}{b \operatorname{arcosh}(dx + c) + a} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{a + b\text{arccosh}(c + dx)} \, dx = \int \frac{1}{b \operatorname{arcosh}(dx + c) + a} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{1}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

input
```
int(1/(a + b*acosh(c + d*x)),x)
```

output
```
int(1/(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{1}{acosh\left(dx+c\right)b+a}dx$$

input
```
int(1/(a+b*acosh(d*x+c)),x)
```

output
```
int(1/(acosh(c + d*x)*b + a),x)
```

3.53 $\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx=\frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c)),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx=\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))}d(c+dx)}{de}$$

$$\Big\downarrow \text{ 6303}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\operatorname{arccosh}\left(dx+c\right)\right)}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arccosh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx = \frac{\int \frac{1}{ac+adx+bc\,\text{acosh}\,(c+dx)+bdx\,\text{acosh}\,(c+dx)}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

output
```
Integral(1/(a*c + a*d*x + b*c*acosh(c + d*x) + b*d*x*acosh(c + d*x)), x)/e
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}\,(c+d\,x)\right)}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))}\,dx = \frac{\int \frac{1}{acosh(dx+c)bc+acosh(dx+c)bdx+ac+adx}dx}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

output
```
int(1/(acosh(c + d*x)*b*c + acosh(c + d*x)*b*d*x + a*c + a*d*x),x)/e
```

3.54 $\int \frac{(ce+dex)^4}{(a+b\,\text{arccosh}(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 263

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\,\text{arccosh}(c+dx))^2}\,dx = &-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd(a+b\,\text{arccosh}(c+dx))}\\ &+\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\,\text{arccosh}(c+dx)}{b}\right)}{8b^2d}\\ &+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\,\text{arccosh}(c+dx))}{b}\right)}{16b^2d}\\ &+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\,\text{arccosh}(c+dx))}{b}\right)}{16b^2d}\\ &-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\,\text{arccosh}(c+dx)}{b}\right)}{8b^2d}\\ &-\frac{9e^4\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\,\text{arccosh}(c+dx))}{b}\right)}{16b^2d}\\ &-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\,\text{arccosh}(c+dx))}{b}\right)}{16b^2d}\end{aligned}$$

output
```
-e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))+1/
8*e^4*cosh(a/b)*Chi((a+b*arccosh(d*x+c))/b)/b^2/d+9/16*e^4*cosh(3*a/b)*Chi
(3*(a+b*arccosh(d*x+c))/b)/b^2/d+5/16*e^4*cosh(5*a/b)*Chi(5*(a+b*arccosh(d
*x+c))/b)/b^2/d-1/8*e^4*sinh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^2/d-9/16*e
^4*sinh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^2/d-5/16*e^4*sinh(5*a/b)*Sh
i(5*(a+b*arccosh(d*x+c))/b)/b^2/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.92 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.11

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^2}\,dx = \frac{e^4\left(-\frac{16b(c+dx)^4\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\text{arccosh}(c+dx)}-16\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arccos}\right.\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^4*((-16*b*(c + d*x)^4*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))
/(a + b*ArcCosh[c + d*x]) - 16*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c +
 d*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a
/b + ArcCosh[c + d*x])]) + 5*(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c +
d*x]] + 5*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5
*a)/b]*CoshIntegral[5*(a/b + ArcCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegra
l[a/b + ArcCosh[c + d*x]] - 5*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[
c + d*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(16
*b^2*d)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6411, 27, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^2}dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 6300$$

$$\frac{e^4\left(-\frac{\int \left(-\frac{5\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16(a+b\text{arccosh}(c+dx))}-\frac{9\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16(a+b\text{arccosh}(c+dx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+d}{b^2}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$e^4\left(-\frac{-\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\frac{9}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)-\frac{5}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{8}}{b^2}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^4*(-((Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*(a + b*ArcCo
sh[c + d*x]))) - (-1/8*(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]
) - (9*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/16 - (5
*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcCosh[c + d*x]))/b])/16 + (Sinh[a/
b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/8 + (9*Sinh[(3*a)/b]*SinhInte
gral[(3*(a + b*ArcCosh[c + d*x]))/b])/16 + (5*Sinh[(5*a)/b]*SinhIntegral[(
5*(a + b*ArcCosh[c + d*x]))/b])/16)/b^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(247) = 494$.

Time = 0.25 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.53

method	result
derivativedivides	$\frac{\left(-16\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^4+12\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+16(dx+c)^5-20(dx+c)^3+5dx+5c\right)e^4}{32b(a+b\,\mathrm{arccosh}(dx+c))}-$
default	$\frac{\left(-16\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^4+12\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+16(dx+c)^5-20(dx+c)^3+5dx+5c\right)e^4}{32b(a+b\,\mathrm{arccosh}(dx+c))}-$

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/32*(-16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^4+12*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-2
0*(d*x+c)^3+5*d*x+5*c)*e^4/b/(a+b*arccosh(d*x+c))-5/32*e^4/b^2*exp(5*a/b)*
Ei(1,5*arccosh(d*x+c)+5*a/b)+3/32*(-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x
+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4/b/(a+b*ar
ccosh(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*(-
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^4/b/(a+b*arccosh(d*x+c))-1/16*e^4
/b^2*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arccosh(
d*x+c)-a/b)-3/32/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/32
/b^2*e^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/32/b*e^4*(16*(d*x+c)^
5-20*(d*x+c)^3+16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^4+5*d*x+5*c-12*(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/
(a+b*arccosh(d*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b
))
```

Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arccosh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^4}{(b\,\mathrm{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^2}\,dx = e^4\left(\int \frac{c^4}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx + \int \frac{d^4x^4}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx + \int \frac{4cd^3x^3}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx + \int \frac{6c^2d^2x^2}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx + \int \frac{4c^3dx}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**2,x)
```

output
```
e**4*(Integral(c**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2)
, x) + Integral(d**4*x**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*
x)**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*ac
osh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*acosh(c + d
*x) + b**2*acosh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*acos
h(c + d*x) + b**2*acosh(c + d*x)**2), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*
d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*
d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^
3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*
c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d
*e^4 - 5*c^4*d*e^4)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a
*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2
*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*s
qrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + in
tegrate((5*d^8*e^4*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 - 10*c^6*e^4 + 5*c^4
*e^4 + 10*(14*c^2*d^6*e^4 - d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 - 3*c*d^5*e^
4)*x^5 + 5*(70*c^4*d^4*e^4 - 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^
3*e^4 - 10*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^
5 + 5*c^6*e^4 - 3*c^4*e^4 + 3*(25*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(25*c^3*d
^3*e^4 - 3*c*d^3*e^4)*x^3 + 3*(25*c^4*d^2*e^4 - 6*c^2*d^2*e^4)*x^2 + 6*(5*
c^5*d*e^4 - 2*c^3*d*e^4)*x)*(d*x + c + 1)*(d*x + c - 1) + 10*(14*c^6*d^2*e
^4 - 15*c^4*d^2*e^4 + 3*c^2*d^2*e^4)*x^2 + (10*d^7*e^4*x^7 + 70*c*d^6*e^4*
x^6 + 10*c^7*e^4 - 13*c^5*e^4 + 4*c^3*e^4 + (210*c^2*d^5*e^4 - 13*d^5*e^4)
*x^5 + 5*(70*c^3*d^4*e^4 - 13*c*d^4*e^4)*x^4 + 2*(175*c^4*d^3*e^4 - 65*...
```

Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^4}{(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^2}\,dx\\
&=e^4\left(\left(\int \frac{x^4}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)d^4\right.\\
&\quad+4\left(\int \frac{x^3}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)c\,d^3\\
&\quad+6\left(\int \frac{x^2}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)c^2d^2\\
&\quad+4\left(\int \frac{x}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)c^3d\\
&\quad\left.+\left(\int \frac{1}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)c^4\right)
\end{aligned}$$

input
```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^2,x)
```

output
```
e**4*(int(x**4/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*d
**4 + 4*int(x**3/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)
*c*d**3 + 6*int(x**2/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2
),x)*c**2*d**2 + 4*int(x/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b +
a**2),x)*c**3*d + int(1/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a
**2),x)*c**4)
```

3.55 $\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2} \, dx = -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd(a+b\text{arccosh}(c+dx))} + \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2b^2d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{2b^2d} - \frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2b^2d} - \frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{2b^2d}$$

output
```
-e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))+1/
2*e^3*cosh(2*a/b)*Chi(2*(a+b*arccosh(d*x+c))/b)/b^2/d+1/2*e^3*cosh(4*a/b)*
Chi(4*(a+b*arccosh(d*x+c))/b)/b^2/d-1/2*e^3*sinh(2*a/b)*Shi(2*(a+b*arccosh
(d*x+c))/b)/b^2/d-1/2*e^3*sinh(4*a/b)*Shi(4*(a+b*arccosh(d*x+c))/b)/b^2/d
```

Mathematica [A] (warning: unable to verify)

Time = 2.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.18

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx$$
$$= \frac{e^3\left(-\frac{2b(c+dx)^3\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\text{arccosh}(c+dx)}+4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arccos}\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^3*((-2*b*(c + d*x)^3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/
(a + b*ArcCosh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c
 + d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c + d*x])] + 3*Log
[a + b*ArcCosh[c + d*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c
 + d*x])] - 3*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Lo
g[a + b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c
+ d*x])]) - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c + d*x])]))/(2*b^
2*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6411, 27, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$\Big\downarrow$ 6300

$$\frac{e^3\left(-\frac{\int \left(-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{2(a+b\text{arccosh}(c+dx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{e^3\left(-\frac{-\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-\frac{1}{2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{2}}{b^2}\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^3*(-((Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*(a + b*ArcCo
sh[c + d*x]))) - (-1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d
*x]))/b]) - (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/2
 + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/2 + (Sinh[
(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/2)/b^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(183) = 366$.

Time = 0.20 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.14

method	result
derivativedivides	$\frac{\left(-8\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3+4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3}{16b(a+b\operatorname{arccosh}(dx+c))}-\frac{e^3 e^{\frac{4a}{b}}\operatorname{expIntegral}_1\left(4\operatorname{arccosh}(dx-\right.}{4b^2}$
default	$\frac{\left(-8\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3+4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3}{16b(a+b\operatorname{arccosh}(dx+c))}-\frac{e^3 e^{\frac{4a}{b}}\operatorname{expIntegral}_1\left(4\operatorname{arccosh}(dx-\right.}{4b^2}$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16*(-8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3+4*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3/b/(a+b*arccosh(d*x+
c))-1/4*e^3/b^2*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*(-2*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3/b/(a+b*arccosh(d*x+c))-1/
4*e^3/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8/b*e^3*(2*(d*x+c)^2-1
+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4/b^2*e
^3*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/16/b*e^3*(8*(d*x+c)^4-8*(d*
x+c)^2+8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-4*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,-4
*arccosh(d*x+c)-4*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*ar
ccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx=e^3\left(\int \frac{c^3}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx+\int \frac{d^3x^3}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx+\int \frac{3cd^2x^2}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx+\int \frac{3c^2dx}{a^2+2ab\,\text{acosh}\,(c+dx)+b^2\,\text{acosh}^2\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**2,x)
```

output
```
e**3*(Integral(c**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2)
, x) + Integral(d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*
x)**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*ac
osh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*acosh(c + d*x) +
b**2*acosh(c + d*x)**2), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 - c^4*e^3 + (15*c^2*d^4*e^3 - d^
4*e^3)*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 - 2*c^2*
d^2*e^3)*x^2 + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^
2*d^3*e^3 - d^3*e^3)*x^3 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (5*c^4*d*e
^3 - 3*c^2*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(3*c^5*d*e^3
- 2*c^3*d*e^3)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^
2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*
c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d
*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integra
te((4*d^7*e^3*x^7 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 - 8*c^5*e^3 + 4*c^3*e^3 +
 4*(21*c^2*d^5*e^3 - 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 - 2*c*d^4*e^3)*x^4
 + 4*(35*c^4*d^3*e^3 - 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 2*(2*d^5*e^3*x^5 +
10*c*d^4*e^3*x^4 + 2*c^5*e^3 - c^3*e^3 + (20*c^2*d^3*e^3 - d^3*e^3)*x^3 +
(20*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (10*c^4*d*e^3 - 3*c^2*d*e^3)*x)*(d*x
+ c + 1)*(d*x + c - 1) + 4*(21*c^5*d^2*e^3 - 20*c^3*d^2*e^3 + 3*c*d^2*e^3)
*x^2 + (8*d^6*e^3*x^6 + 48*c*d^5*e^3*x^5 + 8*c^6*e^3 - 10*c^4*e^3 + 3*c^2*
e^3 + 10*(12*c^2*d^4*e^3 - d^4*e^3)*x^4 + 40*(4*c^3*d^3*e^3 - c*d^3*e^3)*x
^3 + 3*(40*c^4*d^2*e^3 - 20*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(24*c^5*d*e^3 -
 20*c^3*d*e^3 + 3*c*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 4*(7*c
^6*d*e^3 - 10*c^4*d*e^3 + 3*c^2*d*e^3)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^...
```

Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^2}\,dx &= e^3\Bigg(\left(\int \frac{x^3}{acosh\,(dx+c)^2\,b^2+2acosh\,(dx+c)\,ab+a^2}dx\right)d^3\\ &\quad+3\left(\int \frac{x^2}{acosh\,(dx+c)^2\,b^2+2acosh\,(dx+c)\,ab+a^2}dx\right)c\,d^2\\ &\quad+3\left(\int \frac{x}{acosh\,(dx+c)^2\,b^2+2acosh\,(dx+c)\,ab+a^2}dx\right)c^2d\\ &\quad+\left(\int \frac{1}{acosh\,(dx+c)^2\,b^2+2acosh\,(dx+c)\,ab+a^2}dx\right)c^3\Bigg)\end{aligned}$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^2,x)
```

output
```
e**3*(int(x**3/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*d
**3 + 3*int(x**2/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)
*c*d**2 + 3*int(x/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x
)*c**2*d + int(1/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)
*c**3)
```

3.56 $\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^2}\,dx = -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd(a+b\text{arccosh}(c+dx))} + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4b^2d} + \frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4b^2d} - \frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4b^2d} - \frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4b^2d}$$

output
```
-e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))+1/
4*e^2*cosh(a/b)*Chi((a+b*arccosh(d*x+c))/b)/b^2/d+3/4*e^2*cosh(3*a/b)*Chi(
3*(a+b*arccosh(d*x+c))/b)/b^2/d-1/4*e^2*sinh(a/b)*Shi((a+b*arccosh(d*x+c))
/b)/b^2/d-3/4*e^2*sinh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^2/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx$$
$$= \frac{e^2\left(-\frac{4b(c+dx)^2\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\operatorname{arccosh}(c+dx)}+\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arccosh}(c\right.\right.\right.}{4b^2d}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^2*((-4*b*(c + d*x)^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/
(a + b*ArcCosh[c + d*x]) + Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
+ 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[a/b]*Sin
hIntegral[a/b + ArcCosh[c + d*x]] - 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
ArcCosh[c + d*x])]))/(4*b^2*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6411, 27, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\operatorname{arccosh}(c+dx))^2}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\operatorname{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$\downarrow$ 6300

$$e^2\left(-\frac{\int\left(-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4(a+b\text{arccosh}(c+dx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 2009

$$e^2\left(-\frac{-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\frac{3}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)+\frac{3}{4}\sinh\left(\frac{3}{b}\right.}{b^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e^2*(-((Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*(a + b*ArcCo
sh[c + d*x]))) - (-1/4*(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]
) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/4 + (Si
nh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/4 + (3*Sinh[(3*a)/b]*Sin
hIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/4)/b^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(179) = 358.

Time = 0.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{\left(-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2}{8b(a+b\,\operatorname{arccosh}(dx+c))}-\frac{3e^2 e^{\frac{3a}{b}}\operatorname{expIntegral}_1\left(3\,\operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b^2}+$
default	$\frac{\left(-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2}{8b(a+b\,\operatorname{arccosh}(dx+c))}-\frac{3e^2 e^{\frac{3a}{b}}\operatorname{expIntegral}_1\left(3\,\operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b^2}+$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*(-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2/b/(a+b*arccosh(d*x+c))-3/8*e^2/b^2
*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)+d*x+c)*e^2/b/(a+b*arccosh(d*x+c))-1/8*e^2/b^2*exp(a/b)*Ei(1,arccosh(d
*x+c)+a/b)-1/8/b*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(
d*x+c))-1/8/b^2*e^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8/b*e^2*(4*(d*x+
c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*arc
cosh(d*x+c)-3*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^2}{(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arccosh(d*x + c)^2 + 2
*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^2}\,dx = e^2\left(\int \frac{c^2}{a^2+2ab\operatorname{acosh}(c+dx)+b^2\operatorname{acosh}^2(c+dx)}\,dx + \int \frac{d^2x^2}{a^2+2ab\operatorname{acosh}(c+dx)+b^2\operatorname{acosh}^2(c+dx)}\,dx + \int \frac{2cdx}{a^2+2ab\operatorname{acosh}(c+dx)+b^2\operatorname{acosh}^2(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**2,x)
```

output
```
e**2*(Integral(c**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2)
, x) + Integral(d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*
x)**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c
+ d*x)**2), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 - c^3*e^2 + (10*c^2*d^3*e^2 - d^
3*e^2)*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)*x^2 + (d^4*e^2*x^4 + 4*c*d^3*e
^2*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(2*c^3*d*e^
2 - c*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^2 - 3*c^2
*d*e^2)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a
*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c
- 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((3*d
^6*e^2*x^6 + 18*c*d^5*e^2*x^5 + 3*c^6*e^2 - 6*c^4*e^2 + 3*(15*c^2*d^4*e^2
- 2*d^4*e^2)*x^4 + 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 - 2*c*d^3*e^2)*x^3 + (3*d
^4*e^2*x^4 + 12*c*d^3*e^2*x^3 + 3*c^4*e^2 - c^2*e^2 + (18*c^2*d^2*e^2 - d^
2*e^2)*x^2 + 2*(6*c^3*d*e^2 - c*d*e^2)*x)*(d*x + c + 1)*(d*x + c - 1) + 3*
(15*c^4*d^2*e^2 - 12*c^2*d^2*e^2 + d^2*e^2)*x^2 + (6*d^5*e^2*x^5 + 30*c*d^
4*e^2*x^4 + 6*c^5*e^2 - 7*c^3*e^2 + (60*c^2*d^3*e^2 - 7*d^3*e^2)*x^3 + 2*c
*e^2 + 3*(20*c^3*d^2*e^2 - 7*c*d^2*e^2)*x^2 + (30*c^4*d*e^2 - 21*c^2*d*e^2
 + 2*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 6*(3*c^5*d*e^2 - 4*c^
3*d*e^2 + c*d*e^2)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)
*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(
d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*
b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sq...
```

Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{(a+b\operatorname{acosh}(c+d\,x))^2}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx \\
&= e^2\Bigg(\left(\int \frac{x^2}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)d^2 \\
&\quad +2\left(\int \frac{x}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)cd \\
&\quad +\left(\int \frac{1}{acosh(dx+c)^2 b^2+2acosh(dx+c)ab+a^2}dx\right)c^2\Bigg)
\end{aligned}$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^2,x)
```

output
```
e**2*(int(x**2/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*d
**2 + 2*int(x/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*c*
d + int(1/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*c**2)
```

3.57 $\int \frac{ce+dex}{(a+b\,\textbf{arccosh}(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{ce+dex}{(a+b\,\text{arccosh}(c+dx))^2}\,dx = -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd(a+b\,\text{arccosh}(c+dx))} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{b^2d} - \frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{b^2d}$$

output
```
-e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))+e*cosh
(2*a/b)*Chi(2*(a+b*arccosh(d*x+c))/b)/b^2/d-e*sinh(2*a/b)*Shi(2*(a+b*arcco
sh(d*x+c))/b)/b^2/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx$$
$$= \frac{e\left(-\frac{b\sqrt{\frac{-1+c+dx}{1+c+dx}}(c+c^2+2cdx+dx(1+dx))}{a+b\operatorname{arccosh}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\left(\frac{a}{b}+\operatorname{arcco}\right.\right.\right.}{b^2d}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e*(-((b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^2 + 2*c*d*x + d*x*(1 +
d*x)))/(a + b*ArcCosh[c + d*x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + Arc
Cosh[c + d*x])] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]))
/(b^2*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6411, 27, 6300, 25, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx$$
$$\Big\downarrow \ 6411$$
$$\frac{\int \frac{e(c+dx)}{(a+b\operatorname{arccosh}(c+dx))^2}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{e\int \frac{c+dx}{(a+b\operatorname{arccosh}(c+dx))^2}d(c+dx)}{d}$$
$$\Big\downarrow \ 6300$$

$$\frac{e\left(-\frac{\int -\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}\right)}{d}$$

$\Big\downarrow$ 25

$$\frac{e\left(\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}\right)}{d}$$

$\Big\downarrow$ 3042

$$\frac{e\left(-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{d}$$

$\Big\downarrow$ 3784

$$\frac{e\left(-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{-\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+i\sinh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arccos}\cdots}{b}\right)}{a+b\text{arccosh}(\cdots}\cdots}{b^2}\cdots\right)}{d}$$

$\Big\downarrow$ 26

$$\frac{e\left(-\frac{\sinh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\cdots\right)}{d}$$

$\Big\downarrow$ 3042

$$\frac{e\left(-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{\sinh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arccosh}(c\cdots}{b}\right)}{a+b\text{arccosh}(c+\cdots}\cdots}{b^2}\cdots\right)}{d}$$

$\Big\downarrow$ 26

$$\frac{e\left(-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+\cdots}{b}\right.}{a+b\text{arccosh}(c+\cdots}}{b^2}\right.}{d}$$

$\Big\downarrow$ 3779

$$\frac{e\left(-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c\cdots}{b^2}\right.}{d}$$

$\Big\downarrow$ 3782

$$\frac{e\left(-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}\right)}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(e*(-((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c
 + d*x]))) - (-(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]
) + Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/b^2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{\left(-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+2(dx+c)^2-1\right)e}{4b(a+b\,\operatorname{arccosh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\text{expIntegral}_1\left(2\,\operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2-1+2\sqrt{dx+c-1}\,\sqrt{dx+}\right.}{4b(a+b\,\operatorname{arccosh}(dx+c)}}{d}$
default	$\frac{\frac{\left(-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+2(dx+c)^2-1\right)e}{4b(a+b\,\operatorname{arccosh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\text{expIntegral}_1\left(2\,\operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2-1+2\sqrt{dx+c-1}\,\sqrt{dx+}\right.}{4b(a+b\,\operatorname{arccosh}(dx+c)}}{d}$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*(-2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e/b/(a
+b*arccosh(d*x+c))-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4/b
*e*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(
d*x+c))-1/2/b^2*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

Fricas [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{dex+ce}{\left(b\operatorname{arcosh}\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) +
a^2), x)
```

Sympy [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^2} dx = e\left(\int \frac{c}{a^2 + 2ab\,\text{acosh}\,(c + dx) + b^2\,\text{acosh}^2\,(c + dx)} dx + \int \frac{dx}{a^2 + 2ab\,\text{acosh}\,(c + dx) + b^2\,\text{acosh}^2\,(c + dx)} dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

output
```
e*(Integral(c/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x) +
 Integral(d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x))
```

Maxima [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^2} dx = \int \frac{dex + ce}{(b\,\text{arcosh}\,(dx + c) + a)^2} dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e - c^2*e + (6*c^2*d^2*e - d^2*e)*x^2 +
(d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e - c*e + (3*c^2*d*e - d*e)*x)*sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + 2*(2*c^3*d*e - c*d*e)*x)/(a*b*d^3*x^2 + 2*a*b*
c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x
+ b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1
)*sqrt(d*x + c - 1) + c)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^
5*e - 4*c^3*e + 4*(5*c^2*d^3*e - d^3*e)*x^3 + 2*(d^3*e*x^3 + 3*c*d^2*e*x^2
 + 3*c^2*d*e*x + c^3*e)*(d*x + c + 1)*(d*x + c - 1) + 4*(5*c^3*d^2*e - 3*c
*d^2*e)*x^2 + (4*d^4*e*x^4 + 16*c*d^3*e*x^3 + 4*c^4*e - 4*c^2*e + 4*(6*c^2
*d^2*e - d^2*e)*x^2 + 8*(2*c^3*d*e - c*d*e)*x + e)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + 2*c*e + 2*(5*c^4*d*e - 6*c^2*d*e + d*e)*x)/(a*b*d^4*x^4 + 4
*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*
b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*
c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c
^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*
d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1
)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)
*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*...
```

Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{dex+ce}{(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{c\,e+d\,e\,x}{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^2}\,dx = e\left(\left(\int \frac{x}{acosh\left(dx+c\right)^2b^2+2acosh\left(dx+c\right)ab+a^2}dx\right)d+\left(\int \frac{1}{acosh\left(dx+c\right)^2b^2+2acosh\left(dx+c\right)ab+a^2}dx\right)c\right)$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^2,x)
```

output
```
e*(int(x/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*d + int
(1/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)*c)
```

3.58 $\int \frac{1}{(a+b\,\mathbf{arccosh}(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{1}{(a+b\,\mathrm{arccosh}(c+dx))^2}\,dx = -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd(a+b\,\mathrm{arccosh}(c+dx))} + \frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\,\mathrm{arccosh}(c+dx)}{b}\right)}{b^2d} - \frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arccosh}(c+dx)}{b}\right)}{b^2d}$$

output
```
-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))+cosh(a/b)*Chi((a
+b*arccosh(d*x+c))/b)/b^2/d-sinh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^2/d
```

Mathematica [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b\,\mathrm{arccosh}(c+dx))^2}\,dx = \frac{-\frac{b\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\,\mathrm{arccosh}(c+dx)} + \cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arccosh}(c+dx)\right) - \sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{arccosh}(c+dx)\right)}{b^2d}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-2),x]
```

output
```
(-((b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(a + b*ArcCosh[c +
 d*x])) + Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] - Sinh[a/b]*SinhI
ntegral[a/b + ArcCosh[c + d*x]])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6410, 6295, 6368, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6295}$$

$$\frac{\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}d(c+dx)}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{d}$$

$$\Big\downarrow \text{6368}$$

$$\frac{\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \text{3784}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\sinh\left(\frac{a}{b}\right)\int-\frac{i\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b}{b^2}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}}{b(a+b\text{arccos}}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(}{b^2}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a}{b^2}}{d}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(-2),x]
```

output
```
(-((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) +
(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b] - Sinh[a/b]*SinhIntegr
al[(a + b*ArcCosh[c + d*x])/b])/b^2)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{\frac{-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c}{2b(a+b\,\text{arccosh}(dx+c))}-\frac{e^{\frac{a}{b}}\,\text{expIntegral}_1\left(\text{arccosh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2b(a+b\,\text{arccosh}(dx+c))}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}_1\left(-\text{arccosh}\right.}{2b^2}}{d}$
default	$\frac{\frac{-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c}{2b(a+b\,\text{arccosh}(dx+c))}-\frac{e^{\frac{a}{b}}\,\text{expIntegral}_1\left(\text{arccosh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2b(a+b\,\text{arccosh}(dx+c))}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}_1\left(-\text{arccosh}\right.}{2b^2}}{d}$

input
```
int(1/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)/b/(a+b*arccosh(d*x+c))-1
/2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a
/b))
```

Fricas [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(a+b\,\text{acosh}\,(c+dx))^2}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**2,x)
```

output
```
Integral((a + b*acosh(c + d*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*
x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*
c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)) + integrate((d^4*x^4 + 4*c*d^3*x^3 + c^4 + (d^2*x^2 +
2*c*d*x + c^2 + 1)*(d*x + c + 1)*(d*x + c - 1) + 2*(3*c^2*d^2 - d^2)*x^2 +
 (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d - d)*x - c)*sqrt(d*x + c + 1)
*sqrt(d*x + c - 1) - 2*c^2 + 4*(c^3*d - c*d)*x + 1)/(a*b*d^4*x^4 + 4*a*b*c
*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*
x^2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 +
1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c
)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2
+ 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx=\int \frac{1}{\left(b\operatorname{arcosh}\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acosh(c + d*x))^2,x)
```

output
```
int(1/(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^2}\,dx=\int \frac{1}{acosh\left(dx+c\right)^2 b^2+2acosh\left(dx+c\right)ab+a^2}dx$$

input
```
int(1/(a+b*acosh(d*x+c))^2,x)
```

output
```
int(1/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2),x)
```

3.59 $\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2} dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^2},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^2,x)/e
```

Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2} dx = \int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2} dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6303}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arccosh}\left(dx+c\right)\right)^2}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arccosh(d*x + c)^2
 + 2*(a*b*d*e*x + a*b*c*e)*arccosh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}\,dx = \frac{\int \frac{1}{a^2c+a^2dx+2abc\,\text{acosh}\,(c+dx)+2abdx\,\text{acosh}\,(c+dx)+b^2c\,\text{acosh}^2\,(c+dx)+b^2dx\,\text{acosh}^2\,(c+dx)}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

output
```
Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*acosh(c + d*x) + 2*a*b*d*x*acosh(c
 + d*x) + b**2*c*acosh(c + d*x)**2 + b**2*d*x*acosh(c + d*x)**2), x)/e
```

Maxima [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 1077, normalized size of antiderivative = 46.83

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^
3*e*x^2 + (3*c^2*d^2*e - d^2*e)*a*b*x + (c^3*d*e - c*d*e)*a*b + (a*b*d^3*e
*x^2 + 2*a*b*c*d^2*e*x + a*b*c^2*d*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ (b^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*c^2*d^2*e - d^2*e)*b^2*x + (c^3*
d*e - c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*c*d^2*e*x + b^2*c^2*d*e)*sqrt(d*
x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)) + integrate((2*(d*x + c + 1)*(d*x + c)*(d*x + c - 1) + (2*d^2*x^2
+ 4*c*d*x + 2*c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))/(a*b*d^6*e*x^6
 + 6*a*b*c*d^5*e*x^5 + (15*c^2*d^4*e - 2*d^4*e)*a*b*x^4 + 4*(5*c^3*d^3*e -
 2*c*d^3*e)*a*b*x^3 + (15*c^4*d^2*e - 12*c^2*d^2*e + d^2*e)*a*b*x^2 + 2*(3
*c^5*d*e - 4*c^3*d*e + c*d*e)*a*b*x + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 +
 6*a*b*c^2*d^2*e*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d*x + c + 1)*(d*x + c
 - 1) + (c^6*e - 2*c^4*e + c^2*e)*a*b + 2*(a*b*d^5*e*x^5 + 5*a*b*c*d^4*e*x
^4 + (10*c^2*d^3*e - d^3*e)*a*b*x^3 + (10*c^3*d^2*e - 3*c*d^2*e)*a*b*x^2 +
 (5*c^4*d*e - 3*c^2*d*e)*a*b*x + (c^5*e - c^3*e)*a*b)*sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + (b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e - 2*d
^4*e)*b^2*x^4 + 4*(5*c^3*d^3*e - 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e - 12*c
^2*d^2*e + d^2*e)*b^2*x^2 + 2*(3*c^5*d*e - 4*c^3*d*e + c*d*e)*b^2*x + (b^2
*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 + 4*b^2*c^3*d*e*x ...
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}\,(c+d\,x)\right)^2}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^2),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^2}\,dx = \frac{\int \frac{1}{acosh(dx+c)^2b^2c+acosh(dx+c)^2b^2dx+2acosh(dx+c)abc+2acosh(dx+c)abdx+a^2c+a^2dx}dx}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^2,x)
```

output
```
int(1/(acosh(c + d*x)**2*b**2*c + acosh(c + d*x)**2*b**2*d*x + 2*acosh(c +
 d*x)*a*b*c + 2*acosh(c + d*x)*a*b*d*x + a**2*c + a**2*d*x),x)/e
```

3.60 $\int \frac{(ce+dex)^4}{(a+b\,\mathrm{arccosh}(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 327

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\,\mathrm{arccosh}(c+dx))^3}\,dx &= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd(a+b\,\mathrm{arccosh}(c+dx))^2} \\ &+ \frac{2e^4(c+dx)^3}{b^2d(a+b\,\mathrm{arccosh}(c+dx))} \\ &- \frac{5e^4(c+dx)^5}{2b^2d(a+b\,\mathrm{arccosh}(c+dx))} \\ &- \frac{e^4\mathrm{Chi}\left(\frac{a+b\,\mathrm{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{16b^3d} \\ &- \frac{27e^4\mathrm{Chi}\left(\frac{3(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32b^3d} \\ &- \frac{25e^4\mathrm{Chi}\left(\frac{5(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{32b^3d} \\ &+ \frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arccosh}(c+dx)}{b}\right)}{16b^3d} \\ &+ \frac{27e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{32b^3d} \\ &+ \frac{25e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{32b^3d}\end{aligned}$$

output
```
-1/2*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^2+2*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+
b*arccosh(d*x+c))-1/16*e^4*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^3/d-27/
32*e^4*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^3/d-25/32*e^4*Chi(5*(a+
b*arccosh(d*x+c))/b)*sinh(5*a/b)/b^3/d+1/16*e^4*cosh(a/b)*Shi((a+b*arccosh
(d*x+c))/b)/b^3/d+27/32*e^4*cosh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^3/
d+25/32*e^4*cosh(5*a/b)*Shi(5*(a+b*arccosh(d*x+c))/b)/b^3/d
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.99

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^3}\,dx = \frac{e^4\left(-\frac{16b^2\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{(a+b\text{arccosh}(c+dx))^2}+\frac{16b\left(4(c+dx)^3-5(c+dx)^5\right)}{a+b\text{arccosh}(c+dx)}+48\left(\text{Chi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)+\text{Chi}(3\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^4*((-16*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Ar
cCosh[c + d*x])^2 + (16*b*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[
c + d*x]) + 48*(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] + CoshInteg
ral[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b
 + ArcCosh[c + d*x]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x
])]) + 25*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshInteg
ral[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcC
osh[c + d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c +
d*x]] + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5
*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(32*b^3*d)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6411, 27, 6301, 6366, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^3}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6301}$$

$$\frac{e^4\left(-\frac{2\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}+\frac{5\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(\cdots}{2b(a+b\text{arccosh}(c+\cdots}\right.}{d}$$

$$\Big\downarrow \text{ 6366}$$

$$\frac{e^4\left(-\frac{2\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right)}{b}+\frac{5\left(\frac{5\int \frac{(c+dx)^4}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^5}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{\sqrt{c+\cdots}}{2b(\cdots}\right.}{d}$$

$$\Big\downarrow \text{ 6302}$$

$$e^4\left(\frac{5\left(\frac{5\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^5}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{2\left(\frac{3\int -\frac{\cosh^2}{}}{}\right.}{}\right.$$

$\downarrow$ 25

$$e^4\left(\frac{5\left(-\frac{5\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^5}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{2\left(-\frac{3\int \frac{\cosh^2}{}}{}\right.}{}\right.$$

$\downarrow$ 5971

$$e^4\left(\frac{5\left(-\frac{5\int\left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16(a+b\text{arccosh}(c+dx))}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16(a+b\text{arccosh}(c+dx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{b^2}\right.}{2b}\right.$$

$\downarrow$ 2009

$$e^4\left(-\frac{2\left(\frac{3\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)+\frac{1}{4}\cosh(\right.}{b^2}\right.}{b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^4*(-1/2*(Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*(a + b*Ar
cCosh[c + d*x])^2) - (2*(-((c + d*x)^3/(b*(a + b*ArcCosh[c + d*x]))) + (3*
(-1/4*(CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b]) - (CoshIntegral
[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/4 + (Cosh[a/b]*SinhIntegra
l[(a + b*ArcCosh[c + d*x])/b])/4 + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcCosh[c + d*x]))/b])/4))/b^2))/b + (5*(-((c + d*x)^5/(b*(a + b*ArcCosh[c
+ d*x]))) + (5*(-1/8*(CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])
- (3*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/16 - (Cos
hIntegral[(5*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*S
inhIntegral[(a + b*ArcCosh[c + d*x])/b])/8 + (3*Cosh[(3*a)/b]*SinhIntegral
[(3*(a + b*ArcCosh[c + d*x]))/b])/16 + (Cosh[(5*a)/b]*SinhIntegral[(5*(a +
 b*ArcCosh[c + d*x]))/b])/16))/b^2))/(2*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(307) = 614$.

Time = 0.26 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.04

method	result	size
derivativedivides	Expression too large to display	993
default	Expression too large to display	993

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/64*(-16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^4+12*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-
20*(d*x+c)^3+5*d*x+5*c)*e^4*(5*b*arccosh(d*x+c)+5*a-b)/b^2/(b^2*arccosh(d*
x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+25/64*e^4/b^3*exp(5*a/b)*Ei(1,5*arccosh(d
*x+c)+5*a/b)-3/64*(-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(3*b*arccosh(d*x+c)+3*a-b
)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+27/64*e^4/b^3*exp(3*
a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)-1/32*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d
*x+c)*e^4*(b*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d
*x+c)+a^2)+1/32*e^4/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/32/b*e^4*(d*x+
c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/32/b^2*e^4*(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/32/b^3*e^4*exp
(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-3/64/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))^2-9/64/b^2*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x
+c))-27/64/b^3*e^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/64/b*e^4*(1
6*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^4+5*d*
x+5*c-12*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))/(a+b*arccosh(d*x+c))^2-5/64/b^2*e^4*(16*(d*x+c)^5-20*(d*x+c)^...
```

Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*
b*arccosh(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^3}\,dx = e^4\left(\int \frac{c^4}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx + \int \frac{d^4x^4}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx + \int \frac{4cd^3x^3}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx + \int \frac{6c^2d^2x^2}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx + \int \frac{4c^3dx}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**3,x)
```

output
```
e**4*(Integral(c**4/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d
*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b
*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x)
 + Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh
(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a*
*3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c +
 d*x)**3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*
b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((5*a*d^11*e^4 + b*d^11*e^4)*x^11 + 11*(5*a*c*d^10*e^4 + b*c*d^10*e^4
)*x^10 + (5*(55*c^2*d^9*e^4 - 3*d^9*e^4)*a + (55*c^2*d^9*e^4 - 3*d^9*e^4)*
b)*x^9 + 3*(5*(55*c^3*d^8*e^4 - 9*c*d^8*e^4)*a + (55*c^3*d^8*e^4 - 9*c*d^8
*e^4)*b)*x^8 + 3*(5*(110*c^4*d^7*e^4 - 36*c^2*d^7*e^4 + d^7*e^4)*a + (110*
c^4*d^7*e^4 - 36*c^2*d^7*e^4 + d^7*e^4)*b)*x^7 + 21*(5*(22*c^5*d^6*e^4 - 1
2*c^3*d^6*e^4 + c*d^6*e^4)*a + (22*c^5*d^6*e^4 - 12*c^3*d^6*e^4 + c*d^6*e^
4)*b)*x^6 + (5*(462*c^6*d^5*e^4 - 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 - d^5*e
^4)*a + (462*c^6*d^5*e^4 - 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 - d^5*e^4)*b)*
x^5 + (5*(330*c^7*d^4*e^4 - 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 - 5*c*d^4*e^
4)*a + (330*c^7*d^4*e^4 - 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 - 5*c*d^4*e^4)
*b)*x^4 + ((5*a*d^8*e^4 + b*d^8*e^4)*x^8 + 8*(5*a*c*d^7*e^4 + b*c*d^7*e^4)
*x^7 + (4*(35*c^2*d^6*e^4 - 2*d^6*e^4)*a + (28*c^2*d^6*e^4 - d^6*e^4)*b)*x
^6 + 2*(4*(35*c^3*d^5*e^4 - 6*c*d^5*e^4)*a + (28*c^3*d^5*e^4 - 3*c*d^5*e^4
)*b)*x^5 + ((350*c^4*d^4*e^4 - 120*c^2*d^4*e^4 + 3*d^4*e^4)*a + 5*(14*c^4*
d^4*e^4 - 3*c^2*d^4*e^4)*b)*x^4 + 4*((70*c^5*d^3*e^4 - 40*c^3*d^3*e^4 + 3*
c*d^3*e^4)*a + (14*c^5*d^3*e^4 - 5*c^3*d^3*e^4)*b)*x^3 + (2*(70*c^6*d^2*e^
4 - 60*c^4*d^2*e^4 + 9*c^2*d^2*e^4)*a + (28*c^6*d^2*e^4 - 15*c^4*d^2*e^4)*
b)*x^2 + (5*c^8*e^4 - 8*c^6*e^4 + 3*c^4*e^4)*a + (c^8*e^4 - c^6*e^4)*b + 2
*(2*(10*c^7*d*e^4 - 12*c^5*d*e^4 + 3*c^3*d*e^4)*a + (4*c^7*d*e^4 - 3*c^5*d
*e^4)*b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + (5*(165*c^8*d^3*e...
```

Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^4}{(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\operatorname{acosh}(c+d\,x))^3}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^4}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx \\
&=e^4\left(\left(\int \frac{x^4}{acosh(dx+c)^3 b^3+3acosh(dx+c)^2 a b^2+3acosh(dx+c)a^2b+a^3}dx\right)d^4\right. \\
&\quad+4\left(\int \frac{x^3}{acosh(dx+c)^3 b^3+3acosh(dx+c)^2 a b^2+3acosh(dx+c)a^2b+a^3}dx\right)c\,d^3 \\
&\quad+6\left(\int \frac{x^2}{acosh(dx+c)^3 b^3+3acosh(dx+c)^2 a b^2+3acosh(dx+c)a^2b+a^3}dx\right)c^2d^2 \\
&\quad+4\left(\int \frac{x}{acosh(dx+c)^3 b^3+3acosh(dx+c)^2 a b^2+3acosh(dx+c)a^2b+a^3}dx\right)c^3d \\
&\quad\left.+\left(\int \frac{1}{acosh(dx+c)^3 b^3+3acosh(dx+c)^2 a b^2+3acosh(dx+c)a^2b+a^3}dx\right)c^4\right)
\end{aligned}$$

input

```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^3,x)
```

output

```
e**4*(int(x**4/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*ac
osh(c + d*x)*a**2*b + a**3),x)*d**4 + 4*int(x**3/(acosh(c + d*x)**3*b**3 +
 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)*c*d**3 +
6*int(x**2/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(
c + d*x)*a**2*b + a**3),x)*c**2*d**2 + 4*int(x/(acosh(c + d*x)**3*b**3 + 3
*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)*c**3*d + in
t(1/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x
)*a**2*b + a**3),x)*c**4)
```

3.61 $\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 254

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx = &-\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd(a+b\text{arccosh}(c+dx))^2} \\ &+\frac{3e^3(c+dx)^2}{2b^2d(a+b\text{arccosh}(c+dx))} \\ &-\frac{2e^3(c+dx)^4}{b^2d(a+b\text{arccosh}(c+dx))} \\ &-\frac{e^3\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2b^3d} \\ &-\frac{e^3\text{Chi}\left(\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{b^3d} \\ &+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2b^3d} \\ &+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{b^3d}\end{aligned}$$

output
```
-1/2*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^2+3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+
b*arccosh(d*x+c))-1/2*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*sinh(2*a/b)/b^3/d-
e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b^3/d+1/2*e^3*cosh(2*a/b)*Sh
i(2*(a+b*arccosh(d*x+c))/b)/b^3/d+e^3*cosh(4*a/b)*Shi(4*(a+b*arccosh(d*x+c
))/b)/b^3/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.73

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx = \frac{e^3\left(-\frac{b^2\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{(a+b\text{arccosh}(c+dx))^2}+\frac{b(3(c+dx)^2-4(c+dx)^4)}{a+b\text{arccosh}(c+dx)}-\text{Chi}\left(2\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2\text{Chi}(4(\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^3*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^2) + (b*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c +
d*x]) - CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshInt
egral[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + Cosh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcCosh[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + Arc
Cosh[c + d*x])]))/(2*b^3*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {6411, 27, 6301, 6366, 6302, 25, 5971, 27, 2009, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}dx$$

$\big\downarrow$ 6411

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$\big\downarrow$ 27

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$\big\downarrow$ 6301

$$\frac{e^3\left(-\frac{3\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}+\frac{2\int \frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(}{2b(a+b\text{arccosh}(c+}\right.}{d}$$

$\big\downarrow$ 6366

$$\frac{e^3\left(-\frac{3\left(\frac{2\int \frac{c+dx}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}+\frac{2\left(\frac{4\int \frac{(c+dx)^3}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^4}{b(a+b\text{arccosh}(c+dx))}\right)}{b}-\frac{\sqrt{c+}}{2b(}\right.}{d}$$

$\big\downarrow$ 6302

$$\frac{e^3\left(\frac{2\left(\frac{4\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^4}{b(a+b\text{arccosh}(c+dx))}\right)}{b}-\frac{3\left(\frac{2\int -\frac{\cosh(}{}}{}\right.}{}\right.}{d}$$

$\big\downarrow$ 25

$$\frac{e^3\left(\frac{2\left(-\frac{4\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^4}{b(a+b\text{arccosh}(c+dx))}\right)}{b}-\frac{3\left(-\frac{2\int \frac{\cosh \cdots}{\cdots}}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{d\cdots}$$

$\downarrow$ 5971

$$\frac{e^3\left(-\frac{3\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2(a+b\text{arccosh}(c+dx))}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}+\frac{2\left(-\frac{4\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+\cdots}{b}\cdots\right)}{8(a+b\text{arccosh}(c+dx))}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{d\cdots}$$

$\downarrow$ 27

$$\frac{e^3\left(-\frac{3\left(-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}+\frac{2\left(-\frac{4\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+dx\cdots}{b}\cdots\right)}{8(a+b\text{arccosh}(c+dx))}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{d\cdots}$$

$\downarrow$ 2009

$$e^3\left(-\frac{3\left(-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}+\frac{2\left(\frac{4\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}}{b}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{2b}+\frac{2\left(\frac{4\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcco}}{b}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 26

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{2b}+\frac{2\left(\frac{4\left(-\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccos}}{b}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3784

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sinh\left(\frac{2(a+b\text{arccosh}(c+\right.}{b}}{a+b\text{arccosh}(c+dx}\right.}{b^2}\right.}{2b}\right.$$

$$\Big\downarrow \textbf{26}$$

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arccosh}(c+d\right.}{b}}{a+b\text{arccosh}(c+dx)}\right.}{b^2}\right.}{2b}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arccosh}\right.}{b}}{a+b\text{arccosh}(c}\right.}{b^2}\right.}{2b}\right.$$

$$\Big\downarrow \textbf{26}$$

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+\cdots}{a+b\text{arccosh}(c+d\cdots}\cdots\right.}{b^2}\cdots\right.}{2b}\cdots\right.$$

$$\Big\downarrow \ 3779$$

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+\cdots}{b}\cdots\right.\right.}{b^2}\cdots\right.}{2b}\cdots\right.$$

$$\Big\downarrow \ 3782$$

$$e^3\left(-\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)\right)}{b^2}\right)}{2b}+\frac{2\left(\frac{4\left(-\frac{1}{4}\sinh\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^3*(-1/2*(Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*(a + b*Ar
cCosh[c + d*x])^2) - (3*(-((c + d*x)^2/(b*(a + b*ArcCosh[c + d*x]))) + (I*
(I*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b] - I*Cosh[(2*
a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]))/b^2))/(2*b) + (2*(-((
c + d*x)^4/(b*(a + b*ArcCosh[c + d*x]))) + (4*(-1/4*(CoshIntegral[(2*(a +
b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b]) - (CoshIntegral[(4*(a + b*ArcCosh[c
 + d*x]))/b]*Sinh[(4*a)/b])/8 + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcC
osh[c + d*x]))/b])/4 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c + d
*x]))/b])/8))/b^2))/b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(242) = 484$.

Time = 0.20 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.46

method	result
derivativedivides	$-\frac{\left(-8\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3+4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3(4b\,\mathrm{arccosh}(dx+c)+4a-b)}{32b^2\left(b^2\,\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}+\frac{e^3 e^{\frac{4a}{b}}}{}$
default	$-\frac{\left(-8\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3+4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3(4b\,\mathrm{arccosh}(dx+c)+4a-b)}{32b^2\left(b^2\,\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}+\frac{e^3 e^{\frac{4a}{b}}}{}$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/32*(-8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3+4*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(4*b*arccosh(d*x+c
)+4*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/2*e^3/b^3*e
xp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)-1/16*(-2*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arcco
sh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4*e^3/b^3*exp(2*a/b)*Ei(1,2*arccos
h(d*x+c)+2*a/b)-1/16/b*e^3*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/8/b^2*e^3*(2*(d*x+c)^2-1+2*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4/b^3*e^3*exp(-2*a/b)
*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*
x+c)+1)/(a+b*arccosh(d*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d
*x+c)+1)/(a+b*arccosh(d*x+c))-1/2/b^3*e^3*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+
c)-4*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arccosh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^3}{(b\,\mathrm{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*ar
ccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) +
 a^3), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arccosh}(c+dx))^3}\,dx = e^3\left(\int \frac{c^3}{a^3+3a^2b\,\mathrm{acosh}\,(c+dx)+3ab^2\,\mathrm{acosh}^2\,(c+dx)+b^3\,\mathrm{acosh}^3\,(c+dx)}\,dx + \int \frac{d^3x^3}{a^3+3a^2b\,\mathrm{acosh}\,(c+dx)+3ab^2\,\mathrm{acosh}^2\,(c+dx)+b^3\,\mathrm{acosh}^3\,(c+dx)}\,dx + \int \frac{3cd^2x^2}{a^3+3a^2b\,\mathrm{acosh}\,(c+dx)+3ab^2\,\mathrm{acosh}^2\,(c+dx)+b^3\,\mathrm{acosh}^3\,(c+dx)}\,dx + \int \frac{3c^2dx}{a^3+3a^2b\,\mathrm{acosh}\,(c+dx)+3ab^2\,\mathrm{acosh}^2\,(c+dx)+b^3\,\mathrm{acosh}^3\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**3,x)
```

output
```
e**3*(Integral(c**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d
*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b
*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x)
 + Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh
(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3
*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)*
*3), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((4*a*d^10*e^3 + b*d^10*e^3)*x^10 + 10*(4*a*c*d^9*e^3 + b*c*d^9*e^3)*
x^9 + 3*(4*(15*c^2*d^8*e^3 - d^8*e^3)*a + (15*c^2*d^8*e^3 - d^8*e^3)*b)*x^
8 + 24*(4*(5*c^3*d^7*e^3 - c*d^7*e^3)*a + (5*c^3*d^7*e^3 - c*d^7*e^3)*b)*x
^7 + 3*(4*(70*c^4*d^6*e^3 - 28*c^2*d^6*e^3 + d^6*e^3)*a + (70*c^4*d^6*e^3
- 28*c^2*d^6*e^3 + d^6*e^3)*b)*x^6 + 6*(4*(42*c^5*d^5*e^3 - 28*c^3*d^5*e^3
 + 3*c*d^5*e^3)*a + (42*c^5*d^5*e^3 - 28*c^3*d^5*e^3 + 3*c*d^5*e^3)*b)*x^5
 + (4*(210*c^6*d^4*e^3 - 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 - d^4*e^3)*a + (
210*c^6*d^4*e^3 - 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 - d^4*e^3)*b)*x^4 + ((4
*a*d^7*e^3 + b*d^7*e^3)*x^7 + 7*(4*a*c*d^6*e^3 + b*c*d^6*e^3)*x^6 + (6*(14
*c^2*d^5*e^3 - d^5*e^3)*a + (21*c^2*d^5*e^3 - d^5*e^3)*b)*x^5 + 5*(2*(14*c
^3*d^4*e^3 - 3*c*d^4*e^3)*a + (7*c^3*d^4*e^3 - c*d^4*e^3)*b)*x^4 + (2*(70*
c^4*d^3*e^3 - 30*c^2*d^3*e^3 + d^3*e^3)*a + 5*(7*c^4*d^3*e^3 - 2*c^2*d^3*e
^3)*b)*x^3 + (6*(14*c^5*d^2*e^3 - 10*c^3*d^2*e^3 + c*d^2*e^3)*a + (21*c^5*
d^2*e^3 - 10*c^3*d^2*e^3)*b)*x^2 + 2*(2*c^7*e^3 - 3*c^5*e^3 + c^3*e^3)*a +
 (c^7*e^3 - c^5*e^3)*b + (2*(14*c^6*d*e^3 - 15*c^4*d*e^3 + 3*c^2*d*e^3)*a
+ (7*c^6*d*e^3 - 5*c^4*d*e^3)*b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2
) + 4*(4*(30*c^7*d^3*e^3 - 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 - c*d^3*e^3)*a
+ (30*c^7*d^3*e^3 - 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 - c*d^3*e^3)*b)*x^3 +
(3*(4*a*d^8*e^3 + b*d^8*e^3)*x^8 + 24*(4*a*c*d^7*e^3 + b*c*d^7*e^3)*x^7 +
(24*(14*c^2*d^6*e^3 - d^6*e^3)*a + (84*c^2*d^6*e^3 - 5*d^6*e^3)*b)*x^6 ...
```

Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{acosh}\,(c+d\,x))^3}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^3}\,dx \\
&= e^3\left(\left(\int \frac{x^3}{acosh\,(dx+c)^3\,b^3+3acosh\,(dx+c)^2\,a\,b^2+3acosh\,(dx+c)\,a^2b+a^3}dx\right)d^3\right. \\
&\quad+3\left(\int \frac{x^2}{acosh\,(dx+c)^3\,b^3+3acosh\,(dx+c)^2\,a\,b^2+3acosh\,(dx+c)\,a^2b+a^3}dx\right)c\,d^2 \\
&\quad+3\left(\int \frac{x}{acosh\,(dx+c)^3\,b^3+3acosh\,(dx+c)^2\,a\,b^2+3acosh\,(dx+c)\,a^2b+a^3}dx\right)c^2d \\
&\quad\left.+\left(\int \frac{1}{acosh\,(dx+c)^3\,b^3+3acosh\,(dx+c)^2\,a\,b^2+3acosh\,(dx+c)\,a^2b+a^3}dx\right)c^3\right)
\end{aligned}$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^3,x)
```

output
```
e**3*(int(x**3/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*ac
osh(c + d*x)*a**2*b + a**3),x)*d**3 + 3*int(x**2/(acosh(c + d*x)**3*b**3 +
 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)*c*d**2 +
3*int(x/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c +
 d*x)*a**2*b + a**3),x)*c**2*d + int(1/(acosh(c + d*x)**3*b**3 + 3*acosh(c
 + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)*c**3)
```

3.62 $\int \frac{(ce+dex)^2}{(a+b\,\text{arccosh}(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}\,dx = -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd(a+b\text{arccosh}(c+dx))^2} + \frac{e^2(c+dx)}{b^2d(a+b\text{arccosh}(c+dx))} - \frac{3e^2(c+dx)^3}{2b^2d(a+b\text{arccosh}(c+dx))} - \frac{e^2\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^3d} - \frac{9e^2\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{8b^3d} + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8b^3d} + \frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{8b^3d}$$

output
```
-1/2*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^2+e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*ar
ccosh(d*x+c))-1/8*e^2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^3/d-9/8*e^2*
Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^3/d+1/8*e^2*cosh(a/b)*Shi((a+b
*arccosh(d*x+c))/b)/b^3/d+9/8*e^2*cosh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b
)/b^3/d
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}\,dx = \frac{e^2\left(-\frac{4b^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{(a+b\text{arccosh}(c+dx))^2}+\frac{4b(2(c+dx)-3(c+dx)^3)}{a+b\text{arccosh}(c+dx)}+8\text{Chi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-8\cosh\left(\frac{a}{b}\right)\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^2*((-4*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^2 + (4*b*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c +
d*x]) + 8*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 8*Cosh[a/b]*Sin
hIntegral[a/b + ArcCosh[c + d*x]] + 9*(-(CoshIntegral[a/b + ArcCosh[c + d*
x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] +
Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegra
l[3*(a/b + ArcCosh[c + d*x])])))/(8*b^3*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {6411, 27, 6301, 6366, 6296, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{6301}$$

$$\frac{e^2\left(-\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}+\frac{3\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+\cdots}{2b(a+b\text{arccosh}(c+\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \text{6366}$$

$$\frac{e^2\left(-\frac{\frac{\int \frac{1}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{\sqrt{c+dx-1}\cdots}{2b(a+b\text{ar}\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \text{6296}$$

$$\frac{e^2\left(-\frac{\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx\cdots}{b(a+b\text{arccos}\cdots}\cdots\right.}{2b}\cdots\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\left(-\frac{-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx}{b(a+b\text{arccos}}\right.}{2b}\right.}{d}$$

$\Big\downarrow$ 3042

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(}{b(a+b\text{ar}}\right.}{2b}\right.}{d}$$

$\Big\downarrow$ 26

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c-}{b(a+b\text{arcc}}\right.}{2b}\right.}{d}$$

$\Big\downarrow$ 3784

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+\right.}{b^2}}{b}\right.}{d}$$

$\Big\downarrow$ 26

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\ldots}{b^2}}{b}\ldots\right.}{d\ldots}$$

$\big\downarrow$ 3042

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arccosh}(c+dx)\ldots}{b}\right.}{a+b\text{arccosh}(c+dx)}\ldots\right.}{b^2}}{b}\ldots\right.}{d\ldots}$$

$\big\downarrow$ 26

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d\ldots\right.}{b^2}}{b}\ldots\right.}{d\ldots}$$

$\big\downarrow$ 3779

$$\frac{e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)}{b^2}}{b}\ldots\right.}{d\ldots}$$

$\big\downarrow$ 3782

$$\frac{e^2\left(\frac{3\left(\frac{3\int \frac{(c+dx)^2}{\frac{a+b\text{arccosh}(c+dx)}{b}}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-i\cosh}{b^2}}{b}\right.}{d}$$

$\downarrow$ 6302

$$\frac{e^2\left(\frac{3\left(\frac{3\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{-\frac{c+}{b(a+b\text{arcc}}}{}\right.}{d}$$

$\downarrow$ 25

$$\frac{e^2\left(\frac{3\left(-\frac{3\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{-\frac{c+}{b(a+b\text{arcc}}}{}\right.}{d}$$

$\downarrow$ 5971

$$e^2\left(\frac{3\left(-\frac{3\int\left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4(a+b\text{arccosh}(c+dx))}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^3}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}-\frac{-\ldots}{b(a+\ldots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad d}$$

$\downarrow$ 2009

$$e^2\left(-\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)}{b^2}}{b}+\frac{3\left(\frac{3\left(-\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a\ldots}{}\right.\right.}{}\right.}{}\right.$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e^2*(-1/2*(Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*(a + b*Ar
cCosh[c + d*x])^2) - (-((c + d*x)/(b*(a + b*ArcCosh[c + d*x]))) + (I*(I*Co
shIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegra
l[(a + b*ArcCosh[c + d*x])/b]))/b^2)/b + (3*(-((c + d*x)^3/(b*(a + b*ArcCo
sh[c + d*x]))) + (3*(-1/4*(CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a
/b]) - (CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/4 + (C
osh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/4 + (Cosh[(3*a)/b]*Sinh
Integral[(3*(a + b*ArcCosh[c + d*x]))/b])/4))/b^2))/(2*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(236) = 472$.

Time = 0.14 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.21

method	result
derivativedivides	$-\frac{\left(-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2(3b\,\mathrm{arccosh}(dx+c)+3a-b)}{16b^2\left(b^2\,\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}+\frac{9e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{expIntegral}$
default	$-\frac{\left(-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2(3b\,\mathrm{arccosh}(dx+c)+3a-b)}{16b^2\left(b^2\,\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}+\frac{9e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{expIntegral}$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16*(-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(3*b*arccosh(d*x+c)+3*a-b)/b^2/(
b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+9/16*e^2/b^3*exp(3*a/b)*Ei(
1,3*arccosh(d*x+c)+3*a/b)-1/16*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^
2*(b*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^
2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16/b*e^2*(d*x+c+(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/16/b^2*e^2*(d*x+c+(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/16/b^3*e^2*exp(-a/b)*E
i(1,-arccosh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh
(d*x+c))^2-3/16/b^2*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/1
6/b^3*e^2*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arccosh(d*x + c)^3 + 3
*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx$$
$$= e^2\left(\int \frac{c^2}{a^3+3a^2b\operatorname{acosh}(c+dx)+3ab^2\operatorname{acosh}^2(c+dx)+b^3\operatorname{acosh}^3(c+dx)}\,dx + \int \frac{d^2x^2}{a^3+3a^2b\operatorname{acosh}(c+dx)+3ab^2\operatorname{acosh}^2(c+dx)+b^3\operatorname{acosh}^3(c+dx)}\,dx + \int \frac{2cdx}{a^3+3a^2b\operatorname{acosh}(c+dx)+3ab^2\operatorname{acosh}^2(c+dx)+b^3\operatorname{acosh}^3(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**3,x)
```

output
```
e**2*(Integral(c**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d
*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b
*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x)
 + Integral(2*c*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d
*x)**2 + b**3*acosh(c + d*x)**3), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^2}{(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((3*a*d^9*e^2 + b*d^9*e^2)*x^9 + 9*(3*a*c*d^8*e^2 + b*c*d^8*e^2)*x^8
+ 3*(3*(12*c^2*d^7*e^2 - d^7*e^2)*a + (12*c^2*d^7*e^2 - d^7*e^2)*b)*x^7 +
21*(3*(4*c^3*d^6*e^2 - c*d^6*e^2)*a + (4*c^3*d^6*e^2 - c*d^6*e^2)*b)*x^6 +
 3*(3*(42*c^4*d^5*e^2 - 21*c^2*d^5*e^2 + d^5*e^2)*a + (42*c^4*d^5*e^2 - 21
*c^2*d^5*e^2 + d^5*e^2)*b)*x^5 + 3*(3*(42*c^5*d^4*e^2 - 35*c^3*d^4*e^2 + 5
*c*d^4*e^2)*a + (42*c^5*d^4*e^2 - 35*c^3*d^4*e^2 + 5*c*d^4*e^2)*b)*x^4 + (
(3*a*d^6*e^2 + b*d^6*e^2)*x^6 + 6*(3*a*c*d^5*e^2 + b*c*d^5*e^2)*x^5 + ((45
*c^2*d^4*e^2 - 4*d^4*e^2)*a + (15*c^2*d^4*e^2 - d^4*e^2)*b)*x^4 + 4*((15*c
^3*d^3*e^2 - 4*c*d^3*e^2)*a + (5*c^3*d^3*e^2 - c*d^3*e^2)*b)*x^3 + ((45*c^
4*d^2*e^2 - 24*c^2*d^2*e^2 + d^2*e^2)*a + 3*(5*c^4*d^2*e^2 - 2*c^2*d^2*e^2
)*b)*x^2 + (3*c^6*e^2 - 4*c^4*e^2 + c^2*e^2)*a + (c^6*e^2 - c^4*e^2)*b + 2
*((9*c^5*d*e^2 - 8*c^3*d*e^2 + c*d*e^2)*a + (3*c^5*d*e^2 - 2*c^3*d*e^2)*b)
*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + (3*(84*c^6*d^3*e^2 - 105*c^4
*d^3*e^2 + 30*c^2*d^3*e^2 - d^3*e^2)*a + (84*c^6*d^3*e^2 - 105*c^4*d^3*e^2
 + 30*c^2*d^3*e^2 - d^3*e^2)*b)*x^3 + (3*(3*a*d^7*e^2 + b*d^7*e^2)*x^7 + 2
1*(3*a*c*d^6*e^2 + b*c*d^6*e^2)*x^6 + ((189*c^2*d^5*e^2 - 17*d^5*e^2)*a +
(63*c^2*d^5*e^2 - 5*d^5*e^2)*b)*x^5 + 5*((63*c^3*d^4*e^2 - 17*c*d^4*e^2)*a
 + (21*c^3*d^4*e^2 - 5*c*d^4*e^2)*b)*x^4 + (5*(63*c^4*d^3*e^2 - 34*c^2*d^3
*e^2 + 2*d^3*e^2)*a + (105*c^4*d^3*e^2 - 50*c^2*d^3*e^2 + 2*d^3*e^2)*b)*x^
3 + ((189*c^5*d^2*e^2 - 170*c^3*d^2*e^2 + 30*c*d^2*e^2)*a + (63*c^5*d^2...
```

Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^3}\,dx\\&=e^2\left(\left(\int \frac{x^2}{acosh\left(dx+c\right)^3b^3+3acosh\left(dx+c\right)^2a\,b^2+3acosh\left(dx+c\right)a^2b+a^3}dx\right)d^2\right.\\&\quad+2\left(\int \frac{x}{acosh\left(dx+c\right)^3b^3+3acosh\left(dx+c\right)^2a\,b^2+3acosh\left(dx+c\right)a^2b+a^3}dx\right)cd\\&\quad\left.+\left(\int \frac{1}{acosh\left(dx+c\right)^3b^3+3acosh\left(dx+c\right)^2a\,b^2+3acosh\left(dx+c\right)a^2b+a^3}dx\right)c^2\right)\end{aligned}$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^3,x)
```

output
```
e**2*(int(x**2/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*ac
osh(c + d*x)*a**2*b + a**3),x)*d**2 + 2*int(x/(acosh(c + d*x)**3*b**3 + 3*
acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)*c*d + int(1/
(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a*
*2*b + a**3),x)*c**2)
```

3.63 $\int \frac{ce+dex}{(a+b\,\mathrm{arccosh}(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{ce+dex}{(a+b\,\mathrm{arccosh}(c+dx))^3}\,dx = -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd(a+b\,\mathrm{arccosh}(c+dx))^2} + \frac{e}{2b^2d(a+b\,\mathrm{arccosh}(c+dx))} - \frac{e(c+dx)^2}{b^2d(a+b\,\mathrm{arccosh}(c+dx))} - \frac{e\,\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^3d} + \frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{b^3d}$$

output
```
-1/2*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^2+
1/2*e/b^2/d/(a+b*arccosh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))-e*
Chi(2*(a+b*arccosh(d*x+c))/b)*sinh(2*a/b)/b^3/d+e*cosh(2*a/b)*Shi(2*(a+b*a
rccosh(d*x+c))/b)/b^3/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.78

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx = \frac{e\left(-\frac{b^2\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{(a+b\operatorname{arccosh}(c+dx))^2}+\frac{b(1-2(c+dx)^2)}{a+b\operatorname{arccosh}(c+dx)}-2\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)+2\cosh\left(\frac{2a}{b}\right.}{2b^3d}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[
c + d*x])^2) + (b*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x]) - 2*CoshIn
tegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcCosh[c + d*x])]))/(2*b^3*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {6411, 27, 6301, 6308, 6366, 6302, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\operatorname{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{(a+b\operatorname{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \; 6301$$

$$\frac{e\left(-\frac{\int \frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}+\frac{\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+d}{2b(a+b\text{arccosh}(c+dx))}\right.}{d}$$

$$\Big\downarrow \; 6308$$

$$\frac{e\left(\frac{\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+dx))}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \; 6366$$

$$\frac{e\left(\frac{\frac{2\int \frac{c+dx}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+dx))}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \; 6302$$

$$\frac{e\left(\frac{\frac{2\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{e\left(\frac{-\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c-}\right.}{d}$$

$$\Big\downarrow \; 5971$$

$$e\left(\frac{-\frac{-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2(a+b\text{arccosh}(c+dx))}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+dx))}-\frac{\sqrt{c+dx-1}\sqrt{c+dx}}{2b(a+b\text{arccosh}}}{d}\right.$$

$$\Big\downarrow 27$$

$$e\left(\frac{-\frac{-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+dx))}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+}}{2b(a+b\text{arccosh}(}}{d}\right.$$

$$\Big\downarrow 3042$$

$$e\left(\frac{-\frac{-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+dx))}-\frac{\sqrt{c+dx-1}\sqrt{c+c}}{2b(a+b\text{arccos}}}{d}\right.$$

$$\Big\downarrow 26$$

$$e\left(\frac{-\frac{-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+dx))}-\frac{\sqrt{c+dx-1}\sqrt{c+dx}}{2b(a+b\text{arccosh}}}{d}\right.$$

$$\Big\downarrow 3784$$

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sinh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}\cdots\right.}{b^2}}{b}\cdots\right.}{d\cdots}$$

$\downarrow$ 26

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(\cdots\right.}{b^2}}{b}\cdots\right.}{d\cdots}$$

$\downarrow$ 3042

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(c+dx\cdots}{b}\right.}{a+b\text{arccosh}(c+dx)}\cdots\right.}{b^2}}{b}\cdots\right.}{d\cdots}$$

$\downarrow$ 26

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}\cdots\right.}{b^2}}{b}\cdots\right.}{d\cdots}$$

$\downarrow$ 3779

$$\frac{e\left(-\frac{\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)\right)}{b^2}}{b}\right.}{d}$$

$$\Big\downarrow \; 3782$$

$$\frac{e\left(-\frac{\frac{(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)\right)}{b^2}}{b}+\frac{1}{2b^2(a+b\text{arccosh}(c+\right.}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
(e*(-1/2*(Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*(a + b*ArcCos
h[c + d*x])^2) + 1/(2*b^2*(a + b*ArcCosh[c + d*x])) + (-((c + d*x)^2/(b*(a
 + b*ArcCosh[c + d*x]))) + (I*(I*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))
/b]*Sinh[(2*a)/b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x
]))/b]))/b^2)/b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.56

method	result
derivativedivides	$-\frac{\left(-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+2(dx+c)^2-1\right)e(2b\ \operatorname{arccosh}(dx+c)+2a-b)}{8b^2\left(b^2\ \operatorname{arccosh}(dx+c)^2+2ab\ \operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\ \text{expIntegral}_1\left(2\ \operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(d\cdots\right.}{\cdots}$
default	$-\frac{\left(-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+2(dx+c)^2-1\right)e(2b\ \operatorname{arccosh}(dx+c)+2a-b)}{8b^2\left(b^2\ \operatorname{arccosh}(dx+c)^2+2ab\ \operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\ \text{expIntegral}_1\left(2\ \operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(d\cdots\right.}{\cdots}$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*(-2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*
b*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2
)+1/2*e/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2-1
+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/4/b^2
*e*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(
d*x+c))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

Fricas [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{dex+ce}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^
2 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F]

$$\begin{aligned}&\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^3}\,dx\\&= e\left(\int \frac{c}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx\right.\\&\quad\left.+\int \frac{dx}{a^3+3a^2b\,\text{acosh}\,(c+dx)+3ab^2\,\text{acosh}^2\,(c+dx)+b^3\,\text{acosh}^3\,(c+dx)}\,dx\right)\end{aligned}$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

output
```
e*(Integral(c/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2
 + b**3*acosh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*acosh(c + d
*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{dex+ce}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a*d^8*e + b*d^8*e)*x^8 + 8*(2*a*c*d^7*e + b*c*d^7*e)*x^7 + (2*(28
*c^2*d^6*e - 3*d^6*e)*a + (28*c^2*d^6*e - 3*d^6*e)*b)*x^6 + 2*(2*(28*c^3*d
^5*e - 9*c*d^5*e)*a + (28*c^3*d^5*e - 9*c*d^5*e)*b)*x^5 + (2*(70*c^4*d^4*e
 - 45*c^2*d^4*e + 3*d^4*e)*a + (70*c^4*d^4*e - 45*c^2*d^4*e + 3*d^4*e)*b)*
x^4 + ((2*a*d^5*e + b*d^5*e)*x^5 + 5*(2*a*c*d^4*e + b*c*d^4*e)*x^4 + (2*(1
0*c^2*d^3*e - d^3*e)*a + (10*c^2*d^3*e - d^3*e)*b)*x^3 + (2*(10*c^3*d^2*e
- 3*c*d^2*e)*a + (10*c^3*d^2*e - 3*c*d^2*e)*b)*x^2 + 2*(c^5*e - c^3*e)*a +
 (c^5*e - c^3*e)*b + (2*(5*c^4*d*e - 3*c^2*d*e)*a + (5*c^4*d*e - 3*c^2*d*e
)*b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 4*(2*(14*c^5*d^3*e - 15*
c^3*d^3*e + 3*c*d^3*e)*a + (14*c^5*d^3*e - 15*c^3*d^3*e + 3*c*d^3*e)*b)*x^
3 + (3*(2*a*d^6*e + b*d^6*e)*x^6 + 18*(2*a*c*d^5*e + b*c*d^5*e)*x^5 + 5*(2
*(9*c^2*d^4*e - d^4*e)*a + (9*c^2*d^4*e - d^4*e)*b)*x^4 + 20*(2*(3*c^3*d^3
*e - c*d^3*e)*a + (3*c^3*d^3*e - c*d^3*e)*b)*x^3 + (5*(18*c^4*d^2*e - 12*c
^2*d^2*e + d^2*e)*a + (45*c^4*d^2*e - 30*c^2*d^2*e + 2*d^2*e)*b)*x^2 + (6*
c^6*e - 10*c^4*e + 5*c^2*e - e)*a + (3*c^6*e - 5*c^4*e + 2*c^2*e)*b + 2*((
18*c^5*d*e - 20*c^3*d*e + 5*c*d*e)*a + (9*c^5*d*e - 10*c^3*d*e + 2*c*d*e)*
b)*x)*(d*x + c + 1)*(d*x + c - 1) + (2*(28*c^6*d^2*e - 45*c^4*d^2*e + 18*c
^2*d^2*e - d^2*e)*a + (28*c^6*d^2*e - 45*c^4*d^2*e + 18*c^2*d^2*e - d^2*e)
*b)*x^2 + (3*(2*a*d^7*e + b*d^7*e)*x^7 + 21*(2*a*c*d^6*e + b*c*d^6*e)*x^6
+ 7*(2*(9*c^2*d^5*e - d^5*e)*a + (9*c^2*d^5*e - d^5*e)*b)*x^5 + 35*(2*(...
```

Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{dex+ce}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^3}\,dx = \int \frac{c\,e + d\,e\,x}{(a + b\,\text{acosh}\,(c + d\,x))^3}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^3}\,dx$$
$$= e\left(\left(\int \frac{x}{acosh\,(dx + c)^3\,b^3 + 3acosh\,(dx + c)^2\,a\,b^2 + 3acosh\,(dx + c)\,a^2b + a^3}dx\right)d + \left(\int \frac{1}{acosh\,(dx + c)^3\,b^3 + 3acosh\,(dx + c)^2\,a\,b^2 + 3acosh\,(dx + c)\,a^2b + a^3}dx\right)c\right)$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^3,x)
```

output
```
e*(int(x/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c
+ d*x)*a**2*b + a**3),x)*d + int(1/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d
*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)*c)
```

3.64 $\int \frac{1}{(a+b\mathbf{arccosh}(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 132

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^3}\,dx=-\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd(a+b\text{arccosh}(c+dx))^2}-\frac{c+dx}{2b^2d(a+b\text{arccosh}(c+dx))}-\frac{\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{2b^3d}$$

output
```
-1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^2-1/2*(d*x+c
)/b^2/d/(a+b*arccosh(d*x+c))-1/2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^3
/d+1/2*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^3/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^3} dx = -\frac{\frac{b\left(ac+adx+b\sqrt{-1+c+dx}\sqrt{1+c+dx}+b(c+dx)\text{arccosh}(c+dx)\right)}{(a+b\text{arccosh}(c+dx))^2}+\text{Chi}\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}(}{2b^3d}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-3),x]
```

output
```
-1/2*((b*(a*c + a*d*x + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*(c + d*
x)*ArcCosh[c + d*x]))/(a + b*ArcCosh[c + d*x])^2 + CoshIntegral[a/b + ArcC
osh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/
(b^3*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6410, 6295, 6366, 6296, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^3} dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{6295}$$

$$\frac{\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}}{d}$$

$\Big\downarrow$ 6366

$$\frac{\frac{\frac{\int \frac{1}{a+b\text{arccosh}(c+dx)}d(c+dx)}{b}-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}}{d}$$

$\Big\downarrow$ 6296

$$\frac{\frac{\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}}{d}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{2b}}{d}$$

$\Big\downarrow$ 26

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{2b}}{d}$$

$\Big\downarrow$ 3784

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\right.}{a+}\right.}{b^2}}{2b}}{d}$$

$\Big\downarrow$ 26

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+\cdots}{}\right)}{a+b\cdots}\right.}{b^2}}{2b}}{d}$$

$\downarrow$ 3042

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\cdots}{}\right.}{b^2}}{2b}}{d}$$

$\downarrow$ 26

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\cdots\right)}{}\right.}{b^2}}{2b}}{d}$$

$\downarrow$ 3779

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\cdots\right)\right.}{b^2}}{2b}}{d}$$

$\downarrow$ 3782

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arccosh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)\right)}{b^2}}{2b}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(-3),x]
```

output
```
(-1/2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x])^2
) + (-((c + d*x)/(b*(a + b*ArcCosh[c + d*x]))) + (I*(I*CoshIntegral[(a + b
*ArcCosh[c + d*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[
c + d*x])/b]))/b^2)/(2*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{-\frac{(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c)(b\,\operatorname{arccosh}(dx+c)+a-b)}{4b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\,\operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e^{\frac{a}{b}}\,\operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4b(a+b\,\operatorname{arccosh}(dx+c))^2}-\cdots}{d}$
default	$\frac{-\frac{(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c)(b\,\operatorname{arccosh}(dx+c)+a-b)}{4b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\,\operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e^{\frac{a}{b}}\,\operatorname{expIntegral}_1\left(\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4b(a+b\,\operatorname{arccosh}(dx+c))^2}-\cdots}{d}$

input
```
int(1/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b*arccosh(d*x+c)+a-b)/
b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4/b^3*exp(a/b)*Ei(1,
arccosh(d*x+c)+a/b)-1/4/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arc
cosh(d*x+c))^2-1/4/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccos
h(d*x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

Fricas [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx = \int \frac{1}{(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*
arccosh(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx = \int \frac{1}{(a+b\operatorname{acosh}(c+dx))^3}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**3,x)
```

output
```
Integral((a + b*acosh(c + d*x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^3}\,dx = \int \frac{1}{(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 - d^
5)*a + (7*c^2*d^5 - d^5)*b)*x^5 + 5*((7*c^3*d^4 - 3*c*d^4)*a + (7*c^3*d^4
- 3*c*d^4)*b)*x^4 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c*d^3)*x^3 + (6*
a*c^2*d^2 + (6*c^2*d^2 - d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 - c^2)*b + 2*(2*
a*c^3*d + (2*c^3*d - c*d)*b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) +
((35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*b
)*x^3 + (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d
^3 - d^3)*a + 5*(6*c^2*d^3 - d^3)*b)*x^3 + 3*((10*c^3*d^2 - 3*c*d^2)*a + 5
*(2*c^3*d^2 - c*d^2)*b)*x^2 + 3*(c^5 - c^3)*a + (3*c^5 - 5*c^3 + 2*c)*b +
(3*(5*c^4*d - 3*c^2*d)*a + (15*c^4*d - 15*c^2*d + 2*d)*b)*x)*(d*x + c + 1)
*(d*x + c - 1) + 3*((7*c^5*d^2 - 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d^2 - 10
*c^3*d^2 + 3*c*d^2)*b)*x^2 + (3*(a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^
5)*x^5 + (3*(15*c^2*d^4 - 2*d^4)*a + (45*c^2*d^4 - 7*d^4)*b)*x^4 + 4*(3*(5
*c^3*d^3 - 2*c*d^3)*a + (15*c^3*d^3 - 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 - 36*
c^2*d^2 + 4*d^2)*a + (45*c^4*d^2 - 42*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 - 6
*c^4 + 4*c^2 - 1)*a + (3*c^6 - 7*c^4 + 5*c^2 - 1)*b + 2*((9*c^5*d - 12*c^3
*d + 4*c*d)*a + (9*c^5*d - 14*c^3*d + 5*c*d)*b)*x)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + (c^7 - 3*c^5 + 3*c^3 - c)*a + (c^7 - 3*c^5 + 3*c^3 - c)*b +
 ((7*c^6*d - 15*c^4*d + 9*c^2*d - d)*a + (7*c^6*d - 15*c^4*d + 9*c^2*d - d
)*b)*x + (b*d^7*x^7 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 - d^5)*b*x^5 + 5*(7*...
```

Giac [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{1}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\,\text{acosh}\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acosh(c + d*x))^3,x)
```

output
```
int(1/(a + b*acosh(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{1}{acosh\left(dx+c\right)^3 b^3+3acosh\left(dx+c\right)^2 a\,b^2+3acosh\left(dx+c\right)a^2b+a^3}dx$$

input
```
int(1/(a+b*acosh(d*x+c))^3,x)
```

output
```
int(1/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d
*x)*a**2*b + a**3),x)
```

3.65 $\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3} \, dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^3},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^3,x)/e
```

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3} \, dx = \int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3} \, dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3} \, dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{de}$$

$$\Big\downarrow \text{ 6303}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arccosh}\left(dx+c\right)\right)^{3}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arccosh(d*x + c)^3
 + 3*(a*b^2*d*e*x + a*b^2*c*e)*arccosh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b
*c*e)*arccosh(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 10.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.87

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3}\,dx = \frac{\int \frac{1}{a^3c+a^3dx+3a^2bc\,\text{acosh}\,(c+dx)+3a^2bdx\,\text{acosh}\,(c+dx)+3ab^2c\,\text{acosh}^2\,(c+dx)+3ab^2dx\,\text{acosh}^2\,(c+dx)+b^3c\,\text{acosh}^3\,(c+dx)+b^3dx\,\text{acosh}^3\,(c+dx}}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

output
```
Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*acosh(c + d*x) + 3*a**2*b*d*x*a
cosh(c + d*x) + 3*a*b**2*c*acosh(c + d*x)**2 + 3*a*b**2*d*x*acosh(c + d*x)
**2 + b**3*c*acosh(c + d*x)**3 + b**3*d*x*acosh(c + d*x)**3), x)/e
```

Maxima [N/A]

Not integrable

Time = 122.57 (sec) , antiderivative size = 6669, normalized size of antiderivative = 289.96

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(b*d^8*x^8 + 8*b*c*d^7*x^7 + (28*c^2*d^6 - 3*d^6)*b*x^6 + 2*(28*c^3*d
^5 - 9*c*d^5)*b*x^5 + (70*c^4*d^4 - 45*c^2*d^4 + 3*d^4)*b*x^4 + 4*(14*c^5*
d^3 - 15*c^3*d^3 + 3*c*d^3)*b*x^3 + (b*d^5*x^5 + 5*b*c*d^4*x^4 + (2*a*d^3
+ (10*c^2*d^3 - d^3)*b)*x^3 + (6*a*c*d^2 + (10*c^3*d^2 - 3*c*d^2)*b)*x^2 +
 2*(c^3 - c)*a + (c^5 - c^3)*b + (2*(3*c^2*d - d)*a + (5*c^4*d - 3*c^2*d)*
b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + (28*c^6*d^2 - 45*c^4*d^2 +
 18*c^2*d^2 - d^2)*b*x^2 + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + (4*a*d^4 + 5*(9
*c^2*d^4 - d^4)*b)*x^4 + 4*(4*a*c*d^3 + 5*(3*c^3*d^3 - c*d^3)*b)*x^3 + ((2
4*c^2*d^2 - 5*d^2)*a + (45*c^4*d^2 - 30*c^2*d^2 + 2*d^2)*b)*x^2 + (4*c^4 -
 5*c^2 + 1)*a + (3*c^6 - 5*c^4 + 2*c^2)*b + 2*((8*c^3*d - 5*c*d)*a + (9*c^
5*d - 10*c^3*d + 2*c*d)*b)*x)*(d*x + c + 1)*(d*x + c - 1) + 2*(4*c^7*d - 9
*c^5*d + 6*c^3*d - c*d)*b*x + (3*b*d^7*x^7 + 21*b*c*d^6*x^6 + (2*a*d^5 + 7
*(9*c^2*d^5 - d^5)*b)*x^5 + 5*(2*a*c*d^4 + 7*(3*c^3*d^4 - c*d^4)*b)*x^4 +
((20*c^2*d^3 - 3*d^3)*a + 5*(21*c^4*d^3 - 14*c^2*d^3 + d^3)*b)*x^3 + ((20*
c^3*d^2 - 9*c*d^2)*a + (63*c^5*d^2 - 70*c^3*d^2 + 15*c*d^2)*b)*x^2 + (2*c^
5 - 3*c^3 + c)*a + (3*c^7 - 7*c^5 + 5*c^3 - c)*b + ((10*c^4*d - 9*c^2*d +
d)*a + (21*c^6*d - 35*c^4*d + 15*c^2*d - d)*b)*x)*sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) + (c^8 - 3*c^6 + 3*c^4 - c^2)*b + (2*(b*d^3*x^3 + 3*b*c*d^2*x^
2 + (3*c^2*d - d)*b*x + (c^3 - c)*b)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/
2) + (4*b*d^4*x^4 + 16*b*c*d^3*x^3 + (24*c^2*d^2 - 5*d^2)*b*x^2 + 2*(8*...
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3), x)
```

Mupad [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3}\,dx=\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^3),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.57

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^3}\,dx$$
$$=\frac{\int \frac{1}{acosh(dx+c)^3b^3c+acosh(dx+c)^3b^3dx+3acosh(dx+c)^2a\,b^2c+3acosh(dx+c)^2a\,b^2dx+3acosh(dx+c)a^2bc+3acosh(dx+c)a^2bdx+a^3c+a^3dx}d}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^3,x)
```

output
```
int(1/(acosh(c + d*x)**3*b**3*c + acosh(c + d*x)**3*b**3*d*x + 3*acosh(c +
 d*x)**2*a*b**2*c + 3*acosh(c + d*x)**2*a*b**2*d*x + 3*acosh(c + d*x)*a**2
*b*c + 3*acosh(c + d*x)*a**2*b*d*x + a**3*c + a**3*d*x),x)/e
```

3.66 $\int \frac{(ce+dex)^4}{(a+b\,\mathbf{arccosh}(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 431

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx = -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd(a+b\operatorname{arccosh}(c+dx))^3} + \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arccosh}(c+dx))^2} - \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arccosh}(c+dx))^2} + \frac{2e^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{b^3d(a+b\operatorname{arccosh}(c+dx))} - \frac{25e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{6b^3d(a+b\operatorname{arccosh}(c+dx))} + \frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(c+dx)}{b}\right)}{48b^4d} + \frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{32b^4d} + \frac{125e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{96b^4d} - \frac{e^4\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(c+dx)}{b}\right)}{48b^4d} - \frac{27e^4\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{32b^4d} - \frac{125e^4\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{96b^4d}$$

output

```
-1/3*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^3+2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d
/(a+b*arccosh(d*x+c))^2+2*e^4*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b^
3/d/(a+b*arccosh(d*x+c))-25/6*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2
)/b^3/d/(a+b*arccosh(d*x+c))+1/48*e^4*cosh(a/b)*Chi((a+b*arccosh(d*x+c))/b
)/b^4/d+27/32*e^4*cosh(3*a/b)*Chi(3*(a+b*arccosh(d*x+c))/b)/b^4/d+125/96*e
^4*cosh(5*a/b)*Chi(5*(a+b*arccosh(d*x+c))/b)/b^4/d-1/48*e^4*sinh(a/b)*Shi(
(a+b*arccosh(d*x+c))/b)/b^4/d-27/32*e^4*sinh(3*a/b)*Shi(3*(a+b*arccosh(d*x
+c))/b)/b^4/d-125/96*e^4*sinh(5*a/b)*Shi(5*(a+b*arccosh(d*x+c))/b)/b^4/d
```

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.98

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^4}\,dx = \frac{e^4\left(-\frac{32b^3\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{(a+b\text{arccosh}(c+dx))^3}+\frac{16b^2(4(c+dx)^3-5(c+dx)^5)}{(a+b\text{arccosh}(c+dx))^2}-\frac{16b\sqrt{-1+c+dx}\sqrt{1+c+dx}(-12(c+dx)^2+25(c+dx)^4)}{a+b\text{arccosh}(c+dx)}+384\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^4*((-32*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Ar
cCosh[c + d*x])^3 + (16*b^2*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCos
h[c + d*x])^2 - (16*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-12*(c + d*x)^
2 + 25*(c + d*x)^4))/(a + b*ArcCosh[c + d*x]) + 384*(Cosh[a/b]*CoshIntegra
l[a/b + ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]
) - 544*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*
CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b +
ArcCosh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]
) + 125*(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + 5*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5
*(a/b + ArcCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d
*x]] - 5*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[(5*
a)/b]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(96*b^4*d)
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6411, 27, 6301, 6366, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^4}\,dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$\downarrow$ 27

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$\downarrow$ 6301

$$\frac{e^4\left(-\frac{4\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}+\frac{5\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(}{3b(a+b\text{arccosh}(c+}\right.}{d}$$

$\downarrow$ 6366

$$\frac{e^4\left(-\frac{4\left(\frac{3\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^3}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\frac{5\left(\frac{5\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^5}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}\right.}{d}$$

$\downarrow$ 6300

$$e^4\left(-\frac{4\left(\frac{3\left(-\frac{\int\left(-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4(a+b\text{arccosh}(c+dx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4(a+b\text{arccosh}(c+dx))}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{b(a+b\text{arccosh}(c+dx))}\right)}{2b}\right)}{3b}\right.$$

$\downarrow$ 2009

$$e^4\left(-\frac{4\left(\frac{3\left(-\frac{-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\frac{3}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)+\frac{3}{4}\sin}{b^2}\right.}{2b}\right.}{3b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^4*(-1/3*(Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*(a + b*Ar
cCosh[c + d*x])^3) - (4*(-1/2*(c + d*x)^3/(b*(a + b*ArcCosh[c + d*x])^2) +
 (3*(-((Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*(a + b*ArcCos
h[c + d*x]))) - (-1/4*(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])
 - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/4 + (Sin
h[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/4 + (3*Sinh[(3*a)/b]*Sinh
Integral[(3*(a + b*ArcCosh[c + d*x]))/b])/4)/b^2))/(2*b)))/(3*b) + (5*(-1/
2*(c + d*x)^5/(b*(a + b*ArcCosh[c + d*x])^2) + (5*(-((Sqrt[-1 + c + d*x]*(
c + d*x)^4*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) - (-1/8*(Cosh[
a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]) - (9*Cosh[(3*a)/b]*CoshInte
gral[(3*(a + b*ArcCosh[c + d*x]))/b])/16 - (5*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcCosh[c + d*x]))/b])/16 + (Sinh[a/b]*SinhIntegral[(a + b*ArcCos
h[c + d*x])/b])/8 + (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*
x]))/b])/16 + (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c + d*x]))/b
])/16)/b^2))/(2*b)))/(3*b)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(399) = 798$.

Time = 0.27 (sec) , antiderivative size = 1375, normalized size of antiderivative = 3.19

method	result	size
derivativedivides	Expression too large to display	1375
default	Expression too large to display	1375

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/192*(-16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^4+12*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-
20*(d*x+c)^3+5*d*x+5*c)*e^4*(25*b^2*arccosh(d*x+c)^2+50*a*b*arccosh(d*x+c)
-5*b^2*arccosh(d*x+c)+25*a^2-5*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^
2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-125/192*e^4/b^4*exp(5*a/b)*
Ei(1,5*arccosh(d*x+c)+5*a/b)+1/64*(-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x
+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(9*b^2*ar
ccosh(d*x+c)^2+18*a*b*arccosh(d*x+c)-3*b^2*arccosh(d*x+c)+9*a^2-3*a*b+2*b^
2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+
c)+a^3)-27/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/96*(-(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^4*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(
d*x+c)-b^2*arccosh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2
*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/96*e^4/b^4*exp(a/b)*Ei(1,a
rccosh(d*x+c)+a/b)-1/48/b*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))^3-1/96/b^2*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a
+b*arccosh(d*x+c))^2-1/96/b^3*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/
(a+b*arccosh(d*x+c))-1/96/b^4*e^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/32
/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-3/64/b^2*e^4*(4*(d
*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)...
```

Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*
b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F]

$$\int \frac{(ce + dex)^4}{(a + b\operatorname{arccosh}(c + dx))^4}\,dx$$
$$= e^4 \left(\int \frac{c^4}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{d^4x^4}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{4cd^3x^3}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{6c^2d^2x^2}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{4c^3dx}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**4,x)
```

output
```
e**4*(Integral(c**4/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c
+ d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Int
egral(d**4*x**4/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*
x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integra
l(4*c*d**3*x**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*
x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integra
l(6*c**2*d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Inte
gral(4*c**3*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*
x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(ce + dex)^4}{(a + b\operatorname{arccosh}(c + dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{acosh}\,(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx$$
$$= e^4\Bigg(\Bigg(\Bigg(\int \frac{x^4}{acosh(dx+c)^4 b^4+4acosh(dx+c)^3 a b^3+6acosh(dx+c)^2 a^2b^2+4acosh(dx+c)a^3b+a^4}dx$$
$$+4\Bigg(\int \frac{x^3}{acosh(dx+c)^4 b^4+4acosh(dx+c)^3 a b^3+6acosh(dx+c)^2 a^2b^2+4acosh(dx+c)a^3b+a^4}dx$$
$$+6\Bigg(\int \frac{x^2}{acosh(dx+c)^4 b^4+4acosh(dx+c)^3 a b^3+6acosh(dx+c)^2 a^2b^2+4acosh(dx+c)a^3b+a^4}dx$$
$$+4\Bigg(\int \frac{x}{acosh(dx+c)^4 b^4+4acosh(dx+c)^3 a b^3+6acosh(dx+c)^2 a^2b^2+4acosh(dx+c)a^3b+a^4}dx$$
$$+\Bigg(\int \frac{1}{acosh(dx+c)^4 b^4+4acosh(dx+c)^3 a b^3+6acosh(dx+c)^2 a^2b^2+4acosh(dx+c)a^3b+a^4}dx\Bigg)$$

input
```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^4,x)
```

output
```
e**4*(int(x**4/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*ac
osh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*d**4 + 4*in
t(x**3/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c +
d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c*d**3 + 6*int(x**2
/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**
2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c**2*d**2 + 4*int(x/(acos
h(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2
*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c**3*d + int(1/(acosh(c + d*x)*
*4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2 + 4*a
cosh(c + d*x)*a**3*b + a**4),x)*c**4)
```

3.67 $\int \frac{(ce+dex)^3}{(a+b\,\mathrm{arccosh}(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 360

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\,\mathrm{arccosh}(c+dx))^4}\,dx &= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd(a+b\,\mathrm{arccosh}(c+dx))^3} \\ &+ \frac{e^3(c+dx)^2}{2b^2d(a+b\,\mathrm{arccosh}(c+dx))^2} \\ &- \frac{2e^3(c+dx)^4}{3b^2d(a+b\,\mathrm{arccosh}(c+dx))^2} \\ &+ \frac{e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{b^3d(a+b\,\mathrm{arccosh}(c+dx))} \\ &- \frac{8e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3b^3d(a+b\,\mathrm{arccosh}(c+dx))} \\ &+ \frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{3b^4d} \\ &+ \frac{4e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{3b^4d} \\ &- \frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{3b^4d} \\ &- \frac{4e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\,\mathrm{arccosh}(c+dx))}{b}\right)}{3b^4d}\end{aligned}$$

output
```
-1/3*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^3+1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d
/(a+b*arccosh(d*x+c))^2+e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b^3/d/
(a+b*arccosh(d*x+c))-8/3*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b^3
/d/(a+b*arccosh(d*x+c))+1/3*e^3*cosh(2*a/b)*Chi(2*(a+b*arccosh(d*x+c))/b)/
b^4/d+4/3*e^3*cosh(4*a/b)*Chi(4*(a+b*arccosh(d*x+c))/b)/b^4/d-1/3*e^3*sinh
(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b^4/d-4/3*e^3*sinh(4*a/b)*Shi(4*(a+b
*arccosh(d*x+c))/b)/b^4/d
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.92

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^4}\,dx = \frac{e^3\left(-\frac{2b^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{(a+b\text{arccosh}(c+dx))^3}+\frac{b^2(3(c+dx)^2-4(c+dx)^4)}{(a+b\text{arccosh}(c+dx))^2}-\frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}(-3(c+dx)+8(c+dx)^3)}{a+b\text{arccosh}(c+dx)}+6\log(a+\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^3*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^3 + (b^2*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c
+ d*x])^2 - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-3*(c + d*x) + 8*(c
 + d*x)^3))/(a + b*ArcCosh[c + d*x]) + 6*Log[a + b*ArcCosh[c + d*x]] - 30*
(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a + b*ArcCos
h[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]) + 8*
(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*
CoshIntegral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a + b*ArcCosh[c + d*x]] -
 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])] - Sinh[(4*a)/b]*
SinhIntegral[4*(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {6411, 27, 6301, 6366, 6300, 25, 2009, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^4}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6301}$$

$$\frac{e^3\left(-\frac{\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{b}+\frac{4\int \frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c\ldots}{3b(a+b\text{arccosh}(c+\ldots}\right.}{d}$$

$$\Big\downarrow \text{ 6366}$$

$$\frac{e^3\left(-\frac{\frac{\int \frac{c+dx}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}}{b}+\frac{4\left(\frac{2\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}-\frac{(c+dx)^4}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}-\ldots\right.}{d}$$

$$\Big\downarrow \text{ 6300}$$

$$e^3\left(-\frac{\frac{-\frac{\int -\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{a+b\,\text{arccosh}(c+dx)}d(a+b\,\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\,\text{arccosh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\,\text{arccosh}(c+dx))^2}}{b}+\cdots\right.$$

$\downarrow$ 25

$$e^3\left(-\frac{\frac{\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}}{b}+\frac{4\left(\frac{2\left(-\frac{\int\left(-\frac{\cosh\left(\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

$$e^3\left(-\frac{\frac{\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}}{b}+\frac{4\left(\frac{2\left(-\frac{-\frac{1}{2}\cosh\left(\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$e^3\left(-\frac{-\frac{(c+dx)^2}{2b(a+b\,\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\,\text{arccosh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\,\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\,\text{arccosh}(c+dx)}d(a+b\,\text{arccosh}(c+dx))}{b^2}}{b}}{b}+\frac{4\left(2\left(-\frac{-\frac{1}{2}\cdots}{\cdots}\right.\right.}{\cdots}\right.$$

$\downarrow$ 3784

$$e^3\left(-\frac{-\frac{(c+dx)^2}{2b(a+b\,\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\,\text{arccosh}(c+dx))}-\frac{-\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{a+b\,\text{arccosh}(c+dx)}d(a+b\,\text{arccosh}(c+dx))+i\sinh\left(\frac{2a}{b}\right)\int\cdots}{b^2}}{b}}{b}\cdots\right.$$

$\downarrow$ 26

$$e^3\left(-\frac{\frac{-\frac{\sinh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{a+b\,\text{arccosh}(c+dx)}d(a+b\,\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{a+b\,\text{arccosh}(c+dx)}d(a+b\,\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c\cdots}}{b(\cdots}}{b}}{b}\cdots\right.$$

$\downarrow$ 3042

$$e^3\left(-\frac{-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{\frac{\sinh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int}{b^2}}{b}}{b}\right.$$

$$\downarrow \text{26}$$

$$e^3\left(-\frac{-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{\frac{-i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int}{b^2}}{b}}{b}\right.$$

$$\downarrow \text{3779}$$

$$e^3\left(-\frac{-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx)}{b}\right.}{a+b\text{arccosh}(c+dx)}}{b^2}}{b}}{b}\right.$$

$$\downarrow \text{3782}$$

$$e^3\left(-\frac{-\frac{-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{b^2}}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}}{b}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^3*(-1/3*(Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*(a + b*Ar
cCosh[c + d*x])^3) - (-1/2*(c + d*x)^2/(b*(a + b*ArcCosh[c + d*x])^2) + (-
((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*
x]))) - (-(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]) + S
inh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/b^2)/b)/b + (4*
(-1/2*(c + d*x)^4/(b*(a + b*ArcCosh[c + d*x])^2) + (2*(-((Sqrt[-1 + c + d*
x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) - (-1/2*(C
osh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]) - (Cosh[(4*a)/b
]*CoshIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/2 + (Sinh[(2*a)/b]*SinhInt
egral[(2*(a + b*ArcCosh[c + d*x]))/b])/2 + (Sinh[(4*a)/b]*SinhIntegral[(4*
(a + b*ArcCosh[c + d*x]))/b])/2)/b^2))/b))/(3*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(332) = 664$.

Time = 0.22 (sec) , antiderivative size = 860, normalized size of antiderivative = 2.39

method	result
derivativedivides	$\frac{\left(-8\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3+4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3\left(8b^2\operatorname{arccosh}(dx+c)^2+16ab\operatorname{arccosh}(d}{48b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}$
default	$\frac{\left(-8\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3+4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3\left(8b^2\operatorname{arccosh}(dx+c)^2+16ab\operatorname{arccosh}(d}{48b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/48*(-8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3+4*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(8*b^2*arccosh(d*x+
c)^2+16*a*b*arccosh(d*x+c)-2*b^2*arccosh(d*x+c)+8*a^2-2*a*b+b^2)/b^3/(b^3*
arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2/3*
e^3/b^4*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/24*(-2*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b^2*arccosh(d*x+c)^2+4*a*b*ar
ccosh(d*x+c)-b^2*arccosh(d*x+c)+2*a^2-a*b+b^2)/b^3/(b^3*arccosh(d*x+c)^3+3
*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/6*e^3/b^4*exp(2*a/b)
*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/24/b*e^3*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/24/b^2*e^3*(2*(d*x+c)^2-
1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/12/b
^3*e^3*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arcc
osh(d*x+c))-1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/48/b*e
^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-4*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^3-1/24/b^2
*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-
4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^2-1/6/b^
3*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3
-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-2/3/b^4
*e^3*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input

```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output

```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*ar
ccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)
^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F]

$$\int \frac{(ce + dex)^3}{(a + b\operatorname{arccosh}(c + dx))^4}\,dx$$
$$= e^3\left(\int \frac{c^3}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{d^3x^3}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{3cd^2x^2}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{3c^2dx}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**4,x)
```

output
```
e**3*(Integral(c**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c
+ d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Int
egral(d**3*x**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*
x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integra
l(3*c*d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*
x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integra
l(3*c**2*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(ce + dex)^3}{(a + b\operatorname{arccosh}(c + dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{acosh}\,(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^4}\,dx\\&=e^3\Bigg(\Bigg(\bigg(\int \frac{x^3}{acosh\,(dx+c)^4\,b^4+4acosh\,(dx+c)^3\,a\,b^3+6acosh\,(dx+c)^2\,a^2b^2+4acosh\,(dx+c)\,a^3b+a^4}dx\\&\quad+3\bigg(\int \frac{x^2}{acosh\,(dx+c)^4\,b^4+4acosh\,(dx+c)^3\,a\,b^3+6acosh\,(dx+c)^2\,a^2b^2+4acosh\,(dx+c)\,a^3b+a^4}dx\\&\quad+3\bigg(\int \frac{x}{acosh\,(dx+c)^4\,b^4+4acosh\,(dx+c)^3\,a\,b^3+6acosh\,(dx+c)^2\,a^2b^2+4acosh\,(dx+c)\,a^3b+a^4}dx\\&\quad+\bigg(\int \frac{1}{acosh\,(dx+c)^4\,b^4+4acosh\,(dx+c)^3\,a\,b^3+6acosh\,(dx+c)^2\,a^2b^2+4acosh\,(dx+c)\,a^3b+a^4}dx\bigg)\end{aligned}$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^4,x)
```

output
```
e**3*(int(x**3/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*ac
osh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*d**3 + 3*in
t(x**2/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c +
d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c*d**2 + 3*int(x/(a
cosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a
**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c**2*d + int(1/(acosh(c + d*
x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2 +
4*acosh(c + d*x)*a**3*b + a**4),x)*c**3)
```

3.68 $\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 352

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx = &-\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^3}\\ &+\frac{e^2(c+dx)}{3b^2d(a+b\text{arccosh}(c+dx))^2}\\ &-\frac{e^2(c+dx)^3}{2b^2d(a+b\text{arccosh}(c+dx))^2}\\ &+\frac{e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3b^3d(a+b\text{arccosh}(c+dx))}\\ &-\frac{3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2b^3d(a+b\text{arccosh}(c+dx))}\\ &+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{24b^4d}\\ &+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{8b^4d}\\ &-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{24b^4d}\\ &-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{8b^4d}\end{aligned}$$

output
```
-1/3*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^3+1/3*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(
a+b*arccosh(d*x+c))^2+1/3*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*a
rccosh(d*x+c))-3/2*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b^3/d/(a+
b*arccosh(d*x+c))+1/24*e^2*cosh(a/b)*Chi((a+b*arccosh(d*x+c))/b)/b^4/d+9/8
*e^2*cosh(3*a/b)*Chi(3*(a+b*arccosh(d*x+c))/b)/b^4/d-1/24*e^2*sinh(a/b)*Sh
i((a+b*arccosh(d*x+c))/b)/b^4/d-9/8*e^2*sinh(3*a/b)*Shi(3*(a+b*arccosh(d*x
+c))/b)/b^4/d
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.77

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx = \frac{e^2\left(-\frac{8b^3\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{(a+b\text{arccosh}(c+dx))^3}+\frac{4b^2(2(c+dx)-3(c+dx)^3)}{(a+b\text{arccosh}(c+dx))^2}-\frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}(-2+9(c+dx)^2)}{a+b\text{arccosh}(c+dx)}-80\cosh\left(\frac{a}{b}\right)\text{Chi}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^2*((-8*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^3 + (4*b^2*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c
+ d*x])^2 - (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-2 + 9*(c + d*x)^2)
)/(a + b*ArcCosh[c + d*x]) - 80*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d
*x]] + 80*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + 27*(3*Cosh[a/b]
*CoshIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b
+ ArcCosh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] -
Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])))/(24*b^4*d)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6411, 27, 6301, 6366, 6295, 6300, 2009, 6368, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6301}$$

$$\frac{e^2\left(-\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}+\frac{\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c\cdots}{3b(a+b\text{arccosh}(c+\cdots}\right.}{d}$$

$$\Big\downarrow \text{ 6366}$$

$$\frac{e^2\left(-\frac{2\left(\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\frac{\frac{3\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^3}{2b(a+b\text{arccosh}(c+dx))^2}}{b}-\cdots\right.}{d}$$

$$\Big\downarrow \text{ 6295}$$

$$e^2 \left(\frac{\frac{\frac{3\int \frac{(c+dx)^2}{(a+b\,\text{arccosh}(c+dx))^2}d(c+dx)}{2b} - \frac{(c+dx)^3}{2b(a+b\,\text{arccosh}(c+dx))^2}}{b} - \frac{2\left(\frac{\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\,\text{arccosh}(c+dx))}d(c+dx)}{b}}{2b} - \frac{\sqrt{c+dx-1}\sqrt{c+dx-}}{b(a+b\,\text{arccosh}(c+}\right.}{3b}}{d}$$

$\downarrow$ 6300

$$e^2 \left(\frac{\frac{3\left(-\frac{\int \left(-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\,\text{arccosh}(c+dx))}{b}\right)}{4(a+b\,\text{arccosh}(c+dx))} - \frac{\cosh\left(\frac{a}{b}-\frac{a+b\,\text{arccosh}(c+dx)}{b}\right)}{4(a+b\,\text{arccosh}(c+dx))}\right)d(a+b\,\text{arccosh}(c+dx))}{b^2} - \frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{b(a+b\,\text{arccosh}(c+dx))}\right)}{2b} - \frac{}{2b(a}}{b}\right.$$

$\downarrow$ 2009

$$e^2 \left(-\frac{2\left(\frac{\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\,\text{arccosh}(c+dx))}d(c+dx)}{b}}{2b} - \frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\,\text{arccosh}(c+dx))} - \frac{c+dx}{2b(a+b\,\text{arccosh}(c+dx))^2}\right)}{3b} + \frac{3\left(-\frac{-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a-}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 6368

$$e^2\left(-\frac{2\left(\frac{\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{2b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\frac{3\left(-\frac{-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{C}\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$e^2\left(-\frac{2\left(-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{2b}\right)}{3b}+\frac{3\left(-\frac{-\frac{1}{4}\text{c}\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3784

$$e^2\left(-\frac{2\left(-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\cdots}{\cdots}}{b^2}}{2b}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 26

$$e^2\left(-\frac{2\left(\frac{\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx\cdots}}{b(a+b\text{arc}\cdots}\cdots}{2b}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 3042

$$e^2\left(-\frac{2\left(-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int-\cdots}{b^2}\cdots}{2b}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 26

$$e^2\left(-\frac{2\left(-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\cdots\right)}{\cdots}\cdots}{b^2}\cdots}{2b}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 3779

$$e^2\left(-\frac{2\left(-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\right.}{a+b\text{arccosh}(c+dx)}}{b^2}}{2b}\right.}{3b}\right.$$

$\downarrow$ 3782

$$e^2\left(-\frac{2\left(\frac{\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\right.$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e^2*(-1/3*(Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*(a + b*Ar
cCosh[c + d*x])^3) - (2*(-1/2*(c + d*x)/(b*(a + b*ArcCosh[c + d*x])^2) + (
-((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) + (
Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b] - Sinh[a/b]*SinhIntegra
l[(a + b*ArcCosh[c + d*x])/b])/b^2)/(2*b)))/(3*b) + (-1/2*(c + d*x)^3/(b*(
a + b*ArcCosh[c + d*x])^2) + (3*(-((Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
+ c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) - (-1/4*(Cosh[a/b]*CoshIntegral[
(a + b*ArcCosh[c + d*x])/b]) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Arc
Cosh[c + d*x]))/b])/4 + (Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b
])/4 + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/4)/b
^2))/(2*b))/b))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6300

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6366

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6368

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(322) = 644$.

Time = 0.16 (sec) , antiderivative size = 777, normalized size of antiderivative = 2.21

method	result
derivativedivides	$\frac{\left(-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2\left(9b^2\operatorname{arccosh}(dx+c)^2+18ab\operatorname{arccosh}(dx+c)-3b^2\operatorname{arc}\right.}{48b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}$
default	$\frac{\left(-4\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2\left(9b^2\operatorname{arccosh}(dx+c)^2+18ab\operatorname{arccosh}(dx+c)-3b^2\operatorname{arc}\right.}{48b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/48*(-4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(9*b^2*arccosh(d*x+c)^2+18*a*b*ar
ccosh(d*x+c)-3*b^2*arccosh(d*x+c)+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+
c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-9/16*e^2/b^4*exp
(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/48*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+d*x+c)*e^2*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-b^2*arccosh(d*x+c)
+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b
*arccosh(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/24/b
*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-1/48/b
^2*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/48
/b^3*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/48
/b^4*e^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3-3*d*x
-3*c+4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))/(a+b*arccosh(d*x+c))^3-1/16/b^2*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))^2-3/16/b^3*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x
+c))-9/16/b^4*e^2*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^2}{\left(b\operatorname{arcosh}\left(dx+c\right)+a\right)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arccosh(d*x + c)^4 + 4
*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh
(d*x + c) + a^4), x)
```

Sympy [F]

$$\begin{aligned}&\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx\\&=e^2\left(\int \frac{c^2}{a^4+4a^3b\operatorname{acosh}\left(c+dx\right)+6a^2b^2\operatorname{acosh}^2\left(c+dx\right)+4ab^3\operatorname{acosh}^3\left(c+dx\right)+b^4\operatorname{acosh}^4\left(c+dx\right)}\,dx\right.\\&\quad+\int \frac{d^2x^2}{a^4+4a^3b\operatorname{acosh}\left(c+dx\right)+6a^2b^2\operatorname{acosh}^2\left(c+dx\right)+4ab^3\operatorname{acosh}^3\left(c+dx\right)+b^4\operatorname{acosh}^4\left(c+dx\right)}\,dx\\&\quad\left.+\int \frac{2cdx}{a^4+4a^3b\operatorname{acosh}\left(c+dx\right)+6a^2b^2\operatorname{acosh}^2\left(c+dx\right)+4ab^3\operatorname{acosh}^3\left(c+dx\right)+b^4\operatorname{acosh}^4\left(c+dx\right)}\,dx\right)\end{aligned}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**4,x)
```

output
```
e**2*(Integral(c**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c
+ d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Int
egral(d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*
x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integra
l(2*c*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2
+ 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^4}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{acosh}\,(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx$$
$$= e^2\left(\left(\int \frac{x^2}{acosh(dx+c)^4 b^4 + 4acosh(dx+c)^3 a b^3 + 6acosh(dx+c)^2 a^2b^2 + 4acosh(dx+c)\,a^3b + a^4}dx\right)d^2 + 2\left(\int \frac{x}{acosh(dx+c)^4 b^4 + 4acosh(dx+c)^3 a b^3 + 6acosh(dx+c)^2 a^2b^2 + 4acosh(dx+c)\,a^3b + a^4}dx\right)cd + \left(\int \frac{1}{acosh(dx+c)^4 b^4 + 4acosh(dx+c)^3 a b^3 + 6acosh(dx+c)^2 a^2b^2 + 4acosh(dx+c)\,a^3b + a^4}dx\right)c^2\right)$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^4,x)
```

output
```
e**2*(int(x**2/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*ac
osh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*d**2 + 2*in
t(x/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x
)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c*d + int(1/(acosh(c +
 d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2
 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c**2)
```

3.69 $\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^4}\,dx = -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^3} + \frac{e}{6b^2d(a+b\text{arccosh}(c+dx))^2} - \frac{e(c+dx)^2}{3b^2d(a+b\text{arccosh}(c+dx))^2} - \frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3b^3d(a+b\text{arccosh}(c+dx))} + \frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{3b^4d} - \frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{3b^4d}$$

output
```
-1/3*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3+
1/6*e/b^2/d/(a+b*arccosh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+
c))^2-2/3*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x
+c))+2/3*e*cosh(2*a/b)*Chi(2*(a+b*arccosh(d*x+c))/b)/b^4/d-2/3*e*sinh(2*a/
b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b^4/d
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx$$
$$= \frac{e\left(-\frac{2b^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{(a+b\operatorname{arccosh}(c+dx))^3}+\frac{b^2(1-2(c+dx)^2)}{(a+b\operatorname{arccosh}(c+dx))^2}-\frac{4b\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{a+b\operatorname{arccosh}(c+dx)}-4\log(a+b\operatorname{arccosh}(c+\cdots$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh
[c + d*x])^3 + (b^2*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x])^2 - (4*b
*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])
- 4*Log[a + b*ArcCosh[c + d*x]] + 4*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + A
rcCosh[c + d*x])] + Log[a + b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6411, 27, 6301, 6308, 6366, 6300, 25, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int \frac{e(c+dx)}{(a+b\operatorname{arccosh}(c+dx))^4}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e\int \frac{c+dx}{(a+b\operatorname{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{6301}$$

$$\frac{e\left(-\frac{\int \frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}+\frac{2\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{3b(a+b\text{arccosh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{6308}$$

$$\frac{e\left(\frac{2\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^3}d(c+dx)}{3b}+\frac{1}{6b^2(a+b\text{arccosh}(c+dx))^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{3b(a+b\text{arccosh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{6366}$$

$$\frac{e\left(\frac{2\left(\frac{\int \frac{c+dx}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{b}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\frac{1}{6b^2(a+b\text{arccosh}(c+dx))^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{3b(a+b\text{arccosh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{6300}$$

$$\frac{e\left(\frac{2\left(\frac{-\frac{\int -\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\frac{1}{6b^2(a+b\text{arcc}}\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\left(\frac{2\left(\frac{\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{b}-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}\right)}{3b}+\frac{1}{6b^2(a+b\text{arccos}\cdots}\cdots\right.}{d}$$

$\downarrow$ 3042

$$\frac{e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}}{b}\right)}{3b}+\frac{\cdots}{6b^2(a+ba\cdots}\cdots\right.}{d}$$

$\downarrow$ 3784

$$\frac{e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{-\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))+i\sinh\left(\frac{2a}{b}\right)\int\cdots}{b^2}}{b}\cdots\right.}{3b}\cdots\right.}{d}$$

$\downarrow$ 26

$$e\left(\frac{2\left(\frac{-\frac{\sinh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c-\cdots}}{b(\cdots}}{b}\cdots\right.}{3b}\cdots\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\Big\downarrow$ 3042

$$e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{\sinh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\cdots}{b^2}\cdots}{b}\cdots\right.}{3b}\cdots\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\Big\downarrow$ 26

$$e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\text{arccosh}(c+dx))}-\frac{-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\cdots}{b^2}\cdots}{b}\cdots\right.}{3b}\cdots\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\Big\downarrow$ 3779

$$e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\,\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\,\text{arccosh}(c+dx))}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\,\text{arccosh}(c+dx))}{b}\right.}{a+b\,\text{arccosh}(c+dx)}}{b^2}}{b}\right.}{3b}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\downarrow \text{3782}$$

$$e\left(\frac{2\left(-\frac{\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\,\text{arccosh}(c+dx))}{b}\right)}{b^2}}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b(a+b\,\text{arccosh}(c+dx))}-\frac{(c+dx)^2}{2b(a+b\,\text{arccosh}(c+dx))^2}\right.}{3b}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
(e*(-1/3*(Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*(a + b*ArcCos
h[c + d*x])^3) + 1/(6*b^2*(a + b*ArcCosh[c + d*x])^2) + (2*(-1/2*(c + d*x)
^2/(b*(a + b*ArcCosh[c + d*x])^2) + (-((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[
1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) - (-(Cosh[(2*a)/b]*CoshIntegra
l[(2*(a + b*ArcCosh[c + d*x]))/b]) + Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*
ArcCosh[c + d*x]))/b])/b^2)/b))/(3*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{\left(-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+2(dx+c)^2-1\right)e\left(2b^2\operatorname{arccosh}(dx+c)^2+4ab\operatorname{arccosh}(dx+c)-b^2\operatorname{arccosh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)} - e\,\mathrm{e}\cdots$
default	$\frac{\left(-2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)+2(dx+c)^2-1\right)e\left(2b^2\operatorname{arccosh}(dx+c)^2+4ab\operatorname{arccosh}(dx+c)-b^2\operatorname{arccosh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)} - e\,\mathrm{e}\cdots$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/12*(-2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*
b^2*arccosh(d*x+c)^2+4*a*b*arccosh(d*x+c)-b^2*arccosh(d*x+c)+2*a^2-a*b+b^2
)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c
)+a^3)-1/3*e/b^4*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+
c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1
/12/b^2*e*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c))/(a+b*a
rccosh(d*x+c))^2-1/6/b^3*e*(2*(d*x+c)^2-1+2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)*(d*x+c))/(a+b*arccosh(d*x+c))-1/3/b^4*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+
c)-2*a/b))
```

Fricas [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^4} \, dx = \int \frac{dex + ce}{(b \text{arcosh}\,(dx + c) + a)^4} \, dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^
3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^4} \, dx = e\left(\int \frac{c}{a^4 + 4a^3b \operatorname{acosh}(c + dx) + 6a^2b^2 \operatorname{acosh}^2(c + dx) + 4ab^3 \operatorname{acosh}^3(c + dx) + b^4 \operatorname{acosh}^4(c + dx)} \, dx + \int \frac{dx}{a^4 + 4a^3b \operatorname{acosh}(c + dx) + 6a^2b^2 \operatorname{acosh}^2(c + dx) + 4ab^3 \operatorname{acosh}^3(c + dx) + b^4 \operatorname{acosh}^4(c + dx)} \, dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

output
```
e*(Integral(c/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)
**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(
d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*a*
b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx = \int \frac{dex+ce}{(b\operatorname{arcosh}(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^4}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\operatorname{acosh}(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^4}\, dx$$
$$= e\left(\left(\int \frac{x}{acosh\,(dx+c)^4\, b^4 + 4acosh\,(dx+c)^3\, a\, b^3 + 6acosh\,(dx+c)^2\, a^2b^2 + 4acosh\,(dx+c)\, a^3b + a^4}dx\right)\right.$$
$$+\left(\int \frac{1}{acosh\,(dx+c)^4\, b^4 + 4acosh\,(dx+c)^3\, a\, b^3 + 6acosh\,(dx+c)^2\, a^2b^2 + 4acosh\,(dx+c)\, a^3b + a^4}dx\right)$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^4,x)
```

output
```
e*(int(x/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c
+ d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*d + int(1/(acosh(
c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b
**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)*c)
```

3.70 $\int \frac{1}{(a+b\text{arccosh}(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 174

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^4}\,dx = -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\text{arccosh}(c+dx))^2} - \frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6b^3d(a+b\text{arccosh}(c+dx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{6b^4d} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{6b^4d}$$

output
```
-1/3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3-1/6*(d*x+c
)/b^2/d/(a+b*arccosh(d*x+c))^2-1/6*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(
a+b*arccosh(d*x+c))+1/6*cosh(a/b)*Chi((a+b*arccosh(d*x+c))/b)/b^4/d-1/6*si
nh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^4/d
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a + b\,\text{arccosh}(c + dx))^4}\,dx = -\frac{\frac{2b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}}{(a+b\,\text{arccosh}(c+dx))^3} + \frac{b^2(c+dx)}{(a+b\,\text{arccosh}(c+dx))^2} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{a+b\,\text{arccosh}(c+dx)} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b} + \text{arccosh}(c+dx)\right) +}{6b^4d}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-4),x]
```

output
```
-1/6*((2*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x]
)^3 + (b^2*(c + d*x))/(a + b*ArcCosh[c + d*x])^2 + (b*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b +
ArcCosh[c + d*x]] + Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(b^4*d
)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6410, 6295, 6366, 6295, 6368, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a + b\,\text{arccosh}(c + dx))^4}\,dx$$

$$\Big\downarrow \; 6410$$

$$\frac{\int \frac{1}{(a+b\,\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 6295$$

$$\frac{\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\,\text{arccosh}(c+dx))^3}d(c+dx)}{3b} - \frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\,\text{arccosh}(c+dx))^3}}{d}$$

$$\Big\downarrow \text{6366}$$

$$\frac{\frac{\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{2b}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}}{3b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}}{d}$$

$$\Big\downarrow \text{6295}$$

$$\frac{\frac{\frac{\frac{\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))}d(c+dx)}{b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}}{3b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}}{d}$$

$$\Big\downarrow \text{6368}$$

$$\frac{\frac{\frac{\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}}{3b}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c\cdots}{b^2}}{2b}}{3b}}{d}$$

$$\Big\downarrow \text{3784}$$

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c\cdots}{\cdots}}{2b}}{3b}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+d}}{2b}}{3b}}{d}$$

$\downarrow$ 3042

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccos}}{}}{2b}}{3b}}{d}$$

$\downarrow$ 26

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{a+b\text{arccosh}(c+dx)}d(a+b\text{arccosh}(}{}}{2b}}{3b}}{d}$$

$\downarrow$ 3779

$$\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}+\frac{-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}+\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(}{}\right.}{}}{b^2}}{2b}}{3b}}{d}$$

$\downarrow$ 3782

$$\frac{\frac{\frac{\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{b^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{b(a+b\text{arccosh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arccosh}(c+dx))^2}}{3b}-\frac{\sqrt{c+dx-1}\sqrt{}}{3b(a+b\text{arcco}}}{d}$$

input

```
Int[(a + b*ArcCosh[c + d*x])^(-4),x]
```

output
```
(-1/3*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x])^3
) + (-1/2*(c + d*x)/(b*(a + b*ArcCosh[c + d*x])^2) + (-((Sqrt[-1 + c + d*x
]*Sqrt[1 + c + d*x])/(b*(a + b*ArcCosh[c + d*x]))) + (Cosh[a/b]*CoshIntegr
al[(a + b*ArcCosh[c + d*x])/b] - Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c +
 d*x])/b])/b^2)/(2*b))/(3*b))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)\left(b^{2} \operatorname{arccosh}\left(dx+c\right)^{2}+2ab \operatorname{arccosh}\left(dx+c\right)-b^{2} \operatorname{arccosh}\left(dx+c\right)+a^{2}-ab+2b^{2}\right)}{12b^{3}\left(b^{3} \operatorname{arccosh}\left(dx+c\right)^{3}+3a\,b^{2} \operatorname{arccosh}\left(dx+c\right)^{2}+3a^{2}b \operatorname{arccosh}\left(dx+c\right)+a^{3}\right)}-\frac{e^{\frac{a}{b}} \operatorname{expIntegral}_{1}\left(\operatorname{arcco}\right.}{12b^{4}}}{}$
default	$\frac{\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)\left(b^{2} \operatorname{arccosh}\left(dx+c\right)^{2}+2ab \operatorname{arccosh}\left(dx+c\right)-b^{2} \operatorname{arccosh}\left(dx+c\right)+a^{2}-ab+2b^{2}\right)}{12b^{3}\left(b^{3} \operatorname{arccosh}\left(dx+c\right)^{3}+3a\,b^{2} \operatorname{arccosh}\left(dx+c\right)^{2}+3a^{2}b \operatorname{arccosh}\left(dx+c\right)+a^{3}\right)}-\frac{e^{\frac{a}{b}} \operatorname{expIntegral}_{1}\left(\operatorname{arcco}\right.}{12b^{4}}}{}$

input
```
int(1/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/12*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b^2*arccosh(d*x+c)^2+2
*a*b*arccosh(d*x+c)-b^2*arccosh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x
+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/12/b^4*exp(a/
b)*Ei(1,arccosh(d*x+c)+a/b)-1/6/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/
(a+b*arccosh(d*x+c))^3-1/12/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a
+b*arccosh(d*x+c))^2-1/12/b^3*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

Fricas [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^4}\,dx = \int \frac{1}{\left(b\,\text{arcosh}\,(dx+c)+a\right)^4}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral(1/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^
2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^4}\,dx = \int \frac{1}{\left(a+b\,\text{acosh}\,(c+dx)\right)^4}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**4,x)
```

output
```
Integral((a + b*acosh(c + d*x))**(-4), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^4} \, dx = \text{Timed out}$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^4} \, dx = \int \frac{1}{\left(b \operatorname{arcosh}\left(dx + c\right) + a\right)^4} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^4} \, dx = \int \frac{1}{\left(a + b \operatorname{acosh}\left(c + d\,x\right)\right)^4} \, dx$$

input
```
int(1/(a + b*acosh(c + d*x))^4,x)
```

output
```
int(1/(a + b*acosh(c + d*x))^4, x)
```

Reduce [F]

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^4} \, dx$$
$$= \int \frac{1}{acosh\left(dx+c\right)^{4} b^{4}+4acosh\left(dx+c\right)^{3} a\, b^{3}+6acosh\left(dx+c\right)^{2} a^{2} b^{2}+4acosh\left(dx+c\right) a^{3} b+a^{4}} dx$$

input
```
int(1/(a+b*acosh(d*x+c))^4,x)
```

output
```
int(1/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d
*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4),x)
```

3.71 $\int \frac{1}{(ce+dex)(a+b\textbf{arccosh}(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx=\frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^4},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^4,x)/e
```

Mathematica [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx=\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6303}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^4}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\text{arccosh}\,(dx+c))^4}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 5.26

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arccosh(d*x + c)^4
 + 4*(a*b^3*d*e*x + a*b^3*c*e)*arccosh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2
*b^2*c*e)*arccosh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arccosh(d*x + c
)), x)
```

Sympy [N/A]

Not integrable

Time = 33.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.57

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx$$
$$= \frac{\int \frac{1}{a^4c+a^4dx+4a^3bc\,\text{acosh}\,(c+dx)+4a^3bdx\,\text{acosh}\,(c+dx)+6a^2b^2c\,\text{acosh}^2\,(c+dx)+6a^2b^2dx\,\text{acosh}^2\,(c+dx)+4ab^3c\,\text{acosh}^3\,(c+dx)+4ab^3dx\,\text{acosh}^3}}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

output
```
Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*acosh(c + d*x) + 4*a**3*b*d*x*a
cosh(c + d*x) + 6*a**2*b**2*c*acosh(c + d*x)**2 + 6*a**2*b**2*d*x*acosh(c
+ d*x)**2 + 4*a*b**3*c*acosh(c + d*x)**3 + 4*a*b**3*d*x*acosh(c + d*x)**3
+ b**4*c*acosh(c + d*x)**4 + b**4*d*x*acosh(c + d*x)**4), x)/e
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^4}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^4}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^4}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^4),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.09

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^4}\,dx$$
$$= \frac{\int \frac{1}{acosh(dx+c)^4 b^4 c+acosh(dx+c)^4 b^4 dx+4acosh(dx+c)^3 a\,b^3 c+4acosh(dx+c)^3 a\,b^3 dx+6acosh(dx+c)^2 a^2 b^2 c+6acosh(dx+c)^2 a^2 b^2 dx+4acosh(}}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^4,x)
```

output
```
int(1/(acosh(c + d*x)**4*b**4*c + acosh(c + d*x)**4*b**4*d*x + 4*acosh(c +
 d*x)**3*a*b**3*c + 4*acosh(c + d*x)**3*a*b**3*d*x + 6*acosh(c + d*x)**2*a
**2*b**2*c + 6*acosh(c + d*x)**2*a**2*b**2*d*x + 4*acosh(c + d*x)*a**3*b*c
 + 4*acosh(c + d*x)*a**3*b*d*x + a**4*c + a**4*d*x),x)/e
```

3.72 $\int (ce+dex)^4\sqrt{a+b\,\mathrm{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 361

$$\int (ce+dex)^4\sqrt{a+b\,\mathrm{arccosh}(c+dx)}\,dx=\frac{e^4(c+dx)^5\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{5d}-\frac{\sqrt{b}e^4e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{b}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{b}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{320d}-\frac{\sqrt{b}e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{b}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{b}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\mathrm{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

output
```
1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))^(1/2)/d-1/32*b^(1/2)*e^4*exp(a/b)*P
i^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/192*b^(1/2)*e^4*exp(3*
a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/
1600*b^(1/2)*e^4*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arccosh(d*x+
c))^(1/2)/b^(1/2))/d-1/32*b^(1/2)*e^4*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))/d/exp(a/b)-1/192*b^(1/2)*e^4*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(
a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(3*a/b)-1/1600*b^(1/2)*e^4*5^(1/2)
*Pi^(1/2)*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(5*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}\,dx$$
$$= \frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\text{arccosh}(c+dx)}\left(150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\text{arcc}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^4*Sqrt[a + b*ArcCosh[c + d*x]]*(150*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + Arc
Cosh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcCosh[c + d*x]))/b] + 25*Sqrt[3]*E^
((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d
*x]))/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a +
b*ArcCosh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcCosh[c +
 d*x])/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c + d*x]))/b] + 3*Sqrt[5]*E^((10*a
)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcCosh[c +
d*x]))/b]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6411, 27, 6299, 6368, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}dx$$

$$\Big\downarrow \ 6411$$

$$\frac{\int e^4(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e^4\int (c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \ 6299$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{10}b\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \ 6368$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{10}\int \frac{\cosh^5\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{10}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)^5}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$$\Big\downarrow \ 3793$$

$$\frac{e^4\left(\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{10}\int\left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{5\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}+\right.\right.\right.}{d}$$

$\downarrow$ 2009

$$e^4\left(\frac{1}{10}\left(-\frac{5}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{5}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^4*(((c + d*x)^5*Sqrt[a + b*ArcCosh[c + d*x]])/5 + ((-5*Sqrt[b]*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/16 - (5*Sqrt[b]*E^((3*
a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 -
 (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]])/32 - (5*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqr
t[b]])/(16*E^(a/b)) - (5*Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(32*E^((3*a)/b)) - (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[
5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((5*a)/b)))/10))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^4 \sqrt{a + b \operatorname{arccosh}(dx + c)}dx$$

input
```
int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}\,dx=e^4\left(\int c^4\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx+\int d^4x^4\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx+\int 4cd^3x^3\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx+\int 6c^2d^2x^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx+\int 4c^3dx\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e**4*(Integral(c**4*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4*sq
rt(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*acosh(c +
 d*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*acosh(c + d*x)), x) + In
tegral(4*c**3*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (dex+ce)^4\sqrt{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4*sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (dex+ce)^4\sqrt{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (c\,e+d\,e\,x)^4\,\sqrt{a+b\,\text{acosh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int (ce+dex)^4\sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx = e^4\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)c^4 + \left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^4dx\right)d^4 + 4\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^3dx\right)c\,d^3 + 6\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^2dx\right)c^2d^2 + 4\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)c^3d\right)$$

input
```
int((d*e*x+c*e)^4*(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e**4*(int(sqrt(acosh(c + d*x)*b + a),x)*c**4 + int(sqrt(acosh(c + d*x)*b +
 a)*x**4,x)*d**4 + 4*int(sqrt(acosh(c + d*x)*b + a)*x**3,x)*c*d**3 + 6*int
(sqrt(acosh(c + d*x)*b + a)*x**2,x)*c**2*d**2 + 4*int(sqrt(acosh(c + d*x)*
b + a)*x,x)*c**3*d)
```

3.73 $\int (ce+dex)^3\sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 272

$$\int (ce+dex)^3\sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx=-\frac{3e^3\sqrt{a+b\operatorname{arccosh}(c+dx)}}{32d}+\frac{e^3(c+dx)^4\sqrt{a+b\operatorname{arccosh}(c+dx)}}{4d}-\frac{\sqrt{b}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{b}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{b}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{b}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

output
```
-3/32*e^3*(a+b*arccosh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+
c))^(1/2)/d-1/256*b^(1/2)*e^3*exp(4*a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(d*x+c
))^(1/2)/b^(1/2))/d-1/64*b^(1/2)*e^3*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/
2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/256*b^(1/2)*e^3*Pi^(1/2)*erfi(2
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(4*a/b)-1/64*b^(1/2)*e^3*2^(1/2)
*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx = \frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\text{arccosh}(c+dx)}\left(\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)+4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcc}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^3*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2,
 (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCo
sh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqr
t[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c
 + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6411, 27, 6299, 6368, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^3(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int (c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}b\int \frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6368}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3793}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{8\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{}{8\sqrt{}}\right.\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$e^3\left(\frac{1}{8}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^3*(((c + d*x)^4*Sqrt[a + b*ArcCosh[c + d*x]])/4 + ((-3*Sqrt[a + b*ArcCo
sh[c + d*x]])/4 - (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/8))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^3 \sqrt{a + b \operatorname{arccosh}(dx + c)}dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 \sqrt{a + b\text{arccosh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx = e^3\left(\int c^3\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx + \int d^3x^3\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx + \int 3cd^2x^2\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx + \int 3c^2dx\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e**3*(Integral(c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3*sq
rt(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*acosh(c +
 d*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx = \int (dex+ce)^3\sqrt{b\operatorname{arcosh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (dex+ce)^3\sqrt{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (c\,e+d\,e\,x)^3\,\sqrt{a+b\,\text{acosh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int (ce+dex)^3\sqrt{a+b\text{arccosh}(c+dx)}\,dx=e^3\Bigg(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)c^3\\+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^3dx\right)d^3\\+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^2dx\right)c\,d^2\\+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)c^2d\Bigg)\end{aligned}$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e**3*(int(sqrt(acosh(c + d*x)*b + a),x)*c**3 + int(sqrt(acosh(c + d*x)*b +
 a)*x**3,x)*d**3 + 3*int(sqrt(acosh(c + d*x)*b + a)*x**2,x)*c*d**2 + 3*int
(sqrt(acosh(c + d*x)*b + a)*x,x)*c**2*d)
```

3.74 $\int (ce+dex)^2 \sqrt{a+b\text{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\int (ce+dex)^2 \sqrt{a+b\text{arccosh}(c+dx)}\,dx = \frac{e^2(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}}{3d} - \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{48d} - \frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{48d}$$

output
```
1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(1/2)/d-1/16*b^(1/2)*e^2*exp(a/b)*P
i^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/144*b^(1/2)*e^2*exp(3*
a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/
16*b^(1/2)*e^2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(a/b
)-1/144*b^(1/2)*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))/d/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int (ce+dex)^2\sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx$$
$$= \frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arccosh}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arccosh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arccosh}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c +
 d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh
[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqr
t[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt
[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2
/b^2)])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6411, 27, 6299, 6368, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2\sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int e^2(c+dx)^2\sqrt{a+b\operatorname{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int (c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$\big\downarrow$ 6299

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}b\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)\right)}{d}$$

$\big\downarrow$ 6368

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$\big\downarrow$ 3042

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$\big\downarrow$ 3793

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{3\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+\right.}{d}$$

$\big\downarrow$ 2009

$$\frac{e^2\left(\frac{1}{6}\left(-\frac{3}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{3}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+}}{}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^2*(((c + d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/3 + ((-3*Sqrt[b]*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/8 - (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/8 - (3*
Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) -
 (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(8*E^((3*a)/b)))/6))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^2 \sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^2 \sqrt{a + b\mathrm{arccosh}(c + dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce + dex)^2 \sqrt{a + b\mathrm{arccosh}(c + dx)}\,dx = e^2 \left(\int c^2 \sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int d^2x^2 \sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int 2cdx \sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx \right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e**2*(Integral(c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2*sq
rt(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*acosh(c + d*x))
, x))
```

Maxima [F]

$$\int (ce + dex)^2 \sqrt{a + b\text{arccosh}(c + dx)}\, dx = \int (dex + ce)^2 \sqrt{b\,\text{arcosh}\,(dx + c) + a}\, dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int (ce + dex)^2 \sqrt{a + b\text{arccosh}(c + dx)}\, dx = \int (dex + ce)^2 \sqrt{b\,\text{arcosh}\,(dx + c) + a}\, dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2\sqrt{a+b\text{arccosh}(c+dx)}\,dx = \int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\mathrm{acosh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int (ce+dex)^2\sqrt{a+b\text{arccosh}(c+dx)}\,dx = e^2\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)c^2 + \left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^2dx\right)d^2 + 2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)cd\right)$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e**2*(int(sqrt(acosh(c + d*x)*b + a),x)*c**2 + int(sqrt(acosh(c + d*x)*b +
 a)*x**2,x)*d**2 + 2*int(sqrt(acosh(c + d*x)*b + a)*x,x)*c*d)
```

3.75 $\int (ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int (ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}\,dx = -\frac{e\sqrt{a+b\text{arccosh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{2d} - \frac{\sqrt{b}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

output
```
-1/4*e*(a+b*arccosh(d*x+c))^(1/2)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(
1/2)/d-1/32*b^(1/2)*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh
(d*x+c))^(1/2)/b^(1/2))/d-1/32*b^(1/2)*e*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+
b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int (ce + dex)\sqrt{a + b\text{arccosh}(c + dx)}\,dx$$
$$= \frac{e\left(8\sqrt{a + b\text{arccosh}(c + dx)}\cosh(2\text{arccosh}(c + dx)) - \sqrt{b}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right) - \text{s}\right.\right.}{32d}$$

input
```
Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e*(8*Sqrt[a + b*ArcCosh[c + d*x]]*Cosh[2*ArcCosh[c + d*x]] - Sqrt[b]*Sqrt
[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 - Sinh[(2*a)/b]) - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])))/(32*d)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6411, 27, 6299, 6368, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce + dex)\sqrt{a + b\text{arccosh}(c + dx)}\,dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int e(c + dx)\sqrt{a + b\text{arccosh}(c + dx)}d(c + dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{e\int (c + dx)\sqrt{a + b\text{arccosh}(c + dx)}d(c + dx)}{d}$$
$$\Big\downarrow \text{ 6299}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}b\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)\right)}{d}$$

$\Big\downarrow$ 6368

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$\Big\downarrow$ 3042

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{d}$$

$\Big\downarrow$ 3793

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{1}{2\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccos}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{e\left(\frac{1}{4}\left(-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\sqrt{a+b\text{arccosh}(c\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e*(((c + d*x)^2*Sqrt[a + b*ArcCosh[c + d*x]])/2 + (-Sqrt[a + b*ArcCosh[c
+ d*x]] - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcC
osh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/4))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)\sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}dx$$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)\sqrt{a + b\mathrm{arccosh}(c + dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce + dex)\sqrt{a + b\mathrm{arccosh}(c + dx)}\,dx = e\left(\int c\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int dx\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e*(Integral(c*sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x*sqrt(a + b*aco
sh(c + d*x)), x))
```

Maxima [F]

$$\int (ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (dex+ce)\sqrt{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int (ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (dex+ce)\sqrt{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int (c\,e+d\,e\,x)\ \sqrt{a+b\,\text{acosh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int (ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}\,dx = e\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)c + \left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)d\right)$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e*(int(sqrt(acosh(c + d*x)*b + a),x)*c + int(sqrt(acosh(c + d*x)*b + a)*x,
x)*d)
```

3.76 $\int \sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx = \frac{(c+dx)\sqrt{a+b\operatorname{arccosh}(c+dx)}}{d} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4d}$$

output
```
(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d-1/4*b^(1/2)*exp(a/b)*Pi^(1/2)*erf((a+
b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/4*b^(1/2)*Pi^(1/2)*erfi((a+b*arccosh(
d*x+c))^(1/2)/b^(1/2))/d/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\text{arccosh}(c+dx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arccosh}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}}\right)}{2d}$$

input
```
Integrate[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d
*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])
/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6410, 6294, 6368, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arccosh}(c+dx)}\,dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6294}$$

$$\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}b\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6368}$$

$$\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{d}$$

$$\Big\downarrow \text{3788}$$

$$\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}+\frac{1}{2}\left(\frac{1}{2}i\int\frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int-\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\left(-\frac{1}{2}\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)+(c+d}{d}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)+(c+dx)\sqrt{}}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)+(c+dx)\sqrt{a+ba}}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{1}{2}\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)+(c+dx)\sqrt{a+b\text{arccosh}(c}}{d}$$

input
```
Int[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*
Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}dx$$

input
```
int((a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\mathrm{arccosh}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int \sqrt{a+b\operatorname{acosh}{(c+dx)}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acosh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int \sqrt{b\operatorname{arcosh}{(dx+c)}+a}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\text{arccosh}(c+dx)}\,dx=\int \sqrt{b\operatorname{arcosh}{(dx+c)}+a}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx = \int \sqrt{a+b\,\mathrm{acosh}\,(c+d\,x)}\,dx$$

input
```
int((a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\operatorname{arccosh}(c+dx)}\,dx = \int \sqrt{acosh\,(dx+c)\,b+a}dx$$

input
```
int((a+b*acosh(d*x+c))^(1/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a),x)
```

3.77 $\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}\,dx=\frac{\text{Int}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{c+dx},x\right)}{e}$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^(1/2)/(d*x+c),x)/e
```

Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]
```

output
```
Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{c+dx}d(c+dx)}{de}$$
$$\Big\downarrow \text{6303}$$
$$\frac{\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{c+dx}d(c+dx)}{de}$$

input
```
Int[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\,\mathrm{arccosh}\left(dx+c\right)}}{dex+ce}dx$$

input
```
int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\mathrm{arccosh}(c+dx)}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+b\mathrm{arccosh}(c+dx)}}{ce+dex}\,dx = \frac{\int \frac{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*acosh(d*x+c))**(1/2)/(d*e*x+c*e),x)
```

output
```
Integral(sqrt(a + b*acosh(c + d*x))/(c + d*x), x)/e
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Giac [N/A]

Not integrable

Time = 11.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\operatorname{arccosh}(c + dx)}}{ce + dex}\,dx = \int \frac{\sqrt{a + b\,\mathrm{acosh}\,(c + d\,x)}}{c\,e + d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^(1/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\operatorname{arccosh}(c + dx)}}{ce + dex}\,dx = \frac{\int \frac{\sqrt{acosh(dx+c)b+a}}{dx+c}dx}{e}$$

input
```
int((a+b*acosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(c + d*x),x)/e
```

3.78 $\int (ce+dex)^3(a+b\,\mathrm{arccosh}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 374

$$\begin{aligned}
\int (ce+dex)^3(a+b\,\mathrm{arccosh}(c+dx))^{3/2}\,dx =&-\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{64d}\\
&-\frac{3be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{32d}\\
&-\frac{3e^3(a+b\,\mathrm{arccosh}(c+dx))^{3/2}}{32d}+\frac{e^3(c+dx)^4(a+b\,\mathrm{arccosh}(c+dx))^{3/2}}{4d}\\
&-\frac{3b^{3/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2048d}\\
&-\frac{3b^{3/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{128d}\\
&+\frac{3b^{3/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2048d}\\
&+\frac{3b^{3/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{128d}
\end{aligned}$$

output
```
-9/64*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(
1/2)/d-3/32*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)*(a+b*arccosh(d
*x+c))^(1/2)/d-3/32*e^3*(a+b*arccosh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+
b*arccosh(d*x+c))^(3/2)/d-3/2048*b^(3/2)*e^3*exp(4*a/b)*Pi^(1/2)*erf(2*(a+
b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-3/256*b^(3/2)*e^3*exp(2*a/b)*2^(1/2)*Pi
^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d+3/2048*b^(3/2)*e^
3*Pi^(1/2)*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(4*a/b)+3/256*b
^(3/2)*e^3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2
))/d/exp(2*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.99 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.49

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{3/2}\,dx = e^3\left(\frac{ae^{-\frac{4a}{b}}\sqrt{a+b\text{arccosh}(c+dx)}\left(\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\Gamma\left(\frac{3}{2},-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)+4\sqrt{2}\cdots\right.}{\cdots}\right.$$
$$+\frac{\sqrt{b}\left((8a+3b)\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{4a}{b}\right)-\sinh\left(\frac{4a}{b}\right)\right)+(8a-3b)\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+d\cdots}}{\sqrt{b}}\right.\cdots}{\cdots}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
e^3*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3
/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*
Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCos
h[c + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b]))
)/(128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (Sqrt[b]*(
(8*a + 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[
(4*a)/b] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt
[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 - Sinh[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a +
b*ArcCosh[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[
2*ArcCosh[c + d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[
c + d*x]*Cosh[4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(2048*d)
)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.39 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.24, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6411, 27, 6299, 6354, 6302, 25, 5971, 2009, 6354, 6302, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\int\frac{(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{1}{8}b\int\frac{(c+dx)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)+\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+b\text{arccosh}(c+}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.}{d}$$

$$\Big\downarrow \text{6302}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{8}\int-\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccc}}{b}\right.}{}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\int\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}}{b}\right.}{\sqrt{c}}\right.\right.$$

$$\Big\downarrow \text{5971}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcc}}{}\right.}{8\sqrt{a+b\text{arccos}}}\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{6354}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(-\frac{1}{4}b\int\frac{c+dx}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.$$

$\downarrow$ 6302

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{4}\int-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c}{b}\right.}{\sqrt{a+}}\right.\right.\right.$$

$\downarrow$ 25

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+d}{b}\right.}{\sqrt{a+ba}}\right.\right.\right.$$

$\downarrow$ 5971

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c}{b}\right.}{2\sqrt{a+b\text{arccosh}(c+d}}\right.\right.\right.$$

$\downarrow$ 27

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c}{b}\right.}{\sqrt{a+b\text{arccosh}(c+dx}}\right.\right.\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\int-\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccos}}{b}\right.}{\sqrt{a+b\text{arccosh}(c-}}\right.\right.\right.$$

$\downarrow$ 26

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{8}i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}}{b}\right.}{\sqrt{a+b\text{arccosh}(c+c}}\right.\right.\right.$$

$\downarrow$ 3789

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(-\frac{1}{8}i\left(\frac{1}{2}i\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int\right.\right.\right.\right.$$

$\big\downarrow$ 2611

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e\right.\right.\right.\right.$$

$\big\downarrow$ 2633

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{}\right.\right.\right.\right.$$

$\big\downarrow$ 2634

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 6308

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{1}{8}\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcCosh[c + d*x])^(3/2))/4 - (3*b*((Sqrt[-1 + c
+ d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/4 + ((S
qrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/32 + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(32*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((2*a)/b)))/8 + (3*((Sqrt[-1 + c + d*
x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/2 + (a + b*Ar
cCosh[c + d*x])^(3/2)/(3*b) - (I/8)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*
Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)))
/4))/8))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^3 (a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 (a + b\operatorname{arccosh}(c + dx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{3/2}\,dx = e^3\Bigg(\int ac^3\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx + \int ad^3x^3\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx + \int bc^3\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)\,dx + \int 3acd^2x^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx + \int 3ac^2dx\sqrt{a+b\,\text{acosh}\,(c+dx)}\,dx + \int bd^3x^3\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)\,dx + \int 3bcd^2x^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)\,dx + \int 3bc^2dx\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)\,dx\Bigg)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e**3*(Integral(a*c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**3*x**
3*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*acosh(c + d*
x))*acosh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*acosh(c + d*x
)), x) + Integral(3*a*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x) + Integral(b
*d**3*x**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integral(3*b*c*
d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integral(3*b*c**
2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{3/2}\,dx = \int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{3/2}\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\text{acosh}(c+d\,x))^{3/2}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (ce+dex)^3(a+b\operatorname{arccosh}(c+dx))^{3/2}\,dx = e^3\left(\left(\int \sqrt{acosh\left(dx+c\right)b+a}dx\right)a\,c^3+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,x^3dx\right)b\,d^3+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,x^2dx\right)bc\,d^2+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,xdx\right)b\,c^2d+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,dx\right)b\,c^3+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,x^3dx\right)a\,d^3+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,x^2dx\right)ac\,d^2+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,xdx\right)a\,c^2d\right)$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c))^(3/2),x)
```

output
```
e**3*(int(sqrt(acosh(c + d*x)*b + a),x)*a*c**3 + int(sqrt(acosh(c + d*x)*b
 + a)*acosh(c + d*x)*x**3,x)*b*d**3 + 3*int(sqrt(acosh(c + d*x)*b + a)*aco
sh(c + d*x)*x**2,x)*b*c*d**2 + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c +
d*x)*x,x)*b*c**2*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x),x)*b*c*
*3 + int(sqrt(acosh(c + d*x)*b + a)*x**3,x)*a*d**3 + 3*int(sqrt(acosh(c +
d*x)*b + a)*x**2,x)*a*c*d**2 + 3*int(sqrt(acosh(c + d*x)*b + a)*x,x)*a*c**
2*d)
```

3.79 $\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 342

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx = -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\text{arccosh}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\sqrt{a+b\text{arccosh}(c+dx)}}{6d} + \frac{e^2(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}}{3d} - \frac{3b^{3/2}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d} - \frac{b^{3/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{96d} + \frac{3b^{3/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{b^{3/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{96d}$$

output
```
-1/3*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d-1/
6*b*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/
2)/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(3/2)/d-3/32*b^(3/2)*e^2*exp(a
/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-1/288*b^(3/2)*e^2*e
xp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))
/d+3/32*b^(3/2)*e^2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/ex
p(a/b)+1/288*b^(3/2)*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))/d/exp(3*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.99 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.73

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx = e^2\left(\frac{ae^{-\frac{3a}{b}}\sqrt{a+b\text{arccosh}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)+\sqrt{}\right.}{}\right.$$
$$+\frac{\sqrt{b}\left(9\left(-12\sqrt{b}\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\sqrt{a+b\text{arccosh}(c+dx)}+8\sqrt{b}(c+dx)\text{arccosh}(c+dx)\sqrt{a+b\text{arcc}}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
e^2*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[
c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*
Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + S
qrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a +
b*ArcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x]
)^2/b^2)]) + (Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(
1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c
+ d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]
*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a
 + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cos
h[(3*a)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b
*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*
Sqrt[a + b*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]]
- Sinh[3*ArcCosh[c + d*x]])))/(288*d))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.21, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {6411, 27, 6299, 6354, 6302, 25, 5971, 2009, 6330, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \ 6299$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \ 6354$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{1}{6}b\int\frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)+\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.}{d}$$

$$\Big\downarrow \ 6302$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{6}\int-\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccos}}{b}\right.}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \ 25$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{6}\int\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}}{b}\right.}{\sqrt{a-}}\right.\right.$$

$$\Big\downarrow \ 5971$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{6}\int\left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcc}}{b}\right.}{4\sqrt{a+b\text{arccosh}}}\right.\right.\right.$$

$$\Big\downarrow \ 2009$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{6}\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \ 6330$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}b\int\frac{}{\sqrt{a+b}}\right.\right.\right.$$

$\downarrow$ 6296

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\int -\frac{\sinh}{}\right.\right.\right.$$

$\downarrow$ 25

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\frac{1}{2}\int\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))+\sqrt{c+}\right.\right.\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}+\frac{1}{2}\int -\frac{i\sin}{}\right.\right.\right.$$

$\downarrow$ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\int\frac{\sin\left(\right.}{}\right.\right.\right.$$

$\downarrow$ 3789

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\int -\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}}\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}}\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\sqrt{\pi}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^(3/2))/3 - (b*((Sqrt[-1 + c +
d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/3 + (2*(S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] - (I/2)*(
(I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] -
 ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/
b))))/3 + ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt
[b]])/8 + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x
]]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b)))/6))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex+ce)^2 (a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}} dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^2(a+b\mathrm{arccosh}(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce+dex)^2(a+b\mathrm{arccosh}(c+dx))^{3/2}\,dx = e^2\left(\int ac^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx + \int ad^2x^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx + \int bc^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)\,dx + \int 2acdx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx + \int bd^2x^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)\,dx + \int 2bcdx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e**2*(Integral(a*c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**2*x**
2*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*acosh(c + d*
x))*acosh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*acosh(c + d*x)), x)
 + Integral(b*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + In
tegral(2*b*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx = \int (dex+ce)^2(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx = \int (dex+ce)^2(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^{3/2}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (ce+dex)^2(a\\&+b\text{arccosh}(c+dx))^{3/2}\,dx=e^2\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)a\,c^2\right.\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,x^2dx\right)b\,d^2\\&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,xdx\right)bcd\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,dx\right)b\,c^2\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^2dx\right)a\,d^2\\&\left.+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)acd\right)\end{aligned}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^(3/2),x)
```

output
```
e**2*(int(sqrt(acosh(c + d*x)*b + a),x)*a*c**2 + int(sqrt(acosh(c + d*x)*b
 + a)*acosh(c + d*x)*x**2,x)*b*d**2 + 2*int(sqrt(acosh(c + d*x)*b + a)*aco
sh(c + d*x)*x,x)*b*c*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x),x)*
b*c**2 + int(sqrt(acosh(c + d*x)*b + a)*x**2,x)*a*d**2 + 2*int(sqrt(acosh(
c + d*x)*b + a)*x,x)*a*c*d)
```

3.80 $\int (ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 212

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}\,dx = -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\text{arccosh}(c+dx)}}{8d} - \frac{e(a+b\text{arccosh}(c+dx))^{3/2}}{4d} + \frac{e(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}}{2d} - \frac{3b^{3/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{64d} + \frac{3b^{3/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{64d}$$

output
```
-3/8*b*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2
)/d-1/4*e*(a+b*arccosh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c)
)^(3/2)/d-3/128*b^(3/2)*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arc
cosh(d*x+c))^(1/2)/b^(1/2))/d+3/128*b^(3/2)*e*2^(1/2)*Pi^(1/2)*erfi(2^(1/2
)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}\,dx = \frac{e\left(3b^{3/2}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)-3b^{3/2}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcc}}}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 3*b^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sq
rt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*S
qrt[a + b*ArcCosh[c + d*x]]*(4*a*Cosh[2*ArcCosh[c + d*x]] + 4*b*ArcCosh[c
+ d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*b*Sinh[2*ArcCosh[c + d*x]])))/(128*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6411, 27, 6299, 6354, 6302, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$\big\downarrow$ 6299

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\int \frac{(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$\big\downarrow$ 6354

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{4}b\int \frac{c+dx}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)+\frac{1}{2}\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+\right.\right.}{d}$$

$\big\downarrow$ 6302

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{4}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcc}}}\right.\right.}{}$$

$\big\downarrow$ 25

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\text{s}}{\sqrt{a+b\text{arcco}}}\right.\right.}{}$$

$\big\downarrow$ 5971

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{4}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right.}{2\sqrt{a+b\text{arccosh}(c+dx)}}\right.\right.}{d}$$

$\big\downarrow$ 27

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int \frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right.}{\sqrt{a+b\text{arccosh}(c+dx)}}\right.\right.}{d}$$

$\big\downarrow$ 3042

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{1}{8}\int-\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+\cdots}{b}\right.}{\sqrt{a+b\text{arccosh}(c+dx)}}\cdots\right.\right.$$
$$\overline{\qquad\qquad\qquad\qquad d\qquad}$$

$\downarrow$ 26

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{8}i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx)}{b}\right.}{\sqrt{a+b\text{arccosh}(c+dx)}}\cdots\right.\right.$$
$$\overline{\qquad\qquad\qquad\qquad d\qquad}$$

$\downarrow$ 3789

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int\frac{e\cdots}{\sqrt{a+\cdots}}\right.\right.\right.$$

$\downarrow$ 2611

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{2(a+\cdots}{}}\right.\right.\right.$$

$\downarrow$ 2633

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{\cdots}\right.\right.\right.$$

$\downarrow$ 2634

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{ar}\cdots}}{}\right.\right.\right.\right.$$

$\downarrow$ 6308

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erf}\cdots\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^(3/2))/2 - (3*b*((Sqrt[-1 + c +
d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/2 + (a + b*
ArcCosh[c + d*x])^(3/2)/(3*b) - (I/8)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2
]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b))
))/4))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (ce+dex)(a+b\operatorname{arccosh}(c+dx))^{3/2}\,dx = e\left(\int ac\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx + \int adx\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx + \int bc\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx + \int bdx\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e*(Integral(a*c*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b
*acosh(c + d*x)), x) + Integral(b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x), x) + Integral(b*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)(a+b\operatorname{arccosh}(c+dx))^{3/2}\,dx = \int (dex+ce)(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (ce+dex)(a+b\operatorname{arccosh}(c+dx))^{3/2}\,dx = \int (dex+ce)(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)\;\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^{3/2}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}\,dx=e\left(\left(\int \sqrt{acosh\left(dx+c\right)b+a}dx\right)ac+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,xdx\right)bd+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,dx\right)bc+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,xdx\right)ad\right)$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^(3/2),x)
```

output
```
e*(int(sqrt(acosh(c + d*x)*b + a),x)*a*c + int(sqrt(acosh(c + d*x)*b + a)*
acosh(c + d*x)*x,x)*b*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x),x)
*b*c + int(sqrt(acosh(c + d*x)*b + a)*x,x)*a*d)
```

3.81 $\int (a+b\text{arccosh}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 157

$$\int (a+b\text{arccosh}(c+dx))^{3/2}\,dx = -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\text{arccosh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}}{d} - \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

output

```
-3/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d+(d*x+c
)*(a+b*arccosh(d*x+c))^(3/2)/d-3/8*b^(3/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))/d+3/8*b^(3/2)*Pi^(1/2)*erfi((a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))/d/exp(a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.88

$$\int (a+b\text{arccosh}(c+dx))^{3/2}\,dx = \frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arccosh}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}}\right)}{2d} + \frac{b\left(-12\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\sqrt{a+b\text{arccosh}(c+dx)}+8(c+dx)\text{arccosh}(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}\right.}{8d}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c +
 d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x
])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b)) + (b*(-12*Sqrt[
(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] +
 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*b)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]
))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b
]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6410, 6294, 6330, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6410}$$

$$\frac{\int (a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6294}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\int \frac{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6330}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}b\int \frac{1}{\sqrt{a+b\text{arccosh}(c\cdots}}\right.}{d}$$

$$\Big\downarrow \text{ 6296}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arc}\cdots}{}\right)}{\sqrt{a+b\text{arcco}\cdots}}\right.}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))+\sqrt{c+dx-1}\sqrt{\cdots}\right.}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}+\frac{1}{2}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{a}\cdots}{}\right)}{\sqrt{a+b\text{arcc}\cdots}}\right.}{d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arc}\cdots}{}\right)}{\sqrt{a+b\text{arcco}\cdots}}\right.}{d}$$

$\downarrow$ 3789

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\int\frac{e^{\frac{a-c-}{b}}}{\sqrt{a+b\text{arcco}}}\right.\right.}{d}$$

$\downarrow$ 2611

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}}{b}}\right.\right.}{d}$$

$\downarrow$ 2633

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}}{b}}\right.\right.}{d}$$

$\downarrow$ 2634

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\right.\right.}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2) - (3*b*(Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b))))/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int (a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}\,dx$$

input
```
int((a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\mathrm{arccosh}(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a + b\text{arccosh}(c + dx))^{3/2}\, dx = \int (a + b\operatorname{acosh}(c + dx))^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*acosh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (a + b\text{arccosh}(c + dx))^{3/2}\, dx = \int (b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a + b\text{arccosh}(c + dx))^{3/2}\, dx = \int (b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arccosh}(c + dx))^{3/2} dx = \int (a + b\,\text{acosh}(c + d\,x))^{3/2} dx$$

input
```
int((a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a + b\text{arccosh}(c + dx))^{3/2} dx = \left(\int \sqrt{acosh\,(dx + c)\,b + a}dx\right) a + \left(\int \sqrt{acosh\,(dx + c)\,b + a}\,acosh(dx + c)\,dx\right) b$$

input
```
int((a+b*acosh(d*x+c))^(3/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a),x)*a + int(sqrt(acosh(c + d*x)*b + a)*acosh
(c + d*x),x)*b
```

3.82 $\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx = \frac{\text{Int}\left(\frac{(a+b\text{arccosh}(c+dx))^{3/2}}{c+dx},x\right)}{e}$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^(3/2)/(d*x+c),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

output
```
Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{ce+dex} \, dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6303}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{3/2}}{c+dx}d(c+dx)}{de}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}{dex+ce}dx$$

input
```
int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 8.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx=\frac{\int \frac{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}{c+dx}\,dx+\int \frac{b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*acosh(d*x+c))**(3/2)/(d*e*x+c*e),x)
```

output
```
(Integral(a*sqrt(a + b*acosh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
 b*acosh(c + d*x))*acosh(c + d*x)/(c + d*x), x))/e
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arccosh}(c + dx))^{3/2}}{ce + dex} \, dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}{dex + ce} \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

Giac [N/A]

Not integrable

Time = 17.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arccosh}(c + dx))^{3/2}}{ce + dex} \, dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}{dex + ce} \, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

Mupad [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx=\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^{3/2}}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^(3/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^{3/2}}{ce+dex}\,dx=\frac{\left(\int \frac{\sqrt{acosh(dx+c)b+a}}{dx+c}dx\right)a+\left(\int \frac{\sqrt{acosh(dx+c)b+a}\,acosh(dx+c)}{dx+c}dx\right)b}{e}$$

input
```
int((a+b*acosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

output
```
(int(sqrt(acosh(c + d*x)*b + a)/(c + d*x),x)*a + int((sqrt(acosh(c + d*x)*
b + a)*acosh(c + d*x))/(c + d*x),x)*b)/e
```

3.83 $\int (ce+dex)^3(a+b\,\mathbf{arccosh}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 469

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx = -\frac{225b^2e^3\sqrt{a+b\text{arccosh}(c+dx)}}{2048d}+\frac{45b^2e^3(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{256d}+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}}{256d}-\frac{15be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^{3/2}}{64d}-\frac{5be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^{3/2}}{32d}-\frac{3e^3(a+b\text{arccosh}(c+dx))^{5/2}}{32d}+\frac{e^3(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}}{4d}-\frac{15b^{5/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15b^{5/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15b^{5/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15b^{5/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{512d}$$

output

```
-225/2048*b^2*e^3*(a+b*arccosh(d*x+c))^(1/2)/d+45/256*b^2*e^3*(d*x+c)^2*(a
+b*arccosh(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^(
1/2)/d-15/64*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*
x+c))^(3/2)/d-5/32*b*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)*(a+b*ar
ccosh(d*x+c))^(3/2)/d-3/32*e^3*(a+b*arccosh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c
)^4*(a+b*arccosh(d*x+c))^(5/2)/d-15/16384*b^(5/2)*e^3*exp(4*a/b)*Pi^(1/2)*
erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-15/1024*b^(5/2)*e^3*exp(2*a/b)
*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-15/163
84*b^(5/2)*e^3*Pi^(1/2)*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(4
*a/b)-15/1024*b^(5/2)*e^3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c
))^(1/2)/b^(1/2))/d/exp(2*a/b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 968 vs. $2(469) = 938$.

Time = 8.39 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.06

$$\int (ce + dex)^3 (a + b\text{arccosh}(c + dx))^{5/2} \, dx = \text{Too large to display}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
e^3*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma
[3/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b +
ArcCosh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b
)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcC
osh[c + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b]
)))/(128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (a*Sqrt[
b]*((8*a + 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(C
osh[(4*a)/b] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*Arc
Cosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*
Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a
)/b] - Sinh[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*Arc
Cosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[
a + b*ArcCosh[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*S
inh[2*ArcCosh[c + d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcC
osh[c + d*x]*Cosh[4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(102
4*d) + (-(Sqrt[b]*(64*a^2 + 48*a*b + 15*b^2)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*A
rcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b] - Sinh[(4*a)/b])) - Sqrt[b]*(64*
a^2 - 48*a*b + 15*b^2)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) - 16*(Sqrt[b]*(16*a^2 + 24*a*b + 15*b^
2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cos...
```

Rubi [A] (verified)

Time = 5.56 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {6411, 27, 6299, 6354, 6299, 6354, 6299, 6308, 6368, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\int \frac{(c+dx)^4(a+b\text{arccosh}(c+dx))^{3/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\int (c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)+\frac{3}{4}\int \frac{(c+dx)^2(a+}{\sqrt{c+d}}\right.\right.}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}b\int \frac{(c-}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}b\int \frac{(c-}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{6299}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}b\int\frac{(c-}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.$$

$\downarrow$ 6308

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}b\int\frac{}{\sqrt{c+dx-1}\sqrt{c+d}}\right.\right.\right.\right.$$

$\downarrow$ 6368

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{8}\int\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccc}}{b}\right.}{\sqrt{a+b\text{arccosh}}}\right.\right.\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{ar}}{}\right.}{\sqrt{a+b\text{ar}}}\right.\right.\right.\right.$$

$\downarrow$ 3793

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int\left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a}{}\right.}{2\sqrt{a+b\text{a}}}\right.\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{8}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcCosh[c + d*x])^(5/2))/4 - (5*b*((Sqrt[-1 + c
+ d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/4 - (
3*b*(((c + d*x)^4*Sqrt[a + b*ArcCosh[c + d*x]])/4 + ((-3*Sqrt[a + b*ArcCos
h[c + d*x]])/4 - (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/8))/8
+ (3*((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d
*x])^(3/2))/2 + (a + b*ArcCosh[c + d*x])^(5/2)/(5*b) - (3*b*(((c + d*x)^2*
Sqrt[a + b*ArcCosh[c + d*x]])/2 + (-Sqrt[a + b*ArcCosh[c + d*x]] - (Sqrt[b
]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]])/4 - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]])/(4*E^((2*a)/b)))/4))/4))/4))/8))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^3 (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}} \, dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (dex+ce)^3(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\text{acosh}(c+d\,x))^{5/2}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (ce+dex)^3(a+b\text{arccosh}(c+dx))^{5/2}\,dx = e^3\left(\left(\int \sqrt{acosh\left(dx+c\right)b+a}dx\right)a^2c^3\right.$$
$$+2\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,x^3dx\right)ab\,d^3$$
$$+6\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,x^2dx\right)abc\,d^2$$
$$+6\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,xdx\right)ab\,c^2d$$
$$+2\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,dx\right)ab\,c^3$$
$$+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)^2\,x^3dx\right)b^2d^3$$
$$+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)^2\,x^2dx\right)b^2c\,d^2$$
$$+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)^2\,xdx\right)b^2c^2d$$
$$+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)^2\,dx\right)b^2c^3$$
$$+\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,x^3dx\right)a^2d^3+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,x^2dx\right)a^2c\,d^2$$
$$\left.+3\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,xdx\right)a^2c^2d\right)$$

input
```
int((d*e*x+c*e)^3*(a+b*acosh(d*x+c))^(5/2),x)
```

output

```
e**3*(int(sqrt(acosh(c + d*x)*b + a),x)*a**2*c**3 + 2*int(sqrt(acosh(c + d
*x)*b + a)*acosh(c + d*x)*x**3,x)*a*b*d**3 + 6*int(sqrt(acosh(c + d*x)*b +
 a)*acosh(c + d*x)*x**2,x)*a*b*c*d**2 + 6*int(sqrt(acosh(c + d*x)*b + a)*a
cosh(c + d*x)*x,x)*a*b*c**2*d + 2*int(sqrt(acosh(c + d*x)*b + a)*acosh(c +
 d*x),x)*a*b*c**3 + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x**3,
x)*b**2*d**3 + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x**2,x)*
b**2*c*d**2 + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x,x)*b**2
*c**2*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x)*b**2*c**3 +
int(sqrt(acosh(c + d*x)*b + a)*x**3,x)*a**2*d**3 + 3*int(sqrt(acosh(c + d*
x)*b + a)*x**2,x)*a**2*c*d**2 + 3*int(sqrt(acosh(c + d*x)*b + a)*x,x)*a**2
*c**2*d)
```

3.84 $\int (ce+dex)^2(a+b\,\mathrm{arccosh}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 408

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^{5/2}\,dx = \frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arccosh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arccosh}(c+dx)}}{36d} - \frac{5be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^{3/2}}{9d} - \frac{5be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^{3/2}}{18d} + \frac{e^2(c+dx)^3(a+b\operatorname{arccosh}(c+dx))^{5/2}}{3d} - \frac{15b^{5/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{5b^{5/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{576d} - \frac{15b^{5/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{5b^{5/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{576d}$$

output
```
5/6*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d+5/36*b^2*e^2*(d*x+c)^3*(a
+b*arccosh(d*x+c))^(1/2)/d-5/9*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*
arccosh(d*x+c))^(3/2)/d-5/18*b*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/
2)*(a+b*arccosh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(5/
2)/d-15/64*b^(5/2)*e^2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))/d-5/1728*b^(5/2)*e^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*a
rccosh(d*x+c))^(1/2)/b^(1/2))/d-15/64*b^(5/2)*e^2*Pi^(1/2)*erfi((a+b*arcco
sh(d*x+c))^(1/2)/b^(1/2))/d/exp(a/b)-5/1728*b^(5/2)*e^2*3^(1/2)*Pi^(1/2)*e
rfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(3*a/b)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1008 vs. $2(408) = 816$.

Time = 8.70 (sec) , antiderivative size = 1008, normalized size of antiderivative = 2.47

$$\int (ce + dex)^2 (a + b\text{arccosh}(c + dx))^{5/2} \, dx = \text{Too large to display}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
e^2*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCos
h[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b
)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*
x])^2/b^2)]) + (a*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x
)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCos
h[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) +
(2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*
(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt
[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*
x]] - Sinh[3*ArcCosh[c + d*x]])))/(144*d) + (-27*(-4*b*Sqrt[a + b*ArcCosh[
c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*Arc
Cosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2
 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(C
osh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[...
```

Rubi [A] (verified)

Time = 4.66 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {6411, 27, 6299, 6354, 6299, 6330, 6294, 6368, 3042, 3788, 26, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\int \frac{(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\int (c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)+\frac{2}{3}\int \frac{(c+dx)(a+b\dots}{\sqrt{c+d\dots}}\right.\right.}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}b\int \frac{(c\dots}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{\dots}$$

$$\Big\downarrow \text{6330}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}b\int \frac{(c\dots}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.\right.}{\dots}$$

$$\Big\downarrow \; 6294$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left((c\right.\right.\right.\right.$$

$$\Big\downarrow \; 6368$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}\int\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arcco}}{b}\right.}{\sqrt{a+b\text{arccosh}}}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left((c\right.\right.\right.\right.$$

$$\Big\downarrow \; 3788$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left((c\right.\right.\right.\right.$$

$$\Big\downarrow \; 26$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2611$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2633$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(\right.\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcco}}{b}\right.}{\sqrt{a+b\text{arccos}}}\right.\right.\right.$$

$\downarrow$ 3793

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{6}\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{ar}}{}\right.}{4\sqrt{a+b\text{arcc}}}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(\right.\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^(5/2))/3 - (5*b*((Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/3 + (
2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2) - (
3*b*((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[
Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/2))/3 - (b*(((c + d*
x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/3 + ((-3*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/8 - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*
Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/8 - (3*Sqrt[b]*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b]*Sqrt
[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b
)))/6))/2))/6))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^2 (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}} dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^2 (a + b\operatorname{arccosh}(c + dx))^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^{5/2}\,dx=\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^{5/2}\,dx=\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{acosh}(c+d\,x))^{5/2}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (ce+dex)^2(a\\&+b\text{arccosh}(c+dx))^{5/2}\,dx=e^2\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)a^2c^2\right.\\&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,x^2dx\right)ab\,d^2\\&+4\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,xdx\right)abcd\\&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,dx\right)ab\,c^2\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,x^2dx\right)b^2d^2\\&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,xdx\right)b^2cd\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,dx\right)b^2c^2\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^2dx\right)a^2d^2\\&\left.+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)a^2cd\right)\end{aligned}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
e**2*(int(sqrt(acosh(c + d*x)*b + a),x)*a**2*c**2 + 2*int(sqrt(acosh(c + d
*x)*b + a)*acosh(c + d*x)*x**2,x)*a*b*d**2 + 4*int(sqrt(acosh(c + d*x)*b +
 a)*acosh(c + d*x)*x,x)*a*b*c*d + 2*int(sqrt(acosh(c + d*x)*b + a)*acosh(c
 + d*x),x)*a*b*c**2 + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x**
2,x)*b**2*d**2 + 2*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x,x)*b
**2*c*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x)*b**2*c**2 +
int(sqrt(acosh(c + d*x)*b + a)*x**2,x)*a**2*d**2 + 2*int(sqrt(acosh(c + d*
x)*b + a)*x,x)*a**2*c*d)
```

3.85 $\int (ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 269

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}\,dx = -\frac{15b^2e\sqrt{a+b\text{arccosh}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}}{32d} - \frac{5be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^{3/2}}{8d} - \frac{e(a+b\text{arccosh}(c+dx))^{5/2}}{4d}+\frac{e(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}}{2d} - \frac{15b^{5/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{256d} - \frac{15b^{5/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{256d}$$

output
```
-15/64*b^2*e*(a+b*arccosh(d*x+c))^(1/2)/d+15/32*b^2*e*(d*x+c)^2*(a+b*arcco
sh(d*x+c))^(1/2)/d-5/8*b*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*ar
ccosh(d*x+c))^(3/2)/d-1/4*e*(a+b*arccosh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(
a+b*arccosh(d*x+c))^(5/2)/d-15/512*b^(5/2)*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*e
rf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d-15/512*b^(5/2)*e*2^(1/2)*
Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.85

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}\,dx = \frac{e\left(-15b^{5/2}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)-15b^{5/2}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqr
t[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 15*b^(5/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]
*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) +
8*Sqrt[a + b*ArcCosh[c + d*x]]*((16*a^2 + 15*b^2)*Cosh[2*ArcCosh[c + d*x]]
 + 16*b^2*ArcCosh[c + d*x]^2*Cosh[2*ArcCosh[c + d*x]] - 20*a*b*Sinh[2*ArcC
osh[c + d*x]] + 4*b*ArcCosh[c + d*x]*(8*a*Cosh[2*ArcCosh[c + d*x]] - 5*b*S
inh[2*ArcCosh[c + d*x]]))))/(512*d)
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6411, 27, 6299, 6354, 6299, 6308, 6368, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int e(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e\int (c+dx)(a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\downarrow \text{6299}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\int \frac{(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\downarrow \text{6354}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\int (c+dx)\sqrt{a+b\text{arccosh}(c+dx)}d(c+dx)+\frac{1}{2}\int \frac{(a+b\text{arccosh}(}{\sqrt{c+dx-1}\sqrt{}}\right.\right.}{d}$$

$$\downarrow \text{6299}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}b\int \frac{(c+}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{}}\right.\right.\right.$$

$$\downarrow \text{6308}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}b\int \frac{(c+}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{}}\right.\right.\right.$$

$$\downarrow \text{6368}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccos}}{b}\right.}{\sqrt{a+b\text{arccosh}(}}\right.\right.\right.$$

$$\downarrow \text{3042}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccos}}{b}\right.}{\sqrt{a+b\text{arccosh}}}\right.\right.\right.$$

$$\downarrow \text{3793}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{4}\int \left(\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arc}}{}\right.}{2\sqrt{a+b\text{arcco}}}\right.\right.\right.\right.$$

$$\downarrow \text{2009}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{4}\left(-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2}{}}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^(5/2))/2 - (5*b*((Sqrt[-1 + c +
d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/2 + (a +
b*ArcCosh[c + d*x])^(5/2)/(5*b) - (3*b*(((c + d*x)^2*Sqrt[a + b*ArcCosh[c
+ d*x]])/2 + (-Sqrt[a + b*ArcCosh[c + d*x]] - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt
[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b
)))/4))/4))/4))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}} dx$$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{5/2}\,dx = \int (dex + ce)(b\operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}(c+d\,x)\right)^{5/2}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (ce+dex)(a\\&+b\text{arccosh}(c+dx))^{5/2}\,dx=e\Bigg(\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)a^2c\right.\\&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\;acosh(dx+c)\,xdx\right)abd\\&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\;acosh(dx+c)\,dx\right)abc\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\;acosh(dx+c)^2\,xdx\right)b^2d\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\;acosh(dx+c)^2\,dx\right)b^2c\\&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)a^2d\Bigg)\end{aligned}$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
e*(int(sqrt(acosh(c + d*x)*b + a),x)*a**2*c + 2*int(sqrt(acosh(c + d*x)*b
+ a)*acosh(c + d*x)*x,x)*a*b*d + 2*int(sqrt(acosh(c + d*x)*b + a)*acosh(c
+ d*x),x)*a*b*c + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x,x)*b*
*2*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x)*b**2*c + int(sq
rt(acosh(c + d*x)*b + a)*x,x)*a**2*d)
```

3.86 $\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 186

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx=\frac{15b^2(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^{3/2}}{2d}+\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}}{d}-\frac{15b^{5/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

output
```
15/4*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d-5/2*b*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)*(a+b*arccosh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arccosh(d*x+c))^(5/2)/
d-15/16*b^(5/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/
d-15/16*b^(5/2)*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(a/
b)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 494 vs. $2(186) = 372$.

Time = 2.29 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.66

$$\int (a + b\text{arccosh}(c+dx))^{5/2}\,dx = \frac{4b\sqrt{a+b\text{arccosh}(c+dx)}\left(2\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)(a-5b\text{arccosh}(c+dx))+b(c+dx)(15\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d
*x]^2)) + (8*a^2*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b
 + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*A
rcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) - Sqrt[
b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sq
rt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 4*a*
b*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh
[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (
(2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b]
 - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(16*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6410, 6294, 6330, 6294, 6368, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx$$

$\big\downarrow$ 6410

$$\frac{\int (a+b\text{arccosh}(c+dx))^{5/2}d(c+dx)}{d}$$

$\big\downarrow$ 6294

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\int \frac{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{d}$$

$\big\downarrow$ 6330

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Big(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\int \sqrt{a+b\text{arcco}}}{d}$$

$\big\downarrow$ 6294

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a}}{d}$$

$\big\downarrow$ 6368

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a}}{d}$$

$\big\downarrow$ 3042

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a}}{d}$$

$\big\downarrow$ 3788

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a}}{}$$

$\big\downarrow$ 26

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{}{\sqrt{a+}}\right.\right.\right.}{}$$

$\downarrow$ 2611

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+}{}}\right.\right.\right.}{}$$

$\downarrow$ 2633

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a}{}}\right.\right.\right.}{}$$

$\downarrow$ 2634

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
((c + d*x)*(a + b*ArcCosh[c + d*x])^(5/2) - (5*b*(Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2) - (3*b*((c + d*x)*Sqrt[a + b*A
rcCosh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(2*E^(a/b)))/2))/2))/2)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int (a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{5}{2}}\,dx$$

input
```
int((a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\mathrm{arccosh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx=\int (a+b\,\text{acosh}(c+d\,x))^{5/2}\,dx$$

input
```
int((a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\text{arccosh}(c+dx))^{5/2}\,dx=\left(\int \sqrt{acosh\left(dx+c\right)b+a}dx\right)a^2 +2\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)\,dx\right)ab +\left(\int \sqrt{acosh\left(dx+c\right)b+a}\,acosh(dx+c)^2\,dx\right)b^2$$

input
```
int((a+b*acosh(d*x+c))^(5/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a),x)*a**2 + 2*int(sqrt(acosh(c + d*x)*b + a)*
acosh(c + d*x),x)*a*b + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x
)*b**2
```

3.87 $\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\frac{\text{Int}\left(\frac{(a+b\text{arccosh}(c+dx))^{5/2}}{c+dx},x\right)}{e}$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^(5/2)/(d*x+c),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

output
```
Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6303}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{c+dx}d(c+dx)}{de}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{5}{2}}}{dex+ce}dx$$

input
```
int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

Giac [N/A]

Not integrable

Time = 28.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

Mupad [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(a+b\,\text{acosh}(c+d\,x))^{5/2}}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^(5/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.76

$$\int \frac{(a+b\text{arccosh}(c+dx))^{5/2}}{ce+dex}\,dx = \frac{\left(\int \frac{\sqrt{acosh(dx+c)b+a}}{dx+c}dx\right)a^2+2\left(\int \frac{\sqrt{acosh(dx+c)b+a}\,acosh(dx+c)}{dx+c}dx\right)ab+\left(\int \frac{\sqrt{}}{}\right.}{e}$$

input
```
int((a+b*acosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

output
```
(int(sqrt(acosh(c + d*x)*b + a)/(c + d*x),x)*a**2 + 2*int((sqrt(acosh(c +
d*x)*b + a)*acosh(c + d*x))/(c + d*x),x)*a*b + int((sqrt(acosh(c + d*x)*b
+ a)*acosh(c + d*x)**2)/(c + d*x),x)*b**2)/e
```

3.88 $\int (ce+dex)^2(a+b\,\mathbf{arccosh}(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 509

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arccosh}(c+dx))^{7/2}\,dx = \\
&-\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{54d} \\
&-\frac{35b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{216d} \\
&+\frac{35b^2e^2(c+dx)(a+b\operatorname{arccosh}(c+dx))^{3/2}}{18d} \\
&+\frac{35b^2e^2(c+dx)^3(a+b\operatorname{arccosh}(c+dx))^{3/2}}{108d} \\
&-\frac{7be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^{5/2}}{9d} \\
&-\frac{7be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^{5/2}}{18d} \\
&+\frac{e^2(c+dx)^3(a+b\operatorname{arccosh}(c+dx))^{7/2}}{3d} \\
&-\frac{105b^{7/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&-\frac{35b^{7/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3456d} \\
&+\frac{105b^{7/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&+\frac{35b^{7/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3456d}
\end{aligned}$$

output
```
-175/54*b^3*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)
/d-35/216*b^3*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)*(a+b*arccosh(d
*x+c))^(1/2)/d+35/18*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d+35/108*b
^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(3/2)/d-7/9*b*e^2*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(5/2)/d-7/18*b*e^2*(d*x+c-1)^(1/2)*(d*x
+c)^2*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(5/2)/d+1/3*e^2*(d*x+c)^3*(a+b*
arccosh(d*x+c))^(7/2)/d-105/128*b^(7/2)*e^2*exp(a/b)*Pi^(1/2)*erf((a+b*arc
cosh(d*x+c))^(1/2)/b^(1/2))/d-35/10368*b^(7/2)*e^2*exp(3*a/b)*3^(1/2)*Pi^(
1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d+105/128*b^(7/2)*e^2
*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(a/b)+35/10368*b^(
7/2)*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))
/d/exp(3*a/b)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1523 vs. 2(509) = 1018.

Time = 10.21 (sec) , antiderivative size = 1523, normalized size of antiderivative = 2.99

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
e^2*((a^3*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCos
h[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b
)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*
x])^2/b^2)]) + (a^2*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d
*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcC
osh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt
[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))
+ (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]
]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sq
rt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c +
d*x]] - Sinh[3*ArcCosh[c + d*x]])))/(96*d) + (a*(-27*(-4*b*Sqrt[a + b*ArcC
osh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b
*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4
*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]
]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 8.16 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.37, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6411, 27, 6299, 6354, 6299, 6330, 6294, 6330, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634, 6354, 6302, 25, 5971, 2009, 6330, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 6299$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\int \frac{(c+dx)^3(a+b\text{arccosh}(c+dx))^{5/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \; 6354$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\int (c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)+\frac{2}{3}\int \frac{(c+dx)(a\cdots}{\sqrt{c\cdots}}\right.\right.}{d}$$

$$\Big\downarrow \; 6299$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int \frac{(c+dx)^3\sqrt{a+b\text{arc}\cdots}}{\sqrt{c+dx-1}\sqrt{\cdots}}\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \; 6330$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int \frac{(c+dx)^3\sqrt{a+b\text{arc}\cdots}}{\sqrt{c+dx-1}\sqrt{\cdots}}\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \; 6294$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c\cdots\right.\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \; 6330$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c\cdots\right.\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \; 6296$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{ar}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.$$

$\downarrow$ 25

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{ar}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{ar}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.$$

$\downarrow$ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{ar}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.$$

$\downarrow$ 3789

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c-\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c-\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{ar}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.$$

$\downarrow$ 6354

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{1}{6}b\int\frac{(c}{\sqrt{a+b\text{arc}}}\right.\right.\right.\right.$$

$\downarrow$ 6302

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+}}{\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 25

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+}}{\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 5971

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+}}{\sqrt{c+dx}}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b}}{\sqrt{c+dx-}}\right.\right.\right.\right.$$

$\downarrow$ 6330

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 6296

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 25

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\frac{1}{2}\int\frac{\sinh\left(\frac{a}{b}\right.}{\sqrt{a}}\right.\right.\right.\right.\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 3789

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{3/2}-\frac{1}{2}b\left(\frac{2}{3}\left(\sqrt{c+dx-}\right.\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{6}b\left(\frac{2}{3}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^(7/2))/3 - (7*b*((Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/3 + (
2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2) - (
5*b*((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2) - (3*b*(Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/
b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b))))/2))/2))/3 - (
5*b*(((c + d*x)^3*(a + b*ArcCosh[c + d*x])^(3/2))/3 - (b*((Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/3 + (2*(Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] - (I/2)*((
I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] -
((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b
))))/3 + ((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[
b]])/8 + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*Arc
Cosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b)))/6))/2))/6))/6))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^2 (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}} dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^2 (a + b\operatorname{arccosh}(c + dx))^{7/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^2 (a + b\text{arccosh}(c + dx))^{7/2}\, dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ce + dex)^2 (a + b\text{arccosh}(c + dx))^{7/2}\, dx = \int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int (ce + dex)^2 (a + b\text{arccosh}(c + dx))^{7/2}\, dx = \int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{7/2}\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{acosh}(c+d\,x))^{7/2}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\begin{aligned}\int (ce+dex)^2(a+b\text{arccosh}(c+dx))^{7/2}\,dx=&\,e^2\Bigg(\left(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)a^3c^2\right.\\
&+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,x^2dx\right)a^2b\,d^2\\
&+6\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,xdx\right)a^2bcd\\
&+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,dx\right)a^2b\,c^2\\
&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^3\,x^2dx\right)b^3d^2\\
&+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^3\,xdx\right)b^3cd\\
&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^3\,dx\right)b^3c^2\\
&+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,x^2dx\right)a\,b^2d^2\\
&+6\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,xdx\right)a\,b^2cd\\
&+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,dx\right)a\,b^2c^2\\
&+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,x^2dx\right)a^3d^2+2\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)a^3cd\Bigg)\end{aligned}$$

input
```
int((d*e*x+c*e)^2*(a+b*acosh(d*x+c))^(7/2),x)
```

output
```
e**2*(int(sqrt(acosh(c + d*x)*b + a),x)*a**3*c**2 + 3*int(sqrt(acosh(c + d
*x)*b + a)*acosh(c + d*x)*x**2,x)*a**2*b*d**2 + 6*int(sqrt(acosh(c + d*x)*
b + a)*acosh(c + d*x)*x,x)*a**2*b*c*d + 3*int(sqrt(acosh(c + d*x)*b + a)*a
cosh(c + d*x),x)*a**2*b*c**2 + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*
x)**3*x**2,x)*b**3*d**2 + 2*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)*
*3*x,x)*b**3*c*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**3,x)*b**
3*c**2 + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x**2,x)*a*b**2
*d**2 + 6*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x,x)*a*b**2*c*d
 + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x)*a*b**2*c**2 + int
(sqrt(acosh(c + d*x)*b + a)*x**2,x)*a**3*d**2 + 2*int(sqrt(acosh(c + d*x)*
b + a)*x,x)*a**3*c*d)
```

3.89 $\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^{7/2}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 319

$$\begin{aligned}
\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^{7/2}\, dx =
&-\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{128d}\\
&-\frac{35b^2e(a+b\operatorname{arccosh}(c+dx))^{3/2}}{64d}+\frac{35b^2e(c+dx)^2(a+b\operatorname{arccosh}(c+dx))^{3/2}}{32d}\\
&-\frac{7be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}(a+b\operatorname{arccosh}(c+dx))^{5/2}}{8d}\\
&-\frac{e(a+b\operatorname{arccosh}(c+dx))^{7/2}}{4d}+\frac{e(c+dx)^2(a+b\operatorname{arccosh}(c+dx))^{7/2}}{2d}\\
&-\frac{105b^{7/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{1024d}\\
&+\frac{105b^{7/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{1024d}
\end{aligned}$$

output
```
-105/128*b^3*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c)
)^(1/2)/d-35/64*b^2*e*(a+b*arccosh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(
a+b*arccosh(d*x+c))^(3/2)/d-7/8*b*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2
)*(a+b*arccosh(d*x+c))^(5/2)/d-1/4*e*(a+b*arccosh(d*x+c))^(7/2)/d+1/2*e*(d
*x+c)^2*(a+b*arccosh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*2^(1/2)
*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d+105/2048*b^(7/
2)*e*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/e
xp(2*a/b)
```

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.90

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{7/2}\,dx = \frac{e\left(105b^{7/2}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right) - \sinh\left(\frac{2a}{b}\right)\right) - 105b^{7/2}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+\ldots}}{\ldots}\right.\right.}{\ldots}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e*(105*b^(7/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqr
t[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 105*b^(7/2)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) +
 8*Sqrt[a + b*ArcCosh[c + d*x]]*(4*a*(16*a^2 + 35*b^2)*Cosh[2*ArcCosh[c +
d*x]] + 64*b^3*ArcCosh[c + d*x]^3*Cosh[2*ArcCosh[c + d*x]] - 7*b*(16*a^2 +
 15*b^2)*Sinh[2*ArcCosh[c + d*x]] + 16*b^2*ArcCosh[c + d*x]^2*(12*a*Cosh[2
*ArcCosh[c + d*x]] - 7*b*Sinh[2*ArcCosh[c + d*x]]) + 4*b*ArcCosh[c + d*x]*
((48*a^2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]] - 56*a*b*Sinh[2*ArcCosh[c + d*
x]]))))/(2048*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {6411, 27, 6299, 6354, 6299, 6308, 6354, 6302, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int e(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arccosh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\int \frac{(c+dx)^2(a+b\text{arccosh}(c+dx))^{5/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6354}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\int (c+dx)(a+b\text{arccosh}(c+dx))^{3/2}d(c+dx)+\frac{1}{2}\int \frac{(a+b\text{arccos}}{\sqrt{c+dx-}}\right.\right.}{d}$$

$$\Big\downarrow \text{6299}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\int \frac{(c+dx)^2\sqrt{a+b\text{arc}}}{\sqrt{c+dx-1}\sqrt{c}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{6308}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+b\text{arc}}}{\sqrt{c+dx-1}\sqrt{c}}\right.\right.\right.$$

$\downarrow$ 6354

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{4}b\int\frac{c+}{\sqrt{a+b\text{arcc}}}\right.\right.\right.\right.$$

$\downarrow$ 6302

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccos}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 25

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccos}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 5971

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccos}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 27

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccos}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 3042

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccos}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 26

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccos}}}{\sqrt{c+dx-1}\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 3789

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\int\frac{}{\sqrt{a+b}}\right.\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(}{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(}{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(\frac{1}{2}\int\frac{\sqrt{a+b\text{arccosh}}}{\sqrt{c+dx-1}\sqrt{c}}\right.\right.\right.\right.$$

$\downarrow$ 6308

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arccosh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}\right.\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x])^(7/2))/2 - (7*b*((Sqrt[-1 + c +
d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/2 + (a +
b*ArcCosh[c + d*x])^(7/2)/(7*b) - (5*b*(((c + d*x)^2*(a + b*ArcCosh[c + d*
x])^(3/2))/2 - (3*b*((Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[
a + b*ArcCosh[c + d*x]])/2 + (a + b*ArcCosh[c + d*x])^(3/2)/(3*b) - (I/8)*
((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/E^((2*a)/b))))/4))/4))/4))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6299
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x
], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6354
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))   Int[(f*x)^(m
 - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Simp[b*f
*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(
-1 + c*x)^p]   Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && N
eQ[m + 2*p + 1, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}} dx$$

input
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)(a + b\operatorname{arccosh}(c + dx))^{7/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{7/2} \, dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{7/2} \, dx = \int (dex + ce)(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}} \, dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int (ce + dex)(a + b\text{arccosh}(c + dx))^{7/2} \, dx = \int (dex + ce)(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}} \, dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}(c+d\,x)\right)^{7/2}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}\,dx=e\Bigg(\left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)a^3c+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,xdx\right)a^2bd+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,dx\right)a^2bc+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^3\,xdx\right)b^3d+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^3\,dx\right)b^3c+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,xdx\right)a\,b^2d+3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,dx\right)a\,b^2c+\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,xdx\right)a^3d\Bigg)$$

input
```
int((d*e*x+c*e)*(a+b*acosh(d*x+c))^(7/2),x)
```

output
```
e*(int(sqrt(acosh(c + d*x)*b + a),x)*a**3*c + 3*int(sqrt(acosh(c + d*x)*b
+ a)*acosh(c + d*x)*x,x)*a**2*b*d + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh
(c + d*x),x)*a**2*b*c + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**3*x
,x)*b**3*d + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**3,x)*b**3*c +
3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2*x,x)*a*b**2*d + 3*int(s
qrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x)*a*b**2*c + int(sqrt(acosh(c
 + d*x)*b + a)*x,x)*a**3*d)
```

3.90 $\int (a+b\text{arccosh}(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 230

$$\int (a+b\text{arccosh}(c+dx))^{7/2}\,dx = -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\text{arccosh}(c+dx)}}{8d} + \frac{35b^2(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}}{4d} - \frac{7b\sqrt{-1+c+dx}\sqrt{1+c+dx}(a+b\text{arccosh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}}{d} - \frac{105b^{7/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{105b^{7/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

output
```
-105/8*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d+35
/4*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d-7/2*b*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)*(a+b*arccosh(d*x+c))^(5/2)/d+(d*x+c)*(a+b*arccosh(d*x+c))^(7/2)/d-
105/32*b^(7/2)*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d
+105/32*b^(7/2)*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/d/exp(a/
b)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 748 vs. $2(230) = 460$.

Time = 4.74 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.25

$$\int (a + b\text{arccosh}(c + dx))^{7/2} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(-2*b*(c + d*x)*(-10*a + 35*b*ArcCosh[c
 + d*x] + 4*b*ArcCosh[c + d*x]^3) + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15 + 4*ArcCosh[c + d*x
]^2))) + (16*a^3*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b
 + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*A
rcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) + Sqrt[
b]*(8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(-8*a^3 + 36*a^2*b
- 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(
Cosh[a/b] + Sinh[a/b]) + 12*a^2*b*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*
(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*
Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCo
sh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sq
rt[b]) + 6*a*(4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 +
 c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*
ArcCosh[c + d*x]^2)) - Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^
2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(C
osh[a/b] + Sinh[a/b])))/(32*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6410, 6294, 6330, 6294, 6330, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \ 6410$$

$$\frac{\int (a+b\text{arccosh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \ 6294$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\int \frac{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{d}$$

$$\Big\downarrow \ 6330$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\int(a+b\text{arccosh}}{d}$$

$$\Big\downarrow \ 6294$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.}{d}$$

$$\Big\downarrow \ 6330$$

$$\frac{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.}{}$$

$$\Big\downarrow \ 6296$$

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$\downarrow$ 25

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$\downarrow$ 3042

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$\downarrow$ 26

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$\downarrow$ 3789

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$\downarrow$ 2611

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$\downarrow$ 2633

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

$$\downarrow 2634$$

$$(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}-\frac{7}{2}b\Bigg(\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg((c+dx)(a+$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
((c + d*x)*(a + b*ArcCosh[c + d*x])^(7/2) - (7*b*(Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2) - (5*b*((c + d*x)*(a + b*ArcCo
sh[c + d*x])^(3/2) - (3*b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b
*ArcCosh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcCosh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCo
sh[c + d*x]]/Sqrt[b]])/E^(a/b))))/2))/2))/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6330
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Simp[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p]   Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && E
qQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}} \, dx$$

input
```
int((a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a + b\operatorname{arccosh}(c + dx))^{7/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\operatorname{arccosh}(c + dx))^{7/2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\text{arccosh}(c + dx))^{7/2}\, dx = \int (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int (a + b\text{arccosh}(c + dx))^{7/2}\, dx = \int (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arccosh}(c + dx))^{7/2}\, dx = \int (a + b \operatorname{acosh}(c + d\,x))^{7/2}\, dx$$

input
```
int((a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (a+b\operatorname{arccosh}(c+dx))^{7/2}\,dx = \left(\int \sqrt{acosh\,(dx+c)\,b+a}dx\right)a^3 + 3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)\,dx\right)a^2b + \left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{acosh\,(dx+c)\,b+a}\,acosh(dx+c)^2\,dx\right)a\,b^2$$

input
```
int((a+b*acosh(d*x+c))^(7/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a),x)*a**3 + 3*int(sqrt(acosh(c + d*x)*b + a)*
acosh(c + d*x),x)*a**2*b + int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**
3,x)*b**3 + 3*int(sqrt(acosh(c + d*x)*b + a)*acosh(c + d*x)**2,x)*a*b**2
```

3.91 $\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx=\frac{\text{Int}\left(\frac{(a+b\text{arccosh}(c+dx))^{7/2}}{c+dx},x\right)}{e}$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^(7/2)/(d*x+c),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

output
```
Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex} \, dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{c+dx}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6303}$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{c+dx}d(c+dx)}{de}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}{dex + ce} dx$$

input
```
int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^{7/2}}{ce + dex} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^{7/2}}{ce + dex} dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

Giac [N/A]

Not integrable

Time = 39.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

Mupad [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{(a+b\,\text{acosh}(c+d\,x))^{7/2}}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*acosh(c + d*x))^(7/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*acosh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

Reduce [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.24

$$\int \frac{(a+b\text{arccosh}(c+dx))^{7/2}}{ce+dex}\,dx = \frac{\left(\int \frac{\sqrt{acosh(dx+c)b+a}}{dx+c}dx\right)a^3+3\left(\int \frac{\sqrt{acosh(dx+c)b+a}\,acosh(dx+c)}{dx+c}dx\right)a^2b+\left(\int \cdots\right.}{e}$$

input
```
int((a+b*acosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

output
```
(int(sqrt(acosh(c + d*x)*b + a)/(c + d*x),x)*a**3 + 3*int((sqrt(acosh(c +
d*x)*b + a)*acosh(c + d*x))/(c + d*x),x)*a**2*b + int((sqrt(acosh(c + d*x)
*b + a)*acosh(c + d*x)**3)/(c + d*x),x)*b**3 + 3*int((sqrt(acosh(c + d*x)*
b + a)*acosh(c + d*x)**2)/(c + d*x),x)*a*b**2)/e
```

3.92 $\int \frac{(ce+dex)^4}{\sqrt{a+b\,\mathbf{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 326

$$\begin{aligned}\int \frac{(ce+dex)^4}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}\,dx &= -\frac{e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} \\ &\quad - \frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} \\ &\quad - \frac{e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} \\ &\quad + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} \\ &\quad + \frac{e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} \\ &\quad + \frac{e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}\end{aligned}$$

output
```
-1/16*e^4*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2
)/d-1/32*e^4*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^
(1/2)/b^(1/2))/b^(1/2)/d-1/160*e^4*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d+1/16*e^4*Pi^(1/2)*erfi((a+b
*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(a/b)+1/32*e^4*3^(1/2)*Pi^(1/
2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(3*a/b)+1
/160*e^4*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))
/b^(1/2)/d/exp(5*a/b)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.98

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx$$
$$= \frac{e^4e^{-\frac{5a}{b}}\left(10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{5(a+b\text{ar}}{}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^4*(10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh
[c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a
 + b*ArcCosh[c + d*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c
 + d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 10*E^((4*a)/b)*
Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/
b)] + 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a
+ b*ArcCosh[c + d*x]))/b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*
x]]*Gamma[1/2, (5*(a + b*ArcCosh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a
 + b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6411, 27, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^4(c+dx)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6302}$$

$$\frac{e^4\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{e^4\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{e^4\int \left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccos}}{bd}$$

$$\Big\downarrow \text{2009}$$

$$e^4\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{}\right.\right.$$

input
```
Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^4*(-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqr
t[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sq
rt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((5*a
)/b))))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex + ce)^4}{\sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^4}{\sqrt{a + b\mathrm{arccosh}(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\mathrm{arccosh}(c+dx)}}\,dx = e^4\left(\int \frac{c^4}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx + \int \frac{d^4x^4}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx + \int \frac{4cd^3x^3}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx + \int \frac{6c^2d^2x^2}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx + \int \frac{4c^3dx}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e**4*(Integral(c**4/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/sq
rt(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*acosh(c +
 d*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*acosh(c + d*x)), x) + In
tegral(4*c**3*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\mathrm{arccosh}(c+dx)}}\,dx = \int \frac{(dex+ce)^4}{\sqrt{b\,\mathrm{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \int \frac{(dex+ce)^4}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{\sqrt{a+b\,\text{acosh}\,(c+d\,x)}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = e^4\Bigg(&\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}}{acosh\,(dx+c)\,b+a}dx\right)c^4\\&+\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x^4}{acosh\,(dx+c)\,b+a}dx\right)d^4\\&+4\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x^3}{acosh\,(dx+c)\,b+a}dx\right)c\,d^3\\&+6\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x^2}{acosh\,(dx+c)\,b+a}dx\right)c^2d^2\\&+4\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x}{acosh\,(dx+c)\,b+a}dx\right)c^3d\Bigg)\end{aligned}$$

input
```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e**4*(int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b + a),x)*c**4 + int(
(sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)*b + a),x)*d**4 + 4*int((
sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)*b + a),x)*c*d**3 + 6*int(
(sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)*b + a),x)*c**2*d**2 + 4*
int((sqrt(acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)*b + a),x)*c**3*d)
```

3.93 $\int \frac{(ce+dex)^3}{\sqrt{a+b\mathbf{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

output
```
-1/32*e^3*exp(4*a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^
(1/2)/d-1/16*e^3*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+
c))^(1/2)/b^(1/2))/b^(1/2)/d+1/32*e^3*Pi^(1/2)*erfi(2*(a+b*arccosh(d*x+c))
^(1/2)/b^(1/2))/b^(1/2)/d/exp(4*a/b)+1/16*e^3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2
)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)+2\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcc}}{b}\right.\right.}{32d\sqrt{a+b\text{ar}}}$$

input
```
Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c
+ d*x]))/b] + 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Ga
mma[1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcCosh
[c + d*x]]*(2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((2*a
)/b)*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[
a + b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6411, 27, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^3(c+dx)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$\Big\downarrow$ 6302

$$\frac{e^3\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 25

$$-\frac{e^3\int \frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 5971

$$-\frac{e^3\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{8\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 2009

$$\frac{e^3\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a}\right.}{}\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^3*(-1/32*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]]) - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcCosh[c + d*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[
2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((2*a)/b))))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^3}{\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = e^3\left(\int \frac{c^3}{\sqrt{a+b\,\text{acosh}\,(c+dx)}}\,dx + \int \frac{d^3x^3}{\sqrt{a+b\,\text{acosh}\,(c+dx)}}\,dx + \int \frac{3cd^2x^2}{\sqrt{a+b\,\text{acosh}\,(c+dx)}}\,dx + \int \frac{3c^2dx}{\sqrt{a+b\,\text{acosh}\,(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e**3*(Integral(c**3/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/sq
rt(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*acosh(c +
 d*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{(dex+ce)^3}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{(dex+ce)^3}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{(c\,e+d\,e\,x)^3}{\sqrt{a+b\,\text{acosh}\,(c+d\,x)}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}\,dx = e^3\left(\left(\int \frac{\sqrt{acosh\left(dx+c\right)b+a}}{acosh\left(dx+c\right)b+a}dx\right)c^3 + \left(\int \frac{\sqrt{acosh\left(dx+c\right)b+a}\,x^3}{acosh\left(dx+c\right)b+a}dx\right)d^3 + 3\left(\int \frac{\sqrt{acosh\left(dx+c\right)b+a}\,x^2}{acosh\left(dx+c\right)b+a}dx\right)c\,d^2 + 3\left(\int \frac{\sqrt{acosh\left(dx+c\right)b+a}\,x}{acosh\left(dx+c\right)b+a}dx\right)c^2d\right)$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e**3*(int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b + a),x)*c**3 + int(
(sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)*b + a),x)*d**3 + 3*int((
sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)*b + a),x)*c*d**2 + 3*int(
(sqrt(acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)*b + a),x)*c**2*d)
```

3.94 $\int \frac{(ce+dex)^2}{\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx = -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\,\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\,\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

output
```
-1/8*e^2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)
/d-1/24*e^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))/b^(1/2)/d+1/8*e^2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b
^(1/2))/b^(1/2)/d/exp(a/b)+1/24*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arc
cosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(3*a/b)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\text{arc}}{}\right.\right.}{24d}$$

input
```
Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[
c + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcCosh[c + d*x]))/b] + 3*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/
b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a
/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/(24*d*
E^((3*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6411, 27, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^2(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2 \int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$\Big\downarrow$ 6302

$$\frac{e^2 \int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 25

$$-\frac{e^2 \int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 5971

$$-\frac{e^2 \int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 2009

$$\frac{e^2\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{ar}}}{}\right.\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(e^2*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt
[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
+ d*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]
/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcC
osh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^2}{\sqrt{a+b\ \mathrm{arccosh}\,(dx+c)}}dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\mathrm{arccosh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\mathrm{arccosh}(c+dx)}}\,dx = e^2\left(\int \frac{c^2}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx + \int \frac{d^2x^2}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx + \int \frac{2cdx}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e**2*(Integral(c**2/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/sq
rt(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*acosh(c + d*x))
, x))
```

Maxima [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{(dex+ce)^2}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{(dex+ce)^2}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{\sqrt{a+b\,\text{acosh}\,(c+d\,x)}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = e^2\left(\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}}{acosh\,(dx+c)\,b+a}dx\right)c^2 + \left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x^2}{acosh\,(dx+c)\,b+a}dx\right)d^2 + 2\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x}{acosh\,(dx+c)\,b+a}dx\right)cd\right)$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e**2*(int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b + a),x)*c**2 + int(
(sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)*b + a),x)*d**2 + 2*int((
sqrt(acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)*b + a),x)*c*d)
```

3.95 $\int \frac{ce+dex}{\sqrt{a+b\,\mathbf{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{ce+dex}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=-\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}+\frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

output

```
-1/8*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/
b^(1/2))/b^(1/2)/d+1/8*e*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))/b^(1/2)/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}\,dx = -\frac{e\sqrt{\frac{\pi}{2}}\left(\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(-\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)\right.\right.}{4\sqrt{b}d}$$

input
```
Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
-1/4*(e*Sqrt[Pi/2]*(Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(
-Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])))/(Sqrt[b]*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6411, 27, 6302, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{e(c+dx)}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{c+dx}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6302$$

$$\frac{e\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \; 25$$

$$-\frac{e\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \; 5971$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \; 27$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{2bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{e\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{2bd}$$

$$\Big\downarrow \; 26$$

$$\frac{ie\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{2bd}$$

$$\Big\downarrow \; 3789$$

$$\frac{ie\left(\frac{1}{2}i\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{2bd}$$

$$\Big\downarrow \; 2611$$

$$\frac{ie\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{2(a+b\text{arccosh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{2bd}$$

$$\Big\downarrow \textbf{2633}$$

$$\frac{ie\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{2bd}$$

$$\Big\downarrow \textbf{2634}$$

$$\frac{ie\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{2bd}$$

input
```
Int[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
((I/2)*e*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*Arc
Cosh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{dex + ce}{\sqrt{a + b \operatorname{arccosh}(dx + c)}} dx$$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{\sqrt{a + b\text{arccosh}(c + dx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{ce + dex}{\sqrt{a + b\text{arccosh}(c + dx)}} dx = e\left(\int \frac{c}{\sqrt{a + b \operatorname{acosh}(c + dx)}} dx + \int \frac{dx}{\sqrt{a + b \operatorname{acosh}(c + dx)}} dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
e*(Integral(c/sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x/sqrt(a + b*aco
sh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{ce+dex}{\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx=\int \frac{dex+ce}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{ce+dex}{\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx=\int \frac{dex+ce}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx=\int \frac{c\,e+d\,e\,x}{\sqrt{a+b\,\text{acosh}\,(c+d\,x)}}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}\,dx = e\left(\left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}}{acosh\,(dx+c)\,b+a}dx\right)c + \left(\int \frac{\sqrt{acosh\,(dx+c)\,b+a}\,x}{acosh\,(dx+c)\,b+a}dx\right)d\right)$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
e*(int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b + a),x)*c + int((sqrt(
acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)*b + a),x)*d)
```

3.96 $\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=-\frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

output
```
-1/2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d+1
/2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d/exp(a/b)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arccosh}(c+dx)\right)+\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\text{arccosh}(c+dx)}{b}\right.\right.}{2d\sqrt{a+b\text{arccosh}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x
]] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d
*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6410, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6296}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 26

$$\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{bd}$$

$\Big\downarrow$ 3789

$$\frac{i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{bd}$$

$\Big\downarrow$ 2611

$$\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{bd}$$

$\Big\downarrow$ 2633

$$\frac{i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{bd}$$

$\Big\downarrow$ 2634

$$\frac{i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{bd}$$

input
```
Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

output
```
(I*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b
]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E
^(a/b)))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b\,\operatorname{arccosh}\left(dx + c\right)}}dx$$

input
```
int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\operatorname{arccosh}(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\operatorname{arccosh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + b\operatorname{acosh}\left(c + dx\right)}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{arccosh}(c + dx)}} \, dx = \int \frac{1}{\sqrt{b \operatorname{arcosh}(dx + c) + a}} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{arccosh}(c + dx)}} \, dx = \int \frac{1}{\sqrt{b \operatorname{arcosh}(dx + c) + a}} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{arccosh}(c + dx)}} \, dx = \int \frac{1}{\sqrt{a + b \operatorname{acosh}(c + d x)}} \, dx$$

input
```
int(1/(a + b*acosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*acosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}\,dx=\int \frac{\sqrt{acosh\left(dx+c\right)b+a}}{acosh\left(dx+c\right)b+a}dx$$

input
```
int(1/(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b + a),x)
```

3.97 $\int \frac{1}{(ce+dex)\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx=\frac{\text{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\,\text{arccosh}(c+dx)}},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^(1/2),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx=\int \frac{1}{(ce+dex)\sqrt{a+b\,\text{arccosh}(c+dx)}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]
```

output
```
Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}}dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{1}{e(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6303}$$
$$\frac{\int \frac{1}{(c+dx)\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\mathrm{arccosh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{(ce+dex)\sqrt{a+b\mathrm{arccosh}(c+dx)}}\,dx = \frac{\int \frac{1}{c\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+dx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/(c*sqrt(a + b*acosh(c + d*x)) + d*x*sqrt(a + b*acosh(c + d*x)))
, x)/e
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \int \frac{1}{(dex+ce)\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

Giac [N/A]

Not integrable

Time = 11.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \int \frac{1}{(dex+ce)\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\sqrt{a+b\,\text{acosh}\,(c+d\,x)}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arccosh}(c+dx)}}\,dx = \frac{\int \frac{\sqrt{acosh(dx+c)b+a}}{acosh(dx+c)bc+acosh(dx+c)bdx+ac+adx}dx}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^(1/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)*b*c + acosh(c + d*x)*b*d*x
+ a*c + a*d*x),x)/e
```

3.98 $\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 374

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx =& -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\text{arccosh}(c+dx)}} \\ &+ \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} \\ &+ \frac{e^4e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} \\ &+ \frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} \\ &+ \frac{e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}\end{aligned}$$

output
```
-2*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^
(1/2)+1/8*e^4*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^
(3/2)/d+3/16*e^4*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+
c))^(1/2)/b^(1/2))/b^(3/2)/d+1/16*e^4*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1
/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+1/8*e^4*Pi^(1/2)*erfi((a
+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d/exp(a/b)+3/16*e^4*3^(1/2)*Pi^(
1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d/exp(3*a/b)
+1/16*e^4*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2)
)/b^(3/2)/d/exp(5*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.06

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \frac{e^4e^{-\frac{5a}{b}}\left(-4e^{\frac{5a}{b}}\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)-2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\Gamma\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^4*(-4*E^((5*a)/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - 2*
E^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]
] + Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a + b*ArcC
osh[c + d*x]))/b] + 3*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/
b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a
+ b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - 3*Sq
rt[3]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCos
h[c + d*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[
1/2, (5*(a + b*ArcCosh[c + d*x]))/b] - 6*E^((5*a)/b)*Sinh[3*ArcCosh[c + d*
x]] - 2*E^((5*a)/b)*Sinh[5*ArcCosh[c + d*x]]))/(16*b*d*E^((5*a)/b)*Sqrt[a
+ b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6411, 27, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6300}$$

$$\frac{e^4\left(-\frac{2\int \left(-\frac{5\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{9\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+\right.}{b^2}}{d}$$

$$\Big\downarrow \text{2009}$$

$$e^4\left(-\frac{2\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{3}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{5\pi}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(\right.}{\sqrt{b}}\right.}{}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^4*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*
ArcCosh[c + d*x]]) - (2*(-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]) - (3*Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[5*
Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) - (3*Sqrt[b
]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^(
(3*a)/b)) - (Sqrt[b]*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]])/(32*E^((5*a)/b))))/b^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex + ce)^4}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}} dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^4}{(a + b\text{arccosh}(c + dx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce + dex)^4}{(a + b\text{arccosh}(c + dx))^{3/2}}\, dx = e^4 \Bigg(\int \frac{c^4}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)}$$

$$+ \int \frac{d^4x^4}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)}\, dx$$

$$+ \int \frac{4cd^3x^3}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)}\, dx$$

$$+ \int \frac{6c^2d^2x^2}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)}\, dx$$

$$+ \int \frac{4c^3dx}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)}\, dx \Bigg)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e**4*(Integral(c**4/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*acosh(c + d*
x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c*d**3
*x**3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c
 + d*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)) + b
*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(a*s
qrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)),
x))
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\text{too large to display}$$

input
```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^(3/2),x)
```

output

```
(e**4*(acosh(c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**6)/(acosh(c + d*x
)**2*b**2*c**2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d
**2*x**2 - acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c
+ d*x)*a*b*c*d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b +
 a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*d**7 + 6*acosh(
c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**5)/(acosh(c + d*x)**2*b**2*c**
2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - ac
osh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*
d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 +
2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*c*d**6 + 15*acosh(c + d*x)*i
nt((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)**2*b**2*c**2 + 2*acos
h(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*
x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*ac
osh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d
*x + a**2*d**2*x**2 - a**2),x)*b**2*c**2*d**5 - acosh(c + d*x)*int((sqrt(a
cosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)**2*b**2*c**2 + 2*acosh(c + d*x)
**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*x)**2*b**2
 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*acosh(c + d*
x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d*x + a**2*
d**2*x**2 - a**2),x)*b**2*d**5 + 20*acosh(c + d*x)*int((sqrt(acosh(c + ...
```

3.99 $\int \frac{(ce+dex)^3}{(a+b\,\mathrm{arccosh}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 269

$$\int \frac{(ce+dex)^3}{(a+b\,\mathrm{arccosh}(c+dx))^{3/2}}\,dx = -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\,\mathrm{arccosh}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}$$

output
```
-2*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^
(1/2)+1/4*e^3*exp(4*a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2)
)/b^(3/2)/d+1/4*e^3*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d
*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+1/4*e^3*Pi^(1/2)*erfi(2*(a+b*arccosh(d*x+c
))^(1/2)/b^(1/2))/b^(3/2)/d/exp(4*a/b)+1/4*e^3*2^(1/2)*Pi^(1/2)*erfi(2^(1/
2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d/exp(2*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.99

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arccosh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)+\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a\cdots}{}}\right.}{}$$

input

```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output

```
(e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c
+ d*x]))/b] + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] - E^((4*a)/b)*(8*(c + d*x)^3*Sqrt[
(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b
 + ArcCosh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((4*a)
/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b
])))/(4*b*d*E^((4*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6411, 27, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6300}$$

$$\frac{e^3\left(-\frac{2\int\left(-\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+d}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^3\left(-\frac{2\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(c+}{\sqrt{b}}\right.\right.}{b^2}\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^3*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*
ArcCosh[c + d*x]]) - (2*(-1/8*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]]) - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi]*Erfi[(
2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((4*a)/b)) - (Sqrt[b]*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)
)))/b^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arccosh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}\,dx = e^3\left(\int \frac{c^3}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx\right.$$
$$+\int \frac{d^3x^3}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx$$
$$+\int \frac{3cd^2x^2}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx$$
$$\left.+\int \frac{3c^2dx}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e**3*(Integral(c**3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*acosh(c + d*
x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(3*c*d**2
*x**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c
 + d*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(
a + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^3}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{acosh}\,(c+d\,x))^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\text{too large to display}$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^(3/2),x)
```

output

```
(e**3*(acosh(c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**5)/(acosh(c + d*x
)**2*b**2*c**2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d
**2*x**2 - acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c
+ d*x)*a*b*c*d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b +
 a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*d**6 + 5*acosh(
c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)**2*b**2*c**
2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - ac
osh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*
d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 +
2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*c*d**5 + 10*acosh(c + d*x)*i
nt((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**2*b**2*c**2 + 2*acos
h(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*
x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*ac
osh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d
*x + a**2*d**2*x**2 - a**2),x)*b**2*c**2*d**4 - acosh(c + d*x)*int((sqrt(a
cosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**2*b**2*c**2 + 2*acosh(c + d*x)
**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*x)**2*b**2
 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*acosh(c + d*
x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d*x + a**2*
d**2*x**2 - a**2),x)*b**2*d**4 + 9*acosh(c + d*x)*int((sqrt(acosh(c + d...
```

3.100 $\int \frac{(ce+dex)^2}{(a+b\,\text{arccosh}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\text{arccosh}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}$$

output
```
-2*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^
(1/2)+1/4*e^2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^
(3/2)/d+1/4*e^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c
))^(1/2)/b^(1/2))/b^(3/2)/d+1/4*e^2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))/b^(3/2)/d/exp(a/b)+1/4*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*
arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d/exp(3*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.01

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}\,dx = \frac{e^2e^{-\frac{3a}{b}}\left(-e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arccosh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)+\sqrt{3}\sqrt{\cdot}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^2*(-(E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[
c + d*x]]) + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a
 + b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b
)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/
b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b] - 2*E^((3
*a)/b)*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sinh[3*ArcCosh[
c + d*x]])))/(4*b*d*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6411, 27, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6300$$

$$\frac{e^2\left(-\frac{2\int\left(-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^2\left(-\frac{2\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right.}{b^2}\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e^2*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*
ArcCosh[c + d*x]]) - (2*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Cosh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sq
rt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}} dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^2}{(a + b\operatorname{arccosh}(c + dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = e^2\left(\int \frac{c^2}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx + \int \frac{d^2x^2}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx + \int \frac{2cdx}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e**2*(Integral(c**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*acosh(c + d*
x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(2*c*d*x/
(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{acosh}\,(c+d\,x))^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^(3/2),x)
```

output

```
(e**2*(acosh(c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x
)**2*b**2*c**2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d
**2*x**2 - acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c
+ d*x)*a*b*c*d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b +
 a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*d**5 + 4*acosh(
c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**2*b**2*c**
2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - ac
osh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*
d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 +
2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*c*d**4 + 5*acosh(c + d*x)*in
t((sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**2*b**2*c**2 + 2*acosh
(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*x
)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*aco
sh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d*
x + a**2*d**2*x**2 - a**2),x)*b**2*c**2*d**3 - acosh(c + d*x)*int((sqrt(ac
osh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**2*b**2*c**2 + 2*acosh(c + d*x)*
*2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*x)**2*b**2
+ 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*acosh(c + d*x
)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d*x + a**2*d
**2*x**2 - a**2),x)*b**2*d**3 + 2*acosh(c + d*x)*int((sqrt(acosh(c + d*...
```

3.101 $\int \frac{ce+dex}{(a+b\,\mathbf{arccosh}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\text{arccosh}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

output
```
-2*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2
)+1/2*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)
/b^(1/2))/b^(3/2)/d+1/2*e*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c
))^(1/2)/b^(1/2))/b^(3/2)/d/exp(2*a/b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(155) = 310.

Time = 4.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.03

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \frac{e\left(-2ce^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right) + e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}}}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e*((-2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b) + (
Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/
b) - 2*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + S
inh[a/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]
*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*(c*E^((2*a)/b)*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] - c*Sqrt[-((a + b*ArcCo
sh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^(a/b)*Sinh[
2*ArcCosh[c + d*x]]))/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(2*b^(3/2)*
d)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6411, 27, 6300, 25, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6300}$$

$$\frac{e\left(-\frac{2\int -\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{e\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{e\left(-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{d}$$

$$\Big\downarrow \ 3788$$

$$\frac{e\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\left(\frac{1}{2}i\int \frac{ie^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int -\frac{ie^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+\cdots\right)}{b^2}\right)}{d}$$

$$\Big\downarrow \ 26$$

$$\frac{e\left(-\frac{2\left(-\frac{1}{2}\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c\cdots}{b\sqrt{a+b\text{arccosh}(c+\cdots}}\right)}{d}$$

$$\Big\downarrow \ 2611$$

$$\frac{e\left(-\frac{2\left(-\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\int e^{\frac{2(a+b\text{arccosh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+d\cdots}}{b\sqrt{a+b\text{arccos}\cdots}}\right)}{d}$$

$$\Big\downarrow \ 2633$$

$$\frac{e\left(-\frac{2\left(-\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c}{b\sqrt{a+b\text{arccosh}(c+}}\right.}{d}$$

$\Big\downarrow$ 2634

$$\frac{e\left(-\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

output
```
(e*((-2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*ArcC
osh[c + d*x]]) - (2*(-1/2*(Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*S
qrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*E^((2*a)/b))))/b^2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{dex + ce}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}} dx$$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^{3/2}} dx = e\left(\int \frac{c}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)} \right.$$

$$\left. + \int \frac{dx}{a\sqrt{a + b \operatorname{acosh}(c + dx)} + b\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)} dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
e*(Integral(c/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))
*acosh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt
(a + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

Maxima [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{3/2}}\,dx = \int \frac{dex + ce}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{3/2}}\,dx = \int \frac{dex + ce}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{3/2}}\,dx = \int \frac{c\,e + d\,e\,x}{(a + b\operatorname{acosh}(c + d\,x))^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\text{too large to display}$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^(3/2),x)
```

output
```
(e*(acosh(c + d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**
2*b**2*c**2 + 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2
*x**2 - acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d
*x)*a*b*c*d*x + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a*
*2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*d**4 + 2*acosh(c +
 d*x)*int((sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**2*b**2*c**2 +
 2*acosh(c + d*x)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh
(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x
 + 2*acosh(c + d*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a
**2*c*d*x + a**2*d**2*x**2 - a**2),x)*b**2*c*d**3 + acosh(c + d*x)*int((sq
rt(acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)**2*b**2*c**2 + 2*acosh(c + d*x
)**2*b**2*c*d*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*x)**2*b**
2 + 2*acosh(c + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*acosh(c + d
*x)*a*b*d**2*x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d*x + a**2
*d**2*x**2 - a**2),x)*b**2*c**2*d**2 - acosh(c + d*x)*int((sqrt(acosh(c +
d*x)*b + a)*x)/(acosh(c + d*x)**2*b**2*c**2 + 2*acosh(c + d*x)**2*b**2*c*d
*x + acosh(c + d*x)**2*b**2*d**2*x**2 - acosh(c + d*x)**2*b**2 + 2*acosh(c
 + d*x)*a*b*c**2 + 4*acosh(c + d*x)*a*b*c*d*x + 2*acosh(c + d*x)*a*b*d**2*
x**2 - 2*acosh(c + d*x)*a*b + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 -
a**2),x)*b**2*d**2 + 2*acosh(c + d*x)*int((sqrt(c + d*x + 1)*sqrt(c + d...
```

3.102 $\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}} \, dx = -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\sqrt{a+b\text{arccosh}(c+dx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

output
```
-2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)+exp(a/b)
*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+Pi^(1/2)*erfi(
(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d/exp(a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}} \, dx = \frac{e^{-\frac{a}{b}}\left(-2e^{a/b}\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx) - e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}\right.\right.}{bd\sqrt{a+b\text{arcc}}}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

output
```
(-2*E^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - E^((2*a)/b)
*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] + Sqrt[-(
(a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/(b
*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6410, 6295, 6368, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6295}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{6368}$$

$$\frac{\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \text{3788}$$

$$\frac{-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}}{d}$$

$$\Big\downarrow \; 2611$$

$$\frac{\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}+\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}}{d}$$

$$\Big\downarrow \; 2633$$

$$\frac{\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}}{d}$$

$$\Big\downarrow \; 2634$$

$$\frac{\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

output
```
((-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*ArcCosh[c + d*x]]
) + (2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]
])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*E^
(a/b))))/b^2)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arccosh}(c+dx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{acosh}\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*acosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*acosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{acosh\left(dx+c\right)b+a}}{acosh\left(dx+c\right)^{2}b^{2}+2acosh\left(dx+c\right)ab+a^{2}}dx$$

input
```
int(1/(a+b*acosh(d*x+c))^(3/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*
a*b + a**2),x)
```

3.103 $\int \frac{1}{(ce+dex)(a+b\,\text{arccosh}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\,\text{arccosh}(c+dx))^{3/2}}\,dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\,\text{arccosh}(c+dx))^{3/2}},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^(3/2),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\,\text{arccosh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\,\text{arccosh}(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{de}$$

$$\Big\downarrow \text{ 6303}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex + ce)\left(a + b\,\mathrm{arccosh}\left(dx + c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce + dex)(a + b\,\mathrm{arccosh}(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.52

$$\int \frac{1}{(ce + dex)(a + b\,\mathrm{arccosh}(c + dx))^{3/2}}\,dx = \frac{\int \frac{1}{ac\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+adx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+bc\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c}}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
```

output
```
Integral(1/(a*c*sqrt(a + b*acosh(c + d*x)) + a*d*x*sqrt(a + b*acosh(c + d*
x)) + b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b*d*x*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)), x)/e
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(c\,e+d\,e\,x\right)\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.28

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{3/2}}\,dx=\frac{\int \frac{\sqrt{acosh(dx+c)b+a}}{acosh(dx+c)^{2} b^{2} c+acosh(dx+c)^{2} b^{2} dx+2acosh(dx+c)abc+2acosh(dx+c)abdx+a}}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^(3/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)**2*b**2*c + acosh(c + d*x)*
*2*b**2*d*x + 2*acosh(c + d*x)*a*b*c + 2*acosh(c + d*x)*a*b*d*x + a**2*c +
 a**2*d*x),x)/e
```

3.104 $\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 444

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx &= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^{3/2}} \\ &+\frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\text{arccosh}(c+dx)}} \\ &-\frac{e^4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} \\ &-\frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} \\ &+\frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} \\ &+\frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}\end{aligned}$$

output
```
-2/3*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^(3/2)+16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-20/3*e^4*(d*x+
c)^5/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-1/12*e^4*exp(a/b)*Pi^(1/2)*erf((a+b*
arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d-3/8*e^4*exp(3*a/b)*3^(1/2)*Pi^(1/
2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d-5/24*e^4*exp(
5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^
(5/2)/d+1/12*e^4*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)
/d/exp(a/b)+3/8*e^4*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d/exp(3*a/b)+5/24*e^4*5^(1/2)*Pi^(1/2)*erfi(5^(1/2)*(a
+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d/exp(5*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 2.22 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.39

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \frac{e^4e^{-5\left(\frac{a}{b}+\text{arccosh}(c+dx)\right)}\left(-10\sqrt{5}be^{5\text{arccosh}(c+dx)}\left(-\frac{a+b\text{arccosh}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^4*(-10*Sqrt[5]*b*E^(5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))
^(3/2)*Gamma[1/2, (-5*(a + b*ArcCosh[c + d*x]))/b] - 18*Sqrt[3]*b*E^((2*a)
/b + 5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2,
(-3*(a + b*ArcCosh[c + d*x]))/b] + 2*E^(4*(a/b + ArcCosh[c + d*x]))*(2*E^(
(2*a)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c
+ d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c + d
*x]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCosh[c
 + d*x]))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*ArcCo
sh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])) + 3*E^(
(5*a)/b + 2*ArcCosh[c + d*x])*(b - 6*a*(1 + E^(6*ArcCosh[c + d*x])) - 6*b*
ArcCosh[c + d*x] - b*E^(6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*S
qrt[3]*E^(3*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*
ArcCosh[c + d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]) + 2*E^((5*a)
/b)*(-1/2*(b*(-1 + E^(10*ArcCosh[c + d*x]))) - 5*(1 + E^(10*ArcCosh[c + d*
x]))*(a + b*ArcCosh[c + d*x]) + 5*Sqrt[5]*E^(5*(a/b + ArcCosh[c + d*x]))*S
qrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*
ArcCosh[c + d*x]))/b])))/(48*b^2*d*E^(5*(a/b + ArcCosh[c + d*x]))*(a + b*A
rcCosh[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6411, 27, 6301, 6366, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 6411$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\downarrow 6301$$

$$\frac{e^4\left(-\frac{8\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{10\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+\cdots}}{3b(a+b\text{arccos}\cdots}\cdots\right.}{d}$$

$$\downarrow 6366$$

$$\frac{e^4\left(-\frac{8\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{10\left(\frac{10\int \frac{(c+dx)^4}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^5}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\cdots\right.}{d}$$

$$\downarrow 6302$$

$$\frac{e^4\left(\frac{10\left(\frac{10\int -\frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^5}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{8\left(\frac{6\int -\frac{\cos\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\downarrow 25$$

$$\frac{e^4\left(\frac{10\left(-\frac{10\int \frac{\cosh^4\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^5}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{8\left(-\frac{6\int \frac{\text{co}\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\downarrow 5971$$

$$e^4\left(\frac{10\left(-\frac{10\int\left(\frac{\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{8\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+d}{b^2}\right.}{3b}\right.$$

$\downarrow$ 2009

$$e^4\left(-\frac{8\left(\frac{6\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right.}{b^2}\right.}{3b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^4*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*(a + b*Ar
cCosh[c + d*x])^(3/2)) - (8*((-2*(c + d*x)^3)/(b*Sqrt[a + b*ArcCosh[c + d*
x]]) + (6*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]
/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c +
d*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b
*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/(3*b) + (10*((-2*(c
+ d*x)^5)/(b*Sqrt[a + b*ArcCosh[c + d*x]]) + (10*(-1/16*(Sqrt[b]*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 - (Sqrt
[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(
16*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/b^2))/(3*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\operatorname{arccosh}(dx+c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = e^4\left(\int \frac{c^4}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+\right.$$
$$+\int \frac{d^4x^4}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$
$$+\int \frac{4cd^3x^3}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$
$$+\int \frac{6c^2d^2x^2}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$
$$+\int \frac{4c^3dx}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
e**4*(Integral(c**4/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*ac
osh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*
sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*acosh(c
 + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a +
 b*acosh(c + d*x))*acosh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**
2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c +
d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x) + Integral(4
*c**3*d*x/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*
x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x
))
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\text{too large to display}$$

input
```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
(e**4*(3*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**6)/(acosh(c
+ d*x)**3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b
**3*d**2*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 +
 6*acosh(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 -
 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d
*x)*a**2*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a*
*2*b + a**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*d**7 + 18
*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**5)/(acosh(c + d*x)**
3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2
*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh
(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh
(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2
*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a
**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*c*d**6 + 45*acosh
(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)**3*b**3
*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2*x**2
- acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh(c + d
*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh(c + d
*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2*b*c*d
*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a**3...
```

3.105 $\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 333

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx &= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^{3/2}} \\ &+ \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\text{arccosh}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\text{arccosh}(c+dx)}} \\ &- \frac{2e^3e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} \\ &+ \frac{2e^3e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}\end{aligned}$$

output
```
-2/3*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^(3/2)+4*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^
4/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-2/3*e^3*exp(4*a/b)*Pi^(1/2)*erf(2*(a+b*
arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d-1/3*e^3*exp(2*a/b)*2^(1/2)*Pi^(1/
2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+2/3*e^3*Pi^(1
/2)*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d/exp(4*a/b)+1/3*e^
3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2
)/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.17

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = \frac{e^3e^{-4\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)}\left(-16be^{4\operatorname{arccosh}(c+dx)}\left(-\frac{a+b\operatorname{arccosh}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^3*(-16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*G
amma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] - 8*Sqrt[2]*b*E^((2*a)/b + 4*Ar
cCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a +
b*ArcCosh[c + d*x]))/b] + E^((4*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))^2*(b
*(-1 + E^(4*ArcCosh[c + d*x])) + 8*a*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*Ar
cCosh[c + d*x])) + 8*b*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh[c + d*x]
))*ArcCosh[c + d*x])) + 8*Sqrt[2]*E^((2*a)/b + 4*ArcCosh[c + d*x])*Sqrt[a/
b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*ArcCos
h[c + d*x]))/b] + 16*E^(4*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c +
 d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b]
)))/(24*b^2*d*E^(4*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2
))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.31, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6411, 27, 6301, 6366, 6302, 25, 5971, 27, 2009, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx$$

$\big\downarrow$ 6411

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$\big\downarrow$ 27

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$\big\downarrow$ 6301

$$\frac{e^3\left(-\frac{2\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{b}+\frac{8\int \frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+d}}{3b(a+b\text{arccosh}}\right.}{d}$$

$\big\downarrow$ 6366

$$\frac{e^3\left(-\frac{2\left(\frac{4\int \frac{c+dx}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}+\frac{8\left(\frac{8\int \frac{(c+dx)^3}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^4}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\right.}{d}$$

$\big\downarrow$ 6302

$$\frac{e^3\left(\frac{8\left(\frac{8\int -\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^4}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2\left(\frac{4\int -\frac{\cosh}{}}{}\right.}{}\right.}{d}$$

$\big\downarrow$ 25

$$e^3\left(\frac{8\left(-\frac{8\int\frac{\cosh^3\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^4}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2\left(-\frac{4\int\frac{\cosh}{}}{}\right.}{}\right.}{e}$$

$\big\downarrow$ 5971

$$e^3\left(-\frac{2\left(-\frac{4\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}+\frac{8\left(-\frac{8\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+}{b}\right.}{8\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{}\right.}{}\right.}{d}$$

$\big\downarrow$ 27

$$e^3\left(-\frac{2\left(-\frac{2\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}+\frac{8\left(-\frac{8\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arccosh}(c+}{b}\right.}{8\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{}\right.}{}\right.}{d}$$

$\big\downarrow$ 2009

$$e^3\left(-\frac{2\left(-\frac{2\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}+\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcco}\cdots}}{\sqrt{b}}\right.\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 3042

$$e^3\left(-\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\int-\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{b}+\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arc}\cdots}}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 26

$$e^3\left(-\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{b}+\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcc}\cdots}}{\sqrt{b}}\right.\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 3789

$$e^3\left(-\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}\right.}{b}\right.$$

$$\Big\downarrow \text{2611}$$

$$e^3\left(-\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{2(a+b\text{arccosh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right.}{b^2}\right.}{b}\right.$$

$$\Big\downarrow \text{2633}$$

$$e^3\left(-\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right.}{b}\right.$$

$$\Big\downarrow \text{2634}$$

$$e^3\left(-\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{b}+\frac{8}{\cdots}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^3*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*(a + b*Ar
cCosh[c + d*x])^(3/2)) - (2*((-2*(c + d*x)^2)/(b*Sqrt[a + b*ArcCosh[c + d*
x]]) + ((2*I)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sq
rt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)))/b^2))/b + (8*((-2*(c +
 d*x)^4)/(b*Sqrt[a + b*ArcCosh[c + d*x]]) + (8*(-1/32*(Sqrt[b]*E^((4*a)/b)
*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]) - (Sqrt[b]*E^((2*
a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/8 +
(Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^((
4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]])/(8*E^((2*a)/b))))/b^2))/(3*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arccosh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = e^3\left(\int \frac{c^3}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+\right.$$
$$+\int \frac{d^3x^3}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$
$$+\int \frac{3cd^2x^2}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$
$$+\int \frac{3c^2dx}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}}$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
e**3*(Integral(c**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*ac
osh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*
sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*acosh(c
 + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a +
 b*acosh(c + d*x))*acosh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt
(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) +
 b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{acosh}\,(c+d\,x))^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
(e**3*(3*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**5)/(acosh(c
+ d*x)**3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b
**3*d**2*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 +
 6*acosh(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 -
 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d
*x)*a**2*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a*
*2*b + a**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*d**6 + 15
*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)**
3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2
*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh
(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh
(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2
*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a
**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*c*d**5 + 30*acosh
(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**3*b**3
*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2*x**2
- acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh(c + d
*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh(c + d
*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2*b*c*d
*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a**3...
```

3.106 $\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 328

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\text{arccosh}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\text{arccosh}(c+dx)}} - \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}$$

output
```
-2/3*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^(3/2)+8/3*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-4*e^2*(d*x+c)^3/b
^2/d/(a+b*arccosh(d*x+c))^(1/2)-1/6*e^2*exp(a/b)*Pi^(1/2)*erf((a+b*arccosh
(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d-1/2*e^2*exp(3*a/b)*3^(1/2)*Pi^(1/2)*erf(
3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+1/6*e^2*Pi^(1/2)*erf
i((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d/exp(a/b)+1/2*e^2*3^(1/2)*P
i^(1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d/exp(3*a
/b)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.19

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = \frac{e^2 e^{-3\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)}\left(2e^{\frac{4a}{b}+3\operatorname{arccosh}(c+dx)}\sqrt{\frac{a}{b}+\operatorname{arccosh}(c+dx)}(a+b\operatorname{arc}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^2*(2*E^((4*a)/b + 3*ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a +
 b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 6*Sqrt[3]*b*E^(3
*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a
 + b*ArcCosh[c + d*x]))/b] - 2*b*E^((2*a)/b + 3*ArcCosh[c + d*x])*(-((a +
b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] +
E^((3*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))*(a*(6 - 4*E^(2*ArcCosh[c + d*x
]) + 6*E^(4*ArcCosh[c + d*x])) + b*(-1 + 6*ArcCosh[c + d*x] - 4*E^(2*ArcCo
sh[c + d*x])*ArcCosh[c + d*x] + E^(4*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c +
d*x])))) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c +
 d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]
)))/(12*b^2*d*E^(3*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2
))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.31 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.26, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {6411, 27, 6301, 6366, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634, 6302, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6301$$

$$\frac{e^2\left(-\frac{4\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{2\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2\sqrt{c+dx-1}\sqrt{c+\cdots}}{3b(a+b\text{arccosh}\cdots}\right.}{d}$$

$$\Big\downarrow \; 6366$$

$$\frac{e^2\left(-\frac{4\left(\frac{2\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}-\cdots\right.}{d}$$

$$\Big\downarrow \; 6296$$

$$\frac{e^2\left(-\frac{4\left(\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{\cdots}{b\sqrt{a\cdots}}\right.}{b}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$e^2\left(-\frac{4\left(-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{\ldots}{b\sqrt{a\ldots}}\right)}{b}\ldots\right)$$
$$\overline{\qquad d \qquad}$$

$\downarrow$ 3042

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}+\frac{2\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\ldots\right)}{b}\ldots\right)$$
$$\overline{\qquad d \qquad}$$

$\downarrow$ 26

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}+\frac{2\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{\ldots}{b\ldots}\right)}{b}\ldots\right)$$
$$\overline{\qquad d \qquad}$$

$\downarrow$ 3789

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}\right)}{3b}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 2611

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}\right)}{3b}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 2633

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{3b}+\frac{2}{}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 2634

$$e^2\left(\frac{2\left(\frac{6\int\frac{(c+dx)^2}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+}}{\sqrt{b}}\right.\right.}{}\right.}{3b}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 6302

$$e^2\left(\frac{2\left(\frac{6\int -\frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}-\frac{4\left(-\frac{\cdots}{b\sqrt{a+b\cdots}}\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$e^2\left(\frac{2\left(-\frac{6\int \frac{\cosh^2\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}-\frac{4\left(-\frac{\cdots}{b\sqrt{a+b\cdots}}\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 5971$$

$$e^2\left(\frac{2\left(-\frac{6\int \left(\frac{\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{b}-\frac{4\left(-\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 2009$$

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\operatorname{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{3b}+\frac{2\left(\frac{6\left(-\frac{1}{8}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e^2*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*(a + b*Ar
cCosh[c + d*x])^(3/2)) - (4*((-2*(c + d*x))/(b*Sqrt[a + b*ArcCosh[c + d*x]
]) + ((2*I)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x
]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sq
rt[b]])/E^(a/b)))/b^2))/(3*b) + (2*((-2*(c + d*x)^3)/(b*Sqrt[a + b*ArcCosh
[c + d*x]]) + (6*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCo
sh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex + ce)^2}{(a + b\,\text{arccosh}\,(dx + c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = e^2\Bigg(\int \frac{c^2}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+$$
$$+\int \frac{d^2x^2}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}$$
$$+\int \frac{2cdx}{a^2\sqrt{a+b\,\text{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}\,(c+dx)+b^2\sqrt{a+b\,\text{acosh}\,(c+dx)}\,\text{acosh}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
e**2*(Integral(c**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*ac
osh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*
sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*acosh(c + d*x
)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)**2), x))
```

Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{(dex+ce)^2}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{(dex+ce)^2}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{(a+b\operatorname{acosh}(c+d\,x))^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(ce + dex)^2}{(a + b\text{arccosh}(c + dx))^{5/2}} \, dx = \text{too large to display}$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
(e**2*(3*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c
+ d*x)**3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b
**3*d**2*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 +
 6*acosh(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 -
 3*acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d
*x)*a**2*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a*
*2*b + a**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*d**5 + 12
*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**
3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2
*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh
(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh
(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2
*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a
**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*c*d**4 + 15*acosh
(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**3*b**3
*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2*x**2
- acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh(c + d
*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh(c + d
*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2*b*c*d
*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a**3...
```

3.107 $\int \frac{ce+dex}{(a+b\,\mathbf{arccosh}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 216

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd(a+b\operatorname{arccosh}(c+dx))^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\operatorname{arccosh}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\operatorname{arccosh}(c+dx)}} - \frac{2ee^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2ee^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

output
```
-2/3*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3
/2)+4/3*e/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-8/3*e*(d*x+c)^2/b^2/d/(a+b*arcc
osh(d*x+c))^(1/2)-2/3*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arcco
sh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+2/3*e*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a
+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d/exp(2*a/b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 687 vs. $2(216) = 432$.

Time = 3.79 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.18

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^{5/2}} \, dx = \frac{e\left(4a\sqrt{b}c(c + dx) + 4b^{3/2}c(c + dx)\text{arccosh}(c + dx) - 2\sqrt{b}ce^{-\text{arccosh}(c+dx}\right.}{}$$

input

```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output

```
(e*(4*a*Sqrt[b]*c*(c + d*x) + 4*b^(3/2)*c*(c + d*x)*ArcCosh[c + d*x] - (2*
Sqrt[b]*c*(1 + E^(2*ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x]))/E^ArcCosh
[c + d*x] - 4*a*Sqrt[b]*Cosh[2*ArcCosh[c + d*x]] - 4*b^(3/2)*ArcCosh[c + d
*x]*Cosh[2*ArcCosh[c + d*x]] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)
*Cosh[a/b]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - 2*Sqrt[2*Pi]*(a + b
*ArcCosh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]] - 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cosh[a/b]*E
rfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 2*Sqrt[2*Pi]*(a + b*ArcCosh[c
+ d*x])^(3/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]] + 2*Sqrt[b]*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c
 + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - (2*b^(3/2)*c*(-((a + b*ArcCo
sh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/E^(a/b)
+ 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erf[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]]*Sinh[a/b] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 2*Sqrt[2*Pi]*(a + b*Arc
Cosh[c + d*x])^(3/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*S
inh[(2*a)/b] - 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[(Sqrt[2]*S
qrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - b^(3/2)*Sinh[2*ArcCo
sh[c + d*x]]))/(3*b^(5/2)*d*(a + b*ArcCosh[c + d*x])^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6411, 27, 6301, 6308, 6366, 6302, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6301}$$

$$\frac{e\left(-\frac{2\int \frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\operatorname{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{4\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\operatorname{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx}}{3b(a+b\operatorname{arccosh}(}\right.}{d}$$

$$\Big\downarrow \text{6308}$$

$$\frac{e\left(\frac{4\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\operatorname{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{4}{3b^2\sqrt{a+b\operatorname{arccosh}(c+dx)}}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{3b(a+b\operatorname{arccosh}(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{6366}$$

$$\frac{e\left(\frac{4\left(\frac{4\int \frac{c+dx}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{d}$$

$\downarrow$ 6302

$$\frac{e\left(\frac{4\left(\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arcc}}}\right.}{d}$$

$\downarrow$ 25

$$\frac{e\left(\frac{4\left(-\frac{4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arcc}}}\right.}{d}$$

$\downarrow$ 5971

$$\frac{e\left(\frac{4\left(-\frac{4\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\sqrt{c+dx-1}}{3b(a+b\text{arc}}\right.}{d}$$

$\downarrow$ 27

$$\frac{e\left(\frac{4\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\sqrt{c+dx-1}}{3b(a+b\text{arc}}\right.}{d}$$

$\downarrow$ 3042

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\sqrt{c+dx}}{3b(a+b\text{a}}\right.}{d}$$

$\downarrow$ 26

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}+\frac{4}{3b^2\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\sqrt{c+dx-}}{3b(a+b\text{ar}}\right.}{d}$$

$\downarrow$ 3789

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int\frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}\right)}{3b}+\right.}{d}$$

$$\Big\downarrow \text{2611}$$

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{2(a+b\text{arccosh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}\right.}{3b}\right.}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{3b}\right.}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{3b}+\frac{}{3b^2\sqrt{}}\right.}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

output
```
(e*((-2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*(a + b*ArcCos
h[c + d*x])^(3/2)) + 4/(3*b^2*Sqrt[a + b*ArcCosh[c + d*x]]) + (4*((-2*(c +
 d*x)^2)/(b*Sqrt[a + b*ArcCosh[c + d*x]]) + ((2*I)*((I/2)*Sqrt[b]*E^((2*a)
/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - ((I/2
)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/E^((2*a)/b)))/b^2))/(3*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6302
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
 a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{dex + ce}{(a + b\operatorname{arccosh}(dx + c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{5/2}}\,dx = e\left(\int \frac{c}{a^2\sqrt{a + b\operatorname{acosh}(c + dx)} + 2ab\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + a\right.$$
$$+ \int \frac{dx}{a^2\sqrt{a + b\operatorname{acosh}(c + dx)} + 2ab\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx) + b^2\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}}$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
e*(Integral(c/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c
+ d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2
), x) + Integral(d*x/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*a
cosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c +
d*x)**2), x))
```

Maxima [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^{5/2}} \, dx = \int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^{5/2}} \, dx = \int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\text{too large to display}$$

input
```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
(e*(3*acosh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d
*x)**3*b**3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3
*d**2*x**2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*
acosh(c + d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*
acosh(c + d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)
*a**2*b*c*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*
b + a**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*d**4 + 6*aco
sh(c + d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**3*b*
*3*c**2 + 2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2*x**
2 - acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh(c +
 d*x)**2*a*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh(c +
 d*x)**2*a*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2*b*c
*d*x + 3*acosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a**3*
c**2 + 2*a**3*c*d*x + a**3*d**2*x**2 - a**3),x)*b**3*c*d**3 + 3*acosh(c +
d*x)**2*int((sqrt(acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)**3*b**3*c**2 +
2*acosh(c + d*x)**3*b**3*c*d*x + acosh(c + d*x)**3*b**3*d**2*x**2 - acosh(
c + d*x)**3*b**3 + 3*acosh(c + d*x)**2*a*b**2*c**2 + 6*acosh(c + d*x)**2*a
*b**2*c*d*x + 3*acosh(c + d*x)**2*a*b**2*d**2*x**2 - 3*acosh(c + d*x)**2*a
*b**2 + 3*acosh(c + d*x)*a**2*b*c**2 + 6*acosh(c + d*x)*a**2*b*c*d*x + 3*a
cosh(c + d*x)*a**2*b*d**2*x**2 - 3*acosh(c + d*x)*a**2*b + a**3*c**2 + ...
```

3.108 $\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 165

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}} \, dx = -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd(a+b\text{arccosh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arccosh}(c+dx)}} - \frac{2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

output
```
-2/3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)-4/3*(d
*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-2/3*exp(a/b)*Pi^(1/2)*erf((a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+2/3*Pi^(1/2)*erfi((a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))/b^(5/2)/d/exp(a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.33

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\text{arccosh}(c+dx)}{b}}\left(2e^{\frac{2a}{b}+\text{arccosh}(c+dx)}\sqrt{\frac{a}{b}+\text{arccosh}(c+dx)}(a+b\text{arccosh}(\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

output
```
(2*E^((2*a)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcC
osh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh
[c + d*x]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*Arc
Cosh[c + d*x]))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b
*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(
3*b^2*d*E^((a + b*ArcCosh[c + d*x])/b)*(a + b*ArcCosh[c + d*x])^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6410, 6295, 6366, 6296, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6295}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6366}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6296}$$

$$\frac{\frac{2\left(\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\left(-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}}{d}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+d}}\right.}{b^2}\right.}{3b}}{d}$$

$\downarrow$ 2611

$$-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-i\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}\right.}{b^2}\right.}{3b}}{d}$$

$\downarrow$ 2633

$$-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{a}}}{}\right.\right.}{b^2}\right.}{3b}}{d}$$

$\downarrow$ 2634

$$-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccos}}}{\sqrt{b}}\right.\right.}{b^2}\right.}{3b}}{d}$$

input

```
Int[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

output

```
((-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*(a + b*ArcCosh[c + d*x])^(
3/2)) + (2*((-2*(c + d*x))/(b*Sqrt[a + b*ArcCosh[c + d*x]]) + ((2*I)*((I/2
)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - ((I
/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b)))
/b^2))/(3*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6296
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \frac{1}{\left(a + b \operatorname{arccosh}\left(dx + c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\operatorname{acosh}(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*acosh(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*acosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{acosh\left(dx+c\right)b+a}}{acosh\left(dx+c\right)^{3}b^{3}+3acosh\left(dx+c\right)^{2}a\,b^{2}+3acosh\left(dx+c\right)a^{2}b+a^{3}}$$

input
```
int(1/(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)**3*b**3 + 3*acosh(c + d*x)*
*2*a*b**2 + 3*acosh(c + d*x)*a**2*b + a**3),x)
```

3.109 $\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^(5/2),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6303}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\,(a+b\,\operatorname{arccosh}\,(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 67.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.20

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^{5/2}}\,dx = \frac{\int \frac{}{a^2c\sqrt{a+b\operatorname{acosh}(c+dx)}+a^2dx\sqrt{a+b\operatorname{acosh}(c+dx)}+2abc\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{aco}}}{}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

output
```
Integral(1/(a**2*c*sqrt(a + b*acosh(c + d*x)) + a**2*d*x*sqrt(a + b*acosh(
c + d*x)) + 2*a*b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + 2*a*b*d*x*
sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*c*sqrt(a + b*acosh(c + d*
x))*acosh(c + d*x)**2 + b**2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)
**2), x)/e
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.68

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sqrt{acosh(dx+c)b+a}}{acosh(dx+c)^3b^3c+acosh(dx+c)^3b^3dx+3acosh(dx+c)^2a\,b^2c+3acosh(dx+c)^2a\,b^2}}{e}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^(5/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)**3*b**3*c + acosh(c + d*x)*
*3*b**3*d*x + 3*acosh(c + d*x)**2*a*b**2*c + 3*acosh(c + d*x)**2*a*b**2*d*
x + 3*acosh(c + d*x)*a**2*b*c + 3*acosh(c + d*x)*a**2*b*d*x + a**3*c + a**
3*d*x),x)/e
```

3.110 $\int \frac{(ce+dex)^4}{(a+b\,\mathrm{arccosh}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 552

$$
\begin{aligned}
\int \frac{(ce+dex)^4}{(a+b\,\mathrm{arccosh}(c+dx))^{7/2}}\,dx =& -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd(a+b\,\mathrm{arccosh}(c+dx))^{5/2}} \\
&+\frac{16e^4(c+dx)^3}{15b^2d(a+b\,\mathrm{arccosh}(c+dx))^{3/2}}-\frac{4e^4(c+dx)^5}{3b^2d(a+b\,\mathrm{arccosh}(c+dx))^{3/2}} \\
&+\frac{32e^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5b^3d\sqrt{a+b\,\mathrm{arccosh}(c+dx)}} \\
&-\frac{40e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3b^3d\sqrt{a+b\,\mathrm{arccosh}(c+dx)}} \\
&+\frac{e^4e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\
&+\frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\,\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} \\
&+\frac{9e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\
&+\frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\,\mathrm{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\,\mathrm{arccosh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}
\end{aligned}
$$

output
```
-2/5*e^4*(d*x+c-1)^(1/2)*(d*x+c)^4*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^(5/2)+16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-4/3*e^4*(d*x+
c)^5/b^2/d/(a+b*arccosh(d*x+c))^(3/2)+32/5*e^4*(d*x+c-1)^(1/2)*(d*x+c)^2*(
d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)-40/3*e^4*(d*x+c-1)^(1/2)*(
d*x+c)^4*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)+1/30*e^4*exp(a/b
)*Pi^(1/2)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+9/20*e^4*exp(
3*a/b)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^
(7/2)/d+5/12*e^4*exp(5*a/b)*5^(1/2)*Pi^(1/2)*erf(5^(1/2)*(a+b*arccosh(d*x+
c))^(1/2)/b^(1/2))/b^(7/2)/d+1/30*e^4*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))/b^(7/2)/d/exp(a/b)+9/20*e^4*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a
+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d/exp(3*a/b)+5/12*e^4*5^(1/2)*Pi
^(1/2)*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d/exp(5*a/
b)
```

Mathematica [A] (warning: unable to verify)

Time = 3.06 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.18

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \frac{e^4\left(-4\left(3b^2\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)+e^{-\text{arccosh}(c+dx)}(a+b\text{arccosh}(c+dx\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^4*(-4*(3*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + ((a + b
*ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c
 + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2,
a/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] + ((a + b*ArcCosh[c + d*x])*(
E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a +
b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/
E^(a/b)) - 9*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c +
 d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b)
+ (2*(b + 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(
6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCo
sh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[
1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 5*(a + b*
ArcCosh[c + d*x])*((2*(b + 10*a*(-1 + E^(10*ArcCosh[c + d*x])) - 10*b*ArcC
osh[c + d*x] + b*E^(10*ArcCosh[c + d*x])*(1 + 10*ArcCosh[c + d*x])))/E^(5*
ArcCosh[c + d*x]) + (20*Sqrt[5]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Ga
mma[1/2, (-5*(a + b*ArcCosh[c + d*x]))/b])/E^((5*a)/b) + 20*Sqrt[5]*E^((5*
a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*
(a + b*ArcCosh[c + d*x]))/b]) - 18*b^2*Sinh[3*ArcCosh[c + d*x]] - 6*b^2*Si
nh[5*ArcCosh[c + d*x]]))/(240*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.61 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6411, 27, 6301, 6366, 6300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$\Big\downarrow$ 6301

$$e^4\left(-\frac{8\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{2\int \frac{(c+dx)^5}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{b}-\frac{2\sqrt{c+dx-1}\sqrt{c+d}}{5b(a+b\text{arccosh}}\right.}{d}$$

$\Big\downarrow$ 6366

$$e^4\left(-\frac{8\left(\frac{2\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2(c+dx)^3}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{2\left(\frac{10\int \frac{(c+dx)^4}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^5}{3b(a+b\text{arccosh}(c-}\right.}{b}\right.}{d}$$

$\Big\downarrow$ 6300

$$e^4\left(-\frac{8\left(\frac{2\left(-\frac{2\int\left(-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arccosh}(c+dx)}}\right)d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{b}\right.}{5b}\right.$$

$\Big\downarrow$ 2009

$$e^4\left(-\frac{8\left(\frac{2\left(-\frac{2\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(c+}{\sqrt{b}}\right.}{b^2}\right.}{b}\right.}{5b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^4*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(5*b*(a + b*Ar
cCosh[c + d*x])^(5/2)) - (8*((-2*(c + d*x)^3)/(3*b*(a + b*ArcCosh[c + d*x]
)^(3/2)) + (2*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*Sq
rt[a + b*ArcCosh[c + d*x]]) - (2*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(S
qrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*E
rfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2
))/b))/(5*b) + (2*((-2*(c + d*x)^5)/(3*b*(a + b*ArcCosh[c + d*x])^(3/2)) +
 (10*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*
ArcCosh[c + d*x]]) - (2*(-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]) - (3*Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[5*
Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) - (3*Sqrt[b
]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*E^(
(3*a)/b)) - (Sqrt[b]*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]])/(32*E^((5*a)/b))))/b^2))/(3*b)))/b))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{7}{2}}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arccosh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arccosh}(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{acosh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input

```
int((d*e*x+c*e)^4/(a+b*acosh(d*x+c))^(7/2),x)
```

output

```
(e**4*(5*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**6)/(acosh(c
+ d*x)**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b
**4*d**2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 +
 8*acosh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 -
 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acos
h(c + d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 -
6*acosh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c +
 d*x)*a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*
a**3*b + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*d**7 +
30*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**5)/(acosh(c + d*x)
**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d*
*2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 + 8*aco
sh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 - 4*aco
sh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acosh(c +
d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 - 6*acos
h(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c + d*x)*
a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*a**3*b
 + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*c*d**6 + 75*a
cosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)**4*
b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d**...
```

3.111 $\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 441

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx &= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd(a+b\text{arccosh}(c+dx))^{5/2}} \\
&+\frac{4e^3(c+dx)^2}{5b^2d(a+b\text{arccosh}(c+dx))^{3/2}}-\frac{16e^3(c+dx)^4}{15b^2d(a+b\text{arccosh}(c+dx))^{3/2}} \\
&+\frac{16e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5b^3d\sqrt{a+b\text{arccosh}(c+dx)}} \\
&-\frac{128e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{15b^3d\sqrt{a+b\text{arccosh}(c+dx)}} \\
&+\frac{16e^3e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4e^3e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\
&+\frac{16e^3e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\
&+\frac{4e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}\end{aligned}$$

output
```
-2/5*e^3*(d*x+c-1)^(1/2)*(d*x+c)^3*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^(5/2)+4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-16/15*e^3*(d*x+
c)^4/b^2/d/(a+b*arccosh(d*x+c))^(3/2)+16/5*e^3*(d*x+c-1)^(1/2)*(d*x+c)*(d*
x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)-128/15*e^3*(d*x+c-1)^(1/2)*(
d*x+c)^3*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)+16/15*e^3*exp(4*
a/b)*Pi^(1/2)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+4/15*e^3
*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2
))/b^(7/2)/d+16/15*e^3*Pi^(1/2)*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))
/b^(7/2)/d/exp(4*a/b)+4/15*e^3*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(a+b*arccosh(
d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d/exp(2*a/b)
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.01

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arccosh}(c+dx))^{7/2}}\,dx = \frac{e^3\left(-4e^{-4\left(\frac{a}{b}+\operatorname{arccosh}(c+dx)\right)}(a+b\operatorname{arccosh}(c+dx))\left(16be^{4\operatorname{arccosh}(c+dx)}\left(-\frac{a}{}\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^3*((-4*(a + b*ArcCosh[c + d*x])*(16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*
ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] +
E^((4*a)/b)*(b + 8*a*(-1 + E^(8*ArcCosh[c + d*x])) - 8*b*ArcCosh[c + d*x]
+ b*E^(8*ArcCosh[c + d*x])*(1 + 8*ArcCosh[c + d*x]) + 16*E^(4*(a/b + ArcCo
sh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[
1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/E^(4*(a/b + ArcCosh[c + d*x])) - 2
*((a + b*ArcCosh[c + d*x])*((2*(b + 4*a*(-1 + E^(4*ArcCosh[c + d*x])) - 4*
b*ArcCosh[c + d*x] + b*E^(4*ArcCosh[c + d*x])*(1 + 4*ArcCosh[c + d*x])))/E
^(2*ArcCosh[c + d*x]) + (8*Sqrt[2]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)
*Gamma[1/2, (-2*(a + b*ArcCosh[c + d*x]))/b])/E^((2*a)/b) + 8*Sqrt[2]*E^((
2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (
2*(a + b*ArcCosh[c + d*x]))/b]) + 3*b^2*Sinh[2*ArcCosh[c + d*x]]) - 3*b^2*
Sinh[4*ArcCosh[c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {6411, 27, 6301, 6366, 6300, 25, 2009, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6301}$$

$$\frac{e^3\left(-\frac{6\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{8\int \frac{(c+dx)^4}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+d}}{5b(a+b\text{arccosh}}\right.}{d}$$

$$\Big\downarrow \text{6366}$$

$$\frac{e^3\left(-\frac{6\left(\frac{4\int \frac{c+dx}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{8\left(\frac{8\int \frac{(c+dx)^3}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^4}{3b(a+b\text{arccosh}(c+}\right.}{5b}\right.}{d}$$

$$\Big\downarrow \text{6300}$$

$$e^3\left(-\frac{6\left(\frac{4\left(-\frac{2\int -\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}d(a+b\operatorname{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\operatorname{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\operatorname{arccosh}(c+dx))^{3/2}}\right)}{5b}+ \cdots 8 \cdots\right.$$

$\downarrow$ 25

$$e^3\left(-\frac{6\left(\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{8\left(\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2009

$$e^3\left(-\frac{6\left(\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{8\left(\cdots\right.}{\cdots}\right.$$

$\Big\downarrow$ 3042

$$e^3\left(-\frac{6\left(-\frac{2(c+dx)^2}{3b(a+b\,\text{arccosh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\,\text{arccosh}(c+dx)}}+\frac{2\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\,\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\,\text{arccosh}(c+dx)}}d(a+b\,\text{arccosh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}\right.$$

$\Big\downarrow$ 3788

$$e^3\left(-\frac{6\left(-\frac{2(c+dx)^2}{3b(a+b\,\text{arccosh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\,\text{arccosh}(c+dx)}}-\frac{2\left(\frac{1}{2}i\int\frac{ie^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\,\text{arccosh}(c+dx)}}d(a+b\,\text{arccosh}(c+dx))-\frac{1}{2}i\int-\frac{ie^{\frac{2(a-}{}}}{\sqrt{a+b\,\text{arc}}}\right.}{b^2}\right.}{3b}\right.}{5b}\right.$$

$\Big\downarrow$ 26

$$e^3\left(-\frac{6\left(\frac{4\left(-\frac{2\left(-\frac{1}{2}\int\frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+\cdots}{b\sqrt{a+b\text{arccosh}(c+\cdots}}\cdots\right)}{3b}\cdots\right)}{5b}\cdots\right.$$

$\downarrow$ 2611

$$e^3\left(-\frac{6\left(\frac{4\left(-\frac{2\left(-\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\int e^{\frac{2(a+b\text{arccosh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx\cdots}}{b\sqrt{a+b\text{arccosh}\cdots}}\cdots\right)}{3b}\cdots\right)}{5b}\cdots\right.$$

$\downarrow$ 2633

$$e^3\left(-\frac{6\left(\frac{4\left(-\frac{2\left(-\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+}{b\sqrt{a+b\text{arccosh}(c+}}\right.}{3b}\right.}{5b}\right.$$

$$\Big\downarrow \textbf{2634}$$

$$e^3\left(-\frac{6\left(\frac{4\left(-\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\right.}{5b}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^3*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(5*b*(a + b*Ar
cCosh[c + d*x])^(5/2)) - (6*((-2*(c + d*x)^2)/(3*b*(a + b*ArcCosh[c + d*x]
)^(3/2)) + (4*((-2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*Sqrt
[a + b*ArcCosh[c + d*x]]) - (2*(-1/2*(Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi/2]*Erfi
[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*E^((2*a)/b))))/b^2))/
(3*b)))/(5*b) + (8*((-2*(c + d*x)^4)/(3*b*(a + b*ArcCosh[c + d*x])^(3/2))
+ (8*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*
ArcCosh[c + d*x]]) - (2*(-1/8*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]]) - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi]*Erfi[(
2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*E^((4*a)/b)) - (Sqrt[b]*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)
)))/b^2))/(3*b)))/(5*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex+ce)^3}{(a+b\,\operatorname{arccosh}(dx+c))^{\frac{7}{2}}}dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{acosh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int((d*e*x+c*e)^3/(a+b*acosh(d*x+c))^(7/2),x)
```

output

```
(e**3*(5*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**5)/(acosh(c
+ d*x)**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b
**4*d**2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 +
 8*acosh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 -
 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acos
h(c + d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 -
6*acosh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c +
 d*x)*a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*
a**3*b + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*d**6 +
25*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c + d*x)
**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d*
*2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 + 8*aco
sh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 - 4*aco
sh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acosh(c +
d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 - 6*acos
h(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c + d*x)*
a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*a**3*b
 + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*c*d**5 + 50*a
cosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)**4*
b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d**...
```

3.112 $\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 431

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = &-\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd(a+b\text{arccosh}(c+dx))^{5/2}} \\ &+\frac{8e^2(c+dx)}{15b^2d(a+b\text{arccosh}(c+dx))^{3/2}}-\frac{4e^2(c+dx)^3}{5b^2d(a+b\text{arccosh}(c+dx))^{3/2}} \\ &+\frac{16e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15b^3d\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{24e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5b^3d\sqrt{a+b\text{arccosh}(c+dx)}} \\ &+\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} \\ &+\frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}\end{aligned}$$

output
```
-2/5*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c)
)^(5/2)+8/15*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^
3/b^2/d/(a+b*arccosh(d*x+c))^(3/2)+16/15*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)-24/5*e^2*(d*x+c-1)^(1/2)*(d*x+c)^2*(d*
x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)+1/15*e^2*exp(a/b)*Pi^(1/2)*e
rf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+3/5*e^2*exp(3*a/b)*3^(1/2
)*Pi^(1/2)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+1/15*
e^2*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d/exp(a/b)+3
/5*e^2*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b
^(7/2)/d/exp(3*a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.87 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.05

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \frac{e^2\left(-6b^2\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)-2e^{-\text{arccosh}(c+dx)}(a+b\text{arccosh}(c+dx))\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^2*(-6*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*
ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c
+ d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a
/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*(a + b*ArcCosh[c + d*x])*
(E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a +
 b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))
/E^(a/b) - 3*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c +
 d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b)
+ (2*(b + 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(
6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCo
sh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[
1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 6*b^2*Sin
h[3*ArcCosh[c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.91 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {6411, 27, 6301, 6366, 6295, 6300, 2009, 6368, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6301}$$

$$\frac{e^2\left(-\frac{4\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{6\int \frac{(c+dx)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+d}}{5b(a+b\text{arccosh}}\right.}{d}$$

$$\Big\downarrow \text{ 6366}$$

$$\frac{e^2\left(-\frac{4\left(\frac{2\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\int \frac{(c+dx)^2}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2(c+dx)^3}{3b(a+b\text{arccosh}(c+}\right.}{5b}\right.}{d}$$

$$\Big\downarrow \text{ 6295}$$

$$\frac{e^2\left(\frac{6\left(\frac{2\int \frac{(c+dx)^2}{(a+b\operatorname{arccosh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2(c+dx)^3}{3b(a+b\operatorname{arccosh}(c+dx))^{3/2}}\right)}{5b}-\frac{4\left(\frac{2\left(\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\operatorname{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{\cdots}}{b\sqrt{a\cdots}}\right.}{3b}\right.}{5b}\right.}{d}$$

$\downarrow$ 6300

$$e^2\left(\frac{6\left(\frac{2\left(-\frac{2\int\left(-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\operatorname{arccosh}(c+dx))}{b}\right)}{4\sqrt{a+b\operatorname{arccosh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\operatorname{arccosh}(c+dx)}{b}\right)}{4\sqrt{a+b\operatorname{arccosh}(c+dx)}}\right)d(a+b\operatorname{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{b\sqrt{a+b\operatorname{arccosh}(c+dx)}}\right)}{b}\right.}{5b}\right.\cdots$$

$\downarrow$ 2009

$$e^2\left(-\frac{4\left(\frac{2\left(\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\left(-\frac{2\left(-\frac{1}{8}\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 6368

$$e^2\left(-\frac{4\left(\frac{2\left(\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\left(-\frac{2\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{3b(a+b\,\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\,\text{arccosh}(c+dx)}}+\frac{2\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\,\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\,\text{arccosh}(c+dx)}}d(a+b\,\text{arccosh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}+-\ \cdots 6 \cdots\right.$$

$\downarrow$ 3788

$$e^2\left(-\frac{4\left(-\frac{2(c+dx)}{3b(a+b\,\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\,\text{arccosh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int-\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\,\text{arccosh}(c+dx)}}d(a+b\,\text{arccosh}(c+dx))-\frac{1}{2}i\int\frac{ie^{-\frac{a-c\cdots}{b}}}{\sqrt{a+b\,\text{arccos}\cdots}}\cdots\right.}{b^2}\cdots\right.}{3b}\cdots\right.}{5b}\cdots\right.$$

$\downarrow$ 26

$$e^2\left(-\frac{4\left(\frac{2\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}\right.}{5b}\right.$$

$\downarrow$ 2611

$$e^2\left(-\frac{4\left(\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}+\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\right.}{5b}\right.$$

$\downarrow$ 2633

$$e^2\left(-\frac{4\left(\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{}{3b(a+b}\right.}{5b}\right.$$

$$\Big\downarrow \color{blue}{2634}$$

$$e^2\left(-\frac{4\left(\frac{2\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c}{3b(a+b\text{arcc}}\right.}{5b}\right.$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e^2*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b*(a + b*Ar
cCosh[c + d*x])^(5/2)) - (4*((-2*(c + d*x))/(3*b*(a + b*ArcCosh[c + d*x])^
(3/2)) + (2*((-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*ArcCo
sh[c + d*x]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqr
t[b]])/(2*E^(a/b))))/b^2))/(3*b)))/(5*b) + (6*((-2*(c + d*x)^3)/(3*b*(a +
b*ArcCosh[c + d*x])^(3/2)) + (2*((-2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
 + c + d*x])/(b*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*(-1/8*(Sqrt[b]*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*E^((3*a)/b)
*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/8 - (Sqrt
[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) - (Sq
rt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*
E^((3*a)/b))))/b^2))/b))/(5*b)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(dex + ce)^2}{(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{7}{2}}}dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^2}{(a + b\text{arccosh}(c + dx))^{7/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ce + dex)^2}{(a + b\text{arccosh}(c + dx))^{7/2}} \, dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ce + dex)^2}{(a + b\text{arccosh}(c + dx))^{7/2}} \, dx = \int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}} \, dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{acosh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\text{too large to display}$$

input
```
int((d*e*x+c*e)^2/(a+b*acosh(d*x+c))^(7/2),x)
```

output
```
(e**2*(5*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**4)/(acosh(c
+ d*x)**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b
**4*d**2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 +
 8*acosh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 -
 4*acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acos
h(c + d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 -
6*acosh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c +
 d*x)*a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*
a**3*b + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*d**5 +
20*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d*x)
**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d*
*2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 + 8*aco
sh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 - 4*aco
sh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acosh(c +
d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 - 6*acos
h(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c + d*x)*
a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*a**3*b
 + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*c*d**4 + 25*a
cosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**4*
b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d**...
```

3.113 $\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 266

$$\begin{aligned}\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx &= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd(a+b\text{arccosh}(c+dx))^{5/2}}\\ &+\frac{4e}{15b^2d(a+b\text{arccosh}(c+dx))^{3/2}}-\frac{8e(c+dx)^2}{15b^2d(a+b\text{arccosh}(c+dx))^{3/2}}\\ &-\frac{32e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{15b^3d\sqrt{a+b\text{arccosh}(c+dx)}}\\ &+\frac{8ee^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8ee^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}\end{aligned}$$

output
```
-2/5*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5
/2)+4/15*e/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*ar
ccosh(d*x+c))^(3/2)-32/15*e*(d*x+c-1)^(1/2)*(d*x+c)*(d*x+c+1)^(1/2)/b^3/d/
(a+b*arccosh(d*x+c))^(1/2)+8/15*e*exp(2*a/b)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*
(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+8/15*e*2^(1/2)*Pi^(1/2)*erfi
(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d/exp(2*a/b)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 916 vs. $2(266) = 532$.

Time = 3.24 (sec) , antiderivative size = 916, normalized size of antiderivative = 3.44

$$\int \frac{ce + dex}{(a + b\text{arccosh}(c + dx))^{7/2}} \, dx = \text{Too large to display}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e*(4*a*b^(3/2)*c*(c + d*x) + 8*a^2*Sqrt[b]*c*Sqrt[(-1 + c + d*x)/(1 + c +
 d*x)]*(1 + c + d*x) + 4*b^(5/2)*c*(c + d*x)*ArcCosh[c + d*x] + 16*a*b^(3/
2)*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x] + 8
*b^(5/2)*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*
x]^2 - 4*a*b^(3/2)*Cosh[2*ArcCosh[c + d*x]] - 4*b^(5/2)*ArcCosh[c + d*x]*C
osh[2*ArcCosh[c + d*x]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh
[a/b]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcC
osh[c + d*x])^(5/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*Erfi[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x
])^(5/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]
] - (2*Sqrt[b]*c*(a + b*ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x]
 + 2*E^(a/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCos
h[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*S
qrt[b]*c*(a + b*ArcCosh[c + d*x])*(E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2
*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2
, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c
 + d*x])^(5/2)*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 4*c*S
qrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/S
qrt[b]]*Sinh[a/b] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Erf[(Sq...
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6411, 27, 6301, 6308, 6366, 6300, 25, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6301}$$

$$\frac{e\left(-\frac{2\int \frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{4\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx\cdots}}{5b(a+b\text{arccosh}(\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \text{ 6308}$$

$$\frac{e\left(\frac{4\int \frac{(c+dx)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{4}{15b^2(a+b\text{arccosh}(c+dx))^{3/2}}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{5b(a+b\text{arccosh}(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{ 6366}$$

$$\frac{e\left(\frac{4\left(\frac{4\int \frac{c+dx}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{4}{15b^2(a+b\text{arccosh}(c+dx))^{3/2}}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{5b(a+b\text{arccosh}(c+dx))^{5/2}}\cdots\right.}{d}$$

$$\Big\downarrow \text{ 6300}$$

$$\frac{e\left(\frac{4\left(\frac{4\left(-\frac{2\int -\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{\cdots}{15b^2\cdots}\right.}{d}$$

$\downarrow$ 25

$$\frac{e\left(\frac{4\left(\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}+\frac{\cdots}{15b^2(a+\cdots}\right.}{d}$$

$\downarrow$ 3042

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}+ \cdots\right)}{d}$$

$\downarrow$ 3788

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}-\frac{2\left(\frac{1}{2}i\int \frac{ie^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))-\frac{1}{2}i\int -\frac{ie^{\frac{2(a-c-\cdots}{b}}}{\sqrt{a+b\text{arccos}\cdots}}\cdots\right)}{b^2}\cdots\right)}{3b}\cdots\right)}{5b}\cdots\right)}{d}$$

$\downarrow$ 26

$$\frac{e\left(\frac{4\left(\frac{4\left(-\frac{2\left(-\frac{1}{2}\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}d(a+b\operatorname{arccosh}(c+dx))-\frac{1}{2}\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\operatorname{arccosh}(c+dx)}}d(a+b\operatorname{arccosh}(c+dx))\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\operatorname{arccosh}(c+dx)}} \cdots}{3b} \cdots}{5b} \cdots\right)}{d \cdots}$$

$\downarrow$ 2611

$$\frac{e\left(\frac{4\left(\frac{4\left(-\frac{2\left(-\int e^{\frac{2a}{b}-\frac{2(a+b\operatorname{arccosh}(c+dx))}{b}}d\sqrt{a+b\operatorname{arccosh}(c+dx)}-\int e^{\frac{2(a+b\operatorname{arccosh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\operatorname{arccosh}(c+dx)}\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c \cdots}{b\sqrt{a+b\operatorname{arccosh}(c+ \cdots}} \cdots}{3b} \cdots}{5b} \cdots\right)}{d \cdots}$$

$\downarrow$ 2633

$$e\left(\frac{4\left(\frac{4\left(-\frac{2\left(-\int e^{\frac{2a}{b}-\frac{2(a+b\text{arccosh}(c+dx))}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right.}{3b}}{5b}\right.}{d}$$

$\downarrow$ 2634

$$e\left(\frac{4\left(\frac{4\left(-\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{\ldots}{3b(\ldots}}{5b}\right.}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

output
```
(e*((-2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b*(a + b*ArcCos
h[c + d*x])^(5/2)) + 4/(15*b^2*(a + b*ArcCosh[c + d*x])^(3/2)) + (4*((-2*(
c + d*x)^2)/(3*b*(a + b*ArcCosh[c + d*x])^(3/2)) + (4*((-2*Sqrt[-1 + c + d
*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*(-1
/2*(Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d
*x]])/Sqrt[b]])/(2*E^((2*a)/b))))/b^2))/(3*b)))/(5*b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6300
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Simp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n +
 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2]
&& LtQ[n, -1]
```

rule 6301
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCosh[c*x
])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[m/(b*c*(n + 1))
 Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])
), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{dex + ce}{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}} dx$$

input
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{7/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{7/2}} dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{dex+ce}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{dex+ce}{(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{c\,e+d\,e\,x}{(a+b\,\text{acosh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arccosh}(c + dx))^{7/2}}\,dx = \text{too large to display}$$

input

```
int((d*e*x+c*e)/(a+b*acosh(d*x+c))^(7/2),x)
```

output

```
(e*(5*acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**3)/(acosh(c + d
*x)**4*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4
*d**2*x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 + 8*
acosh(c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 - 4*
acosh(c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acosh(c
 + d*x)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 - 6*a
cosh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c + d*
x)*a**3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*a**
3*b + a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*d**4 + 10*
acosh(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x**2)/(acosh(c + d*x)**4
*b**4*c**2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d**2*
x**2 - acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)**3*a*b**3*c**2 + 8*acosh(
c + d*x)**3*a*b**3*c*d*x + 4*acosh(c + d*x)**3*a*b**3*d**2*x**2 - 4*acosh(
c + d*x)**3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2*c**2 + 12*acosh(c + d*x
)**2*a**2*b**2*c*d*x + 6*acosh(c + d*x)**2*a**2*b**2*d**2*x**2 - 6*acosh(c
 + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b*c**2 + 8*acosh(c + d*x)*a**
3*b*c*d*x + 4*acosh(c + d*x)*a**3*b*d**2*x**2 - 4*acosh(c + d*x)*a**3*b +
a**4*c**2 + 2*a**4*c*d*x + a**4*d**2*x**2 - a**4),x)*b**4*c*d**3 + 5*acosh
(c + d*x)**3*int((sqrt(acosh(c + d*x)*b + a)*x)/(acosh(c + d*x)**4*b**4*c*
*2 + 2*acosh(c + d*x)**4*b**4*c*d*x + acosh(c + d*x)**4*b**4*d**2*x**2 ...
```

3.114 $\int \frac{1}{(a+b\mathbf{arccosh}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 209

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd(a+b\text{arccosh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\text{arccosh}(c+dx))^{3/2}} - \frac{8\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15b^3d\sqrt{a+b\text{arccosh}(c+dx)}} + \frac{4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

output

```
-2/5*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)-4/15*(
d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-8/15*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)+4/15*exp(a/b)*Pi^(1/2)*erf((a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+4/15*Pi^(1/2)*erfi((a+b*arccosh(d*x+c))
^(1/2)/b^(1/2))/b^(7/2)/d/exp(a/b)
```

Mathematica [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \frac{-6\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)-\frac{2e^{-\text{arccosh}(c+dx)}(a+b\text{arccosh}(c+dx))\left(-2a+b-2b\text{arc}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

output
```
(-6*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*ArcCosh[c
 + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c + d*x])*S
qrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcC
osh[c + d*x]]))/(b^2*E^ArcCosh[c + d*x]) - (2*(a + b*ArcCosh[c + d*x])*(E^
(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*
ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(b
^2*E^(a/b)))/(15*b*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6410, 6295, 6366, 6295, 6368, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 6410}$$

$$\frac{\int \frac{1}{(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6295}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}(a+b\text{arccosh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{5b(a+b\text{arccosh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \ 6366$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{(a+b\text{arccosh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{5b(a+b\text{arccosh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \ 6295$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int \frac{c+dx}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{arccosh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{5b(a+b\text{arccosh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \ 6368$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{5b(a+b\text{arccosh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{5b(a+b\text{arccosh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arccosh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}}{d}$$

$$\Big\downarrow \ 3788$$

$$-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{5b(a+b\text{arccosh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arccosh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}\right.}{}\right.}{3b}\right.}{5b}$$

$$d$$

$\downarrow$ 26

$$\frac{2\left(\frac{2\left(\frac{2\left(\frac{1}{2}\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))+\frac{1}{2}\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arccosh}(c+dx)}}d(a+b\text{arccosh}(c+dx))\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{}{3b(a+}\right.}{5b}$$

$$d$$

$\downarrow$ 2611

$$\frac{2\left(\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}+\int e^{\frac{a+b\text{arccosh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{}{3b(a+b\text{a}}\right.}{5b}$$

$$d$$

$\downarrow$ 2633

$$\frac{2\left(\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arccosh}(c+dx)}{b}}d\sqrt{a+b\text{arccosh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+}{3b(a+b\text{arccos}}\right.}{5b}$$

$$d$$

$\downarrow$ 2634

$$\frac{2\left(\frac{2\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\,\text{arccosh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\,\text{arccosh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{b\sqrt{a+b\,\text{arccosh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\,\text{arccosh}(c+d}\right)}{5b}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

output
```
((-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(5*b*(a + b*ArcCosh[c + d*x])^(
5/2)) + (2*((-2*(c + d*x))/(3*b*(a + b*ArcCosh[c + d*x])^(3/2)) + (2*((-2*
Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*Sqrt[a + b*ArcCosh[c + d*x]]) + (
2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/2
+ (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)
)))/b^2))/(3*b)))/(5*b))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6295
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c
/(b*(n + 1))   Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
 c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

rule 6366
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Simp[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]   Int[
(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e
1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arccosh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\mathrm{arccosh}(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^{7/2}}\, dx = \int \frac{1}{(b\,\text{arcosh}\,(dx + c) + a)^{\frac{7}{2}}}\, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^{7/2}}\, dx = \int \frac{1}{(b\,\text{arcosh}\,(dx + c) + a)^{\frac{7}{2}}}\, dx$$

input
```
integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arccosh}(c + dx))^{7/2}}\, dx = \int \frac{1}{(a + b\,\text{acosh}\,(c + d\,x))^{7/2}}\, dx$$

input
```
int(1/(a + b*acosh(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b*acosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{acosh\,(dx+c)\,b+a}}{acosh\,(dx+c)^4\,b^4+4acosh\,(dx+c)^3\,a\,b^3+6acosh\,(dx+c)^2\,a^2b^2+4}$$

input
```
int(1/(a+b*acosh(d*x+c))^(7/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)**4*b**4 + 4*acosh(c + d*x)*
*3*a*b**3 + 6*acosh(c + d*x)**2*a**2*b**2 + 4*acosh(c + d*x)*a**3*b + a**4
),x)
```

3.115 $\int \frac{1}{(ce+dex)(a+b\,\mathrm{arccosh}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^{7/2}}\,dx=\frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arccosh}(c+dx))^{7/2}},x\right)}{e}$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*arccosh(d*x+c))^(7/2),x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(ce+dex)(a+b\operatorname{arccosh}(c+dx))^{7/2}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{ 6411}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6303}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arccosh}(c+dx))^{7/2}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)(a+b\mathrm{arccosh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ce+dex)(a+b\mathrm{arccosh}(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arcosh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

Mupad [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{acosh}\,(c+d\,x)\right)^{7/2}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 6.08

$$\int \frac{1}{(ce+dex)(a+b\text{arccosh}(c+dx))^{7/2}}\,dx = \int \frac{}{acosh(dx+c)^4 b^4 c + acosh(dx+c)^4 b^4 dx + 4 acosh(dx+c)^3 a\, b^3 c + 4 acosh(dx+c)^3 a\, b^3}$$

input
```
int(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))^(7/2),x)
```

output
```
int(sqrt(acosh(c + d*x)*b + a)/(acosh(c + d*x)**4*b**4*c + acosh(c + d*x)*
*4*b**4*d*x + 4*acosh(c + d*x)**3*a*b**3*c + 4*acosh(c + d*x)**3*a*b**3*d*
x + 6*acosh(c + d*x)**2*a**2*b**2*c + 6*acosh(c + d*x)**2*a**2*b**2*d*x +
4*acosh(c + d*x)*a**3*b*c + 4*acosh(c + d*x)*a**3*b*d*x + a**4*c + a**4*d*
x),x)/e
```

3.116 $\int (ce+dex)^{5/2}(a+b\operatorname{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\begin{aligned}\int (ce+dex)^{5/2}(a+b\operatorname{arccosh}(c+dx))\,dx= & -\frac{20be^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{147d} \\ & -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d} \\ & +\frac{2(e(c+dx))^{7/2}(a+b\operatorname{arccosh}(c+dx))}{7de} \\ & -\frac{20be^{5/2}\sqrt{1-c-dx}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{147d\sqrt{-1+c+dx}}\end{aligned}$$

output
```
-20/147*b*e^2*(d*x+c-1)^(1/2)*(e*(d*x+c))^(1/2)*(d*x+c+1)^(1/2)/d-4/49*b*(
d*x+c-1)^(1/2)*(e*(d*x+c))^(5/2)*(d*x+c+1)^(1/2)/d+2/7*(e*(d*x+c))^(7/2)*(
a+b*arccosh(d*x+c))/d/e-20/147*b*e^(5/2)*(-d*x-c+1)^(1/2)*EllipticF((e*(d*
x+c))^(1/2)/e^(1/2),I)/d/(d*x+c-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))\,dx = \frac{2(e(c+dx))^{5/2}\left((c+dx)^{7/2}(a+b\text{arccosh}(c+dx))+\frac{2b\left(5(1-(c+dx)^2)+3(c+dx)^2(1-(c+\cdots\right.}{\cdots}\right.}{7d(c+dx)^{5/2}}$$

input
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(2*(e*(c + d*x))^(5/2)*((c + d*x)^(7/2)*(a + b*ArcCosh[c + d*x]) + (2*b*(5
*(1 - (c + d*x)^2) + 3*(c + d*x)^2*(1 - (c + d*x)^2) - 5*Sqrt[1 - (c + d*x
)^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2]))/(21*Sqrt[(-1 + c + d*
x)/(c + d*x)]*Sqrt[1 + c + d*x])))/(7*d*(c + d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6411, 6298, 113, 27, 113, 27, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int (e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\int \frac{(e(c+dx))^{7/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{7e}}{d}$$

$$\Big\downarrow \; 113$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\left(\frac{2}{7}\int\frac{5e^2(e(c+dx))^{3/2}}{2\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{7}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{5/2}\right)}{7e}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\left(\frac{5}{7}e^2\int\frac{(e(c+dx))^{3/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{7}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{5/2}\right)}{7e}}{d}$$

$$\Big\downarrow \; 113$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\left(\frac{5}{7}e^2\left(\frac{2}{3}\int\frac{e^2}{2\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)+\frac{2}{7}e\sqrt{c+dx-}\right.}{7e}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)+\frac{2}{7}e\sqrt{c+dx}\right.}{7e}}{d}$$

$$\Big\downarrow \; 127$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{-c-dx+1}\int\frac{1}{\sqrt{-c-dx+1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{3\sqrt{c+dx-1}}+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)+\right.}{7e}}{d}$$

$$\Big\downarrow \; 126$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{7e}-\frac{2b\left(\frac{5}{7}e^2\left(\frac{2e^{3/2}\sqrt{-c-dx+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{3\sqrt{c+dx-1}}+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)+\frac{2}{7}e\right.}{7e}}{d}$$

input

```
Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

output
```
((2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x]))/(7*e) - (2*b*((2*e*Sqrt[
-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqrt[1 + c + d*x])/7 + (5*e^2*((2*e*Sqrt
[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/3 + (2*e^(3/2)*Sqrt[1
- c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(3*Sqrt[-1 +
c + d*x])))/7))/(7*e))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 6.45 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{2a(dex+ce)^{\frac{7}{2}}}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(3\sqrt{-\frac{1}{e}}(dex+ce)^{\frac{9}{2}}+2\sqrt{-\frac{1}{e}}e^2(dex+ce)^{\frac{5}{2}}+5e^4\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+}{e}}\right.}{147e\sqrt{-\frac{1}{e}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{-de}{}}}\right.}{de}$
default	$\frac{\frac{2a(dex+ce)^{\frac{7}{2}}}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(3\sqrt{-\frac{1}{e}}(dex+ce)^{\frac{9}{2}}+2\sqrt{-\frac{1}{e}}e^2(dex+ce)^{\frac{5}{2}}+5e^4\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+}{e}}\right.}{147e\sqrt{-\frac{1}{e}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{-de}{}}}\right.}{de}$
parts	$\frac{2a(dex+ce)^{\frac{7}{2}}}{7de}+\frac{2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(3\sqrt{-\frac{1}{e}}(dex+ce)^{\frac{9}{2}}+2\sqrt{-\frac{1}{e}}e^2(dex+ce)^{\frac{5}{2}}+5e^4\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex+}{e}}\right.}{147e\sqrt{-\frac{1}{e}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{}}\right.}{de}$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/7*a*(d*e*x+c*e)^(7/2)+b*(1/7*(d*e*x+c*e)^(7/2)*arccosh((d*e*x+c*e
)/e)-2/147/e*(3*(-1/e)^(1/2)*(d*e*x+c*e)^(9/2)+2*(-1/e)^(1/2)*e^2*(d*e*x+c
*e)^(5/2)+5*e^4*((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*EllipticF
((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-5*(-1/e)^(1/2)*e^4*(d*e*x+c*e)^(1/2))/(
-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.50

$$\int (ce+dex)^{5/2}(a+b\operatorname{arccosh}(c+dx))\,dx =$$

$$-\frac{2\left(10\sqrt{d^3e}be^2\text{weierstrassPInverse}\left(\frac{4}{d^2},0,\frac{dx+c}{d}\right)-21\left(bd^5e^2x^3+3bcd^4e^2x^2+3bc^2d^3e^2x+bc^3d^2e^2\right)\sqrt{dex}\right.}{}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
-2/147*(10*sqrt(d^3*e)*b*e^2*weierstrassPInverse(4/d^2, 0, (d*x + c)/d) -
21*(b*d^5*e^2*x^3 + 3*b*c*d^4*e^2*x^2 + 3*b*c^2*d^3*e^2*x + b*c^3*d^2*e^2)
*sqrt(d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*(3
*b*d^4*e^2*x^2 + 6*b*c*d^3*e^2*x + (3*b*c^2 + 5*b)*d^2*e^2)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*sqrt(d*e*x + c*e) - 21*(a*d^5*e^2*x^3 + 3*a*c*d^4*e^2*
x^2 + 3*a*c^2*d^3*e^2*x + a*c^3*d^2*e^2)*sqrt(d*e*x + c*e))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^{5/2}(a + b\operatorname{arccosh}(c + dx))\, dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (ce + dex)^{5/2}(a + b\operatorname{arccosh}(c + dx))\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))\,dx=\int (dex+ce)^{\frac{5}{2}}(b\,\text{arcosh}\,(dx+c)+a)\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\text{acosh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))\,dx=\frac{\sqrt{e}\,e^2\left(2\sqrt{dx+c}\,a\,c^3+6\sqrt{dx+c}\,a\,c^2dx+6\sqrt{dx+c}\,ac\,d^2x^2+2\sqrt{dx+c}\,a\,d^3x^3+\right.}{}$$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*acosh(d*x+c)),x)
```

output
```
(sqrt(e)*e**2*(2*sqrt(c + d*x)*a*c**3 + 6*sqrt(c + d*x)*a*c**2*d*x + 6*sqr
t(c + d*x)*a*c*d**2*x**2 + 2*sqrt(c + d*x)*a*d**3*x**3 + 7*int(sqrt(c + d*
x)*acosh(c + d*x)*x**2,x)*b*d**3 + 14*int(sqrt(c + d*x)*acosh(c + d*x)*x,x
)*b*c*d**2 + 7*int(sqrt(c + d*x)*acosh(c + d*x),x)*b*c**2*d))/(7*d)
```

3.117 $\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\begin{aligned}\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx &= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} \\ &+ \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arccosh}(c+dx))}{5de} \\ &+ \frac{12be\sqrt{1-c-dx}\sqrt{e(c+dx)}E\left(\arcsin\left(\frac{\sqrt{1-c-dx}}{\sqrt{2}}\right)\middle|2\right)}{25d\sqrt{-1+c+dx}\sqrt{c+dx}}\end{aligned}$$

output
```
-4/25*b*(d*x+c-1)^(1/2)*(e*(d*x+c))^(3/2)*(d*x+c+1)^(1/2)/d+2/5*(e*(d*x+c)
)^(5/2)*(a+b*arccosh(d*x+c))/d/e+12/25*b*e*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1
/2)*EllipticE(1/2*(-d*x-c+1)^(1/2)*2^(1/2),2^(1/2))/d/(d*x+c-1)^(1/2)/(d*x
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx = \frac{2(e(c+dx))^{3/2}\left(5(c+dx)(a+b\operatorname{arccosh}(c+dx)) - \frac{2b\left(-1+c^2+2cdx+d^2x^2+\sqrt{1-(c+dx)^2}\right.}{\sqrt{-1+c+dx}}\right.}{25d}$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(2*(e*(c + d*x))^(3/2)*(5*(c + d*x)*(a + b*ArcCosh[c + d*x]) - (2*b*(-1 +
c^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4
, 7/4, (c + d*x)^2]))/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(25*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6411, 6298, 113, 27, 124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \ 6411$$

$$\frac{\int (e(c+dx))^{3/2}(a+b\operatorname{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \ 6298$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\operatorname{arccosh}(c+dx))}{5e}-\frac{2b\int \frac{(e(c+dx))^{5/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \ 113$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}\int\frac{3e^2\sqrt{e(c+dx)}}{2\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{5}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}\right)}{5e}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))}{5e}-\frac{2b\left(\frac{3}{5}e^2\int\frac{\sqrt{e(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{5}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}\right)}{5e}}{d}$$

$$\Big\downarrow \text{124}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))}{5e}-\frac{2b\left(\frac{3e^2\sqrt{-c-dx+1}\sqrt{e(c+dx)}\int\frac{\sqrt{2}\sqrt{-c-dx}}{\sqrt{-c-dx+1}\sqrt{c+dx+1}}d(c+dx)}{5\sqrt{2}\sqrt{-c-dx}\sqrt{c+dx-1}}+\frac{2}{5}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}\right)}{5e}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))}{5e}-\frac{2b\left(\frac{3e^2\sqrt{-c-dx+1}\sqrt{e(c+dx)}\int\frac{\sqrt{-c-dx}}{\sqrt{-c-dx+1}\sqrt{c+dx+1}}d(c+dx)}{5\sqrt{-c-dx}\sqrt{c+dx-1}}+\frac{2}{5}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}\right)}{5e}}{d}$$

$$\Big\downarrow \text{123}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))}{5e}-\frac{2b\left(\frac{6e^2\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\arcsin\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{5\sqrt{-c-dx}\sqrt{c+dx-1}}+\frac{2}{5}e\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}\right)}{5e}}{d}$$

input

```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

output

```
((2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x]))/(5*e) - (2*b*((2*e*Sqrt[
-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/5 + (6*e^2*Sqrt[1 - c
 - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])
/(5*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])))/(5*e))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}+3\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\,e^{3}\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\right.\right.}{25e\sqrt{-\frac{1}{e}}\,\sqrt{}}\right.}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}+3\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\,e^{3}\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\right.\right.}{25e\sqrt{-\frac{1}{e}}\,\sqrt{}}\right.}{de}$
parts	$\frac{2a(dex+ce)^{\frac{5}{2}}}{5de}+\frac{2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}+3\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,e^{3}\operatorname{EllipticF}\left(\sqrt{dex+\right.}{25e\sqrt{}}\right.}{de}$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arccosh((d*e*x+c*e
)/e)-2/25/e*((-1/e)^(1/2)*(d*e*x+c*e)^(7/2)+3*((d*e*x+c*e+e)/e)^(1/2)*((-d
*e*x-c*e+e)/e)^(1/2)*e^3*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-3*e^3
*((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*EllipticE((d*e*x+c*e)^(1
/2)*(-1/e)^(1/2),I)-(-1/e)^(1/2)*e^2*(d*e*x+c*e)^(3/2))/(-1/e)^(1/2)/((d*e
*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx = \frac{2\left(6\sqrt{d^3e}be\operatorname{weierstrassZeta}\left(\frac{4}{d^2},0,\operatorname{weierstrassPInverse}\left(\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)+5\left(bd^3ex^2+\right.\right.}{}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
2/25*(6*sqrt(d^3*e)*b*e*weierstrassZeta(4/d^2, 0, weierstrassPInverse(4/d^
2, 0, (d*x + c)/d)) + 5*(b*d^3*e*x^2 + 2*b*c*d^2*e*x + b*c^2*d*e)*sqrt(d*e
*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^2*e*x
+ b*c*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*e*x + c*e) + 5*(a*d^3*
e*x^2 + 2*a*c*d^2*e*x + a*c^2*d*e)*sqrt(d*e*x + c*e))/d^2
```

Sympy [F]

$$\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx = \int (e(c+dx))^{\frac{3}{2}}\,(a+b\operatorname{acosh}(c+dx))\,dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\operatorname{arccosh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (ce + dex)^{3/2}(a + b\operatorname{arccosh}(c + dx))\,dx = \int (dex + ce)^{\frac{3}{2}}(b\operatorname{arcosh}(dx + c) + a)\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^{3/2}(a + b\operatorname{arccosh}(c + dx))\,dx = \int (c\,e + d\,e\,x)^{3/2}\,(a + b\operatorname{acosh}(c + d\,x))\,dx$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int (ce + dex)^{3/2}(a + b\operatorname{arccosh}(c+dx))\,dx = \frac{\sqrt{e}\,e\big(2\sqrt{dx + c}\,a\,c^2 + 4\sqrt{dx + c}\,acdx + 2\sqrt{dx + c}\,a\,d^2x^2 + 5\big(\int \sqrt{dx + c}\,acosh(dx}{5d}$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*acosh(d*x+c)),x)
```

output
```
(sqrt(e)*e*(2*sqrt(c + d*x)*a*c**2 + 4*sqrt(c + d*x)*a*c*d*x + 2*sqrt(c +
d*x)*a*d**2*x**2 + 5*int(sqrt(c + d*x)*acosh(c + d*x)*x,x)*b*d**2 + 5*int(
sqrt(c + d*x)*acosh(c + d*x),x)*b*c*d))/(5*d)
```

3.118 $\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx = -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d}+\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))}{3de} - \frac{4b\sqrt{e}\sqrt{1-c-dx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{9d\sqrt{-1+c+dx}}$$

output
```
-4/9*b*(d*x+c-1)^(1/2)*(e*(d*x+c))^(1/2)*(d*x+c+1)^(1/2)/d+2/3*(e*(d*x+c))
^(3/2)*(a+b*arccosh(d*x+c))/d/e-4/9*b*e^(1/2)*(-d*x-c+1)^(1/2)*EllipticF((
e*(d*x+c))^(1/2)/e^(1/2),I)/d/(d*x+c-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx = \frac{\sqrt{e(c+dx)}\left(\frac{2}{3}(c+dx)^{3/2}(a+b\text{arccosh}(c+dx)) - \frac{4b\left(-1+c^2+2cdx+d^2x^2+\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+\right.\right.}{9\sqrt{\frac{-1+c+dx}{c+dx}}\sqrt{1+c+dx}}\right.}{d\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(Sqrt[e*(c + d*x)]*((2*(c + d*x)^(3/2)*(a + b*ArcCosh[c + d*x]))/3 - (4*b*
(-1 + c^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/
4, 1/2, 5/4, (c + d*x)^2]))/(9*Sqrt[(-1 + c + d*x)/(c + d*x)]*Sqrt[1 + c +
 d*x])))/(d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6411, 6298, 113, 27, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))}{3e}-\frac{2b\int \frac{(e(c+dx))^{3/2}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{3e}}{d}$$

$$\Big\downarrow \text{113}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))}{3e}-\frac{2b\left(\frac{2}{3}\int \frac{e^2}{2\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)}{3e}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))}{3e}-\frac{2b\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)}{3e}}{d}$$

$$\Big\downarrow \text{127}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))}{3e}-\frac{2b\left(\frac{e^2\sqrt{-c-dx+1}\int\frac{1}{\sqrt{-c-dx+1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{3\sqrt{c+dx-1}}+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)}{3e}}{d}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))}{3e}-\frac{2b\left(\frac{2e^{3/2}\sqrt{-c-dx+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{3\sqrt{c+dx-1}}+\frac{2}{3}e\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}\right)}{3e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

output
```
((2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x]))/(3*e) - (2*b*((2*e*Sqrt[
-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/3 + (2*e^(3/2)*Sqrt[1 -
 c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(3*Sqrt[-1 + c
 + d*x])))/(3*e))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}+\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\ \mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}}\right.\right.}{9e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right.}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}+\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\ \mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}}\right.\right.}{9e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right.}{de}$
parts	$\frac{2a(dex+ce)^{\frac{3}{2}}}{3de}+\frac{2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}+\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\ \mathrm{EllipticF}\left(\sqrt{dex+ce}\right.\right.}{9e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right.}{de}$

input
```
int((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arccosh((d*e*x+c*e
)/e)-2/9/e*((-1/e)^(1/2)*(d*e*x+c*e)^(5/2)+((d*e*x+c*e+e)/e)^(1/2)*((-d*e*
x-c*e+e)/e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e^2-(-1/e)^(
1/2)*e^2*(d*e*x+c*e)^(1/2))/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x
-c*e+e)/e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx = -\frac{2\left(2\sqrt{d^2x^2+2\,cdx+c^2-1}\sqrt{dex+ce}bd^2-3\,(bd^3x+bcd^2)\sqrt{dex+ce}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+\right.\right.}{9\,d^3}$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
-2/9*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*e*x + c*e)*b*d^2 - 3*(b*d
^3*x + b*c*d^2)*sqrt(d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) + 2*sqrt(d^3*e)*b*weierstrassPInverse(4/d^2, 0, (d*x + c)/d) - 3*
(a*d^3*x + a*c*d^2)*sqrt(d*e*x + c*e))/d^3
```

Sympy [F]

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx = \int \sqrt{e\,(c+dx)}(a+b\operatorname{acosh}\,(c+dx))\;dx$$

input
```
integrate((d*e*x+c*e)**(1/2)*(a+b*acosh(d*x+c)),x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\,\mathrm{arccosh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \sqrt{ce+dex}(a+b\,\mathrm{arccosh}(c+dx))\,dx = \int \sqrt{dex+ce}(b\,\mathrm{arcosh}\,(dx+c)+a)\,dx$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ce+dex}(a+b\,\mathrm{arccosh}(c+dx))\,dx = \int \sqrt{c\,e+d\,e\,x}\,(a+b\,\mathrm{acosh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))\,dx$$
$$=\frac{\sqrt{e}\,\left(2\sqrt{dx+c}\,ac+2\sqrt{dx+c}\,adx+3\left(\int \sqrt{dx+c}\,acosh(dx+c)\,dx\right)bd\right)}{3d}$$

input
```
int((d*e*x+c*e)^(1/2)*(a+b*acosh(d*x+c)),x)
```

output
```
(sqrt(e)*(2*sqrt(c + d*x)*a*c + 2*sqrt(c + d*x)*a*d*x + 3*int(sqrt(c + d*x
)*acosh(c + d*x),x)*b*d))/(3*d)
```

3.119 $\int \frac{a+b\operatorname{arccosh}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arccosh}(c+dx))}{de} + \frac{4b\sqrt{1-c-dx}\sqrt{e(c+dx)}E\left(\arcsin\left(\frac{\sqrt{1-c-dx}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-1+c+dx}\sqrt{c+dx}}$$

output
```
2*(e*(d*x+c))^(1/2)*(a+b*arccosh(d*x+c))/d/e+4*b*(-d*x-c+1)^(1/2)*(e*(d*x+
c))^(1/2)*EllipticE(1/2*(-d*x-c+1)^(1/2)*2^(1/2),2^(1/2))/d/e/(d*x+c-1)^(1
/2)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}\left(3(a+b\operatorname{arccosh}(c+dx))-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right)}{3de}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*(3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 -
 (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2])/(Sqrt[-1 + c
+ d*x]*Sqrt[1 + c + d*x])))/(3*d*e)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6411, 6298, 124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{ce+dex}}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))}{e}-\frac{2b\int \frac{\sqrt{e(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{ 124}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))}{e}-\frac{\sqrt{2}b\sqrt{-c-dx+1}\sqrt{e(c+dx)}\int \frac{\sqrt{2}\sqrt{-c-dx}}{\sqrt{-c-dx+1}\sqrt{c+dx+1}}d(c+dx)}{e\sqrt{-c-dx}\sqrt{c+dx-1}}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))}{e}-\frac{2b\sqrt{-c-dx+1}\sqrt{e(c+dx)}\int \frac{\sqrt{-c-dx}}{\sqrt{-c-dx+1}\sqrt{c+dx+1}}d(c+dx)}{e\sqrt{-c-dx}\sqrt{c+dx-1}}}{d}$$

$$\Big\downarrow \text{ 123}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))}{e}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\arcsin\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{e\sqrt{-c-dx}\sqrt{c+dx-1}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

output
```
((2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x]))/e - (4*b*Sqrt[1 - c - d*x]
*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(e*Sqr
t[-c - d*x]*Sqrt[-1 + c + d*x]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{2\sqrt{dex+ce}\,a+2b\left(\sqrt{dex+ce}\,\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\right)\sqrt{\frac{-dex-ce-}{e}}}{\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right.}{de}$
default	$\frac{2\sqrt{dex+ce}\,a+2b\left(\sqrt{dex+ce}\,\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\right)\sqrt{\frac{-dex-ce-}{e}}}{\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right.}{de}$
parts	$\frac{2a\sqrt{dex+ce}}{de}+\frac{2b\left(\sqrt{dex+ce}\,\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\right)\sqrt{-\frac{dex+}{e}}}{\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right.}{de}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*((d*e*x+c*e)^(1/2)*a+b*((d*e*x+c*e)^(1/2)*arccosh((d*e*x+c*e)/e)-2*(
EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/2)*(-
1/e)^(1/2),I))*((-d*e*x-c*e+e)/e)^(1/2)/(-1/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{2\left(\sqrt{dex+ce}bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\sqrt{dex+ce}ad+2\,\sqrt{d^3e}b\mathrm{weierstrassZeta}\left(\frac{4}{d^2},0,\right.\right.}{d^2e}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(d*e*x + c*e)*b*d*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
+ sqrt(d*e*x + c*e)*a*d + 2*sqrt(d^3*e)*b*weierstrassZeta(4/d^2, 0, weiers
trassPInverse(4/d^2, 0, (d*x + c)/d)))/(d^2*e)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{\sqrt{ce + dex}}\,dx = \int \frac{a + b\,\text{acosh}\,(c + dx)}{\sqrt{e\,(c + dx)}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))/sqrt(e*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(c + dx)}{\sqrt{ce + dex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{\sqrt{ce + dex}}\, dx = \int \frac{b\,\text{arcosh}\,(dx + c) + a}{\sqrt{dex + ce}}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arccosh}(c + dx)}{\sqrt{ce + dex}}\, dx = \int \frac{a + b\,\text{acosh}(c + d\,x)}{\sqrt{c\,e + d\,e\,x}}\, dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{\sqrt{ce + dex}}\, dx = \frac{2\sqrt{dx + c}\,a + \left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}} dx\right) bd}{\sqrt{e}\,d}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*a + int(acosh(c + d*x)/sqrt(c + d*x),x)*b*d)/(sqrt(e)*d)
```

3.120 $\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\text{arccosh}(c+dx))}{de\sqrt{e(c+dx)}} + \frac{4b\sqrt{1-c-dx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{de^{3/2}\sqrt{-1+c+dx}}$$

output
```
(-2*a-2*b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(1/2)+4*b*(-d*x-c+1)^(1/2)*Ellip
ticF((e*(d*x+c))^(1/2)/e^(1/2),I)/d/e^(3/2)/(d*x+c-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{2\left(-a-b\text{arccosh}(c+dx)+\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right)}{de\sqrt{e(c+dx)}}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

output
```
(2*(-a - b*ArcCosh[c + d*x] + (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hyperge
ometric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d
*x])))/(d*e*Sqrt[e*(c + d*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6411, 6298, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2b\int \frac{1}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{e}-\frac{2(a+b\text{arccosh}(c+dx))}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{127}$$

$$\frac{\frac{2b\sqrt{-c-dx+1}\int \frac{1}{\sqrt{-c-dx+1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{e\sqrt{c+dx-1}}-\frac{2(a+b\text{arccosh}(c+dx))}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{4b\sqrt{-c-dx+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{e^{3/2}\sqrt{c+dx-1}}-\frac{2(a+b\text{arccosh}(c+dx))}{e\sqrt{e(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

output
```
((-2*(a + b*ArcCosh[c + d*x]))/(e*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[1 - c - d
*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(e^(3/2)*Sqrt[-1 + c
 + d*x]))/d
```

Defintions of rubi rules used

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\,\mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\sqrt{\frac{-dex-ce+e}{e}}}{e\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right)}{de}$	119
default	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\,\mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\sqrt{\frac{-dex-ce+e}{e}}}{e\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right)}{de}$	119
parts	$-\frac{2a}{\sqrt{dex+ce}\,de}+\frac{2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\,\mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\sqrt{-\frac{dex+ce-e}{e}}}{e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right)}{de}$	124

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arccosh((d*e*x+c*e)/e)
+2/e*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*((-d*e*x-c*e+e)/e)^(1/2)/
(-1/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{dex+ce}bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\sqrt{dex+ce}ad^2-2\sqrt{d^3e}(bdx+bc)\mathrm{weierstrass}\right.}{d^4e^2x+cd^3e^2}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(d*e*x + c*e)*b*d^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)) + sqrt(d*e*x + c*e)*a*d^2 - 2*sqrt(d^3*e)*(b*d*x + b*c)*weierstrassPInv
erse(4/d^2, 0, (d*x + c)/d))/(d^4*e^2*x + c*d^3*e^2)
```

Sympy [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx=\int \frac{a+b\,\text{acosh}\,(c+dx)}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx=\int \frac{b\,\text{arcosh}\,(dx+c)+a}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{3/2}}\,dx=\frac{\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx\right)bd-2a}{\sqrt{e}\,\sqrt{dx+c}\,de}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^(3/2),x)
```

output
```
(sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
*b*d - 2*a)/(sqrt(e)*sqrt(c + d*x)*d*e)
```

3.121 $\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2(a+b\text{arccosh}(c+dx))}{3de(e(c+dx))^{3/2}} + \frac{4b\sqrt{1-c-dx}\sqrt{e(c+dx)}E\left(\arcsin\left(\frac{\sqrt{1-c-dx}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-1+c+dx}\sqrt{c+dx}}$$

output
```
4/3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^2/(e*(d*x+c))^(1/2)-2/3*(a+b*arc
cosh(d*x+c))/d/e/(e*(d*x+c))^(3/2)+4/3*b*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2
)*EllipticE(1/2*(-d*x-c+1)^(1/2)*2^(1/2),2^(1/2))/d/e^3/(d*x+c-1)^(1/2)/(d
*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{2\left(-a-b\text{arccosh}(c+dx)-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right)}{3de(e(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

output
```
(2*(-a - b*ArcCosh[c + d*x] - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hyperge
ometric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c +
d*x])))/(3*d*e*(e*(c + d*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6411, 6298, 115, 8, 27, 124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2b\int \frac{1}{\sqrt{c+dx-1}(e(c+dx))^{3/2}\sqrt{c+dx+1}}d(c+dx)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{115}$$

$$\frac{\frac{2b\left(\frac{2\int -\frac{e(c+dx)}{2\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{e^2}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{8}$$

$$\frac{\frac{2b\left(\frac{2\int -\frac{e\sqrt{e(c+dx)}}{2\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e^3}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2b\left(\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{e\sqrt{e(c+dx)}}-\frac{\int \frac{\sqrt{e(c+dx)}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e^2}\right)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \; 124$$

$$\frac{\frac{2b\left(\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{e\sqrt{e(c+dx)}}-\frac{\sqrt{-c-dx+1}\sqrt{e(c+dx)}\int \frac{\sqrt{2}\sqrt{-c-dx}}{\sqrt{-c-dx+1}\sqrt{c+dx+1}}d(c+dx)}{\sqrt{2}e^2\sqrt{-c-dx}\sqrt{c+dx-1}}\right)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2b\left(\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{e\sqrt{e(c+dx)}}-\frac{\sqrt{-c-dx+1}\sqrt{e(c+dx)}\int \frac{\sqrt{-c-dx}}{\sqrt{-c-dx+1}\sqrt{c+dx+1}}d(c+dx)}{e^2\sqrt{-c-dx}\sqrt{c+dx-1}}\right)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \; 123$$

$$\frac{\frac{2b\left(\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{e\sqrt{e(c+dx)}}-\frac{2\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\arcsin\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{e^2\sqrt{-c-dx}\sqrt{c+dx-1}}\right)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

output
```
((-2*(a + b*ArcCosh[c + d*x]))/(3*e*(e*(c + d*x))^(3/2)) + (2*b*((2*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(e*Sqrt[e*(c + d*x)]) - (2*Sqrt[1 - c - d*
x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(e^2
*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])))/(3*e))/d
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\sqrt{dex+ce}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)e}{3}+\frac{2\sqrt{\frac{dex+ce}{e}}\cdots}{\cdots}}{e^3\sqrt{-\frac{1}{e}}\sqrt{dex+ce}\sqrt{\cdots}}\cdots\right.}{de}$
default	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\sqrt{dex+ce}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)e}{3}+\frac{2\sqrt{\frac{dex+ce}{e}}\cdots}{\cdots}}{e^3\sqrt{-\frac{1}{e}}\sqrt{dex+ce}\sqrt{\cdots}}\cdots\right.}{de}$
parts	$-\frac{2a}{3(dex+ce)^{\frac{3}{2}}de}+\frac{2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex+ce-e}{e}}\sqrt{dex+ce}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)e}{3}+\frac{2\sqrt{\cdots}}{\cdots}}{e^3\sqrt{-\frac{1}{e}}\sqrt{d\cdots}}\cdots\right.}{de}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arccosh((d*e*x+c
*e)/e)+2/3/e^3*(-((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*(d*e*x+c
*e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e+((d*e*x+c*e+e)/e)^
(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*(d*e*x+c*e)^(1/2)*EllipticE((d*e*x+c*e)^(1/
2)*(-1/e)^(1/2),I)*e+(-1/e)^(1/2)*(d*e*x+c*e)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(
1/2)/(d*e*x+c*e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx =$$
$$-\frac{2\left(\sqrt{dex+ce}bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\sqrt{dex+ce}ad-2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{d^3e}v\cdots\right.}{3\left(d^4e^3x^2+2\cdots\right.}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(d*e*x + c*e)*b*d*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)) + sqrt(d*e*x + c*e)*a*d - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(d^3*e)
*weierstrassZeta(4/d^2, 0, weierstrassPInverse(4/d^2, 0, (d*x + c)/d)) - 2
*(b*d^2*x + b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*e*x + c*e))/(d
^4*e^3*x^2 + 2*c*d^3*e^3*x + c^2*d^2*e^3)
```

Sympy [F]

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^{5/2}}\,dx = \int \frac{a + b\,\text{acosh}\,(c + dx)}{(e\,(c + dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arccosh}(c + dx)}{(ce + dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \int \frac{a+b\,\text{acosh}(c+d\,x)}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{3\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx\right)bcd+3\sqrt{dx+c}\left(\int \frac{}{\sqrt{dx+c}\,c^2+2}\right.}{3\sqrt{e}\sqrt{dx+c}\,d\,e^2\,(dx+c)}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^(5/2),x)
```

output
```
(3*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*
c*d*x + sqrt(c + d*x)*d**2*x**2),x)*b*c*d + 3*sqrt(c + d*x)*int(acosh(c +
d*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2*x**2
),x)*b*d**2*x - 2*a)/(3*sqrt(e)*sqrt(c + d*x)*d*e**2*(c + d*x))
```

3.122 $\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}}-\frac{2(a+b\text{arccosh}(c+dx))}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{1-c-dx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{15de^{7/2}\sqrt{-1+c+dx}}$$

output

```
4/15*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^2/(e*(d*x+c))^(3/2)-2/5*(a+b*ar
ccosh(d*x+c))/d/e/(e*(d*x+c))^(5/2)+4/15*b*(-d*x-c+1)^(1/2)*EllipticF((e*(
d*x+c))^(1/2)/e^(1/2),I)/d/e^(7/2)/(d*x+c-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\frac{2\left(-3(a+b\text{arccosh}(c+dx))-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},(c+dx)\right.}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right.}{15de(e(c+dx))^{5/2}}$$

input

```
Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

output
```
(2*(-3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hyp
ergeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x])))/(15*d*e*(e*(c + d*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6411, 6298, 115, 8, 27, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{a+b\text{arccosh}(c+dx)}{(e(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2b\int \frac{1}{\sqrt{c+dx-1}(e(c+dx))^{5/2}\sqrt{c+dx+1}}d(c+dx)}{5e}-\frac{2(a+b\text{arccosh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{115}$$

$$\frac{\frac{2b\left(\frac{2\int \frac{e(c+dx)}{2\sqrt{c+dx-1}(e(c+dx))^{3/2}\sqrt{c+dx+1}}d(c+dx)}{3e^2}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arccosh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{8}$$

$$\frac{\frac{2b\left(\frac{2\int \frac{e}{2\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{3e^3}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arccosh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2b\left(\frac{\int \frac{1}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{3e^2}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arccosh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 127$$

$$\frac{\frac{2b\left(\frac{\sqrt{-c-dx+1}\int \frac{1}{\sqrt{-c-dx+1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{3e^2\sqrt{c+dx-1}}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arccosh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 126$$

$$\frac{\frac{2b\left(\frac{2\sqrt{-c-dx+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{3e^{5/2}\sqrt{c+dx-1}}+\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arccosh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

output
```
((-2*(a + b*ArcCosh[c + d*x]))/(5*e*(e*(c + d*x))^(5/2)) + (2*b*((2*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(3*e*(e*(c + d*x))^(3/2)) + (2*Sqrt[1 - c
- d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(3*e^(5/2)*Sqrt[-
1 + c + d*x])))/(5*e))/d
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)(dex+ce)^{\frac{3}{2}}}{15}+\frac{2\sqrt{-\frac{1}{e}}(dex\cdots}{15}}{e^3\sqrt{-\frac{1}{e}}(dex+ce)^{\frac{3}{2}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{-dex-ce+e}{e}}}\cdots\right.}{de}$
default	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)(dex+ce)^{\frac{3}{2}}}{15}+\frac{2\sqrt{-\frac{1}{e}}(dex\cdots}{15}}{e^3\sqrt{-\frac{1}{e}}(dex+ce)^{\frac{3}{2}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{-dex-ce+e}{e}}}\cdots\right.}{de}$
parts	$-\frac{2a}{5(dex+ce)^{\frac{5}{2}}de}+\frac{2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex+ce-e}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)(dex+ce)^{\frac{3}{2}}}{15}+2\sqrt{-\cdots}}{e^3\sqrt{-\frac{1}{e}}(dex+ce)^{\frac{3}{2}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{dex+ce-e}{e}}}\cdots\right.}{de}$

input
```
int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arccosh((d*e*x+c
*e)/e)+2/15/e^3*(((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*Elliptic
F((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*(d*e*x+c*e)^(3/2)+(-1/e)^(1/2)*(d*e*x+
c*e)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2)/(d*e*x+c*e)^(3/2)/((d*e*x+c*e+e)/e)^
(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.60

$$\int \frac{a+b\operatorname{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx =$$
$$-\frac{2\left(3\sqrt{dex+ce}bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+3\sqrt{dex+ce}ad^2-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc\cdots\right.\right.}{15\left(d^6e^4x^3+3\,cd^5e^4x^2+\cdots\right.}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*sqrt(d*e*x + c*e)*b*d^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)) + 3*sqrt(d*e*x + c*e)*a*d^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*sqrt(d^3*e)*weierstrassPInverse(4/d^2, 0, (d*x + c)/d) - 2
*(b*d^3*x + b*c*d^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*e*x + c*e))/
(d^6*e^4*x^3 + 3*c*d^5*e^4*x^2 + 3*c^2*d^4*e^4*x + c^3*d^3*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arccosh}(c + dx)}{(ce + dex)^{7/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\int \frac{b\,\text{arcosh}\,(dx+c)+a}{(dex+ce)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\int \frac{a+b\,\text{acosh}(c+d\,x)}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

input
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(7/2),x)
```

output
```
int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\frac{5\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx\right)b\,c^2d+10\sqrt{dx+}}{}$$

input
```
int((a+b*acosh(d*x+c))/(d*e*x+c*e)^(7/2),x)
```

output

```
(5*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*
c**2*d*x + 3*sqrt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)*b*c**
2*d + 10*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c**3 + 3*sqrt(c +
 d*x)*c**2*d*x + 3*sqrt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
*b*c*d**2*x + 5*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c**3 + 3*s
qrt(c + d*x)*c**2*d*x + 3*sqrt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x
**3),x)*b*d**3*x**2 - 2*a)/(5*sqrt(e)*sqrt(c + d*x)*d*e**3*(c**2 + 2*c*d*x
 + d**2*x**2))
```

3.123 $\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx=\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))^2}{7de}$$
$$-\frac{8b\sqrt{1-c-dx}(e(c+dx))^{9/2}(a+b\text{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},(c+dx)^2\right)}{63de^2\sqrt{-1+c+dx}}$$
$$-\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)}{693de^3}$$

output
```
2/7*(e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))^2/d/e-8/63*b*(-d*x-c+1)^(1/2)*(
e*(d*x+c))^(9/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 9/4],[13/4],(d*x+c)^
2)/d/e^2/(d*x+c-1)^(1/2)-16/693*b^2*(e*(d*x+c))^(11/2)*hypergeom([1, 11/4,
 11/4],[13/4, 15/4],(d*x+c)^2)/d/e^3
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{7/2}\left(99(a+b\text{arccosh}(c+dx))^2-4b(c+dx)\left(\frac{11\sqrt{1-(c+dx)^2}(a+b\text{arccos}}{\sqrt{}}\right.\right.}{69}$$

input
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((11
*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 9/4
, 13/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*
x)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])))/(693*d
*e)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int (e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))^2}{7e}-\frac{4b\int \frac{(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{7e}}{d}$$

$$\Big\downarrow \text{6364}$$

$$\frac{2(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))^2}{7e} - \frac{4b\left(\frac{4b(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)}{99e^2}+\frac{2\sqrt{-c-dx+1}(e(c+dx))^{9/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},\right.}{9e\sqrt{c+dx-1}}}{7e}}{d}$$

input
```
Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])^2)/(7*e) - (4*b*((2*Sqrt[
1 - c - d*x]*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F
1[1/2, 9/4, 13/4, (c + d*x)^2])/(9*e*Sqrt[-1 + c + d*x]) + (4*b*(e*(c + d*
x))^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])/
(99*e^2)))/(7*e))/d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex+ce)^{\frac{5}{2}} (a+b \operatorname{arccosh}(dx+c))^2 \, dx$$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

Fricas [F]

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{5}{2}}(b \operatorname{arcosh}(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x
^2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arccosh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^
2 + 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{5}{2}}(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\text{arccosh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\text{acosh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce+dex)^{5/2}(a+b\operatorname{arccosh}(c+dx))^2\,dx = \frac{\sqrt{e}\,e^2\left(2\sqrt{dx+c}\,a^2c^3+6\sqrt{dx+c}\,a^2c^2dx+6\sqrt{dx+c}\,a^2c\,d^2x^2+2\sqrt{dx+c}\,a^2d^3x^3\right.}{}$$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*acosh(d*x+c))^2,x)
```

output
```
(sqrt(e)*e**2*(2*sqrt(c + d*x)*a**2*c**3 + 6*sqrt(c + d*x)*a**2*c**2*d*x +
 6*sqrt(c + d*x)*a**2*c*d**2*x**2 + 2*sqrt(c + d*x)*a**2*d**3*x**3 + 14*in
t(sqrt(c + d*x)*acosh(c + d*x)*x**2,x)*a*b*d**3 + 28*int(sqrt(c + d*x)*aco
sh(c + d*x)*x,x)*a*b*c*d**2 + 14*int(sqrt(c + d*x)*acosh(c + d*x),x)*a*b*c
**2*d + 7*int(sqrt(c + d*x)*acosh(c + d*x)**2*x**2,x)*b**2*d**3 + 14*int(s
qrt(c + d*x)*acosh(c + d*x)**2*x,x)*b**2*c*d**2 + 7*int(sqrt(c + d*x)*acos
h(c + d*x)**2,x)*b**2*c**2*d))/(7*d)
```

3.124 $\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx=\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^2}{5de}-\frac{8b\sqrt{1-c-dx}(e(c+dx))^{7/2}(a+b\text{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},(c+dx)^2\right)}{35de^2\sqrt{-1+c+dx}}-\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)}{315de^3}$$

output
```
2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^2/d/e-8/35*b*(-d*x-c+1)^(1/2)*(
e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 7/4],[11/4],(d*x+c)^
2)/d/e^2/(d*x+c-1)^(1/2)-16/315*b^2*(e*(d*x+c))^(9/2)*hypergeom([1, 9/4, 9
/4],[11/4, 13/4],(d*x+c)^2)/d/e^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{5/2}\left(63(a+b\text{arccosh}(c+dx))^2-4b(c+dx)\left(\frac{9\sqrt{1-(c+dx)^2}(a+b\text{arccosh}}{\sqrt{}}\right.\right.}{315}$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((9*
Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 7/4,
 11/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x
)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])))/(315*d*e)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int (e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^2}{5e}-\frac{4b\int \frac{(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \text{6364}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^2}{5e}-\frac{4b\left(\frac{4b(e(c+dx))^{9/2}{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)}{63e^2}+\frac{2\sqrt{-c-dx+1}(e(c+dx))^{7/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},(c+\right.}{7e\sqrt{c+dx-1}}\right.}{5e}}{d}$$

input
```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^2)/(5*e) - (4*b*((2*Sqrt[
1 - c - d*x]*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F
1[1/2, 7/4, 11/4, (c + d*x)^2])/(7*e*Sqrt[-1 + c + d*x]) + (4*b*(e*(c + d*
x))^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])/(63
*e^2)))/(5*e))/d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex+ce)^{\frac{3}{2}} (a+b \operatorname{arccosh}(dx+c))^2 \, dx$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

Fricas [F]

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2 \, dx = \int (dex+ce)^{\frac{3}{2}}(b \operatorname{arcosh}(dx+c)+a)^2 \, dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arccosh(d*x + c)^2 +
 2*(a*b*d*e*x + a*b*c*e)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F]

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2 \, dx = \int (e(c+dx))^{\frac{3}{2}} (a+b \operatorname{acosh}(c+dx))^2 \, dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**2,x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\text{acosh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce + dex)^{3/2}(a + b\text{arccosh}(c+dx))^2\,dx = \frac{\sqrt{e}\,e\left(2\sqrt{dx+c}\,a^2c^2 + 4\sqrt{dx+c}\,a^2cdx + 2\sqrt{dx+c}\,a^2d^2x^2 + 10\left(\int \sqrt{dx+c}\,acosh\right.\right.}{}$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*acosh(d*x+c))^2,x)
```

output
```
(sqrt(e)*e*(2*sqrt(c + d*x)*a**2*c**2 + 4*sqrt(c + d*x)*a**2*c*d*x + 2*sqr
t(c + d*x)*a**2*d**2*x**2 + 10*int(sqrt(c + d*x)*acosh(c + d*x)*x,x)*a*b*d
**2 + 10*int(sqrt(c + d*x)*acosh(c + d*x),x)*a*b*c*d + 5*int(sqrt(c + d*x)
*acosh(c + d*x)**2*x,x)*b**2*d**2 + 5*int(sqrt(c + d*x)*acosh(c + d*x)**2,
x)*b**2*c*d))/(5*d)
```

3.125 $\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^2}{3de} - \frac{8b\sqrt{1-c-dx}(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},(c+dx)^2\right)}{15de^2\sqrt{-1+c+dx}} - \frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)}{105de^3}$$

output
```
2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^2/d/e-8/15*b*(-d*x-c+1)^(1/2)*(
e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 5/4],[9/4],(d*x+c)^2
)/d/e^2/(d*x+c-1)^(1/2)-16/105*b^2*(e*(d*x+c))^(7/2)*hypergeom([1, 7/4, 7/
4],[9/4, 11/4],(d*x+c)^2)/d/e^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \sqrt{ce+dex}(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$= \frac{2(e(c+dx))^{3/2}\left(35(a+b\operatorname{arccosh}(c+dx))^2-4b(c+dx)\left(\frac{7\sqrt{1-(c+dx)^2}(a+b\operatorname{arccosh}(c+dx))\operatorname{Hypergeometric2F1}(\ldots}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right.\right.}{105de}$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((7*
Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 5/4,
 9/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)
*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])))/(105*d*e)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\operatorname{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \; 6411$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\operatorname{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 6298$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\operatorname{arccosh}(c+dx))^2}{3e}-\frac{4b\int \frac{(e(c+dx))^{3/2}(a+b\operatorname{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{3e}}{d}$$

$$\Big\downarrow \; 6364$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^2}{3e}-\frac{4b\left(\frac{4b(e(c+dx))^{7/2}{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)}{35e^2}+\frac{2\sqrt{-c-dx+1}(e(c+dx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},(c+dx\right.}{5e\sqrt{c+dx-1}}\right.}{3e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^2)/(3*e) - (4*b*((2*Sqrt[
1 - c - d*x]*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F
1[1/2, 5/4, 9/4, (c + d*x)^2])/(5*e*Sqrt[-1 + c + d*x]) + (4*b*(e*(c + d*x
))^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])/(35*e
^2)))/(3*e))/d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \sqrt{dex+ce}\,(a+b\,\mathrm{arccosh}\,(dx+c))^2\,dx$$

input
```
int((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^2,x)
```

Fricas [F]

$$\int \sqrt{ce+dex}(a+b\mathrm{arccosh}(c+dx))^2\,dx=\int \sqrt{dex+ce}(b\,\mathrm{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*
x + c*e), x)
```

Sympy [F]

$$\int \sqrt{ce+dex}(a+b\mathrm{arccosh}(c+dx))^2\,dx=\int \sqrt{e\,(c+dx)}(a+b\,\mathrm{acosh}\,(c+dx))^2\,dx$$

input
```
integrate((d*e*x+c*e)**(1/2)*(a+b*acosh(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^2\,dx = \int \sqrt{dex+ce}(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^2\,dx = \int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^2\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int \sqrt{ce+dex}(a+b\operatorname{arccosh}(c+dx))^2\,dx$$
$$=\frac{\sqrt{e}\left(2\sqrt{dx+c}\,a^2c+2\sqrt{dx+c}\,a^2dx+6\left(\int \sqrt{dx+c}\,acosh(dx+c)\,dx\right)abd+3\left(\int \sqrt{dx+c}\,acosh(dx+c)\right.\right.}{3d}$$

input
```
int((d*e*x+c*e)^(1/2)*(a+b*acosh(d*x+c))^2,x)
```

output
```
(sqrt(e)*(2*sqrt(c + d*x)*a**2*c + 2*sqrt(c + d*x)*a**2*d*x + 6*int(sqrt(c
 + d*x)*acosh(c + d*x),x)*a*b*d + 3*int(sqrt(c + d*x)*acosh(c + d*x)**2,x)
*b**2*d))/(3*d)
```

3.126 $\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arccosh}(c+dx))^2}{de} - \frac{8b\sqrt{1-c-dx}(e(c+dx))^{3/2}(a+b\operatorname{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},(c+dx)^2\right)}{3de^2\sqrt{-1+c+dx}} - \frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)}{15de^3}$$

output

```
2*(e*(d*x+c))^(1/2)*(a+b*arccosh(d*x+c))^2/d/e-8/3*b*(-d*x-c+1)^(1/2)*(e*(
d*x+c))^(3/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 3/4],[7/4],(d*x+c)^2)/d
/e^2/(d*x+c-1)^(1/2)-16/15*b^2*(e*(d*x+c))^(5/2)*hypergeom([1, 5/4, 5/4],[
7/4, 9/4],(d*x+c)^2)/d/e^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{\sqrt{ce + dex}} dx = \frac{2\sqrt{e(c + dx)}\left(15(a + b\text{arccosh}(c + dx))^2 - 4b(c + dx)\left(\frac{5\sqrt{1-(c+dx)^2}(a+b\text{arccosh}(c+dx))\text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right.\right.}{15de}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((5*Sq
rt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7
/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*H
ypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])))/(15*d*e)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{\sqrt{ce + dex}} dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{e(c+dx)}}d(c + dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^2}{e} - \frac{4b\int \frac{\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e}}{d}$$

$$\downarrow 6364$$

$$\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^2}{e}-\frac{4b\left(\frac{\frac{4b(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)}{15e^2}+\frac{2\sqrt{-c-dx+1}(e(c+dx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},(c+dx)^2\right)}{3e\sqrt{c+dx-1}}}{e}\right.}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

output
```
((2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^2)/e - (4*b*((2*Sqrt[1 - c
- d*x]*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2,
 3/4, 7/4, (c + d*x)^2])/(3*e*Sqrt[-1 + c + d*x]) + (4*b*(e*(c + d*x))^(5/
2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])/(15*e^2)))/e
)/d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^2}{\sqrt{dex+ce}}dx$$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \int \frac{(b\operatorname{arcosh}(dx+c)+a)^2}{\sqrt{dex+ce}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/sqrt(d*e*
x + c*e), x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\operatorname{acosh}(c+dx))^2}{\sqrt{e(c+dx)}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{\sqrt{ce + dex}} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{\sqrt{ce + dex}} dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^2}{\sqrt{dex + ce}} dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{\sqrt{ce + dex}} dx = \int \frac{(a + b \operatorname{acosh}(c + d x))^2}{\sqrt{c e + d e x}} dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a + b\text{arccosh}(c + dx))^2}{\sqrt{ce + dex}}\,dx$$
$$= \frac{2\sqrt{dx + c}\,a^2 + 2\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}}dx\right)abd + \left(\int \frac{acosh(dx+c)^2}{\sqrt{dx+c}}dx\right)b^2d}{\sqrt{e}\,d}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*a**2 + 2*int(acosh(c + d*x)/sqrt(c + d*x),x)*a*b*d + int(
acosh(c + d*x)**2/sqrt(c + d*x),x)*b**2*d)/(sqrt(e)*d)
```

3.127 $\int \frac{(a+b\,\mathbf{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\text{arccosh}(c+dx))^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{1-c-dx}\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right)}{de^2\sqrt{-1+c+dx}} + \frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)}{3de^3}$$

output
```
-2*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(-d*x-c+1)^(1/2)*(e*(d
*x+c))^(1/2)*(a+b*arccosh(d*x+c))*hypergeom([1/4, 1/2],[5/4],(d*x+c)^2)/d/
e^2/(d*x+c-1)^(1/2)+16/3*b^2*(e*(d*x+c))^(3/2)*hypergeom([3/4, 3/4, 1],[5/
4, 7/4],(d*x+c)^2)/d/e^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx=\frac{2\left(-3(a+b\text{arccosh}(c+dx))^2+4b(c+dx)\left(\frac{3\sqrt{1-(c+dx)^2}(a+b\text{arccosh}(c+dx)}{\sqrt{-1+c+d}}\right.\right.}{3de\sqrt{e(}}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

output
```
(2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*((3*Sqrt[1 - (c + d*x)^2
]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*HypergeometricPFQ[{
3/4, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{4b\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{e}-\frac{2(a+b\text{arccosh}(c+dx))^2}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{6364}$$

$$\frac{\frac{4b\left(\frac{4b(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)}{3e^2}+\frac{2\sqrt{-c-dx+1}\sqrt{e(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right)(a+b\,\text{arccosh}(c+dx))}{e\sqrt{c+dx-1}}\right)}{e}-\frac{2(a+b\,\text{arccosh}}{e\sqrt{e(c+}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

output
```
((-2*(a + b*ArcCosh[c + d*x])^2)/(e*Sqrt[e*(c + d*x)]) + (4*b*((2*Sqrt[1 -
 c - d*x]*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4
, 1/2, 5/4, (c + d*x)^2])/(e*Sqrt[-1 + c + d*x]) + (4*b*(e*(c + d*x))^(3/2
)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2])/(3*e^2)))/e)/
d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^2}{(dex + ce)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^2}{(ce + dex)^{3/2}} dx = \int \frac{(b \operatorname{arcosh}(dx + c) + a)^2}{(dex + ce)^{\frac{3}{2}}} dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*
x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a + b\operatorname{arccosh}(c + dx))^2}{(ce + dex)^{3/2}} dx = \int \frac{(a + b \operatorname{acosh}(c + dx))^2}{(e(c + dx))^{\frac{3}{2}}} dx$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{acosh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\,\text{arccosh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \frac{2\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx\right)abd+\sqrt{dx+c}\left(\int \frac{acosh(dx+c)^2}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx\right)b}{\sqrt{e}\,\sqrt{dx+c}\,de}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

output
```
(2*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),
x)*a*b*d + sqrt(c + d*x)*int(acosh(c + d*x)**2/(sqrt(c + d*x)*c + sqrt(c +
 d*x)*d*x),x)*b**2*d - 2*a**2)/(sqrt(e)*sqrt(c + d*x)*d*e)
```

3.128 $\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\operatorname{arccosh}(c+dx))^2}{3de(e(c+dx))^{3/2}} - \frac{8b\sqrt{1-c-dx}(a+b\operatorname{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},(c+dx)^2\right)}{3de^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}} - \frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)}{3de^3}$$

output

```
-2/3*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(-d*x-c+1)^(1/2)*(
a+b*arccosh(d*x+c))*hypergeom([-1/4, 1/2],[3/4],(d*x+c)^2)/d/e^2/(d*x+c-1)
^(1/2)/(e*(d*x+c))^(1/2)-16/3*b^2*(e*(d*x+c))^(1/2)*hypergeom([1/4, 1/4, 1
],[3/4, 5/4],(d*x+c)^2)/d/e^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \frac{2\left(-(a+b\text{arccosh}(c+dx))^2+4b(c+dx)\left(-\frac{\sqrt{1-(c+dx)^2}(a+b\text{arccosh}(c+dx)}{\sqrt{-1+c+}}\right.\right.}{3de(e(c}$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

output
```
(2*(-(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]*
(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(
Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])) - 2*b*(c + d*x)*HypergeometricPFQ[{
1/4, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^2}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{\frac{4b\int \frac{a+b\text{arccosh}(c+dx)}{\sqrt{c+dx-1}(e(c+dx))^{3/2}\sqrt{c+dx+1}}d(c+dx)}{3e}-\frac{2(a+b\text{arccosh}(c+dx))^2}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{ 6364}$$

$$\frac{\frac{4b\left(-\frac{4b\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)}{e^2}-\frac{2\sqrt{-c-dx+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},(c+dx)^2\right)(a+b\,\text{arccosh}(c+dx))}{e\sqrt{c+dx-1}\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\,\text{arccosh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

output
```
((-2*(a + b*ArcCosh[c + d*x])^2)/(3*e*(e*(c + d*x))^(3/2)) + (4*b*((-2*Sqr
t[1 - c - d*x]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4,
(c + d*x)^2])/(e*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]) - (4*b*Sqrt[e*(c +
d*x)]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])/e^2))/(3*
e))/d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(a + b\ \mathrm{arccosh}\,(dx + c))^2}{(dex + ce)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a + b\mathrm{arccosh}(c + dx))^2}{(ce + dex)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcosh}\,(dx + c) + a)^2}{(dex + ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*
x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F]

$$\int \frac{(a + b\mathrm{arccosh}(c + dx))^2}{(ce + dex)^{5/2}}\,dx = \int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^2}{(e\,(c + dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2}{\left(c\,e+d\,e\,x\right)^{5/2}}\,dx$$

input
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arccosh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \frac{6\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx\right)abcd+6\sqrt{dx+c}\left(\int \frac{}{\sqrt{dx+c}}\right.}{}$$

input
```
int((a+b*acosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

output
```
(6*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*
c*d*x + sqrt(c + d*x)*d**2*x**2),x)*a*b*c*d + 6*sqrt(c + d*x)*int(acosh(c
+ d*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2*x*
*2),x)*a*b*d**2*x + 3*sqrt(c + d*x)*int(acosh(c + d*x)**2/(sqrt(c + d*x)*c
**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2*x**2),x)*b**2*c*d + 3*sqr
t(c + d*x)*int(acosh(c + d*x)**2/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d
*x + sqrt(c + d*x)*d**2*x**2),x)*b**2*d**2*x - 2*a**2)/(3*sqrt(e)*sqrt(c +
 d*x)*d*e**2*(c + d*x))
```

3.129 $\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx = \text{Int}\left((e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^3,x\right)$$

output
```
Defer(Int)((e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 32.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx = \int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int (e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^3}{5e}-\frac{6b\int \frac{(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \text{6376}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^3}{5e}-\frac{6b\int \frac{(e(c+dx))^{5/2}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{5e}}{d}$$

input
```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{3}{2}} (a+b\operatorname{arccosh}(dx+c))^3\,dx$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{3}{2}}(b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arccosh(d*x + c)^3 +
 3*(a*b^2*d*e*x + a*b^2*c*e)*arccosh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c
*e)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [N/A]

Not integrable

Time = 72.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx = \int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{acosh}(c+dx))^3\,dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**3,x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arcosh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{acosh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 204, normalized size of antiderivative = 8.16

$$\int (ce+dex)^{3/2}(a+b\text{arccosh}(c+dx))^3\,dx=\frac{\sqrt{e}\,e\left(2\sqrt{dx+c}\,a^3c^2+4\sqrt{dx+c}\,a^3cdx+2\sqrt{dx+c}\,a^3d^2x^2+15\left(\int\sqrt{dx+c}\,acosh\right.\right.}{}$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*acosh(d*x+c))^3,x)
```

output
```
(sqrt(e)*e*(2*sqrt(c + d*x)*a**3*c**2 + 4*sqrt(c + d*x)*a**3*c*d*x + 2*sqr
t(c + d*x)*a**3*d**2*x**2 + 15*int(sqrt(c + d*x)*acosh(c + d*x)*x,x)*a**2*
b*d**2 + 15*int(sqrt(c + d*x)*acosh(c + d*x),x)*a**2*b*c*d + 5*int(sqrt(c
+ d*x)*acosh(c + d*x)**3*x,x)*b**3*d**2 + 5*int(sqrt(c + d*x)*acosh(c + d*
x)**3,x)*b**3*c*d + 15*int(sqrt(c + d*x)*acosh(c + d*x)**2*x,x)*a*b**2*d**
2 + 15*int(sqrt(c + d*x)*acosh(c + d*x)**2,x)*a*b**2*c*d))/(5*d)
```

3.130 $\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx=\text{Int}\left(\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^3,x\right)$$

output
```
Defer(Int)((e*(d*x+c))^(1/2)*(a+b*arccosh(d*x+c))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 94.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx=\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^3}{3e}-\frac{2b\int \frac{(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{6376}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^3}{3e}-\frac{2b\int \frac{(e(c+dx))^{3/2}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt{dex+ce}\,(a+b\,\mathrm{arccosh}\,(dx+c))^3\,dx$$

input
```
int((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \sqrt{ce+dex}(a+b\mathrm{arccosh}(c+dx))^3\,dx = \int \sqrt{dex+ce}(b\,\mathrm{arcosh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

Sympy [N/A]

Not integrable

Time = 6.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{ce+dex}(a+b\mathrm{arccosh}(c+dx))^3\,dx = \int \sqrt{e\,(c+dx)}(a+b\,\mathrm{acosh}\,(c+dx))^3\,dx$$

input
```
integrate((d*e*x+c*e)**(1/2)*(a+b*acosh(d*x+c))**3,x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce + dex}(a + b\operatorname{arccosh}(c + dx))^3 \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce + dex}(a + b\operatorname{arccosh}(c + dx))^3 \, dx = \int \sqrt{dex + ce}(b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

input
```
integrate((d*e*x+c*e)^(1/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx=\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^3\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.08

$$\int \sqrt{ce+dex}(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{\sqrt{e}\left(2\sqrt{dx+c}\,a^3c+2\sqrt{dx+c}\,a^3dx+9\left(\int \sqrt{dx+c}\,acosh(dx+c)\,dx\right)a^2bd+3\left(\int \sqrt{dx+c}\,acosh(dx+c\right.\right.}{3d}$$

input
```
int((d*e*x+c*e)^(1/2)*(a+b*acosh(d*x+c))^3,x)
```

output
```
(sqrt(e)*(2*sqrt(c + d*x)*a**3*c + 2*sqrt(c + d*x)*a**3*d*x + 9*int(sqrt(c
 + d*x)*acosh(c + d*x),x)*a**2*b*d + 3*int(sqrt(c + d*x)*acosh(c + d*x)**3
,x)*b**3*d + 9*int(sqrt(c + d*x)*acosh(c + d*x)**2,x)*a*b**2*d))/(3*d)
```

3.131 $\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\text{Int}\left(\frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{e(c+dx)}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^3/(e*(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 44.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

output
```
Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^3}{e}-\frac{6b\int \frac{\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{6376}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^3}{e}-\frac{6b\int \frac{\sqrt{e(c+dx)}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ \operatorname{arccosh}(dx+c))^3}{\sqrt{dex+ce}}dx$$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{\sqrt{dex+ce}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

Sympy [N/A]

Not integrable

Time = 4.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\operatorname{acosh}(c+dx))^3}{\sqrt{e(c+dx)}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{\sqrt{ce + dex}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{\sqrt{ce + dex}}\, dx = \int \frac{(b\,\text{arcosh}\,(dx + c) + a)^3}{\sqrt{dex + ce}}\, dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

Mupad [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}{\sqrt{c\,e+d\,e\,x}}\,dx$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.76

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{dx+c}\,a^3+3\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}}dx\right)a^2bd+\left(\int \frac{acosh(dx+c)^3}{\sqrt{dx+c}}dx\right)b^3d+3\left(\int \frac{acosh(dx+c)^2}{\sqrt{dx+c}}dx\right)a\,b^2d}{\sqrt{e}\,d}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*a**3 + 3*int(acosh(c + d*x)/sqrt(c + d*x),x)*a**2*b*d + i
nt(acosh(c + d*x)**3/sqrt(c + d*x),x)*b**3*d + 3*int(acosh(c + d*x)**2/sqr
t(c + d*x),x)*a*b**2*d)/(sqrt(e)*d)
```

3.132 $\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\text{arccosh}(c+dx))^3}{(e(c+dx))^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^3/(e*(d*x+c))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 17.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

output
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}dx$$
$$\Big\downarrow \text{6411}$$
$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{6298}$$
$$\frac{\frac{6b\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{e}-\frac{2(a+b\text{arccosh}(c+dx))^3}{e\sqrt{e(c+dx)}}}{d}$$
$$\Big\downarrow \text{6376}$$
$$\frac{\frac{6b\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{e(c+dx)}\sqrt{c+dx+1}}d(c+dx)}{e}-\frac{2(a+b\text{arccosh}(c+dx))^3}{e\sqrt{e(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\ \mathrm{arccosh}\,(dx+c))^{3}}{(dex+ce)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^{3}}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e
^2), x)
```

Sympy [N/A]

Not integrable

Time = 8.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\mathrm{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acosh}\,(c+dx))^{3}}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.12

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \frac{3\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx\right)a^2bd+\sqrt{dx+c}\left(\int \frac{acosh(dx+c)^3}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx\right)}{\sqrt{e}\,\sqrt{dx+c}\,de}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

output
```
(3*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),
x)*a**2*b*d + sqrt(c + d*x)*int(acosh(c + d*x)**3/(sqrt(c + d*x)*c + sqrt(
c + d*x)*d*x),x)*b**3*d + 3*sqrt(c + d*x)*int(acosh(c + d*x)**2/(sqrt(c +
d*x)*c + sqrt(c + d*x)*d*x),x)*a*b**2*d - 2*a**3)/(sqrt(e)*sqrt(c + d*x)*d
*e)
```

3.133 $\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}} dx = \text{Int}\left(\frac{(a+b\text{arccosh}(c+dx))^3}{(e(c+dx))^{5/2}}, x\right)$$

output
```
Defer(Int)((a+b*arccosh(d*x+c))^3/(e*(d*x+c))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 15.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}} dx = \int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}} dx$$

input
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

output
```
Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}} \, dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(a+b\text{arccosh}(c+dx))^3}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{2b\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(e(c+dx))^{3/2}\sqrt{c+dx+1}}d(c+dx)}{e}-\frac{2(a+b\text{arccosh}(c+dx))^3}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6376}$$

$$\frac{\frac{2b\int \frac{(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}(e(c+dx))^{3/2}\sqrt{c+dx+1}}d(c+dx)}{e}-\frac{2(a+b\text{arccosh}(c+dx))^3}{3e(e(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

output
```
int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.88

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3
*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [N/A]

Not integrable

Time = 28.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acosh}(c+dx))^3}{(e(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{(ce + dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arccosh}(c + dx))^3}{(ce + dex)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^3}{{\left(c\,e+d\,e\,x\right)}^{5/2}} \,d x$$

input
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 405, normalized size of antiderivative = 16.20

$$\int \frac{(a+b\text{arccosh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\frac{9\sqrt{dx+c}\left(\int \frac{acosh(dx+c)}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx\right)a^2bcd+9\sqrt{dx+c}\left(\int \frac{}{\sqrt{dx+}}\right.}{}$$

input
```
int((a+b*acosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

output
```
(9*sqrt(c + d*x)*int(acosh(c + d*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*
c*d*x + sqrt(c + d*x)*d**2*x**2),x)*a**2*b*c*d + 9*sqrt(c + d*x)*int(acosh
(c + d*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2
*x**2),x)*a**2*b*d**2*x + 3*sqrt(c + d*x)*int(acosh(c + d*x)**3/(sqrt(c +
d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2*x**2),x)*b**3*c*d +
 3*sqrt(c + d*x)*int(acosh(c + d*x)**3/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*
x)*c*d*x + sqrt(c + d*x)*d**2*x**2),x)*b**3*d**2*x + 9*sqrt(c + d*x)*int(a
cosh(c + d*x)**2/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*
x)*d**2*x**2),x)*a*b**2*c*d + 9*sqrt(c + d*x)*int(acosh(c + d*x)**2/(sqrt(
c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2*x**2),x)*a*b**2
*d**2*x - 2*a**3)/(3*sqrt(e)*sqrt(c + d*x)*d*e**2*(c + d*x))
```

3.134 $\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx=\text{Int}\left((e(c+dx))^m(a+b\text{arccosh}(c+dx))^4,x\right)$$

output
```
Defer(Int)((e*(d*x+c))^m*(a+b*arccosh(d*x+c))^4,x)
```

Mathematica [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx=\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arccosh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^4}{e(m+1)}-\frac{4b\int \frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{6376}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^4}{e(m+1)}-\frac{4b\int \frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e(m+1)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (dex+ce)^m (a+b\operatorname{arccosh}(dx+c))^4\,dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int (ce+dex)^m(a+b\mathrm{arccosh}(c+dx))^4\,dx = \int (b\operatorname{arcosh}(dx+c)+a)^4(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*
arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

Sympy [N/A]

Not integrable

Time = 94.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m(a+b\mathrm{arccosh}(c+dx))^4\,dx = \int (e(c+dx))^m\,(a+b\operatorname{acosh}(c+dx))^4\,dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**4,x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**4, x)
```

Maxima [N/A]

Not integrable

Time = 6.37 (sec) , antiderivative size = 935, normalized size of antiderivative = 40.65

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx = \int (b\operatorname{arcosh}(dx+c)+a)^4(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
(b^4*d*e^m*x + b^4*c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + c)^4/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^4/(d*e*(m + 1)) + i
ntegrate(-2*(2*((b^4*c^2*e^m - (c^2*e^m*(m + 1) - e^m*(m + 1))*a*b^3 - (a*
b^3*d^2*e^m*(m + 1) - b^4*d^2*e^m)*x^2 - 2*(a*b^3*c*d*e^m*(m + 1) - b^4*c*
d*e^m)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - ((c^3*e^m*(m +
 1) - c*e^m*(m + 1))*a*b^3 - (c^3*e^m - c*e^m)*b^4 + (a*b^3*d^3*e^m*(m + 1
) - b^4*d^3*e^m)*x^3 + 3*(a*b^3*c*d^2*e^m*(m + 1) - b^4*c*d^2*e^m)*x^2 + (
(3*c^2*d*e^m*(m + 1) - d*e^m*(m + 1))*a*b^3 - (3*c^2*d*e^m - d*e^m)*b^4)*x
)*(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 - 3*((
a^2*b^2*d^2*e^m*(m + 1)*x^2 + 2*a^2*b^2*c*d*e^m*(m + 1)*x + (c^2*e^m*(m +
1) - e^m*(m + 1))*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m
 + (a^2*b^2*d^3*e^m*(m + 1)*x^3 + 3*a^2*b^2*c*d^2*e^m*(m + 1)*x^2 + (3*c^2
*d*e^m*(m + 1) - d*e^m*(m + 1))*a^2*b^2*x + (c^3*e^m*(m + 1) - c*e^m*(m +
1))*a^2*b^2)*(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
c)^2 - 2*((a^3*b*d^2*e^m*(m + 1)*x^2 + 2*a^3*b*c*d*e^m*(m + 1)*x + (c^2*e^
m*(m + 1) - e^m*(m + 1))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x +
 c)^m + (a^3*b*d^3*e^m*(m + 1)*x^3 + 3*a^3*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^
2*d*e^m*(m + 1) - d*e^m*(m + 1))*a^3*b*x + (c^3*e^m*(m + 1) - c*e^m*(m + 1
))*a^3*b)*(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^...
```

Giac [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx = \int (b\operatorname{arcosh}(dx+c)+a)^4(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

Mupad [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^m\,(a+b\,\mathrm{acosh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 11.52

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^4\,dx$$
$$= \frac{(dex+ce)^m\,a^4c+(dex+ce)^m\,a^4dx+4\left(\int (dex+ce)^m\,acosh(dx+c)\,dx\right)a^3bdm+4\left(\int (dex+ce)^m\,aco\right.}{}$$

input
```
int((d*e*x+c*e)^m*(a+b*acosh(d*x+c))^4,x)
```

output
```
((c*e + d*e*x)**m*a**4*c + (c*e + d*e*x)**m*a**4*d*x + 4*int((c*e + d*e*x)
**m*acosh(c + d*x),x)*a**3*b*d*m + 4*int((c*e + d*e*x)**m*acosh(c + d*x),x
)*a**3*b*d + int((c*e + d*e*x)**m*acosh(c + d*x)**4,x)*b**4*d*m + int((c*e
 + d*e*x)**m*acosh(c + d*x)**4,x)*b**4*d + 4*int((c*e + d*e*x)**m*acosh(c
+ d*x)**3,x)*a*b**3*d*m + 4*int((c*e + d*e*x)**m*acosh(c + d*x)**3,x)*a*b*
*3*d + 6*int((c*e + d*e*x)**m*acosh(c + d*x)**2,x)*a**2*b**2*d*m + 6*int((
c*e + d*e*x)**m*acosh(c + d*x)**2,x)*a**2*b**2*d)/(d*(m + 1))
```

3.135 $\int (ce+dex)^m(a+b\mathbf{arccosh}(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx=\text{Int}\left((e(c+dx))^m(a+b\text{arccosh}(c+dx))^3,x\right)$$

output
```
Defer(Int)((e*(d*x+c))^m*(a+b*arccosh(d*x+c))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx=\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arccosh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^3}{e(m+1)}-\frac{3b\int \frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{ 6376}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^3}{e(m+1)}-\frac{3b\int \frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e(m+1)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (dex+ce)^m (a+b\operatorname{arccosh}(dx+c))^3\,dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int (ce+dex)^m (a+b\text{arccosh}(c+dx))^3\,dx = \int (b\operatorname{arcosh}(dx+c)+a)^3 (dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*ar
ccosh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

Sympy [N/A]

Not integrable

Time = 36.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m (a+b\text{arccosh}(c+dx))^3\,dx = \int (e(c+dx))^m (a+b\operatorname{acosh}(c+dx))^3\,dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**3,x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 713, normalized size of antiderivative = 31.00

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx = \int (b\,\text{arcosh}\,(dx+c)+a)^3(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(b^3*d*e^m*x + b^3*c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + c)^3/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^3/(d*e*(m + 1)) + i
ntegrate(-3*(((b^3*c^2*e^m - (c^2*e^m*(m + 1) - e^m*(m + 1))*a*b^2 - (a*b^
2*d^2*e^m*(m + 1) - b^3*d^2*e^m)*x^2 - 2*(a*b^2*c*d*e^m*(m + 1) - b^3*c*d*
e^m)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - ((c^3*e^m*(m + 1
) - c*e^m*(m + 1))*a*b^2 - (c^3*e^m - c*e^m)*b^3 + (a*b^2*d^3*e^m*(m + 1)
- b^3*d^3*e^m)*x^3 + 3*(a*b^2*c*d^2*e^m*(m + 1) - b^3*c*d^2*e^m)*x^2 + ((3
*c^2*d*e^m*(m + 1) - d*e^m*(m + 1))*a*b^2 - (3*c^2*d*e^m - d*e^m)*b^3)*x)*
(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - ((a^2*
b*d^2*e^m*(m + 1)*x^2 + 2*a^2*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) - e^m
*(m + 1))*a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m + (a^2*b*
d^3*e^m*(m + 1)*x^3 + 3*a^2*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1)
 - d*e^m*(m + 1))*a^2*b*x + (c^3*e^m*(m + 1) - c*e^m*(m + 1))*a^2*b)*(d*x
+ c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*(m + 1)*x
^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*
x + c^2*(m + 1) - m - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) +
 (3*c^2*d*(m + 1) - d*(m + 1))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx=\int (b\,\text{arcosh}\,(dx+c)+a)^3(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

Mupad [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^m\,(a+b\,\text{acosh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 8.87

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^3\,dx$$
$$=\frac{(dex+ce)^m\,a^3c+(dex+ce)^m\,a^3dx+3\left(\int (dex+ce)^m\,acosh(dx+c)\,dx\right)a^2bdm+3\left(\int (dex+ce)^m\,aco\right.}{}$$

input
```
int((d*e*x+c*e)^m*(a+b*acosh(d*x+c))^3,x)
```

output
```
((c*e + d*e*x)**m*a**3*c + (c*e + d*e*x)**m*a**3*d*x + 3*int((c*e + d*e*x)
**m*acosh(c + d*x),x)*a**2*b*d*m + 3*int((c*e + d*e*x)**m*acosh(c + d*x),x
)*a**2*b*d + int((c*e + d*e*x)**m*acosh(c + d*x)**3,x)*b**3*d*m + int((c*e
 + d*e*x)**m*acosh(c + d*x)**3,x)*b**3*d + 3*int((c*e + d*e*x)**m*acosh(c
+ d*x)**2,x)*a*b**2*d*m + 3*int((c*e + d*e*x)**m*acosh(c + d*x)**2,x)*a*b*
*2*d)/(d*(m + 1))
```

3.136 $\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 206

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx = \frac{(e(c+dx))^{1+m}(a+b\text{arccosh}(c+dx))^2}{de(1+m)} - \frac{2b\sqrt{1-c-dx}(e(c+dx))^{2+m}(a+b\text{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},(c+dx)^2\right)}{de^2(1+m)(2+m)\sqrt{-1+c+dx}} - \frac{2b^2(e(c+dx))^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};(c+dx)^2\right)}{de^3(1+m)\left(6+5m+m^2\right)}$$

output
```
(e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^2/d/e/(1+m)-2*b*(-d*x-c+1)^(1/2)*(e
*(d*x+c))^(2+m)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],(d
*x+c)^2)/d/e^2/(1+m)/(2+m)/(d*x+c-1)^(1/2)-2*b^2*(e*(d*x+c))^(3+m)*hyperge
om([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],(d*x+c)^2)/d/e^3/(1+m)/(
m^2+5*m+6)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.86

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx$$

$$= \frac{(c+dx)(e(c+dx))^m\left((a+b\text{arccosh}(c+dx))^2 - \frac{2b(c+dx)\left(\frac{\sqrt{1-(c+dx)^2}(a+b\text{arccosh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\right.}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right.}{2+m}\right.}{d(1+m)}$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
((c + d*x)*(e*(c + d*x))^m*((a + b*ArcCosh[c + d*x])^2 - (2*b*(c + d*x)*((
Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, (2 +
 m)/2, (4 + m)/2, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + (
b*(c + d*x)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m
/2}, (c + d*x)^2])/(3 + m)))/(2 + m)))/(d*(1 + m))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6411, 6298, 6364}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arccosh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6298}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^2}{e(m+1)}-\frac{2b\int\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{6364}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))^2}{e(m+1)}-\frac{2b\left(\frac{b(e(c+dx))^{m+3}{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};(c+dx)^2\right)}{e^2(m+2)(m+3)}+\frac{\sqrt{-c-dx+1}(e(c+dx))^{m+2}\text{Hypergeometric2}}{e(m}\right.}{e(m+1)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
(((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^2)/(e*(1 + m)) - (2*b*((S
qrt[1 - c - d*x]*(e*(c + d*x))^(2 + m)*(a + b*ArcCosh[c + d*x])*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/(e*(2 + m)*Sqrt[-1 + c +
d*x]) + (b*(e*(c + d*x))^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/
2}, {2 + m/2, 5/2 + m/2}, (c + d*x)^2])/(e^2*(2 + m)*(3 + m))))/(e*(1 + m)
))/d
```

Defintions of rubi rules used

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6364
```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^m (a + b \operatorname{arccosh}(dx + c))^2 \, dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

Fricas [F]

$$\int (ce + dex)^m (a + b\text{arccosh}(c + dx))^2 \, dx = \int (b \operatorname{arcosh}(dx + c) + a)^2 (dex + ce)^m \, dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*(d*e*x +
c*e)^m, x)
```

Sympy [F]

$$\int (ce + dex)^m (a + b\text{arccosh}(c + dx))^2 \, dx = \int (e(c + dx))^m \left(a + b \operatorname{acosh}(c + dx)\right)^2 \, dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**2,x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**2, x)
```

Maxima [F]

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx=\int (b\,\text{arcosh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
(b^2*d*e^m*x + b^2*c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + c)^2/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^2/(d*e*(m + 1)) + i
ntegrate(-2*((b^2*c^2*e^m - (c^2*e^m*(m + 1) - e^m*(m + 1))*a*b - (a*b*d^2
*e^m*(m + 1) - b^2*d^2*e^m)*x^2 - 2*(a*b*c*d*e^m*(m + 1) - b^2*c*d*e^m)*x)
*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - ((a*b*d^3*e^m*(m + 1) -
 b^2*d^3*e^m)*x^3 + (c^3*e^m*(m + 1) - c*e^m*(m + 1))*a*b - (c^3*e^m - c*e
^m)*b^2 + 3*(a*b*c*d^2*e^m*(m + 1) - b^2*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*(m
 + 1) - d*e^m*(m + 1))*a*b - (3*c^2*d*e^m - d*e^m)*b^2)*x)*(d*x + c)^m)*lo
g(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*(m + 1)*x^3 + 3*c*d^
2*(m + 1)*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m
+ 1) - m - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(
m + 1) - d*(m + 1))*x), x)
```

Giac [F]

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx=\int (b\,\text{arcosh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^m\,(a+b\,\text{acosh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^2, x)
```

Reduce [F]

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))^2\,dx$$
$$=\frac{(dex+ce)^m\,a^2c+(dex+ce)^m\,a^2dx+2\left(\int (dex+ce)^m\,acosh(dx+c)\,dx\right)abdm+2\left(\int (dex+ce)^m\,acos\right.}{d\,(m\,}$$

input
```
int((d*e*x+c*e)^m*(a+b*acosh(d*x+c))^2,x)
```

output
```
((c*e + d*e*x)**m*a**2*c + (c*e + d*e*x)**m*a**2*d*x + 2*int((c*e + d*e*x)
**m*acosh(c + d*x),x)*a*b*d*m + 2*int((c*e + d*e*x)**m*acosh(c + d*x),x)*a
*b*d + int((c*e + d*e*x)**m*acosh(c + d*x)**2,x)*b**2*d*m + int((c*e + d*e
*x)**m*acosh(c + d*x)**2,x)*b**2*d)/(d*(m + 1))
```

3.137 $\int (ce+dex)^m(a+b\,\text{arccosh}(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int (ce+dex)^m(a+b\,\text{arccosh}(c+dx))\,dx = \frac{(e(c+dx))^{1+m}(a+b\,\text{arccosh}(c+dx))}{de(1+m)} - \frac{b\sqrt{1-c-dx}(e(c+dx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},(c+dx)^2\right)}{de^2(1+m)(2+m)\sqrt{-1+c+dx}}$$

output

```
(e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))/d/e/(1+m)-b*(-d*x-c+1)^(1/2)*(e*(d*
x+c))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],(d*x+c)^2)/d/e^2/(1+m)/(2+m
)/(d*x+c-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int (ce+dex)^m(a+b\,\text{arccosh}(c+dx))\,dx = \frac{(c+dx)(e(c+dx))^m\left(a+b\,\text{arccosh}(c+dx)-\frac{b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},(c+dx)^2\right)}{(2+m)\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right)}{d(1+m)}$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

output
```
((c + d*x)*(e*(c + d*x))^m*(a + b*ArcCosh[c + d*x] - (b*(c + d*x)*Sqrt[1 -
 (c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/(
(2 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(d*(1 + m))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6411, 6298, 136, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))\,dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arccosh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6298}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))}{e(m+1)}-\frac{b\int \frac{(e(c+dx))^{m+1}}{\sqrt{c+dx-1}\sqrt{c+dx+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{136}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))}{e(m+1)}-\frac{b\sqrt{(c+dx)^2-1}\int \frac{(e(c+dx))^{m+1}}{\sqrt{(c+dx)^2-1}}d(c+dx)}{e(m+1)\sqrt{c+dx-1}\sqrt{c+dx+1}}}{d}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arccosh}(c+dx))}{e(m+1)}-\frac{b\sqrt{1-(c+dx)^2}\int \frac{(e(c+dx))^{m+1}}{\sqrt{1-(c+dx)^2}}d(c+dx)}{e(m+1)\sqrt{c+dx-1}\sqrt{c+dx+1}}}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\,\text{arccosh}(c+dx))}{e(m+1)} - \frac{b\sqrt{1-(c+dx)^2}(e(c+dx))^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},(c+dx)^2\right)}{e^2(m+1)(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

output
```
(((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x]))/(e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, (c + d*x)^2])/(e^2*(1 + m)*(2 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c +
 d*x]))/d
```

Defintions of rubi rules used

rule 136
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^Fr
acPart[m])   Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b, c, d, f,
 m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 6298
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& NeQ[m, -1]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [F]

$$\int (dex + ce)^m (a + b \operatorname{arccosh}(dx + c))\, dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

Fricas [F]

$$\int (ce + dex)^m (a + b\text{arccosh}(c + dx))\, dx = \int (b \operatorname{arcosh}(dx + c) + a)(dex + ce)^m\, dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

Sympy [F]

$$\int (ce + dex)^m (a + b\text{arccosh}(c + dx))\, dx = \int (e(c + dx))^m\, (a + b \operatorname{acosh}(c + dx))\, dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x)), x)
```

Maxima [F]

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))\,dx = \int (b\,\text{arcosh}\,(dx+c)+a)(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
b*((d*e^m*x + c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + c)/(d*(m + 1)) - integrate((d^2*e^m*x^2 + 2*c*d*e^m*x + c^2*e^m)*(d
*x + c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m - 1), x) +
integrate((d*e^m*x + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)
*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m
- 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1) -
d*(m + 1))*x), x)) + (d*e*x + c*e)^(m + 1)*a/(d*e*(m + 1))
```

Giac [F]

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))\,dx = \int (b\,\text{arcosh}\,(dx+c)+a)(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^m\,(a+b\,\text{acosh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^m*(a + b*acosh(c + d*x)), x)
```

Reduce [F]

$$\int (ce+dex)^m(a+b\text{arccosh}(c+dx))\,dx$$
$$=\frac{(dex+ce)^m\,ac+(dex+ce)^m\,adx+\left(\int (dex+ce)^m\,acosh(dx+c)\,dx\right)bdm+\left(\int (dex+ce)^m\,acosh(dx\right.}{d\,(m+1)}$$

input
```
int((d*e*x+c*e)^m*(a+b*acosh(d*x+c)),x)
```

output
```
((c*e + d*e*x)**m*a*c + (c*e + d*e*x)**m*a*d*x + int((c*e + d*e*x)**m*acos
h(c + d*x),x)*b*d*m + int((c*e + d*e*x)**m*acosh(c + d*x),x)*b*d)/(d*(m +
1))
```

3.138 $\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx = \text{Int}\left(\frac{(e(c+dx))^m}{a+b\text{arccosh}(c+dx)},x\right)$$

output
```
Defer(Int)((e*(d*x+c))^m/(a+b*arccosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx = \int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx$$

input
```
Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]
```

output
```
Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{(e(c+dx))^m}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6303}$$

$$\frac{\int \frac{(e(c+dx))^m}{a+b\text{arccosh}(c+dx)}d(c+dx)}{d}$$

input
```
Int[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(dex+ce)^m}{a+b\,\text{arccosh}\,(dx+c)}dx$$

input
```
int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

output
```
int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^m}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(e(c+dx))^m}{a+b\,\text{acosh}\,(c+dx)}\,dx$$

input
```
integrate((d*e*x+c*e)**m/(a+b*acosh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**m/(a + b*acosh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^m}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(dex+ce)^m}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx=\int \frac{(c\,e+d\,e\,x)^m}{a+b\,\text{acosh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^m/(a + b*acosh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^m/(a + b*acosh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arccosh}(c+dx)}\,dx = \int \frac{(dex+ce)^m}{acosh\,(dx+c)\,b+a}dx$$

input
```
int((d*e*x+c*e)^m/(a+b*acosh(d*x+c)),x)
```

output
```
int((c*e + d*e*x)**m/(acosh(c + d*x)*b + a),x)
```

3.139 $\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2} \, dx = \text{Int}\left(\frac{(e(c+dx))^m}{(a+b\text{arccosh}(c+dx))^2}, x\right)$$

output
```
Defer(Int)((e*(d*x+c))^m/(a+b*arccosh(d*x+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2} \, dx = \int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2} \, dx$$

input
```
Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}dx$$

$$\Big\downarrow \text{ 6411}$$

$$\frac{\int \frac{(e(c+dx))^m}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6303}$$

$$\frac{\int \frac{(e(c+dx))^m}{(a+b\text{arccosh}(c+dx))^2}d(c+dx)}{d}$$

input
```
Int[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(dex+ce)^m}{(a+b\ \text{arccosh}\,(dx+c))^2}dx$$

input
```
int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^m}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^m/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c)
+ a^2), x)
```

Sympy [N/A]

Not integrable

Time = 14.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(e(c+dx))^m}{(a+b\,\text{acosh}\,(c+dx))^2}\,dx$$

input
```
integrate((d*e*x+c*e)**m/(a+b*acosh(d*x+c))**2,x)
```

output
```
Integral((e*(c + d*x))**m/(a + b*acosh(c + d*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 1180, normalized size of antiderivative = 51.30

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^m}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((d^2*e^m*x^2 + 2*c*d*e^m*x + c^2*e^m - e^m)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1)*(d*x + c)^m + (d^3*e^m*x^3 + 3*c*d^2*e^m*x^2 + c^3*e^m - c*e^m + (
3*c^2*d*e^m - d*e^m)*x)*(d*x + c)^m)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d
 - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b
^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)) + integrate(((d^3*e^m*(m + 1)*x^3 + 3*c*d^2*e^m*(m + 1)*x^2 + c^
3*e^m*(m + 1) - c*e^m*(m - 1) + (3*c^2*d*e^m*(m + 1) - d*e^m*(m - 1))*x)*(
d*x + c + 1)*(d*x + c - 1)*(d*x + c)^m + (2*d^4*e^m*(m + 1)*x^4 + 8*c*d^3*
e^m*(m + 1)*x^3 + 2*c^4*e^m*(m + 1) - c^2*e^m*(3*m + 1) + (12*c^2*d^2*e^m*
(m + 1) - d^2*e^m*(3*m + 1))*x^2 + e^m*m + 2*(4*c^3*d*e^m*(m + 1) - c*d*e^
m*(3*m + 1))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m + (d^5*e^m
*(m + 1)*x^5 + 5*c*d^4*e^m*(m + 1)*x^4 + c^5*e^m*(m + 1) - 2*c^3*e^m*(m +
1) + 2*(5*c^2*d^3*e^m*(m + 1) - d^3*e^m*(m + 1))*x^3 + c*e^m*(m + 1) + 2*(
5*c^3*d^2*e^m*(m + 1) - 3*c*d^2*e^m*(m + 1))*x^2 + (5*c^4*d*e^m*(m + 1) -
6*c^2*d*e^m*(m + 1) + d*e^m*(m + 1))*x)*(d*x + c)^m)/(a*b*d^5*x^5 + 5*a*b*
c*d^4*x^4 + 2*(5*c^2*d^3 - d^3)*a*b*x^3 + 2*(5*c^3*d^2 - 3*c*d^2)*a*b*x^2
+ (5*c^4*d - 6*c^2*d + d)*a*b*x + (a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c
^2*d*x + a*b*c^3)*(d*x + c + 1)*(d*x + c - 1) + (c^5 - 2*c^3 + c)*a*b + 2*
(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + (6*c^2*d^2 - d^2)*a*b*x^2 + 2*(2*c^3*d...
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^m}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^m}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((c*e + d*e*x)^m/(a + b*acosh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^m/(a + b*acosh(c + d*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{(ce+dex)^m}{(a+b\text{arccosh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^m}{acosh\,(dx+c)^2\,b^2+2acosh\,(dx+c)\,ab+a^2}dx$$

input
```
int((d*e*x+c*e)^m/(a+b*acosh(d*x+c))^2,x)
```

output
```
int((c*e + d*e*x)**m/(acosh(c + d*x)**2*b**2 + 2*acosh(c + d*x)*a*b + a**2
),x)
```

3.140 $\int \frac{\text{arccosh}(ax^5)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\text{arccosh}(ax^5)}{x} \, dx = -\frac{1}{10}\text{arccosh}(ax^5)^2 + \frac{1}{5}\text{arccosh}(ax^5)\log\left(1+e^{2\text{arccosh}(ax^5)}\right) + \frac{1}{10}\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax^5)}\right)$$

output
```
-1/10*arccosh(a*x^5)^2+1/5*arccosh(a*x^5)*ln(1+(a*x^5+(a*x^5-1)^(1/2)*(a*x
^5+1)^(1/2))^2)+1/10*polylog(2,-(a*x^5+(a*x^5-1)^(1/2)*(a*x^5+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\text{arccosh}(ax^5)}{x} \, dx = \frac{1}{10}\left(\text{arccosh}(ax^5)\left(\text{arccosh}(ax^5)+2\log\left(1+e^{-2\text{arccosh}(ax^5)}\right)\right) - \text{PolyLog}\left(2,-e^{-2\text{arccosh}(ax^5)}\right)\right)$$

input
```
Integrate[ArcCosh[a*x^5]/x,x]
```

output
```
(ArcCosh[a*x^5]*(ArcCosh[a*x^5] + 2*Log[1 + E^(-2*ArcCosh[a*x^5])]) - Poly
Log[2, -E^(-2*ArcCosh[a*x^5])])/10
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6426, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax^5)}{x} dx$$

$$\Big\downarrow \text{6426}$$

$$\frac{1}{5}\int \frac{\sqrt{\frac{ax^5-1}{ax^5+1}}(ax^5+1)\,\text{arccosh}(ax^5)}{ax^5}d\text{arccosh}(ax^5)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int -i\text{arccosh}(ax^5)\tan\left(i\text{arccosh}(ax^5)\right)d\text{arccosh}(ax^5)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{5}i\int \text{arccosh}(ax^5)\tan\left(i\text{arccosh}(ax^5)\right)d\text{arccosh}(ax^5)$$

$$\Big\downarrow \text{4201}$$

$$-\frac{1}{5}i\left(2i\int \frac{e^{2\text{arccosh}(ax^5)}\text{arccosh}(ax^5)}{1+e^{2\text{arccosh}(ax^5)}}d\text{arccosh}(ax^5)-\frac{1}{2}i\text{arccosh}(ax^5)^2\right)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{5}i\left(2i\left(\frac{1}{2}\text{arccosh}(ax^5)\log\left(e^{2\text{arccosh}(ax^5)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arccosh}(ax^5)}\right)d\text{arccosh}(ax^5)\right)-\frac{1}{2}i\text{arccosh}(ax$$

$$\Big\downarrow \text{2715}$$

$$-\frac{1}{5}i\left(2i\left(\frac{1}{2}\text{arccosh}(ax^5)\log\left(e^{2\text{arccosh}(ax^5)}+1\right)-\frac{1}{4}\int e^{-2\text{arccosh}(ax^5)}\log\left(1+e^{2\text{arccosh}(ax^5)}\right)de^{2\text{arccosh}(ax^5)}\right)-\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{1}{5}i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax^5)}\right)+\frac{1}{2}\text{arccosh}(ax^5)\log\left(e^{2\text{arccosh}(ax^5)}+1\right)\right)-\frac{1}{2}i\text{arccosh}(ax^5)^2\right)$$

input
```
Int[ArcCosh[a*x^5]/x,x]
```

output
```
(-1/5*I)*((-1/2*I)*ArcCosh[a*x^5]^2 + (2*I)*((ArcCosh[a*x^5]*Log[1 + E^(2*
ArcCosh[a*x^5])])/2 + PolyLog[2, -E^(2*ArcCosh[a*x^5])]/4))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6426
```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\operatorname{arccosh}\left(a\,x^{5}\right)}{x}dx$$

input
```
int(arccosh(a*x^5)/x,x)
```

output
```
int(arccosh(a*x^5)/x,x)
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcosh}\left(ax^{5}\right)}{x}\,dx$$

input
```
integrate(arccosh(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arccosh(a*x^5)/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax^5)}{x}\,dx = \int \frac{\operatorname{acosh}\left(ax^5\right)}{x}\,dx$$

input
```
integrate(acosh(a*x**5)/x,x)
```

output
```
Integral(acosh(a*x**5)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcosh}\left(ax^5\right)}{x}\,dx$$

input
```
integrate(arccosh(a*x^5)/x,x, algorithm="maxima")
```

output
```
integrate(arccosh(a*x^5)/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcosh}\left(ax^5\right)}{x}\,dx$$

input
```
integrate(arccosh(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arccosh(a*x^5)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax^5)}{x}\,dx = \int \frac{\operatorname{acosh}(a\,x^5)}{x}\,dx$$

input
```
int(acosh(a*x^5)/x,x)
```

output
```
int(acosh(a*x^5)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax^5)}{x}\,dx = \int \frac{acosh(a\,x^5)}{x}dx$$

input
```
int(acosh(a*x^5)/x,x)
```

output
```
int(acosh(a*x**5)/x,x)
```

3.141 $\int x^2 \text{arccosh}(\sqrt{x})\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int x^2 \text{arccosh}(\sqrt{x})\,dx = -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}-\frac{5\text{arccosh}(\sqrt{x})}{48}+\frac{1}{3}x^3\text{arccosh}(\sqrt{x})$$

output
```
-5/48*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)-5/72*(-1+x^(1/2))^(1/2)
*(1+x^(1/2))^(1/2)*x^(3/2)-1/18*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/
2)-5/48*arccosh(x^(1/2))+1/3*x^3*arccosh(x^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int x^2 \text{arccosh}(\sqrt{x})\,dx = \frac{1}{144}\left(-\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}(15+10x+8x^2)+48x^3\text{arccosh}(\sqrt{x})-30\text{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

input
```
Integrate[x^2*ArcCosh[Sqrt[x]],x]
```

output
```
(-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2)) + 48*
x^3*ArcCosh[Sqrt[x]] - 30*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/144
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6432, 27, 845, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arccosh}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{6432}$$

$$\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)-\frac{1}{6}\int \frac{x^{5/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{6}\left(-\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{6}\left(-\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{6}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-\right.$$
$$\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow 852$$

$$\frac{1}{6}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\right.$$
$$\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow 43$$

$$\frac{1}{6}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)$$
$$\frac{1}{3}x^3\text{arccosh}\left(\sqrt{x}\right)$$

input

```
Int[x^2*ArcCosh[Sqrt[x]],x]
```

output

```
(x^3*ArcCosh[Sqrt[x]])/3 + (-1/3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2)) - (5*((Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (3*(Sqrt[-1
 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]))/4))/6)/6
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

rule 6432
```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
 + u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFu
nctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOf
ExponentialQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result	siz
derivativedivides	$\frac{x^3 \operatorname{arccosh}(\sqrt{x})}{3}-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\left(8\sqrt{-1+x}\,x^{\frac{5}{2}}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{144\sqrt{-1+x}}$	75
default	$\frac{x^3 \operatorname{arccosh}(\sqrt{x})}{3}-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\left(8\sqrt{-1+x}\,x^{\frac{5}{2}}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{144\sqrt{-1+x}}$	75
parts	$\frac{x^3 \operatorname{arccosh}(\sqrt{x})}{3}-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\left(8\sqrt{-1+x}\,x^{\frac{5}{2}}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{144\sqrt{-1+x}}$	75

input
```
int(x^2*arccosh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccosh(x^(1/2))-1/144*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*(-1
+x)^(1/2)*x^(5/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^
(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.34

$$\int x^2\text{arccosh}\left(\sqrt{x}\right)\,dx = -\frac{1}{144}\left(8\,x^2+10\,x+15\right)\sqrt{x-1}\sqrt{x} + \frac{1}{48}\left(16\,x^3-5\right)\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

input
```
integrate(x^2*arccosh(x^(1/2)),x, algorithm="fricas")
```

output
```
-1/144*(8*x^2 + 10*x + 15)*sqrt(x - 1)*sqrt(x) + 1/48*(16*x^3 - 5)*log(sqr
t(x - 1) + sqrt(x))
```

Sympy [F]

$$\int x^2\text{arccosh}\left(\sqrt{x}\right)\,dx = \int x^2\,\text{acosh}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x**2*acosh(x**(1/2)),x)
```

output
```
Integral(x**2*acosh(sqrt(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48

$$\int x^2 \operatorname{arccosh}(\sqrt{x})\,dx = \frac{1}{3}\,x^3 \operatorname{arcosh}\left(\sqrt{x}\right) - \frac{1}{18}\,\sqrt{x-1}x^{\frac{5}{2}} - \frac{5}{72}\,\sqrt{x-1}x^{\frac{3}{2}} - \frac{5}{48}\,\sqrt{x-1}\sqrt{x} - \frac{5}{48}\,\log\left(2\,\sqrt{x-1} + 2\,\sqrt{x}\right)$$

input
```
integrate(x^2*arccosh(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccosh(sqrt(x)) - 1/18*sqrt(x - 1)*x^(5/2) - 5/72*sqrt(x - 1)*x^(
3/2) - 5/48*sqrt(x - 1)*sqrt(x) - 5/48*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

$$\int x^2 \operatorname{arccosh}(\sqrt{x})\,dx = \frac{1}{3}\,x^3 \log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \sqrt{x}\right) - \frac{1}{144}\,(2\,(4\,x+5)x+15)\sqrt{x-1}\sqrt{x} + \frac{5}{48}\,\log\left(-\sqrt{x-1} + \sqrt{x}\right)$$

input
```
integrate(x^2*arccosh(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/144*(2*(4*x
 + 5)*x + 15)*sqrt(x - 1)*sqrt(x) + 5/48*log(-sqrt(x - 1) + sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arccosh}\left(\sqrt{x}\right)\,dx = \int x^2\,\text{acosh}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^2*acosh(x^(1/2)),x)
```

output
```
int(x^2*acosh(x^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int x^2 \text{arccosh}\left(\sqrt{x}\right)\,dx = \frac{acosh\left(\sqrt{x}\right)x^3}{3} - \frac{\sqrt{x}\,\sqrt{x-1}\,x^2}{18} - \frac{5\sqrt{x}\,\sqrt{x-1}\,x}{72} - \frac{5\sqrt{x}\,\sqrt{x-1}}{48} - \frac{5\log\left(\sqrt{x-1}+\sqrt{x}\right)}{48}$$

input
```
int(x^2*acosh(x^(1/2)),x)
```

output
```
(48*acosh(sqrt(x))*x**3 - 8*sqrt(x)*sqrt(x - 1)*x**2 - 10*sqrt(x)*sqrt(x -
 1)*x - 15*sqrt(x)*sqrt(x - 1) - 15*log(sqrt(x - 1) + sqrt(x)))/144
```

3.142 $\int x\text{arccosh}\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 86

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx = -\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{3\text{arccosh}\left(\sqrt{x}\right)}{16}+\frac{1}{2}x^2\text{arccosh}\left(\sqrt{x}\right)$$

output
```
-3/16*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)-1/8*(-1+x^(1/2))^(1/2)*
(1+x^(1/2))^(1/2)*x^(3/2)-3/16*arccosh(x^(1/2))+1/2*x^2*arccosh(x^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx = \frac{1}{16}\left(-\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}(3+2x)+8x^2\text{arccosh}\left(\sqrt{x}\right) - 6\text{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

input
```
Integrate[x*ArcCosh[Sqrt[x]],x]
```

output
```
(-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x)) + 8*x^2*ArcCosh
[Sqrt[x]] - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/16
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6432, 27, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{6432}$$

$$\frac{1}{2}x^2\text{arccosh}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{x^{3/2}}{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\text{arccosh}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{4}\left(-\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{2}x^2\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{2}x^2\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{852}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{2}x^2\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow 43$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\operatorname{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{2}x^2\operatorname{arccosh}\left(\sqrt{x}\right)$$

input
```
Int[x*ArcCosh[Sqrt[x]],x]
```

output
```
(x^2*ArcCosh[Sqrt[x]])/2 + (-1/2*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
3/2)) - (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]
]))/4)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

rule 6432
```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
 + u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFu
nctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOf
ExponentialQ[u, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{x^{2} \operatorname{arccosh}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(2 x^{\frac{3}{2}} \sqrt{-1+x}+3 \sqrt{x}\, \sqrt{-1+x}+3 \ln \left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{16 \sqrt{-1+x}}$	65
default	$\frac{x^{2} \operatorname{arccosh}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(2 x^{\frac{3}{2}} \sqrt{-1+x}+3 \sqrt{x}\, \sqrt{-1+x}+3 \ln \left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{16 \sqrt{-1+x}}$	65
parts	$\frac{x^{2} \operatorname{arccosh}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(2 x^{\frac{3}{2}} \sqrt{-1+x}+3 \sqrt{x}\, \sqrt{-1+x}+3 \ln \left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{16 \sqrt{-1+x}}$	65

input
```
int(x*arccosh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccosh(x^(1/2))-1/16*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x^(3
/2)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx = -\frac{1}{16}\,(2\,x+3)\sqrt{x-1}\sqrt{x} + \frac{1}{16}\,(8\,x^2-3)\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

input
```
integrate(x*arccosh(x^(1/2)),x, algorithm="fricas")
```

output
```
-1/16*(2*x + 3)*sqrt(x - 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x - 1) + s
qrt(x))
```

Sympy [F]

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx = \int x\,\text{acosh}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x*acosh(x**(1/2)),x)
```

output
```
Integral(x*acosh(sqrt(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\,\text{arcosh}\left(\sqrt{x}\right) - \frac{1}{8}\,\sqrt{x-1}x^{\frac{3}{2}} - \frac{3}{16}\,\sqrt{x-1}\sqrt{x} - \frac{3}{16}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

input
```
integrate(x*arccosh(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccosh(sqrt(x)) - 1/8*sqrt(x - 1)*x^(3/2) - 3/16*sqrt(x - 1)*sqrt
(x) - 3/16*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.64

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx=\frac{1}{2}\,x^2\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{16}\,(2\,x+3)\sqrt{x-1}\sqrt{x}+\frac{3}{16}\,\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

input
```
integrate(x*arccosh(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/16*(2*x + 3
)*sqrt(x - 1)*sqrt(x) + 3/16*log(-sqrt(x - 1) + sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx=\int x\,\text{acosh}\left(\sqrt{x}\right)\,dx$$

input
```
int(x*acosh(x^(1/2)),x)
```

output
```
int(x*acosh(x^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

$$\int x\text{arccosh}\left(\sqrt{x}\right)\,dx=\frac{acosh\left(\sqrt{x}\right)x^2}{2}-\frac{\sqrt{x}\,\sqrt{x-1}\,x}{8}-\frac{3\sqrt{x}\,\sqrt{x-1}}{16}-\frac{3\log\left(\sqrt{x-1}+\sqrt{x}\right)}{16}$$

input
```
int(x*acosh(x^(1/2)),x)
```

output
```
(8*acosh(sqrt(x))*x**2 - 2*sqrt(x)*sqrt(x - 1)*x - 3*sqrt(x)*sqrt(x - 1) -
 3*log(sqrt(x - 1) + sqrt(x)))/16
```

3.143 $\int \operatorname{arccosh}\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 50

$$\int \operatorname{arccosh}\left(\sqrt{x}\right)\,dx=-\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2}+x\operatorname{arccosh}\left(\sqrt{x}\right)$$

output
```
-1/2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)-1/2*arccosh(x^(1/2))+x*a
rccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 273 vs. $2(50) = 100$.

Time = 4.60 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.46

$$\int \operatorname{arccosh}\left(\sqrt{x}\right)\,dx=$$

$$-\frac{2\left(4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\right.}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1}\right.}$$

$$+x\operatorname{arccosh}\left(\sqrt{x}\right)+2\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[ArcCosh[Sqrt[x]],x]
```

output
```
(-2*(4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sq
rt[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(
28 + 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 -
8*Sqrt[x] + 5*x + 3*x^(3/2)))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*
Sqrt[x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqr
t[x]) + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + x*ArcCosh[Sqrt[x]] + 2*ArcTanh
[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6431, 27, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arccosh}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{ 6431}$$

$$x\text{arccosh}\left(\sqrt{x}\right)-\int \frac{\sqrt{x}}{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$x\text{arccosh}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{ 845}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+x\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 852}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+x\text{arccosh}\left(\sqrt{x}\right)$$

$$\Big\downarrow 43$$

$$\frac{1}{2}\left(-\text{arccosh}\left(\sqrt{x}\right)-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+x\text{arccosh}\left(\sqrt{x}\right)$$

input
```
Int[ArcCosh[Sqrt[x]],x]
```

output
```
(-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) - ArcCosh[Sqrt[x]])/2 + x
*ArcCosh[Sqrt[x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

rule 6431
```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1 + u])), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$x \operatorname{arccosh}\left(\sqrt{x}\right)-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{2\sqrt{-1+x}}$	49
default	$x \operatorname{arccosh}\left(\sqrt{x}\right)-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{2\sqrt{-1+x}}$	49
parts	$x \operatorname{arccosh}\left(\sqrt{x}\right)-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{2\sqrt{-1+x}}$	49

input
```
int(arccosh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccosh(x^(1/2))-1/2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x^(1/2)*(-1+x
)^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

$$\int \operatorname{arccosh}\left(\sqrt{x}\right)\,dx=\frac{1}{2}\,(2\,x-1)\log\left(\sqrt{x-1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x-1}\sqrt{x}$$

input
```
integrate(arccosh(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*x - 1)*log(sqrt(x - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x)
```

Sympy [F]

$$\int \operatorname{arccosh}(\sqrt{x})\,dx = \int \operatorname{acosh}\left(\sqrt{x}\right)dx$$

input
```
integrate(acosh(x**(1/2)),x)
```

output
```
Integral(acosh(sqrt(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int \operatorname{arccosh}(\sqrt{x})\,dx = x\operatorname{arcosh}\left(\sqrt{x}\right) - \frac{1}{2}\sqrt{x-1}\sqrt{x} - \frac{1}{2}\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

input
```
integrate(arccosh(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arccosh(sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x) - 1/2*log(2*sqrt(x - 1) + 2*s
qrt(x))
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \operatorname{arccosh}(\sqrt{x})\,dx = x\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \sqrt{x}\right) - \frac{1}{2}\sqrt{x-1}\sqrt{x} + \frac{1}{2}\log\left(-\sqrt{x-1} + \sqrt{x}\right)$$

input
```
integrate(arccosh(x^(1/2)),x, algorithm="giac")
```

output
```
x*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqr
t(x) + 1/2*log(-sqrt(x - 1) + sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \operatorname{arccosh}\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\,\mathrm{acosh}\left(\sqrt{x}\right)\left(\frac{1}{4\sqrt{x}}-\frac{\sqrt{x}}{2}\right)-\frac{\sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{2}$$

input
```
int(acosh(x^(1/2)),x)
```

output
```
- 2*x^(1/2)*acosh(x^(1/2))*(1/(4*x^(1/2)) - x^(1/2)/2) - (x^(1/2)*(x^(1/2)
 - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.48

$$\int \operatorname{arccosh}\left(\sqrt{x}\right)\,dx = acosh\left(\sqrt{x}\right)x-\frac{\sqrt{x}\,\sqrt{x-1}}{2}-\frac{\log\left(\sqrt{x-1}+\sqrt{x}\right)}{2}$$

input
```
int(acosh(x^(1/2)),x)
```

output
```
(2*acosh(sqrt(x))*x - sqrt(x)*sqrt(x - 1) - log(sqrt(x - 1) + sqrt(x)))/2
```

3.144 $\int \frac{\operatorname{arccosh}(\sqrt{x})}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x}\,dx = -\operatorname{arccosh}(\sqrt{x})^2 + 2\operatorname{arccosh}(\sqrt{x})\log\left(1+e^{2\operatorname{arccosh}(\sqrt{x})}\right) + \operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(\sqrt{x})}\right)$$

output
```
-arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/
2))^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x}\,dx = \operatorname{arccosh}(\sqrt{x})\left(\operatorname{arccosh}(\sqrt{x}) + 2\log\left(1+e^{-2\operatorname{arccosh}(\sqrt{x})}\right)\right) - \operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}(\sqrt{x})}\right)$$

input
```
Integrate[ArcCosh[Sqrt[x]]/x,x]
```

output
```
ArcCosh[Sqrt[x]]*(ArcCosh[Sqrt[x]] + 2*Log[1 + E^(-2*ArcCosh[Sqrt[x]])]) -
 PolyLog[2, -E^(-2*ArcCosh[Sqrt[x]])]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6426, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(\sqrt{x})}{x} \, dx$$

$$\Big\downarrow \text{ 6426}$$

$$2 \int \frac{\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}} (\sqrt{x}+1) \text{arccosh}(\sqrt{x})}{\sqrt{x}} d\text{arccosh}(\sqrt{x})$$

$$\Big\downarrow \text{ 3042}$$

$$2 \int -i\text{arccosh}(\sqrt{x}) \tan \left(i\text{arccosh}(\sqrt{x})\right) d\text{arccosh}(\sqrt{x})$$

$$\Big\downarrow \text{ 26}$$

$$-2i \int \text{arccosh}(\sqrt{x}) \tan \left(i\text{arccosh}(\sqrt{x})\right) d\text{arccosh}(\sqrt{x})$$

$$\Big\downarrow \text{ 4201}$$

$$-2i \left(2i \int \frac{e^{2\text{arccosh}(\sqrt{x})} \text{arccosh}(\sqrt{x})}{1+e^{2\text{arccosh}(\sqrt{x})}} d\text{arccosh}(\sqrt{x}) - \frac{1}{2} i\text{arccosh}(\sqrt{x})^2\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-2i \left(2i \left(\frac{1}{2}\text{arccosh}(\sqrt{x}) \log \left(e^{2\text{arccosh}(\sqrt{x})}+1\right) - \frac{1}{2} \int \log \left(1+e^{2\text{arccosh}(\sqrt{x})}\right) d\text{arccosh}(\sqrt{x})\right) - \frac{1}{2} i\text{arccosh}(\sqrt{x})\right.$$

$$\Big\downarrow \text{ 2715}$$

$$-2i\left(2i\left(\frac{1}{2}\text{arccosh}(\sqrt{x})\log\left(e^{2\text{arccosh}(\sqrt{x})}+1\right)-\frac{1}{4}\int e^{-2\text{arccosh}(\sqrt{x})}\log\left(1+e^{2\text{arccosh}(\sqrt{x})}\right)de^{2\text{arccosh}(\sqrt{x})}\right)-\frac{1}{2}i$$

$$\Big\downarrow \text{2838}$$

$$-2i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}(\sqrt{x})}\right)+\frac{1}{2}\text{arccosh}(\sqrt{x})\log\left(e^{2\text{arccosh}(\sqrt{x})}+1\right)\right)-\frac{1}{2}i\text{arccosh}(\sqrt{x})^2\right)$$

input
```
Int[ArcCosh[Sqrt[x]]/x,x]
```

output
```
(-2*I)*((-1/2*I)*ArcCosh[Sqrt[x]]^2 + (2*I)*((ArcCosh[Sqrt[x]]*Log[1 + E^(
2*ArcCosh[Sqrt[x]])])/2 + PolyLog[2, -E^(2*ArcCosh[Sqrt[x]])]/4))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6426
```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
derivativedivides	$-\operatorname{arccosh}\left(\sqrt{x}\right)^{2}+2\operatorname{arccosh}\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\right)^{2}\right)+\mathrm{poly}$
default	$-\operatorname{arccosh}\left(\sqrt{x}\right)^{2}+2\operatorname{arccosh}\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\right)^{2}\right)+\mathrm{poly}$

input
```
int(arccosh(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/
2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x}\,dx=\int \frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccosh(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arccosh(sqrt(x))/x, x)
```

Sympy [F]

$$\int \frac{\text{arccosh}(\sqrt{x})}{x} \, dx = \int \frac{\text{acosh}\left(\sqrt{x}\right)}{x} \, dx$$

input
```
integrate(acosh(x**(1/2))/x,x)
```

output
```
Integral(acosh(sqrt(x))/x, x)
```

Maxima [F]

$$\int \frac{\text{arccosh}(\sqrt{x})}{x} \, dx = \int \frac{\text{arcosh}\left(\sqrt{x}\right)}{x} \, dx$$

input
```
integrate(arccosh(x^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arccosh(sqrt(x))/x, x)
```

Giac [F]

$$\int \frac{\text{arccosh}(\sqrt{x})}{x} \, dx = \int \frac{\text{arcosh}\left(\sqrt{x}\right)}{x} \, dx$$

input
```
integrate(arccosh(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arccosh(sqrt(x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{acosh}(\sqrt{x})}{x}\,dx$$

input
```
int(acosh(x^(1/2))/x,x)
```

output
```
int(acosh(x^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x}\,dx = \int \frac{acosh(\sqrt{x})}{x}dx$$

input
```
int(acosh(x^(1/2))/x,x)
```

output
```
int(acosh(sqrt(x))/x,x)
```

3.145 $\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^2} \, dx = \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}} - \frac{\operatorname{arccosh}(\sqrt{x})}{x}$$

output
```
(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)-arccosh(x^(1/2))/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^2} \, dx = \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}} - \frac{\operatorname{arccosh}(\sqrt{x})}{x}$$

input
```
Integrate[ArcCosh[Sqrt[x]]/x^2,x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6432, 27, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}\left(\sqrt{x}\right)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6432}$$

$$\int \frac{1}{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx-\frac{\text{arccosh}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx-\frac{\text{arccosh}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 797}$$

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\text{arccosh}\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcCosh[Sqrt[x]]/x^2,x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 6432
```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
 + u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFu
nctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOf
ExponentialQ[u, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x}$	29
default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x}$	29
parts	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x}$	29

input
```
int(arccosh(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)-arccosh(x^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^2}\,dx = \frac{\sqrt{x-1}\sqrt{x} - \log\left(\sqrt{x-1} + \sqrt{x}\right)}{x}$$

input
```
integrate(arccosh(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
(sqrt(x - 1)*sqrt(x) - log(sqrt(x - 1) + sqrt(x)))/x
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^2}\,dx$$

input
```
integrate(acosh(x**(1/2))/x**2,x)
```

output
```
Integral(acosh(sqrt(x))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^2}\,dx = \frac{\sqrt{x-1}}{\sqrt{x}} - \frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}$$

input
```
integrate(arccosh(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
sqrt(x - 1)/sqrt(x) - arccosh(sqrt(x))/x
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{x} + \frac{2}{\left(\sqrt{x-1}-\sqrt{x}\right)^2+1}$$

input
```
integrate(arccosh(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x + 2/((sqrt(x - 1) -
sqrt(x))^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^2}\,dx$$

input
```
int(acosh(x^(1/2))/x^2,x)
```

output
```
int(acosh(x^(1/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{-acosh\left(\sqrt{x}\right)-\sqrt{x}\,\sqrt{x-1}-x}{x}$$

input
```
int(acosh(x^(1/2))/x^2,x)
```

output
```
( - (acosh(sqrt(x)) + sqrt(x)*sqrt(x - 1) + x))/x
```

3.146 $\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx=\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{6x^{3/2}}+\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}-\frac{\operatorname{arccosh}(\sqrt{x})}{2x^2}$$

output
```
1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+1/3*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)/x^(1/2)-1/2*arccosh(x^(1/2))/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx=\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}(1+2x)-3\operatorname{arccosh}(\sqrt{x})}{6x^2}$$

input
```
Integrate[ArcCosh[Sqrt[x]]/x^3,x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 + 2*x) - 3*ArcCosh[Sqrt[x
]])/(6*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6432, 27, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x^3} \, dx$$

$$\Big\downarrow \text{6432}$$

$$\frac{1}{2}\int \frac{1}{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}}dx-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}}dx-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{804}$$

$$\frac{1}{4}\left(\frac{2}{3}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}\right)-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{797}$$

$$\frac{1}{4}\left(\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}\right)-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcCosh[Sqrt[x]]/x^3,x]
```

output
```
((2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[
x]]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x]))/4 - ArcCosh[Sqrt[x]]/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

rule 6432
```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
 + u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFu
nctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOf
ExponentialQ[u, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

method	result	size
derivativedivides	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2 x^{2}}+\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(2 x +1\right)}{6 x^{\frac{3}{2}}}$	35
default	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2 x^{2}}+\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(2 x +1\right)}{6 x^{\frac{3}{2}}}$	35
parts	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2 x^{2}}+\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(2 x +1\right)}{6 x^{\frac{3}{2}}}$	35

input
```
int(arccosh(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccosh(x^(1/2))/x^2+1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x+1)
/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.42

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3} \, dx = \frac{(2x+1)\sqrt{x-1}\sqrt{x} - 3 \log\left(\sqrt{x-1} + \sqrt{x}\right)}{6x^2}$$

input
```
integrate(arccosh(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*((2*x + 1)*sqrt(x - 1)*sqrt(x) - 3*log(sqrt(x - 1) + sqrt(x)))/x^2
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3} \, dx = \int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^3} \, dx$$

input
```
integrate(acosh(x**(1/2))/x**3,x)
```

output
```
Integral(acosh(sqrt(x))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.39

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx = \frac{\sqrt{x-1}}{3\sqrt{x}} + \frac{\sqrt{x-1}}{6\,x^{\frac{3}{2}}} - \frac{\operatorname{arcosh}(\sqrt{x})}{2\,x^2}$$

input
```
integrate(arccosh(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/3*sqrt(x - 1)/sqrt(x) + 1/6*sqrt(x - 1)/x^(3/2) - 1/2*arccosh(sqrt(x))/x
^2
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx = -\frac{\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{2\,x^2} + \frac{2\left(3\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)}{3\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)^3}$$

input
```
integrate(arccosh(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt
(x - 1) - sqrt(x))^2 + 1)/((sqrt(x - 1) - sqrt(x))^2 + 1)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{acosh}(\sqrt{x})}{x^3}\,dx$$

input
```
int(acosh(x^(1/2))/x^3,x)
```

output
```
int(acosh(x^(1/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arccosh}(\sqrt{x})}{x^3}\,dx = \frac{-3acosh(\sqrt{x}) - 2\sqrt{x}\,\sqrt{x-1}\,x - \sqrt{x}\,\sqrt{x-1} + 2x^2}{6x^2}$$

input
```
int(acosh(x^(1/2))/x^3,x)
```

output
```
( - 3*acosh(sqrt(x)) - 2*sqrt(x)*sqrt(x - 1)*x - sqrt(x)*sqrt(x - 1) + 2*x
**2)/(6*x**2)
```

3.147 $\int \operatorname{arccosh}\left(\frac{1}{x}\right) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right) \, dx = x\operatorname{sech}^{-1}(x) + \sqrt{\frac{1}{1+x}}\sqrt{1+x}\arcsin(x)$$

output
```
x*arcsech(x)+(1/(1+x))^(1/2)*(1+x)^(1/2)*arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 59 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right) \, dx = x\operatorname{arccosh}\left(\frac{1}{x}\right) + \frac{2\sqrt{-1+\frac{1}{x}}\sqrt{1+\frac{1}{x}}x\arctan\left(\frac{\sqrt{1-x^2}}{1-x}\right)}{\sqrt{1-x^2}}$$

input
```
Integrate[ArcCosh[x^(-1)],x]
```

output
```
x*ArcCosh[x^(-1)] + (2*Sqrt[-1 + x^(-1)]*Sqrt[1 + x^(-1)]*x*ArcTan[Sqrt[1
- x^2]/(1 - x)])/Sqrt[1 - x^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6427, 6831, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arccosh}\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \text{6427}$$

$$\int \text{sech}^{-1}(x)dx$$

$$\Big\downarrow \text{6831}$$

$$\sqrt{\frac{1}{x+1}}\sqrt{x+1}\int \frac{1}{\sqrt{1-x^2}}dx + x\text{sech}^{-1}(x)$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\frac{1}{x+1}}\sqrt{x+1}\arcsin(x) + x\text{sech}^{-1}(x)$$

input
```
Int[ArcCosh[x^(-1)],x]
```

output
```
x*ArcSech[x] + Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcSin[x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6427
```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6831
```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Simp[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)]   Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$x \operatorname{arccosh}\left(\frac{1}{x}\right)+\frac{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}\right)}{\sqrt{\frac{1}{x^2}-1}}$	38
default	$x \operatorname{arccosh}\left(\frac{1}{x}\right)+\frac{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}\right)}{\sqrt{\frac{1}{x^2}-1}}$	38
parts	$x \operatorname{arccosh}\left(\frac{1}{x}\right)+\frac{\sqrt{-\frac{-1+x}{x}}\,x\sqrt{\frac{1+x}{x}}\,\arcsin(x)}{\sqrt{-x^2+1}}$	40

input
```
int(arccosh(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arccosh(1/x)+(1/x-1)^(1/2)*(1/x+1)^(1/2)/(1/x^2-1)^(1/2)*arctan(1/(1/x^2
-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(7) = 14$.

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right)\,dx=(x-2)\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}+1}{x}\right)-2\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)-2\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

input
```
integrate(arccosh(1/x),x, algorithm="fricas")
```

output
```
(x - 2)*log((x*sqrt(-(x^2 - 1)/x^2) + 1)/x) - 2*arctan((x*sqrt(-(x^2 - 1)/
x^2) - 1)/x) - 2*log((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [F]

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right) dx = \int \operatorname{acosh}\left(\frac{1}{x}\right) dx$$

input
```
integrate(acosh(1/x),x)
```

output
```
Integral(acosh(1/x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right) dx = x \operatorname{arcosh}\left(\frac{1}{x}\right) - \arctan\left(\sqrt{\frac{1}{x^2} - 1}\right)$$

input
```
integrate(arccosh(1/x),x, algorithm="maxima")
```

output
```
x*arccosh(1/x) - arctan(sqrt(1/x^2 - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right)\,dx = x\log\left(\sqrt{\frac{1}{x^2}-1}+\frac{1}{x}\right)+\frac{\arcsin(x)}{\operatorname{sgn}(x)}$$

input
```
integrate(arccosh(1/x),x, algorithm="giac")
```

output
```
x*log(sqrt(1/x^2 - 1) + 1/x) + arcsin(x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right)\,dx = \operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}}\right)+x\operatorname{acosh}\left(\frac{1}{x}\right)$$

input
```
int(acosh(1/x),x)
```

output
```
atan(1/((1/x - 1)^(1/2)*(1/x + 1)^(1/2))) + x*acosh(1/x)
```

Reduce [F]

$$\int \operatorname{arccosh}\left(\frac{1}{x}\right)\,dx = \int acosh\left(\frac{1}{x}\right)dx$$

input
```
int(acosh(1/x),x)
```

output
```
int(acosh(1/x),x)
```

3.148 $\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = -\frac{\operatorname{arccosh}(ax^n)^2}{2n}+\frac{\operatorname{arccosh}(ax^n)\log\left(1+e^{2\operatorname{arccosh}(ax^n)}\right)}{n}+\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(ax^n)}\right)}{2n}$$

output
```
-1/2*arccosh(a*x^n)^2/n+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+
1)^(1/2))^2)/n+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2)/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(60) = 120.

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.98

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \operatorname{arccosh}(ax^n)\log(x)+\frac{a\sqrt{1-a^2x^{2n}}\left(\operatorname{arcsinh}\left(\sqrt{-a^2}x^n\right)^2+2\operatorname{arcsinh}\left(\sqrt{-a^2}x^n\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\sqrt{-a^2}x^n\right)}\right)-2n\log(x)\log\left(\right.\right.}{2\sqrt{-a^2}n\sqrt{-1+ax^n}\sqrt{1+ax^n}}$$

input
```
Integrate[ArcCosh[a*x^n]/x,x]
```

output
```
ArcCosh[a*x^n]*Log[x] + (a*Sqrt[1 - a^2*x^(2*n)]*(ArcSinh[Sqrt[-a^2]*x^n]^
2 + 2*ArcSinh[Sqrt[-a^2]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] - 2*
n*Log[x]*Log[Sqrt[-a^2]*x^n + Sqrt[1 - a^2*x^(2*n)]] - PolyLog[2, E^(-2*Ar
cSinh[Sqrt[-a^2]*x^n])]))/(2*Sqrt[-a^2]*n*Sqrt[-1 + a*x^n]*Sqrt[1 + a*x^n]
)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6426, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(ax^n)}{x}dx$$

$$\Big\downarrow \text{6426}$$

$$\frac{\int \frac{x^{-n}\sqrt{\frac{ax^n-1}{ax^n+1}}(ax^n+1)\text{arccosh}(ax^n)}{a}d\text{arccosh}(ax^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\text{arccosh}(ax^n)\tan(i\text{arccosh}(ax^n))d\text{arccosh}(ax^n)}{n}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \text{arccosh}(ax^n)\tan(i\text{arccosh}(ax^n))d\text{arccosh}(ax^n)}{n}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{2\text{arccosh}(ax^n)}\text{arccosh}(ax^n)}{1+e^{2\text{arccosh}(ax^n)}}d\text{arccosh}(ax^n)-\frac{1}{2}i\text{arccosh}(ax^n)^2\right)}{n}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}(ax^n)\log\left(e^{2\text{arccosh}(ax^n)}+1\right)-\frac{1}{2}\int\log\left(1+e^{2\text{arccosh}(ax^n)}\right)d\text{arccosh}(ax^n)\right)-\frac{1}{2}i\text{arccosh}(ax^n)^2\right)}{n}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}(ax^n)\log\left(e^{2\text{arccosh}(ax^n)}+1\right)-\frac{1}{4}\int e^{-2\text{arccosh}(ax^n)}\log\left(1+e^{2\text{arccosh}(ax^n)}\right)de^{2\text{arccosh}(ax^n)}\right)-\frac{1}{2}i\text{arcc}\right.}{n}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}(ax^n)}\right)+\frac{1}{2}\text{arccosh}(ax^n)\log\left(e^{2\text{arccosh}(ax^n)}+1\right)\right)-\frac{1}{2}i\text{arccosh}(ax^n)^2\right)}{n}$$

input
```
Int[ArcCosh[a*x^n]/x,x]
```

output
```
((-I)*((-1/2*I)*ArcCosh[a*x^n]^2 + (2*I)*((ArcCosh[a*x^n]*Log[1 + E^(2*Arc
Cosh[a*x^n])])/2 + PolyLog[2, -E^(2*ArcCosh[a*x^n])]/4)))/n
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6426
```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{arccosh}(a x^n)^2}{2}+\operatorname{arccosh}(a x^n)\ln\left(1+\left(a x^n+\sqrt{a x^n-1}\,\sqrt{a x^n+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(a x^n+\sqrt{a x^n-1}\,\sqrt{a x^n+1}\right)^2\right)}{2}}{n}$	86
default	$\frac{-\frac{\operatorname{arccosh}(a x^n)^2}{2}+\operatorname{arccosh}(a x^n)\ln\left(1+\left(a x^n+\sqrt{a x^n-1}\,\sqrt{a x^n+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(a x^n+\sqrt{a x^n-1}\,\sqrt{a x^n+1}\right)^2\right)}{2}}{n}$	86

input
```
int(arccosh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/2*arccosh(a*x^n)^2+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x
^n+1)^(1/2))^2)+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccosh(a*x^n)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \int \frac{\operatorname{acosh}(ax^n)}{x}\,dx$$

input
```
integrate(acosh(a*x**n)/x,x)
```

output
```
Integral(acosh(a*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcosh}(ax^n)}{x}\,dx$$

input
```
integrate(arccosh(a*x^n)/x,x, algorithm="maxima")
```

output
```
a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) - a*x*x^n + (a^2*x*x^(2*n) - x)*sq
rt(a*x^n + 1)*sqrt(a*x^n - 1)), x) - 1/2*n*log(x)^2 + n*integrate(1/2*log(
x)/(a*x*x^n + x), x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(a*x^
n + sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

Giac [F]

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcosh}\left(ax^n\right)}{x}\,dx$$

input
```
integrate(arccosh(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arccosh(a*x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \int \frac{\operatorname{acosh}(a\,x^n)}{x}\,dx$$

input
```
int(acosh(a*x^n)/x,x)
```

output
```
int(acosh(a*x^n)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(ax^n)}{x}\,dx = \int \frac{acosh(x^n a)}{x}dx$$

input
```
int(acosh(a*x^n)/x,x)
```

output
```
int(acosh(x**n*a)/x,x)
```

3.149 $\int (a + b\text{arccosh}(1 + dx^2))^4 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 145

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^4 \, dx = 384b^4x - \frac{192b^3(2x^2+dx^4)\left(a+b\text{arccosh}(1+dx^2)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}} + 48b^2x\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2 - \frac{8b(2x^2+dx^4)\left(a+b\text{arccosh}(1+dx^2)\right)^3}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^4$$

output
```
384*b^4*x-192*b^3*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))/x/(d*x^2)^(1/2)/(d*
x^2+2)^(1/2)+48*b^2*x*(a+b*arccosh(d*x^2+1))^2-8*b*(d*x^4+2*x^2)*(a+b*arcc
osh(d*x^2+1))^3/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)+x*(a+b*arccosh(d*x^2+1))^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.82

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^4 \, dx$$

$$= \frac{(a^4+48a^2b^2+384b^4)\,dx^2 - 8ab(a^2+24b^2)\,\sqrt{dx^2}\sqrt{2+dx^2} + 4b\Big(a^3dx^2 + 24ab^2dx^2 - 6a^2b\sqrt{dx^2}\sqrt{2+d}}{}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

output
```
((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqr
t[2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[
2 + d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2] + 6*b^
2*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1
+ d*x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 +
 d*x^2]^3 + b^4*d*x^2*ArcCosh[1 + d*x^2]^4)/(d*x)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6416, 6416, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^4 dx$$

$$\Big\downarrow \text{6416}$$

$$48b^2\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2dx+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^4-\frac{8b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

$$\Big\downarrow \text{6416}$$

$$48b^2\left(8b^2\int 1dx+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2-\frac{4b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}\right)+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^4-\frac{8b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

$$\Big\downarrow \text{24}$$

$$48b^2\left(x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2-\frac{4b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+8b^2x\right)+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^4-\frac{8b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

output
```
(-8*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[2
+ d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*(2*x^2
 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x*
(a + b*ArcCosh[1 + d*x^2])^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(137) = 274$.

Time = 0.21 (sec) , antiderivative size = 1044, normalized size of antiderivative = 7.20

method	result	size
orering	Expression too large to display	1044

input
```
int((a+b*arccosh(d*x^2+1))^4,x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2+1))^4-32*(a+b*arccosh(d*x^2+1))^3*b*x/(d*x^2)^(1/2)/(
d*x^2+2)^(1/2)+1/d*x*(5*d*x^2+4)*(48*b^2*(a+b*arccosh(d*x^2+1))^2*d/(d*x^2
+2)+8*(a+b*arccosh(d*x^2+1))^3*b*d/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-8*(a+b*ar
ccosh(d*x^2+1))^3*b*d^2*x^2/(d*x^2)^(3/2)/(d*x^2+2)^(1/2)-8*(a+b*arccosh(d
*x^2+1))^3*b*d^2*x^2/(d*x^2)^(1/2)/(d*x^2+2)^(3/2))+(5*d^2*x^4+8*d*x^2-4)/
d^2*(192*b^3*(a+b*arccosh(d*x^2+1))*d^2/(d*x^2+2)^(3/2)*x/(d*x^2)^(1/2)-14
4*b^2*(a+b*arccosh(d*x^2+1))^2*d^2/(d*x^2+2)^2*x-24*(a+b*arccosh(d*x^2+1))
^3*b*d^2/(d*x^2)^(3/2)/(d*x^2+2)^(1/2)*x-24*(a+b*arccosh(d*x^2+1))^3*b*d^2
/(d*x^2)^(1/2)/(d*x^2+2)^(3/2)*x+24*(a+b*arccosh(d*x^2+1))^3*b*d^3*x^3/(d*
x^2)^(5/2)/(d*x^2+2)^(1/2)+16*(a+b*arccosh(d*x^2+1))^3*b*d^3*x^3/(d*x^2)^(
3/2)/(d*x^2+2)^(3/2)+24*(a+b*arccosh(d*x^2+1))^3*b*d^3*x^3/(d*x^2)^(1/2)/(
d*x^2+2)^(5/2))+1/d^2*x*(d*x^2+2)^2*(-72*(a+b*arccosh(d*x^2+1))^3*b*d^4*x^
4/(d*x^2)^(3/2)/(d*x^2+2)^(5/2)-1152*b^3*(a+b*arccosh(d*x^2+1))*d^3/(d*x^2
+2)^(5/2)*x^2/(d*x^2)^(1/2)-192*b^3*(a+b*arccosh(d*x^2+1))*d^3/(d*x^2+2)^(
3/2)*x^2/(d*x^2)^(3/2)-120*(a+b*arccosh(d*x^2+1))^3*b*d^4*x^4/(d*x^2)^(1/2
)/(d*x^2+2)^(7/2)+384*b^4*d^2/(d*x^2+2)^2-192*b^2*(a+b*arccosh(d*x^2+1))^2
*d^2/(d*x^2+2)^2+720*b^2*(a+b*arccosh(d*x^2+1))^2*d^3/(d*x^2+2)^3*x^2-24*(
a+b*arccosh(d*x^2+1))^3*b*d^2/(d*x^2)^(1/2)/(d*x^2+2)^(3/2)+144*(a+b*arcco
sh(d*x^2+1))^3*b*d^3/(d*x^2)^(1/2)/(d*x^2+2)^(5/2)*x^2-120*(a+b*arccosh(d*
x^2+1))^3*b*d^4*x^4/(d*x^2)^(7/2)/(d*x^2+2)^(1/2)-72*(a+b*arccosh(d*x^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(137) = 274$.

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.06

$$\int \left(a + b\operatorname{arccosh}\left(1 + dx^2\right)\right)^4 dx$$
$$= \frac{b^4 dx^2 \log\left(dx^2 + \sqrt{d^2x^4 + 2\,dx^2} + 1\right)^4 + \left(a^4 + 48\,a^2b^2 + 384\,b^4\right)dx^2 + 4\left(ab^3dx^2 - 2\sqrt{d^2x^4 + 2\,dx^2}b^4\right)\log \cdots}{}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="fricas")
```

output
```
(b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^4 + (a^4 + 48*a^2*b^2
+ 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^4)*log(d*x
^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 - 6*(4*sqrt(d^2*x^4 + 2*d*x^2)*a*b^3 -
 (a^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + 4*(
(a^3*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 + 2*d*x^2)*(a^2*b^2 + 8*b^4))*lo
g(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 8*sqrt(d^2*x^4 + 2*d*x^2)*(a^3*b
+ 24*a*b^3))/(d*x)
```

Sympy [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^4\,dx=\int \left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^4\,dx$$

input
```
integrate((a+b*acosh(d*x**2+1))**4,x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**4, x)
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^4\,dx=\int \left(b\,\text{arcosh}\left(dx^2+1\right)+a\right)^4\,dx$$

input
```
integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^4 + 6*a^2*b^2*x*arccosh(d
*x^2 + 1)^2 + 24*a^2*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*log(d*x^2 +
sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 4*(x*arccosh(d*x
^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a^3*b + a^4*x +
 integrate(4*((a*b^3*d^2 - 2*b^4*d^2)*x^4 + 2*a*b^3 + (3*a*b^3*d - 4*b^4*d
)*x^2 + ((a*b^3*d - 2*b^4*d)*sqrt(d)*x^3 + 2*(a*b^3 - b^4)*sqrt(d)*x)*sqrt
(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^3/(d^2*x^4 + 3*d*x
^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\operatorname{arccosh}\left(1+dx^2\right)\right)^4\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{arccosh}\left(1+dx^2\right)\right)^4\,dx=\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^4\,dx$$

input
```
int((a + b*acosh(d*x^2 + 1))^4,x)
```

output
```
int((a + b*acosh(d*x^2 + 1))^4, x)
```

Reduce [F]

$$\begin{aligned}\int \left(a+b\operatorname{arccosh}\left(1+dx^2\right)\right)^4\,dx=&\,4\left(\int acosh\left(d\,x^2+1\right)dx\right)a^3b\\&+\left(\int acosh\left(d\,x^2+1\right)^4dx\right)b^4\\&+4\left(\int acosh\left(d\,x^2+1\right)^3dx\right)a\,b^3\\&+6\left(\int acosh\left(d\,x^2+1\right)^2dx\right)a^2b^2+a^4x\end{aligned}$$

input
```
int((a+b*acosh(d*x^2+1))^4,x)
```

output
```
4*int(acosh(d*x**2 + 1),x)*a**3*b + int(acosh(d*x**2 + 1)**4,x)*b**4 + 4*i
nt(acosh(d*x**2 + 1)**3,x)*a*b**3 + 6*int(acosh(d*x**2 + 1)**2,x)*a**2*b**
2 + a**4*x
```

3.150 $\int \left(a+b\text{arccosh}(1+dx^2)\right)^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 120

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3 \, dx = 24ab^2x-\frac{48b^3\sqrt{dx^2}\sqrt{2+dx^2}}{dx}+24b^3x\text{arccosh}\left(1+dx^2\right) -\frac{6b(2x^2+dx^4)\left(a+b\text{arccosh}(1+dx^2)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} +x\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3$$

output
```
24*a*b^2*x-48*b^3*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/d/x+24*b^3*x*arccosh(d*x^2
+1)-6*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^2/x/(d*x^2)^(1/2)/(d*x^2+2)^(
1/2)+x*(a+b*arccosh(d*x^2+1))^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3 \, dx = \frac{a(a^2+24b^2)\,dx^2-6b(a^2+8b^2)\,\sqrt{dx^2}\sqrt{2+dx^2}+3b\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2}\sqrt{2+dx^2}\right)\text{arccosh}(1+}{dx}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

output
```
(a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[2 + d*x^2] +
3*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[
1 + d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 +
 d*x^2]^2 + b^3*d*x^2*ArcCosh[1 + d*x^2]^3)/(d*x)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6416, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^3 dx$$

$$\Big\downarrow \text{ 6416}$$

$$24b^2\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)dx+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^3-\frac{6b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

$$\Big\downarrow \text{ 2009}$$

$$24b^2\left(ax+bx\text{arccosh}\left(dx^2+1\right)-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}\right)+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^3-\frac{6b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

output
```
(-6*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^2)/(x*Sqrt[d*x^2]*Sqrt[2
+ d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^3 + 24*b^2*(a*x - (2*b*Sqrt[(d*x^
2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + b*x*ArcCosh[1 + d*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(112) = 224$.

Time = 0.13 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.94

method	result
orering	$x(a+b\,\operatorname{arccosh}(d\,x^2+1))^3-\frac{6(d\,x^2+4)(a+b\,\operatorname{arccosh}(d\,x^2+1))^2bx}{\sqrt{d\,x^2}\,\sqrt{d\,x^2+2}}-\frac{2(d\,x^2+2)x\left(\frac{24b^2d(a+b\,\operatorname{arccosh}(d\,x^2+1))}{d\,x^2+2}+\frac{6}{}\right.}{}$

input
```
int((a+b*arccosh(d*x^2+1))^3,x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2+1))^3-6*(d*x^2+4)*(a+b*arccosh(d*x^2+1))^2*b*x/(d*x^2
)^(1/2)/(d*x^2+2)^(1/2)-2/d*(d*x^2+2)*x*(24*b^2*d*(a+b*arccosh(d*x^2+1))/(
d*x^2+2)+6*(a+b*arccosh(d*x^2+1))^2*b*d/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-6*(a
+b*arccosh(d*x^2+1))^2*b*d^2*x^2/(d*x^2)^(3/2)/(d*x^2+2)^(1/2)-6*(a+b*arcc
osh(d*x^2+1))^2*b*d^2*x^2/(d*x^2)^(1/2)/(d*x^2+2)^(3/2))-1/d^2*(d*x^2+2)^2
*(48*b^3*d^2*x/(d*x^2)^(1/2)/(d*x^2+2)^(3/2)-72*b^2*d^2*(a+b*arccosh(d*x^2
+1))/(d*x^2+2)^2*x-18*(a+b*arccosh(d*x^2+1))^2*b*d^2/(d*x^2)^(3/2)/(d*x^2+
2)^(1/2)*x-18*(a+b*arccosh(d*x^2+1))^2*b*d^2/(d*x^2)^(1/2)/(d*x^2+2)^(3/2)
*x+18*(a+b*arccosh(d*x^2+1))^2*b*d^3*x^3/(d*x^2)^(5/2)/(d*x^2+2)^(1/2)+12*
(a+b*arccosh(d*x^2+1))^2*b*d^3*x^3/(d*x^2)^(3/2)/(d*x^2+2)^(3/2)+18*(a+b*a
rccosh(d*x^2+1))^2*b*d^3*x^3/(d*x^2)^(1/2)/(d*x^2+2)^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3\,dx$$
$$=\frac{b^3dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^3+\left(a^3+24\,ab^2\right)dx^2+3\left(ab^2dx^2-2\sqrt{d^2x^4+2\,dx^2}b^3\right)\log\left(dx^2+\sqrt{\right.}\right.}{}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

output
```
(b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 + (a^3 + 24*a*b^2)*d
*x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^
2*x^4 + 2*d*x^2) + 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 + 2*d*
x^2)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 6*sqrt(d^2*x^4 + 2*
d*x^2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3\,dx=\int \left(a+b\,\text{acosh}\left(dx^2+1\right)\right)^3\,dx$$

input
```
integrate((a+b*acosh(d*x**2+1))**3,x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**3, x)
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3\,dx=\int \left(b\,\text{arcosh}\left(dx^2+1\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

output
```
3*a*b^2*x*arccosh(d*x^2 + 1)^2 + 12*a*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt
(d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) +
 3*(x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d)
)*a^2*b + (x*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^3 - integrate(6*(d
^2*x^4 + 2*d*x^2 + (d^(3/2)*x^3 + sqrt(d)*x)*sqrt(d*x^2 + 2))*log(d*x^2 +
sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^2/(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqr
t(d)*x)*sqrt(d*x^2 + 2) + 2), x))*b^3 + a^3*x
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3 \, dx = \int \left(a+b\,\text{acosh}\left(d\,x^2+1\right)\right)^3 \,dx$$

input
```
int((a + b*acosh(d*x^2 + 1))^3,x)
```

output
```
int((a + b*acosh(d*x^2 + 1))^3, x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3 \, dx = 3\left(\int acosh\left(d\,x^2+1\right) dx\right) a^2 b + \left(\int acosh\left(d\,x^2+1\right)^3 dx\right) b^3 + 3\left(\int acosh\left(d\,x^2+1\right)^2 dx\right) a\,b^2 + a^3 x$$

input
```
int((a+b*acosh(d*x^2+1))^3,x)
```

output
```
3*int(acosh(d*x**2 + 1),x)*a**2*b + int(acosh(d*x**2 + 1)**3,x)*b**3 + 3*i
nt(acosh(d*x**2 + 1)**2,x)*a*b**2 + a**3*x
```

3.151 $\int (a+b\text{arccosh}(1+dx^2))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2\,dx=8b^2x-\frac{4b(2x^2+dx^4)\left(a+b\text{arccosh}(1+dx^2)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2$$

output
```
8*b^2*x-4*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))/x/(d*x^2)^(1/2)/(d*x^2+2)
^(1/2)+x*(a+b*arccosh(d*x^2+1))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2\,dx=\left(a^2+8b^2\right)x-\frac{4ab\sqrt{dx^2}\sqrt{2+dx^2}}{dx}+\frac{2b\left(adx^2-2b\sqrt{dx^2}\sqrt{2+dx^2}\right)\text{arccosh}(1+dx^2)}{dx}+b^2x\text{arccosh}\left(1+dx^2\right)^2$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

output
```
(a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*x) + (2*b*(a*d*x^
2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2])/(d*x) + b^2*x*Arc
Cosh[1 + d*x^2]^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6416, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2 dx$$

$$\Big\downarrow \ 6416$$

$$8b^2\int 1dx+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2-\frac{4b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

$$\Big\downarrow \ 24$$

$$x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2-\frac{4b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+8b^2x$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

output
```
8*b^2*x - (4*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*x^2]*
Sqrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.56

method	result
orering	$x(a+b\operatorname{arccosh}(d\,x^2+1))^2-\frac{8(a+b\operatorname{arccosh}(d\,x^2+1))bx}{\sqrt{d\,x^2}\,\sqrt{d\,x^2+2}}+\frac{x(d\,x^2+2)\left(\frac{8b^2d}{d\,x^2+2}+\frac{4(a+b\operatorname{arccosh}(d\,x^2+1))bd}{\sqrt{d\,x^2}\,\sqrt{d\,x^2+2}}-\frac{4(a+b}{}\right.}{}$

input
```
int((a+b*arccosh(d*x^2+1))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2+1))^2-8*(a+b*arccosh(d*x^2+1))*b*x/(d*x^2)^(1/2)/(d*x
^2+2)^(1/2)+1/d*x*(d*x^2+2)*(8*b^2*d/(d*x^2+2)+4*(a+b*arccosh(d*x^2+1))*b*
d/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-4*(a+b*arccosh(d*x^2+1))*b*d^2*x^2/(d*x^2)
^(3/2)/(d*x^2+2)^(1/2)-4*(a+b*arccosh(d*x^2+1))*b*d^2*x^2/(d*x^2)^(1/2)/(d
*x^2+2)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.82

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2\,dx$$

$$=\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^2+(a^2+8\,b^2)dx^2-4\,\sqrt{d^2x^4+2\,dx^2}ab+2\,\left(abdx^2-2\,\sqrt{d^2x^4+2\,d\right.}{dx}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + (a^2 + 8*b^2)*d*x^
2 - 4*sqrt(d^2*x^4 + 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^
2)*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1))/(d*x)
```

Sympy [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2\,dx=\int \left(a+b\,\text{acosh}\left(dx^2+1\right)\right)^2\,dx$$

input
```
integrate((a+b*acosh(d*x**2+1))**2,x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2 dx$$
$$= b^2x \operatorname{arcosh}\left(dx^2+1\right)^2$$
$$+ 4b^2d\left(\frac{2x}{d} - \frac{\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2+2}\sqrt{dx^2}+1\right)}{\sqrt{dx^2+2}d^2}\right)$$
$$+ 2\left(x\operatorname{arcosh}\left(dx^2+1\right) - \frac{2\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)}{\sqrt{dx^2+2}d}\right)ab + a^2x$$

input
```
integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

output
```
b^2*x*arccosh(d*x^2 + 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*lo
g(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 2*(x*a
rccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a*b +
 a^2*x
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2 dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^2\,dx$$

input
```
int((a + b*acosh(d*x^2 + 1))^2,x)
```

output
```
int((a + b*acosh(d*x^2 + 1))^2, x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2\,dx=2\left(\int acosh\left(d\,x^2+1\right)dx\right)ab+\left(\int acosh\left(d\,x^2+1\right)^2dx\right)b^2+a^2x$$

input
```
int((a+b*acosh(d*x^2+1))^2,x)
```

output
```
2*int(acosh(d*x**2 + 1),x)*a*b + int(acosh(d*x**2 + 1)**2,x)*b**2 + a**2*x
```

3.152 $\int (a + b\text{arccosh}(1 + dx^2))\ dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int (a + b\text{arccosh}(1 + dx^2))\ dx = ax - \frac{2b\sqrt{dx^2}\sqrt{2 + dx^2}}{dx} + bx\text{arccosh}(1 + dx^2)$$

output
```
a*x-2*b*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/d/x+b*x*arccosh(d*x^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (a + b\text{arccosh}(1 + dx^2))\ dx = ax - \frac{2bx}{\sqrt{\frac{dx^2}{2+dx^2}}} + bx\text{arccosh}(1 + dx^2)$$

input
```
Integrate[a + b*ArcCosh[1 + d*x^2],x]
```

output
```
a*x - (2*b*x)/Sqrt[(d*x^2)/(2 + d*x^2)] + b*x*ArcCosh[1 + d*x^2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+bx\text{arccosh}\left(dx^2+1\right)-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}$$

input
```
Int[a + b*ArcCosh[1 + d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + b*x*ArcCosh[1 +
d*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
orering	$x\left(a+b\,\operatorname{arccosh}\left(d\,x^{2}+1\right)\right)-\frac{2\sqrt{d\,x^{2}+2}\,b x}{\sqrt{d\,x^{2}}}$	36
default	$ax+b\left(x\,\operatorname{arccosh}\left(d\,x^{2}+1\right)-\frac{2x\sqrt{d\,x^{2}+2}}{\sqrt{d\,x^{2}}}\right)$	37
parts	$ax+b\left(x\,\operatorname{arccosh}\left(d\,x^{2}+1\right)-\frac{2x\sqrt{d\,x^{2}+2}}{\sqrt{d\,x^{2}}}\right)$	37

input
```
int(a+b*arccosh(d*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2+1))-2*(d*x^2+2)^(1/2)*b*x/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)\,dx = \frac{bdx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)+adx^2-2\,\sqrt{d^2x^4+2\,dx^2}b}{dx}$$

input
```
integrate(a+b*arccosh(d*x^2+1),x, algorithm="fricas")
```

output
```
(b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) + a*d*x^2 - 2*sqrt(d^2*x
^4 + 2*d*x^2)*b)/(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)\,dx = ax+b\left(\begin{cases} x\,\text{acosh}\left(dx^2+1\right)-\frac{2x\sqrt{dx^2+2}}{\sqrt{dx^2}} & \text{for}\ d\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*acosh(d*x**2+1),x)
```

output
```
a*x + b*Piecewise((x*acosh(d*x**2 + 1) - 2*x*sqrt(d*x**2 + 2)/sqrt(d*x**2)
, Ne(d, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right) \, dx = \left(x \operatorname{arcosh}\left(dx^2+1\right) - \frac{2\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)}{\sqrt{dx^2+2}d}\right)b + ax$$

input
```
integrate(a+b*arccosh(d*x^2+1),x, algorithm="maxima")
```

output
```
(x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*b
 + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right) \, dx$$
$$= \left(x \log\left(dx^2+\sqrt{\left(dx^2+1\right)^2-1}+1\right) + \frac{2\sqrt{2}\text{sgn}(x)}{\sqrt{d}} - \frac{2\sqrt{d^2x^2+2d}}{d\text{sgn}\left(x\right)}\right)b + ax$$

input
```
integrate(a+b*arccosh(d*x^2+1),x, algorithm="giac")
```

output
```
(x*log(d*x^2 + sqrt((d*x^2 + 1)^2 - 1) + 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2
*sqrt(d^2*x^2 + 2*d)/(d*sgn(x)))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)\,dx = a\,x+b\,x\,\mathrm{acosh}\left(d\,x^2+1\right)-\frac{2\,b\,\mathrm{sign}(x)\,\sqrt{d\,x^2+2}}{\sqrt{d}}$$

input
```
int(a + b*acosh(d*x^2 + 1),x)
```

output
```
a*x + b*x*acosh(d*x^2 + 1) - (2*b*sign(x)*(d*x^2 + 2)^(1/2))/d^(1/2)
```

Reduce [F]

$$\int \left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)\,dx = \left(\int acosh\left(d\,x^2+1\right)dx\right)b+ax$$

input
```
int(a+b*acosh(d*x^2+1),x)
```

output
```
int(acosh(d*x**2 + 1),x)*b + a*x
```

3.153 $\int \frac{1}{a+b\text{arccosh}(1+dx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \frac{1}{a+b\text{arccosh}(1+dx^2)}\,dx=\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

output
```
1/2*x*cosh(1/2*a/b)*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*2^(1/2)/b/(d*x^2)^(1
/2)-1/2*x*sinh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2+1))/b)*2^(1/2)/b/(d*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{arccosh}(1+dx^2)}\,dx=\frac{x\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(1+dx^2)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(1+dx^2)}{2b}\right)\right)}{b\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

output
```
(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1
 + d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/
(2*b)]))/(b*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6417}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arccosh}\left(dx^2+1\right)}dx$$

$$\Big\downarrow \text{ 6417}$$

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

output
```
(x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*
b*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/
(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 6417
```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos
h[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[
d*x^2])), x] - Simp[x*Sinh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Maple [F]

$$\int \frac{1}{a+b\ \text{arccosh}\left(d\,x^{2}+1\right)}dx$$

input
```
int(1/(a+b*arccosh(d*x^2+1)),x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1)),x)
```

Fricas [F]

$$\int \frac{1}{a+b\text{arccosh}\left(1+dx^{2}\right)}\,dx=\int \frac{1}{b\,\text{arcosh}\left(dx^{2}+1\right)+a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\text{arccosh}\left(1+dx^{2}\right)}\,dx=\int \frac{1}{a+b\,\text{acosh}\left(dx^{2}+1\right)}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2+1)),x)
```

output
```
Integral(1/(a + b*acosh(d*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\operatorname{arccosh}(1+dx^2)}\,dx=\int \frac{1}{b\operatorname{arcosh}(dx^2+1)+a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\operatorname{arccosh}(1+dx^2)}\,dx=\int \frac{1}{b\operatorname{arcosh}(dx^2+1)+a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\operatorname{arccosh}(1+dx^2)}\,dx=\int \frac{1}{a+b\operatorname{acosh}(d\,x^2+1)}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1)),x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1)), x)
```

Reduce [F]

$$\int \frac{1}{a + b\text{arccosh}\left(1 + dx^2\right)}\,dx = \int \frac{1}{acosh\left(d\,x^2 + 1\right)b + a}dx$$

input
```
int(1/(a+b*acosh(d*x^2+1)),x)
```

output
```
int(1/(acosh(d*x**2 + 1)*b + a),x)
```

3.154 $\int \frac{1}{\left(a+b\,\mathrm{arccosh}(1+dx^2)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{(a+b\,\mathrm{arccosh}(1+dx^2))^2}\,dx = -\frac{\sqrt{dx^2}\sqrt{2+dx^2}}{2bdx\,(a+b\,\mathrm{arccosh}(1+dx^2))} - \frac{x\mathrm{Chi}\left(\frac{a+b\mathrm{arccosh}(1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} + \frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\mathrm{arccosh}(1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}$$

output
```
-1/2*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2+1))-1/4*x*Chi(
1/2*(a+b*arccosh(d*x^2+1))/b)*sinh(1/2*a/b)*2^(1/2)/b^2/(d*x^2)^(1/2)+1/4*
x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2+1))/b)*2^(1/2)/b^2/(d*x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\text{arccosh}(1+dx^2))^2}\,dx = -\frac{\frac{2b\sqrt{dx^2}\sqrt{2+dx^2}}{ad+bd\text{arccosh}(1+dx^2)}+x^2\text{csch}\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(\text{Chi}\left(\frac{a+b\text{arccosh}(1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\right.\right.}{4b^2x}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

output
```
-1/4*((2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(a*d + b*d*ArcCosh[1 + d*x^2]) + x
^2*Csch[ArcCosh[1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*
b)]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/
(2*b)]))/(b^2*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6423}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2+1))^2}dx$$

$$\Big\downarrow \text{6423}$$

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2+2}}{2bdx(a+b\text{arccosh}(dx^2+1))}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

output
```
-1/2*(Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*(a + b*ArcCosh[1 + d*x^2])) - (x
*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(2*Sqrt[2]*
b^2*Sqrt[d*x^2]) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 6423
```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[(-Sqr
t[d*x^2])*(Sqrt[2 + d*x^2]/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2]))), x] + (-Si
mp[x*Sinh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]/(2*Sqrt[
2]*b^2*Sqrt[d*x^2])), x] + Simp[x*Cosh[a/(2*b)]*(SinhIntegral[(a + b*ArcCos
h[1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2])), x]) /; FreeQ[{a, b, d},
x]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)\right)^{2}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+b\text{arccosh}(1+dx^2))^2}\,dx=\int \frac{1}{(b\,\text{arcosh}\left(dx^2+1\right)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccosh(d*x^2 + 1)^2 + 2*a*b*arccosh(d*x^2 + 1) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2+1))**2,x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2+1\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

output
```
-1/2*(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2)
/(a*b*d^2*x^3 + 2*a*b*d*x + (b^2*d^2*x^3 + 2*b^2*d*x + (b^2*d^(3/2)*x^2 +
b^2*sqrt(d))*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) +
 (a*b*d^(3/2)*x^2 + a*b*sqrt(d))*sqrt(d*x^2 + 2)) + integrate(1/2*(d^3*x^6
 + 3*d^2*x^4 + (d^2*x^4 + d*x^2 + 2)*(d*x^2 + 2) + (2*d^(5/2)*x^5 + 4*d^(3
/2)*x^3 + sqrt(d)*x)*sqrt(d*x^2 + 2) - 4)/(a*b*d^3*x^6 + 4*a*b*d^2*x^4 + 4
*a*b*d*x^2 + (a*b*d^2*x^4 + 2*a*b*d*x^2 + a*b)*(d*x^2 + 2) + (b^2*d^3*x^6
+ 4*b^2*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 + 2*b^2*d*x^2 + b^2)*(d*x^2 +
 2) + 2*(b^2*d^(5/2)*x^5 + 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2
 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 2*(a*b*d^(5/2)*x^5 + 3
*a*b*d^(3/2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{(a + b\text{arccosh}(1 + dx^2))^2} dx = \int \frac{1}{(b\,\text{arcosh}(dx^2 + 1) + a)^2} dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arccosh}(1 + dx^2))^2} dx = \int \frac{1}{(a + b\,\text{acosh}(d\,x^2 + 1))^2} dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1))^2,x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a + b\text{arccosh}(1 + dx^2))^2} dx = \int \frac{1}{acosh(d\,x^2 + 1)^2 b^2 + 2acosh(d\,x^2 + 1)\,ab + a^2} dx$$

input
```
int(1/(a+b*acosh(d*x^2+1))^2,x)
```

output
```
int(1/(acosh(d*x**2 + 1)**2*b**2 + 2*acosh(d*x**2 + 1)*a*b + a**2),x)
```

3.155 $\int \frac{1}{\left(a+b\,\mathbf{arccosh}(1+dx^2)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 180

$$\int \frac{1}{\left(a+b\operatorname{arccosh}\left(1+dx^2\right)\right)^3}\,dx = -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\operatorname{arccosh}\left(1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\operatorname{arccosh}\left(1+dx^2\right)\right)} + \frac{x\cosh\left(\frac{a}{2b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} - \frac{x\sinh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}$$

output

```
-1/4*(d*x^4+2*x^2)/b/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)/(a+b*arccosh(d*x^2+1)
)^2-1/8*x/b^2/(a+b*arccosh(d*x^2+1))+1/16*x*cosh(1/2*a/b)*Chi(1/2*(a+b*arc
cosh(d*x^2+1))/b)*2^(1/2)/b^3/(d*x^2)^(1/2)-1/16*x*sinh(1/2*a/b)*Shi(1/2*(
a+b*arccosh(d*x^2+1))/b)*2^(1/2)/b^3/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\text{arccosh}(1+dx^2))^3}\,dx$$

$$= \frac{-\frac{2b^2\sqrt{dx^2}\sqrt{2+dx^2}}{d(a+b\text{arccosh}(1+dx^2))^2}-\frac{bx^2}{a+b\text{arccosh}(1+dx^2)}+\frac{\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(1+dx^2)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\ldots\right)}{d}}{8b^3x}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

output
```
((-2*b^2*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*(a + b*ArcCosh[1 + d*x^2])^2) - (
b*x^2)/(a + b*ArcCosh[1 + d*x^2]) + (Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2
*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhIn
tegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6425, 6417}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2+1))^3}\,dx$$

$$\Big\downarrow \; 6425$$

$$\frac{\int \frac{1}{a+b\text{arccosh}(dx^2+1)}dx}{8b^2}-\frac{x}{8b^2(a+b\text{arccosh}(dx^2+1))}-\frac{dx^4+2x^2}{4bx\sqrt{dx^2}\sqrt{dx^2+2}(a+b\text{arccosh}(dx^2+1))^2}$$

$$\Big\downarrow \; 6417$$

$$\frac{\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}}{8b^2}-\frac{x}{8b^2\left(a+b\text{arccosh}\left(dx^2+1\right)\right)}-\frac{dx^4+2x^2}{4bx\sqrt{dx^2}\sqrt{dx^2+2}\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2}$$

input

```
Int[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

output

```
-1/4*(2*x^2 + d*x^4)/(b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d
*x^2])^2) - x/(8*b^2*(a + b*ArcCosh[1 + d*x^2])) + ((x*Cosh[a/(2*b)]*CoshI
ntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]) - (x*Si
nh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqr
t[d*x^2]))/(8*b^2)
```

Defintions of rubi rules used

rule 6417

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos
h[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[
d*x^2])), x] - Simp[x*Sinh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

rule 6425

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp
[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x*Sqrt[
-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x] + Simp[1/(4*b^2*(n + 1)*(n + 2))
  Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]) /; FreeQ[{a, b, c, d}, x]
 && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,\operatorname{arccosh}\left(d\,x^{2}+1\right)\right)^{3}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

Fricas [F]

$$\int \frac{1}{\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2+1\right)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arccosh(d*x^2 + 1)^3 + 3*a*b^2*arccosh(d*x^2 + 1)^2 + 3*a^
2*b*arccosh(d*x^2 + 1) + a^3), x)
```

Sympy [F]

$$\int \frac{1}{\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^3}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2+1))**3,x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\text{arccosh}(1 + dx^2))^3} \, dx = \int \frac{1}{(b \operatorname{arcosh}(dx^2 + 1) + a)^3} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

output
```
-1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 + 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
 (11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 + 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*
(3*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 + (3*a*d^3 + 8*b*d^3)
*x^5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 + 4*(a*d + b*d)*x)*(d*x^2 + 2)^(3/2) + (3
*(a*d^4 + 2*b*d^4)*sqrt(d)*x^8 + 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*
a*d^2 + 25*b*d^2)*sqrt(d)*x^4 + 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*s
qrt(d))*(d*x^2 + 2) + (b*d^(11/2)*x^10 + 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^
6 + 2*b*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 + 3*b*d^3*x^5 + 4*b*d^
2*x^3 + 4*b*d*x)*(d*x^2 + 2)^(3/2) + (3*b*d^(9/2)*x^8 + 12*b*d^(7/2)*x^6 +
 16*b*d^(5/2)*x^4 + 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2) + (3*b*d^5
*x^9 + 15*b*d^4*x^7 + 23*b*d^3*x^5 + 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2
) - 8*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (3*(a*d^5 +
2*b*d^5)*x^9 + 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 + (7
*a*d^2 + 64*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2) - 8*a*sqrt(d
))/(a^2*b^2*d^(11/2)*x^9 + 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5
+ 8*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*b^4*d^(9/2)*x^7 + 12*b^4*d
^(7/2)*x^5 + 8*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*b^4*d^3*x^4 + 3*b^4*d^2*
x^2 + b^4*d)*(d*x^2 + 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 + 4*b^4*d^(7/2)*x^5 +
5*b^4*d^(5/2)*x^3 + 2*b^4*d^(3/2)*x)*(d*x^2 + 2) + 3*(b^4*d^5*x^8 + 5*b^4*
d^4*x^6 + 8*b^4*d^3*x^4 + 4*b^4*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + s...
```

Giac [F]

$$\int \frac{1}{(a + b\text{arccosh}(1 + dx^2))^3} \, dx = \int \frac{1}{(b \operatorname{arcosh}(dx^2 + 1) + a)^3} \, dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1))^3,x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1))^3, x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^3}\,dx$$
$$=\int \frac{1}{acosh\left(d\,x^2+1\right)^3 b^3+3acosh\left(d\,x^2+1\right)^2 a\,b^2+3acosh\left(d\,x^2+1\right)a^2b+a^3}dx$$

input
```
int(1/(a+b*acosh(d*x^2+1))^3,x)
```

output
```
int(1/(acosh(d*x**2 + 1)**3*b**3 + 3*acosh(d*x**2 + 1)**2*a*b**2 + 3*acosh
(d*x**2 + 1)*a**2*b + a**3),x)
```

3.156 $\int (a+b\text{arccosh}(-1+dx^2))^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 147

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4\,dx=384b^4x+\frac{192b^3(2x^2-dx^4)\left(a+b\text{arccosh}(-1+dx^2)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+48b^2x\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2+\frac{8b(2x^2-dx^4)\left(a+b\text{arccosh}(-1+dx^2)\right)^3}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4$$

output

```
384*b^4*x+192*b^3*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))/x/(d*x^2)^(1/2)/(d
*x^2-2)^(1/2)+48*b^2*x*(a+b*arccosh(d*x^2-1))^2+8*b*(-d*x^4+2*x^2)*(a+b*ar
ccosh(d*x^2-1))^3/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)+x*(a+b*arccosh(d*x^2-1))
^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.80

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4 dx$$
$$=\frac{(a^4+48a^2b^2+384b^4)\,dx^2-8ab(a^2+24b^2)\sqrt{dx^2}\sqrt{-2+dx^2}+4b\left(a^3dx^2+24ab^2dx^2-6a^2b\sqrt{dx^2}\sqrt{-2}\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

output
```
((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqr
t[-2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2] +
6*b^2*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCo
sh[-1 + d*x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcC
osh[-1 + d*x^2]^3 + b^4*d*x^2*ArcCosh[-1 + d*x^2]^4)/(d*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6416, 6416, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)^4 dx$$
$$\Big\downarrow \text{ 6416}$$
$$48b^2\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2dx+x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^4+\frac{8b(2x^2-dx^4)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$
$$\Big\downarrow \text{ 6416}$$

$$48b^2\left(8b^2\int 1dx + x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2 + \frac{4b(2x^2-dx^4)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}\right) + x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^4 + \frac{8b(2x^2-dx^4)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

$$\Big\downarrow \text{24}$$

$$48b^2\left(x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2 + \frac{4b(2x^2-dx^4)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}} + 8b^2x\right) + x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^4 + \frac{8b(2x^2-dx^4)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

output
```
(8*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[-2
 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^4 + 48*b^2*(8*b^2*x + (4*b*(2*x
^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])
+ x*(a + b*ArcCosh[-1 + d*x^2])^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(139) = 278$.

Time = 0.21 (sec) , antiderivative size = 1044, normalized size of antiderivative = 7.10

method	result	size
orering	Expression too large to display	1044

input
```
int((a+b*arccosh(d*x^2-1))^4,x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2-1))^4+32*(a+b*arccosh(d*x^2-1))^3*b*x/(d*x^2-2)^(1/2)
/(d*x^2)^(1/2)+1/d*x*(5*d*x^2-4)*(48*b^2*(a+b*arccosh(d*x^2-1))^2*d/(d*x^2
-2)+8*(a+b*arccosh(d*x^2-1))^3*b*d/(d*x^2-2)^(1/2)/(d*x^2)^(1/2)-8*(a+b*ar
ccosh(d*x^2-1))^3*b*d^2*x^2/(d*x^2-2)^(3/2)/(d*x^2)^(1/2)-8*(a+b*arccosh(d
*x^2-1))^3*b*d^2*x^2/(d*x^2-2)^(1/2)/(d*x^2)^(3/2))+(5*d^2*x^4-8*d*x^2-4)/
d^2*(192*b^3*(a+b*arccosh(d*x^2-1))*d^2/(d*x^2-2)^(3/2)*x/(d*x^2)^(1/2)-14
4*b^2*(a+b*arccosh(d*x^2-1))^2*d^2/(d*x^2-2)^2*x-24*(a+b*arccosh(d*x^2-1))
^3*b*d^2/(d*x^2-2)^(3/2)/(d*x^2)^(1/2)*x-24*(a+b*arccosh(d*x^2-1))^3*b*d^2
/(d*x^2-2)^(1/2)/(d*x^2)^(3/2)*x+24*(a+b*arccosh(d*x^2-1))^3*b*d^3*x^3/(d*
x^2-2)^(5/2)/(d*x^2)^(1/2)+16*(a+b*arccosh(d*x^2-1))^3*b*d^3*x^3/(d*x^2-2)
^(3/2)/(d*x^2)^(3/2)+24*(a+b*arccosh(d*x^2-1))^3*b*d^3*x^3/(d*x^2-2)^(1/2)
/(d*x^2)^(5/2))+1/d^2*x*(d*x^2-2)^2*(96*(a+b*arccosh(d*x^2-1))^3*b*d^3/(d*
x^2-2)^(3/2)/(d*x^2)^(3/2)*x^2+192*b^3*(a+b*arccosh(d*x^2-1))*d^2/(d*x^2-2
)^(3/2)/(d*x^2)^(1/2)-1152*b^3*(a+b*arccosh(d*x^2-1))*d^3/(d*x^2-2)^(5/2)*
x^2/(d*x^2)^(1/2)-192*b^3*(a+b*arccosh(d*x^2-1))*d^3/(d*x^2-2)^(3/2)*x^2/(
d*x^2)^(3/2)-24*(a+b*arccosh(d*x^2-1))^3*b*d^2/(d*x^2-2)^(3/2)/(d*x^2)^(1/
2)+144*(a+b*arccosh(d*x^2-1))^3*b*d^3/(d*x^2-2)^(5/2)/(d*x^2)^(1/2)*x^2+38
4*b^4*d^2/(d*x^2-2)^2-192*b^2*(a+b*arccosh(d*x^2-1))^2*d^2/(d*x^2-2)^2+720
*b^2*(a+b*arccosh(d*x^2-1))^2*d^3/(d*x^2-2)^3*x^2-120*(a+b*arccosh(d*x^2-1
))^3*b*d^4*x^4/(d*x^2-2)^(7/2)/(d*x^2)^(1/2)-72*(a+b*arccosh(d*x^2-1))^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(137) = 274$.

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.03

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4\,dx$$
$$= \frac{b^4dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^4+\left(a^4+48\,a^2b^2+384\,b^4\right)dx^2+4\left(ab^3dx^2-2\sqrt{d^2x^4-2\,dx^2}b^4\right)\log\cdots}{}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="fricas")
```

output
```
(b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^4 + (a^4 + 48*a^2*b^2
+ 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^4)*log(d*x
^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 - 6*(4*sqrt(d^2*x^4 - 2*d*x^2)*a*b^3 -
 (a^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + 4*(
(a^3*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 - 2*d*x^2)*(a^2*b^2 + 8*b^4))*lo
g(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 8*sqrt(d^2*x^4 - 2*d*x^2)*(a^3*b
+ 24*a*b^3))/(d*x)
```

Sympy [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4\,dx = \int \left(a+b\,\text{acosh}\left(dx^2-1\right)\right)^4\,dx$$

input
```
integrate((a+b*acosh(d*x**2-1))**4,x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**4, x)
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4\,dx=\int \left(b\operatorname{arcosh}\left(dx^2-1\right)+a\right)^4dx$$

input
```
integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^4 + 6*a^2*b^2*x*arccosh(d
*x^2 - 1)^2 + 24*a^2*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*log(d*x^2 +
sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 4*(x*arccosh(d*x
^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a^3*b + a^4*x +
 integrate(4*((a*b^3*d^2 - 2*b^4*d^2)*x^4 + 2*a*b^3 - (3*a*b^3*d - 4*b^4*d
)*x^2 + ((a*b^3*d - 2*b^4*d)*sqrt(d)*x^3 - 2*(a*b^3 - b^4)*sqrt(d)*x)*sqrt
(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^3/(d^2*x^4 - 3*d*x
^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^4\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)^4\,dx=\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^4\,dx$$

input
```
int((a + b*acosh(d*x^2 - 1))^4,x)
```

output
```
int((a + b*acosh(d*x^2 - 1))^4, x)
```

Reduce [F]

$$\int \left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)^4\,dx=4\left(\int \mathit{acosh}\left(d\,x^2-1\right)dx\right)a^3b+\left(\int \mathit{acosh}\left(d\,x^2-1\right)^4dx\right)b^4+4\left(\int \mathit{acosh}\left(d\,x^2-1\right)^3dx\right)a\,b^3+6\left(\int \mathit{acosh}\left(d\,x^2-1\right)^2dx\right)a^2b^2+a^4x$$

input
```
int((a+b*acosh(d*x^2-1))^4,x)
```

output
```
4*int(acosh(d*x**2 - 1),x)*a**3*b + int(acosh(d*x**2 - 1)**4,x)*b**4 + 4*i
nt(acosh(d*x**2 - 1)**3,x)*a*b**3 + 6*int(acosh(d*x**2 - 1)**2,x)*a**2*b**
2 + a**4*x
```

3.157 $\int (a+b\,\text{arccosh}(-1+dx^2))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int (a+b\,\text{arccosh}(-1+dx^2))^3\,dx = 24ab^2x-48b^3\sqrt{1-\frac{2}{dx^2}}\,x+24b^3x\,\text{arccosh}(-1+dx^2)+\frac{6b(2x^2-dx^4)(a+b\,\text{arccosh}(-1+dx^2))^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x(a+b\,\text{arccosh}(-1+dx^2))^3$$

output
```
24*a*b^2*x-48*b^3*(1-2/d/x^2)^(1/2)*x+24*b^3*x*arccosh(d*x^2-1)+6*b*(-d*x^
4+2*x^2)*(a+b*arccosh(d*x^2-1))^2/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)+x*(a+b*a
rccosh(d*x^2-1))^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int (a+b\,\text{arccosh}(-1+dx^2))^3\,dx = \frac{a(a^2+24b^2)\,dx^2-6b(a^2+8b^2)\sqrt{dx^2}\sqrt{-2+dx^2}+3b\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2}\sqrt{-2+dx^2}\right)\text{arccosh}}{dx}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

output
```
(a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[-2 + d*x^2] +
 3*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCos
h[-1 + d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh
[-1 + d*x^2]^2 + b^3*d*x^2*ArcCosh[-1 + d*x^2]^3)/(d*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6416, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)^3 dx$$

$$\Big\downarrow \text{6416}$$

$$24b^2\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)dx+x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^3+\frac{6b\left(2x^2-dx^4\right)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

$$\Big\downarrow \text{2009}$$

$$24b^2\left(ax+bx\text{arccosh}\left(dx^2-1\right)-2bx\sqrt{1-\frac{2}{dx^2}}\right)+x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^3+\frac{6b\left(2x^2-dx^4\right)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

output
```
(6*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^2)/(x*Sqrt[d*x^2]*Sqrt[-2
 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^3 + 24*b^2*(a*x - 2*b*Sqrt[1 -
2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(104) = 208$.

Time = 0.14 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.30

method	result
orering	$x(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right))^{3}-\frac{6\left(d\,x^{2}-4\right)\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)^{2} b x}{\sqrt{d\,x^{2}-2}\,\sqrt{d\,x^{2}}}-\frac{2\left(d\,x^{2}-2\right) x\left(\frac{24 b^{2} d\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)}{d\,x^{2}-2}+6\right.}{}$

input
```
int((a+b*arccosh(d*x^2-1))^3,x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2-1))^3-6*(d*x^2-4)*(a+b*arccosh(d*x^2-1))^2*b*x/(d*x^2
-2)^(1/2)/(d*x^2)^(1/2)-2/d*(d*x^2-2)*x*(24*b^2*d*(a+b*arccosh(d*x^2-1))/(
d*x^2-2)+6*(a+b*arccosh(d*x^2-1))^2*b*d/(d*x^2-2)^(1/2)/(d*x^2)^(1/2)-6*(a
+b*arccosh(d*x^2-1))^2*b*d^2*x^2/(d*x^2-2)^(3/2)/(d*x^2)^(1/2)-6*(a+b*arcc
osh(d*x^2-1))^2*b*d^2*x^2/(d*x^2-2)^(1/2)/(d*x^2)^(3/2))-1/d^2*(d*x^2-2)^2
*(48*b^3*d^2*x/(d*x^2-2)^(3/2)/(d*x^2)^(1/2)-72*b^2*d^2*(a+b*arccosh(d*x^2
-1))/(d*x^2-2)^2*x-18*(a+b*arccosh(d*x^2-1))^2*b*d^2/(d*x^2-2)^(3/2)/(d*x^
2)^(1/2)*x-18*(a+b*arccosh(d*x^2-1))^2*b*d^2/(d*x^2-2)^(1/2)/(d*x^2)^(3/2)
*x+18*(a+b*arccosh(d*x^2-1))^2*b*d^3*x^3/(d*x^2-2)^(5/2)/(d*x^2)^(1/2)+12*
(a+b*arccosh(d*x^2-1))^2*b*d^3*x^3/(d*x^2-2)^(3/2)/(d*x^2)^(3/2)+18*(a+b*a
rccosh(d*x^2-1))^2*b*d^3*x^3/(d*x^2-2)^(1/2)/(d*x^2)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(103) = 206$.

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.91

$$\int \left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^3 \, dx$$
$$= \frac{b^3dx^2 \log\left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right)^3 + (a^3 + 24\,ab^2)dx^2 + 3\left(ab^2dx^2 - 2\sqrt{d^2x^4 - 2\,dx^2}b^3\right)\log\left(dx^2 + \sqrt{\ldots}\right.}{\ldots}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

output
```
(b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 + (a^3 + 24*a*b^2)*d
*x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^
2*x^4 - 2*d*x^2) - 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 - 2*d*
x^2)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 6*sqrt(d^2*x^4 - 2*
d*x^2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F]

$$\int \left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^3 \, dx = \int \left(a + b\,\text{acosh}\left(dx^2 - 1\right)\right)^3 \, dx$$

input
```
integrate((a+b*acosh(d*x**2-1))**3,x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**3, x)
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^3\,dx=\int \left(b\,\text{arcosh}\left(dx^2-1\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

output
```
3*a*b^2*x*arccosh(d*x^2 - 1)^2 + 12*a*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt
(d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) +
 3*(x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d)
)*a^2*b + (x*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^3 - integrate(6*(d
^2*x^4 - 2*d*x^2 + (d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2))*log(d*x^2 +
sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^2/(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqr
t(d)*x)*sqrt(d*x^2 - 2) + 2), x))*b^3 + a^3*x
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^3\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^3\,dx=\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^3\,dx$$

input
```
int((a + b*acosh(d*x^2 - 1))^3,x)
```

output
```
int((a + b*acosh(d*x^2 - 1))^3, x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^3\,dx=3\left(\int acosh\left(d\,x^2-1\right)dx\right)a^2b+\left(\int acosh\left(d\,x^2-1\right)^3dx\right)b^3+3\left(\int acosh\left(d\,x^2-1\right)^2dx\right)a\,b^2+a^3x$$

input
```
int((a+b*acosh(d*x^2-1))^3,x)
```

output
```
3*int(acosh(d*x**2 - 1),x)*a**2*b + int(acosh(d*x**2 - 1)**3,x)*b**3 + 3*i
nt(acosh(d*x**2 - 1)**2,x)*a*b**2 + a**3*x
```

3.158 $\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2 dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2 dx = 8b^2x+\frac{4b(2x^2-dx^4)\left(a+b\text{arccosh}(-1+dx^2)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2$$

output
```
8*b^2*x+4*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))/x/(d*x^2)^(1/2)/(d*x^2-2
)^(1/2)+x*(a+b*arccosh(d*x^2-1))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

$$\begin{aligned}&\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2 dx\\&=\left(a^2+8b^2\right)x-\frac{4ab\sqrt{dx^2}\sqrt{-2+dx^2}}{dx}\\&\quad+\frac{2b\left(adx^2-2b\sqrt{dx^2}\sqrt{-2+dx^2}\right)\text{arccosh}(-1+dx^2)}{dx}+b^2x\text{arccosh}\left(-1+dx^2\right)^2\end{aligned}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

output
```
(a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*x) + (2*b*(a*d*x
^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2])/(d*x) + b^2*x*
ArcCosh[-1 + d*x^2]^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6416, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2 dx$$

$$\Big\downarrow \text{6416}$$

$$8b^2\int 1dx+x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2+\frac{4b\left(2x^2-dx^4\right)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

$$\Big\downarrow \text{24}$$

$$x\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2+\frac{4b\left(2x^2-dx^4\right)\left(a+b\text{arccosh}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+8b^2x$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

output
```
8*b^2*x + (4*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d*x^2]
*Sqrt[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(69) = 138$.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.52

method	result
orering	$x\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)^{2}+\frac{8\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)bx}{\sqrt{d\,x^{2}-2}\,\sqrt{d\,x^{2}}}+\frac{x\left(d\,x^{2}-2\right)\left(\frac{8b^{2}d}{d\,x^{2}-2}+\frac{4\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)bd}{\sqrt{d\,x^{2}-2}\,\sqrt{d\,x^{2}}}-\frac{4\left(a+b\right.}{}\right.}{}$

input
```
int((a+b*arccosh(d*x^2-1))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2-1))^2+8*(a+b*arccosh(d*x^2-1))*b*x/(d*x^2-2)^(1/2)/(d
*x^2)^(1/2)+1/d*x*(d*x^2-2)*(8*b^2*d/(d*x^2-2)+4*(a+b*arccosh(d*x^2-1))*b*
d/(d*x^2-2)^(1/2)/(d*x^2)^(1/2)-4*(a+b*arccosh(d*x^2-1))*b*d^2*x^2/(d*x^2-
2)^(3/2)/(d*x^2)^(1/2)-4*(a+b*arccosh(d*x^2-1))*b*d^2*x^2/(d*x^2-2)^(1/2)/
(d*x^2)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2\,dx$$
$$=\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^2+(a^2+8\,b^2)dx^2-4\,\sqrt{d^2x^4-2\,dx^2}ab+2\left(abdx^2-2\,\sqrt{d^2x^4-2\,d\cdots}\right.}{dx}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + (a^2 + 8*b^2)*d*x^
2 - 4*sqrt(d^2*x^4 - 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^
2)*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1))/(d*x)
```

Sympy [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2\,dx=\int \left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^2\,dx$$

input
```
integrate((a+b*acosh(d*x**2-1))**2,x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int \left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)^2\,dx$$
$$= b^2x\operatorname{arcosh}\left(dx^2-1\right)^2 + 4\,b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2-2}\sqrt{dx^2}-1\right)}{\sqrt{dx^2-2}d^2}\right) + 2\left(x\operatorname{arcosh}\left(dx^2-1\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)}{\sqrt{dx^2-2}d}\right)ab+a^2x$$

input
```
integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

output
```
b^2*x*arccosh(d*x^2 - 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*lo
g(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 2*(x*a
rccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a*b +
 a^2*x
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^2\,dx$$

input
```
int((a + b*acosh(d*x^2 - 1))^2,x)
```

output
```
int((a + b*acosh(d*x^2 - 1))^2, x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^2\,dx=2\left(\int acosh\left(d\,x^2-1\right)dx\right)ab+\left(\int acosh\left(d\,x^2-1\right)^2dx\right)b^2+a^2x$$

input
```
int((a+b*acosh(d*x^2-1))^2,x)
```

output
```
2*int(acosh(d*x**2 - 1),x)*a*b + int(acosh(d*x**2 - 1)**2,x)*b**2 + a**2*x
```

3.159 $\int (a + b\text{arccosh}(-1 + dx^2)) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int (a + b\text{arccosh}(-1 + dx^2)) \, dx = ax - 2b\sqrt{1 - \frac{2}{dx^2}}x + bx\text{arccosh}(-1 + dx^2)$$

output
```
a*x-2*b*(1-2/d/x^2)^(1/2)*x+b*x*arccosh(d*x^2-1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (a + b\text{arccosh}(-1 + dx^2)) \, dx = ax - 2b\sqrt{1 - \frac{2}{dx^2}}x + bx\text{arccosh}(-1 + dx^2)$$

input
```
Integrate[a + b*ArcCosh[-1 + d*x^2],x]
```

output
```
a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+bx\text{arccosh}\left(dx^2-1\right)-2bx\sqrt{1-\frac{2}{dx^2}}$$

input
```
Int[a + b*ArcCosh[-1 + d*x^2],x]
```

output
```
a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
orering	$x\left(a+b\,\operatorname{arccosh}\left(d\,x^{2}-1\right)\right)-\frac{2\sqrt{d\,x^{2}-2}\,b x}{\sqrt{d\,x^{2}}}$	36
default	$a x+b\left(x\,\operatorname{arccosh}\left(d\,x^{2}-1\right)-\frac{2x\sqrt{d\,x^{2}-2}}{\sqrt{d\,x^{2}}}\right)$	37
parts	$a x+b\left(x\,\operatorname{arccosh}\left(d\,x^{2}-1\right)-\frac{2x\sqrt{d\,x^{2}-2}}{\sqrt{d\,x^{2}}}\right)$	37

input
```
int(a+b*arccosh(d*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
x*(a+b*arccosh(d*x^2-1))-2*(d*x^2-2)^(1/2)*b*x/(d*x^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \left(a + b\text{arccosh}\left(-1 + dx^2\right)\right) dx = \frac{bdx^2 \log\left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right) + adx^2 - 2\sqrt{d^2x^4 - 2\,dx^2}b}{dx}$$

input
```
integrate(a+b*arccosh(d*x^2-1),x, algorithm="fricas")
```

output
```
(b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) + a*d*x^2 - 2*sqrt(d^2*x
^4 - 2*d*x^2)*b)/(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \left(a + b\text{arccosh}\left(-1 + dx^2\right)\right) dx = ax + b \left(\begin{cases} x \operatorname{acosh}\left(dx^2 - 1\right) - \frac{2x\sqrt{dx^2-2}}{\sqrt{dx^2}} & \text{for}\ d \neq 0 \\ i\pi x & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*acosh(d*x**2-1),x)
```

output
```
a*x + b*Piecewise((x*acosh(d*x**2 - 1) - 2*x*sqrt(d*x**2 - 2)/sqrt(d*x**2)
, Ne(d, 0)), (I*pi*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)\,dx=\left(x\,\text{arcosh}\left(dx^2-1\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)}{\sqrt{dx^2-2}d}\right)b+ax$$

input
```
integrate(a+b*arccosh(d*x^2-1),x, algorithm="maxima")
```

output
```
(x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*b
 + a*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)\,dx$$
$$=\left(x\log\left(dx^2+\sqrt{\left(dx^2-1\right)^2-1}-1\right)+\frac{2\sqrt{2}\sqrt{-d}\text{sgn}(x)}{d}-\frac{2\sqrt{d^2x^2-2\,d}}{d\text{sgn}\left(x\right)}\right)b+ax$$

input
```
integrate(a+b*arccosh(d*x^2-1),x, algorithm="giac")
```

output
```
(x*log(d*x^2 + sqrt((d*x^2 - 1)^2 - 1) - 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d
- 2*sqrt(d^2*x^2 - 2*d)/(d*sgn(x)))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)\,dx=a\,x+b\,x\,\mathrm{acosh}\left(d\,x^2-1\right)-\frac{2\,b\,\mathrm{sign}(x)\,\sqrt{d\,x^2-2}}{\sqrt{d}}$$

input
```
int(a + b*acosh(d*x^2 - 1),x)
```

output
```
a*x + b*x*acosh(d*x^2 - 1) - (2*b*sign(x)*(d*x^2 - 2)^(1/2))/d^(1/2)
```

Reduce [F]

$$\int \left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)\,dx=\left(\int acosh\left(d\,x^2-1\right)dx\right)b+ax$$

input
```
int(a+b*acosh(d*x^2-1),x)
```

output
```
int(acosh(d*x**2 - 1),x)*b + a*x
```

3.160 $\int \frac{1}{a+b\text{arccosh}(-1+dx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \frac{1}{a+b\text{arccosh}(-1+dx^2)}\,dx = -\frac{x\text{Chi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}} + \frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

output
```
-1/2*x*Chi(1/2*(a+b*arccosh(d*x^2-1))/b)*sinh(1/2*a/b)*2^(1/2)/b/(d*x^2)^(
1/2)+1/2*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2-1))/b)*2^(1/2)/b/(d*x^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\text{arccosh}(-1+dx^2)}\,dx = -\frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\text{Chi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)\right)}{bdx}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

output
```
-((Cosh[ArcCosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(
2*b)]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2
])/(2*b)]))/(b*d*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{6418\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a + b\text{arccosh}\left(dx^2 - 1\right)} \, dx$$

$$\Big\downarrow \text{ 6418}$$

$$\frac{x \cosh\left(\frac{a}{2b}\right) \text{Shi}\left(\frac{a+b\text{arccosh}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}} - \frac{x \sinh\left(\frac{a}{2b}\right) \text{Chi}\left(\frac{a+b\text{arccosh}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

output
```
-((x*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(Sqrt[
2]*b*Sqrt[d*x^2])) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x
^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 6418
```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x)
*Sinh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*
Sqrt[d*x^2])), x] + Simp[x*Cosh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[-1 +
d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Maple [F]

$$\int \frac{1}{a+b\,\mathrm{arccosh}\,(d\,x^2-1)}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1)),x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1)),x)
```

Fricas [F]

$$\int \frac{1}{a+b\mathrm{arccosh}\,(-1+dx^2)}\,dx=\int \frac{1}{b\,\mathrm{arcosh}\,(dx^2-1)+a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\mathrm{arccosh}\,(-1+dx^2)}\,dx=\int \frac{1}{a+b\,\mathrm{acosh}\,(dx^2-1)}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2-1)),x)
```

output
```
Integral(1/(a + b*acosh(d*x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{a + b\text{arccosh}\left(-1 + dx^2\right)}\,dx = \int \frac{1}{b\,\text{arcosh}\left(dx^2 - 1\right) + a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Giac [F]

$$\int \frac{1}{a + b\text{arccosh}\left(-1 + dx^2\right)}\,dx = \int \frac{1}{b\,\text{arcosh}\left(dx^2 - 1\right) + a}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\text{arccosh}\left(-1 + dx^2\right)}\,dx = \int \frac{1}{a + b\,\text{acosh}\left(d\,x^2 - 1\right)}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1)),x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{1}{a + b\text{arccosh}\left(-1 + dx^2\right)}\,dx = \int \frac{1}{acosh\left(d\,x^2 - 1\right)b + a}dx$$

input
```
int(1/(a+b*acosh(d*x^2-1)),x)
```

output
```
int(1/(acosh(d*x**2 - 1)*b + a),x)
```

3.161 $\int \frac{1}{\left(a+b\,\mathbf{arccosh}(-1+dx^2)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{(a+b\,\text{arccosh}\,(-1+dx^2))^2}\,dx = -\frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{2bdx\,(a+b\,\text{arccosh}\,(-1+dx^2))} + \frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}$$

output
```
-1/2*(d*x^2)^(1/2)*(d*x^2-2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2-1))+1/4*x*cosh
(1/2*a/b)*Chi(1/2*(a+b*arccosh(d*x^2-1))/b)*2^(1/2)/b^2/(d*x^2)^(1/2)-1/4*
x*sinh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2-1))/b)*2^(1/2)/b^2/(d*x^2)^(1/2
)
```

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+b\text{arccosh}(-1+dx^2))^2}\,dx$$
$$= \frac{-\frac{b\sqrt{dx^2}\sqrt{-2+dx^2}}{a+b\text{arccosh}(-1+dx^2)} + \frac{\sinh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(-1+dx^2)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(-1+}{2b}\right.\right.}{\sqrt{1-\frac{2}{dx^2}}}}{2b^2dx}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

output
```
(-((b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(a + b*ArcCosh[-1 + d*x^2])) + (Sinh[A
rcCosh[-1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^
2])/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]
))/Sqrt[1 - 2/(d*x^2)])/(2*b^2*d*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6424}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2-1))^2}\,dx$$
$$\Big\downarrow \text{6424}$$
$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} - \frac{\sqrt{dx^2}\sqrt{dx^2-2}}{2bdx\,(a+b\text{arccosh}(dx^2-1))}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

output
```
-1/2*(Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*(a + b*ArcCosh[-1 + d*x^2])) +
(x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqrt[
2]*b^2*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*
x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 6424
```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[(-Sq
rt[d*x^2])*(Sqrt[-2 + d*x^2]/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2]))), x] + (
Simp[x*Cosh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]/(2*Sq
rt[2]*b^2*Sqrt[d*x^2])), x] - Simp[x*Sinh[a/(2*b)]*(SinhIntegral[(a + b*Arc
Cosh[-1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2])), x]) /; FreeQ[{a, b,
d}, x]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)^{2}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+b\text{arccosh}(-1+dx^2))^2}\,dx=\int \frac{1}{\left(b\,\text{arcosh}\left(dx^2-1\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccosh(d*x^2 - 1)^2 + 2*a*b*arccosh(d*x^2 - 1) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a + b\text{arccosh}\,(-1 + dx^2))^2}\,dx = \int \frac{1}{(a + b\,\text{acosh}\,(dx^2 - 1))^2}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2-1))**2,x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\text{arccosh}\,(-1 + dx^2))^2}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx^2 - 1) + a)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

output
```
-1/2*(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2)
/(a*b*d^2*x^3 - 2*a*b*d*x + (b^2*d^2*x^3 - 2*b^2*d*x + (b^2*d^(3/2)*x^2 -
b^2*sqrt(d))*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) +
 (a*b*d^(3/2)*x^2 - a*b*sqrt(d))*sqrt(d*x^2 - 2)) + integrate(1/2*(d^3*x^6
 - 3*d^2*x^4 + (d^2*x^4 - d*x^2 + 2)*(d*x^2 - 2) + (2*d^(5/2)*x^5 - 4*d^(3
/2)*x^3 + sqrt(d)*x)*sqrt(d*x^2 - 2) + 4)/(a*b*d^3*x^6 - 4*a*b*d^2*x^4 + 4
*a*b*d*x^2 + (a*b*d^2*x^4 - 2*a*b*d*x^2 + a*b)*(d*x^2 - 2) + (b^2*d^3*x^6
- 4*b^2*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 - 2*b^2*d*x^2 + b^2)*(d*x^2 -
 2) + 2*(b^2*d^(5/2)*x^5 - 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2
 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 2*(a*b*d^(5/2)*x^5 - 3
*a*b*d^(3/2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 - 2)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^2}\,dx=\int \frac{1}{(b\operatorname{arcosh}(dx^2-1)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^2}\,dx=\int \frac{1}{(a+b\operatorname{acosh}(d\,x^2-1))^2}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1))^2,x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^2}\,dx=\int \frac{1}{acosh(d\,x^2-1)^2\,b^2+2acosh(d\,x^2-1)\,ab+a^2}dx$$

input
```
int(1/(a+b*acosh(d*x^2-1))^2,x)
```

output
```
int(1/(acosh(d*x**2 - 1)**2*b**2 + 2*acosh(d*x**2 - 1)*a*b + a**2),x)
```

3.162 $\int \frac{1}{\left(a+b\textbf{arccosh}(-1+dx^2)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 181

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^3}\,dx = \frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{-2+dx^2}\,(a+b\operatorname{arccosh}(-1+dx^2))^2} - \frac{x}{8b^2(a+b\operatorname{arccosh}(-1+dx^2))} - \frac{x\operatorname{Chi}\left(\frac{a+b\operatorname{arccosh}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} + \frac{x\cosh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arccosh}(-1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}$$

output
```
1/4*(-d*x^4+2*x^2)/b/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)/(a+b*arccosh(d*x^2-1)
)^2-1/8*x/b^2/(a+b*arccosh(d*x^2-1))-1/16*x*Chi(1/2*(a+b*arccosh(d*x^2-1))
/b)*sinh(1/2*a/b)*2^(1/2)/b^3/(d*x^2)^(1/2)+1/16*x*cosh(1/2*a/b)*Shi(1/2*(
a+b*arccosh(d*x^2-1))/b)*2^(1/2)/b^3/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+b\text{arccosh}(-1+dx^2))^3}\,dx = -\frac{\frac{2b^2\sqrt{dx^2}\sqrt{-2+dx^2}}{d\left(a+b\text{arccosh}(-1+dx^2)\right)^2}+\frac{bx^2}{a+b\text{arccosh}(-1+dx^2)}+\frac{1}{2}\sqrt{1-\frac{2}{dx^2}}x^2\text{csch}\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\text{Chi}\left(\frac{a+b\text{arcc}}{}\right.\right.}{8b^3x}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

output
```
-1/8*((2*b^2*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*(a + b*ArcCosh[-1 + d*x^2])^
2) + (b*x^2)/(a + b*ArcCosh[-1 + d*x^2]) + (Sqrt[1 - 2/(d*x^2)]*x^2*Csch[A
rcCosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sin
h[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]
))/2)/(b^3*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6425, 6418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2-1))^3}dx$$

$$\Big\downarrow \text{6425}$$

$$\frac{\int \frac{1}{a+b\text{arccosh}(dx^2-1)}dx}{8b^2}-\frac{x}{8b^2(a+b\text{arccosh}(dx^2-1))}+\frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{dx^2-2}(a+b\text{arccosh}(dx^2-1))^2}$$

$$\Big\downarrow \text{6418}$$

$$\frac{\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\text{arccosh}\left(dx^2-1\right)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\text{arccosh}\left(dx^2-1\right)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}}{8b^2}-\frac{x}{8b^2\left(a+b\text{arccosh}\left(dx^2-1\right)\right)}+\frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{dx^2-2}\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^2}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

output
```
(2*x^2 - d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*
x^2])^2) - x/(8*b^2*(a + b*ArcCosh[-1 + d*x^2])) + (-((x*CoshIntegral[(a +
 b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])) + (
x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(Sqrt[2]*
b*Sqrt[d*x^2]))/(8*b^2)
```

Defintions of rubi rules used

rule 6418
```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x)
*Sinh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*
Sqrt[d*x^2])), x] + Simp[x*Cosh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[-1 +
d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

rule 6425
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp
[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x*Sqrt[
-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x] + Simp[1/(4*b^2*(n + 1)*(n + 2))
  Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]) /; FreeQ[{a, b, c, d}, x]
 && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Maple [F]

$$\int \frac{1}{(a+b\ \mathrm{arccosh}\,(d\,x^2-1))^3}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+b\mathrm{arccosh}\,(-1+dx^2))^3}\,dx = \int \frac{1}{(b\,\mathrm{arcosh}\,(dx^2-1)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arccosh(d*x^2 - 1)^3 + 3*a*b^2*arccosh(d*x^2 - 1)^2 + 3*a^
2*b*arccosh(d*x^2 - 1) + a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arccosh}\,(-1+dx^2))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{acosh}\,(dx^2-1))^3}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2-1))**3,x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arccosh}\,(-1+dx^2))^3}\,dx=\int \frac{1}{(b\,\text{arcosh}\,(dx^2-1)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

output
```
-1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 - 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
 (11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 - 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*
(3*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*a*d^3 + 8*b*d^3)
*x^5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(a*d + b*d)*x)*(d*x^2 - 2)^(3/2) + (3
*(a*d^4 + 2*b*d^4)*sqrt(d)*x^8 - 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*
a*d^2 + 25*b*d^2)*sqrt(d)*x^4 - 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*s
qrt(d))*(d*x^2 - 2) + (b*d^(11/2)*x^10 - 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^
6 - 2*b*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 - 3*b*d^3*x^5 + 4*b*d^
2*x^3 - 4*b*d*x)*(d*x^2 - 2)^(3/2) + (3*b*d^(9/2)*x^8 - 12*b*d^(7/2)*x^6 +
 16*b*d^(5/2)*x^4 - 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2) + (3*b*d^5
*x^9 - 15*b*d^4*x^7 + 23*b*d^3*x^5 - 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2
) + 8*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (3*(a*d^5 +
2*b*d^5)*x^9 - 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 - (7
*a*d^2 + 64*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2) + 8*a*sqrt(d
))/(a^2*b^2*d^(11/2)*x^9 - 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5
- 8*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*b^4*d^(9/2)*x^7 + 12*b^4*d
^(7/2)*x^5 - 8*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*b^4*d^3*x^4 + 3*b^4*d^2*
x^2 - b^4*d)*(d*x^2 - 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 - 4*b^4*d^(7/2)*x^5 +
5*b^4*d^(5/2)*x^3 - 2*b^4*d^(3/2)*x)*(d*x^2 - 2) + 3*(b^4*d^5*x^8 - 5*b^4*
d^4*x^6 + 8*b^4*d^3*x^4 - 4*b^4*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + s...
```

Giac [F]

$$\int \frac{1}{(a+b\text{arccosh}\,(-1+dx^2))^3}\,dx=\int \frac{1}{(b\,\text{arcosh}\,(dx^2-1)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\operatorname{acosh}\left(d\,x^2-1\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1))^3,x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1))^3, x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\operatorname{arccosh}\left(-1+dx^2\right)\right)^3}\,dx = \int \frac{1}{acosh\left(d\,x^2-1\right)^3 b^3+3acosh\left(d\,x^2-1\right)^2 a\,b^2+3acosh\left(d\,x^2-1\right)a^2b+a^3}dx$$

input
```
int(1/(a+b*acosh(d*x^2-1))^3,x)
```

output
```
int(1/(acosh(d*x**2 - 1)**3*b**3 + 3*acosh(d*x**2 - 1)**2*a*b**2 + 3*acosh
(d*x**2 - 1)*a**2*b + a**3),x)
```

3.163 $\int (a+b\text{arccosh}(1+dx^2))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 280

$$\int (a+b\text{arccosh}(1+dx^2))^{5/2}\,dx = -\frac{5b(2x^2+dx^4)\,(a+b\text{arccosh}(1+dx^2))^{3/2}}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2} - \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{dx} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{dx} + \frac{30b^2\sqrt{a+b\text{arccosh}\left(1+dx^2\right)}\sinh^2\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{dx}$$

output

```
-5*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^(3/2)/x/(d*x^2)^(1/2)/(d*x^2+2)^
(1/2)+x*(a+b*arccosh(d*x^2+1))^(5/2)-15/2*b^(5/2)*2^(1/2)*Pi^(1/2)*erfi(1/
2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/
b))*sinh(1/2*arccosh(d*x^2+1))/d/x+15/2*b^(5/2)*2^(1/2)*Pi^(1/2)*erf(1/2*(
a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))
*sinh(1/2*arccosh(d*x^2+1))/d/x+30*b^2*(a+b*arccosh(d*x^2+1))^(1/2)*sinh(1
/2*arccosh(d*x^2+1))^2/d/x
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.11

$$\int (a+b\text{arccosh}(1+dx^2))^{5/2}\,dx = \frac{x\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(-15b^{5/2}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

output
```
(x*Sinh[ArcCosh[1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcC
osh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 15*b^
(5/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Co
sh[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-5*a*b*Co
sh[ArcCosh[1 + d*x^2]/2] + (a^2 + 15*b^2)*Sinh[ArcCosh[1 + d*x^2]/2] + b^2
*ArcCosh[1 + d*x^2]^2*Sinh[ArcCosh[1 + d*x^2]/2] - b*ArcCosh[1 + d*x^2]*(5
*b*Cosh[ArcCosh[1 + d*x^2]/2] - 2*a*Sinh[ArcCosh[1 + d*x^2]/2]))))/(2*Sqrt
[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6416, 6414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(dx^2+1))^{5/2}\,dx$$

$$\Big\downarrow \text{6416}$$

$$15b^2\int\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}dx+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{5/2}-\frac{5b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

$\Big\downarrow$ 6414

$$15b^2\left(\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\right.}{}\right.$$
$$x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{5/2}-\frac{5b\left(dx^4+2x^2\right)\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

output
```
(-5*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqr
t[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(5/2) + 15*b^2*(-((Sqrt[b]*Sq
rt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2
*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)) + (Sqrt[b]*Sqrt[P
i/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)]
+ Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (2*Sqrt[a + b*ArcCosh
[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x))
```

Defintions of rubi rules used

rule 6414
```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCosh[1 + d*x^2]]*(Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2/(d*x)), x] + (
Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh[
1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x] -
 Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x]
) /; FreeQ[{a, b, d}, x]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [F]

$$\int \left(a+b \,\mathrm{arccosh}\left(d \,x^{2}+1\right)\right)^{\frac{5}{2}}d x$$

input
```
int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

output
```
int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh(d*x**2+1))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2}\,dx=\int \left(b\,\text{arcosh}\left(dx^2+1\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2}\,dx=\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^{5/2}\,dx$$

input
```
int((a + b*acosh(d*x^2 + 1))^(5/2),x)
```

output
```
int((a + b*acosh(d*x^2 + 1))^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{5/2}\,dx=\left(\int \sqrt{acosh\left(d\,x^2+1\right)b+a}dx\right)a^2+2\left(\int \sqrt{acosh\left(d\,x^2+1\right)b+a}\,acosh\left(d\,x^2+1\right)dx\right)ab+\left(\int \sqrt{acosh\left(d\,x^2+1\right)b+a}\,acosh\left(d\,x^2+1\right)^2dx\right)b^2$$

input
```
int((a+b*acosh(d*x^2+1))^(5/2),x)
```

output
```
int(sqrt(acosh(d*x**2 + 1)*b + a),x)*a**2 + 2*int(sqrt(acosh(d*x**2 + 1)*b
 + a)*acosh(d*x**2 + 1),x)*a*b + int(sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d
*x**2 + 1)**2,x)*b**2
```

3.164 $\int (a + b\operatorname{arccosh}(1 + dx^2))^{3/2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 238

$$\int (a + b\operatorname{arccosh}(1 + dx^2))^{3/2} \, dx = -\frac{3b(2x^2 + dx^4)\sqrt{a + b\operatorname{arccosh}(1 + dx^2)}}{x\sqrt{dx^2}\sqrt{2 + dx^2}} + x(a + b\operatorname{arccosh}(1 + dx^2))^{3/2} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right) - \sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\operatorname{arccosh}(1 + dx^2)\right)}{dx} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right) + \sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\operatorname{arccosh}(1 + dx^2)\right)}{dx}$$

output
```
-3*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^(1/2)/x/(d*x^2)^(1/2)/(d*x^2+2)^
(1/2)+x*(a+b*arccosh(d*x^2+1))^(3/2)+3/2*b^(3/2)*2^(1/2)*Pi^(1/2)*erfi(1/2
*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b
))*sinh(1/2*arccosh(d*x^2+1))/d/x+3/2*b^(3/2)*2^(1/2)*Pi^(1/2)*erf(1/2*(a+
b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*s
inh(1/2*arccosh(d*x^2+1))/d/x
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx=\frac{x\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(3b^{3/2}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

output
```
(x*Sinh[ArcCosh[1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCos
h[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 3*b^(3/
2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[
a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-3*b*Cosh[Ar
cCosh[1 + d*x^2]/2] + a*Sinh[ArcCosh[1 + d*x^2]/2] + b*ArcCosh[1 + d*x^2]*
Sinh[ArcCosh[1 + d*x^2]/2])))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqr
t[2 + d*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6416, 6419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{6416}$$

$$3b^2\int \frac{1}{\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}dx+x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{3/2}-\frac{3b\left(dx^4+2x^2\right)\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

$$\Big\downarrow \; 6419$$

$$3b^2\left(\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right.}{}\right.$$

$$x\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{3/2}-\frac{3b\left(dx^4+2x^2\right)\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

output
```
(-3*b*(2*x^2 + d*x^4)*Sqrt[a + b*ArcCosh[1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[
2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(3/2) + 3*b^2*((Sqrt[Pi/2]*Erfi
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt[Pi/2]*Erf[Sqrt[
a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x))
```

Defintions of rubi rules used

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

rule 6419
```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqrt
[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] + Simp[Sqrt[Pi/2]
*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d}, x]
```

Maple [F]

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

output
```
int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx = \int \left(a+b\,\mathrm{acosh}\left(dx^{2}+1\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*acosh(d*x**2+1))**(3/2),x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx=\int \left(b\,\text{arcosh}\left(dx^2+1\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx=\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^{3/2}\,dx$$

input
```
int((a + b*acosh(d*x^2 + 1))^(3/2),x)
```

output
```
int((a + b*acosh(d*x^2 + 1))^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(1+dx^2\right)\right)^{3/2}\,dx = \left(\int \sqrt{acosh\left(d\,x^2+1\right)b+a}dx\right)a + \left(\int \sqrt{acosh\left(d\,x^2+1\right)b+a}\,acosh\left(d\,x^2+1\right)dx\right)b$$

input
```
int((a+b*acosh(d*x^2+1))^(3/2),x)
```

output
```
int(sqrt(acosh(d*x**2 + 1)*b + a),x)*a + int(sqrt(acosh(d*x**2 + 1)*b + a)
*acosh(d*x**2 + 1),x)*b
```

3.165 $\int \sqrt{a+b\text{arccosh}\left(1+dx^2\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 205

$$\begin{aligned}\int \sqrt{a+b\text{arccosh}\left(1+dx^2\right)}\,dx &= -\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{dx}\\&\quad+\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{dx}\\&\quad+\frac{2\sqrt{a+b\text{arccosh}\left(1+dx^2\right)}\sinh^2\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{dx}\end{aligned}$$

output
```
-1/2*b^(1/2)*2^(1/2)*Pi^(1/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))/d/x+1/
2*b^(1/2)*2^(1/2)*Pi^(1/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^
(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))/d/x+2*(a+b
*arccosh(d*x^2+1))^(1/2)*sinh(1/2*arccosh(d*x^2+1))^2/d/x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

$$\int \sqrt{a+b\operatorname{arccosh}(1+dx^2)}\,dx$$
$$=\frac{x\sinh\left(\frac{1}{2}\operatorname{arccosh}(1+dx^2)\right)\left(\sqrt{b}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{b}\sqrt{2\pi}\operatorname{erf}\left(\right.\right.}{2\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx}}}$$

input
```
Integrate[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

output
```
(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[
1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[b]*
Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 +
 d*x^2]/2]))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\operatorname{arccosh}(dx^2+1)}\,dx$$
$$\Big\downarrow \text{ 6414}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(dx^2+1)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(dx^2+1)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{2\sinh^2\left(\frac{1}{2}\text{arccosh}(dx^2+1)\right)\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}{dx}$$

input
```
Int[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

output
```
-((Sqrt[b]*Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b]
)]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)) + (S
qrt[b]*Sqrt[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(C
osh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (2*Sqrt[
a + b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Defintions of rubi rules used

rule 6414
```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCosh[1 + d*x^2]]*(Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2/(d*x)), x] + (
Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh[
1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x] -
 Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x]
) /; FreeQ[{a, b, d}, x]
```

Maple [F]

$$\int \sqrt{a+b\,\text{arccosh}\left(d\,x^2+1\right)}dx$$

input
```
int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

output
```
int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\text{arccosh}\left(1 + dx^2\right)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a + b\text{arccosh}\left(1 + dx^2\right)}\, dx = \int \sqrt{a + b\,\text{acosh}\left(dx^2 + 1\right)}\, dx$$

input
```
integrate((a+b*acosh(d*x**2+1))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acosh(d*x**2 + 1)), x)
```

Maxima [F]

$$\int \sqrt{a + b\text{arccosh}\left(1 + dx^2\right)}\, dx = \int \sqrt{b\,\text{arcosh}\left(dx^2 + 1\right) + a}\, dx$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\operatorname{arccosh}\left(1+dx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{arccosh}\left(1+dx^2\right)}\,dx = \int \sqrt{a+b\operatorname{acosh}\left(d\,x^2+1\right)}\,dx$$

input
```
int((a + b*acosh(d*x^2 + 1))^(1/2),x)
```

output
```
int((a + b*acosh(d*x^2 + 1))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\operatorname{arccosh}\left(1+dx^2\right)}\,dx = \int \sqrt{acosh\left(d\,x^2+1\right)b+a}dx$$

input
```
int((a+b*acosh(d*x^2+1))^(1/2),x)
```

output
```
int(sqrt(acosh(d*x**2 + 1)*b + a),x)
```

3.166 $\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}(1+dx^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(1+dx^2\right)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{\sqrt{b}dx} + \frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{\sqrt{b}dx}$$

output
```
1/2*2^(1/2)*Pi^(1/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2)
)*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))/b^(1/2)/d/x+1/2
*2^(1/2)*Pi^(1/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(c
osh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))/b^(1/2)/d/x
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(1+dx^2\right)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}x\left(\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\right.\right.\right.}{\sqrt{b}\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}}$$

input
```
Integrate[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

output
```
(Sqrt[Pi/2]*x*(Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cos
h[a/(2*b)] - Sinh[a/(2*b)]) + Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*
Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(Sq
rt[b]*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}dx$$

$$\Big\downarrow \text{6419}$$

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}$$

input
```
Int[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

output
```
(Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a
/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt
[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)
] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x)
```

Defintions of rubi rules used

rule 6419
```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqrt
[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] + Simp[Sqrt[Pi/2]
*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\mathrm{arccosh}\left(1+dx^2\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}\,dx=\int \frac{1}{\sqrt{a+b\operatorname{acosh}(dx^2+1)}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2+1))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acosh(d*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}\,dx=\int \frac{1}{\sqrt{b\operatorname{arcosh}(dx^2+1)+a}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{arccosh}\,(1 + dx^2)}}\,dx = \int \frac{1}{\sqrt{a + b\,\mathrm{acosh}\,(d\,x^2 + 1)}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1))^(1/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + b\text{arccosh}\,(1 + dx^2)}}\,dx = \int \frac{\sqrt{acosh\,(d\,x^2 + 1)\,b + a}}{acosh\,(d\,x^2 + 1)\,b + a}\,dx$$

input
```
int(1/(a+b*acosh(d*x^2+1))^(1/2),x)
```

output
```
int(sqrt(acosh(d*x**2 + 1)*b + a)/(acosh(d*x**2 + 1)*b + a),x)
```

3.167 $\int \frac{1}{\left(a+b\,\mathbf{arccosh}\left(1+dx^2\right)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{1}{(a+b\operatorname{arccosh}(1+dx^2))^{3/2}}\,dx = -\frac{\sqrt{dx^2}\sqrt{2+dx^2}}{bdx\sqrt{a+b\operatorname{arccosh}(1+dx^2)}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\operatorname{arccosh}(1+dx^2)\right)}{b^{3/2}dx} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\operatorname{arccosh}(1+dx^2)\right)}{b^{3/2}dx}$$

output
```
-(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2+1))^(1/2)+1/2*2^(1
/2)*Pi^(1/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(
1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))/b^(3/2)/d/x-1/2*2^(1/2)
*Pi^(1/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*
a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))/b^(3/2)/d/x
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\text{arccosh}\,(1+dx^2))^{3/2}}\,dx =$$

$$-\frac{x\left(4\sqrt{b}\cosh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)+\sqrt{2\pi}\sqrt{a+b\text{arccosh}\,(1+dx^2)}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)\right.\right.}{2b^{3/2}\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

output
```
-1/2*(x*(4*Sqrt[b]*Cosh[ArcCosh[1 + d*x^2]/2] + Sqrt[2*Pi]*Sqrt[a + b*ArcC
osh[1 + d*x^2]]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-C
osh[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[1 + d*x^2]]*
Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sin
h[a/(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)
/(2 + d*x^2)]*Sqrt[2 + d*x^2]*Sqrt[a + b*ArcCosh[1 + d*x^2]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}\,(dx^2+1))^{3/2}}\,dx$$

$$\Big\downarrow \; 6421$$

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}-\frac{\sqrt{dx^2}\sqrt{dx^2+2}}{bdx\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

output
```
-((Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])) +
(Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a
/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x) - (Sqrt
[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)
] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x)
```

Defintions of rubi rules used

rule 6421
```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-S
qrt[d*x^2])*(Sqrt[2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])), x] +
(-Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2
]*(Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + Simp[
Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; FreeQ[{a,
 b, d}, x]
```

Maple [F]

$$\int \frac{1}{\left(a+b\ \text{arccosh}\left(d\,x^2+1\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b\operatorname{arccosh}(1 + dx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a + b\operatorname{arccosh}(1 + dx^2))^{3/2}}\,dx = \int \frac{1}{(a + b\operatorname{acosh}(dx^2 + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2+1))**(3/2),x)
```

output
```
Integral((a + b*acosh(d*x**2 + 1))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\operatorname{arccosh}(1 + dx^2))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arcosh}(dx^2 + 1) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(1+dx^2))^{3/2}}\,dx=\int \frac{1}{(b\operatorname{arcosh}(dx^2+1)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arccosh}(1+dx^2))^{3/2}}\,dx=\int \frac{1}{(a+b\operatorname{acosh}(d\,x^2+1))^{3/2}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1))^(3/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(1+dx^2))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*acosh(d*x^2+1))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1)
- sqrt(d)*acosh(d*x**2 + 1)*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b
 + a)*acosh(d*x**2 + 1)*x)/(acosh(d*x**2 + 1)**2*b**2*d*x**2 + 2*acosh(d*x
**2 + 1)**2*b**2 + 2*acosh(d*x**2 + 1)*a*b*d*x**2 + 4*acosh(d*x**2 + 1)*a*
b + a**2*d*x**2 + 2*a**2),x)*a*b*d - sqrt(d)*acosh(d*x**2 + 1)*int((sqrt(d
*x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1)**2*x)/(acosh(d*
x**2 + 1)**2*b**2*d*x**2 + 2*acosh(d*x**2 + 1)**2*b**2 + 2*acosh(d*x**2 +
1)*a*b*d*x**2 + 4*acosh(d*x**2 + 1)*a*b + a**2*d*x**2 + 2*a**2),x)*b**2*d
- acosh(d*x**2 + 1)*int((sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1)*x
**2)/(acosh(d*x**2 + 1)**2*b**2*d*x**2 + 2*acosh(d*x**2 + 1)**2*b**2 + 2*a
cosh(d*x**2 + 1)*a*b*d*x**2 + 4*acosh(d*x**2 + 1)*a*b + a**2*d*x**2 + 2*a*
*2),x)*b**2*d**2 - 2*acosh(d*x**2 + 1)*int((sqrt(acosh(d*x**2 + 1)*b + a)*
acosh(d*x**2 + 1))/(acosh(d*x**2 + 1)**2*b**2*d*x**2 + 2*acosh(d*x**2 + 1)
**2*b**2 + 2*acosh(d*x**2 + 1)*a*b*d*x**2 + 4*acosh(d*x**2 + 1)*a*b + a**2
*d*x**2 + 2*a**2),x)*b**2*d - sqrt(d)*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x
**2 + 1)*b + a)*acosh(d*x**2 + 1)*x)/(acosh(d*x**2 + 1)**2*b**2*d*x**2 + 2
*acosh(d*x**2 + 1)**2*b**2 + 2*acosh(d*x**2 + 1)*a*b*d*x**2 + 4*acosh(d*x*
*2 + 1)*a*b + a**2*d*x**2 + 2*a**2),x)*a**2*d - sqrt(d)*int((sqrt(d*x**2 +
 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1)**2*x)/(acosh(d*x**2 +
1)**2*b**2*d*x**2 + 2*acosh(d*x**2 + 1)**2*b**2 + 2*acosh(d*x**2 + 1)*a...
```

3.168 $\int \frac{1}{\left(a+b\text{arccosh}(1+dx^2)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 252

$$\int \frac{1}{(a+b\text{arccosh}(1+dx^2))^{5/2}}\,dx = -\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\sqrt{2+dx^2}\,(a+b\text{arccosh}(1+dx^2))^{3/2}} - \frac{x}{3b^2\sqrt{a+b\text{arccosh}(1+dx^2)}} + \frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{3b^{5/2}dx} + \frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)}{3b^{5/2}dx}$$

output
```
-1/3*(d*x^4+2*x^2)/b/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)/(a+b*arccosh(d*x^2+1)
)^(3/2)-1/3*x/b^2/(a+b*arccosh(d*x^2+1))^(1/2)+1/6*2^(1/2)*Pi^(1/2)*erfi(1
/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a
/b))*sinh(1/2*arccosh(d*x^2+1))/b^(5/2)/d/x+1/6*2^(1/2)*Pi^(1/2)*erf(1/2*(
a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))
*sinh(1/2*arccosh(d*x^2+1))/b^(5/2)/d/x
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+b\text{arccosh}\left(1+dx^2\right))^{5/2}}\,dx = \frac{x\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(\sqrt{2\pi}(a+b\text{arccosh}(1+dx^2))^{3/2}\,\text{erfi}\left(\frac{\sqrt{a+}}{}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

output
```
(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)
*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - S
inh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) +
4*Sqrt[b]*(-(b*Cosh[ArcCosh[1 + d*x^2]/2]) - (a + b*ArcCosh[1 + d*x^2])*Si
nh[ArcCosh[1 + d*x^2]/2])))/(6*b^(5/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2
)]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^2])^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6425, 6419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}\left(dx^2+1\right))^{5/2}}dx$$

$$\Big\downarrow \text{6425}$$

$$\frac{\int \frac{1}{\sqrt{a+b\text{arccosh}(dx^2+1)}}dx}{3b^2} - \frac{x}{3b^2\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}} - \frac{dx^4+2x^2}{3bx\sqrt{dx^2}\sqrt{dx^2+2}\,(a+b\text{arccosh}\left(dx^2+1\right))^{3/2}}$$

$$\Big\downarrow \text{6419}$$

$$\frac{\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}\left(dx^2+1\right)\right)}{\sqrt{b}dx}}{3b^2}$$

$$\frac{x}{3b^2\sqrt{a+b\text{arccosh}\left(dx^2+1\right)}}-\frac{dx^4+2x^2}{3bx\sqrt{dx^2}\sqrt{dx^2+2}\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{3/2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

output
```
-1/3*(2*x^2 + d*x^4)/(b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d
*x^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + ((Sqrt[Pi/2]*Er
fi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh
[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt[Pi/2]*Erf[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*
b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x))/(3*b^2)
```

Defintions of rubi rules used

rule 6419
```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqrt
[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] + Simp[Sqrt[Pi/2]
*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d}, x]
```

rule 6425
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp
[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x*Sqrt[
-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x] + Simp[1/(4*b^2*(n + 1)*(n + 2))
  Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]) /; FreeQ[{a, b, c, d}, x]
 && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Maple [F]

$$\int \frac{1}{\left(a+b \,\mathrm{arccosh}\left(d \,x^{2}+1\right)\right)^{\frac{5}{2}}}d x$$

input
```
int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arccosh}(1+dx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arccosh}(1+dx^2))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*acosh(d*x**2+1))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arccosh}\,(1+dx^2))^{5/2}}\,dx=\int \frac{1}{(b\,\text{arcosh}\,(dx^2+1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arccosh}\,(1+dx^2))^{5/2}}\,dx=\int \frac{1}{(b\,\text{arcosh}\,(dx^2+1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arccosh}\,(1+dx^2))^{5/2}}\,dx=\int \frac{1}{(a+b\,\text{acosh}\,(d\,x^2+1))^{5/2}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1))^(5/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + b\text{arccosh}\,(1 + dx^2))^{5/2}}\,dx = \text{too large to display}$$

input
```
int(1/(a+b*acosh(d*x^2+1))^(5/2),x)
```

output
```
( - sqrt(d)*acosh(d*x**2 + 1)**2*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 +
 1)*b + a)*acosh(d*x**2 + 1)*x)/(acosh(d*x**2 + 1)**3*b**3*d*x**2 + 2*acos
h(d*x**2 + 1)**3*b**3 + 3*acosh(d*x**2 + 1)**2*a*b**2*d*x**2 + 6*acosh(d*x
**2 + 1)**2*a*b**2 + 3*acosh(d*x**2 + 1)*a**2*b*d*x**2 + 6*acosh(d*x**2 +
1)*a**2*b + a**3*d*x**2 + 2*a**3),x)*a*b**2*d - sqrt(d)*acosh(d*x**2 + 1)*
*2*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1)**
2*x)/(acosh(d*x**2 + 1)**3*b**3*d*x**2 + 2*acosh(d*x**2 + 1)**3*b**3 + 3*a
cosh(d*x**2 + 1)**2*a*b**2*d*x**2 + 6*acosh(d*x**2 + 1)**2*a*b**2 + 3*acos
h(d*x**2 + 1)*a**2*b*d*x**2 + 6*acosh(d*x**2 + 1)*a**2*b + a**3*d*x**2 + 2
*a**3),x)*b**3*d + acosh(d*x**2 + 1)**2*int((sqrt(acosh(d*x**2 + 1)*b + a)
*acosh(d*x**2 + 1)*x**2)/(acosh(d*x**2 + 1)**3*b**3*d*x**2 + 2*acosh(d*x**
2 + 1)**3*b**3 + 3*acosh(d*x**2 + 1)**2*a*b**2*d*x**2 + 6*acosh(d*x**2 + 1
)**2*a*b**2 + 3*acosh(d*x**2 + 1)*a**2*b*d*x**2 + 6*acosh(d*x**2 + 1)*a**2
*b + a**3*d*x**2 + 2*a**3),x)*b**3*d**2 + 2*acosh(d*x**2 + 1)**2*int((sqrt
(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1))/(acosh(d*x**2 + 1)**3*b**3*d*
x**2 + 2*acosh(d*x**2 + 1)**3*b**3 + 3*acosh(d*x**2 + 1)**2*a*b**2*d*x**2
+ 6*acosh(d*x**2 + 1)**2*a*b**2 + 3*acosh(d*x**2 + 1)*a**2*b*d*x**2 + 6*ac
osh(d*x**2 + 1)*a**2*b + a**3*d*x**2 + 2*a**3),x)*b**3*d + sqrt(d)*sqrt(d*
x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1) - 2*sqrt(d)*acos
h(d*x**2 + 1)*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh...
```

3.169 $\int \frac{1}{\left(a+b\,\mathrm{arccosh}(1+dx^2)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 301

$$\int \frac{1}{(a+b\,\mathrm{arccosh}(1+dx^2))^{7/2}}\,dx = -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\,(a+b\,\mathrm{arccosh}(1+dx^2))^{5/2}} - \frac{x}{15b^2(a+b\,\mathrm{arccosh}(1+dx^2))^{3/2}} - \frac{\sqrt{dx^2}\sqrt{2+dx^2}}{15b^3dx\sqrt{a+b\,\mathrm{arccosh}(1+dx^2)}} + \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\mathrm{arccosh}(1+dx^2)\right)}{15b^{7/2}dx} - \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\mathrm{arccosh}(1+dx^2)\right)}{15b^{7/2}dx}$$

output
```
-1/5*(d*x^4+2*x^2)/b/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)/(a+b*arccosh(d*x^2+1)
)^(5/2)-1/15*x/b^2/(a+b*arccosh(d*x^2+1))^(3/2)-1/15*(d*x^2)^(1/2)*(d*x^2+
2)^(1/2)/b^3/d/x/(a+b*arccosh(d*x^2+1))^(1/2)+1/30*2^(1/2)*Pi^(1/2)*erfi(1
/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a
/b))*sinh(1/2*arccosh(d*x^2+1))/b^(7/2)/d/x-1/30*2^(1/2)*Pi^(1/2)*erf(1/2*
(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b)
)*sinh(1/2*arccosh(d*x^2+1))/b^(7/2)/d/x
```

Mathematica [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+b\text{arccosh}\,(1+dx^2))^{7/2}}\,dx =$$

$$-\frac{x\sinh\left(\frac{1}{2}\text{arccosh}(1+dx^2)\right)\left(\sqrt{2\pi}(a+b\text{arccosh}(1+dx^2))^{5/2}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\text{si}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

output
```
-1/30*(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])
^(5/2)*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*
b)] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(5/2)*Erf[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*
b)]) + 4*Sqrt[b]*((3*b^2 + (a + b*ArcCosh[1 + d*x^2])^2)*Cosh[ArcCosh[1 +
d*x^2]/2] + b*(a + b*ArcCosh[1 + d*x^2])*Sinh[ArcCosh[1 + d*x^2]/2])))/(b^
(7/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2]*(a + b*ArcCosh
[1 + d*x^2])^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6425, 6421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2+1))^{7/2}}dx$$

$$\Big\downarrow \text{ 6425}$$

$$\frac{\int \frac{1}{(a+b\text{arccosh}(dx^2+1))^{3/2}}dx}{15b^2}-\frac{x}{15b^2(a+b\text{arccosh}(dx^2+1))^{3/2}}-\frac{dx^4+2x^2}{5bx\sqrt{dx^2}\sqrt{dx^2+2}(a+b\text{arccosh}(dx^2+1))^{5/2}}$$

$$\Big\downarrow \text{ 6421}$$

$$-\frac{x}{15b^2(a+b\text{arccosh}(dx^2+1))^{3/2}}+\frac{-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(dx^2+1)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\text{arccosh}(dx^2+1\right.}{b^{3/2}dx}}{15b^2}-\frac{dx^4+2x^2}{5bx\sqrt{dx^2}\sqrt{dx^2+2}(a+b\text{arccosh}(dx^2+1))^{5/2}}$$

input
```
Int[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

output
```
-1/5*(2*x^2 + d*x^4)/(b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d
*x^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[1 + d*x^2])^(3/2)) + (-((Sqrt[d*x
^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])) + (Sqrt[Pi/2]
*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - S
inh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x) - (Sqrt[Pi/2]*Erf[
Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/
(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x))/(15*b^2)
```

Defintions of rubi rules used

rule 6421
```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-S
qrt[d*x^2])*(Sqrt[2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])), x] +
(-Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2
]*(Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + Simp[
Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; FreeQ[{a,
 b, d}, x]
```

rule 6425
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp
[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x*Sqrt[
-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x] + Simp[1/(4*b^2*(n + 1)*(n + 2))
  Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]) /; FreeQ[{a, b, c, d}, x]
 && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arccosh}\,(d\,x^2+1))^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arccosh}\,(1+dx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*acosh(d*x**2+1))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\,\mathrm{arcosh}\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(1+dx^2\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\,\mathrm{arcosh}\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 + 1) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arccosh}\,(1 + dx^2))^{7/2}}\,dx = \int \frac{1}{(a + b\,\text{acosh}\,(d\,x^2 + 1))^{7/2}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 + 1))^(7/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 + 1))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + b\text{arccosh}\,(1 + dx^2))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(1/(a+b*acosh(d*x^2+1))^(7/2),x)
```

output
```
( - sqrt(d)*acosh(d*x**2 + 1)**3*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 +
 1)*b + a)*acosh(d*x**2 + 1)*x)/(acosh(d*x**2 + 1)**4*b**4*d*x**2 + 2*acos
h(d*x**2 + 1)**4*b**4 + 4*acosh(d*x**2 + 1)**3*a*b**3*d*x**2 + 8*acosh(d*x
**2 + 1)**3*a*b**3 + 6*acosh(d*x**2 + 1)**2*a**2*b**2*d*x**2 + 12*acosh(d*
x**2 + 1)**2*a**2*b**2 + 4*acosh(d*x**2 + 1)*a**3*b*d*x**2 + 8*acosh(d*x**
2 + 1)*a**3*b + a**4*d*x**2 + 2*a**4),x)*a*b**3*d - sqrt(d)*acosh(d*x**2 +
 1)**3*int((sqrt(d*x**2 + 2)*sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 +
1)**2*x)/(acosh(d*x**2 + 1)**4*b**4*d*x**2 + 2*acosh(d*x**2 + 1)**4*b**4 +
 4*acosh(d*x**2 + 1)**3*a*b**3*d*x**2 + 8*acosh(d*x**2 + 1)**3*a*b**3 + 6*
acosh(d*x**2 + 1)**2*a**2*b**2*d*x**2 + 12*acosh(d*x**2 + 1)**2*a**2*b**2
+ 4*acosh(d*x**2 + 1)*a**3*b*d*x**2 + 8*acosh(d*x**2 + 1)*a**3*b + a**4*d*
x**2 + 2*a**4),x)*b**4*d + 3*acosh(d*x**2 + 1)**3*int((sqrt(acosh(d*x**2 +
 1)*b + a)*acosh(d*x**2 + 1)*x**2)/(acosh(d*x**2 + 1)**4*b**4*d*x**2 + 2*a
cosh(d*x**2 + 1)**4*b**4 + 4*acosh(d*x**2 + 1)**3*a*b**3*d*x**2 + 8*acosh(
d*x**2 + 1)**3*a*b**3 + 6*acosh(d*x**2 + 1)**2*a**2*b**2*d*x**2 + 12*acosh
(d*x**2 + 1)**2*a**2*b**2 + 4*acosh(d*x**2 + 1)*a**3*b*d*x**2 + 8*acosh(d*
x**2 + 1)*a**3*b + a**4*d*x**2 + 2*a**4),x)*b**4*d**2 + 6*acosh(d*x**2 + 1
)**3*int((sqrt(acosh(d*x**2 + 1)*b + a)*acosh(d*x**2 + 1))/(acosh(d*x**2 +
 1)**4*b**4*d*x**2 + 2*acosh(d*x**2 + 1)**4*b**4 + 4*acosh(d*x**2 + 1)**3*
a*b**3*d*x**2 + 8*acosh(d*x**2 + 1)**3*a*b**3 + 6*acosh(d*x**2 + 1)**2*...
```

3.170 $\int (a+b\text{arccosh}(-1+dx^2))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 281

$$\int (a+b\text{arccosh}(-1+dx^2))^{5/2}\,dx = \frac{5b(2x^2-dx^4)(a+b\text{arccosh}(-1+dx^2))^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x(a+b\text{arccosh}(-1+dx^2))^{5/2} + \frac{30b^2\sqrt{a+b\text{arccosh}(-1+dx^2)}\cosh^2\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)}{dx} - \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{dx} - \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{dx}$$

output

```
5*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^(3/2)/x/(d*x^2)^(1/2)/(d*x^2-2)^
(1/2)+x*(a+b*arccosh(d*x^2-1))^(5/2)+30*b^2*(a+b*arccosh(d*x^2-1))^(1/2)*c
osh(1/2*arccosh(d*x^2-1))^2/d/x-15/2*b^(5/2)*2^(1/2)*Pi^(1/2)*cosh(1/2*arc
cosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cos
h(1/2*a/b)-sinh(1/2*a/b))/d/x-15/2*b^(5/2)*2^(1/2)*Pi^(1/2)*cosh(1/2*arcco
sh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1
/2*a/b)+sinh(1/2*a/b))/d/x
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.99

$$\int (a+b\text{arccosh}(-1+dx^2))^{5/2}\,dx=\frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(-15b^{5/2}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

output
```
(Cosh[ArcCosh[-1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCo
sh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 15*b^
(5/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(C
osh[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*((a^2 +
15*b^2)*Cosh[ArcCosh[-1 + d*x^2]/2] + b^2*ArcCosh[-1 + d*x^2]^2*Cosh[ArcCo
sh[-1 + d*x^2]/2] - 5*a*b*Sinh[ArcCosh[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x
^2]*(2*a*Cosh[ArcCosh[-1 + d*x^2]/2] - 5*b*Sinh[ArcCosh[-1 + d*x^2]/2]))))
/(2*d*x)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6416, 6415}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(dx^2-1))^{5/2}\,dx$$

$$\Big\downarrow \text{6416}$$

$$15b^2\int \sqrt{a+b\text{arccosh}(dx^2-1)}dx+x(a+b\text{arccosh}(dx^2-1))^{5/2}+\frac{5b(2x^2-dx^4)(a+b\text{arccosh}(dx^2-1))^{3/2}}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

$$\Big\downarrow \text{6415}$$

$$15b^2\left(-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}(dx^2-1)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)\right.}{}\right.$$

$$x\left(a+b\text{arccosh}(dx^2-1)\right)^{5/2}+\frac{5b(2x^2-dx^4)\left(a+b\text{arccosh}(dx^2-1)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

output
```
(5*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqr
t[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(5/2) + 15*b^2*((2*Sqrt[a +
 b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (Sqrt[b]*Sq
rt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/
(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (Sqrt[b]*Sqrt[
Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x))
```

Defintions of rubi rules used

rule 6415
```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqr
t[a + b*ArcCosh[-1 + d*x^2]]*(Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2/(d*x)), x]
+ (-Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*ArcC
osh[-1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d*x))
, x] - Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*A
rcCosh[-1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d
*x)), x]) /; FreeQ[{a, b, d}, x]
```

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Maple [F]

$$\int \left(a + b \operatorname{arccosh}\left(d\,x^{2}-1\right)\right)^{\frac{5}{2}}dx$$

input
```
int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

output
```
int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acosh(d*x**2-1))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{5/2}\,dx=\int \left(b\,\text{arcosh}\left(dx^2-1\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{5/2}\,dx=\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)}^{5/2}\,dx$$

input
```
int((a + b*acosh(d*x^2 - 1))^(5/2),x)
```

output
```
int((a + b*acosh(d*x^2 - 1))^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{5/2}\,dx = \left(\int \sqrt{acosh\left(d\,x^2-1\right)b+a}dx\right)a^2 + 2\left(\int \sqrt{acosh\left(d\,x^2-1\right)b+a}\,acosh\left(d\,x^2-1\right)dx\right)ab + \left(\int \sqrt{acosh\left(d\,x^2-1\right)b+a}\,acosh\left(d\,x^2-1\right)^2 dx\right)b^2$$

input
```
int((a+b*acosh(d*x^2-1))^(5/2),x)
```

output
```
int(sqrt(acosh(d*x**2 - 1)*b + a),x)*a**2 + 2*int(sqrt(acosh(d*x**2 - 1)*b
 + a)*acosh(d*x**2 - 1),x)*a*b + int(sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d
*x**2 - 1)**2,x)*b**2
```

3.171 $\int (a+b\text{arccosh}(-1+dx^2))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 239

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2}\,dx=\frac{3b(2x^2-dx^4)\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2}+\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{dx}-\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{dx}$$

output
```
3*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^(1/2)/x/(d*x^2)^(1/2)/(d*x^2-2)^
(1/2)+x*(a+b*arccosh(d*x^2-1))^(3/2)+3/2*b^(3/2)*2^(1/2)*Pi^(1/2)*cosh(1/2
*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*
(cosh(1/2*a/b)-sinh(1/2*a/b))/d/x-3/2*b^(3/2)*2^(1/2)*Pi^(1/2)*cosh(1/2*ar
ccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cos
h(1/2*a/b)+sinh(1/2*a/b))/d/x
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int (a+b\text{arccosh}(-1+dx^2))^{3/2}\,dx = \frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(3b^{3/2}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

output
```
(Cosh[ArcCosh[-1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh
[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 3*b^(3/
2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*(a*Cosh[Arc
Cosh[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*Cosh[ArcCosh[-1 + d*x^2]/2] -
3*b*Sinh[ArcCosh[-1 + d*x^2]/2])))/(2*d*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6416, 6420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arccosh}(dx^2-1))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6416}$$

$$3b^2\int \frac{1}{\sqrt{a+b\text{arccosh}(dx^2-1)}}dx + x(a+b\text{arccosh}(dx^2-1))^{3/2} + \frac{3b(2x^2-dx^4)\sqrt{a+b\text{arccosh}(dx^2-1)}}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

$$\Big\downarrow \text{ 6420}$$

$$3b^2\left(\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}(dx^2-1)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\right.\right.}{}\right.$$

$$x\left(a+b\text{arccosh}(dx^2-1)\right)^{3/2}+\frac{3b(2x^2-dx^4)\sqrt{a+b\text{arccosh}(dx^2-1)}}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

output
```
(3*b*(2*x^2 - d*x^4)*Sqrt[a + b*ArcCosh[-1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[
-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(3/2) + 3*b^2*((Sqrt[Pi/2]*Co
sh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sqrt[Pi/2]*Cosh[
ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x))
```

Defintions of rubi rules used

rule 6416
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (-Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a +
 b*ArcCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2
])), x] + Simp[4*b^2*n*(n - 1)   Int[(a + b*ArcCosh[c + d*x^2])^(n - 2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

rule 6420
```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] - Simp[Sqrt[Pi
/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d},
 x]
```

Maple [F]

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

output
```
int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2}\,dx=\int \left(a+b\,\mathrm{acosh}\left(dx^{2}-1\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*acosh(d*x**2-1))**(3/2),x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2} dx = \int \left(b\,\text{arcosh}\left(dx^2-1\right)+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2} dx = \int \left(a+b\,\text{acosh}\left(d\,x^2-1\right)\right)^{3/2} dx$$

input
```
int((a + b*acosh(d*x^2 - 1))^(3/2),x)
```

output
```
int((a + b*acosh(d*x^2 - 1))^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2} dx = \left(\int \sqrt{acosh\left(d\,x^2-1\right)b+a}dx\right)a + \left(\int \sqrt{acosh\left(d\,x^2-1\right)b+a}\,acosh\left(d\,x^2-1\right)dx\right)b$$

input
```
int((a+b*acosh(d*x^2-1))^(3/2),x)
```

output
```
int(sqrt(acosh(d*x**2 - 1)*b + a),x)*a + int(sqrt(acosh(d*x**2 - 1)*b + a)
*acosh(d*x**2 - 1),x)*b
```

3.172 $\int \sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 206

$$\int \sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\,dx = \frac{2\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\cosh^2\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)}{dx} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{dx} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{dx}$$

output
```
2*(a+b*arccosh(d*x^2-1))^(1/2)*cosh(1/2*arccosh(d*x^2-1))^2/d/x-1/2*b^(1/2
)*2^(1/2)*Pi^(1/2)*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-
1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))/d/x-1/2*b^(1/2)*2
^(1/2)*Pi^(1/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^
(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))/d/x
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.86

$$\int \sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\,dx$$

$$= \frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(4\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)+\sqrt{b}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{a}}{}\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

output
```
(Cosh[ArcCosh[-1 + d*x^2]/2]*(4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCo
sh[-1 + d*x^2]/2] + Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]
]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) - Sqrt[b]*Sqrt[2*Pi]
*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + S
inh[a/(2*b)])))/(2*d*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{6415\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arccosh}\left(dx^2-1\right)}\,dx$$

$$\Big\downarrow \text{6415}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\mathrm{arccosh}\left(dx^2-1\right)\right)\mathrm{erf}\left(\frac{\sqrt{a+b\mathrm{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\mathrm{arccosh}\left(dx^2-1\right)\right)\mathrm{erfi}\left(\frac{\sqrt{a+b\mathrm{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{2\cosh^2\left(\frac{1}{2}\mathrm{arccosh}\left(dx^2-1\right)\right)\sqrt{a+b\mathrm{arccosh}\left(dx^2-1\right)}}{dx}$$

input
```
Int[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

output
```
(2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) -
(Sqrt[b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1
 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (Sq
rt[b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d
*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Defintions of rubi rules used

rule 6415
```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqr
t[a + b*ArcCosh[-1 + d*x^2]]*(Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2/(d*x)), x]
+ (-Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*ArcC
osh[-1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d*x))
, x] - Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*A
rcCosh[-1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d
*x)), x]) /; FreeQ[{a, b, d}, x]
```

Maple [F]

$$\int \sqrt{a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)}dx$$

input
```
int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

output
```
int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\operatorname{arccosh}\left(-1 + dx^2\right)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a + b\operatorname{arccosh}\left(-1 + dx^2\right)}\, dx = \int \sqrt{a + b\operatorname{acosh}\left(dx^2 - 1\right)}\, dx$$

input
```
integrate((a+b*acosh(d*x**2-1))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acosh(d*x**2 - 1)), x)
```

Maxima [F]

$$\int \sqrt{a + b\operatorname{arccosh}\left(-1 + dx^2\right)}\, dx = \int \sqrt{b\operatorname{arcosh}\left(dx^2 - 1\right) + a}\, dx$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\,dx = \int \sqrt{a+b\,\text{acosh}\left(d\,x^2-1\right)}\,dx$$

input
```
int((a + b*acosh(d*x^2 - 1))^(1/2),x)
```

output
```
int((a + b*acosh(d*x^2 - 1))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\,dx = \int \sqrt{acosh\left(d\,x^2-1\right)b+a}dx$$

input
```
int((a+b*acosh(d*x^2-1))^(1/2),x)
```

output
```
int(sqrt(acosh(d*x**2 - 1)*b + a),x)
```

3.173 $\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}(-1+dx^2)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 166

$$\int \frac{1}{\sqrt{a+b\operatorname{arccosh}(-1+dx^2)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\operatorname{arccosh}(-1+dx^2)\right)\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{b}dx} - \frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\operatorname{arccosh}(-1+dx^2)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{b}dx}$$

output
```
1/2*2^(1/2)*Pi^(1/2)*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^
2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))/b^(1/2)/d/x-1/2
*2^(1/2)*Pi^(1/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1)
)^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))/b^(1/2)/d/x
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx =$$
$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\text{erf}\left(\frac{\sqrt{a+b\text{ar}}}{}\right.\right.}{\sqrt{b}dx}$$

input
```
Integrate[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

output
```
-((Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[Sqrt[a + b*ArcCosh[-1 + d*
x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Erf[Sqrt[a + b
*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])))
/(Sqrt[b]*d*x))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(dx^2-1\right)}}dx$$
$$\Big\downarrow \text{ 6420}$$
$$\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}$$

input
```
Int[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

output
```
(Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sq
rt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(
Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x)
```

Defintions of rubi rules used

rule 6420
```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] - Simp[Sqrt[Pi
/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d},
 x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{a+b\operatorname{acosh}\left(dx^2-1\right)}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2-1))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acosh(d*x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{b\,\text{arcosh}\left(dx^2-1\right)+a}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(d\,x^2-1\right)}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1))^(1/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}}\,dx=\int \frac{\sqrt{acosh\left(d\,x^2-1\right)b+a}}{acosh\left(d\,x^2-1\right)b+a}dx$$

input
```
int(1/(a+b*acosh(d*x^2-1))^(1/2),x)
```

output
```
int(sqrt(acosh(d*x**2 - 1)*b + a)/(acosh(d*x**2 - 1)*b + a),x)
```

3.174 $\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(-1+dx^2\right)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 212

$$\int \frac{1}{(a+b\,\mathrm{arccosh}\,(-1+dx^2))^{3/2}}\,dx=-\frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{bdx\sqrt{a+b\,\mathrm{arccosh}\,(-1+dx^2)}}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\mathrm{arccosh}(-1+dx^2)\right)\mathrm{erfi}\left(\frac{\sqrt{a+b\mathrm{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\mathrm{arccosh}(-1+dx^2)\right)\mathrm{erf}\left(\frac{\sqrt{a+b\mathrm{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{b^{3/2}dx}$$

output
```
-(d*x^2)^(1/2)*(d*x^2-2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2-1))^(1/2)+1/2*2^(1
/2)*Pi^(1/2)*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1
/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))/b^(3/2)/d/x+1/2*2^(1/2)
*Pi^(1/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*
2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))/b^(3/2)/d/x
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{3/2}}\,dx = \frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\sqrt{2\pi}\sqrt{a+b\text{arccosh}\left(-1+dx^2\right)}\text{erfi}\left(\sqrt{\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

output
```
(Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*E
rfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Erf[Sqrt[a + b*A
rcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) - 4
*Sqrt[b]*Sinh[ArcCosh[-1 + d*x^2]/2]))/(2*b^(3/2)*d*x*Sqrt[a + b*ArcCosh[-
1 + d*x^2]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6422}$$

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{bdx\sqrt{a+b\text{arccosh}\left(dx^2-1\right)}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

output
```
-((Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]))
+ (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x
^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(b^(3/2)*d*x) + (
Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]
/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b^(3/2)*d*x)
```

Defintions of rubi rules used

rule 6422
```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-
Sqrt[d*x^2])*(Sqrt[-2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])), x]
 + (Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2
]/2]*(Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + S
imp[Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*
(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; Fre
eQ[{a, b, d}, x]
```

Maple [F]

$$\int \frac{1}{\left(a+b\ \text{arccosh}\left(d\,x^{2}-1\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{acosh}(dx^2-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acosh(d*x**2-1))**(3/2),x)
```

output
```
Integral((a + b*acosh(d*x**2 - 1))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arcosh}(dx^2-1)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + b\text{arccosh}(-1 + dx^2))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx^2 - 1) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{arccosh}(-1 + dx^2))^{3/2}}\,dx = \int \frac{1}{(a + b\,\text{acosh}\,(d\,x^2 - 1))^{3/2}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1))^(3/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + b\text{arccosh}(-1 + dx^2))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*acosh(d*x^2-1))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1)
- sqrt(d)*acosh(d*x**2 - 1)*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b
 + a)*acosh(d*x**2 - 1)*x)/(acosh(d*x**2 - 1)**2*b**2*d*x**2 - 2*acosh(d*x
**2 - 1)**2*b**2 + 2*acosh(d*x**2 - 1)*a*b*d*x**2 - 4*acosh(d*x**2 - 1)*a*
b + a**2*d*x**2 - 2*a**2),x)*a*b*d - sqrt(d)*acosh(d*x**2 - 1)*int((sqrt(d
*x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1)**2*x)/(acosh(d*
x**2 - 1)**2*b**2*d*x**2 - 2*acosh(d*x**2 - 1)**2*b**2 + 2*acosh(d*x**2 -
1)*a*b*d*x**2 - 4*acosh(d*x**2 - 1)*a*b + a**2*d*x**2 - 2*a**2),x)*b**2*d
- acosh(d*x**2 - 1)*int((sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1)*x
**2)/(acosh(d*x**2 - 1)**2*b**2*d*x**2 - 2*acosh(d*x**2 - 1)**2*b**2 + 2*a
cosh(d*x**2 - 1)*a*b*d*x**2 - 4*acosh(d*x**2 - 1)*a*b + a**2*d*x**2 - 2*a*
*2),x)*b**2*d**2 + 2*acosh(d*x**2 - 1)*int((sqrt(acosh(d*x**2 - 1)*b + a)*
acosh(d*x**2 - 1))/(acosh(d*x**2 - 1)**2*b**2*d*x**2 - 2*acosh(d*x**2 - 1)
**2*b**2 + 2*acosh(d*x**2 - 1)*a*b*d*x**2 - 4*acosh(d*x**2 - 1)*a*b + a**2
*d*x**2 - 2*a**2),x)*b**2*d - sqrt(d)*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x
**2 - 1)*b + a)*acosh(d*x**2 - 1)*x)/(acosh(d*x**2 - 1)**2*b**2*d*x**2 - 2
*acosh(d*x**2 - 1)**2*b**2 + 2*acosh(d*x**2 - 1)*a*b*d*x**2 - 4*acosh(d*x*
*2 - 1)*a*b + a**2*d*x**2 - 2*a**2),x)*a**2*d - sqrt(d)*int((sqrt(d*x**2 -
 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1)**2*x)/(acosh(d*x**2 -
1)**2*b**2*d*x**2 - 2*acosh(d*x**2 - 1)**2*b**2 + 2*acosh(d*x**2 - 1)*a...
```

3.175 $\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(-1+dx^2\right)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 253

$$\int \frac{1}{\left(a+b\,\text{arccosh}\left(-1+dx^2\right)\right)^{5/2}}\,dx=\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\,\text{arccosh}\left(-1+dx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+b\,\text{arccosh}\left(-1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{3b^{5/2}dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{3b^{5/2}dx}$$

output
```
1/3*(-d*x^4+2*x^2)/b/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)/(a+b*arccosh(d*x^2-1)
)^(3/2)-1/3*x/b^2/(a+b*arccosh(d*x^2-1))^(1/2)+1/6*2^(1/2)*Pi^(1/2)*cosh(1
/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2)
)*(cosh(1/2*a/b)-sinh(1/2*a/b))/b^(5/2)/d/x-1/6*2^(1/2)*Pi^(1/2)*cosh(1/2*
arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(c
osh(1/2*a/b)+sinh(1/2*a/b))/b^(5/2)/d/x
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+b\text{arccosh}(-1+dx^2))^{5/2}}\,dx =$$

$$-\frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\sqrt{2\pi}(a+b\text{arccosh}(-1+dx^2))^{3/2}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

output
```
-1/6*(Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^
(3/2)*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*
b)] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(3/2)*Erf[Sq
rt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(
2*b)]) + 4*Sqrt[b]*((a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2
] + b*Sinh[ArcCosh[-1 + d*x^2]/2])))/(b^(5/2)*d*x*(a + b*ArcCosh[-1 + d*x^
2])^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6425, 6420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2-1))^{5/2}}dx$$

$$\Big\downarrow \text{6425}$$

$$\frac{\int \frac{1}{\sqrt{a+b\text{arccosh}(dx^2-1)}}dx}{3b^2}-\frac{x}{3b^2\sqrt{a+b\text{arccosh}(dx^2-1)}}+\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{dx^2-2}\,(a+b\text{arccosh}(dx^2-1))^{3/2}}$$

$$\downarrow \textbf{6420}$$

$$\frac{\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arccosh}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right.}{\sqrt{b}dx}}{3b^2}$$

$$\frac{x}{3b^2\sqrt{a+b\text{arccosh}\left(dx^2-1\right)}}+\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{dx^2-2}\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^{3/2}}$$

input

```
Int[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

output

```
(2*x^2 - d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*
x^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + ((Sqrt[Pi/2]*Co
sh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sqrt[Pi/2]*Cosh[
ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x))/(3*b^2)
```

Defintions of rubi rules used

rule 6420

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] - Simp[Sqrt[Pi
/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d},
 x]
```

rule 6425

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp
[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x*Sqrt[
-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x] + Simp[1/(4*b^2*(n + 1)*(n + 2))
  Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]) /; FreeQ[{a, b, c, d}, x]
 && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arccosh}\,(d\,x^2-1))^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arccosh}\,(-1+dx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\mathrm{arccosh}\,(-1+dx^2))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*acosh(d*x**2-1))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arccosh}\,(-1+dx^2))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx^2-1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arccosh}\,(-1+dx^2))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arcosh}\,(dx^2-1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arccosh}\,(-1+dx^2))^{5/2}}\,dx = \int \frac{1}{(a+b\,\text{acosh}\,(d\,x^2-1))^{5/2}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1))^(5/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\text{arccosh}(-1+dx^2))^{5/2}}\,dx = \text{too large to display}$$

input
```
int(1/(a+b*acosh(d*x^2-1))^(5/2),x)
```

output
```
( - sqrt(d)*acosh(d*x**2 - 1)**2*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 -
 1)*b + a)*acosh(d*x**2 - 1)*x)/(acosh(d*x**2 - 1)**3*b**3*d*x**2 - 2*acos
h(d*x**2 - 1)**3*b**3 + 3*acosh(d*x**2 - 1)**2*a*b**2*d*x**2 - 6*acosh(d*x
**2 - 1)**2*a*b**2 + 3*acosh(d*x**2 - 1)*a**2*b*d*x**2 - 6*acosh(d*x**2 -
1)*a**2*b + a**3*d*x**2 - 2*a**3),x)*a*b**2*d - sqrt(d)*acosh(d*x**2 - 1)*
*2*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1)**
2*x)/(acosh(d*x**2 - 1)**3*b**3*d*x**2 - 2*acosh(d*x**2 - 1)**3*b**3 + 3*a
cosh(d*x**2 - 1)**2*a*b**2*d*x**2 - 6*acosh(d*x**2 - 1)**2*a*b**2 + 3*acos
h(d*x**2 - 1)*a**2*b*d*x**2 - 6*acosh(d*x**2 - 1)*a**2*b + a**3*d*x**2 - 2
*a**3),x)*b**3*d + acosh(d*x**2 - 1)**2*int((sqrt(acosh(d*x**2 - 1)*b + a)
*acosh(d*x**2 - 1)*x**2)/(acosh(d*x**2 - 1)**3*b**3*d*x**2 - 2*acosh(d*x**
2 - 1)**3*b**3 + 3*acosh(d*x**2 - 1)**2*a*b**2*d*x**2 - 6*acosh(d*x**2 - 1
)**2*a*b**2 + 3*acosh(d*x**2 - 1)*a**2*b*d*x**2 - 6*acosh(d*x**2 - 1)*a**2
*b + a**3*d*x**2 - 2*a**3),x)*b**3*d**2 - 2*acosh(d*x**2 - 1)**2*int((sqrt
(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1))/(acosh(d*x**2 - 1)**3*b**3*d*
x**2 - 2*acosh(d*x**2 - 1)**3*b**3 + 3*acosh(d*x**2 - 1)**2*a*b**2*d*x**2
- 6*acosh(d*x**2 - 1)**2*a*b**2 + 3*acosh(d*x**2 - 1)*a**2*b*d*x**2 - 6*ac
osh(d*x**2 - 1)*a**2*b + a**3*d*x**2 - 2*a**3),x)*b**3*d + sqrt(d)*sqrt(d*
x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1) - 2*sqrt(d)*acos
h(d*x**2 - 1)*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh...
```

3.176 $\int \frac{1}{\left(a+b\,\mathbf{arccosh}(-1+dx^2)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 302

$$\begin{aligned}\int \frac{1}{(a+b\operatorname{arccosh}(-1+dx^2))^{7/2}}\,dx &= \frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{-2+dx^2}\,(a+b\operatorname{arccosh}(-1+dx^2))^{5/2}} \\ &- \frac{x}{15b^2(a+b\operatorname{arccosh}(-1+dx^2))^{3/2}} - \frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{15b^3dx\sqrt{a+b\operatorname{arccosh}(-1+dx^2)}} \\ &+ \frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\operatorname{arccosh}(-1+dx^2)\right)\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{15b^{7/2}dx} \\ &+ \frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\operatorname{arccosh}(-1+dx^2)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{15b^{7/2}dx}\end{aligned}$$

output
```
1/5*(-d*x^4+2*x^2)/b/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)/(a+b*arccosh(d*x^2-1)
)^(5/2)-1/15*x/b^2/(a+b*arccosh(d*x^2-1))^(3/2)-1/15*(d*x^2)^(1/2)*(d*x^2-
2)^(1/2)/b^3/d/x/(a+b*arccosh(d*x^2-1))^(1/2)+1/30*2^(1/2)*Pi^(1/2)*cosh(1
/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2)
)*(cosh(1/2*a/b)-sinh(1/2*a/b))/b^(7/2)/d/x+1/30*2^(1/2)*Pi^(1/2)*cosh(1/2
*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(
cosh(1/2*a/b)+sinh(1/2*a/b))/b^(7/2)/d/x
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\text{arccosh}(-1+dx^2))^{7/2}}\,dx = \frac{\cosh\left(\frac{1}{2}\text{arccosh}(-1+dx^2)\right)\left(\sqrt{2\pi}(a+b\text{arccosh}(-1+dx^2))^{5/2}\,\text{erfi}\left(\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

output
```
(Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)
*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] -
Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*Erf[Sqrt[a +
 b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])
 + 4*Sqrt[b]*(-(b*(a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2])
 - (3*b^2 + (a + b*ArcCosh[-1 + d*x^2])^2)*Sinh[ArcCosh[-1 + d*x^2]/2])))/
(30*b^(7/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(5/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6425, 6422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arccosh}(dx^2-1))^{7/2}}dx$$

$$\Big\downarrow \text{6425}$$

$$\frac{\int \frac{1}{\left(a+b\text{arccosh}(dx^2-1)\right)^{3/2}}dx}{15b^2}-\frac{x}{15b^2(a+b\text{arccosh}(dx^2-1))^{3/2}}+\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{dx^2-2}\,(a+b\text{arccosh}(dx^2-1))^{5/2}}$$

$$\Big\downarrow \text{6422}$$

$$-\frac{x}{15b^2\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^{3/2}}+\frac{\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right)\text{erf}\left(\frac{\sqrt{a+b\text{arccosh}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\cosh\left(\frac{1}{2}\text{arccosh}\left(dx^2-1\right)\right.}{b^{3/2}dx}}{15b^2}$$

$$\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{dx^2-2}\left(a+b\text{arccosh}\left(dx^2-1\right)\right)^{5/2}}$$

input
```
Int[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

output
```
(2*x^2 - d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*
x^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[-1 + d*x^2])^(3/2)) + (-((Sqrt[d*x
^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])) + (Sqrt[Pi/
2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[
2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(b^(3/2)*d*x) + (Sqrt[Pi/2]*
Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*S
qrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b^(3/2)*d*x))/(15*b^2)
```

Defintions of rubi rules used

rule 6422
```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-
Sqrt[d*x^2])*(Sqrt[-2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])), x]
 + (Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2
]/2]*(Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + S
imp[Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*
(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; Fre
eQ[{a, b, d}, x]
```

rule 6425
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp
[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x*Sqrt[
-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x] + Simp[1/(4*b^2*(n + 1)*(n + 2))
  Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]) /; FreeQ[{a, b, c, d}, x]
 && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Maple [F]

$$\int \frac{1}{\left(a+b \operatorname{arccosh}\left(d\,x^{2}-1\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

output
```
int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\text{arccosh}\left(-1+dx^2\right)\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*acosh(d*x**2-1))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\,\text{arcosh}\left(dx^2 - 1\right) + a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\,\text{arcosh}\left(dx^2 - 1\right) + a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccosh(d*x^2 - 1) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a + b\text{arccosh}\left(-1 + dx^2\right)\right)^{7/2}}\,dx = \int \frac{1}{\left(a + b\,\text{acosh}\left(d\,x^2 - 1\right)\right)^{7/2}}\,dx$$

input
```
int(1/(a + b*acosh(d*x^2 - 1))^(7/2),x)
```

output
```
int(1/(a + b*acosh(d*x^2 - 1))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + b\text{arccosh}\,(-1 + dx^2))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(1/(a+b*acosh(d*x^2-1))^(7/2),x)
```

output
```
( - sqrt(d)*acosh(d*x**2 - 1)**3*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 -
 1)*b + a)*acosh(d*x**2 - 1)*x)/(acosh(d*x**2 - 1)**4*b**4*d*x**2 - 2*acos
h(d*x**2 - 1)**4*b**4 + 4*acosh(d*x**2 - 1)**3*a*b**3*d*x**2 - 8*acosh(d*x
**2 - 1)**3*a*b**3 + 6*acosh(d*x**2 - 1)**2*a**2*b**2*d*x**2 - 12*acosh(d*
x**2 - 1)**2*a**2*b**2 + 4*acosh(d*x**2 - 1)*a**3*b*d*x**2 - 8*acosh(d*x**
2 - 1)*a**3*b + a**4*d*x**2 - 2*a**4),x)*a*b**3*d - sqrt(d)*acosh(d*x**2 -
 1)**3*int((sqrt(d*x**2 - 2)*sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 -
1)**2*x)/(acosh(d*x**2 - 1)**4*b**4*d*x**2 - 2*acosh(d*x**2 - 1)**4*b**4 +
 4*acosh(d*x**2 - 1)**3*a*b**3*d*x**2 - 8*acosh(d*x**2 - 1)**3*a*b**3 + 6*
acosh(d*x**2 - 1)**2*a**2*b**2*d*x**2 - 12*acosh(d*x**2 - 1)**2*a**2*b**2
+ 4*acosh(d*x**2 - 1)*a**3*b*d*x**2 - 8*acosh(d*x**2 - 1)*a**3*b + a**4*d*
x**2 - 2*a**4),x)*b**4*d + 3*acosh(d*x**2 - 1)**3*int((sqrt(acosh(d*x**2 -
 1)*b + a)*acosh(d*x**2 - 1)*x**2)/(acosh(d*x**2 - 1)**4*b**4*d*x**2 - 2*a
cosh(d*x**2 - 1)**4*b**4 + 4*acosh(d*x**2 - 1)**3*a*b**3*d*x**2 - 8*acosh(
d*x**2 - 1)**3*a*b**3 + 6*acosh(d*x**2 - 1)**2*a**2*b**2*d*x**2 - 12*acosh
(d*x**2 - 1)**2*a**2*b**2 + 4*acosh(d*x**2 - 1)*a**3*b*d*x**2 - 8*acosh(d*
x**2 - 1)*a**3*b + a**4*d*x**2 - 2*a**4),x)*b**4*d**2 - 6*acosh(d*x**2 - 1
)**3*int((sqrt(acosh(d*x**2 - 1)*b + a)*acosh(d*x**2 - 1))/(acosh(d*x**2 -
 1)**4*b**4*d*x**2 - 2*acosh(d*x**2 - 1)**4*b**4 + 4*acosh(d*x**2 - 1)**3*
a*b**3*d*x**2 - 8*acosh(d*x**2 - 1)**3*a*b**3 + 6*acosh(d*x**2 - 1)**2*...
```

3.177 $\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Int}\left(\frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Defer(Int)((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

$$\Big\downarrow \text{ 7234}$$

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\text{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 23.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
-integrate((b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\left(\int \frac{\left(acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)b+a\right)^n}{c^2x^2-1}dx\right)$$

input
```
int((a+b*acosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
- int((acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))*b + a)**n/(c**2*x**2 - 1),x)
```

3.178 $\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 265

$$\int \frac{\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \frac{\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1+e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} + \frac{3b^2\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{PolyLog}\left(3,-e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} - \frac{3b^3 \text{PolyLog}\left(4,-e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}$$

output
```
1/4*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/
2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c-3/2
*b*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+1)^(1/2))^2)/c+3/2*b^2*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2
)))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)
-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c-3/4*b^3*polylog(4,-
((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c
```

Mathematica [F]

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {7232, 6297, 25, 3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}dx$$

$$\Big\downarrow \text{ 7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{ 6297}$$

$$-\frac{\int -\frac{(1-cx)^{3/2}\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(1-cx)^{3/2}\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\frac{i(1-cx)^{3/2}\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int \frac{(1-cx)^{3/2}\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{ 4201}$$

$$-\frac{i\left(2i\int \frac{e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}(1-cx)^{3/2}}{\left(1+e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)(cx+1)^{3/2}}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{i(1-cx)^2}{4(cx+1)^2}\right)}{bc}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{i\left(2i\left(\frac{3}{2}b\int \frac{(1-cx)\log\left(1+e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{cx+1}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{b(1-cx)^{3/2}\log\left(e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)}{2(cx+1)^{3/2}}\right)-\right.}{bc}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2(cx+1)}-b\int\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\operatorname{PolyLog}\left(2,-e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}\right)d(}{bc}$$

$\Big\downarrow$ 7163

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2(cx+1)}-b\left(\frac{1}{2}b\int\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)d\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2(cx+1)}-b\left(-\frac{1}{4}b^2\int\frac{\sqrt{cx+1}\operatorname{PolyLog}\left(3,-a-b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}de^{-2\operatorname{arccosh}\left(\frac{\sqrt{}}{\sqrt{}}\right.}\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2(cx+1)}-b\left(-\frac{1}{4}b^2\operatorname{PolyLog}\left(4,-a-b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}b\left(a+ba\right.\right.\right.\right.\right.}{bc}$$

input
```
Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
((-I)*(((-1/4*I)*(1 - c*x)^2)/(1 + c*x)^2 + (2*I)*(-1/2*(b*(1 - c*x)^(3/2)
*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(1 + c*x)^(3/2) + (
3*b*((b*(1 - c*x)*PolyLog[2, -E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])]
)/(2*(1 + c*x)) - b*(-1/2*(b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*
PolyLog[3, -E^((2*(a - Sqrt[1 - c*x]/Sqrt[1 + c*x]))/b)]) - (b^2*PolyLog[4
, -a - b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]])/4))/2))/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(381) = 762$.

Time = 0.42 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.18

method	result
default	$\frac{a^3\ln(cx+1)}{2c} - \frac{a^3\ln(cx-1)}{2c} - b^3\left(-\frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c} + \frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 \ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\,\sqrt{\frac{\sqrt{-cx+}}{\sqrt{cx+1}}}\right.\right.}{c}\right.$
parts	$\frac{a^3\ln(cx+1)}{2c} - \frac{a^3\ln(cx-1)}{2c} - b^3\left(-\frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c} + \frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 \ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\,\sqrt{\frac{\sqrt{-cx+}}{\sqrt{cx+1}}}\right.\right.}{c}\right.$

input
```
int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)-b^3*(-1/4/c*arccosh((-c*x+1)^(1/2)
/(c*x+1)^(1/2))^4+1/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-3/2/c*arccosh((-c*x+1)
^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(
1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/4
/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2))-3*a*b^2*(-1/3*arccosh
((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3/c+1/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2
))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(
1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+1/c*arccosh((-c*x+1)^(1/2)
/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(
c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-1/2/c*pol
ylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/
2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2))-3*a^2*b*(-1/2*arccosh((-c*x
+1)^(1/2)/(c*x+1)^(1/2))^2/c+1/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(
1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+1/2/c*polylog(2,-((-c*x+1)^(1/2...
```

Fricas [F]

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^3*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccosh(s
qrt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a + b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{acosh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{acosh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acosh(sqrt(-c*x + 1)/sq
rt(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acosh(sqrt(-c*x +
1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acosh(sqrt(-c
*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a + b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1)
+ sqrt(-c*x + 1)) + sqrt(-c*x + 1))^3/c + integrate(1/8*(((c*x + 1)*sqrt(-
c*x + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*log(c*x + 1)^3 - 6*((c*x + 1)*sqrt(-c
*x + 1)*a*b^2 - (-c*x + 1)^(3/2)*a*b^2)*log(c*x + 1)^2 - 6*((4*a*b^2 + (b^
3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x + 1))*(c*x + 1)*sqrt(
-c*x + 1) - (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(
-c*x + 1))*(-c*x + 1)^(3/2) + ((4*a*b^2 + (b^3*c*x - b^3)*log(c*x + 1) - (
b^3*c*x - b^3)*log(-c*x + 1))*(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c
*x + 1) - (b^3*c*x + b^3)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^3 + (c
*x - 1)*b^3)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt
(c*x + 1) + sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-c*x + 1)
^(3/2)*b^3)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-s
qrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2 + (((c*x + 1)*b^3 + (c*
x - 1)*b^3)*log(c*x + 1)^3 - 6*((c*x + 1)*a*b^2 + (c*x - 1)*a*b^2)*log(c*x
 + 1)^2 + 12*((c*x + 1)*a^2*b + (c*x - 1)*a^2*b)*log(c*x + 1))*sqrt(sqrt(c
*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + 12*((c*x
 + 1)*sqrt(-c*x + 1)*a^2*b - (-c*x + 1)^(3/2)*a^2*b)*log(c*x + 1) - 6*(4*(
c*x + 1)*sqrt(-c*x + 1)*a^2*b - 4*(-c*x + 1)^(3/2)*a^2*b + ((c*x + 1)*sqrt
(-c*x + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*log(c*x + 1)^2 + (4*(c*x + 1)*a^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\operatorname{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \frac{-6\left(\int \frac{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}dx\right)a^2bc-2\left(\int \frac{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{c^2x^2-1}dx\right)b^3c-6\left(\int \frac{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c^2x^2-1}dx\right)a\,b^2c-\log(c^2x-c}{2c}$$

input
```
int((a+b*acosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

output
```
( - 6*int(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1),x)*a**2*b*
c - 2*int(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))**3/(c**2*x**2 - 1),x)*b**3
*c - 6*int(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1),x)*a*b
**2*c - log(c**2*x - c)*a**3 + log(c**2*x + c)*a**3)/(2*c)
```

3.179 $\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 197

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b^2\,\text{PolyLog}\left(3,-e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
1/3*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/
2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c-b*(
a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(2,-((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)^(1/2))^2)/c+1/2*b^2*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*
x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^
2)/c
```

Mathematica [F]

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7232, 6297, 25, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}dx$$

$$\Big\downarrow \ 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \ 6297$$

$$-\frac{\int -\frac{(1-cx)\tanh\left(\frac{a}{b}-\frac{a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{cx+1}d\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(1-cx)\tanh\left(\frac{a}{b}-\frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{cx+1}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i(1-cx)\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}\right)}{cx+1}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{(1-cx)\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}\right)}{cx+1}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}(1-cx)}{\left(1+e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)(cx+1)}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{i(1-cx)^{3/2}}{3(cx+1)^{3/2}}\right)}{bc}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(b\int \left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(1+e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}\right)d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{b(1-cx)\log\left(e^{-2\text{arccosh}\left(\frac{\sqrt{}}{\sqrt{}}\right.}\right.}{2(cx+1)}\right.\right.}{bc}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\left(2i\left(b\left(\frac{1}{2}b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,-e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}\right)-\frac{1}{2}b\int \text{PolyLog}\left(2,-e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)d\left(\right.\right.\right.\right.}{bc}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int\frac{\sqrt{cx+1}\,\text{PolyLog}\left(2,-a-b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}de^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+\frac{1}{2}b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\right.\right.\right.}{bc}$$

$\big\downarrow$ 7143

$$-\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\,\text{PolyLog}\left(3,-a-b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+\frac{1}{2}b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,-e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}\right)\right.\right.\right.}{bc}$$

input
```
Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
((-I)*(((-1/3*I)*(1 - c*x)^(3/2))/(1 + c*x)^(3/2) + (2*I)*(-1/2*(b*(1 - c*
x)*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(1 + c*x) + b*((b
*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -E^((2*(a - Sqrt[
1 - c*x]/Sqrt[1 + c*x]))/b)])/2 + (b^2*PolyLog[3, -a - b*ArcCosh[Sqrt[1 -
c*x]/Sqrt[1 + c*x]]])/4))))/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.45

method	result
default	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - b^2 \left(-\frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c} + \frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\,\sqrt{\frac{\sqrt{-cx+}}{\sqrt{cx+1}}}\right.\right.}{c}\right.$
parts	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - b^2 \left(-\frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c} + \frac{\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\,\sqrt{\frac{\sqrt{-cx+}}{\sqrt{cx+1}}}\right.\right.}{c}\right.$

input
```
int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2*(-1/3*arccosh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3/c+1/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+1/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*p
olylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(
1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-1/2/c*polylog(3,-((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)^(1/2))^2))+a*b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/
c-2*a*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+1)^(1/2))^2)-a*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^
(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^2*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccosh(sqr
t(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{acosh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acosh(sqrt(-c*x + 1)/sq
rt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acosh(sqrt(-c*x + 1)/
sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*lo
g(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1)
+ sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2/c + integrate(-1/4*(((c*x + 1)*sqrt(
-c*x + 1)*b^2 - (-c*x + 1)^(3/2)*b^2)*log(c*x + 1)^2 + (((c*x + 1)*b^2 + (
c*x - 1)*b^2)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b + (c*x - 1)*a*b)*log(c*x +
 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x
+ 1)) - 4*((c*x + 1)*sqrt(-c*x + 1)*a*b - (-c*x + 1)^(3/2)*a*b)*log(c*x +
1) + 2*((4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-c*x +
 1))*(c*x + 1)*sqrt(-c*x + 1) - (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b
^2*c*x + b^2)*log(-c*x + 1))*(-c*x + 1)^(3/2) + ((4*a*b + (b^2*c*x - b^2)*
log(c*x + 1) - (b^2*c*x - b^2)*log(-c*x + 1))*(c*x + 1) + (4*a*b + (b^2*c*
x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*x - 1) - 2*((c*x
 + 1)*b^2 + (c*x - 1)*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x +
1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^
2 - (-c*x + 1)^(3/2)*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x
 + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2
- 1)*(c*x + 1)*sqrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2
 - 1)*(c*x + 1) + (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x
+ 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \frac{-4\left(\int \frac{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}dx\right)abc-2\left(\int \frac{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c^2x^2-1}dx\right)b^2c-\log(c^2x-c)\,a^2+\log(c^2x+c)\,a^2}{2c}$$

input
```
int((a+b*acosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

output
```
( - 4*int(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1),x)*a*b*c -
 2*int(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1),x)*b**2*c
- log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2)/(2*c)
```

3.180 $\int \frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\,\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
1/2*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)
/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c-1/2*b
*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)
^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c
```

Mathematica [F]

$$\int \frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int \frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {7232, 6297, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}\,dx$$

$$\Big\downarrow \; 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \; 6297$$

$$-\frac{\int -\left(\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tanh\left(\frac{a}{b}-\frac{\sqrt{1-cx}}{b\sqrt{cx+1}}\right)\right)d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tanh\left(\frac{a}{b}-\frac{\sqrt{1-cx}}{b\sqrt{cx+1}}\right)d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -i\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tan\left(\frac{ia}{b}-\frac{i\sqrt{1-cx}}{b\sqrt{cx+1}}\right)d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\int \left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tan\left(\frac{ia}{b}-\frac{i\sqrt{1-cx}}{b\sqrt{cx+1}}\right)d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \; 4201$$

$$-\frac{i\left(2i\int \frac{e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{1+e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}}d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{i(1-cx)}{2(cx+1)}\right)}{bc}$$

$$\Big\downarrow \; 2620$$

$$-\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)d\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(e^{\frac{2\left(a-\frac{\sqrt{1-}}{\sqrt{cx}}\right)}{b}}\right.\right.\right.}{bc}$$

$$\Big\downarrow \; 2715$$

$$-\frac{i\left(2i\left(-\frac{1}{4}b^2\int \frac{\sqrt{cx+1}\log\left(1+e^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{\sqrt{1-cx}}de^{-2\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}-\frac{1}{2}b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}\right.\right.\right.}{bc}$$

$$\Big\downarrow \; 2838$$

$$-\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}\left(2,-a-b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}b\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(e^{\frac{2\left(a-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}}+1\right)\right)-\frac{i(1}{2(c}\right.}{bc}$$

input
```
Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
((-I)*(((-1/2*I)*(1 - c*x))/(1 + c*x) + (2*I)*(-1/2*(b*(a + b*ArcCosh[Sqrt
[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^((2*(a - Sqrt[1 - c*x]/Sqrt[1 + c*x]))
/b)]) + (b^2*PolyLog[2, -a - b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]])/4)))
/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.56

method	result
default	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}+\frac{b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}}{2c}-\frac{b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\,\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}\right)^{2}\right)}{c}$
parts	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}+\frac{b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}}{2c}-\frac{b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\,\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}\right)^{2}\right)}{c}$

input
```
int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

output
```
1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)+1/2*b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(
1/2))^2/c-b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(
c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1)^(1/2))^2)-1/2*b*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x
+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2
)/c
```

Fricas [F]

$$\int \frac{a + b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="fricas")
```

output
```
integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = -\int \frac{a}{c^2x^2 - 1}\,dx - \int \frac{b\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*acosh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{a + b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="maxima")
```

output
```
-1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 +
2*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*
x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqr
t(-c*x + 1)) + sqrt(-c*x + 1)))/c + 8*integrate(1/2*(c*x + 1)*sqrt(-c*x +
1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1)*sqrt(-c*x + 1)
- (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) + (c^2*x^2 - 1
)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sq
rt(-c*x + 1))), x) + 8*integrate(-1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c
*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
- 8*integrate(1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 -
 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x +
1)/c - log(c*x - 1)/c)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\, dx = \text{Timed out}$$

input
```
integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\, dx = \int -\frac{a + b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2 - 1}\, dx$$

input
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$
$$=\frac{-2\left(\int \frac{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}dx\right)bc-\log(c^2x-c)\,a+\log(c^2x+c)\,a}{2c}$$

input
```
int((a+b*acosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

output
```
( - 2*int(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1),x)*b*c - l
og(c**2*x - c)*a + log(c**2*x + c)*a)/(2*c)
```

3.181 $\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x
]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$
$$\Big\downarrow \text{7234}$$
$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - a), x)
```

Sympy [N/A]

Not integrable

Time = 73.55 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acosh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - b*acosh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\text{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\left(\int \frac{1}{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)b\,c^2x^2 - acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)b + a\,c^2x^2 - a}dx\right)$$

input
```
int(1/(-c^2*x^2+1)/(a+b*acosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
 - int(1/(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))*b*c**2*x**2 - acosh(sqrt(
- c*x + 1)/sqrt(c*x + 1))*b + a*c**2*x**2 - a),x)
```

3.182 $\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathbf{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
,x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \ 7234$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\,\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\operatorname{arccosh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}d x$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(sqrt(-c*x + 1)/sqrt
(c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,
x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 1177, normalized size of antiderivative = 29.42

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="maxima")
```

output
```
2*(2*c*x*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-
c*x + 1)) + (c*x + 1)*sqrt(-c*x + 1) - (-c*x + 1)^(3/2))/(2*(c*x + 1)*sqrt
(-c*x + 1)*a*b*c - 2*(-c*x + 1)^(3/2)*a*b*c - ((c*x - 1)*b^2*c*log(c*x + 1
) - 2*(c*x - 1)*a*b*c)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x
 + 1) + sqrt(-c*x + 1)) - ((c*x + 1)*sqrt(-c*x + 1)*b^2*c - (-c*x + 1)^(3/
2)*b^2*c)*log(c*x + 1) + 2*((c*x - 1)*b^2*c*sqrt(sqrt(c*x + 1) + sqrt(-c*x
 + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + (c*x + 1)*sqrt(-c*x + 1)*b^
2*c - (-c*x + 1)^(3/2)*b^2*c)*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqr
t(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))) - integrate(-2*(2*(c
*x + 1)*sqrt(-c*x + 1)*(sqrt(c*x + 1) + sqrt(-c*x + 1))*(sqrt(c*x + 1) - s
qrt(-c*x + 1)) + ((c*x + 1)^2 + 2*(c*x + 1)*(c*x - 1))*sqrt(sqrt(c*x + 1)
+ sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/(2*(a*b*c^2*x^2 -
 a*b)*(c*x + 1)^2*sqrt(-c*x + 1) - 4*(a*b*c^2*x^2 - a*b)*(c*x + 1)*(-c*x +
 1)^(3/2) + 2*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(5/2) + ((b^2*c^2*x^2 - b^2)*
(-c*x + 1)^(3/2)*log(c*x + 1) - 2*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(3/2))*(s
qrt(c*x + 1) + sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) + 2*(2*(a*
b*c^2*x^2 - a*b)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^2*x^2 - a*b)*(c*x - 1)^2 -
 ((b^2*c^2*x^2 - b^2)*(c*x + 1)*(c*x - 1) + (b^2*c^2*x^2 - b^2)*(c*x - 1)^
2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1)
+ sqrt(-c*x + 1)) - ((b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(-c*x + 1) - 2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\text{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x
)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.08

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arccosh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx =$$

$$-\left(\int \frac{1}{acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 b^2c^2x^2 - acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 b^2 + 2acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) ab\,c^2x^2 - 2acosh\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) ab$$

input
```
int(1/(-c^2*x^2+1)/(a+b*acosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
 - int(1/(acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))**2*b**2*c**2*x**2 - acosh(
sqrt( - c*x + 1)/sqrt(c*x + 1))**2*b**2 + 2*acosh(sqrt( - c*x + 1)/sqrt(c*
x + 1))*a*b*c**2*x**2 - 2*acosh(sqrt( - c*x + 1)/sqrt(c*x + 1))*a*b + a**2
*c**2*x**2 - a**2),x)
```

3.183 $\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx = -\frac{\operatorname{arccosh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arccosh}\left(ce^{a+bx}\right)\log\left(1+e^{2\operatorname{arccosh}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}\left(ce^{a+bx}\right)}\right)}{2b}$$

output
```
-1/2*arccosh(c*exp(b*x+a))^2/b+arccosh(c*exp(b*x+a))*ln(1+(c*exp(1)^(b*x+a
)+(c*exp(1)^(b*x+a)-1)^(1/2)*(c*exp(1)^(b*x+a)+1)^(1/2))^2)/b+1/2*polylog(
2,-(c*exp(1)^(b*x+a)+(c*exp(1)^(b*x+a)-1)^(1/2)*(c*exp(1)^(b*x+a)+1)^(1/2)
)^2)/b
```

Mathematica [F]

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx = \int \operatorname{arccosh}\left(ce^{a+bx}\right) dx$$

input
```
Integrate[ArcCosh[c*E^(a + b*x)],x]
```

output
```
Integrate[ArcCosh[c*E^(a + b*x)], x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2720, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\operatorname{arccosh}\left(ce^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{6297}$$

$$\frac{\int \frac{e^{-a-bx}\sqrt{\frac{ce^{a+bx}-1}{e^{a+bx}c+1}}\left(e^{a+bx}c+1\right)\operatorname{arccosh}\left(ce^{a+bx}\right)}{c}d\operatorname{arccosh}\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\operatorname{arccosh}\left(ce^{a+bx}\right)\tan\left(i\operatorname{arccosh}\left(ce^{a+bx}\right)\right)d\operatorname{arccosh}\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \operatorname{arccosh}\left(ce^{a+bx}\right)\tan\left(i\operatorname{arccosh}\left(ce^{a+bx}\right)\right)d\operatorname{arccosh}\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{a+bx+2\operatorname{arccosh}\left(ce^{a+bx}\right)}}{1+e^{2\operatorname{arccosh}\left(ce^{a+bx}\right)}}d\operatorname{arccosh}\left(ce^{a+bx}\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(ce^{a+bx}\right)\log\left(e^{2\text{arccosh}\left(ce^{a+bx}\right)}+1\right)-\frac{1}{2}\int\log\left(1+e^{2\text{arccosh}\left(ce^{a+bx}\right)}\right)d\text{arccosh}\left(ce^{a+bx}\right)\right)-\frac{1}{2}ie^{2a+\cdots}\right.}{b}$$

$$\Big\downarrow \textbf{2715}$$

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(ce^{a+bx}\right)\log\left(e^{2\text{arccosh}\left(ce^{a+bx}\right)}+1\right)-\frac{1}{4}\int e^{-a-bx}\log\left(1+e^{2\text{arccosh}\left(ce^{a+bx}\right)}\right)de^{2\text{arccosh}\left(ce^{a+bx}\right)}\right)-\cdots\right.}{b}$$

$$\Big\downarrow \textbf{2838}$$

$$-\frac{i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(ce^{a+bx}\right)}\right)+\frac{1}{2}\text{arccosh}\left(ce^{a+bx}\right)\log\left(e^{2\text{arccosh}\left(ce^{a+bx}\right)}+1\right)\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

input
```
Int[ArcCosh[c*E^(a + b*x)],x]
```

output
```
((-I)*((-1/2*I)*E^(2*a + 2*b*x) + (2*I)*((ArcCosh[c*E^(a + b*x)]*Log[1 + E
^(2*ArcCosh[c*E^(a + b*x)])])/2 + PolyLog[2, -E^(2*ArcCosh[c*E^(a + b*x)])
]/4)))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccosh}\left(c\,e^{bx+a}\right)^2}{2}+\operatorname{arccosh}\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+a}-1}\,\sqrt{c\,e^{bx+a}+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+}\right.\right.}{2}}{b}$
default	$\frac{-\frac{\operatorname{arccosh}\left(c\,e^{bx+a}\right)^2}{2}+\operatorname{arccosh}\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+a}-1}\,\sqrt{c\,e^{bx+a}+1}\right)^2\right)+\frac{\operatorname{polylog}\left(2,-\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+}\right.\right.}{2}}{b}$

input
```
int(arccosh(c*exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*arccosh(c*exp(b*x+a))^2+arccosh(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)
+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)+1/2*polylog(2,-(c*exp(b
*x+a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2))
```

Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccosh(c*exp(b*x+a)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{acosh}\left(ce^{a+bx}\right)dx$$

input
```
integrate(acosh(c*exp(b*x+a)),x)
```

output
```
Integral(acosh(c*exp(a + b*x)), x)
```

Maxima [F]

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx = \int \operatorname{arcosh}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arccosh(c*exp(b*x+a)),x, algorithm="maxima")
```

output
```
b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) - c*e^(b*x + a) + (c^2*e^
(2*b*x + 2*a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(c*e^(b*x + a) -
 1))), x) + x*log(c*e^(b*x + a) + sqrt(c*e^(b*x + a) + 1)*sqrt(c*e^(b*x +
a) - 1)) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a)))/b - 1/
2*(b*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b
```

Giac [F]

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx = \int \operatorname{arcosh}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arccosh(c*exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccosh(c*e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx = \int \operatorname{acosh}\left(c\,e^{a+b\,x}\right) dx$$

input
```
int(acosh(c*exp(a + b*x)),x)
```

output
```
int(acosh(c*exp(a + b*x)), x)
```

Reduce [F]

$$\int \operatorname{arccosh}\left(ce^{a+bx}\right) dx = \int acosh\left(e^{bx+a}c\right) dx$$

input
```
int(acosh(c*exp(b*x+a)),x)
```

output
```
int(acosh(e**(a + b*x)*c),x)
```

3.184 $\int \frac{\operatorname{arccosh}(a+bx)}{\frac{ad}{b}+dx} dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\operatorname{arccosh}(a+bx)}{\frac{ad}{b}+dx} dx = -\frac{\operatorname{arccosh}(a+bx)^2}{2d} + \frac{\operatorname{arccosh}(a+bx)\log\left(1+e^{2\operatorname{arccosh}(a+bx)}\right)}{d} + \frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{arccosh}(a+bx)}\right)}{2d}$$

output
```
-1/2*arccosh(b*x+a)^2/d+arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+
1)^(1/2))^2)/d+1/2*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arccosh}(a+bx)}{\frac{ad}{b}+dx} dx = \frac{\operatorname{arccosh}(a+bx)\left(\operatorname{arccosh}(a+bx)+2\log\left(1+e^{-2\operatorname{arccosh}(a+bx)}\right)\right)-\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arccosh}(a+bx)}\right)}{2d}$$

input
```
Integrate[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(ArcCosh[a + b*x]*(ArcCosh[a + b*x] + 2*Log[1 + E^(-2*ArcCosh[a + b*x])])
- PolyLog[2, -E^(-2*ArcCosh[a + b*x])])/(2*d)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6411, 27, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \text{6411}$$

$$\frac{\int \frac{b\text{arccosh}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{arccosh}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{6297}$$

$$\frac{\int \frac{\sqrt{\frac{a+bx-1}{a+bx+1}}(a+bx+1)\text{arccosh}(a+bx)}{a+bx}d\text{arccosh}(a+bx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\text{arccosh}(a+bx)\tan(i\text{arccosh}(a+bx))d\text{arccosh}(a+bx)}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \text{arccosh}(a+bx)\tan(i\text{arccosh}(a+bx))d\text{arccosh}(a+bx)}{d}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{2\text{arccosh}(a+bx)}\text{arccosh}(a+bx)}{1+e^{2\text{arccosh}(a+bx)}}d\text{arccosh}(a+bx)-\frac{1}{2}i\text{arccosh}(a+bx)^2\right)}{d}$$

$\Big\downarrow$ 2620

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}(a+bx)\log\left(e^{2\text{arccosh}(a+bx)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arccosh}(a+bx)}\right)d\text{arccosh}(a+bx)\right)-\frac{1}{2}i\text{arccosh}(a\right.}{d}$$

$\Big\downarrow$ 2715

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}(a+bx)\log\left(e^{2\text{arccosh}(a+bx)}+1\right)-\frac{1}{4}\int e^{-2\text{arccosh}(a+bx)}\log\left(1+e^{2\text{arccosh}(a+bx)}\right)de^{2\text{arccosh}(a+bx)}\right)-\right.}{d}$$

$\Big\downarrow$ 2838

$$-\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}(a+bx)\log\left(e^{2\text{arccosh}(a+bx)}+1\right)+\frac{1}{4}\text{PolyLog}(2,-a-bx)\right)-\frac{1}{2}i\text{arccosh}(a+bx)^2\right)}{d}$$

input
```
Int[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-I)*((-1/2*I)*ArcCosh[a + b*x]^2 + (2*I)*((ArcCosh[a + b*x]*Log[1 + E^(2
*ArcCosh[a + b*x])])/2 + PolyLog[2, -a - b*x]/4)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6411
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

method	result	s
derivativedivides	$\frac{-\frac{b\,\mathrm{arccosh}(bx+a)^2}{2d}+\frac{b\,\mathrm{arccosh}(bx+a)\ln\left(1+\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{d}+\frac{b\,\mathrm{polylog}\left(2,-\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{2d}}{b}$	9
default	$\frac{-\frac{b\,\mathrm{arccosh}(bx+a)^2}{2d}+\frac{b\,\mathrm{arccosh}(bx+a)\ln\left(1+\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{d}+\frac{b\,\mathrm{polylog}\left(2,-\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{2d}}{b}$	9

input
```
int(arccosh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*b/d*arccosh(b*x+a)^2+b/d*arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1
/2)*(b*x+a+1)^(1/2))^2)+1/2*b/d*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^2))
```

Fricas [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arcosh}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arccosh(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{acosh}\left(a+bx\right)}{a+bx}\,dx}{d}$$

input
```
integrate(acosh(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(acosh(a + b*x)/(a + b*x), x)/d
```

Maxima [F]

$$\int \frac{\text{arccosh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\text{arcosh}\,(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

Giac [F]

$$\int \frac{\text{arccosh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\text{arcosh}\,(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arccosh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\text{acosh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acosh(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(acosh(a + b*x)/(d*x + (a*d)/b), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\left(\int \frac{acosh(bx+a)}{bx+a}dx\right)b}{d}$$

input
```
int(acosh(b*x+a)/(a*d/b+d*x),x)
```

output
```
(int(acosh(a + b*x)/(a + b*x),x)*b)/d
```

3.185 $\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,\mathrm{arccosh}(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 3

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,\mathrm{arccosh}(x)}\,dx = \mathrm{Chi}(\mathrm{arccosh}(x))$$

output
```
Chi(arccosh(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,\mathrm{arccosh}(x)}\,dx = \mathrm{Chi}(\mathrm{arccosh}(x))$$

input
```
Integrate[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

output
```
CoshIntegral[ArcCosh[x]]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6368, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x-1}\sqrt{x+1}\text{arccosh}(x)}dx$$

$$\Big\downarrow \text{ 6368}$$

$$\int \frac{x}{\text{arccosh}(x)}d\text{arccosh}(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+i\text{arccosh}(x)\right)}{\text{arccosh}(x)}d\text{arccosh}(x)$$

$$\Big\downarrow \text{ 3782}$$

$$\text{Chi}(\text{arccosh}(x))$$

input
```
Int[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

output
```
CoshIntegral[ArcCosh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6368
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p]   Subst[In
t[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c
*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[
e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

method	result	size
default	$\operatorname{csgn}\left(\sinh\left(\frac{\operatorname{arccosh}(x)}{2}\right)\right)\operatorname{csgn}\left(\cosh\left(\frac{\operatorname{arccosh}(x)}{2}\right)\right)\operatorname{Chi}\left(\operatorname{arccosh}\left(x\right)\right)$	17

input
```
int(x/(-1+x)^(1/2)/(1+x)^(1/2)/arccosh(x),x,method=_RETURNVERBOSE)
```

output
```
csgn(sinh(1/2*arccosh(x)))*csgn(cosh(1/2*arccosh(x)))*Chi(arccosh(x))
```

Fricas [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\operatorname{arccosh}(x)}\,dx = \int \frac{x}{\sqrt{x+1}\sqrt{x-1}\operatorname{arcosh}\left(x\right)}\,dx$$

input
```
integrate(x/(-1+x)^(1/2)/(1+x)^(1/2)/arccosh(x),x, algorithm="fricas")
```

output
```
integral(sqrt(x + 1)*sqrt(x - 1)*x/((x^2 - 1)*arccosh(x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\text{arccosh}(x)} \, dx = \int \frac{x}{\sqrt{x-1}\sqrt{x+1}\,\text{acosh}\,(x)} \, dx$$

input
```
integrate(x/(-1+x)**(1/2)/(1+x)**(1/2)/acosh(x),x)
```

output
```
Integral(x/(sqrt(x - 1)*sqrt(x + 1)*acosh(x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\text{arccosh}(x)} \, dx = \int \frac{x}{\sqrt{x+1}\sqrt{x-1}\,\text{arcosh}\,(x)} \, dx$$

input
```
integrate(x/(-1+x)^(1/2)/(1+x)^(1/2)/arccosh(x),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\text{arccosh}(x)} \, dx = \int \frac{x}{\sqrt{x+1}\sqrt{x-1}\,\text{arcosh}\,(x)} \, dx$$

input
```
integrate(x/(-1+x)^(1/2)/(1+x)^(1/2)/arccosh(x),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\text{arccosh}(x)}\,dx = \int \frac{x}{\text{acosh}\,(x)\ \sqrt{x-1}\,\sqrt{x+1}}\,dx$$

input
```
int(x/(acosh(x)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int(x/(acosh(x)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\text{arccosh}(x)}\,dx = \int \frac{x}{\sqrt{x+1}\,\sqrt{x-1}\,acosh\,(x)}dx$$

input
```
int(x/(-1+x)^(1/2)/(1+x)^(1/2)/acosh(x),x)
```

output
```
int(x/(sqrt(x + 1)*sqrt(x - 1)*acosh(x)),x)
```

3.186 $\int x^3 \text{arccosh}(a + bx^4) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int x^3 \text{arccosh}\left(a+bx^4\right) \, dx = -\frac{\sqrt{-1+a+bx^4}\sqrt{1+a+bx^4}}{4b} + \frac{(a+bx^4)\,\text{arccosh}(a+bx^4)}{4b}$$

output
```
-1/4*(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2)/b+1/4*(b*x^4+a)*arccosh(b*x^4+a)/
b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int x^3 \text{arccosh}\left(a+bx^4\right) \, dx = \frac{-\sqrt{-1+a+bx^4}\sqrt{1+a+bx^4} + (a+bx^4)\,\text{arccosh}(a+bx^4)}{4b}$$

input
```
Integrate[x^3*ArcCosh[a + b*x^4],x]
```

output
```
(-(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4]) + (a + b*x^4)*ArcCosh[a + b*x
^4])/(4*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7266, 6410, 6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arccosh}\left(a+bx^4\right)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{4}\int \text{arccosh}\left(bx^4+a\right)dx^4$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \text{arccosh}\left(bx^4+a\right)d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{6294}$$

$$\frac{\left(a+bx^4\right)\text{arccosh}\left(a+bx^4\right)-\int \frac{bx^4+a}{\sqrt{bx^4+a-1}\sqrt{bx^4+a+1}}d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{83}$$

$$\frac{\left(a+bx^4\right)\text{arccosh}\left(a+bx^4\right)-\sqrt{a+bx^4-1}\sqrt{a+bx^4+1}}{4b}$$

input
```
Int[x^3*ArcCosh[a + b*x^4],x]
```

output
```
(-(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4]) + (a + b*x^4)*ArcCosh[a + b*x
^4])/(4*b)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{(b x^4+a) \operatorname{arccosh}(b x^4+a)-\sqrt{b x^4+a-1}\,\sqrt{b x^4+a+1}}{4b}$
default	$\frac{(b x^4+a) \operatorname{arccosh}(b x^4+a)-\sqrt{b x^4+a-1}\,\sqrt{b x^4+a+1}}{4b}$
orering	$\frac{(7b^2x^8+10ab x^4+3a^2-3) \operatorname{arccosh}(b x^4+a)}{16b^2x^4}-\frac{(b x^4+a-1)(b x^4+a+1)\left(3x^2 \operatorname{arccosh}(b x^4+a)+\frac{4x^6 b}{\sqrt{b x^4+a-1}\,\sqrt{b x^4+a+}}\right.}{16b^2x^6}$

input
```
int(x^3*arccosh(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arccosh(b*x^4+a)-(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int x^3 \operatorname{arccosh}(a+bx^4)\,dx = \frac{(bx^4+a)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2-1}}{4\,b}$$

input
```
integrate(x^3*arccosh(b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)) - sq
rt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int x^3 \operatorname{arccosh}(a+bx^4)\,dx = \begin{cases} \frac{a\operatorname{acosh}(a+bx^4)}{4b}+\frac{x^4\operatorname{acosh}(a+bx^4)}{4}-\frac{\sqrt{a+bx^4-1}\sqrt{a+bx^4+1}}{4b} & \text{for } b\neq 0 \\ \frac{x^4\operatorname{acosh}(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acosh(b*x**4+a),x)
```

output
```
Piecewise((a*acosh(a + b*x**4)/(4*b) + x**4*acosh(a + b*x**4)/4 - sqrt(a +
 b*x**4 - 1)*sqrt(a + b*x**4 + 1)/(4*b), Ne(b, 0)), (x**4*acosh(a)/4, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int x^3 \operatorname{arccosh}\left(a+bx^4\right)\,dx=\frac{(bx^4+a)\operatorname{arcosh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2-1}}{4\,b}$$

input
```
integrate(x^3*arccosh(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*((b*x^4 + a)*arccosh(b*x^4 + a) - sqrt((b*x^4 + a)^2 - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(46) = 92.

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.96

$$\int x^3 \operatorname{arccosh}\left(a+bx^4\right)\,dx=\frac{1}{4}\,x^4\log\left(bx^4+a+\sqrt{\left(bx^4+a\right)^2-1}\right)-\frac{1}{4}\,b\left(\frac{a\log\left(\left|-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,abx^4+a^2-1}}{b^2}\right)$$

input
```
integrate(x^3*arccosh(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 - 1)) - 1/4*b*(a*log(abs(-a*b -
 (x^4*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))*abs(b)))/(b*abs(b)) +
sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/b^2)
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 295, normalized size of antiderivative = 5.46

$$\int x^3 \text{arccosh}\left(a+bx^4\right) \, dx = \frac{x^4 \operatorname{acosh}(b\,x^4+a)}{4} - \frac{\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)}{b\,\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)}+\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^3}{b\,\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^3}-\frac{8\,\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^2\,\sqrt{a-1}\,\sqrt{a+1}}{b\,\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^2}}{4\,\left(\frac{\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^4}-\frac{2\,\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^2}+1\right)} + \frac{a\operatorname{atanh}\left(\frac{\sqrt{a-1}-\sqrt{b\,x^4+a-1}}{\sqrt{a+1}-\sqrt{b\,x^4+a+1}}\right)}{b}$$

input
```
int(x^3*acosh(a + b*x^4),x)
```

output
```
(x^4*acosh(a + b*x^4))/4 - ((4*a*((a - 1)^(1/2) - (a + b*x^4 - 1)^(1/2)))/
(b*((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))) + (4*a*((a - 1)^(1/2) - (a + b
*x^4 - 1)^(1/2))^3)/(b*((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^3) - (8*((a
 - 1)^(1/2) - (a + b*x^4 - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a
 + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^2))/(4*(((a - 1)^(1/2) - (a + b*x^4 -
 1)^(1/2))^4/((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^4 - (2*((a - 1)^(1/2)
 - (a + b*x^4 - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^2 + 1
)) + (a*atanh(((a - 1)^(1/2) - (a + b*x^4 - 1)^(1/2))/((a + 1)^(1/2) - (a
+ b*x^4 + 1)^(1/2))))/b
```

Reduce [F]

$$\int x^3 \text{arccosh}\left(a+bx^4\right) \, dx = \int acosh\left(b\,x^4+a\right) x^3 dx$$

input
```
int(x^3*acosh(b*x^4+a),x)
```

output
```
int(acosh(a + b*x**4)*x**3,x)
```

3.187 $\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx=-\frac{\sqrt{-1+a+bx^n}\sqrt{1+a+bx^n}}{bn}+\frac{(a+bx^n)\,\text{arccosh}(a+bx^n)}{bn}$$

output
```
-(-1+a+b*x^n)^(1/2)*(1+a+b*x^n)^(1/2)/b/n+(a+b*x^n)*arccosh(a+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx=\frac{-\sqrt{-1+a+bx^n}\sqrt{1+a+bx^n}+(a+bx^n)\,\text{arccosh}(a+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)*ArcCosh[a + b*x^n],x]
```

output
```
(-(Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n]) + (a + b*x^n)*ArcCosh[a + b*x
^n])/(b*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 6410, 6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\text{arccosh}(a+bx^n)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{\int \text{arccosh}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{6410}$$

$$\frac{\int \text{arccosh}(bx^n+a)\,d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{6294}$$

$$\frac{(a+bx^n)\,\text{arccosh}(a+bx^n)-\int \frac{bx^n+a}{\sqrt{bx^n+a-1}\sqrt{bx^n+a+1}}d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{83}$$

$$\frac{(a+bx^n)\,\text{arccosh}(a+bx^n)-\sqrt{a+bx^n-1}\sqrt{a+bx^n+1}}{bn}$$

input
```
Int[x^(-1 + n)*ArcCosh[a + b*x^n],x]
```

output
```
(-(Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n]) + (a + b*x^n)*ArcCosh[a + b*x
^n])/(b*n)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6410
```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int x^{-1+n} \operatorname{arccosh}\left(a + b\,x^n\right) dx$$

input
```
int(x^(-1+n)*arccosh(a+b*x^n),x)
```

output
```
int(x^(-1+n)*arccosh(a+b*x^n),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(51) = 102$.

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.76

$$\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx$$
$$= \frac{(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a)\log\Big(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a + \sqrt{\frac{2\,ab+(a^2+b^2-}{}}}{bn}$$

input
```
integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="fricas")
```

output
```
((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n
*log(x)) + a + sqrt((2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 -
 1)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a
^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log
(x)) - sinh(n*log(x)))))/(b*n)
```

Sympy [F]

$$\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx = \int x^{n-1}\operatorname{acosh}(a+bx^n)\,dx$$

input
```
integrate(x**(-1+n)*acosh(a+b*x**n),x)
```

output
```
Integral(x**(n - 1)*acosh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx=\frac{(bx^n+a)\operatorname{arcosh}(bx^n+a)-\sqrt{(bx^n+a)^2-1}}{bn}$$

input
```
integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="maxima")
```

output
```
((b*x^n + a)*arccosh(b*x^n + a) - sqrt((b*x^n + a)^2 - 1))/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(51) = 102.

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

$$\int x^{-1+n}\text{arccosh}(a+bx^n)\,dx=$$
$$-\frac{b\left(\frac{a\log\left(\left|-ab-\left(x^n|b|-\sqrt{b^2x^{2n}+2abx^n+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^{2n}+2abx^n+a^2-1}}{b^2}\right)-x^n\log\left(bx^n+a+\sqrt{b^2x^{2n}+2abx^n \cdots}\right)}{n}$$

input
```
integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="giac")
```

output
```
-(b*(a*log(abs(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1
))*abs(b)))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)/b^2) - x^
n*log(b*x^n + a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)))/n
```

Mupad [B] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 5.51

$$\int x^{-1+n}\operatorname{arccosh}(a+bx^n)\,dx = \frac{x^n\,\mathrm{acosh}(a+b\,x^n)}{n} - \frac{\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^3}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^3}+\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}-\frac{8\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^2\,\sqrt{a-1}\,\sqrt{a+1}}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^2}}{n\,\left(\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^4}-\frac{2\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^2}+1\right)} + \frac{4\,a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x^n-1}}{\sqrt{a+1}-\sqrt{a+b\,x^n+1}}\right)}{b\,n}$$

input
```
int(x^(n - 1)*acosh(a + b*x^n),x)
```

output
```
(x^n*acosh(a + b*x^n))/n - ((4*a*((a - 1)^(1/2) - (a + b*x^n - 1)^(1/2))^3
)/(b*((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^3) + (4*a*((a - 1)^(1/2) - (a
 + b*x^n - 1)^(1/2)))/(b*((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))) - (8*((a
 - 1)^(1/2) - (a + b*x^n - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a
 + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^2))/(n*(((a - 1)^(1/2) - (a + b*x^n -
 1)^(1/2))^4/((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^4 - (2*((a - 1)^(1/2)
 - (a + b*x^n - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^2 + 1
)) + (4*a*atanh(((a - 1)^(1/2) - (a + b*x^n - 1)^(1/2))/((a + 1)^(1/2) - (
a + b*x^n + 1)^(1/2))))/(b*n)
```

Reduce [F]

$$\int x^{-1+n}\operatorname{arccosh}(a+bx^n)\,dx = \int \frac{x^n acosh(x^n b+a)}{x}dx$$

input
```
int(x^(-1+n)*acosh(a+b*x^n),x)
```

output
```
int((x**n*acosh(x**n*b + a))/x,x)
```

3.188 $\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right) dx = \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{2c\arctan\left(\sqrt{-1+\frac{2c}{a+c+bx}}\right)}{b}$$

output
```
(b*x+a)*arcsech(a/c+b*x/c)/b-2*c*arctan((-1+2*c/(b*x+a+c))^(1/2))/b
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.47

$$\begin{aligned}&\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right) dx\\&= x\operatorname{arccosh}\left(\frac{c}{a+bx}\right)\\&\quad+\frac{2\sqrt{-\frac{a-c+bx}{a+c+bx}}\sqrt{a+c+bx}\left(a\arctan\left(\frac{\sqrt{a-c+bx}}{\sqrt{a+c+bx}}\right)-c\operatorname{arctanh}\left(\frac{\sqrt{a-c+bx}}{\sqrt{a+c+bx}}\right)\right)}{b\sqrt{a-c+bx}}\end{aligned}$$

input
```
Integrate[ArcCosh[c/(a + b*x)],x]
```

output
```
x*ArcCosh[c/(a + b*x)] + (2*Sqrt[-((a - c + b*x)/(a + c + b*x))]*Sqrt[a +
c + b*x]*(a*ArcTan[Sqrt[a - c + b*x]/Sqrt[a + c + b*x]] - c*ArcTanh[Sqrt[a
 - c + b*x]/Sqrt[a + c + b*x]]))/(b*Sqrt[a - c + b*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 6867, 2055, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right) dx$$

$$\Big\downarrow \; 6427$$

$$\int \operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right) dx$$

$$\Big\downarrow \; 6867$$

$$\int \frac{\sqrt{\frac{-\frac{a}{c}-\frac{bx}{c}+1}{\frac{a}{c}+\frac{bx}{c}+1}}}{-\frac{a}{c}-\frac{bx}{c}+1}dx+\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

$$\Big\downarrow \; 2055$$

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{4b\int \frac{c^2}{2b^2\left(\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}+1\right)}d\sqrt{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\int \frac{1}{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}+1}d\sqrt{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}}}{b}$$

$$\Big\downarrow \; 216$$

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\arctan\left(\sqrt{\frac{c\left(1-\frac{a}{c}\right)-bx}{a+bx+c}}\right)}{b}$$

input
```
Int[ArcCosh[c/(a + b*x)],x]
```

output
```
((a + b*x)*ArcSech[a/c + (b*x)/c])/b - (2*c*ArcTan[Sqrt[((1 - a/c)*c - b*x
)/(a + c + b*x)]])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 6427
```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6867
```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

method	result
derivativedivides	$-\frac{c\left(-\frac{(bx+a)\operatorname{arccosh}\left(\frac{c}{bx+a}\right)}{c}-\frac{\sqrt{\frac{c}{bx+a}-1}\,\sqrt{\frac{c}{bx+a}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{b}$
default	$-\frac{c\left(-\frac{(bx+a)\operatorname{arccosh}\left(\frac{c}{bx+a}\right)}{c}-\frac{\sqrt{\frac{c}{bx+a}-1}\,\sqrt{\frac{c}{bx+a}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{b}$
parts	$x\operatorname{arccosh}\left(\frac{c}{bx+a}\right)+\frac{\sqrt{-\frac{bx+a-c}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+c}{bx+a}}\left(\operatorname{csgn}(b)\ln\left(\frac{2c\left(\operatorname{csgn}(c)\sqrt{-b^2x^2-2abx-a^2+c^2}+c\right)b}{bx+a}\right)a+\operatorname{csgn}(\right.}{b\sqrt{-b^2x^2-2abx-a^2+c^2}}$

input
```
int(arccosh(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*c*(-1/c*(b*x+a)*arccosh(c/(b*x+a))-(c/(b*x+a)-1)^(1/2)*(c/(b*x+a)+1)^
(1/2)/(c^2/(b*x+a)^2-1)^(1/2)*arctan(1/(c^2/(b*x+a)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(45) = 90$.

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 5.87

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right)\,dx$$
$$=\frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)-2\,c\arctan\left(\frac{(b^2x^2+2\,abx+a^2)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-c^2}\right)+a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2 \cdots}{}}}{}\right.}{2\,b}$$

input
```
integrate(arccosh(c/(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 +
 2*a*b*x + a^2)) + c)/(b*x + a)) - 2*c*arctan((b^2*x^2 + 2*a*b*x + a^2)*sq
rt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 +
2*a*b*x + a^2 - c^2)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 -
c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + c)/x) - a*log(((b*x + a)*sqrt(-(b^2*x^2
+ 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - c)/x))/b
```

Sympy [F]

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right)dx = \int \operatorname{acosh}\left(\frac{c}{a+bx}\right)dx$$

input
```
integrate(acosh(c/(b*x+a)),x)
```

output
```
Integral(acosh(c/(a + b*x)), x)
```

Maxima [F]

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right)dx = \int \operatorname{arcosh}\left(\frac{c}{bx+a}\right)dx$$

input
```
integrate(arccosh(c/(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*log(sqrt(b*x + a + c)*sqrt(-b*x - a + c)*b*x + sqrt(b*x + a + c
)*sqrt(-b*x - a + c)*a + (b*x + a)*c) - 2*b*x*log(b*x + a) + (a + c)*log(b
*x + a + c) - 2*(b*x + a)*log(b*x + a) + (a - c)*log(-b*x - a + c))/b + in
tegrate((b^2*c*x^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c - c^3 + (b^2*
x^2 + 2*a*b*x + a^2 - c^2)*e^(1/2*log(b*x + a + c) + 1/2*log(-b*x - a + c)
)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(45) = 90$.

Time = 2.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right) dx = \frac{c \arcsin\left(-\frac{bx+a}{c}\right) \operatorname{sgn}(b) \operatorname{sgn}(c)}{|b|} + x \log\left(\sqrt{\frac{c}{bx+a}+1}\sqrt{\frac{c}{bx+a}-1}+\frac{c}{bx+a}\right) - \frac{a \log\left(\frac{\left|-2\,bc-2\,\sqrt{-b^2x^2-2\,abx-a^2+c^2}|b|\right|}{|-2\,b^2x-2\,ab|}\right)}{|b|}$$

input
```
integrate(arccosh(c/(b*x+a)),x, algorithm="giac")
```

output
```
c*arcsin(-(b*x + a)/c)*sgn(b)*sgn(c)/abs(b) + x*log(sqrt(c/(b*x + a) + 1)*
sqrt(c/(b*x + a) - 1) + c/(b*x + a)) - a*log(abs(-2*b*c - 2*sqrt(-b^2*x^2
- 2*a*b*x - a^2 + c^2)*abs(b))/abs(-2*b^2*x - 2*a*b))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right) dx = \frac{\operatorname{acosh}\left(\frac{c}{a+b\,x}\right)\,(a+b\,x)}{b} + \frac{c\,\operatorname{atan}\left(\frac{1}{\sqrt{\frac{c}{a+b\,x}-1}\,\sqrt{\frac{c}{a+b\,x}+1}}\right)}{b}$$

input
```
int(acosh(c/(a + b*x)),x)
```

output
```
(acosh(c/(a + b*x))*(a + b*x))/b + (c*atan(1/((c/(a + b*x) - 1)^(1/2)*(c/(
a + b*x) + 1)^(1/2))))/b
```

Reduce [F]

$$\int \operatorname{arccosh}\left(\frac{c}{a+bx}\right)\,dx = \int acosh\left(\frac{c}{bx+a}\right)dx$$

input
```
int(acosh(c/(b*x+a)),x)
```

output
```
int(acosh(c/(a + b*x)),x)
```

3.189 $\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

output
```
(-1+(b*x^2+1)^(1/2))^(1/2)*(1+(b*x^2+1)^(1/2))^(1/2)*arccosh((b*x^2+1)^(1/
2))^(1+n)/b/(1+n)/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

input
```
Integrate[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

output
```
(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x
^2]]^(1 + n))/(b*(1 + n)*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6428, 6308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arccosh}\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}dx$$
$$\Big\downarrow \text{6428}$$
$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\int \frac{\text{arccosh}\left(\sqrt{bx^2+1}\right)^n}{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}}d\sqrt{bx^2+1}}{bx}$$
$$\Big\downarrow \text{6308}$$
$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\text{arccosh}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

input
```
Int[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

output
```
(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x
^2]]^(1 + n))/(b*(1 + n)*x)
```

Defintions of rubi rules used

rule 6308
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

rule 6428
```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Simp[Sqrt[-1 + Sqrt[1 + b*x^2]]*(Sqrt[1 + Sqrt[1 + b*x^2]]/(b*x))   Sub
st[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
 /; FreeQ[{b, n}, x]
```

Maple [F]

$$\int \frac{\operatorname{arccosh}\left(\sqrt{x^2b+1}\right)^n}{\sqrt{x^2b+1}}dx$$

input
```
int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

output
```
int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\text{s}}{(bn+b)x}$$

input
```
integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
(sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2))))*log(sqrt(b*x^
2 + 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 + 1) + sqrt(b*x^2))*sin
h(n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

Sympy [F]

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \int \frac{\operatorname{acosh}^n\left(\sqrt{bx^2+1}\right)}{\sqrt{bx^2+1}}\,dx$$

input
```
integrate(acosh((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

output
```
Integral(acosh(sqrt(b*x**2 + 1))**n/sqrt(b*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \int \frac{\operatorname{arcosh}\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}\,dx$$

input
```
integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(arccosh(sqrt(b*x^2 + 1))^n/sqrt(b*x^2 + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx=\int \frac{\operatorname{acosh}\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

input
```
int(acosh((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2),x)
```

output
```
int(acosh((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx=\int \frac{acosh\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}dx$$

input
```
int(acosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

output
```
int(acosh(sqrt(b*x**2 + 1))**n/sqrt(b*x**2 + 1),x)
```

3.190 $\int \frac{1}{\sqrt{1+bx^2}\text{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{1}{\sqrt{1+bx^2}\text{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\log\left(\text{arccosh}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

output
```
(-1+(b*x^2+1)^(1/2))^(1/2)*(1+(b*x^2+1)^(1/2))^(1/2)*ln(arccosh((b*x^2+1)^
(1/2)))/b/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+bx^2}\text{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\log\left(\text{arccosh}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

input
```
Integrate[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

output
```
(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
 b*x^2]]])/(b*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6428, 6306}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2+1}\text{arccosh}\left(\sqrt{bx^2+1}\right)}dx$$

$$\Big\downarrow \text{6428}$$

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\int \frac{1}{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\text{arccosh}\left(\sqrt{bx^2+1}\right)}d\sqrt{bx^2+1}}{bx}$$

$$\Big\downarrow \text{6306}$$

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\text{arccosh}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

input
```
Int[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

output

```
(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
 b*x^2]]])/(b*x)
```

Defintions of rubi rules used

rule 6306
```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c*x]/Sqrt[d
1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*Log[a + b*ArcCosh[c*x]], x]
 /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2
]
```

rule 6428
```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Simp[Sqrt[-1 + Sqrt[1 + b*x^2]]*(Sqrt[1 + Sqrt[1 + b*x^2]]/(b*x))   Sub
st[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
 /; FreeQ[{b, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^2b+1}\,\operatorname{arccosh}\left(\sqrt{x^2b+1}\right)}dx$$

input
```
int(1/(b*x^2+1)^(1/2)/arccosh((b*x^2+1)^(1/2)),x)
```

output
```
int(1/(b*x^2+1)^(1/2)/arccosh((b*x^2+1)^(1/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{1+bx^2}\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)}{bx}$$

input
```
integrate(1/(b*x^2+1)^(1/2)/arccosh((b*x^2+1)^(1/2)),x, algorithm="fricas"
)
```

output
```
sqrt(b*x^2)*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))/(b*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+bx^2}\text{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2+1}\,\text{acosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

input
```
integrate(1/(b*x**2+1)**(1/2)/acosh((b*x**2+1)**(1/2)),x)
```

output
```
Integral(1/(sqrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+bx^2}\text{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2+1}\,\text{arcosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

input
```
integrate(1/(b*x^2+1)^(1/2)/arccosh((b*x^2+1)^(1/2)),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(b*x^2 + 1)*arccosh(sqrt(b*x^2 + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+bx^2}\text{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+1)^(1/2)/arccosh((b*x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+bx^2}\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx=\int \frac{1}{\operatorname{acosh}\left(\sqrt{b\,x^2+1}\right)\,\sqrt{b\,x^2+1}}\,dx$$

input
```
int(1/(acosh((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(acosh((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+bx^2}\operatorname{arccosh}\left(\sqrt{1+bx^2}\right)}\,dx=\int \frac{1}{\sqrt{b\,x^2+1}\,acosh\left(\sqrt{b\,x^2+1}\right)}dx$$

input
```
int(1/(b*x^2+1)^(1/2)/acosh((b*x^2+1)^(1/2)),x)
```

output
```
int(1/(sqrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file